Pricing and Congestion Management
in a Network with Heterogeneous Users

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1. Introduction

We consider an economic model for a communication network with utility-maximizing elastic users who adapt to congestion by adjusting their flows. Users are heterogeneous with respect to both the utility they attach to different levels of flow and their sensitivity to delay or other measures of congestion. Following Kelly at al [4], we introduce dynamical rate-control algorithms, based on the users' utility functions and delay sensitivities, as well as tolls charged by the system, and examine their behavior. We show that allowing heterogeneity with respect to delay sensitivity introduces a *fundamental* non-convexity into the congestion-cost functions. As a result, there are typically multiple stationary points. Hence marginal-cost pricing – equating users' marginal utilities to their marginal costs – may identify a local maximum or even a saddle point, rather than a global maximum. We present a simple example in which the only interior stationary point is a saddlepoint, which is dominated by *all* the single-user optimal allocations. Thus, heterogeneity of users can lead to *class dominance*: a situation in which the system is dominated by a single user or class of users under an optimal flow allocation. The dynamical-system rate-control algorithm may converge to a local rather than global maximum, depending on the starting point.

2. Summary of Results

We consider a variant of an economic model proposed by Kelly [3] and elaborated by Kelly et al [4] for a communication network with utility-maximizing elastic users who adapt to congestion by adjusting their flows. A distinctive feature of our model is that users not only differ with respect to the utility they attach to different levels of flow, but are heterogeneous in their sensitivity to congestion. Following Kelly at al [4], we introduce dynamical rate-control algorithms, based on the users' utility functions and delay sensitivities, as well as tolls charged by the system, and examine their behavior.

We show that allowing heterogeneity among users dramatically alters the economic properties of the system and the performance of the algorithms. In particular, heterogeneity introduces a *fundamental* non-convexity into the congestion-cost functions. As a result, there are typically multiple stationary points, that is, flow allocations that satisfy the usual first-order conditions for optimality. Hence the principles of marginal-cost pricing must be applied with care. Put another way, simply equating users' marginal utilities to their marginal costs may identify a local maximum or even a saddle point, rather than a global maximum. Indeed, we present a simple example in which the only interior stationary point is a saddlepoint, which is dominated by *all* the single-user optimal allocations. Thus, heterogeneity of users can lead to *class dominance*: a situation in which the system is dominated by a single user or class of users under an optimal flow allocation (see Balachandran and Schaefer [2], Balachandran and Radhakrishnan [1], Ma and Rump [6]).

We also study the convergence properties of a dynamical rate-control algorithm in which each user is charged a dynamically varying toll by each resource (link) on that user's route. Standard economic theory dictates that the toll should be set equal to the external effect at the resource. We show that when this is done the algorithm always converges to a local maximum and never to a saddlepoint, but which local maximum is approached depends on the starting point of the algorithm. In the example referred to above, this means that class-dominant solutions are the only possible limit points of the algorithm. Class-dominant solutions fail (dramatically) to satisfy basic fairness criteria such as max-min fairness or proportional fairness, which have been proposed in the literature (see Kelly [3], [5] for a discussion of fairness concepts). Thus, in the case of heterogeneous users there may be a fundamental incompatibility between fairness and net utility maximization.

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