# Modeling and Reasoning About Wireless Networks: A Graph-based Calculus Approach

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Abstract—We propose a graph-based process calculus for modeling and reasoning about wireless networks with local broadcasts. Graphs are used at syntactical level to describe the topological structures of networks. This calculus is equipped with a reduction semantics and a labelled transition semantics. The former is used to define weak barbed congruence. The latter is used to define a parameterized weak bisimulation emphasizing locations and local broadcasts. We prove that weak bisimilarity implies weak barbed congruence. The potential applications are illustrated by some examples and two case studies.

## I. INTRODUCTION

With the widespread use of wireless communication devices, wireless networks are becoming more important in various fields. In wireless networks, local broadcast is one of the most important features. Messages, which are transmitted in a limited area, can only be received by the devices linked with the transmitter in wireless networks. How to ensure that wireless networks can behave in a reasonable manner becomes a critical issue. Assuring the correctness of behaviours of wireless systems is also a difficult task. The goal of this paper is to develop a graph-based calculus to model and reason about wireless networks.

Formalization of wireless systems has attracted the attentions of many researchers. Process calculi, e.g. Milner's CCS [1] and Hoare's CSP [2], provide a good framework to study concurrent systems in a point-to-point approach. Process calculi for broadcast systems were first studied by Prasad in the work of a calculus for broadcast systems (CBS) [3]. However, the broadcast is global in CBS, i.e. messages can be received by all devices in the systems.

Indeed, local broadcast is a challenge for researchers. Recently, several process calculi have been proposed to study wireless systems, e.g. [4]–[9]. These process calculi deal with local broadcasts typically by carrying separate topological structures, e.g. using locations and transmission radii for nodes at the syntactic level [6], [7], or indexing labelled transitions by graphs at the semantic level [8]. Behavioural equivalences, e.g. bisimulation, are important tools in [5], [7], [8] to reason about wireless systems. However, these behavioural equivalences do not take location information and links into account, and, indeed, they straightly use the identity relation of locations

to relate actions (i.e. the challenger and the responser must play the same actions at the same locations). Therefore, these behavioural equivalences cannot relate implementations and specifications with different number of nodes (see Example 4). As pointed out by Lanese and Sangiorgi [6], "in wireless systems, each device – and therefore presumably also the observer – has a location and a transmission cell, and it is not clear how to take them into account". In fact, providing a process calculus with behavioural equivalences by taking locations and links (i.e. transmission cells [6]) into account is non-trivial. This paper makes an effort for this by using graphs, which can concisely specify locations and links of networks.

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In this paper, we propose a Graph-based Calculus for Wireless Networks (called GCWN) and the semantics of GCWN are suitable for wireless links and local broadcasts. Behavioural equivalences of GCWN are also well studied by taking locations and links into account. Graphs play an important role in GCWN, and make it easy to specify and reason about local broadcasts in wireless networks. We hope that the method in this paper can be applied to other process calculi for wireless systems. This paper makes the following contributions.

Firstly, a graph-based calculus for wireless networks is proposed, where the topology of a wireless network is specified by a graph at the syntactic level. In a network, vertices of the associated graph are locations for processes (i.e. nodes) and edges of the graph represent the connections between nodes.

Secondly, in order to capture evolutions of wireless networks, we define both a reduction semantics of the form  $M \to M'$ , and a labelled transition semantics of the form  $M \xrightarrow{p:\alpha} M'$  (observable transition performing action  $\alpha$  at location p) or  $M \xrightarrow{\tau} M'$  (unobservable transition). As the first theoretical result, we prove that the two semantics describe the same behaviours.

Thirdly, two kinds of behavioural equivalences for GCWN are developed. We first adopt the concept of *barb* [10] to define a weak barbed congruence without location information. Barbed congruence is natural to describe that two networks are identical if they exhibit the same barbs during their reductions in any context. However, barbed congruence is hard to handle directly, because one has to consider all possible contexts

by the definition. Instead, labelled transition systems (LTSs) are widely adopted to study behavioural equivalences. LTSs derive the concept of bisimulation, which is more tractable and equipped with powerful proof techniques. We define a weak bisimulation for GCWN by taking locations and links into account. As another theoretical result, we prove that weak bisimilarity implies weak barbed congruence, i.e. *soundness*.

Last, the potential applications of GCWN are illustrate by examples and case studies. Specially, we use GCWN to model and reason about scenarios in protocol ARAN [11] and the Alternating Bit Protocol.

The rest of this paper is organized as follows. Section II provides the syntax of GCWN. Section III presents the operational semantics and two kinds of behavioural equivalences for GCWN. Section IV proves that the two semantics coincide and weak bisimilarity implies weak barbed congruence. Section V provides two case studies. We discuss related work in Section VI, and make a conclusion in Section VII.

For lack of space, most of the proofs are omitted, but can be found in the Appendix.

## II. THE CALCULUS

In this section, we define the syntax of GCWN.

**Graphs.** Let Loc be a countable set of locations ranged over by p,q, etc. A finite (undirected) graph  $G=(|G|, \smallfrown_G)$  consists of a finite set of locations |G| and a set of edges  $\smallfrown_G$  which is a binary relation on |G| such that  $p \smallfrown_G q$  implies  $q \smallfrown_G p$  (symmetric) and  $p \neq q$  (no self-loops). Graphs are used to describe locations and links of networks.

Given disjoint sets E and F with  $p \in E$ , let  $E[F/p] = (E - \{p\}) \cup F$ , by substituting p with F. Let G and H be graphs with  $|G| \cap |H| = \emptyset$  and  $p \in |G|$ . We define a graph G[H/p] by substituting location p with H, and G[H/p] consists of locations |G[H/p]| = |G|[|H|/p] and edges  $q \curvearrowright_{G[H/p]} r$  with  $q \curvearrowright_G r$ , or  $q \curvearrowright_H r$ , or  $q \curvearrowright_G p$  and  $r \in |H|$ , or  $r \curvearrowright_G p$  and  $q \in |H|$ . It will be used when we consider network contexts.

Given two graphs G and H with disjoint locations and  $D \subseteq |G| \times |H|$ , we define a new graph  $K = G \oplus_D H$  such that  $|K| = |G| \cup |H|$  and for any  $p, q \in |K|$  if  $p \curvearrowright_G q$  or  $p \curvearrowright_H q$  or  $(p,q) \in D$  then  $p \curvearrowright_K q$ . The composition of graphs is useful when we define parallel composition of networks.

**Expressions.** We use x, y, etc. for variables, and  $v, v_1$ , etc. for values that can be transmitted via channels (defined later). Moreover, values do not include channels. We use  $e, e_1$ , etc. for arithmetic expressions, which at least include variables and values. Specially, we use  $b, b_1$ , etc. for boolean expressions, which at least include  $\{false, true\}$ . We do not provide a grammar for values and expressions, because they can be constructed with respect to the networks we consider. The substitutions for expressions are defined as usual denoted by  $e\{v/x\}$  and  $b\{v/x\}$ . We say that e is data-closed if e does not contain any variable, and similarly for b. We use  $\vec{x}$  and  $\vec{v}$  for the vectors of variables and values, respectively.

We also assume the existence of an evaluation eval for data-closed arithmetic expressions and boolean expressions, returning values and boolean values, respectively. **Processes and Networks.** We define the syntax of GCWN with two levels: a lower one for processes and an upper one for networks. A network consists of a set of processes, and its topology is specified by a graph.

We use letters c, d, etc. for channel names, and  $\overline{c}$ ,  $\overline{d}$ , etc. for co-names. Let  $\mathcal{K}$  be a set of process constants, ranged over by A, B, etc. For each  $A \in \mathcal{K}$ , we assume that there is an assigned arity, a non-negative integer, representing the number of parameters that A takes. The set of all processes, denoted by  $\mathbf{Pr}$ , is defined as follows:

$$P, Q ::= \mathbf{0} \mid c(x).P \mid \overline{c}(e).P \mid P+Q \mid \mathbf{if} \ b \ \mathbf{then} \ P \ \mathbf{else} \ Q \mid A(\vec{v})$$

Processes are sequential and represent single devices.  $\mathbf{0}$  is the empty process, meaning a termination. In an input process c(x).P, variable x is bound; variables in e are free in an output process  $\overline{c}(e).P$ . A sum process P+Q represents a nondeterministic choice. A conditional process if b then P else Q acts as P if b is true, and as Q otherwise, and variables appearing in b are free in the conditional process.  $A(\vec{v})$  denotes a process defined by a (possibly recursive) definition of the form  $A(\vec{x}) \stackrel{\text{def}}{=} P$ . The length of  $\vec{v}$  and the length of  $\vec{x}$  are consistent with the assigned arity of A. A process is dataclosed if all the variables occurring in the process are bound. The substitution of a value for a variable in processes is denoted by  $P\{v/x\}$ , which means substituting v for every free occurrence of x in process P, and similarly for  $P\{\vec{v}/\vec{x}\}$ .

The set of all networks, denoted by **Net**, is defined as follows:

$$M, N ::= G\langle \Phi \rangle \mid M \backslash c \mid M \oplus_D N$$

where, G is a graph and  $\Phi$  is a function from |G| to Pr.

In general, a network is defined by using a graph and a function from locations to processes. The graph specifies the topology of the network and its edges represent the possible communicating capacities between processes. The network  $G\langle\Phi\rangle$  is the parallel composition of processes  $\Phi(p)\in\mathbf{Pr}$  for each  $p\in |G|$  with communication capabilities specified by  $\frown_G$ . Process  $\Phi(p)$  is called a node of the network  $G\langle\Phi\rangle$ . In  $G\langle\Phi\rangle$ ,  $\Phi(p)$  and  $\Phi(q)$  cannot communicate unless there is an edge between p and q.  $M\backslash c$  is a channel restriction (c is bound in M), and c is private to M. We write  $M\backslash I$  as an abbreviation for  $M\backslash c_1\ldots \backslash c_k$ , with  $I=\{c_1,\ldots,c_k\}$ . Moreover,  $\alpha$ -conversion on channels is defined as usual.

 $M \oplus_D N$  represents that two networks can be composed as a new network. Given  $M = G\langle\Phi\rangle\backslash I$  (I can be  $\emptyset$ ) and  $N = H\langle\Psi\rangle\backslash J$  (J can be  $\emptyset$ ) with  $|G|\cap|H| = \emptyset$ ,  $D\subseteq |G|\times|H|$  and  $I\cap J=\emptyset$  (always possible by  $\alpha$ -conversion on channels), we define the network  $M\oplus_D N$  as  $(G\oplus_D H)\langle\Phi'\rangle\backslash (I\cup J)$  such that  $\Phi'(p) = \Phi(p)$  if  $p\in |G|$  and  $\Phi'(p) = \Psi(p)$  if  $p\in |H|$ . When D is empty, we write it as  $M\oplus N$  for simplicity. The network  $M\oplus_D N$  can be written as  $M\mid N$ , if  $D=|G|\times|H|$ .

If  $M = G\langle \Phi \rangle \setminus c$  (or  $M = G\langle \Phi \rangle$ ), we denote |M| = |G|. Meanwhile, for  $p \in |G|$ , let M(p) represent  $\Phi(p)$  and  $\Phi(m)$  represent  $\Phi(m)$ . In this paper, when we talk about several networks together, we implicitly assume that their locations are pairwise disjoint. A network M is data-closed if all the

Processes:	
$P + 0 \equiv P$	
$P + Q \equiv Q + P$	
$P + (Q + R) \equiv (P + Q) + R$	
if b then P else $Q \equiv P$ ,	if $eval(b) = true$
if b then $P$ else $Q \equiv Q$ ,	if $eval(b) = false$
$A(\vec{v}) \equiv P\{\vec{v}/\vec{x}\},$	if $A(\vec{x}) \stackrel{\text{def}}{=} P$
Networks:	

Networks:  $\alpha\text{-conversion on channels}$   $M \setminus c \setminus d \equiv M \setminus d \setminus c$   $M \oplus_D N \equiv N \oplus_D M$   $(M \oplus_D N) \setminus c \equiv M \oplus_D (N \setminus c), \quad \text{if } c \text{ and } \overline{c} \text{ not in } M$ 

if for all  $p \in |G|$ ,  $\Phi(p) \equiv \Psi(p)$ 

TABLE I: Structural Congruence

$$\begin{split} \frac{M(p) = \overline{c}(e).P + R & \text{eval}(e) = v}{L = \{q_i \mid p \frown_M q_i, M(q_i) = c(x_i).Q_i + R_i\}} \\ \frac{M \to M[p \mapsto P][q_i \mapsto Q_i \{v/x_i\}]_{q_i \in L}}{M \to M' & \text{free}(M, D, N)} \\ \frac{M \to M' & \text{free}(M, D, N)}{M \oplus_D N \to M' \oplus_D N} & \text{(R-Par)} \\ \frac{M \to M'}{M \setminus c \to M' \setminus c} & \text{(R-Res)} & \frac{M \equiv N & N \to N' & N' \equiv M'}{M \to M'} & \text{(R-Struct)} \end{split}$$

Fig. 1: Reduction Semantics

variables occurring in  ${\cal M}$  are bound, i.e. every process in  ${\cal M}$  is data-closed.

In the rest of the paper, we focus on data-closed networks.

## III. OPERATIONAL SEMANTICS

We first define a structural congruence,  $\equiv$ , as an auxiliary relation to state reduction semantics. Structural congruence is defined as the congruence satisfying the rules in Table I.

#### A. Reduction Semantics

 $G\langle\Phi\rangle \equiv G\langle\Psi\rangle$ .

Given a network M, a location  $p \in |M|$  and a process Q, let  $M[p \mapsto Q]$  represent a new network obtained from M by replacing the process at location p by Q. We write  $M[p_1 \mapsto Q_1][p_2 \mapsto Q_2]$  for updating M by replacing processes at  $p_1$  and  $p_2$  by  $Q_1$  and  $Q_2$ , respectively. Given a set of locations  $L \subseteq |M|$ , we write  $M[p_i \mapsto Q_i]_{p_i \in L}$  for updating M by changing the process at location  $p_i$  into  $Q_i$  for each  $p_i \in L$ .

The reduction semantics for networks is defined in Fig. 1 and of the form  $M \to M'$ . In rule (R-Bcast), the process at location p in the network M broadcasts the value of e via the channel c to its adjacent nodes (i.e. all the nodes at  $q_i$  with  $p \curvearrowright_M q_i$ ). All the adjacent nodes can receive the message via channel c. While all the other nodes (not in L), which are not connected with location p, or in which channel c is not available, cannot receive the message. After the broadcast, the sending node is changed into P, the receiving nodes are changed into  $Q_i\{v/x\}$ , and other nodes are unchanged. Moreover, when L is empty, the broadcast is lost. The broadcast does not change the topology of the network. Rule (R-Par) focuses on the parallel composition of two networks, and it describes the situation that no nodes in one network (i.e. N)

- can receive a broadcast in the other network (i.e. M). And the condition  $\operatorname{free}(M,D,N)$  denotes that if nodes in M and N are linked by D then every node in N cannot receive the broadcast messages from M. Rules (R-Res) and (R-Struct) are the standard rules in process calculi, representing restriction reduction and structure reduction, respectively. Let  $\to^*$  denote the reflexive and transitive closure of  $\to$ .

**Example 1:** Network  $N=(\{1,2,3\},\{(1,2),(1,3)\})\langle\Phi\rangle$ , with  $\Phi(1)=\overline{c}(0).\mathbf{0},\ \Phi(2)=c(x).\mathbf{0}+\overline{d}(1).\mathbf{0}$  and  $\Phi(3)=c(y).\mathbf{0}+d(x).\mathbf{0}$ . Therefore,  $\Phi(2)$  and  $\Phi(3)$  can hear  $\Phi(1)$ , but  $\Phi(2)$  and  $\Phi(3)$  cannot hear each other for lack of links. Using rule (R-Bcast), we can get  $N\to N[1\mapsto\mathbf{0}][2\mapsto\mathbf{0}][3\mapsto\mathbf{0}]$  by a broadcast from node  $\Phi(1)$ .

## B. Weak Barbed Congruence

What is a proper observation, or *barb*, in GCWN? Here, we choose to observe channel communications as in standard process calculi. To accommodate the ordinary concept of barb [10], we abandon location information in the following definition. Moreover, we only choose broadcasting communications as the barb. Because, in fact, an observer cannot see whether a node receives a broadcast message, but can detect whether there is a node broadcasting a message by listening.

**Definition 1 (Barb):** Given a network  $M \equiv N \setminus I$ , we say that  $\overline{c}$  is a barb of M, written  $M \downarrow_{\overline{c}}$ , if there is a location  $p \in |M|$  such that  $M(p) \equiv \overline{c}(e).P + R$  and  $c \notin I$ .

**Definition 2 (Weak Barbed Bisimulation):** A binary relation  $\mathcal{B}$  on Net is a weak barbed bisimulation if it is symmetric and whenever  $(M, N) \in \mathcal{B}$  the following conditions hold:

- $M \to M'$  implies  $N \to^* N'$  and  $(M', N') \in \mathcal{B}$  for some N'.
- $M \downarrow_{\overline{c}}$  implies  $N \to^* N'$  and  $N' \downarrow_{\overline{c}}$  for some N'.

Weak barbed bisimilarity, denoted by  $\stackrel{\bullet}{\approx}$ , is the union of all weak barbed bisimulations.

**Example 2:** Recall the network N in Example 1. We have  $N \downarrow_{\overline{c}}$  and  $N \downarrow_{\overline{d}}$ . If we build a network  $M = N \backslash c$ , we only have  $M \downarrow_{\overline{d}}$ . Because, the message from channel c has been restricted and cannot be observed by environments.

**Lemma 1:**  $\stackrel{\bullet}{\approx}$  is an equivalence relation.

**Definition 3:** Network contexts are networks with one hole  $[\cdot]$  (i.e. a special node with a location p), defined by

$$\mathcal{C}[\cdot] ::= [\cdot] \mid M \oplus_D [\cdot] \mid [\cdot] \oplus_D M \mid [\cdot] \setminus c$$

where edges  $D\subseteq \{(p,q)\mid q\in |M|, p \text{ is the location of } [\cdot]\}$ .  $\mathcal{C}[N]$  means putting the network N into the hole at location p. Every edge  $(p,q)\in D$  will be replaced by edges (p,r) with  $r\in |N|$ . For instance, let  $\mathcal{C}[\cdot]=G\langle\Phi\rangle\backslash I,\ N=H\langle\Psi\rangle\backslash J,\ I\cap J=\emptyset$  and assume that the hole's location is p, then  $\mathcal{C}[N]$  represents the network  $G[H/p]\langle\Phi'\rangle\backslash (I\cup J)$  such that  $\Phi'(q)=\Phi(q)$  if  $q\notin |H|$  and  $\Phi'(q)=\Psi(q)$  if  $q\in |H|$ . Here I can be empty, and similarly for J.

**Proposition 1:** For any equivalence relation  $\mathcal{R} \subseteq \mathbf{Net} \times \mathbf{Net}$ , there exists a largest congruence  $\overline{\mathcal{R}}$  contained in  $\mathcal{R}$ . This relation is characterized by  $(M,N) \in \overline{\mathcal{R}}$  if and only if for any context  $\mathcal{C}[\cdot]$  one has  $(\mathcal{C}[M],\mathcal{C}[N]) \in \mathcal{R}$ .

$$\begin{split} \frac{\operatorname{eval}(e) = v}{\overline{c}(e).P \xrightarrow{\overline{c}v} P} & \text{(P-Output)} \quad \frac{c(x).P \xrightarrow{cv} P\{v/x\}}{c(x).P \xrightarrow{\overline{c}v} P\{v/x\}} & \text{(P-Input)} \\ \frac{P \xrightarrow{\alpha} P'}{P + Q \xrightarrow{\alpha} P'} & \text{(P-SumL)} \quad \frac{\operatorname{eval}(b) = \operatorname{true} \quad P \xrightarrow{\alpha} P'}{\operatorname{if} \ b \ \operatorname{then} \ P \ \operatorname{else} \ Q \xrightarrow{\alpha} P'} & \text{(P-IfT)} \\ \frac{Q \xrightarrow{\alpha} Q'}{P + Q \xrightarrow{\alpha} Q'} & \text{(P-SumR)} \quad \frac{\operatorname{eval}(b) = \operatorname{false} \quad Q \xrightarrow{\alpha} Q'}{\operatorname{if} \ b \ \operatorname{then} \ P \ \operatorname{else} \ Q \xrightarrow{\alpha} Q'} & \text{(P-IfF)} \\ \frac{P\{\overrightarrow{v}/\overrightarrow{x}\} \xrightarrow{\alpha} P' \quad A(\overrightarrow{x}) \overset{\operatorname{def}}{=} P}{A(\overrightarrow{v}) \xrightarrow{\alpha} P'} & \text{(P-Rec)} \end{split}$$

Fig. 2: Labelled Transition Semantics for Processes

**Definition 4:** Networks M and N are weakly barbed congruent, denoted by  $M\cong N$ , if  $\mathcal{C}[M]\overset{\bullet}{\approx}\mathcal{C}[N]$  for any context  $\mathcal{C}[\cdot]$ .

 $\cong$  is the largest congruence in  $\stackrel{\bullet}{\approx}$  by Proposition 1.

## C. Labelled Transition Semantics

In this paper, the labelled transition systems of GCWN are divided into two parts: one part for processes and the other part for networks.

Fig. 2 describes the labelled transition semantics for processes. The transitions are of the form  $P \xrightarrow{\alpha} P'$ , and the syntax of action  $\alpha$  is defined as

$$\alpha ::= cv \mid \overline{c}v$$

where action cv stands for receiving a broadcast message v via channel c and action  $\overline{c}v$  stands for broadcasting message v via channel c. The rules in Fig. 2 are self-explanatory.

Fig. 3 describes the labelled transition semantics for networks. The transitions for networks are of the form  $M \xrightarrow{\delta} M'$ . The grammar for  $\delta$  is

$$\delta ::= p : \alpha \mid \tau$$

where  $p \in \mathsf{Loc}$  and  $\alpha$  is an action.  $\xrightarrow{p:\alpha}$  represents that the node at location p performs an action  $\alpha$ . And  $\xrightarrow{\tau}$  represents the unobservable transition.

Rule (N-Send) models the broadcast at location p of value v via channel c. Rule (N-Recv) shows that a value can be received at location p via channel c. Rule (N-Bcast) describes the propagation of a broadcast, and the premise  $(p,q) \in D$  makes sure that only the nodes connected with the transmitter can receive the message. Rule (N-Res1) hides a broadcast in restricted networks. Rule (N-Res2) is standard in process calculi. Rules (N-ParL) and (N-ParR) model parallel composition networks

We write  $M \xrightarrow{\tau^*} M'$  if there exists  $n \ge 1$  such that  $M = M_1, \ M' = M_n, \ M_1 \xrightarrow{\tau} M_2 \xrightarrow{\tau} \cdots \xrightarrow{\tau} M_n. \ M \xrightarrow{p:\alpha} M'$  denotes  $M \xrightarrow{\tau^*} M_1 \xrightarrow{p:\alpha} M'_1 \xrightarrow{\tau^*} M'$  for some  $M_1$  and  $M'_1$ .

**Example 3 (Broadcast):** Let  $N_i = (\{i\}, \emptyset) \langle \Phi_i \rangle$ , i = 1, 2, 3, be networks, i.e. each network only has one node. Let  $\Phi_1(1) =$ 

$$\frac{p \in |M| \quad M(p) = P \quad P \xrightarrow{\overline{c}v} P'}{M \xrightarrow{p:\overline{c}v} M[p \mapsto P']} \quad \text{(N-Send)}$$

$$\frac{p \in |M| \quad M(p) = P \quad P \xrightarrow{cv} P'}{M \xrightarrow{p:cv} M[p \mapsto P']} \quad \text{(N-Recv)}$$

$$\frac{M \xrightarrow{p:\overline{c}v} M' \quad N \xrightarrow{q:cv} N' \quad (p,q) \in D}{M \oplus_D N \xrightarrow{p:\overline{c}v} M' \oplus_D N'} \quad \text{(N-Beast)}$$

$$\frac{M \oplus_D N \xrightarrow{p:\overline{c}v} M'}{N \oplus_D M} \xrightarrow{p:\overline{c}v} N' \oplus_D M'$$

$$\frac{M \xrightarrow{p:\overline{c}v} M'}{M \setminus c \xrightarrow{\tau} M' \setminus c} \quad \text{(N-Res1)} \quad \frac{M \xrightarrow{\delta} M' \quad c \text{ and } \overline{c} \text{ not in } \delta}{M \setminus c \xrightarrow{\delta} M' \setminus c} \quad \text{(N-Res2)}$$

$$\frac{M \xrightarrow{\delta} M'}{M \oplus_D N \xrightarrow{\delta} M' \oplus_D N} \quad \text{(N-ParL)} \quad \frac{N \xrightarrow{\delta} N'}{M \oplus_D N \xrightarrow{\delta} M \oplus_D N'} \quad \text{(N-ParR)}$$

Fig. 3: Labelled Transition Semantics for Networks

 $\overline{c}(0).\mathbf{0}$ ,  $\Phi_2(2) = c(x).\mathbf{0}$  and  $\Phi_3(3) = c(y).\mathbf{0}$ . Using the rules in Fig. 2 and Fig. 3,  $N_1 \mid (N_2 \oplus N_3)$  can evolve as follows:

$$\begin{array}{c} N_1(1) = \overline{c}(0).0 & N_2(2) = c(x).0 \\ \overline{c}(0).0 & \overline{c}0 \to 0 & c(x).0 & \overline{c}0 \to 0 \\ \hline N_3(3) = c(y).0 & c(y).0 & \overline{c}0 \to 0 & N_1 & \overline{1:\overline{c}0} \to N_1[1 \mapsto 0] & N_2 & \overline{2:c}0 \to N_2[2 \mapsto 0] \\ \hline N_3 & \overline{3:c}0 & N_3[3 \mapsto 0] & N_1 & N_2 & \overline{1:\overline{c}0} \to N_1[1 \mapsto 0] & N_2 & \overline{2:c}0 \to N_2[2 \mapsto 0] \\ \hline N_1 & | & (N_2 \oplus N_3) & \overline{1:\overline{c}0} \to N[1 \mapsto 0] & | & (N_2[2 \mapsto 0] \oplus N_3[3 \mapsto 0]) \\ \hline \end{array}$$

#### D. Weak Bisimulation

As explained in the introduction, observers (or environments) should take the links and locations into account. We define a weak bisimulation for GCWN to take them into account, and it makes observers more context-sensitive when the observers interact with networks.

For instance, when networks  $M=G\langle\Phi\rangle$  and  $N=H\langle\Psi\rangle$  are bisimilar, the observer has to point out which node  $\Phi(p)$  of M and which node  $\Psi(q)$  of N should be related in the bisimulation relation. Inspired by [12], [13], we define a localized relation on Net through triples (M,E,N) by taking locations into account, and  $E\subseteq |M|\times |N|$  specifies the pairs of locations of M and N. Let  $E^{-1}=\{(q,p)\mid (p,q)\in E\}$ .

**Definition 5 (Localized Relation):** A localized relation on **Net** is a set  $\mathcal{R} \subseteq \mathbf{Net} \times \mathcal{P}(\mathsf{Loc}^2) \times \mathbf{Net}$  such that, if  $(M, E, N) \in \mathcal{R}$  then  $E \subseteq |M| \times |N|$ .  $\mathcal{R}$  is symmetric if  $(P, E, Q) \in \mathcal{R}$  implies  $(Q, E^{-1}, P) \in \mathcal{R}$ .

**Definition 6 (Weak Bisimulation):** A symmetric localized relation  $\mathcal{R}$  on **Net** is a weak bisimulation such that whenever  $(M, E, N) \in \mathcal{R}$ :

- if  $M \xrightarrow{\tau} M'$ , then there is N' such that  $N \xrightarrow{\tau^*} N'$  and  $(M', E, N') \in \mathcal{R};$
- if  $M \xrightarrow{p:\alpha} M'$ , then there is N' such that  $N \stackrel{q:\alpha}{\Longrightarrow} N'$  with  $(p,q) \in E$ , and  $(M',E,N') \in \mathcal{R}$ .

**Definition 7:** M and N are weakly bisimilar, denoted by  $M \approx N$ , if there exist a weak bisimulation  $\mathcal{R}$  and a relation  $E \subseteq |M| \times |N|$  such that  $(M, E, N) \in \mathcal{R}$ .

In the definitions, E can be taken as a parameter and the weak bisimulation can be called parameterized weak bisimulation, similar to the parameterized location bisimulation in [14]. Moreover, if  $E = |M| \times |N|$ , we obtain a weak bisimulation ignoring the location information.

**Example 4 (Comparisons on Bisimulations):** In GCWN, we define a network Sys for a simple protocol, transferring data from one node to another. We provide a network Spec as a specification for the protocol. We define the sender P and the receiver Q in Sys, and the process R in Spec as follows

$$P \stackrel{\text{def}}{=} \overline{c_1(0)}.\overline{d_1(1)}.d_2(x).P$$

$$Q \stackrel{\text{def}}{=} d_1(x).\overline{c_2(0)}.\overline{d_2(1)}.Q$$

$$R \stackrel{\text{def}}{=} \overline{c_1(0)}.\overline{c_2(0)}.R$$

Channels  $d_1$  and  $d_2$  are used to transfer data and acknowledgements between P and Q. Let  $Sys = G_1\langle\Phi\rangle\backslash\{d_1,d_2\}$  and  $Spec = G_2\langle\Psi\rangle$ , where  $G_1 = (\{1,2\},\{(1,2)\})$ ,  $\Phi(1) = P$ ,  $\Phi(2) = Q$ ,  $G_2 = (\{3\},\emptyset)$  and  $\Psi(3) = R$ . From  $(Sys,\{(1,3),(2,3)\},Spec)$ , we can build a weak bisimulation containing it in GCWN, i.e.  $Sys \approx Spec$ . However, Sys and Spec are not weakly bisimilar in the literature [5], [7], [8], where weakly bisimilar networks must play the same observable actions at the same locations.

#### IV. MAIN RESULT

In this section, we show that reduction semantics and labelled transition semantics model the same behaviours, and prove that weak bisimilarity implies weak barbed congruence.

#### A. Harmony Theorem

We have defined reduction semantics and labelled transition semantics for networks in the previous section. There is a close relation between them, i.e. the internal reduction and the labelled transition describe the same behaviours. Before proving this, we provide two lemmas.

## Lemma 2:

- 1) If  $M \xrightarrow{p:cv} M'$ , then there are v, x, I with  $c \notin I$  and  $p \in |M|$  such that  $M \equiv N \setminus I$ , N(p) = c(x).P + R and  $M' \equiv N[p \mapsto P\{v/x\}] \setminus I$ .
- 2) If  $M \xrightarrow{p:\overline{c}v} M'$ , then there are e with  $\operatorname{eval}(e) = v$ , I with  $c \notin I$ ,  $M(p) = \overline{c}(e).P + R$ ,  $L = \{q_i \mid p \curvearrowright_M q_i, M(q_i) = c(x_i).Q_i + R_i\}$ ,  $|M_1| = L \cup \{p\}$ ,  $|M_2| = |M| \setminus |M_1|$  and  $D \subseteq |M_1| \times |M_2|$  consistent with  $\curvearrowright_M$  such that  $M \equiv (M_1 \oplus_D M_2) \setminus I$ , and  $M' \equiv (M_1[p \mapsto P][q_i \mapsto Q_i\{v/x\}]_{q_i \in L} \oplus_D M_2) \setminus I$ .

*Proof:* Induction on the transition rules for networks.  $\blacksquare$  In the second part of Lemma 2, we can divide the network M into two parts. One part  $M_1$  consists of the broadcasting node at location p and all its adjacent nodes that can receive the message via channel c. The other part  $M_2$  consists of the nodes that are not connected with location p, and the nodes that are connected with p but in which c is not available.

**Lemma 3:** If  $M \xrightarrow{\delta} M'$  and  $M \equiv N$ , then there exists N' such that  $N \xrightarrow{\delta} N'$  and  $M' \equiv N'$ .

By Lemmas 2 and 3, we can prove the following theorem. **Theorem 1 (Harmony Theorem):** 

- If  $M \to M'$ , then
  - either  $M \xrightarrow{\tau} M''$  and  $M'' \equiv M'$  for some M'';
  - or there is  $p:\overline{c}v$  such that  $M\xrightarrow{p:\overline{c}v}M''$  and  $M''\equiv M'$  for some M''.
- If  $M \xrightarrow{p:\overline{c}v} M'$  or  $M \xrightarrow{\tau} M'$ , then  $M \to M'$ .

## B. Soundness

We first prove that weak bisimilarity is an equivalence relation.

**Lemma 4:**  $\approx$  is an equivalence relation.

Then we prove that  $\approx$  is preserved by the operators in networks. In CCS [1], if  $\mathcal{R}$  is a weak bisimulation and P  $\mathcal{R}$  Q, then one can prove that, for any S,  $S \mid P$  and  $S \mid Q$  are weak bisimilar, by showing that a new relation  $\mathcal{R}'$  extending  $\mathcal{R}$  (i.e.  $(S \mid P)$   $\mathcal{R}'$   $(S \mid Q)$ ) is a weak bisimulation. However, we cannot simply do this in GCWN, because we need to record locations of nodes and links between nodes. The main challenge is to extend a localized relation  $\mathcal{R}$  to another localized relation  $\mathcal{R}'$  to accommodate the parallel composition in GCWN. For instance, let  $O \oplus_C M$  be a parallel composition of networks O and M with some  $C \subseteq |O| \times |M|$ . We have to build  $O \oplus_D N$  as a parallel composition of networks O and N with some relation  $D \subseteq |O| \times |N|$ . Meanwhile, the relations C and D should satisfy some constraints.

**Definition 8 (Adapted Triple of Relations):** We say that a triple of relations (D, D', E) with  $D \subseteq A \times B$ ,  $D' \subseteq A \times B'$  and  $E \subseteq B \times B'$  is *adapted*, if for any  $(a, b, b') \in A \times B \times B'$  with  $(b, b') \in E$ ,  $(a, b) \in D$  if and only if  $(a, b') \in D'$ .

**Definition 9 (Parallel Extension):** Let  $\mathcal{R}$  be a localized relation. A localized relation  $\mathcal{R}'$  is a parallel extension of  $\mathcal{R}$ , if for any  $(U, F, V) \in \mathcal{R}'$  the following conditions are satisfied:

- there exist a network O, a triple  $(M, E, N) \in \mathcal{R}$ ,  $C \subseteq |O| \times |M|$ ,  $D \subseteq |O| \times |N|$  such that  $U = O \oplus_C M$  and  $V = O \oplus_D N$ ,
- (C, D, E) is adapted,
- F is the relation  $(\mathrm{Id}_{|O|} \cup E) \subseteq |U| \times |V|$ , and  $\mathrm{Id}_{|O|} = \{(p,p) \mid p \in |O|\}$ .

Intuitively, the premise that (C, D, E) is adapted specifies that network O as an observer should have the same connections with M and N up to E, i.e. taking links into account.

**Proposition 2:** If  $\mathcal{R}$  is a weak bisimulation, then its parallel extension  $\mathcal{R}'$  is also a weak bisimulation.

Now we can prove the following theorem.

**Theorem 2:**  $\approx$  is a congruence.

Structurally congruent networks are weakly bisimilar.

**Proposition 3:**  $M \equiv N$  implies  $M \approx N$ .

*Proof:* Induction on the rules of  $\equiv$ .

Weak bisimulation is reduction closed and barb preserving. **Proposition 4:** If  $M \approx N$  then  $M \stackrel{\bullet}{\approx} N$ .

From Proposition 4 and Theorem 2, we can easily get the following theorem.

**Theorem 3 (Soundness):** If  $M \approx N$ , then  $M \cong N$ .

Though the converse direction of Theorem 3 (i.e. completeness) holds in CCS in a point-to-point approach, e.g. [10], [15], it does not hold in this paper with local broadcast.

$$\begin{array}{lll} A(ip_A,ip_X) \stackrel{\mathrm{def}}{=} & \overline{c}((\{RDP,ip_X,N_A\}_{K_{A-}},[cert_A])).A_1(ip_A) \\ A_1(ip_A) \stackrel{\mathrm{def}}{=} & d(x).(\mathbf{if} \ \operatorname{check2}(x) = ok \ \land \ \operatorname{snd}(x) = ip_A \\ & \operatorname{then} \ \overline{s}(0).\mathbf{0} \ \operatorname{else} \ A_1(ip_A)) \\ Q(ip_Q) \stackrel{\mathrm{def}}{=} & c(x).(\mathbf{if} \ \operatorname{check1}(x) = ok \ \operatorname{then} \\ & \overline{c}(\operatorname{NewMsg1}(x,ip_Q)).Q_1(ip_Q,\operatorname{getIP}(x)) \\ & \operatorname{else} \ Q(ip_Q)) \\ Q_1(ip_Q,ip) \stackrel{\mathrm{def}}{=} & d(y).(\mathbf{if} \ \operatorname{check2}(y) = ok \ \land \ \operatorname{snd}(y) = ip_Q \ \operatorname{then} \\ & \overline{d}((\operatorname{NewMsg2}(y,ip_Q),ip)).\mathbf{0} \\ & \operatorname{else} \ Q_1(ip_Q,ip)) \\ X(ip_X) \stackrel{\mathrm{def}}{=} & c(x).(\mathbf{if} \ \operatorname{check1}(x) = ok \ \operatorname{then} \\ & \overline{d}((\operatorname{NewMsg3}(x,ip_X),\operatorname{getIP}(x))).\mathbf{0} \\ & \operatorname{else} \ X(ip_X)) \\ I \stackrel{\mathrm{def}}{=} & c(x).\overline{c}(x).I + d(x).\overline{d}((\operatorname{fst}(x),ip_A)).I \\ \end{array}$$

Fig. 4: Processes for ARAN

#### V. CASE STUDIES

In this section, we show that GCWN can be used to model and reason about non-trivial networks.

#### A. ARAN

We use behavioural equivalences of GCWN to show an attack scenario in ARAN [11]. ARAN is a secure on-demand routing protocol for ad hoc networks. The goal of ARAN is to ensure message integrity and non-repudiation in the route process using public key cryptography. ARAN requires the use of a trusted server T to send a certification to each node. Each valid node A in the network has a pair of public and private keys  $(K_{A+}, K_{A-})$  and a certification (received from T) to authenticate itself to other nodes. For a node X, let X's certification be  $cert_X = \{ip_X, K_{X+}, t, e\}_{K_{T-}}$ containing IP address of X, the public key of X, a timestamp t when  $cert_X$  was created and a time e at which  $cert_X$ expires. As usual, we use  $\{d\}_{K_{X_-}}$  to encrypt data d with the private key  $K_{X-}$  of node X. To abstract some implementation details, we introduce some auxiliary functions to manipulate the messages. For instance, we use check1 and check2 to check the signature using certifications in the messages at the request and the reply steps, use getIP to extract the IP address of the node that broadcasts the message, and use functions NewMsg1, NewMsg2 and NewMsg3 to construct new messages at different steps of the protocol. We use fst (and snd) to return the first element (and the second element)

We assume that each node has received a certification from T. The protocol proceeds as follows and we also describe the procedure in GCWN in Fig. 4:

- The source node A begins the procedure of route to destination X by broadcasting a route require package,  $(\{RDP, ip_X, N_A\}_{K_{A-}}, [cert_A])$ , to its neighbours, where RDP is the package identifier,  $ip_X$  is the IP address of the destination X,  $N_A$  is a nonce, and  $cert_A$  is the certification of A. See process A in Fig. 4.
- When a node B receives the message, msg, it uses A's
  public key extracted from cert<sub>A</sub> in msg to check the
  message. If the check fails, the message is dropped;

otherwise B sets up a reverse path back to the source by recording the neighbor from which it received the RDP and B signs the received message and appends its certification  $cert_B$ . We use check1(msg) to do this, and use getIP(msg) to get the IP address from which the message received. Then B rebroadcasts the message  $(\{RDP, ip_X, N_A\}_{K_{A-}}\}_{K_{B-}}, [cert_A, cert_B])$ , built by  $NewMsg1(msg, ip_B)$ . See process Q in Fig. 4.

- When B's neighbor C receives the message, it checks the message using the certifications of both A and B. If the check fails, the message is dropped; otherwise C records B to unicast the reply, removes B's signature and certification, signs the message broadcasted by A and appends its certification. Then C rebroadcasts the message  $(\{RDP, ip_X, N_A\}_{K_{A-}}\}_{K_{C-}}, [cert_A, cert_C])$ , constructed by NewMsg1 $(msg, ip_C)$ . Each intermediate node along the path repeats the same actions as C. See process Q in Fig. 4.
- When the destination X first receives the RDP, if all the checks are valid then it sends a REP to the source A along the reverse path to the source as a unicast message.  $(\{REP, ip_A, N_A\}_{K_{X-}}, [cert_X])$  is constructed by NewMsg3( $msg, ip_X$ ). See process X in Fig. 4.
- Let D be the first node that receives the REP, msg, sent by X. After a valid check, node D signs the REP, appends its certification and forwards the message,  $(\{REP, ip_A, N_A\}_{K_{X-}}\}_{K_{D-}}, [cert_X, cert_D])$  built by NewMsg2 $(msg, ip_D)$ , to the node from which it receives the RDP. See process  $Q_1$  in Fig. 4.
- Let C be the next hop of D to A. C validates D's signature on the received message, removes D's signature and certification, signs the message and appends its certification, i.e.  $(\{REP, ip_A, N_A\}_{K_{X_-}}\}_{K_{C_-}}, [cert_X, cert_C])$  constructed by NewMsg2(msg, C). Then C unicasts the message to its next hop, i.e. B here. Each node along the reverse path repeats the same actions as C. See process  $Q_1$  in Fig. 4.

Here, we implement a unicast using a broadcast, where nodes drop the message if they are not mentioned or addressed.

We have a source  $A(ip_A,ip_X)$ , a destination  $X(ip_X)$ , two nodes B and C in the routing path as  $Q(ip_B)$  and  $Q(ip_C)$ , and an intruder I (see Fig. 4) which only relays messages. Let the IP address of a node be its location. Let  $N=H\langle\Psi\rangle$ , where  $H=(\{1,2,3,4\},\{(1,3)\}),\,\Psi(1)=A(1,4),\,\Psi(2)=Q(2),\,\Psi(3)=Q(3)$  and  $\Psi(4)=X(4)$ . Let  $M=G\langle\Phi\rangle$ , where  $G=(\{1,2,3,4,5\},\{(1,5),(2,5),(4,5),(1,3)\}),\,\Phi(1)=A(1,4),\,\Phi(2)=Q(2),\,\Phi(3)=Q(3),\,\Phi(4)=X(4)$  and  $\Phi(5)=I$ . M is an attacked network as a composition of N and I.

Now we show that there is an attack from the intruder I in M, by showing that M and N are not weakly barbed bisimilar. In fact, M can evolve to an incorrect route state through: (1) A broadcasting message  $m_1$  to start the routing procedure and I and C receiving the message; (2) I replaying the message to

M	$\rightarrow$	$M[1 \mapsto A_1(1)][3 \mapsto C'][5 \mapsto \overline{c}(m_1).I] \stackrel{\text{def}}{=} M_1,$	by steps in (1)
$M_1$	$\rightarrow \rightarrow$	$M_1[2 \mapsto Q_1(2,1)][5 \mapsto \overline{c}(m_2).I] \stackrel{\text{def}}{=} M_2,$	by steps in (2)
$M_2$	$\rightarrow \rightarrow$	$M_2[5 \mapsto \overline{d}(fst(m_3), 1).I][4 \mapsto 0] \stackrel{\text{def}}{=} M_3,$	by steps in (3)
$M_3$	$\rightarrow$	$M_3[1 \mapsto \overline{s}(0).0][5 \mapsto I] \stackrel{\text{def}}{=} M_4,$	by steps in (4)

Fig. 5: Reductions in ARAN

B, B rebroadcasting the message  $m_2$  signatured by B and only I receiving the message; (3) I rebroadcasting the message  $m_2$  to X and X sending a reply  $m_3$  to I; (4) I sending the replay to A and A reaching an incorrect route state. See Fig. 5 for details. In  $M_1$  from Fig. 5,  $C' = \overline{c}(\text{NewMsg1}(m_1, 3)).Q_1(3, 1)$ .

In network N, node A cannot reach the state  $\overline{s}(0).0$ . Thus N cannot reach a network N' with  $N' \downarrow_{\overline{s}}$ . Since  $M_4 \downarrow_{\overline{s}}$ , M and N are not weakly barbed bisimilar by the definition.

## B. The Alternating Bit Protocol

The Alternating Bit Protocol (ABP) is a simple data link layer network protocol. ABP is used when a transmitter  $P_1$  wants to send messages to a receiver  $P_2$ , with the assumptions that the channel may corrupt a message and that  $P_1$  and  $P_2$  can decide whether they have received a correct message. Each message from  $P_1$  to  $P_2$  contains a data part and a one-bit sequence number, i.e. a value that is 0 or 1.  $P_2$  can send two acknowledge messages, i.e. (Ack, 0) and (Ack, 1), to  $P_1$ .

In [1], ABP was formalized in CCS with an interleaving semantics. In [6], ABP was investigated in a broadcasting semantics based on transmission radius. In GCWN, we intend to show that graphs can be used to concisely characterize communicating capacities.

In Fig. 6 we provide a specification of ABP in GCWN. send and ack are channels. The transmitter  $P_1$  has a list  $lt_1$ containing the messages to be sent, and the receiver  $P_2$  also has a list  $lt_2$  containing the received messages. The list is equipped with operations head (returning the head of a list), tail (returning a list with the first element removed), append (inserting an element as the last element of the new list) and null (testing whether a list is empty). We use fst (and snd) to return the first element (and the second element) of a pair. End is an entry to indicate that all the messages in  $lt_1$  have been transformed.  $Succ(lt_2)$  indicates that the receiver has successfully received all the messages. We define a network M consisting of two nodes linked by an edge, and  $P_1(lt,b)$ located at one location and  $P_2([],b)$  located at the other. Formally,  $M = (\{p_1, p_2\}, \{(p_1, p_2)\}) \langle \Phi \rangle, \ \Phi(p_1) = P_1(lt, b)$ and  $\Phi(p_2) = P_2([], b)$ .

**Proposition 5:** For any network  $O, M \oplus O \to^* M[p_1 \mapsto \mathbf{0}][p_2 \mapsto Succ(lt)] \oplus O'$  for some O'.

The proposition says that if the transmitter  $P_1$  and the receiver  $P_2$  can communicate with each other but they cannot communicate with the nodes in O (i.e.  $(M \oplus O)$ ), then the whole network can reach a state where all the messages in  $P_1$  are correctly received by  $P_2$  no matter what happens in O.

## VI. RELATED WORK

Inspired by [12], [13], which focus on tree structured concurrent systems with point-to-point communications, we

propose a graph-based calculus to study wireless networks with local broadcasts. Below we only discuss some closely related work on wireless systems.

**Calculi for Wireless Systems.** Several process calculi for wireless systems have been proposed. A brief survey of broadcast calculi can be found in [16].

CBS<sup>‡</sup> [8] was probably the first calculus for wireless systems, and it is an extension of CBS [3]. In CBS<sup>‡</sup>, every node is specified by a location, and system transitions are indexed by graphs which represent the connectivity of nodes. Thus, graphs specify possible behaviors of a system at the semantic level. Behavioural equivalences are defined to identify processes. The final goal of CBS<sup>‡</sup> is to give a framework to specify and analyse communication protocols for wireless networks. Different from CBS<sup>‡</sup>, in GCWN graphs are introduced at the syntactic level and the weak bisimulation of GCWN takes locations and links into account.

CWS (Calculus for Wireless Systems) [6], [17] was developed to model protocols at the data-link layer. In CWS, a node  $n[P]_{l,r}^c$  stands for a node named n, located at l, executing P, with channel c and transmission radius r. CWS deals with static topologies, and the topology of a network can be derived by a distance function to compute the nodes in the transmission range of each node. CWS separates the begin and the end of a transmission to handle interferences. The main result of CWS is a correspondence between a reduction semantics and a labelled transition semantics, called harmony theorem. In GCWN, we use graphs to describe connections in networks. Besides a similar harmony theorem, we also develop behavioural equivalences for GCWN.

In CMN (Calculus of Mobile Ad Hoc Networks) [7], the nodes are similar to the ones in CWS. Both a reduction semantics and a labelled transition semantics are developed, and a harmony theorem is proved for them. The main result is that the labelled bisimilarity coincides with reduction barbed congruence.

CMAN (Calculus for Mobile Ad Hoc Networks) [5] supports local broadcast and dynamic changes of the network topology. CMAN is equipped with a reduction semantics and a reduction congruence, and the weak bisimulation coincides with the reduction congruence in CMAN. CMAN also provides a formalisation of an attack on the cryptographic routing protocol ARAN. However, the bisimulations in [5], [7] do not take locations and links into account. And it is unclear how to define a parameterized weak bisimulation in [5], [7].

Cerone and Hennessy [18] proposed a calculus for distributed systems, using directed graphs and equipped with testing preorders in the style of DeNicola and Hennessy [19]. Directed graphs are more refined than undirected graphs, but

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\begin{array}{lll} P_1(lt_1,b) & \stackrel{\mathrm{def}}{=} & \text{if } \operatorname{null}(lt_1) \text{ then } \overline{\operatorname{send}}((End,b)).\operatorname{ack}(x).( \\ & & \text{if } x = (Ack,b) \text{ then } \mathbf{0} \text{ else } P_1(lt_1,b)) \\ & & \operatorname{else } \overline{\operatorname{send}}((\operatorname{head}(lt_1),b)).\operatorname{ack}(x).( \\ & & \text{if } x = (Ack,b) \text{ then } P_1(\operatorname{tail}(lt_1),\neg b) \text{ else } P_1(lt_1,b)) \\ P_2(lt_2,b) & \stackrel{\mathrm{def}}{=} & \operatorname{send}(x).(\mathbf{if} \operatorname{snd}(x) = b \\ & & \operatorname{then } (\operatorname{if } \operatorname{fst}(x) = \underline{E}nd \text{ then } Succ(lt_2) \\ & & \operatorname{else } \overline{\operatorname{ack}}((Ack,b)).P_2(\operatorname{append}(lt_2,\operatorname{fst}(x)),\neg b)) \\ & & \operatorname{else } \overline{\operatorname{ack}}(Ack,\neg b).P_2(lt_2,b)) \end{array}
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Fig. 6: The Alternating Bit Protocol in GCWN

in most situations undirected graphs are enough to specify the connectivity of networks, e.g. [4]–[9]. The testing preorders in [18] are similar to the barbed congruence in GCWN, but they only consider actions with the same locations.

**Calculi for IoT.** Lanese et al. [20] proposed the first process calculi for Internet of Things (IoT). Recently, a calculus for IoT [21] was proposed with a fully abstract semantics in a point-to-point approach.

**Calculi for CPS.** Vigo et al. [22] proposed a calculus for wireless-based cyber-physical systems (CPSs) to model and reason about cryptographical primitives. Inspired by [22], Wu and Zhu [23] considered a static network topology and proved a harmony theorem to link reduction semantics and labelled transition semantics.

#### VII. CONCLUSIONS

We have proposed a graph-based calculus, called GCWN, to model and reason about wireless networks. In GCWN, we use graphs at syntactical level to specify local broadcast. The calculus is equipped with a reduction semantics and a labelled transition semantics. The former has been used to define weak barbed congruence. The latter has been used to define a parameterized weak bisimulation emphasizing locations and local broadcast. We have proved that the two semantics model the same behaviours and weak bisimilarity implies weak barbed congruence. GCWN also has been used to reason about scenarios in ARAN and ABP.

There are some further issues. Firstly, we are going to extend GCWN to handle dynamic topologies of wireless networks. Secondly, we plan to extend GCWN with directed graphs. Thirdly, we would like to apply our calculus and methods to other wireless-based scenarios, e.g. the ones in IoT and CPSs. Finally, we plan to develop an implementation of the calculus to assist in reasoning about systems.

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#### APPENDIX

#### A. Proofs in Section III

**Lemma 5:** Let  $\mathcal{B}$  a weak barbed bisimulation. If  $(M,N) \in \mathcal{B}$  and  $M \to^* M'$ , then there exists N' such that  $N \to^* N'$  and  $(M',N') \in \mathcal{B}$ .

*Proof:* Induction on the length of the derivation  $M \to^* M'$ .

**Lemma 6:** A symmetric binary relation  $\mathcal{B}$  on **Net** is a weak barbed bisimulation if and only if the following conditions hold:

- if  $(M,N) \in \mathcal{B}$  and  $M \to^* M'$ , then  $N \to^* N'$  and  $(M',N') \in \mathcal{B}$  for some N';
- if  $(M,N) \in \mathcal{B}$  and  $M \downarrow_{\overline{c}}$ , then  $N \to^* N'$  and  $N' \downarrow_{\overline{c}}$  for some N'.

*Proof:* ( $\Leftarrow$ ) Because  $\rightarrow$  is a special case of  $\rightarrow^*$ , this direction is obvious.

 $(\Rightarrow)$  For the first statement, it is straightforward by Lemma 5.

For the second statement, it is obvious by the definition of weak barbed bisimulation.

Let  $\mathcal{B}_1$  and  $\mathcal{B}_2$  are binary relations on Net. Let  $\mathcal{B}_2 \circ \mathcal{B}_1$  be the composition of  $\mathcal{B}_1$  and  $\mathcal{B}_2$  such that if  $(M,N) \in \mathcal{B}_1$  and  $(N,O) \in \mathcal{B}_2$ , then  $(M,O) \in \mathcal{B}_2 \circ \mathcal{B}_1$ .

The proof for Lemma 1.

*Proof:* We need to prove that  $\stackrel{*}{\approx}$  is reflexive, symmetric and transitive. It is straightforward from the definition and Lemma 6.

The proof for Proposition 1.

*Proof:* For the first statement, from the definition of congruence, it is obvious that the identity relation contained in  $\mathcal{R}$  is a congruence. And congruences are closed under arbitrary unions and contexts.

For the second statement, let  $\mathcal E$  be a congruence defined as:  $(M,N)\in\mathcal E$  if and only if for any context  $\mathcal C[\cdot]$  one has  $(\mathcal C[M],\mathcal C[N])\in\mathcal R$ . Therefore,  $\mathcal E$  is a congruence contained in  $\mathcal R$  (because we can take  $\mathcal C[\cdot]=[\cdot]$ ) and hence  $\mathcal E\subseteq\overline{\mathcal R}$ . Conversely, let  $(M,N)\in\overline{\mathcal R}$  and  $\mathcal C[\cdot]$  be a context. Because  $\overline{\mathcal R}$  is a congruence, we have  $(\mathcal C[M],\mathcal C[N])\in\overline{\mathcal R}$ . We have  $(\mathcal C[M],\mathcal C[N])\in\mathcal R$  from  $\overline{\mathcal R}\subseteq\mathcal R$  by definition of  $\overline{\mathcal R}$  and hence  $(M,N)\in\mathcal E$ .

## B. Proofs in Section IV

The proof for Lemma 3.

*Proof:* We prove it by induction on the depth of the inference of  $M \xrightarrow{\delta} M'$ .

First, it is enough to prove the result in the special case that the congruence  $M \equiv N$  is due to a single application of a structural congruence rule. And the general case follows just by iterating the special case.

The full proof must treat all possible cases for the final step of the inference of  $M \xrightarrow{\delta} M'$ . Here we only consider one case, and suppose it uses rule (N-ParL), where M is  $M_1 \oplus_C M_2$ , with  $M_1 \xrightarrow{\delta} M'_1$  inferred by a shorter inference. Now there are many ways in which  $M_1 \oplus_C M_2 \equiv N$  may be due to a

single use of a structural congruence rule, and here we only consider two cases to confirm ourselves.

- (1) Suppose that the commutativity rule (i.e.  $M \oplus_D N \equiv N \oplus_D M$ ) is used, so we have N is  $M_2 \oplus_C M_1$ . In this case, we use the rule (N-ParR) to deduce  $N \xrightarrow{\delta} M_2 \oplus_C M_1'$ . Take N' to be  $M_2 \oplus_C M_1'$ , and we have  $M' \equiv N'$  as required.
- (2) Suppose that a single rule of structural congruence is used in  $M_1$ , so that  $M_1 \equiv N_1$  and N is  $N_1 \oplus_C M_2$ . Since  $M_1 \stackrel{\delta}{\to} M_1'$  is inferred by a shorter inference, by applying the induction we have  $N_1 \stackrel{\delta}{\to} N_1'$  and  $M_1' \equiv N_1'$ . Take N' to be  $N_1' \oplus_C M_2$ , and by using (N-ParL) we can deduce that  $N \stackrel{\delta}{\to} N'$  and  $M' \equiv N'$  as required.

So the result follows by a fairly lengthy case analysis, both for the structural congruence rule used and for the last step of the transition inference.

The proof for Theorem 1.

*Proof:* We first prove that if  $M \to M'$  then either  $M \xrightarrow{\tau} M''$  and  $M'' \equiv M'$  for some M'', or there is  $p : \overline{c}v$  such that  $M \xrightarrow{p:\overline{c}v} M''$  and  $M'' \equiv M'$  for some M''. We prove it by rule induction on the inference of  $M \to M'$ .

Supposed that the transition  $M \to M'$  is inferred by an application of rule (R-Bcast), that is

$$\frac{M(p) = \overline{c}(e).P + R \text{ eval}(e) = v \text{ } L = \{q_i \mid p \frown_M q_i, M(q_i) = c(x_i).Q_i + R_i\}}{M \rightarrow M[p \mapsto P][q_i \mapsto Q_i \{v/x\}]_{q_i \in L}}$$

Then we use the following broadcast transition in Fig. 3.

$$\frac{M_i \xrightarrow{p:\overline{c}v} M_i' \quad N_i \xrightarrow{q:cv} N_i' \quad (p,q) \in D_i}{M_i \oplus_{D_i} N_i \xrightarrow{p:\overline{c}v} M_i' \oplus_{D_i} N_i'}$$

Here, we take  $N_i$  as a network that only has one node with location  $q_i \in L$ , the process on the node is  $c(x_i).Q_i + R_i$ , and  $D_i$  is consistent with the graph of the whole network M, i.e. we add |L| networks  $N_i$  to the network  $M_1$  to obtain the network M. And at the beginning,  $M_1$  only contains the nodes in M on locations in set  $|M| \setminus L$ . We can rewrite M as  $(\cdots ((M_1 \oplus_{D_1} N_1) \oplus_{D_2} N_2) \cdots) \oplus_{D_k} N_k$  with k = |L|.

And we apply this rule |L| times, such that all the potential receivers receive the broadcasting message. We get  $M \xrightarrow{p:\overline{c}v} (\cdots ((M_1'\oplus_{D_1}N_1')\oplus_{D_2}N_2')\cdots)\oplus_{D_k}N_k'$ , and M' is  $(\cdots ((M_1'\oplus_{D_1}N_1')\oplus_{D_2}N_2')\cdots)\oplus_{D_k}N_k'$ . Therefore we have  $M \xrightarrow{p:\overline{c}v} M'$  as required.

Suppose that the transition of  $M \to M'$  is inferred by an application of rule (R-Res), it is similar to the rule (R-Bcast). Moreover, at the last step we use the transition rule (N-Res1), we get  $M \xrightarrow{\tau} M'$  as required.

Suppose that the transition of  $M \to M'$  is inferred by an application of rule (R-Par), it is similar to the rule (R-Bcast).

Suppose that the transition of  $M \to M'$  is inferred by an application of rule (N-Struct), that is

$$\frac{M \equiv N \quad N \to N' \quad N' \equiv M'}{M \to M'}$$

We have that  $N \to N'$  is a shorter inference. The induction hypothesis tells us that there is N'' such that  $N \xrightarrow{\tau} N'' \equiv N'$ .

Therefore we have  $M \xrightarrow{\tau} M'' \equiv M'$  by Lemma 3 and the transitivity of  $\equiv$ . The case for  $N \xrightarrow{p:\overline{c}v} N'' \equiv N'$  is similar.

Now we prove the converse direction. We consider all the possible cases for the last step of the inference  $M \xrightarrow{\tau} M'$  and  $M \xrightarrow{p:\overline{c}v} M'$ .

For  $M \xrightarrow{p:\overline{c}v} M'$ , we have  $M \xrightarrow{p:\overline{c}v} M'$  for some  $p, \overline{c}$  and v. By an application of Lemma 2, we have

$$M \equiv (M_1 \oplus_D M_2) \backslash I$$

and

$$M' \equiv (M_1[p \mapsto P][q_i \mapsto Q_i\{v/x_i\}]_{q_i \in L} \oplus_D M_2) \setminus I$$

where  $\operatorname{eval}(e) = v$ , I with  $c \notin I$ ,  $M(p) = \overline{c}(e).P + R$ ,  $L = \{q_i \mid p \curvearrowright_M q_i, M(q_i) = c(x_i).Q_i + R_i\}$ ,  $|M_1| = L \cup \{p\}$ ,  $|M_2| = |M| \setminus |M_1|$  and  $D \subseteq |M_1| \times |M_2|$  is consistent with  $\curvearrowright_M$ .

We apply rule (R-Bcast) to  $M_1$ , then apply rule (R-Par) to  $M_1[p\mapsto P][q_i\mapsto Q_i\{v/x\}]_{q_i\in L}\oplus_D M_2$  and apply rule (R-Res) to  $(M_1[p\mapsto P][q_i\mapsto Q_i\{v/x\}]_{q_i\in L}\oplus_D M_2)\backslash I$ . Finally, apply rule (R-Struct), we get  $M\to M'$ .

For  $M \xrightarrow{\tau} M'$ , suppose the  $\tau$ -transition is generated by an application of (N-Res1). So we have  $N \xrightarrow{p:\overline{c}v} N'$  for some p,  $\overline{c}$ , v,  $M \equiv N \backslash c$  and  $M' \equiv N' \backslash c$ .  $N \xrightarrow{p:\overline{c}v} N'$  can be proved following the previous case. Then apply rule (R-Struct), we get  $M \to M'$  as require.

The other cases follow from the congruence rules of the reduction semantics.

#### **Next we prove the Soundness Theorem**

**Lemma 7:** Let  $\mathcal{R}$  be a weak bisimulation. If  $(M, E, N) \in \mathcal{R}$  and  $M \xrightarrow{\tau^*} M'$ , then  $N \xrightarrow{\tau^*} N'$  and  $(M', E, N') \in \mathcal{R}$  for some N'.

*Proof:* Induction on the length of the derivation of  $M \xrightarrow{\tau^*} M'$ .

**Lemma 8:** If  $M \xrightarrow{\tau^*} M_1$ ,  $M_1 \stackrel{p:\alpha}{\Longrightarrow} M_1'$  and  $M_1' \xrightarrow{\tau^*} M'$ , then  $M \stackrel{p:\alpha}{\Longrightarrow} M'$ .

**Lemma 9:** A symmetric localized relation  $\mathcal{R} \subseteq \mathbf{Net} \times \mathcal{P}(\mathsf{Loc}^2) \times \mathbf{Net}$  is a weak bisimulation if and only if the following properties hold:

- if  $(M, E, N) \in \mathcal{R}$  and  $M \xrightarrow{p:\alpha} M'$ , then there exists N' such that  $N \xrightarrow{q:\alpha} N'$ ,  $(p,q) \in E$  and  $(M', E, N') \in \mathcal{R}$ ;
- if  $(M, E, N) \in \mathcal{R}$  and  $M \xrightarrow{\tau^*} M'$ , then there exists N' such that  $N \xrightarrow{\tau^*} N'$  and  $(M', E, N') \in \mathcal{R}$ .

*Proof:* ( $\Leftarrow$ ) Because  $\xrightarrow{\tau}$  and  $\xrightarrow{\delta}$  with  $\delta \neq \tau$  are special cases of  $\xrightarrow{\tau^*}$  and  $\xrightarrow{\delta}$  respectively, this direction is obvious.

 $(\Rightarrow) \text{ For the first statement, assume that } (M,E,N) \in \mathcal{R}$  and  $M \stackrel{p:\alpha}{\Longrightarrow} M'$  which is  $M \stackrel{\tau^*}{\longrightarrow} M_1 \stackrel{p:\alpha}{\longrightarrow} M'_1 \stackrel{\tau^*}{\longrightarrow} M'$ , by Lemma 7 we can get  $N \stackrel{\tau^*}{\longrightarrow} N_1$  with  $(M_1,E,N_1) \in \mathcal{R}$ .

From  $M_1 \xrightarrow{p:\alpha} M_1'$  and  $(M_1, E, N_1) \in \mathcal{R}$ , we can get  $N_1 \xrightarrow{q:\alpha} N_1'$  with the conditions that  $(p,q) \in E$  and  $(M_1', E, N_1') \in \mathcal{R}$ .

Since  $M_1' \xrightarrow{\tau^*} M'$  and  $(M_1', E, N_1') \in \mathcal{R}$ , by Lemma 7, we can have  $N_1' \xrightarrow{\tau^*} N'$  with  $(N', E, N') \in \mathcal{R}$ .

With  $N \xrightarrow{\tau^*} N_1$ ,  $N_1 \xrightarrow{q:\alpha} N_1'$  and  $N_1' \xrightarrow{\tau^*} N'$ , by Lemma 8 we can get  $N \Longrightarrow N'$ .

For the second statement, it is straightforward from Lemma

**Lemma 10 (Reflexivity):** Let  $\mathcal{I}$  be the localized relation defined by  $(M, E, N) \in \mathcal{I}$  if M = N and  $E = \mathrm{Id}_{|M|}$ . Then  $\mathcal{I}$  is a weak bisimulation.

*Proof:* Straightforward.

Let  $\mathcal{R}$  and  $\mathcal{S}$  be localized relations. We define a localized relation  $\mathcal{S} \circ \mathcal{R}$  for the composition of  $\mathcal{R}$  and  $\mathcal{S}$ .  $(M,H,O) \in \mathcal{S} \circ \mathcal{R}$  if  $H \subseteq |M| \times |O|$  and there exist N, E and F such that  $(M,E,N) \in \mathcal{R}$ ,  $(N,F,O) \in \mathcal{S}$  and  $F \circ E \subseteq H$ . Let  $F \circ E = \{(p,r) \mid (p,q) \in E, (q,r) \in F\}$ .

**Lemma 11 (Transitivity):** If  $\mathcal{R}$  and  $\mathcal{S}$  are weak bisimulations, then  $\mathcal{S} \circ \mathcal{R}$  is a weak bisimulation.

*Proof:* Obviously,  $S \circ R$  is symmetric. Then the proof just follows the definition of the weak bisimulation using the Lemma 9.

From the hypothesis, let  $(M, E, N) \in \mathcal{R}$ ,  $(N, F, O) \in \mathcal{S}$  and  $(M, H, O) \in \mathcal{S} \circ \mathcal{R}$  with  $F \circ E \subseteq H$ .

- (1) If  $M \stackrel{p:\alpha}{\Longrightarrow} M'$ , then  $N \stackrel{q:\alpha}{\Longrightarrow} N'$ ,  $(p,q) \in E$  and  $(M',E,N') \in \mathcal{R}$ . From  $(N,F,O) \in \mathcal{S}$  and  $N \stackrel{q:\alpha}{\Longrightarrow} N'$ , we have  $O \stackrel{r:\alpha}{\Longrightarrow} O'$ ,  $(q,r) \in F$  and  $(N',F,O') \in \mathcal{S}$ . Therefore, for any pair of labels  $p:\alpha$  and  $q:\alpha$  and pair of labels  $q:\alpha$  and  $r:\alpha$ , we have  $(p,r) \in F \circ E \subseteq H$ . So we have  $(M',F\circ E,O') \in \mathcal{S} \circ \mathcal{R}$ .
- (2) From  $(M, E, N) \in \mathcal{R}$ , if  $M \xrightarrow{\tau^*} M'$ , then we have  $N \xrightarrow{\tau^*} N'$  and  $(M', E, N') \in \mathcal{R}$ . Since  $(N, F, O) \in \mathcal{S}$  and  $N \xrightarrow{\tau^*} N'$ , we have  $O \xrightarrow{\tau^*} O'$  and  $(N', F, O') \in \mathcal{S}$ . We just get  $(M', F \circ E, O') \in \mathcal{S} \circ \mathcal{R}$  as required.

Proof for Lemma 4.

*Proof:* Because  $\approx$  is reflexive by Lemma 10, symmetric from the definition and transitive by Lemma 11.

The proof for Proposition 2.

*Proof:* From the definitions of adapted relations and parallel extension, it is straightforward to show that  $\mathcal{R}'$  is symmetric.

Let  $(U, F, V) \in \mathcal{R}'$  with  $(M, E, N) \in \mathcal{R}$ ,  $U = O \oplus_C M$ ,  $V = O \oplus_D N$ , (C, D, E) is adapted and  $F = \operatorname{Id}_{|O|} \cup E$ .

Case of a sending transition. Given  $U \xrightarrow{\underline{p:\overline{c}v}} U'$  with  $p \in |U|$ , we have to show  $V \xrightarrow{\underline{q:\overline{c}v}} V'$  with  $q \in |V|$ ,  $(p,q) \in F$  and  $(U',F,V') \in \mathcal{R}'$ . There are two cases for the sending transition from  $U = O \oplus_C M$ .

(1) The sending transition occurs in O, and  $O \oplus_C M \xrightarrow{p:\overline{c}v} O' \oplus_C M$ . So, we have  $p \in |O|$  and  $O \xrightarrow{p:\overline{c}v} O'$ . Then we have  $U' = O' \oplus_C M$ .

Similarly, for  $V=O\oplus_D N$  we have  $V\xrightarrow{p:\overline{c}v}V'=O'\oplus_D N$ . By the definition of F, we have  $(p,p)\in \mathrm{Id}_{|O|}$ , i.e.  $(p,p)\in F$ . Then it is obvious that the triple (C,D,E) is adapted. Therefore, we have  $(U',F,V')\in \mathcal{R}'$  and  $F=\mathrm{Id}_{|O|}\cup E$ , as required.

(2) The sending transition occurs in M and  $O \oplus_C M \xrightarrow{p:\overline{c}v} O \oplus_C M'$ . So we have  $p \in |M|$  and  $M \xrightarrow{p:\overline{c}v} M'$ , i.e.  $U' = O \oplus_C M'$ .

Since  $(M,E,N)\in\mathcal{R}$ , from  $M\xrightarrow{p:\overline{c}v}M'$ , we have  $N\xrightarrow{q:\overline{c}v}N'$ ,  $(p,q)\in E$  and  $(M',E,N')\in\mathcal{R}$ . We can decompose  $N\xrightarrow{q:\overline{c}v}N'$  as

$$N \xrightarrow{\tau^*} N_1 \xrightarrow{q:\overline{c}v} N_1' \xrightarrow{\tau^*} N'$$

We have  $V \xrightarrow{\tau^*} V_1$  with  $V_1 = O \oplus_D N_1$ . Similarly,  $V \xrightarrow{q:\overline{c}v} V'$  can be decomposed as

$$O \oplus_D N \xrightarrow{\tau^*} O \oplus_D N_1 \xrightarrow{q:\overline{c}v} O \oplus_D N_1' \xrightarrow{\tau^*} O \oplus_D N'$$

We have  $V'=O\oplus_D N'$ . Since  $F\subseteq |U|\times |V|$  and  $F=\mathrm{Id}_{|O|}\cup E$ , we have  $(p,q)\in F$ . Moreover, the triple (C,D,E) is adapted. Therefore, we have  $(U',F,V')\in \mathcal{R}'$  and  $F=\mathrm{Id}_{|O|}\cup E$ , as required.

Case of a receiving transition. Given  $U \xrightarrow{p:cv} U'$  with  $p \in |U|$ , we have to show  $V \xrightarrow{q:cv} V'$  with  $q \in |V|$ ,  $(p,q) \in F$  and  $(U',F,V') \in \mathcal{R}'$ . There are two cases for the receiving transition from  $U = O \oplus_C M$ . The analysis is similar to the case of a sending transition.

Case of a  $\tau$ -transition. Given  $U \xrightarrow{\tau} U'$ , we have to show  $V \xrightarrow{\tau^*} V'$  and  $(U', F, V') \in \mathcal{R}'$ . There are two cases for a  $\tau$ -transition from  $U = O \oplus_C M$ . Because  $\tau$ -transitions are obtained by restriction rules from outputs, i.e. there exist  $U \equiv U_1 \setminus I$  and  $U_1 \xrightarrow{p:\overline{c}v} U'_1$  with  $p \in |U| = |U_1|$ . Then with an application of rule (N-Res1), we get  $U \xrightarrow{\tau} U'$  and  $U' \equiv U'_1 \setminus I$ . Then we have to analysis the cases for  $U_1 \xrightarrow{p:\overline{c}v} U'_1$  with  $p \in |U| = |U_1|$ , and this is similar to the case of a sending transition.

The proof for Theorem 2.

*Proof:*  $\approx$  is an equivalence by Lemma 4. Here, we just need to prove that if M and N are two networks, and  $M \approx N$ , (i.e.  $(M, E, N) \in \mathcal{R}$  for some weak bisimulation  $\mathcal{R}$ ), then

- (1)  $\mathcal{R}$ 's parallel extension is a weak bisimulation,
- (2)  $(M \setminus c, E, N \setminus c)$  is contained in some weak bisimulation for any channel c.

For the proof of (1), we directly apply the Proposition 2. For the proof of (2), it is sufficient to show that the localized relation

$$\mathcal{S} \stackrel{\mathrm{def}}{=} \{ ((M \backslash c, E, N \backslash c)) \mid (M, E, N) \in \mathcal{R} \text{ for any channel } c \}$$

is a weak bisimulation. It is obvious that  $\mathcal S$  is symmetric. Then we do a case analysis on the possible transition from  $M \setminus c$ . The proof is straightforward.

Proof for Proposition 4.

*Proof:* Let  $\mathcal{B}$  be a binary relation on networks defined by:  $(M,N) \in \mathcal{B}$  if  $M \approx N$ . Then we have to prove that  $\mathcal{B}$  is a weak bared bisimulation. First, we know that  $\mathcal{B}$  is symmetric, because  $\approx$  is symmetric. Then we need to prove  $\mathcal{B}$  is reduction closed and barb preserving.

(1) Let  $(M,N) \in \mathcal{B}$ . If  $M \to M'$  which is  $M \xrightarrow{p:\overline{c}v} M'' \equiv M'$  by Theorem 1. Because  $M \approx N$ , we have  $N \xrightarrow{q:\overline{c}v} N'$  (i.e.  $N \to^* N'$  by by Theorem 1) and  $M'' \approx N'$ . We also have  $M'' \approx M'$  by Proposition 3. Since  $\approx$  is an equivalence, we have  $M' \approx N'$ . Thus, we have  $(M',N') \in \mathcal{B}$ .

(2) Let  $(M,N) \in \mathcal{B}$ . If  $M \downarrow_{\overline{c}}$ , then there exists a transition  $M \xrightarrow{p:\overline{c}v} M'$ . Since  $M \approx N$ , we have  $N \xrightarrow{q:\overline{c}v} N'$  and  $M' \approx N'$ .  $N \xrightarrow{q:\overline{c}v} N'$  means  $N \xrightarrow{\tau^*} N_1$  (i.e.  $N \to^* N_1$  by Theorem 1) with  $N_1 \downarrow_{\overline{c}}$  for some  $N_1$ . From  $M \downarrow_{\overline{c}}$ , we get that  $N \to^* N_1$  with  $N_1 \downarrow_{\overline{c}}$  as required.

# C. Proofs in Section V

The proof for Proposition 5.

*Proof:* Since O does not affect the reductions of M, we only consider the reductions of M, i.e. interactions between  $P_1$  and  $P_2$ . There are two cases for the reduction

- either  $M \to^* M[p_1 \mapsto P_1(lt_1,b)][p_2 \mapsto P_2(lt_2,b)]$  with the concatenation of  $lt_1$  and  $lt_2$  equals to lt, and this is a some stage of the reduction,
- or  $M \to^* M[p_1 \mapsto \mathbf{0}][p_2 \mapsto Succ(lt)]$ , and this is the final successful stage of the reduction.

Induction on the length of  $lt_1$ , we only show some cases and other cases are similar:

- If  $\text{null}(lt_1)$  is satisfied, then a possible reduction sequence is: sending the End message, receiving acknowledge from  $P_2$ , passing the conditional evaluation in the sender and in the receiver respectively, then reducing to the final successful stage.
- If  $\operatorname{null}(lt_1)$  is not satisfied, then a possible reduction sequence is: sending the head of  $lt_1$ , passing the conditional evaluation of the receiver, receiving acknowledge from the receiver, then reducing to the next stage of the reduction.