

Anti-Unwinding Sliding Mode Attitude Maneuver Control for Rigid Spacecraft

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Abstract—In this paper, anti-unwinding attitude maneuver control for rigid spacecraft is considered. First, in order to avoid the unwinding phenomenon when the system states are restricted to the switching surface, a novel switching function is constructed by hyperbolic sine functions such that the switching surface contains two equilibriums. Then, a sliding mode attitude maneuver controller is designed based on the constructed switching function to ensure the robustness of the closed-loop attitude maneuver control system to disturbance. Another important feature of the developed attitude control law is that a dynamic parameter is introduced to guarantee the anti-unwinding performance before the system states reach the switching surface. The simulation results demonstrate that the unwinding problem is settled during attitude maneuver for rigid spacecraft by adopting the newly constructed switching function and proposed attitude control scheme.

Index Terms—Unwinding phenomenon, sliding mode control, attitude maneuver, rigid spacecraft

I. INTRODUCTION

Due to the increasingly challenging requirements of the aerospace control tasks such as high pointing accuracy, fast response and strong robustness, the attitude controller design for a spacecraft has been a hot topic. Then, various control schemes have been proposed to deal with the attitude control issue, such as Proportional-Integral-Differential (PID) control law [1], Linear Parameter Varying (LPV) gain-scheduled controller [2], fuzzy control method [3], velocity-free approach [4], robust H_∞ control technique [5], and so on. Despite all of these efforts, the attitude maneuver control of rigid spacecraft is still challenging.

Sliding mode control (SMC) is a nonlinear control technique that alters the dynamics of a nonlinear system by application of a discontinuous control law (or more rigorously, a set-valued control signal) that forces the system to "slide" along a cross-section of the system's normal behavior. Such a control technique was first proposed in [6] for variable structure systems. Subsequently, it has attracted much attention in handling spacecraft attitude control design because of its strong robustness [7], [8], [8], [9]. In [7], an SMC scheme was developed for a three-axis attitude control of rigid spacecraft with unknown dynamic parameters. Then, the SMC control strategy for the pure rigid spacecraft was extended to a flexible spacecraft, and an SMC strategy for the flexible spacecraft attitude maneuver was proposed in [8]. In [9], an SMC output feedback control law was presented to solve the attitude stabilization problem for the flexible spacecraft

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with uncertainty, disturbances, and control input nonlinearities. In [10], an SMC controller was derived for the attitude maneuver problem of a flexible spacecraft under control input nonlinearities, and only the attitude and angular rate information were used. Due to the existence of the sign function, the traditional SMC controllers suffer from the chattering problem. In order to alleviate such undesirable performance, the sign function was approximated by a saturation function [11], [12]. In [13], a higher-order integral SMC method was presented for the attitude control of rigid spacecraft, which was free of chattering because the nonlinear term was introduced into the first derivative of the control input. It should be pointed out that the aforementioned control methods were developed based on a linear sliding surface, and thus the system states reach their equilibrium point in infinite time rather than finite time [14]. Recently, a terminal sliding mode control methodology has been proposed, in which a nonlinear sliding surface was synthesized to achieve the finite-time control performance [15], [16]. In [15], the developed finite-time SMC law can guarantee the convergence of attitude tracking errors in finite time. In [16], by constructing a nonlinear sliding surface, adaptive finite-time SMC algorithms were presented to stabilize the flexible spacecraft attitude. In addition, the SMC technique was also combined with other control methods to obtain enhanced control performance for spacecraft attitude control, such as backstepping method [17], adaptive control [18].

A typical feature in most of the control approaches mentioned above for spacecraft is that the unwinding issue was ignored when the spacecraft attitude is described by quaternions. The quaternion has a double value property, and thus there are two mathematical representations for a given physical attitude of a rigid body [19]. Accordingly, there are two equilibriums $[1, 0, 0, 0]^T$ and $[-1, 0, 0, 0]^T$. However, in conventional control law design, only one equilibrium is considered. In this case, the system states have to move to the considered equilibrium, even if they are very close to another equilibrium. This is called the unwinding phenomenon, which may cause a spacecraft to perform an unnecessary large-angle maneuver when a small-angle maneuver is sufficient to achieve the control objective. To the best knowledge of the author, there are little research about the unwinding issue for spacecraft. In [20], the term $\text{sign}(q_0(0))$ was introduced into the sliding surface to avoid unwinding phenomenon. In [21], a new attitude error function $(1 - |q_0|)$ was constructed to design attitude controller, which considers two equilibrium $[1, 0, 0, 0]^T$ and $[-1, 0, 0, 0]^T$. But the strict proof of how the designed control laws avoid unwinding phenomenon was not given.

In this paper, the unwinding phenomenon is taken into account, and an anti-unwinding sliding mode attitude maneuver control law for rigid spacecraft is presented. The main contribution of this work can be summarized as follows. First of all, a novel switching function that contains two equilibrium

points is developed. Moreover, the anti-unwinding performance is proven when the system states are on the switching surface by constructing a Lyapunov function. This Lyapunov function is constructed by a hyperbolic cosine function. Secondly, a sliding mode control law is designed to guarantee that all the system trajectories are attracted by the switching surface. Further, the anti-unwinding performance is proven by designing a dynamic parameter for the sliding mode control law.

This paper proceeds as follows. In Section II, an attitude maneuver control problem of rigid spacecraft is stated. In Section III, a novel switching function is first constructed, and the property of the switching surface is analyzed. Furthermore, an anti-unwinding sliding mode controller is presented, and its anti-unwinding performance is proven. In Section V, comparing simulations are conducted to demonstrate the efficiency of the proposed attitude maneuver controller.

Throughout this paper, we use the italic-font notation for a scalar variable (as α), the bold-font notation for a vector (as \boldsymbol{v}), and the capital-letter notation for a matrix (as M). The set of n -dimensional real vectors and the set of m -by- n real matrices are denoted by \mathbb{R}^n and $\mathbb{R}^{m \times n}$, respectively. We use $\|\cdot\|$ to represent the 2-norm of a vector, $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ to represent the minimum and maximum eigenvalues of a matrix, respectively. In addition, the following two hyperbolic functions are used, $\sinh x = \frac{e^x - e^{-x}}{2}$, $\cosh x = \frac{e^x + e^{-x}}{2}$. Moreover, the following derivatives are used, $\frac{d(\sinh x)}{dx} = \cosh x$, $\frac{d(\cosh x)}{dx} = \sinh x$, $\frac{d(\arccos x)}{dt} = -\frac{x}{\sqrt{1-x^2}}$, $x \in \mathbb{R}$, respectively.

II. ATTITUDE MANEUVER CONTROL PROBLEM FORMULATION FOR A RIGID SPACECRAFT

In this paper, we aim to design an attitude maneuver controller to rotate the rigid spacecraft from the body frame \mathcal{F}_b to the desired frame \mathcal{F}_d . For this end, the attitude dynamics of the body frame \mathcal{F}_b is given in the subsequent section.

A. Rigid Spacecraft Attitude Kinematics and Dynamics

The quaternion based kinematic and dynamic equations of a rigid spacecraft can be given by [11]

$$\begin{cases} \dot{\boldsymbol{q}} = \frac{1}{2} \begin{bmatrix} -\boldsymbol{q}_v^T \\ q_0 I_3 + \boldsymbol{q}_v^\times \end{bmatrix} \boldsymbol{\omega}, \\ J\dot{\boldsymbol{\omega}} = -\boldsymbol{\omega}^\times J\boldsymbol{\omega} + \boldsymbol{u} + \boldsymbol{d}, \end{cases} \quad (1)$$

where unit quaternion $\boldsymbol{q} = [q_0 \ \boldsymbol{q}_v^T]^T \in \mathbb{R} \times \mathbb{R}^3$ represents the attitude of body frame \mathcal{F}_b with respect to inertia frame \mathcal{F}_I , $\boldsymbol{\omega} \in \mathbb{R}^3$ denotes the angular velocity of body frame \mathcal{F}_b with respect to inertia frame \mathcal{F}_I ; $J \in \mathbb{R}^{3 \times 3}$ is the inertia matrix (symmetric) of the whole rigid spacecraft, \boldsymbol{u} is the external torque acting on the main body, and \boldsymbol{d} is the external disturbance. In addition, for any vector $\boldsymbol{x} \in \mathbb{R}^3$, \boldsymbol{x}^\times represents a skew-symmetric matrix which can be given by

$$\boldsymbol{x}^\times := \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix}.$$

Based on the attitude dynamics of the body frame \mathcal{F}_b , the error kinematics and dynamics between the body frame \mathcal{F}_b and the desired frame \mathcal{F}_d are given in the next section.

B. Relative Attitude Error Kinematics and Dynamics

1) *Attitude Error Kinematics*: Let unit quaternion $\boldsymbol{q}_d := [q_{d0} \ \boldsymbol{q}_{dv}^T]^T \in \mathbb{R} \times \mathbb{R}^3$ represents the rigid spacecraft attitude of desired frame \mathcal{F}_d with respect to inertia frame \mathcal{F}_I . Let $\boldsymbol{\omega}_d \in \mathbb{R}^3$ denotes the rigid spacecraft angular velocity of \mathcal{F}_b with respect to \mathcal{F}_I and is expressed in \mathcal{F}_b . The attitude error $\boldsymbol{q}_e := [q_{e0} \ \boldsymbol{q}_{ev}^T]^T \in \mathbb{R} \times \mathbb{R}^3$ can be given by

$$\boldsymbol{q}_e = \boldsymbol{q}_d^* \otimes \boldsymbol{q}, \quad (2)$$

where $\boldsymbol{q}_d^* := [q_{d0} \ -\boldsymbol{q}_{dv}^T]^T$, and \otimes is the quaternion multiplication operator. Then, the components of the error quaternion \boldsymbol{q}_e can be obtained from (2),

$$\begin{aligned} q_{e0} &= \boldsymbol{q}_{dv}^T \boldsymbol{q}_v + q_{d0} q_0, \\ \boldsymbol{q}_{ev} &= q_{d0} \boldsymbol{q}_v - \boldsymbol{q}_{dv}^\times \boldsymbol{q}_v - q_0 \boldsymbol{q}_{dv}. \end{aligned} \quad (3)$$

Moreover, it can be derived from (3) that

$$q_{e0}^2 + \boldsymbol{q}_{ev}^T \boldsymbol{q}_{ev} = 1. \quad (4)$$

By taking derivative for (3), the following attitude error kinematics can be obtained as

$$\dot{\boldsymbol{q}}_e = \frac{1}{2} \begin{bmatrix} -\boldsymbol{q}_{ev}^T \\ q_{e0} I_3 + \boldsymbol{q}_{ev}^\times \end{bmatrix} \boldsymbol{\omega}_e, \quad (5)$$

where $\boldsymbol{\omega}_e \in \mathbb{R}^3$ represents the angular velocity error, and is defined as

$$\boldsymbol{\omega}_e := \boldsymbol{\omega} - R\boldsymbol{\omega}_d, \quad (6)$$

with R being the relative rotation matrix from \mathcal{F}_b to \mathcal{F}_d , which is given by

$$R := (q_{e0}^2 - \boldsymbol{q}_{ev}^T \boldsymbol{q}_{ev}) I_3 + 2\boldsymbol{q}_{ev} \boldsymbol{q}_{ev}^T - 2q_{e0} \boldsymbol{q}_{ev}^\times.$$

The rotation matrix R satisfies $\dot{R} = -\boldsymbol{\omega}_e^\times R$. Furthermore, it can be obtained from (6) that

$$\dot{\boldsymbol{\omega}}_e = \dot{\boldsymbol{\omega}} + \boldsymbol{\omega}_e^\times R\boldsymbol{\omega}_d - R\dot{\boldsymbol{\omega}}_d. \quad (7)$$

2) *Attitude Error Dynamics*: For a rest-to-rest attitude maneuver control problem, the desired attitude velocity satisfies $\boldsymbol{\omega}_d = 0$, $\dot{\boldsymbol{\omega}}_d = 0$. Thus, it can be obtained from (6) that $\boldsymbol{\omega}_e = \boldsymbol{\omega}$ holds. With this in mind, by substituting (6) and (7) into the second equation of (1), the following attitude error dynamic equation can be obtained,

$$J\dot{\boldsymbol{\omega}}_e = -\boldsymbol{\omega}_e^\times J\boldsymbol{\omega}_e + \boldsymbol{u} + \boldsymbol{d}. \quad (8)$$

Then, by (5) and (8), the attitude error dynamics for the rigid spacecraft can be obtained as [11]

$$\begin{cases} \dot{\boldsymbol{q}}_e = \frac{1}{2} \begin{bmatrix} -\boldsymbol{q}_{ev}^T \\ q_{e0} I_3 + \boldsymbol{q}_{ev}^\times \end{bmatrix} \boldsymbol{\omega}_e, \\ J\dot{\boldsymbol{\omega}}_e = -\boldsymbol{\omega}_e^\times J\boldsymbol{\omega}_e + \boldsymbol{u} + \boldsymbol{d}. \end{cases} \quad (9)$$

In addition, the error quaternion can also be written as [22],

$$\boldsymbol{q}_e = \begin{bmatrix} q_{e0} \\ \boldsymbol{q}_{ev} \end{bmatrix} = \begin{bmatrix} \cos \frac{\theta(t)}{2} \\ \boldsymbol{e} \sin \frac{\theta(t)}{2} \end{bmatrix}, \quad (10)$$

where $\theta(t) \in [0, 2\pi]$ is the rotation angle and $\boldsymbol{e} \in \mathbb{R}^3$ is the fixed Euler axis. Then, the following relation can be obtained by the first relation of (10),

$$\theta(t) = 2 \arccos q_{e0}. \quad (11)$$

It follows from the first relation of (9) and (4) that

$$\begin{aligned}\dot{\theta}(t) &= -\frac{2\dot{q}_{e0}}{\sqrt{1-q_{e0}^2}} \\ &= \frac{\mathbf{q}_{ev}^T \boldsymbol{\omega}_e}{\sqrt{1-q_{e0}^2}} \\ &= \frac{\mathbf{q}_{ev}^T}{\|\mathbf{q}_{ev}\|} \boldsymbol{\omega}_e.\end{aligned}\quad (12)$$

According to (3), $q_{e0}(0)$ and $\mathbf{q}_{ev}(0)$ can be obtained as long as the initial attitude $\mathbf{q}(0)$ of \mathbf{q} and the desired attitude \mathbf{q}_d are given. Further, the initial value $\theta(0)$ of $\theta(t)$ can be obtained by (11). By designing an attitude maneuver controller, the rigid spacecraft is driven to rotate about the fixed Euler axis \mathbf{e} , such that the rotation angle $\theta(t)$ converges from the initial value $\theta(0)$ to the equilibrium point.

C. Unwinding Phenomenon

It can be obtained from (10) that $q_{e0}|_{\theta(t)=0} = 1$ and $q_{e0}|_{\theta(t)=2\pi} = -1$, while $\theta(t) = 0$ and $\theta(t) = 2\pi$ represent the same position. Thus, $q_{e0} = 1$ and $q_{e0} = -1$ are both the equilibrium point of the attitude error dynamics (9) for a rigid spacecraft. However, in most existing controller design approaches, only $q_{e0} = 1$ is considered as equilibrium point. In this case, when the initial value of q_{e0} is less than 0, the designed controller drives q_{e0} to 0, and finally to 1. This means that the rigid spacecraft needs to rotate a Euler angle $\theta(t)$ larger than π . This is the "unwinding phenomenon". However, the rigid spacecraft can reach the desired attitude by rotating an angle smaller than π .

D. Control Objective

The control task in this work is to design an anti-unwinding attitude controller to accomplish a rest-to-rest attitude maneuver for the rigid spacecraft system (1). By adopting the designed control law for the closed-loop attitude maneuver error dynamics (9) of a rigid spacecraft, the following relations are achieved,

$$\lim_{t \rightarrow \infty} q_{e0} = 1 \text{ or } -1, \quad \lim_{t \rightarrow \infty} \boldsymbol{\omega}_e = 0. \quad (13)$$

Moreover, the unwinding phenomenon is avoided during the rigid spacecraft maneuver.

III. CONTROLLER DESIGN

In this section, we aim to design an anti-unwinding sliding mode control law to accomplish the control objective stated in Section II-D. First, a new switching surface is constructed in Section III-A, which considers both $q_{e0} = 1$ and $q_{e0} = -1$ to be equilibrium points. Then, the anti-unwinding performance when the system states are on the switching surface is proven. In section III-B, an anti-unwinding sliding mode attitude control law is derived based on the constructed switching function. In addition, a dynamic parameter is introduced to guarantee the anti-unwinding performance when the system states are outside the switching surface.

Before preceding, we first give the following lemmas.

Lemma 1: [23] Suppose $V(x)$ is a C^1 smooth positive-definite function (defined on $U \subset \mathbb{R}^n$) and $\dot{V}(x) + \lambda V^\alpha(x)$ is a negative semi-definite function on $U \subset \mathbb{R}^n$ for $\alpha \in (0, 1)$ and $\lambda \in \mathbb{R}^+$, then there exists an area $U_0 \subset \mathbb{R}^n$ such that any $V(x)$

which starts from $U_0 \subset \mathbb{R}^n$ can reach $V(x) \equiv 0$ in finite time. Moreover, if T_s is the time needed to reach $V(x) \equiv 0$, then

$$T_s \leq \frac{V^{1-\alpha}(x_0)}{\lambda(1-\alpha)},$$

where $V(x_0)$ is the initial value of $V(x)$.

Lemma 2: For any unit vector $\mathbf{x} \in \mathbb{R}^n$, the matrix $A = \mathbf{x}\mathbf{x}^T$ is a $n \times n$ idempotent matrix.

Proof. Note that $A = \mathbf{x}\mathbf{x}^T$ and \mathbf{x} is a unit vector, thus

$$\begin{aligned}AA &= \mathbf{x}\mathbf{x}^T \mathbf{x}\mathbf{x}^T \\ &= \mathbf{x}\mathbf{x}^T \\ &= A.\end{aligned}$$

This implies that the matrix A is an idempotent matrix. ■

Lemma 3: For any idempotent matrix $A \in \mathbb{R}^{n \times n}$, its eigenvalues are 1 or 0.

Proof. Suppose that the non-zero vector $\mathbf{y} \in \mathbb{R}^n$ is an eigenvector corresponding to a non-zero eigenvalue λ of the matrix A . Then, we have

$$A\mathbf{y} = \lambda\mathbf{y}. \quad (14)$$

Multiplying A of both sides of the above relation, gives

$$AA\mathbf{y} = \lambda A\mathbf{y}. \quad (15)$$

Because A is an idempotent matrix, thus the left side of (15) can be rewritten as $A\mathbf{y}$. In addition, by (14), the right side of (15) can be rewritten as $\lambda^2\mathbf{y}$. Then, there holds

$$A\mathbf{y} = \lambda^2\mathbf{y}. \quad (16)$$

Combining (14) with (16), yields

$$\lambda\mathbf{y} = \lambda^2\mathbf{y}.$$

Because the vector \mathbf{y} is a non-zero vector, then there holds $\lambda = 1$ or 0. Thus, the proof is completed. ■

A. Switching Surface

For the attitude error dynamics (9) of a rigid spacecraft, we design the following switching function,

$$\mathbf{s} = \boldsymbol{\omega}_e + \lambda\boldsymbol{\sigma}, \quad (17)$$

where λ is a positive constant, and

$$\boldsymbol{\sigma} := \sinh(q_{e0}) \mathbf{q}_{ev}. \quad (18)$$

Next, the fact that the switching surface $\mathbf{s} = 0$ containing two equilibriums $q_{e0} = 1$ and -1 is proven. In addition, the convergence performance of the attitude error variables $\boldsymbol{\omega}_e$ and \mathbf{q}_{ev} on the switching surface $\mathbf{s} = 0$ is analyzed. In addition, the anti-unwinding performance of the designed switching function (17) in the sliding phase is demonstrated.

Before given the theorem, we should give some properties of the functions $\cosh q_{e0}$ and $\sinh \cos \frac{\theta(t)}{2}$. The maximum value of the function $\cosh q_{e0}$ can be obtained when $q_{e0} = 1$ and $q_{e0} = -1$. For $\theta(t) \in (0, \pi]$, $\sinh \cos \frac{\theta(t)}{2} \geq 0$, and for $\theta(t) \in (\pi, 2\pi)$, $\sinh \cos \frac{\theta(t)}{2} \leq 0$.

Theorem 4: If the system states of the attitude error dynamics (9) are restricted to the switching surface $\mathbf{s} = 0$, the following conclusions are achieved:

- (i) The switching surface $\mathbf{s} = 0$ contains two equilibriums $q_{e0} = 1$ and -1 , and the control goal in (13) is guaranteed.
- (ii) The unwinding phenomenon is avoided in the sliding phase.

Proof. First, we choose the following Lyapunov function,

$$V_1(t) := 2(\kappa - \cosh q_{e0}), \quad (19)$$

where $\kappa = \max(\cosh q_{e0})$ for $q_{e0} \in [-1, 1]$. By taking time derivative of (19), and using the first equation of (9), the condition $s = 0$, and (18), we have

$$\begin{aligned} \dot{V}_1(t) &= -2 \sinh(q_{e0}) \dot{q}_{e0} \\ &= \sinh(q_{e0}) \mathbf{q}_{ev}^T \boldsymbol{\omega}_e \\ &= \boldsymbol{\sigma}^T \boldsymbol{\omega}_e \\ &= -\lambda \boldsymbol{\sigma}^T \boldsymbol{\sigma}. \end{aligned} \quad (20)$$

Thus, $\dot{V}_1(t) \leq 0$. Further, it can be derived from (20) that if $\dot{V}_1(t) = 0$, there holds $\boldsymbol{\sigma} = 0$. Then, it follows from (18) that $q_{e0} = 0$ or $\mathbf{q}_{ev} = 0$. According to (4), there holds $q_{e0} = 1$ or -1 when $\mathbf{q}_{ev} = 0$. Moreover, it can be obtained from (19) that $\min(V_1(t)) = V_1(t)|_{q_{e0}=1} = V_1(t)|_{q_{e0}=-1} = 0$, and $V_1(t)|_{q_{e0}=0} \neq 0$. This means that the switching surface $s = 0$ contains two equilibriums $q_{e0} = 1$ and $q_{e0} = -1$. In addition, substituting $\mathbf{q}_{ev} = 0$ into (17) gives $\boldsymbol{\omega}_e = 0$.

Thus, the conclusion (i) is proven.

Next, the anti-unwinding performance of the attitude error dynamics (9) with system states being on the switching surface $s = 0$ is proven. According to (11), the Lyapunov function (19) can be rewritten as

$$V_1(t) := 2 \left(\kappa - \cosh \cos \frac{\theta(t)}{2} \right).$$

Consequently,

$$\dot{V}_1(t) = \sin \frac{\theta(t)}{2} \sinh \cos \frac{\theta(t)}{2} \dot{\theta}(t). \quad (21)$$

In addition, there hold $\sin \frac{\theta(t)}{2} > 0$ for $\theta(t) \in (0, 2\pi)$, $\sinh \cos \frac{\theta(t)}{2} \geq 0$ for $\theta(t) \in (0, \pi]$, and $\sinh \cos \frac{\theta(t)}{2} \leq 0$ for $\theta(t) \in (\pi, 2\pi)$. Note that $\dot{V}_1(t) \leq 0$, it can be derived from (21) that there hold $\dot{\theta}(t) \leq 0$ for $\theta(t) \in (0, \pi]$ and $\dot{\theta}(t) \geq 0$ for $\theta(t) \in (\pi, 2\pi)$. Suppose that the system states reach the switching surface $s = 0$ when $t = t_{s0}$. Then, if $\theta(t_{s0}) \in (0, \pi]$, there holds $\lim_{t \rightarrow \infty} \theta(t) = 0$, and if $\theta(t_{s0}) \in (\pi, 2\pi)$, there holds $\lim_{t \rightarrow \infty} \theta(t) = 2\pi$. This implies that the unwinding phenomenon is avoided when the system states are restricted to the switching surface $s = 0$. ■

B. Anti-Unwinding Sliding Mode Attitude Maneuver Control Law

In this section, we need to construct a control law such that the condition $\mathbf{s}^T \dot{\mathbf{s}} < 0$ is satisfied. This condition assures us that the switching surface $s = 0$ will attract all the system trajectories.

Consider a class of state feedback control for the attitude error dynamics (9) of a rigid spacecraft in the following form,

$$\mathbf{u} = \mathbf{u}_{eq} + \mathbf{u}_n, \quad (22)$$

where the term \mathbf{u}_{eq} is the equivalent control for the nominal system, the term \mathbf{u}_n is designed to compensate the disturbance. Thus, the equivalent control \mathbf{u}_{eq} can be obtained from the nominal system part by setting $\dot{\mathbf{s}}$ to be zero. That is

$$\dot{\mathbf{s}} = \dot{\boldsymbol{\omega}}_e + \lambda \boldsymbol{\sigma} = 0. \quad (23)$$

The nominal part of the attitude error dynamics (9) is

$$\dot{\boldsymbol{\omega}}_e = J^{-1} (-\boldsymbol{\omega}_e^\times J \boldsymbol{\omega}_e + \mathbf{u}_{eq}).$$

Substituting this expression into (23), gives

$$\mathbf{u}_{eq} = \boldsymbol{\omega}_e^\times J \boldsymbol{\omega}_e - \lambda J \boldsymbol{\sigma}. \quad (24)$$

The control term \mathbf{u}_n is designed as

$$\mathbf{u}_n = -(\gamma_1 + \gamma_2(t)) \mathbf{f}(s), \quad (25)$$

where $\gamma_1 \geq \|\mathbf{d}\|_{\max}$, $\gamma_2(t)$ is a positive-valued function which will be given later, and

$$\mathbf{f}(s) = \begin{cases} \text{sgn}(s), & \|s\| \neq 0, \\ 0, & \|s\| = 0, \end{cases} \quad (26)$$

with $\text{sgn}(s) = [\text{sgn}(s_1) \text{sgn}(s_2) \text{sgn}(s_3)]^T$, and

$$\text{sgn}(s_i) = \begin{cases} 1, & s_i > 0 \\ -1, & s_i \leq 0 \end{cases} \quad (i = 1, 2, 3).$$

Then, the following anti-unwinding sliding mode attitude maneuver control (briefly, AUSMAMC) law is presented,

$$\begin{cases} \mathbf{u} = \mathbf{u}_{eq} + \mathbf{u}_n, \\ \mathbf{u}_{eq} = \boldsymbol{\omega}_e^\times J \boldsymbol{\omega}_e - \lambda J \boldsymbol{\sigma}, \\ \mathbf{u}_n = -(\gamma_1 + \gamma_2(t)) \mathbf{f}(s), \\ \mathbf{s} = \boldsymbol{\omega}_e + \lambda \boldsymbol{\sigma}, \\ \boldsymbol{\sigma} = \sinh(q_{e0}) \mathbf{q}_{ev}, \end{cases} \quad (27)$$

where λ is a positive numbers, $\gamma_1 \geq \|\mathbf{d}\|_{\max}$, and $\gamma_2(t)$ is a positive-valued function, which will be given in the following section.

C. Convergence Analysis

In this section, the convergence of the closed-loop system under the developed AUSMAMC law (27) is analyzed. In addition, the anti-unwinding performance is proven in the following theorem.

Theorem 5: Consider a rigid spacecraft described by (9) in the presence of disturbance. If the parameter $\gamma_2(t)$ of the proposed AUSMAMC law (27) is chosen as

$$\gamma_2(t) = \frac{\lambda}{\lambda_{\min}(J^{-1})} |\dot{g}|, \quad (28)$$

where

$$g := \sinh(q_{e0}) \|\mathbf{q}_{ev}\|. \quad (29)$$

Then, the following conclusions are achieved:

(i) The switching function s converges to zero in finite time.

(ii) The unwinding phenomenon is avoided before the system states reach the switching surface $s = 0$.

Proof. To prove the conclusion (i), we choose the following Lyapunov function,

$$V_2(t) = \frac{1}{2} \mathbf{s}^T \mathbf{s}. \quad (30)$$

With the help of (9) and (17), we obtain from (30) that

$$\begin{aligned} \dot{V}_2(t) &= \mathbf{s}^T \dot{\mathbf{s}} \\ &= \mathbf{s}^T (\dot{\boldsymbol{\omega}}_e + \lambda \boldsymbol{\sigma}) \\ &= \mathbf{s}^T (J^{-1} (-\boldsymbol{\omega}_e^\times J \boldsymbol{\omega}_e + \mathbf{u} + \mathbf{d}) + \lambda \boldsymbol{\sigma}). \end{aligned} \quad (31)$$

Substituting the AUSMAMC law (27) with $\gamma_1 \geq \|\mathbf{d}\|_{\max}$ into (31), results in

$$\begin{aligned} \dot{V}_2(t) &= \mathbf{s}^T (J^{-1} (-\boldsymbol{\omega}_e^\times J \boldsymbol{\omega}_e + \mathbf{u}_{eq} + \mathbf{u}_n + \mathbf{d}) + \lambda \boldsymbol{\sigma}) \\ &= \mathbf{s}^T J^{-1} (\mathbf{u}_n + \mathbf{d}) \\ &= -\gamma_2(t) \mathbf{s}^T J^{-1} \text{sgn}(s) + \mathbf{s}^T J^{-1} (\|\mathbf{d}\|_{\max} - \gamma_1) \\ &\leq -\gamma_2(t) \mathbf{s}^T J^{-1} \text{sgn}(s). \end{aligned} \quad (32)$$

It is obvious that there holds

$$\mathbf{s}^T J^{-1} \text{sgn}(\mathbf{s}) \geq \lambda_{\min}(J^{-1}) \|\mathbf{s}\|. \quad (33)$$

Thus, it can be obtained from (32) and (33) that

$$\dot{V}_2(t) \leq -\gamma_2(t) \lambda_{\min}(J^{-1}) \|\mathbf{s}\|.$$

By combining this with (30) and (33), it is easy to obtain that

$$\begin{aligned} \dot{V}_2(t) &\leq -\sqrt{2}\gamma_2(t) \lambda_{\min}(J^{-1}) \left(\frac{1}{2}\mathbf{s}^T \mathbf{s}\right)^{\frac{1}{2}} \\ &= -\sqrt{2}\gamma_2(t) \lambda_{\min}(J^{-1}) V_2^{\frac{1}{2}}(t). \end{aligned} \quad (34)$$

Clearly, $\dot{V}_2(t) \leq 0$. Thus, it can be obtained from Lemma 1 that the switching function \mathbf{s} converges to 0 in finite time. The proof of (i) is completed.

Next, by designing the dynamic parameter $\gamma_2(t)$ for the proposed AUSMAMC law (27), the anti-unwinding performance before the system states reach the switching surface $\mathbf{s} = 0$ is guaranteed.

In view of (34), we get

$$\frac{\dot{V}_2(t)}{V_2^{\frac{1}{2}}(t)} \leq -\sqrt{2}\gamma_2(t) \lambda_{\min}(J^{-1}).$$

Suppose that the initial time is $t_0 = 0$. Then, by taking integral of both sides of the above equation, we arrive at

$$\int_0^t \frac{\dot{V}_2(\tau)}{V_2^{\frac{1}{2}}(\tau)} d\tau \leq -\sqrt{2}\lambda_{\min}(J^{-1}) \int_0^t \gamma_2(\tau) d\tau.$$

A direct calculation gives

$$V_2^{\frac{1}{2}}(t) \leq -\frac{\lambda_{\min}(J^{-1})}{\sqrt{2}} \int_0^t \gamma_2(\tau) d\tau + V_2^{\frac{1}{2}}(0). \quad (35)$$

Let

$$v(t) = \frac{\mathbf{q}_{\text{ev}}^T \mathbf{s}}{\|\mathbf{q}_{\text{ev}}\|}. \quad (36)$$

By (12), (17), and (29), the above relation can be rewritten as

$$\begin{aligned} v(t) &= \left(\frac{\mathbf{q}_{\text{ev}}^T}{\|\mathbf{q}_{\text{ev}}\|} \boldsymbol{\omega}_e + \lambda \sinh(q_{e0}) \frac{\mathbf{q}_{\text{ev}}^T \mathbf{q}_{\text{ev}}}{\|\mathbf{q}_{\text{ev}}\|} \right) \\ &= (\dot{\theta}(t) + \lambda g). \end{aligned} \quad (37)$$

Further, it can be derived from (36) that

$$\begin{aligned} v^2(t) &= \left(\frac{\mathbf{q}_{\text{ev}}^T}{\|\mathbf{q}_{\text{ev}}\|} \mathbf{s} \right)^T \frac{\mathbf{q}_{\text{ev}}^T}{\|\mathbf{q}_{\text{ev}}\|} \mathbf{s} \\ &= \mathbf{s}^T \frac{\mathbf{q}_{\text{ev}} \mathbf{q}_{\text{ev}}^T}{\|\mathbf{q}_{\text{ev}}\|^2} \mathbf{s}. \end{aligned} \quad (38)$$

Note that $\frac{\mathbf{q}_{\text{ev}}}{\|\mathbf{q}_{\text{ev}}\|}$ is a unit vector, thus it follows from (38), Lemmas 2 and 3 that

$$\begin{aligned} v^2(t) &\leq \lambda_{\max} \left(\frac{\mathbf{q}_{\text{ev}} \mathbf{q}_{\text{ev}}^T}{\|\mathbf{q}_{\text{ev}}\|^2} \right) \|\mathbf{s}\|^2 \\ &\leq \|\mathbf{s}\|^2. \end{aligned}$$

Combining this with (30) yields

$$\frac{1}{2}v^2(t) \leq V_2(t). \quad (39)$$

It should be noted that, the rest-to-rest attitude maneuver issue is considered in this paper. Thus, $\boldsymbol{\omega}_e(0) = 0$. Further, it can be obtained from (12) that $\dot{\theta}(0) = 0$. Then, the initial value of $v(t)$ in (37) can be obtained as

$$v(0) = \lambda g(0). \quad (40)$$

Moreover, with the help of (29) and (40), we can get the initial value of $V_2(0)$ in (30) as (recall that $\boldsymbol{\omega}_e = 0$)

$$\begin{aligned} V_2(0) &= \frac{1}{2}\lambda^2 (\sinh(q_{e0}) \mathbf{q}_{\text{ev}})^T (\sinh(q_{e0}) \mathbf{q}_{\text{ev}}) \\ &= \frac{1}{2}\lambda^2 \sinh^2(q_{e0}) \mathbf{q}_{\text{ev}}^T \mathbf{q}_{\text{ev}} \\ &= \frac{1}{2}v^2(0) \end{aligned} \quad (41)$$

Thus, the following relation can be obtained from (35), (39), and (41),

$$\begin{aligned} \left(\frac{1}{2}v^2(t)\right)^{\frac{1}{2}} &\leq V_2^{\frac{1}{2}}(t) \leq -\frac{\lambda_{\min}(J^{-1})}{\sqrt{2}} \int_0^t \gamma_2(\tau) d\tau \\ &\quad + \left(\frac{1}{2}v^2(0)\right)^{\frac{1}{2}}, \end{aligned}$$

which can be further written as,

$$|v(t)| \leq -\lambda_{\min}(J^{-1}) \int_0^t \gamma_2(\tau) d\tau + |v(0)|. \quad (42)$$

Moreover, because $\gamma_2(t) > 0$, then it can be obtained from (42) that $v(t)$ will decrease to 0 when $v(0) > 0$, and $v(t)$ will increase to 0 when $v(0) < 0$.

To prove the anti-unwinding property of the proposed control law (27), we need to prove that $\theta(t) \leq 0$ for $\theta(0) \in (0, \pi]$ and $\dot{\theta}(t) \geq 0$ for $\theta(0) \in (\pi, 2\pi)$. For this end, the following two cases are considered to complete the proof.

(a) When $\theta(0) \in (0, \pi]$, by the first equation of (10), we have $\mathbf{q}_{e0}(0) > 0$. Then, using (40) and (29), we get

$$v(0) = \lambda \sinh(q_{e0}(0)) \|\mathbf{q}_{\text{ev}}(0)\| > 0.$$

Thus, $v(t)$ will decrease to 0 due to (42). In such a case, it can be further obtained from (42) that

$$\dot{\theta}(t) + \lambda g \leq -\lambda_{\min}(J^{-1}) \int_0^t \gamma_2(\tau) d\tau + \lambda g(0).$$

It can be further rewritten as

$$\begin{aligned} \dot{\theta}(t) &\leq -\lambda_{\min}(J^{-1}) \int_0^t \gamma_2(\tau) d\tau + \lambda(g(0) - g) \\ &= -\lambda_{\min}(J^{-1}) \int_0^t \gamma_2(\tau) d\tau - \lambda \int_0^t \frac{dg}{d\tau} d\tau \\ &= -\int_0^t \left(\lambda_{\min}(J^{-1}) \gamma_2(\tau) + \lambda \frac{dg}{d\tau} \right) d\tau. \end{aligned} \quad (43)$$

If $\dot{g} > 0$, then it can be obtained from (28) that $\gamma_2(t) = \frac{\lambda \dot{g}}{\lambda_{\min}(J^{-1})}$. It is followed from (43) that

$$\begin{aligned} \dot{\theta}(t) &\leq -2\lambda \int_0^t \frac{dg(\tau)}{d\tau} d\tau \\ &\leq 0. \end{aligned}$$

If $\dot{g} \leq 0$, then it can be obtained from (28) that $\gamma_2(t) = -\frac{\lambda \dot{g}}{\lambda_{\min}(J^{-1})}$. With this, it can be derived from (43) that $\dot{\theta}(t) \leq 0$.

In conclusion, it can be obtained from above two cases that when $\theta(0) \in (0, \pi]$, the rotation angle $\theta(t)$ will decrease to 0.

(b) When $\theta(0) \in (\pi, 2\pi)$, by the first equation of (10), we have $\mathbf{q}_{e0}(0) < 0$. Then, using (40) and (29), we get

$$v(0) = \lambda \sinh(q_{e0}(0)) \|\mathbf{q}_{\text{ev}}(0)\| < 0.$$

Thus, $v(t)$ will increase to 0 due to (42). In this case, it can be obtained from (42) that,

$$-\dot{\theta}(t) - \lambda g \leq -\lambda_{\min}(J^{-1}) \int_0^t \gamma_2(\tau) d\tau - \lambda g(0),$$

or, equivalently,

$$\begin{aligned}\dot{\theta}(t) &\geq \int_0^t \lambda_{\min}(J^{-1}) \gamma_2(\tau) d\tau + \lambda g(0) - \lambda g \\ &= \int_0^t \lambda_{\min}(J^{-1}) \gamma_2(\tau) d\tau - \lambda \int_0^t \frac{dg}{d\tau} d\tau \\ &= \int_0^t \left(\lambda_{\min}(J^{-1}) \gamma_2(\tau) d\tau - \lambda \frac{dg}{d\tau} \right) d\tau. \quad (44)\end{aligned}$$

If $\dot{g} > 0$, there holds $\gamma_2(t) = \frac{\lambda \dot{g}}{\lambda_{\min}(J^{-1})}$. Substitute it into (44), we have $\dot{\theta}(t) \geq 0$.

If $\dot{g} \leq 0$, there holds $\gamma_2(t) = -\frac{\lambda \dot{g}}{\lambda_{\min}(J^{-1})}$. Substitute it into (44), yields

$$\begin{aligned}\dot{\theta}(t) &\geq -2\lambda \int_0^t \frac{dg}{d\tau} d\tau \\ &\geq 0.\end{aligned}$$

Thus, it can be obtained from above two cases that when $\theta(0) \in (\pi, 2\pi)$, the rotation angle $\theta(t)$ will increase to 2π .

Based on above discussion, we have proven the conclusion that the unwinding phenomenon is successfully avoided under the AUSMAMC law (27) with $\gamma_2(t) = \frac{\lambda |\dot{g}|}{\lambda_{\min}(J^{-1})}$. ■

In Theorem 5, the anti-unwinding performance before the system states reach the switching surface is proven. In Theorem 4, the anti-unwinding performance when the system states are constricted on the switching surface is also shown. The results in these two theorems have illustrated that the proposed AUSMAMC law (27) has the performance of anti-unwinding.

Remark 1: A drawback of the control law (25) is that it is discontinuous about the switching surface $s = 0$. This characteristic may cause an undesirable chattering phenomenon. For practical implementations, the controller must be smoothed. Thus, the discontinuous function $\text{sgn}(s)$ is replaced by the smooth continuous function $l(s) := [l(s_1) \ l(s_2) \ l(s_3)]^T$ with $l(s_i)$ in the following equation,

$$l(s_i) := \begin{cases} \text{sgn}(s_i), & \text{if } |s_i| \geq \varepsilon, \\ \arctan \frac{s_i \tan(1)}{\varepsilon}, & \text{if } |s_i| < \varepsilon, \end{cases} \quad (45)$$

where ε is a small positive value. As ε approaches zero, the performance of this boundary layer can be made arbitrarily close to that of original control law.

The advantage of the proposed AUSMAMC law (27) is that the unwinding phenomenon can be avoided during the rigid spacecraft attitude maneuver, and the disturbance can be compensated by the designed controller. Besides, the developed control law has only two tunable parameters.

IV. EXAMPLE

In this section, simulations are conducted to demonstrate the performance of the presented AUSMAMC law (27) for rest-to-rest attitude maneuvers of a rigid spacecraft. In addition, the existing controller (11) in [24] and (23) in [20] are adopted for comparison.

A. Simulation Settings

1) *Spacecraft parameter values:* The inertia matrix of the rigid spacecraft is $J = [20 \ 0 \ 0.9; 0 \ 17 \ 0; 0.9 \ 0 \ 15] \text{ kg} \cdot \text{m}^2$. The initial value of the attitude velocity ω and quaternion q are given in TABLE I. The disturbance is $d = 10^{-2} \times [\sin(0.05t) \ 0.5 \sin(0.05t) \ -\cos(0.05t)]^T$.

TABLE I: Initial value of the signal of the rigid spacecraft system

Notation	Unit	Meaning	Initial value
$q(0) \in \mathbb{R}^4$	/	Attitude of \mathcal{F}_b , with respect to \mathcal{F}_I	$[1 \ 0 \ 0 \ 0]^T$
$q_0(0) \in \mathbb{R}$	/	Scalar part of q	1
$q_v(0) \in \mathbb{R}^3$	/	Vector part of q	$[0 \ 0 \ 0]^T$
$\omega(0) \in \mathbb{R}^3$	rad/s	Attitude of \mathcal{F}_b , with respect to \mathcal{F}_I	$[0 \ 0 \ 0]^T$

2) *Controller parameter values:* The tuning parameters of the proposed AUSMAMC law (27) are chosen by a trail-to-trail selection. The parameters of the controller (11) [24], and controller (23) [20] are chosen the same as in [24] and [20], respectively. The value of the parameters of the above controllers are shown in TABLE II. In addition, $\gamma_2(t)$ can be obtained from (28).

TABLE II: Control parameters chosen for numerical analysis

Control schemes	control parameters
AUSMAMC (27)	$\lambda = 2, \gamma_1 = 10, \varepsilon = 0.5$
Controller (11) [24]	$k = 1, \tau = 15I_3, \sigma = 0.001I_3, p_0 = 1, p_1 = 1, p_2 = 1, \hat{c}(0) = 1, \hat{k}_1(0) = 0.1, \hat{k}_2(0) = 0.1$
Controller (23) [20]	$v_1 = 5I_3, v_2 = 7I_3, \rho = 0.001I_3, k = 1.5, \eta = 0.14, \theta = 7, \hat{K}_1(0) = 0, \hat{K}_2(0) = 0$

3) *Control goal:* The control goal is to perform two rest-to-rest attitude maneuvers for the rigid spacecraft with system parameters given in Section IV-A1. Two different scenarios of desired attitude value are given in the following.

Scenario A. The initial values of the desired quaternion and angular velocity are $q_d = [0.8832 \ 0.3 \ -0.2 \ -0.3]^T$, and $\omega_d = [0 \ 0 \ 0]^T \text{ rad/s}$, respectively.

Scenario B. The initial values of the desired quaternion and angular velocity are $q_d = [-0.6403 \ -0.5 \ -0.3 \ 0.5]^T$, and $\omega_d = [0 \ 0 \ 0]^T \text{ rad/s}$, respectively.

In Scenario A, $q_{d0} > 0$, thus $q_e = [1 \ 0 \ 0 \ 0]^T$ is the nearest equilibrium. In addition, according to the first equation of (10), the spacecraft needs to rotate 55.93° to reach the equilibrium point. In Scenario B, $q_{d0}(0) < 0$, and the spacecraft needs to tilt 259.62° if only the equilibrium $q_e = [1 \ 0 \ 0 \ 0]^T$ is considered. However, the spacecraft only need to rotate 100.38° if $q_e = [-1 \ 0 \ 0 \ 0]^T$ is also considered as an equilibrium.

B. Simulation results

1) *Simulation results for Scenario A:* The controller (11) [24] and proposed AUSMAMC law (27) are adopted to do simulations for Scenario A. The simulation results are shown in Fig. 1, where the controller A represents the controller (11) [24].

The response of error quaternions $q_{ei}, i = 1, 2, 3, 4$ and angular velocity error $\omega_{ei}, i = 1, 2, 3$ are shown in Fig. 1(a) and Fig. 1(b), respectively. It can be seen from Fig. 1(a) and Fig. 1(b) that the attitude errors of system (9) converge to 0 in about 4s by adopting the proposed AUSMAMC law (27), while the Controller A needs longer time. In addition, it can be easily obtained from these two figures that the steady attitude errors of the developed control law AUSMAMC law (27) are smaller than that of the Controller A. The spacecraft attitude responses using Euler angles ϕ, θ, ψ (ϕ, θ, ψ are the roll, pitch, and yaw angles, respectively) are shown in Fig. 1(c), which indicates that the attitude maneuver problem can be effectively settled by the controller AUSMAMC law (27) and Controller A. The time evolution of control torques $u_i, i = 1, 2, 3$ are shown in Fig.

1(d). The control torque of the proposed AUSMAMC law (27) is smaller than that of Controller A.

The AUSMAMC controller is able to obtain higher pointing accuracy and better stability in a shorter time.

2) *Simulation results for Scenario B:* The controller (11) [24], controller (23) [20], and the proposed AUSMAMC law (27) are adopted to do simulations for Scenario B. The simulation results are summarized in Fig. 2, where the controller A is the controller (11) [24], and controller B is the controller (23) [20].

The response of error quaternions q_{ei} , $i = 1, 2, 3, 4$ are shown in Fig. 2(a), which indicates that q_e converges to the nearest equilibrium $[-1 \ 0 \ 0 \ 0]$ in about 5s by adopting the presented controller AUSMAMC (27) and Controller B. However, q_e converges to $[1 \ 0 \ 0 \ 0]$ in about 14s by adopting the Controller A. Thus, it can be obtained that the presented AUSMAMC law (27) in this paper and (23) [20] avoids unwinding phenomenon successfully, but the Controller A suffers unwinding problem. The behaviour of angular velocity error ω_{ei} , $i = 1, 2, 3$ is shown in Fig. 2(b). It can be observed from Fig. 2(b) that the attitude velocity of the rigid spacecraft (9) converges to 0 in about 5s by using the proposed AUSMAMC law (27) and Controller B, while the Controller A needs longer time. In addition, it can be easily obtained from these two figures that the steady attitude errors of the developed control law AUSMAMC law (27) are smaller than that of the Controller A and Controller B. The spacecraft attitude responses using Euler angles ϕ, θ, ψ (ϕ, θ, ψ are the roll, pitch, and yaw angles, respectively) are shown in Fig. 2(c). The maneuver angle of the AUSMAMC law (27) and Controller B is smaller than that of Controller A. The control torques u_i , $i = 1, 2, 3$ are shown in Fig. 2(d), which indicates that the attitude maneuver is effectively settled by the controller AUSMAMC law (27), the Controllers A and B. It can also be observed that the control torque of the proposed control law is less than that of the Controllers A and B.

In conclusion, the proposed AUSMAMC controller (27) satisfies the control objective described in Section II-D, and it achieves higher pointing accuracy and better stability in a shorter time compared with the controller (11) [24], and controller (23) [20].

V. CONCLUSION

In this paper, an anti-unwinding attitude maneuver control law is presented for rigid spacecraft. By constructing a new switching surface, which contains two equilibriums, the unwinding problem is settled when the system states are on the switching surface. Moreover, by designing a sliding mode control law with a dynamic parameter, the anti-unwinding performance is guaranteed before the system states reach the switching surface. Further, the switching function, the attitude velocity error, and the vector part of error quaternion converge to zero under the designed anti-unwinding sliding mode attitude maneuver control law. Finally, a numerical simulation is conducted to demonstrate the effectiveness of the developed control law. The simulation results show that the unwinding phenomenon is avoided by adopting the designed switching surface and controller.

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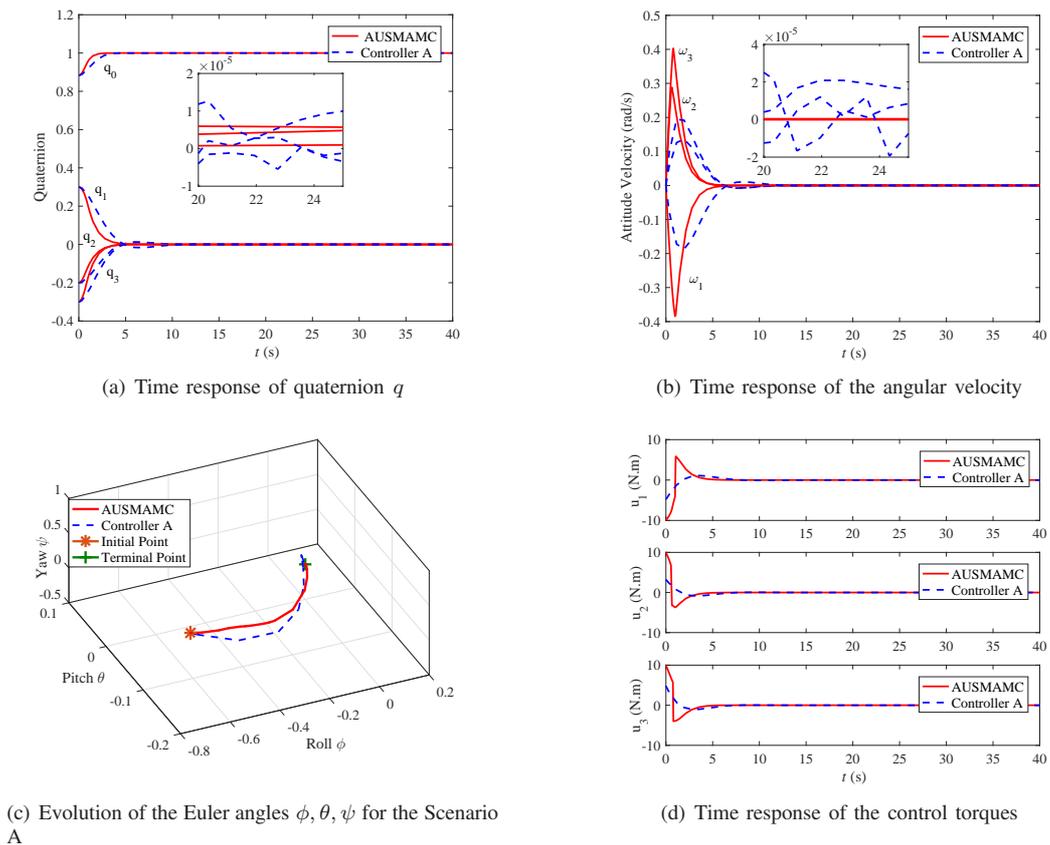


Fig. 1: Comparison results of AUSMAMC law (27) and controller (11) [24] for Scenario A

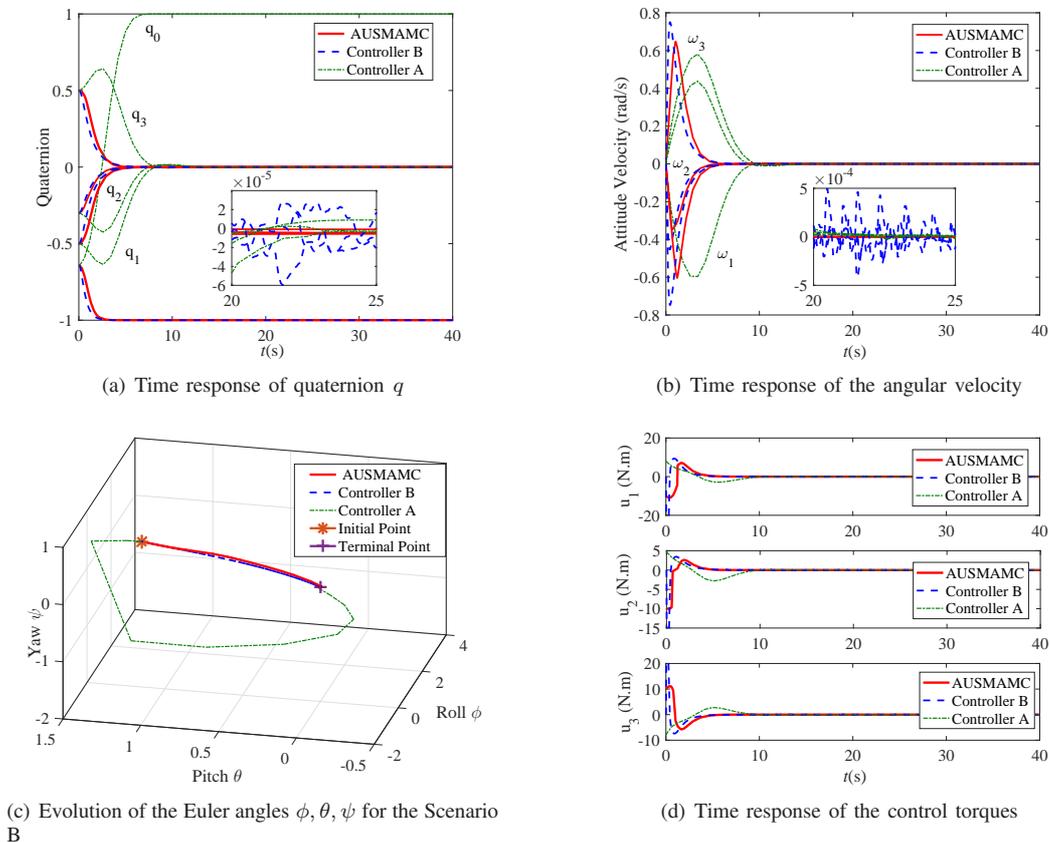


Fig. 2: Comparison results of AUSMAMC law (27) and controllers (11) [24] and (23) [20] for Scenario B