

Distributed Optimal Control with Recovered Robustness for Uncertain Network Systems: A Complementary Design Approach

Zhongkui Li, Junjie Jiao, and Xiang Chen

Abstract—This paper considers the distributed robust suboptimal consensus control problem of linear multi-agent systems, with both H_2 and H_∞ performance requirements. A novel two-step complementary design approach is proposed. In the first step, a distributed control law is designed for the nominal multi-agent system to achieve consensus with a prescribed H_2 performance. In the second step, an extra control input, depending on some carefully chosen residual signals indicating the modeling mismatch, is designed to complement the H_2 performance by providing robustness guarantee in terms of H_∞ requirement with respect to disturbances or uncertainties. The proposed complementary design approach provides an additional degree of freedom for design, having two separate controls to deal with the H_2 performance and the robustness of consensus, respectively. Thereby, it does not need to make much trade-off, and can be expected to be much less conservative than the trade-off design such as the mixed H_2/H_∞ control method. Besides, this complementary approach will recover the achievable H_2 performance when external disturbances or uncertainties do not exist. The effectiveness of the theoretical results and the advantages of the complementary approach are validated via numerical simulations.

Index Terms—Robust control, cooperative control, consensus, distributed control, optimal control, H_∞ control, H_2 control.

I. INTRODUCTION

Optimality and robustness are two main issues and missions in the feedback control theory [1]. The optimality requires an optimal or suboptimal controller to ensure that the closed-loop system satisfies certain predefined performance criteria, with the linear quadratic regulator (LQR) and the linear quadratic Gaussian (LQG)/ H_2 problems as typical examples. The robustness, on the other hand, characterizes the property that a system still works well in the presence of external disturbances and model uncertainties, which can be addressed in the framework of H_∞ control and μ synthesis. For multi-agent network systems, the optimality and robustness problems encounter new inherent challenges, since the control laws need to be distributed in the sense that only local information between neighboring agents can be utilized and meanwhile the control laws are subject to structural constraints imposed by the network topology [2].

In the last decades, many advances have been reported on the optimality and robustness issues of network systems. Distributed optimal and suboptimal LQR problems of multi-agent systems were addressed in [3], [4], [5], [6], [7]. The coherence and centrality of multi-agent networks were formulated and discussed in [8], [9], [10], [11], [12], [13], through studying the H_2 norms of the networks from the white noises to the performance variables. The H_2 consensus problems of multi-agent systems were investigated in [14], [15],

where distributed state and output feedback consensus protocols were designed to satisfy prescribed H_2 performance indices. Disturbance attenuation problems of linear consensus networks were studied from the H_∞ control perspective in [16], [17], [14], [18], [19]. For multi-agent systems, both the agent dynamics and the interactions among neighboring agents could be perturbed by uncertainties. The robust synchronization and consensus problems of multi-agent systems whose agent dynamics are subject to multiplicative and coprime factor uncertainties were investigated in [20], [21], [22]. The authors in [23], [24], [25], [26] considered the robustness of multi-agent consensus in the sense of mean square and almost sure stabilities for the cases where the communication channels among agents are subject to multiplicative stochastic uncertainties. Robust consensus over deterministic uncertain network graphs was studied in [27], [28].

In this paper, we will address the distributed robust optimal consensus problems for linear multi-agent systems, taking into account both the optimality and robustness at the same time. It is well-known that there is an intrinsic conflict between optimality and robustness in the standard feedback framework [1], [29]. Therefore, in the case of multi-objective design, e.g., the mixed H_2/H_∞ control [30] and the H_∞ Gaussian control [31], a trade-off has to be made between the achievable optimal performance and robustness [29]. These trade-off design approaches suffer from a fundamental drawback of severe conservativeness, because a single controller is developed to address the conflicting requirements simultaneously. For example, the mixed H_2/H_∞ control is generally worse than the H_∞ control in terms of the robustness and worse than the LQG control in terms of the optimality [31].

The objective of this paper is to present a novel *non-trade-off* design approach to the robust optimal consensus problems for linear multi-agent systems. Motivated by the structure in [29], a new design paradigm is proposed in [32], consisting of an LQG control designed for the nominal plant and an operator Q as a separate degree of freedom. The operator Q provides an extra control action to recover the robustness performance for the closed-loop system. This new paradigm is shown to be able to avoid trade-off and to reduce the conflict between the robustness and achievable suboptimality/optimality. It should be noted that the design paradigms in [29], [32] are applicable to only single-agent systems. Novel non-trade-off design schemes for multi-agent systems have not witnessed significant progress so far, due to the severe difficulties caused by the requirement of distributed control and structural uncertainties and constraints imposed by the network graphs.

In this paper, motivated by [29], [32], we propose a *complementary design approach* to the H_2 and H_∞ consensus problem of linear multi-agent systems. This complementary approach consists of two steps. In the first step, we design a distributed control law for the nominal multi-agent system, without considering the disturbances or uncertainties, to ensure that consensus is achieved with a prescribed H_2 performance. In the second step, a separate control input, activated by some carefully chosen residual signals indicating the modeling mismatch, will be designed to ensure robustness in terms

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of the H_∞ requirement. Two cases are considered, namely, the case that relative outputs or absolute outputs of neighboring agents are available. Suitable residual signals are chosen for both cases. For the first case with relative outputs, the residual signal is defined as the stacked error between the actual relative outputs of neighboring agents and their observed ones given by a distributed observer. For the second case with absolute outputs, the residual signal quantifies the differences between the actual agent dynamics and the ideal agent dynamics. A distinct feature of this complementary approach is that the design of H_2 consensus control in this first step is independent of the second step and the extra control action in the second step will complement the H_2 performance by providing a robustness guarantee with respect to disturbances or uncertainties.

Compared to the trade-off approach, e.g., the mixed H_2/H_∞ control design, the proposed complementary design approach has at least two main advantages. Firstly, since the extra control provides an additional degree of freedom for design, the complementary approach has two separate controls to deal with the H_2 performance and the robustness of consensus, respectively. Thereby, this approach does not need to make much trade-off, and can be expected to be much less conservative than the trade-off approach where one control tackles two conflicting performances. Secondly, the control action of the second step is proportional to the residual signal which quantifies the modeling mismatch level, thereby having some online ‘‘adaptivity’’ with respect to modeling errors. This complementary approach will yield the same achievable H_2 performance when modeling mismatches do not exist. By contrast, the trade-off approach always considers the *a priori* worst case and still yield the same conservative performance even when disturbances or uncertainties do not exist.

The remainder of this paper is organized as follows. Some mathematical preliminaries including graph theory and results on H_2 and H_∞ performances are summarized in Section II. The H_2 and H_∞ consensus problem is formulated in Section III. A two-step complementary design approach is proposed for the H_2 and H_∞ consensus problem in Section IV. A simulation example that illustrates the proposed theoretical results are presented in Section V. Finally, conclusions are given in Section VI.

II. MATHEMATICAL PRELIMINARIES

A. Notations

The notations used in this paper is standard. $\mathbf{R}^{n \times m}$ denotes the set of $n \times m$ real matrices, I represents the identity matrix of appropriate dimension, and $\mathbf{1}$ denotes a column vector with all entries equal to 1. The matrix inequality $A > B$ means that A and B are symmetric matrices and $A - B$ is positive definite. For a square matrix A , $\text{tr}(A)$ represents its trace. $A \otimes B$ represents the Kronecker product of matrices A and B . The expectation operator is denoted by $\mathbf{E}\{\cdot\}$.

B. Graph Theory

The information flow among the agents can be conveniently modeled by a graph. An undirected graph is defined by $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, \dots, N\}$ is the set of nodes (each node represents an agent) and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ denotes the set of unordered pairs of nodes, called edges. An undirected graph is connected, if there exists a path between every two distinct nodes. For an undirected graph \mathcal{G} , its adjacency matrix, denoted by $\mathcal{A} = [a_{ij}] \in \mathbf{R}^{N \times N}$, is defined such that $a_{ii} = 0$, $a_{ij} = a_{ji} = 1$ if $(i, j) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. The Laplacian matrix $\mathcal{L} = [\mathcal{L}_{ij}] \in \mathbf{R}^{N \times N}$ associated with \mathcal{G} is defined as $\mathcal{L}_{ii} = \sum_{j=1}^N a_{ij}$ and $\mathcal{L}_{ij} = -a_{ij}$, $i \neq j$.

Lemma 1 ([2]). For an undirected graph \mathcal{G} , zero is an eigenvalue of \mathcal{L} with $\mathbf{1}$ as an eigenvector and all nonzero eigenvalues are positive. Moreover, zero is a simple eigenvalue of \mathcal{L} if and only if \mathcal{G} is connected.

C. Results on H_2 and H_∞ performances

In this subsection, we summarize some results on H_2 and H_∞ performances of linear systems. Consider the linear system

$$\begin{aligned} \dot{x} &= Ax + Bw, \\ y &= Cx, \end{aligned} \quad (1)$$

where $x \in \mathbf{R}^n$ is the state, $y \in \mathbf{R}^m$ is the measured output, and $w \in \mathbf{R}^q$ is the external disturbance.

Let $G(s) = C(sI - A)^{-1}B$ be the transfer function matrix of (1). The H_2 norm of G is defined to be

$$\|G\|_2^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} \text{tr}(G^*(j\omega)G(j\omega))d\omega.$$

We then review the following well-known result on the H_2 performance [1].

Lemma 2. Let $\gamma_2 > 0$. The following statements are equivalent:

- i) A is stable and $\|G\|_2 < \gamma_2$.
- ii) There exists $X > 0$ such that

$$AX + XA^T + BB^T < 0, \quad \text{tr}(CXC^T) < \gamma_2^2.$$

- iii) There exist $P > 0$ and $Q > 0$ such that

$$\begin{bmatrix} A^T P + PA & PB \\ B^T P & -I \end{bmatrix} < 0, \quad \begin{bmatrix} P & C^T \\ C & Q \end{bmatrix} > 0, \quad \text{tr}(Q) < \gamma_2^2.$$

Next, we review the H_∞ performance of (1). If A is stable, the H_∞ norm of (1) is then defined to be

$$\|G\|_\infty = \sup_{\omega \in \mathbb{R}} \sigma(G(j\omega)),$$

where $\sigma(G(j\omega))$ is the maximum singular value of $G(j\omega)$.

The following lemma presents a well-known result on the H_∞ performance [1], [33].

Lemma 3. Let $\gamma_\infty > 0$. The following statements are equivalent:

- i) A is stable and $\|G\|_\infty < \gamma_\infty$.
- ii) There exists $X > 0$ such that

$$A^T X + XA + \frac{1}{\gamma_\infty^2} XBB^T X + C^T C < 0.$$

In the next section, we will formulate the problem to be addressed in this paper.

III. FORMULATION OF H_2 AND H_∞ CONSENSUS PROBLEM

Consider a network of N identical linear agents subject to different noises and external disturbances. The dynamics of the i -th agent are described by

$$\begin{aligned} \dot{x}_i &= Ax_i + B_0 w_{0i} + B_1 w_i + B_2 u_i, \\ y_i &= C_2 x_i + D_0 w_{0i} + D_1 w_i, \quad i = 1, \dots, N, \end{aligned} \quad (2)$$

where $x_i \in \mathbf{R}^n$ is the state, $u_i \in \mathbf{R}^p$ is the control input, and $y_i \in \mathbf{R}^m$ is the measurement output of the i -th agent, respectively. In (2), $w_i \in \mathbf{R}^{q_1}$ denotes the external disturbance signal, representing the modeling uncertainty and/or unmodeled dynamics of the i -th agent, $w_{0i} \in \mathbf{R}^{q_2}$ is a white noise signal with $\mathbf{E}\{w_{0i}(t)\} = 0$ and $\mathbf{E}\{w_{0i}(t)w_{0i}(\tau)^T\} = \delta(t - \tau)I$. The matrices A , B_0 , B_1 , B_2 , C_2 , D_0 and D_1 are of suitable dimensions. The pair (A, B_2) is assumed to be stabilizable and the pair (C_2, A) is assumed to be detectable.

The following theorem provides a necessary and sufficient condition for the H_2 suboptimal consensus problem.

Theorem 1. Assume that the graph \mathcal{G} is connected. Let $\gamma_2 > 0$. Then, the distributed protocol (4) achieves H_2 consensus for the network (7) if and only if the following $N - 1$ subsystems

$$\begin{aligned} \dot{\tilde{\xi}}_i &= \begin{bmatrix} A & \lambda_i B_2 F \\ GC_2 & A - GC_2 + \lambda_i B_2 F \end{bmatrix} \tilde{\xi}_i + \begin{bmatrix} B_0 \\ GD_0 \end{bmatrix} \tilde{w}_{0i}, \\ \tilde{z}_i &= [C_1 \quad 0] \tilde{\xi}_i, \quad i = 2, \dots, N, \end{aligned} \quad (9)$$

are internally stable and $\sum_{j=2}^N \|\tilde{T}_{\tilde{w}_{0i} \tilde{z}_i}\|_2^2 < \gamma_2^2$, where $\tilde{T}_{\tilde{w}_{0i} \tilde{z}_i}$ denotes the transfer function matrix of (9) from \tilde{w}_{0i} to \tilde{z}_i .

Proof. The result can be proved by following similar lines in [2], [15]. The key steps are sketched here for clarity. First, we apply the unitary transformation $U \otimes I$ onto the dynamics of the consensus error $(\mathcal{M} \otimes I)\xi$, where U is a unitary matrix such that $U^T \mathcal{L}U = \text{diag}(0, \lambda_2, \dots, \lambda_N)$. Note that $U^T \mathcal{M}U = \text{diag}(0, 1, \dots, 1)$, see e.g., [2]. Next, by observing that the H_2 norm is invariant under unitary transformations, we can get that the H_2 suboptimal consensus problem is solved if and only if the following $N - 1$ subsystems

$$\begin{aligned} \dot{\tilde{\xi}}_i &= \begin{bmatrix} A & B_2 F \\ \lambda_i GC_2 & A - GC_2 + \lambda_i B_2 F \end{bmatrix} \tilde{\xi}_i + \begin{bmatrix} B_0 \\ \lambda_i GD_0 \end{bmatrix} \tilde{w}_{0i}, \\ \tilde{z}_i &= [C_1 \quad 0] \tilde{\xi}_i, \quad i = 2, \dots, N, \end{aligned} \quad (10)$$

are internally stable and $\sum_{j=2}^N \|\tilde{T}_{\tilde{w}_{0i} \tilde{z}_i}\|_2^2 < \gamma_2^2$. Now, by letting $\tilde{\xi}_i = \begin{bmatrix} I & 0 \\ 0 & \frac{1}{\lambda_i} I \end{bmatrix} \tilde{\xi}_i$, evidently the subsystems in (10) are equivalent to those in (9). \square

Before moving forwards, we need to make the following assumption and introduce a lemma.

Assumption 1. The system matrices in (2) satisfy that $D_0 B_0^T = 0$ and $D_0 D_0^T = I$.

Lemma 4 ([15], [37]). Suppose Assumption 1 holds. Consider the i -th subsystem in (9) with $\lambda_i = 1$. Let $P > 0$ and $Q > 0$, respectively, satisfy the following inequalities:

$$(A + B_2 F)^T P + P(A + B_2 F) + C_1^T C_1 < 0, \quad (11)$$

$$AQ + QA^T - QC_2^T C_2 Q + B_0 B_0^T < 0. \quad (12)$$

If the inequality

$$\text{tr}(C_2 Q P Q C_2^T) + \text{tr}(C_1 Q C_1^T) < \gamma^2$$

holds, then $\tilde{T}_{\tilde{w}_{0i} \tilde{z}_i}$, with $G = QC_2^T$ and $\lambda_i = 1$, satisfies that $\|\tilde{T}_{\tilde{w}_{0i} \tilde{z}_i}\|_2 < \gamma$.

The following theorem provides a design method for obtaining distributed protocols (4) that achieves H_2 suboptimal consensus.

Theorem 2. Assume that Assumption 1 holds and the graph \mathcal{G} is connected. Let $\gamma_2 > 0$. Let $Q > 0$ be a solution to (12). Let $P > 0, W > 0, \tau > 0$ be solutions to the following LMIs:

$$\begin{bmatrix} \bar{P}A^T + A\bar{P} - \tau B_2 B_2^T & \bar{P}C_1^T \\ C_1 \bar{P} & -I \end{bmatrix} < 0, \quad (13)$$

$$\begin{bmatrix} \bar{P} & C_2 Q \\ QC_2^T & W \end{bmatrix} > 0, \quad (14)$$

$$\text{tr}(W) + \text{tr}(C_1 Q C_1^T) < \frac{\gamma_2^2}{N-1}. \quad (15)$$

Then, the protocol (4) with $G = QC_2^T$, $F = -cB_2^T \bar{P}^{-1}$ and $c \geq \frac{\tau}{2\lambda_2}$ achieves H_2 consensus.

Proof. In light of Theorem 1, the network (7) achieves H_2 consensus if the $N - 1$ subsystems in (9) are internally stable and $\|\tilde{T}_{\tilde{w}_{0i} \tilde{z}_i}\|_2^2 < \frac{\gamma_2^2}{N-1}$. According to Lemma 4, the i -th subsystem in (9) is internally stable and $\|\tilde{T}_{\tilde{w}_{0i} \tilde{z}_i}\|_2^2 < \frac{\gamma_2^2}{N-1}$, if there exist $Q > 0$ satisfying (12) and $P > 0$ such that

$$(A + \lambda_i B_2 F)^T P + P(A + \lambda_i B_2 F) + C_1^T C_1 < 0, \quad (16)$$

and

$$\text{tr}(C_2 Q P Q C_2^T) + \text{tr}(C_1 Q C_1^T) < \frac{\gamma_2^2}{N-1}. \quad (17)$$

Let $\bar{P} = P^{-1}$. Multiplying on both sides of (16) by \bar{P} and in light of Schur Complement Lemma [38], we obtain that (16) and (17) hold if and only if

$$\begin{bmatrix} \bar{P}(A + \lambda_i B_2 F)^T + (A + \lambda_i B_2 F)\bar{P} & \bar{P}C_1^T \\ C_1 \bar{P} & -I \end{bmatrix} < 0, \quad (18)$$

and the inequalities (14) and (15) hold at the same time. Evidently, if we choose $F = -cB_2^T \bar{P}^{-1}$ and $c \geq \frac{\tau}{2\lambda_2}$, then (13) implies (18) and thereby (16). \square

Remark 2. The separation property of observed-based controllers shown in [15], [37] is employed in this theorem. The observer gain matrix G and the feedback gain F are designed in a decoupled way. Moreover, the feasibility of (13) is equivalent to that of (18). Note that by letting $F\bar{P} = V$ and $\lambda_i = 1$, we know that (18) holds, then

$$\bar{P}A + A^T \bar{P} + B_2 V + V^T B_2^T + \bar{P}C_1^T C_1 \bar{P} < 0,$$

which, in light of Finsler's Lemma [39], [2], is equivalent to that there exist $\bar{P} > 0$ and $\tau > 0$ such that (13) holds. Therefore, (18) implies (13). The converse was shown in the proof.

2) *Step Two:* In the second step, we design an additional regulating control input $u_{\infty i}$ to deal with the external disturbances w_i and to guarantee the H_∞ robustness while not significantly compromising the H_2 performance. Since the noise w_{0i} has been taken care of and filtered out in the first step, we only consider the effect of w_i in the second step.

Under the H_2 consensus protocol (4) in the first step, the augmented agent dynamics are described by

$$\begin{aligned} \dot{x}_i &= Ax_i + B_2 u_i + B_1 w_i, \\ \dot{v}_i &= (A - GC_2)v_i + \sum_{j=1}^N a_{ij} \left(B_2 F(v_i - v_j) \right. \\ &\quad \left. + GC_2(x_i - x_j) + GD_1(w_i - w_j) \right), \\ u_i &= u_{2i} + u_{\infty i}, \\ u_{2i} &= Fv_i, \quad i = 1, \dots, N, \end{aligned} \quad (19)$$

where the gain matrices F and G are designed in the first step.

The residual signal $f = [f_1^T, \dots, f_N^T]^T$ in Fig. 1 is used in the second step to activate the *inner loop*. It builds on the protocol (4) and is given by

$$\begin{aligned} f_i &\triangleq C_2 v_i - \sum_{j=1}^N a_{ij} (y_i - y_j) \\ &= C_2 e_i - D_1 \sum_{j=1}^N a_{ij} (w_i - w_j), \quad i = 1, \dots, N, \end{aligned} \quad (20)$$

where e_i is defined as in (5).

In this step, we consider a distributed protocol of the form

$$\begin{aligned} \dot{n}_i &= A_c n_i + B_c f_i, \\ u_{\infty i} &= C_c n_i + D_c f_i, \end{aligned} \quad (21)$$

where $n_i \in \mathbf{R}^n$ is the state of the protocol, and A_c, B_c, C_c, D_c are protocol matrices to be designed. In this case, $Q(s) = \begin{bmatrix} A_c & B_c \\ C_c & D_c \end{bmatrix}$ in Fig. 1. It should be mentioned that here we assume that $u_{\infty i}$ is a general dynamic controller with f_i as its input. Special forms such as observer-based ones in [20], [21] can be also considered.

Note that the error e_i in the current case satisfies

$$\dot{e}_i = (A - GC_2)e_i + (GD_1 - B_1) \sum_{j=1}^N a_{ij}(w_i - w_j). \quad (22)$$

Evidently, if external disturbances w_i are equal to zero, then e_i and thereby f_i will asymptotically converge to zero. In other words, the inner loop will be activated as f_i are bounded signals and $u_{\infty i}$ will be implemented to recover robustness.

Denote $e = [e_1^T, \dots, e_N^T]^T$, $n = [n_1^T, \dots, n_N^T]^T$ and $\zeta = [x^T \ e^T]^T$. Using (19) and (21), we obtain the closed-loop network dynamics in compact form as

$$\begin{aligned} \begin{bmatrix} \dot{\zeta} \\ \dot{n} \end{bmatrix} &= \begin{bmatrix} \bar{\mathcal{A}} + \mathcal{B}_2 \mathcal{D}_c \mathcal{C}_2 & \mathcal{B}_2 \mathcal{C}_c \\ \mathcal{B}_c \mathcal{C}_2 & \mathcal{A}_c \end{bmatrix} \begin{bmatrix} \zeta \\ n \end{bmatrix} + \begin{bmatrix} \mathcal{B}_1 - \mathcal{B}_2 \mathcal{D}_c \mathcal{D}_1 \\ -\mathcal{B}_c \mathcal{D}_1 \end{bmatrix} w, \\ z &= [\mathcal{C}_1 \ 0] \begin{bmatrix} \zeta \\ n \end{bmatrix}, \end{aligned} \quad (23)$$

where \mathcal{C}_1 is defined in (8), and

$$\begin{aligned} \bar{\mathcal{A}} &= \begin{bmatrix} I \otimes A + \mathcal{L} \otimes B_2 F & I \otimes B_2 F \\ 0 & I \otimes (A - GC_2) \end{bmatrix}, \\ \mathcal{B}_2 &= \begin{bmatrix} I \otimes B_2 \\ 0 \end{bmatrix}, \quad \mathcal{B}_1 = \begin{bmatrix} I \otimes B_1 \\ \mathcal{L} \otimes (GD_1 - B_1) \end{bmatrix}, \\ \mathcal{C}_2 &= I \otimes [0 \ C_2], \quad \mathcal{D}_1 = \mathcal{L} \otimes D_1, \quad \mathcal{A}_c = I \otimes A_c, \\ \mathcal{B}_c &= I \otimes B_c, \quad \mathcal{C}_c = I \otimes C_c, \quad \mathcal{D}_c = I \otimes D_c. \end{aligned} \quad (24)$$

By following similar steps in deriving Theorem 1, it is not difficult to obtain the following result.

Theorem 3. Assume that graph \mathcal{G} is connected. Let $\gamma_\infty > 0$. Then, the network (23) achieves H_∞ consensus if and only if the following $N - 1$ subsystems:

$$\begin{aligned} \begin{bmatrix} \dot{\tilde{\zeta}}_i \\ \dot{\tilde{n}}_i \end{bmatrix} &= (\mathfrak{A}_i + \mathfrak{B}_2 \mathfrak{K} \mathfrak{C}_2) \begin{bmatrix} \tilde{\zeta}_i \\ \tilde{n}_i \end{bmatrix} + (\mathfrak{B}_{1i} + \mathfrak{B}_2 \mathfrak{K} \mathfrak{D}_{1i}) \tilde{w}_i, \\ \tilde{z}_i &= \mathfrak{C}_1 \begin{bmatrix} \tilde{\zeta}_i \\ \tilde{n}_i \end{bmatrix}, \quad i = 2, \dots, N, \end{aligned} \quad (25)$$

are internally stable and the associated transfer functions satisfy $\|\tilde{T}_{\tilde{w}_i \tilde{z}_i}\|_\infty < \gamma_\infty$, where

$$\begin{aligned} \mathfrak{A}_i &= \begin{bmatrix} A + \lambda_i B_2 F & B_2 F & 0 \\ 0 & A - GC_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathfrak{B}_2 = \begin{bmatrix} 0 & B_2 \\ 0 & 0 \\ I & 0 \end{bmatrix}, \\ \mathfrak{K} &= \begin{bmatrix} A_c & B_c \\ C_c & D_c \end{bmatrix}, \quad \mathfrak{C}_2 = \begin{bmatrix} 0 & 0 & I \\ 0 & C_2 & 0 \end{bmatrix}, \quad \mathfrak{C}_1^T = \begin{bmatrix} C_1^T \\ 0 \\ 0 \end{bmatrix}, \\ \mathfrak{B}_{1i} &= \begin{bmatrix} B_1 \\ \lambda_i (GD_1 - B_1) \\ 0 \end{bmatrix}, \quad \mathfrak{D}_{1i} = \begin{bmatrix} 0 \\ -\lambda_i D_1 \end{bmatrix}. \end{aligned}$$

The following theorem provides a design method for obtaining the distributed control law (21).

Theorem 4. Assume that \mathcal{G} is connected and that B_2 is of full column rank. Let $\gamma_\infty > 0$. Then the network (23) achieves H_∞ consensus if there exist positive definite matrices S_{11} and S_{22} , and a matrix Q_1 such that

$$S = \begin{bmatrix} S_{11} & 0 \\ 0 & S_{22} \end{bmatrix}, \quad V = \begin{bmatrix} V_1 \\ 0 \end{bmatrix} \quad (26)$$

satisfying the following LMI's

$$\begin{bmatrix} \Upsilon_{1i} & S \mathfrak{B}_{1i} + V \mathfrak{D}_{1i} \\ (S \mathfrak{B}_{1i} + V \mathfrak{D}_{1i})^T & -\gamma_\infty^2 I \end{bmatrix} < 0, \quad (27)$$

for $i = 2, N$, where

$$\begin{aligned} \Upsilon_{1i} &= \bar{\mathfrak{A}}_i^T S + S \bar{\mathfrak{A}}_i + \bar{\mathfrak{C}}_2^T V^T + V \bar{\mathfrak{C}}_2 + \bar{\mathfrak{C}}_1^T \bar{\mathfrak{C}}_1, \\ \bar{\mathfrak{A}}_i &= T \mathfrak{A}_i T^{-1}, \quad \bar{\mathfrak{B}}_{1i} = T \mathfrak{B}_{1i}, \quad \bar{\mathfrak{C}}_2 = \mathfrak{C}_2 T^{-1}, \quad \bar{\mathfrak{C}}_1 = \mathfrak{C}_1 T^{-1}, \end{aligned}$$

and T is a nonsingular matrix such that $T \mathfrak{B}_2 = \begin{bmatrix} I \\ 0 \end{bmatrix}$. Then, the system matrix \mathfrak{K} of (21) is given by

$$\mathfrak{K} = S_{11}^{-1} V_1. \quad (28)$$

Proof. In virtue of Theorem 3 and Lemma 3, it follows that the $N - 1$ subsystems in (25) are internally stable and $\|\tilde{T}_{\tilde{w}_i \tilde{z}_i}\|_\infty < \gamma_\infty$ if and only if there exist matrices $S_i > 0$ such that

$$\begin{aligned} &(\mathfrak{A}_i + \mathfrak{B}_2 \mathfrak{K} \mathfrak{C}_2)^T S_i + S_i (\mathfrak{A}_i + \mathfrak{B}_2 \mathfrak{K} \mathfrak{C}_2) \\ &+ \frac{1}{\gamma_\infty^2} S_i (\mathfrak{B}_{1i} + \mathfrak{B}_2 \mathfrak{K} \mathfrak{D}_{1i}) (\mathfrak{B}_{1i} + \mathfrak{B}_2 \mathfrak{K} \mathfrak{D}_{1i})^T S_i \\ &+ \mathfrak{C}_1^T \mathfrak{C}_1 < 0, \quad i = 2, \dots, N. \end{aligned} \quad (29)$$

Following the steps in [40, Theorem 2], by using a Schur complement, the above inequalities (29) are equivalent to

$$\begin{bmatrix} \Phi_{1i} & \Phi_{2i} \\ \Phi_{2i}^T & -\gamma_\infty^2 I \end{bmatrix} < 0, \quad i = 2, \dots, N \quad (30)$$

with

$$\begin{aligned} \Phi_{1i} &= (\mathfrak{A}_i + \mathfrak{B}_2 \mathfrak{K} \mathfrak{C}_2)^T S_i + S_i (\mathfrak{A}_i + \mathfrak{B}_2 \mathfrak{K} \mathfrak{C}_2) + \mathfrak{C}_1^T \mathfrak{C}_1, \\ \Phi_{2i} &= S_i (\mathfrak{B}_{1i} + \mathfrak{B}_2 \mathfrak{K} \mathfrak{D}_{1i}). \end{aligned}$$

Since the matrix B_2 is of full column rank, there exists a matrix T such that

$$T \mathfrak{B}_2 = \begin{bmatrix} I \\ 0 \end{bmatrix}.$$

By pre-multiplying $\bar{T} = \begin{bmatrix} T^{-T} & 0 \\ 0 & I \end{bmatrix}$ and post-multiplying \bar{T}^T on (30), it follows that (30) holds if and only if

$$\begin{bmatrix} \bar{\Phi}_{1i} & \bar{\Phi}_{2i} \\ \bar{\Phi}_{2i}^T & -\gamma_\infty^2 I \end{bmatrix} < 0, \quad i = 2, \dots, N, \quad (31)$$

where

$$\begin{aligned} \bar{\Phi}_{1i} &= \bar{\mathfrak{A}}_i^T \bar{S}_i + \bar{S}_i \bar{\mathfrak{A}}_i + \bar{S}_i \bar{\mathfrak{B}}_2 \mathfrak{K} \bar{\mathfrak{C}}_2 + (\bar{S}_i \bar{\mathfrak{B}}_2 \mathfrak{K} \bar{\mathfrak{C}}_2)^T + \bar{\mathfrak{C}}_1^T \bar{\mathfrak{C}}_1, \\ \bar{\Phi}_{2i} &= \bar{S}_i \bar{\mathfrak{B}}_{1i} + \bar{S}_i \bar{\mathfrak{B}}_2 \mathfrak{K} \mathfrak{D}_{1i}, \quad \bar{S}_i = T^{-T} S_i T^{-1}, \\ \bar{\mathfrak{B}}_2 &= T \mathfrak{B}_2 = \begin{bmatrix} I & 0 \end{bmatrix}^T. \end{aligned}$$

Now, let $S = \bar{S}_i$ and $\bar{S}_i \bar{\mathfrak{B}}_2 \mathfrak{K} = V$. Recall that (26), then (31) holds if (27) holds for $i = 2, \dots, N$. In this case, due to

$$\begin{bmatrix} S_{11} & 0 \\ 0 & S_{22} \end{bmatrix} \begin{bmatrix} I \\ 0 \end{bmatrix} \mathfrak{K} = \begin{bmatrix} V_1 \\ 0 \end{bmatrix},$$

the system matrix is given by (28).

Finally, note that the inequalities in (27) are linear matrix inequalities with respect to the unknown variables, we need to check only the two LMIs in (27) for $i = 2$ and N , and the other $N - 3$ LMIs in (27) corresponding to $i = 3, \dots, N - 1$, also hold, with their variables chosen to be some convex combinations of those satisfying the two LMIs in (27) for $i = 2, N$. This completes the proof. \square

Remark 3. In the novel control structure in Fig. 1, the design of H_2 consensus control of the outer loop is independent of the control $Q(s)$ in (21) of the inner loop. The extra control $Q(s)$ relies on the residual

signal f , which is the stacked error between the actual relative outputs of neighboring agents $\sum_{j=1}^N a_{ij}(y_i - y_j)$ and their observed ones $C_2 v_i$ given by the distributed observer (4a). The extra control action of the inner loop, activated by residual signal f in the presence of external disturbances or uncertainties w_i , will complement the H_2 performance by providing H_∞ robustness guarantee with respect to w_i . This is the reason why this approach is called a complementary design approach.

Remark 4. The two-step complementary approach proposed in this section, compared to the trade-off approach, has at least two main advantages:

- i) The extra control $Q(s)$ in (21) provides an additional degree of freedom for design. Therefore, the current complementary approach has two separate control inputs to deal with the H_2 performance and the H_∞ robustness of consensus, respectively, thereby does not need to make much trade-off, and can be expected to be much less conservative, while the trade-off design has only one control to tackle two conflicting performances simultaneously. Although Theorems 2 and 4 in this section are conservative, however, it should be pointed out that the conservatism of these results is not caused by the complementary approach. On the contrary, it in fact highlights the difficulty of distributed control.
- ii) The control action of the inner loop is proportional to the residual signal which quantifies the modeling mismatch level, thereby having some online ‘‘adaptivity’’ with respect to modeling errors. This complementary approach will yield the same achievable H_2 performance when modeling mismatches do not exist, because in this case the inner loop will be de-activated. By contrast, the trade-off approach always considers the *a priori* worst case and produce the same conservative performance even when disturbances or uncertainties disappear.

Remark 5. It should be mentioned that the order of the overall control law designed by the complementary approach is higher than the trade-off approach, since both distributed protocols (4a) and (21) are required in the former approach while only one dynamic controller is needed in the latter. This is the price to provide more degree of design freedom and it is actually not a big issue considering the abundance of cheap storage and computing resources. Besides, the extra control action of the inner loop, providing robustness guarantee, will inject some white noises into the closed-loop network dynamics, and thereby will make certain compromise of the H_2 performance.

B. Absolute Output Feedback Case

In this subsection, we consider the case where *absolute output* information of each agent is available. In this case, we adopt local observers for the agents, instead of the distributed observers as in the previous subsection. The structure of the complementary design approach in this case is depicted in Fig. 2.

1) *Step One:* Based on the absolute output y_i , we propose for each agent the following Luenberger observer:

$$\dot{\check{v}}_i = A\check{v}_i + Bu_i + \check{G}(y_i - C_2\check{v}_i), \quad (32)$$

where \check{G} is the observer gain to be designed. In the first step, we consider only the outer loop, therefore, $u_i = u_{2i}$. For the H_2 consensus problem, we design the following protocol:

$$u_{2i} = \check{F} \sum_{j=1}^N a_{ij}(\check{v}_i - \check{v}_j), \quad (33)$$

where \check{F} is the feedback gain to be designed. Denote $\check{v}_i = [\check{v}_i^T, \dots, \check{v}_i^T]^T$ and $\check{\xi} = [x^T \ \check{v}^T]^T$. The closed-loop network

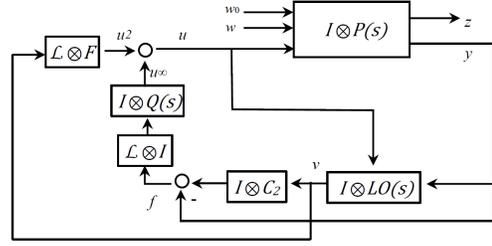


Fig. 2. The controller structure of the complementary approach based on absolute outputs, where $LO(s)$ represents the local observer for each agent, \check{f} is the residual signal, and $Q(s)$ is the additional controller based on f , and the rest of variables are defined as in Fig. 1.

dynamics in this case can be written in compact form as

$$\begin{aligned} \dot{\check{\xi}} &= \begin{bmatrix} I \otimes A & \mathcal{L} \otimes B_2 \check{F} \\ I \otimes \check{G} C_2 & I \otimes (A - \check{G} C_2) + \mathcal{L} \otimes B_2 \check{F} \end{bmatrix} \check{\xi} + \begin{bmatrix} I \otimes B_0 \\ I \otimes \check{G} D_0 \end{bmatrix} w_0, \\ z &= \mathcal{M} \otimes [C_1 \ 0] \check{\xi}. \end{aligned} \quad (34)$$

It is easy to verify that the H_2 consensus problem of (34) can be reduced to the same condition as in Theorem 1. Then, Theorem 2 can also be used to design the protocol (33).

2) *Step Two:* In the second step, we define the residual signals \check{f}_i as follows:

$$\check{f}_i \triangleq C_2 \check{v}_i - y_i, \quad i = 1, \dots, N, \quad (35)$$

which is actually the local estimated output error. Therefore, \check{f}_i quantifies the difference between the actual plant and the ideal plant, and we can see that $\check{f}_i = 0$, if there exist no disturbances or uncertainties.

Since $\sum_{j=1}^N a_{ij}(x_i - x_j)$ is the consensus error and \check{v}_i is the estimate of x_i for agent i , we can see that $\sum_{j=1}^N a_{ij}(\check{f}_i - \check{f}_j)$ denotes the estimated output error of the consensus error. Therefore, in this case we design the control input $u_{\infty i}$ for the inner loop based on $\sum_{j=1}^N a_{ij}(\check{f}_i - \check{f}_j)$, instead of \check{f}_i as in the previous subsection. Specifically, we consider a distributed protocol of the form

$$\begin{aligned} \dot{n}_i &= \check{A}_c n_i + \check{B}_c \sum_{j=1}^N a_{ij}(\check{f}_i - \check{f}_j), \\ u_{\infty i} &= \check{C}_c n_i + \check{D}_c \sum_{j=1}^N a_{ij}(\check{f}_i - \check{f}_j), \end{aligned} \quad (36)$$

where $n_i \in \mathbf{R}^n$ is the state of the protocol, and $\check{A}_c, \check{B}_c, \check{C}_c, \check{D}_c$ are protocol matrices to be designed. In this case, $Q(s) = \begin{bmatrix} \check{A}_c & \check{B}_c \\ \check{C}_c & \check{D}_c \end{bmatrix}$ in Fig. 2.

Let $\check{e}_i = \check{v}_i - x_i$. Denote $\check{e} = [\check{e}_1^T, \dots, \check{e}_N^T]^T$, $\check{\zeta} = [x^T \ \check{e}_i^T]^T$ and $\check{n} = [\check{n}_1^T, \dots, \check{n}_N^T]^T$. Then, it follows from (2), (33) and (36) that the closed-loop network dynamics are given by

$$\begin{aligned} \begin{bmatrix} \dot{\check{\zeta}} \\ \dot{\check{n}} \end{bmatrix} &= \begin{bmatrix} \check{A} + \mathcal{B}_2 \check{D}_c \check{C}_2 & \mathcal{B}_2 \check{C}_c \\ \check{B}_c \check{C}_2 & \check{A}_c \end{bmatrix} \begin{bmatrix} \check{\zeta} \\ \check{n} \end{bmatrix} + \begin{bmatrix} \check{B}_1 - \mathcal{B}_2 \check{D}_c \check{G}_1 \\ -\check{B}_c \check{G}_1 \end{bmatrix} w, \\ z &= [\mathcal{C}_1 \ 0] \begin{bmatrix} \check{\zeta} \\ \check{n} \end{bmatrix}, \end{aligned} \quad (37)$$

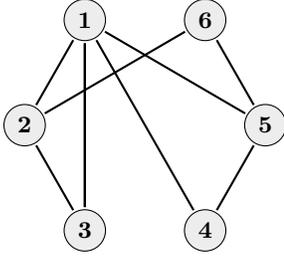


Fig. 3. The communication graph among the six agents.

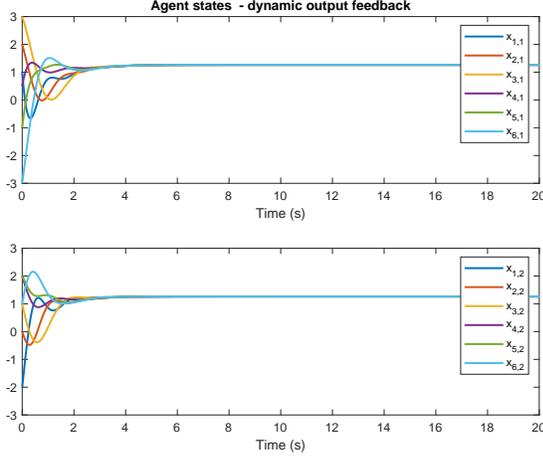


Fig. 4. Complementary H_2 and H_∞ design by output feedback: plots of the agent state vector $x^1 = (x_{1,1}, \dots, x_{6,1})$ (upper plot) and $x^2 = (x_{1,2}, \dots, x_{6,2})$ (lower plot).

where \mathcal{C}_1 , \mathcal{C}_2 , and \mathcal{B}_2 are defined in (8), and

$$\begin{aligned} \check{\mathcal{A}} &= \begin{bmatrix} I \otimes A + \mathcal{L} \otimes B_2 \check{F} & \mathcal{L} \otimes B_2 \check{F} \\ 0 & I \otimes (A - \check{G}C_2) \end{bmatrix}, \\ \check{\mathcal{B}}_1 &= \begin{bmatrix} I \otimes B_1 \\ \mathcal{L} \otimes (\check{G}D_1 - B_1) \end{bmatrix}, \quad \check{\mathcal{D}}_1 = I \otimes D_1, \quad \check{\mathcal{A}}_c = I \otimes \check{A}_c, \\ \check{\mathcal{B}}_c &= I \otimes \check{B}_c, \quad \check{\mathcal{C}}_c = I \otimes \check{C}_c, \quad \check{\mathcal{D}}_c = I \otimes \check{D}_c. \end{aligned}$$

Similarly as in Theorems 3 and 4, the control law (36) can be constructed to achieve H_∞ consensus with prescribed index. The details are omitted here for conciseness.

V. SIMULATION EXAMPLE

In this section we will use a simulation example to illustrate the proposed complementary H_2 and H_∞ design by dynamic output feedback, as in Theorems 2 and 4 in Subsection IV-A, for obtaining distributed protocols.

Consider a multi-agent system that consists of six agents. The dynamics of each agent is given by (2), where

$$\begin{aligned} A &= \begin{bmatrix} -2 & 2 \\ -1 & 1 \end{bmatrix}, \quad B_0 = \begin{bmatrix} 0 & 0 \\ 0.5 & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 \\ 0.6 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \\ C_1 &= [1 \quad 0], \quad C_2 = [1 \quad 0.8], \quad D_1 = 0.1, \quad D_0 = [0 \quad 1]. \end{aligned}$$

The communication graph among the agents is shown in Fig. 3, which is a connected undirected graph with the Laplacian matrix \mathcal{L} . The smallest nonzero and the largest eigenvalues of \mathcal{L} are $\lambda_2 = 1.3820$ and $\lambda_N = 5.3028$.

In the first step of the complementary design, following Theorem 2, we obtain a distributed control law that takes care of the H_2

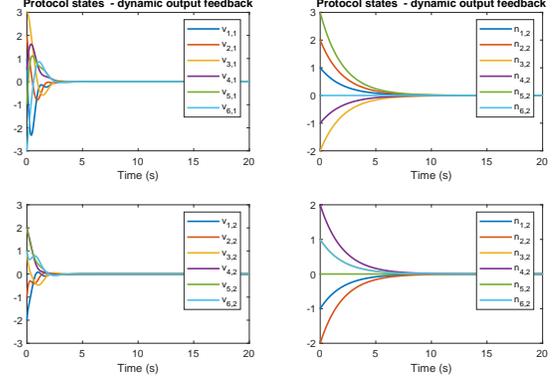


Fig. 5. Complementary H_2 and H_∞ design by output feedback: plots of the protocol state vector $v^1 = (v_{1,1}, \dots, v_{6,1})$ (upper left plot), $v^2 = (v_{1,2}, \dots, v_{6,2})$ (lower left plot), $n^1 = (n_{1,1}, \dots, n_{6,1})$ (upper right plot) and $n^2 = (n_{1,2}, \dots, n_{6,2})$ (lower right plot).

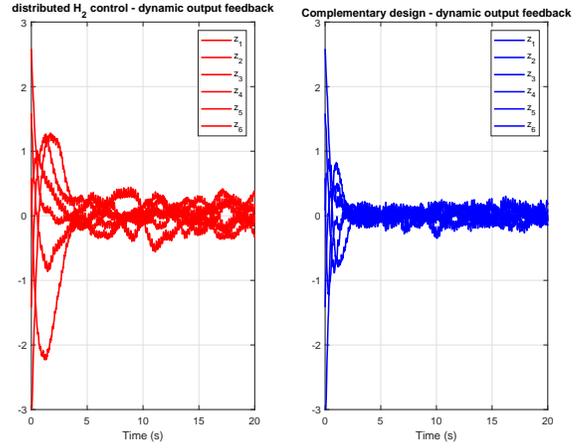


Fig. 6. Complementary H_2 and H_∞ design by output feedback: plots of the agent output vector $z^1 = (z_1, \dots, z_6)$.

performance. We choose $\gamma_2 = 2$ and compute the control gains $F = [-0.1627 \quad 0.7430]$ and $G = [0.5685 \quad 0.7966]^T$. Next in the second step, following Theorem 4, we obtain a distributed control law that deals with the H_∞ consensus. We compute the protocol gains to be $A_c = \begin{bmatrix} -0.5031 & 0 \\ 0 & -0.5031 \end{bmatrix}$, $B_c = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $C_c = [0 \quad 0]$ and $D_c = -0.4045$.

The associated computed upper bound for the H_∞ robustness is $\gamma_{\infty, \min} = 1.5808$. In Fig. 4, we have plotted the state trajectories of the agents, and in Fig. 5 we have plotted the state trajectories of the two proposed distributed protocols. It can be seen that indeed the proposed distributed protocols together achieve consensus for the multi-agent system.

As a comparison, we will next compare, in presence of the external noise and disturbance, the output performance of the proposed complementary approach with that of the distributed H_2 control. In particular, we choose the external noise w_{0i} to be a uniformly distributed signal, generated by Matlab command `30*rand()`. We choose the disturbance w_i to be $w_1 = w_3 = 3 \sin(110t)$, $w_2 = w_4 = 3 \sin(30t)$, $w_5 = w_6 = 3 \sin(60t)$. The plots of the trajectories of the performance outputs z_i are given in Fig. 6. It can be seen that, in the presence of noises and disturbances, indeed the proposed complementary approach guarantees a better performance than the

distributed H_2 control.

VI. CONCLUSIONS

In this paper, we have presented a novel complementary approach to the distributed H_2 and H_∞ consensus problem of multi-agent systems. Through introducing an extra control input that depends on some carefully chosen residual signals, which indicates the modeling mismatch, the complementary approach provides an additional degree of freedom for control design and complements the H_2 performance of consensus by providing the H_∞ robustness guarantee. This complementary approach does not involve much trade-off, and can be expected to be much less conservative than the trade-off design.

Future works include extending the proposed complementary approach to other robust optimal cooperative control problems with different optimal performances under different types of uncertainties.

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