

Fault Diagnosis and Prognosis using a Hybrid Approach combining Structural Analysis and Data-driven Techniques

Xin Fang¹, Vicenç Puig¹ and Shuang Zhang¹

Abstract—This paper presents a fault diagnosis and prognosis based on an hybrid approach that combines structural and data-driven techniques. The proposed method involves two phases. Firstly, the residuals structure is obtained from the structural model of the system using structural analysis without using mathematical models (only the component description of the system). Secondly, the analytical expressions of residuals are obtained from available historical data using a robust identification approach. The diagnosis part consists in checking the evolution of residuals during the process, any inconsistency of residuals can be considered as a fault, so that the thresholds for each residual are introduced. The residuals are obtained using the identified interval model that takes into account the uncertainty and noises affecting the system. Once the fault is detected, also it is possible to determine which fault occurred in the system using the FSM (Fault Signature Matrix) obtained from the structural analysis of the system and residual generation. The prognosis part is developed using the same steps, but instead of considering the actual situation, it evaluates the tendency of deviation respect the nominal operation condition to predict the future residual inconsistency, allowing estimating the RUL (Remaining Useful Life) of the system. The interval model is also introduced for the future prediction of residuals, thus there will be an interval of RUL for each residual which contains the maximum and minimum RUL values. The proposed approach is applied to a brushless DC motor (BLDC) used as a case study. Simulation experiments illustrate the performance of the approach.

I. INTRODUCTION

In the past few years, the research community has gradually realized the industrial interest in fault diagnosis and more recently in fault prognosis. Currently, the majority of the related approaches are based on an exact system model to develop the diagnosis and prognosis. Nevertheless, in real systems is very difficult to obtain a mathematical model of the system that describes precisely its behavior. Thereby, a method that only requires the knowledge of the structural model can be useful to overcome such a difficulty. Once the structure of the resulting analytical redundancy relations have been obtained, they can be identified from historical data of normal operation using robust identification that allow to obtain the model from data altogether with its uncertainty bounds.

In this article, we will propose methods to combine the structural analysis methods and data-driven techniques extending the applicability of conventional model-based diagnosis methods and proposing an extension to prognosis.

Some researchers have already explored similar ideas, for instance, try to use techniques like Grey-box recurrent neural networks to generate residuals in order to develop a hybrid fault diagnosis method [3][4]. Another outstanding research is about combining the state space neural networks and model-decomposition methods for fault diagnosis [1]. Nevertheless, these researches focus more on the diagnosis part not considering the extension to prognosis. On the other hand, most of the existing prognoses are based on application dependent methods that extract features of the measured variables. In this work, we will apply the idea of extending residuals analysis from diagnosis part to prognosis part extending the preliminary results presented [6], including several enhancements as avoiding the need to have the mathematical model of the system and considering modelling uncertainty using interval methods. Many steps of this approach can be dealt with alternative methods. For example, there exist various methods to generate residuals from structural model of the system. In the same way, the identification of the residuals can also be done with other methods.

This paper proposes a fault diagnosis and prognosis approach combining both the structural and data-driven techniques, avoiding the need to know precise mathematical models of system. It is applied to a brushless DC motor as case study, but it can be used for the diagnosis and prognosis of any other system.

The structure of the paper is the following: Section II introduced the problem statement. Section III describes the proposed model-based approach for diagnosis/prognosis. Section IV presents the case study and Section V collects some of the application results of the proposed approach. Section VI draws the conclusions of the paper and suggests future research paths.

II. PROBLEM STATEMENT

A. Problem statement

The proposed method uses the structural analysis of the system; thereby it starts from a graphical or textual description of the system operation. The exact mathematical model of the system to be monitored is not needed but some physical insight could be helpful. Moreover, a set of output observed variables, y_k , and the set of inputs, u_k are available, which are used to generate the structure of the residuals and to estimate the mathematical expression and associated parameters. The set of residual in analytical form can be presented as follows:

¹ X. Fang and V. Puig are with the Institute of Robotics and Industrial Informatics (UPC-CSIC), Carrer Llorens i Artigas, 4, 08028 Barcelona, Spain.

$$\mathcal{R} = \{r_i = \Psi_i(y_k, u_k) = 0, i = 1, \dots, n_r\}, \quad (1)$$

where Ψ_i is the corresponding analytical expression of each residual, n_r is the number of residuals derived from the structural analysis.

In this paper, the proposed approach evaluates the residuals expressions obtained from the structural analysis to perform diagnosis and prognosis problems as follows:

Problem 1. “Fault diagnosis”. Its objective consists in two parts. Fault detection, which means detect the fault as soon as possible when an anomaly failure of system operating appears. Fault isolation, once detected the fault, it is also necessary to find out which part (component) of system is suffering the failure. These objectives can be achieved by checking the time-evolution of the residuals under different system operating conditions.

Therefore, the diagnosis aims at dealing with the faults that have already occurred in the system. On the other hand, the prognosis problem focus on predicting future evolution of the residuals in order to anticipate the appearance of possible faults.

Problem 2. “Fault prognosis”. The prognosis is similar to the diagnosis, the difference lies on the prognosis uses the available data of observed variables and inputs till the actual time instant to estimate the future values of residuals. The concept of RUL is introduced that is used to estimate the remaining time of corresponding residual to be out of bounds.

B. Overview of the presented method

The presented method consists of the following steps:

1. From the accurate study of the provided description of the system, find out the structural model of the system. This step requires information and knowledge about the considered system. The structural model is basically a set of relationships between known and unknown variables, the exact expressions of relations are not needed.
2. By the aid of analysis structural, obtain the corresponding residuals in structural form. There are many methods of generation of analytical redundancy relations (ARRs) from the structural model. This paper uses the MSO method which is well-implemented in Matlab².
3. Starting from the obtained residuals in structural form in previous step and using robust system identification tools identify the residual mathematical expression including the parameters and associated uncertainty. The history data of input variables and measured output variables are required.
4. Based on the calculated residuals, the implementation of diagnosis and prognosis of system can be developed. The thresholds are determined by estimating the corresponding interval model using history data. It is considered that if the value of residuals violates its thresholds, a fault has occurred. The thresholds are determined using the interval model

method. The isolation of the occurred fault can be accomplished by comparing the affected residuals to the FSM (Fault Signature Matrix). As to prognosis part, the RUL (Resting Useful Life) can be estimated once appeared the tendency of deviation of residuals respect to its nominal value.

III. FAULT DIAGNOSIS/PROGNOSIS USING STRUCTURAL ANALYTICAL REDUNDANCY RELATIONS

As already discussed, the approach proposed in this paper is based on the structural model of the monitored system. The structural model consists of a set of relations including known and unknown variables. It has the advantage of no requiring the explicit expressions of them, thus it avoids all the complexity of generation and calibration of accurate explicit model. This is possible to be done since the generation of analytical redundancy relations in structural analysis does not require the explicit model. Afterwards, by the aid of system identification tolls residual expressions can be obtained, thus like any other data-driven technique, a large amount of data (in non-faulty situation) is necessary to correctly estimate the residual expressions.

A. Residual generation in structural analysis

The residuals in structural form are generated from the structural system model finding the ARRs. The structural model is a set of relations between (known and unknown) variables and model relations (constraints) as follows:

Definition 1. Structural model. The structural model of the system $S = (C, Z)$ is a bipartite graph

$$G = (C, Z, \varepsilon),$$

where C is the set of constraints, Z is the set of variables, ε is the set of edges defined as follows:

$$\varepsilon \subset C \times Z \text{ if the variable } z_j \text{ appears in the constraint } c_i$$

Definition 2. Matching. A matching $M \subseteq \varepsilon$ is a set of disjoint edges of a bipartite graph G .

The “size” of a matching M is its cardinality $|M|$.

Definition 3. Complete matching. A matching is called complete with respect to C if $|M| = |C|$ holds. A matching is called complete with respect to Z if $|M| = |Z|$ holds.

Structural analysis is mainly concerned with Z -complete matchings, because such matching shows a way to determine all unknown variables of the system.

In systems with redundancy, one method of matching can be used to generate a set of residuals \mathcal{R} in (1) (see [5] for more details).

B. Residual analysis with interval model

² <https://faultdiagnostoolbox.github.io/>

Considering the structure of each ARR obtained using the structural analysis, a robust system identification approach will be used to obtain the mathematical expressions and parameters bounding their uncertainty. The proposed robust identification approach expresses the ARR in regressor form as follows:

$$y(k) = \boldsymbol{\varphi}(k)\boldsymbol{\theta}(k) + e(k) \quad (2)$$

where $\boldsymbol{\varphi}(k)$ is the regressor vector which can contain any function of inputs and outputs, $e(k)$ is the additive noise bounded by a constant $|e(k)| = \sigma$, $\boldsymbol{\theta}(k) \subset \Theta$ is the parameter vector, with Θ being an interval box centered in the nominal parameter values. It can be parameterized a particular case of a zonotope as follows:

$$\Theta = \{\boldsymbol{\theta}^0 + \mathbf{H}\mathbf{z} : \mathbf{z} \in \mathbf{B}^n\} \quad (3)$$

with nominal value $\boldsymbol{\theta}^0(k)$, matrix uncertainty shape \mathbf{H} which is a diagonal matrix, and \mathbf{B} is a unitary box composed by n unitary ($\mathbf{B}=[-1, 1]$) interval vectors:

$$\begin{aligned} \boldsymbol{\theta}^0 &= (\theta_1, \theta_2, \dots, \theta_{n\theta}) \\ \mathbf{H} &= \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_{n\theta}) \end{aligned} \quad (4)$$

Then, the maximum and minimum values of the prediction are given by

$$\begin{aligned} \bar{y}(k) &= \hat{y}^0(k) + \|\mathbf{H}\boldsymbol{\varphi}(k)\|_1 \\ \underline{y}(k) &= \hat{y}^0(k) - \|\mathbf{H}\boldsymbol{\varphi}(k)\|_1 \end{aligned} \quad (5)$$

where $\hat{y}^0(k)$ is the model output prediction with nominal parameters $\hat{y}^0(k) = \boldsymbol{\varphi}(k)\boldsymbol{\theta}^0$, and it can be estimated with the least square method using regressor in form of straight line: $y = a+bt$.

Then problem has transformed into solving the following optimization problem:

$$\min_{\mathbf{H}} \text{vol}(\Theta(\mathbf{H})) \quad (6)$$

subject to: $\hat{y}^0(k) + \|\mathbf{H}\boldsymbol{\varphi}(k)\|_1 \geq y(k) - \sigma$

$$\hat{y}^0(k) - \|\mathbf{H}\boldsymbol{\varphi}(k)\|_1 \leq y(k) + \sigma \quad k = 1, \dots, N$$

In the case of considering $\mathbf{H} = \lambda\mathbf{H}_0$ with \mathbf{H}_0 being the pre-determined shape, the optimal solution is given:

$$\lambda = \sup_{k \in \{1, \dots, N\}} \left(\frac{|y(k) - \hat{y}^0(k)| - \sigma}{\|\boldsymbol{\varphi}(k)\mathbf{H}_0\|_1} \right) \quad (7)$$

$$r^o(k) = y(k) - \hat{y}^o(k) = y(k) - \boldsymbol{\varphi}(k)\boldsymbol{\theta}^o \quad (8)$$

where $\boldsymbol{\theta}^o$ are the nominal parameters.

Taking into account modelling uncertainty, the detection test consists in evaluating the following condition

$$r_i^o(k) \in \left[\underline{r}_i^o(k), \bar{r}_i^o(k) \right] \quad (9)$$

where: $\underline{r}_i^o(k) = \underline{\hat{y}}_i(k) - \hat{y}_i^o(k)$ and $\bar{r}_i^o(k) = \bar{\hat{y}}_i(k) - \hat{y}_i^o(k)$ while $\underline{\hat{y}}_i(k)$ and $\bar{\hat{y}}_i(k)$ are the bounds of the i^{th} output prediction calculated using (5).

C. Fault diagnosis and prognosis

Once the thresholds of residuals are determined according to (14), the diagnosis consists in checking in every time instant if the residual values are bounded by its corresponding interval model. Any violation of threshold of residuals is considered as a fault, and the isolation process will be developed. The isolation is based on the standard procedure of cross-checking the activated residuals with the theoretical fault signature matrix obtained from the structural analysis.

In the prognosis part, the detection of fault is forecasted by evaluating the time (RUL) in which the threshold will be violated. The RUL of each residual is determined as the time for the current k such that

$$r_i^o(k + RUL_i) \notin \left[\underline{r}_i^o(k + RUL_i), \bar{r}_i^o(k + RUL_i) \right] \quad (10)$$

In this paper, it is proposed the application of interval model in the prognosis part, that is to say, the future evolution of residuals will have associated a corresponding interval. Thus, instead of one estimated value of RUL, there will be an interval of RUL estimated. In this case, the future intervals are estimated using the data till current instant. The predicted interval is starting to be estimated once the tendency of the deviation of residuals is appeared, and the regressor used is also a straight line in this case. Thereby, the extremes of intervals RUL_i are interesting since they represent the earliest and latest instant in which the fault can be considered has appeared.

IV. CASE STUDY DESCRIPTION

A. BLDC motor model

To illustrate the proposed approach, a BLDC motor is used as case study. The BLDC motor is simulated using the equivalent circuit under the 2-phase conduction mode presented in Figure 1. The model dynamics of a BLDC motor under the 2-phase conduction model is given by the electrical equation and by the mechanical equations, giving the following matrix system

written in the state-space equation form, where the state-space variables are the current (i) and the rotor speed (ω_r)

$$\frac{d}{dt} \begin{bmatrix} i \\ \omega_r \end{bmatrix} = \begin{bmatrix} -\frac{R_{eq}}{L_{eq}} & -\frac{k_e}{L_{eq}} \\ \frac{k_T}{J} & -\frac{B_r}{J} \end{bmatrix} \begin{bmatrix} i \\ \omega_r \end{bmatrix} + \begin{bmatrix} \frac{1}{L_{eq}} & 0 \\ 0 & -\frac{1}{J} \end{bmatrix} \begin{bmatrix} V_{dc} \\ T_L \end{bmatrix} \quad (12)$$

where R_{eq} is the equivalent resistance of the simplified circuit, V_{dc} is the DC voltage, L_{eq} is the equivalent inductance of the simplified circuit, T_L is the resisting (or load) torque, J is the moment of inertia of the rotational system (BLDC motor and load), B_r is the damping (or viscous friction) coefficient, k_T is the torque coefficient and k_e is the coefficient of the back electromotive force (emf).

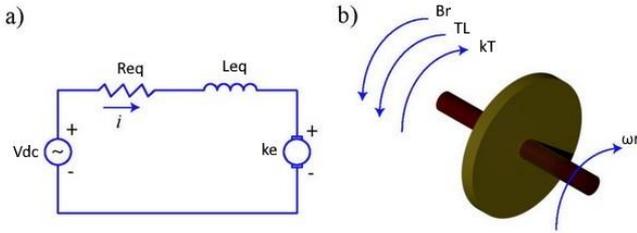


Figure 1. (a) equivalent circuit under the 2-phase conduction mode (each $\pi/3$ rad). (b) the simplified mechanical part of the motor,

In this case study, it is considered that an accurate physical-based model is not available for the diagnosis system design. As an alternative, a structural model is able to be used

$$\begin{aligned} e_1 : i' &= f_1(i, \omega_r, v_{dc}) \\ e_2 : \omega_r' &= f_2(i, \omega_r, T_L) \\ e_3 : y_1 &= i \\ e_4 : y_2 &= \omega_r \\ e_5 : i' &= \frac{di}{dt} \\ e_6 : \omega_r' &= \frac{d\omega_r}{dt} \end{aligned} \quad (14)$$

This behavioural model of the BLDC motor leads to the structural graph presented in Figure 2.

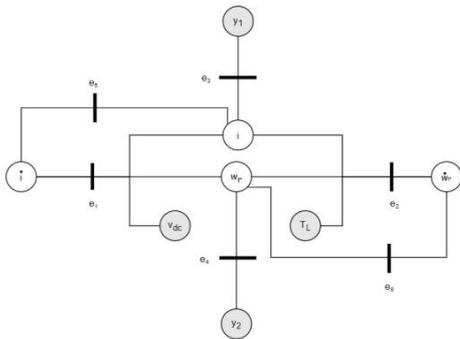


Figure 2: Structure graph of the BLDC motor system

The grey circles represent the inputs and outputs which are known variables, and the rest of circles symbolize all the unknown internal variables.

B. Structural Analysis

The basic tool for the structural analysis is the concept of matching in bipartite graphs as discussed in Section III.A. In loose terms, a matching is a causal assignment which associates with every unknown system variable a constraint that can be used to determine the variable.

In this case study, the matching process is done firstly with an intuitive simple algorithm, referred to as “ranking” (see [5] for more details). This algorithm uses the causal interpretation of matchings, but it cannot handle with the cases where subsystems are so closely coupled that a set of constraints and variables need be solved simultaneously. The obtained matching is the following one:

TABLE I. OBTAINED MATCHING

	Internal variables				Input/Output			
	i	i'	ω_r	ω_r'	V_{dc}	T_L	y_1	y_2
e_1	1	1	1		1			
e_2	1		1	1		1		
e_3	Ⓚ						1	
e_4			Ⓚ					1
e_5	1	Ⓚ						
e_6			1	Ⓚ				

As the ranking algorithm may stop when encountering strongly connected subgraphs, which consist of constraints and unknown variables that need be solved simultaneously, another more generic approach to matching is introduced later, which is called the Minimal Structurally Over-determined (MSO) set approach (see [5] for more details). This approach consists of finding all subsets of an over-constrained structure graph, which have exactly one constraint more than the just-constrained subsystem.

Algorithm determines the four MSO sets listed below.

TABLE II. MSO SETS

	e_1	e_2	e_3	e_4	e_5	e_6
M_1	-	0	i	ω_r	-	ω_r'
M_2	0	-	i	ω_r	i'	-
M_3	0	i	-	ω_r	i'	ω_r'
M_4	ω_r	0	i	-	i'	ω_r'

The table has to be interpreted as follows: The MSO set M_1 includes the constraint $e_2(i, \omega_r, \omega_r') = 0$ as an ARR and uses e_3 to calculate i , e_4 to calculate ω_r , and e_6 to calculate ω_r' . Each of the MSO sets is, by the definition of MSO subsystem, also an ARR. The MSO approach finds four MSO sets for this case. By comparison, the Ranking Algorithm finds one complete matching of the unknown variables and two ARRs, e_1 and e_2 in the Table II.

V. RESULTS

A. BLDC motor residuals

Considering the structural model, and using the second method of generation of structured residuals “the MSO approach” proposed in the previous section, the following residuals can be generated:

$$\begin{aligned} r_1(k) &= f_1(i(k-1), \omega(k), \omega(k-1), T_L(k-1)) \\ r_2(k) &= f_2(i(k), i(k-1), \omega(k-1), V_{dc}(k-1)) \\ r_3(k) &= f_3(\omega(k), \omega(k-1), \omega(k-2), T_L(k-1), T_L(k-2), V_{dc}(k-2)) \\ r_4(k) &= f_4(i(k), i(k-1), i(k-2), T_L(k-2), V_{dc}(k-1), V_{dc}(k-2)) \end{aligned} \quad (15)$$

The next step is to find the analytical expressions of these residuals from the input and measured data by model fitting of (15) the obtained expressions are the following:

$$\begin{aligned} r_1(k) &= \omega(k) - 0,9487 \omega(k-1) - 0,1902 i(k-1) + 2,552 T_L(k-1) \\ r_2(k) &= i(k) - 0,9088 i(k-1) + 0,02477 \omega(k-1) - 0,05296 V_{dc}(k-1) \\ r_3(k) &= \omega(k) - 1,891 \omega(k-1) + 0,8965 \omega(k-2) + 0,1318 T_L(k-2) - 0,003886 V_{dc}(k-1) \\ &\quad - 0,001822 V_{dc}(k-2) \\ r_4(k) &= i(k) + 0,3416 i(k-1) - 0,3795 i(k-2) - 1,88 T_L(k-2) + 0,02705 V_{dc}(k-1) \\ &\quad - 0,1078 V_{dc}(k-2) \end{aligned} \quad (16)$$

These coefficients in expressions are nominal values $\theta^0(k)$. These residuals have been discretized in time for real-time implementation. Threshold values associated to residuals are computed using the interval model of computed residuals in fault free scenarios.

From the previous four residuals, considering the faults described in the following [6],

- faults in sensors: position (f_θ) and current (f_i)
- parametric faults: resistance (f_R), inductance (f_L), friction (f_B) and inertia (f_j)
- systems faults: voltage supply ($f_{V_{dc}}$), load (f_{Load})

the fault signature matrix (FSM) presented in Table IV can be obtained.

TABLE III. FAULT SIGNATURE MATRIX

	f_R	f_L	$f_{V_{dc}}$	f_B	f_j	f_θ	f_i	f_{Load}
r_1	0	0	0	1	1	1	1	1
r_2	1	1	1	0	0	1	1	0
r_3	1	1	1	1	1	1	0	1
r_4	1	1	1	1	1	0	1	1

B. Results of diagnosis

Scenario 1: “Encoder stuck fault”[6]

The position encoder stuck fault scenario consists in losing 20000 pulses of the encoder (with a resolution of 1024 pulses per revolution) during a small time interval,[6] in this case from 10 to 11 seconds (see Figure 3). From the FSM, it can be observed that this scenario will cause the activation of the residuals r_1 , r_2 and r_3 .

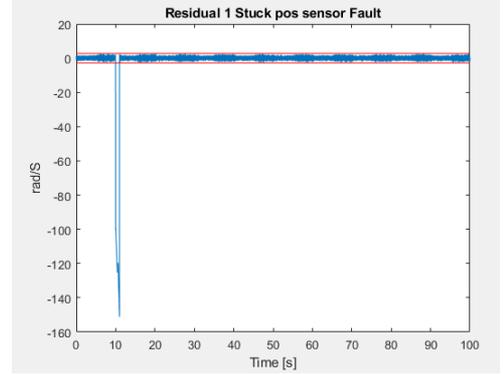


Figure 11: Residual r_1 evolution

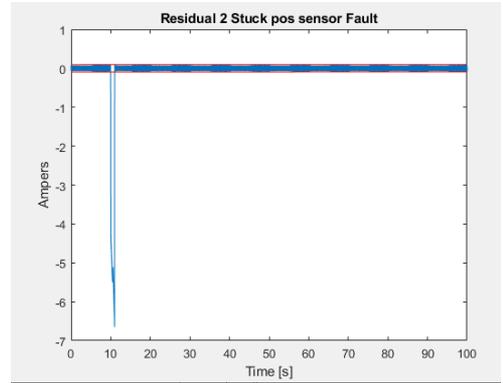


Figure 12: Residual r_2 evolution

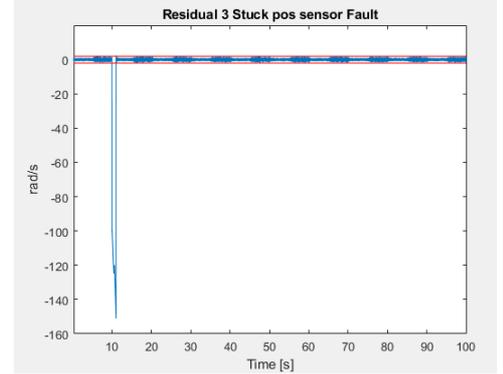


Figure 13: Residual r_3 evolution

C. Results of prognosis

Scenario 2: “Incipient leak in the current sensor”[6]

An incipient fault in the current sensor produces a big error until 0.9A after 100 seconds of the simulation scenario [6]. This fault causes a linear deviation on residuals r_1 , r_2 and r_4 as we can see in the FSM. In the Table V, it is seen that the RULs of residuals have its corresponding intervals thanks to the

introduction of interval model concept. It contributes the considerable reduction of fault detection time and isolation time.

TABLE IV. PROGNOSIS RESULTS

Residual Prognosis	r_1	r_2	r_3	r_4
RUL	[1.07, 23.4] s	[1.01, 18.8] s	Not affected	[0.8, 19] s
Fault detection	Anticipation time of 0.8 seconds to the real fault detection			
Fault isolation	Anticipation time of 1.07 seconds to the real fault isolation			
Current instant	10 s			

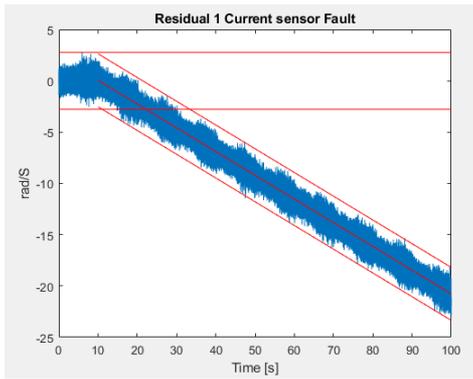


Figure 15: Evolution of r_1 current fault

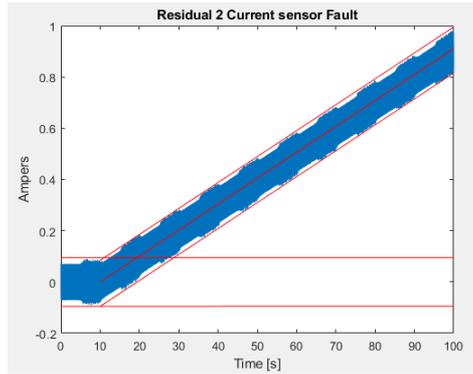


Figure 16: Evolution of r_2 current fault

VI. CONCLUSION

The current work has proposed a hybrid approach that combines data-driven techniques and structural analysis methods. The proposed method involves two phases. Firstly, the residuals structure is obtained from the structural model of the system using structural analysis without using mathematical models. Secondly, the analytical expressions of residuals are obtained from available historical data using a robust identification approach. The diagnosis part consists in checking the evolution of residuals during the process, any inconsistency of residuals can be considered as a fault, so that

the thresholds for each residual are introduced. The residuals are obtained using interval model that takes into account the uncertainty and noises affecting the system. Once the fault is detected, also it is possible to determine which fault occurred in the system using the FSM (Fault Signature Matrix) obtained from the structural analysis of the system and residual generation. The prognosis part is developed by the same steps, but instead of considering the actual situation, it evaluates the tendency of deviation respect the nominal operation condition to predict the future residual inconsistency, allowing estimating the RUL (Remaining Useful Life) of the system. The interval model is also introduced for the future prediction of residuals, thus there will be an interval of RUL for each residual which contains the maximum and minimum RUL values. The proposed approach has satisfactory applied to a brushless DC motor (BLDC) used as a case study in this paper. As future research, the proposed approach will be improved by using machine learning techniques to obtain the expressions of the ARR from data.

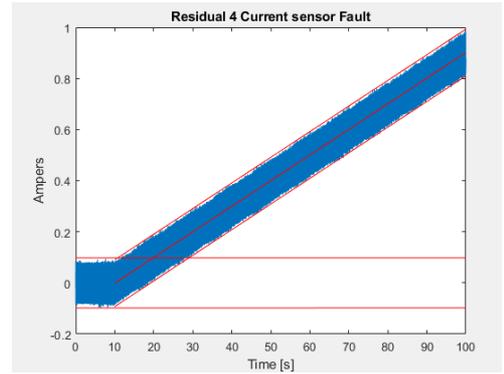


Figure 17: Evolution of r_4 current fault

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