

Modal Dispersion Effects on Coherent Signal Processing Parameters

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Abstract—Acoustic signals propagated in deep ocean ducts are geometrically (or modally) dispersed. The effects of the dispersion on coherent signal processing is examined in this paper. Using a model, which for long range-deep ocean propagation has been shown to adequately predict dispersion measures, it is determined that the effects of dispersion on the bandwidth of the signal are small. However, in coherently correlating spectrally separated but related signals, the effect on the allowable integration time is significant. Curves relating the allowable integration time as functions of frequency and range rate between the source and receiver are presented.

I. INTRODUCTION

ACOUSTIC signals, when propagated in ducts or refractive media, are geometrically or modally dispersed. For signals propagated in the deep ocean the effects of dispersion are small and for many applications can be ignored. However, if one wishes to coherently process these signals, then dispersion may have a significant effect. This is especially true in coherently processing broadband signal transmissions and signal transmissions received on spatially separated sensors. Assuming that broadband signals can be decomposed into a band of discrete spectral components, the effects of dispersion can be studied by measuring the effects on spectral pairs of components. In this paper we use a standard ocean acoustic propagation model [1] to estimate dispersion induced degradations on coherent processing of spectrally separated signals. This model has previously been used to study the effects of dispersion on CW signals [2], [3] and has been shown experimentally to adequately predict dispersion in long-range deep-ocean acoustic propagation [4].

II. COHERENT PROCESSING

To determine the effects of dispersion, we will assume that a distant source transmits two sinusoidal signals that are spectrally separated but have a fixed frequency relationship ($f_2 = qf_1$, $q > 1$). Since we are only interested in dispersion, the received signals are assumed noise free and of the form

$$R_1(t) = A_1(t) \exp [i(\omega_1 t + \phi_1(t))], \text{ and}$$

$$R_2(t) = A_2(t) \exp [i(\omega_2 t + \phi_2(t))].$$

We assume that $\omega_j = 2\pi(1 + \alpha)f_j$ for $j = 1, 2$ where α is the Doppler ratio due to source-receiver motion. Hence $\omega_2 = q\omega_1$, $q > 1$ also holds for the received signals. The quantity of

interest is the correlation of these two signals [2]

$$C(T) = \frac{\left| \int dt R_1(t|q) R_2^*(t) \right|}{\left[\int dt |R_1|^2 \int dt |R_2|^2 \right]^{1/2}} \quad (1)$$

where $t|q$ means that the phase of R_1 has been multiplied by q and T is the integration interval.

Two parameters of interest when calculating the correlation are the bandwidths of the received signals and the integration time T . In the following we will obtain estimates of the effects of dispersion on each parameter and the resulting degradation of the correlation function.

III. MODEL

All model calculations used in this paper are fully reported in [2] and [3]. They use the range independent sound speed profile and geometry depicted in Fig. 1. The NRL normal mode model [1] was used where the surface was assumed flat and perfectly reflecting and the bottom was assumed flat and perfectly absorbing. Fig. 2 (Fig. 2 of Ref. 1) is the normal mode evaluation of the acoustic field over the range 1000–1100 km for the four frequencies 10, 20, 40 and 80 Hz. An estimate of the linear phase variation with range has been removed in order to clearly show the nonlinear phase effects caused by modal dispersion. The model calculations have assumed the source to be at a set of fixed ranges (i.e., no motion). The received signal is then assumed to be equivalent to moving a receiver through the calculated field. Calculations made using modified wave numbers [5] corrected to first order in the ratio of source speed to mode group velocity resulted in negligible changes in the variations of the acoustic field for reasonable source speeds.

IV. ANALYSIS

The instantaneous frequency of the received signals is of the form

$$\omega' = \omega + \dot{\phi}(t) = \omega + \dot{R} \frac{d\phi}{dR} = \omega \left[1 + \frac{\dot{R}}{\omega} \frac{d\phi}{dR} \right].$$

From Fig. 2 we estimate the maximum of $(1/\omega)d\phi/dR$ (excluding “instantaneous” jumps of π) to be on the order of 1×10^{-5} s/m. Hence, $(\delta f)_{\max} \approx f\dot{R} \times 10^{-5}$ which results in a small dispersion bandwidth for realistic values of f and \dot{R} . For example, in long-range deep-ocean propagation, maximum

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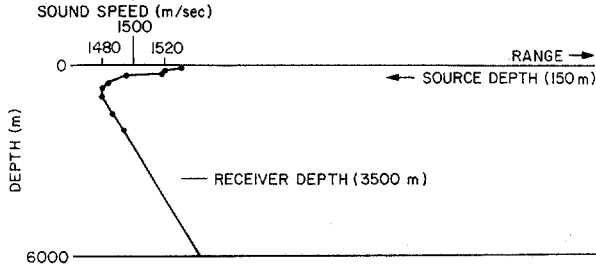


Fig. 1. The geometry and depth dependent sound speed profile used in the acoustic field calculations.

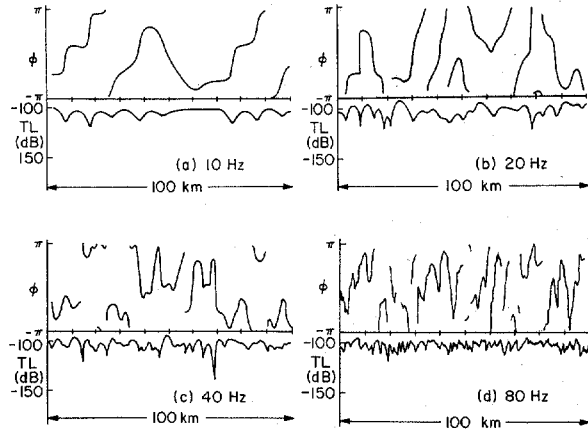


Fig. 2. Calculations of the acoustic field for frequencies of (a) 10, (b) 20, (c) 40, and (d) 80 Hz. The phase of the field with a linear portion removed is illustrated in the top half of each graph. The amplitude of the field (expressed as transmission loss TL in dB) is illustrated in the lower half of each graph. The abscissa is range in km from 1000 km to 1100 km.

frequencies are on the order of 100 Hz and expected maximum range rates are of the order of 15 m/s (≈ 30 knots), resulting in a maximum bandwidth of 0.03 Hz.

The instantaneous observed frequency ratio q' can be written

$$\begin{aligned} q' &= \frac{\omega_2 + \dot{\phi}_2}{\omega_1 + \dot{\phi}_1} = \frac{\omega_2}{\omega_1} \frac{[1 + (\dot{R}/\omega_2)d\phi_2/dR]}{[1 + (\dot{R}/\omega_1)d\phi_1/dR]} \\ &\approx q[1 + (\dot{R}/\omega_2)d\phi_2/dR][1 - (\dot{R}/\omega_1)d\phi_1/dR] \\ &\approx q[1 + (\dot{R}/\omega_2)(d/dR)(\phi_2 - q\phi_1)] \end{aligned}$$

where we have used the observed smallness of $(\dot{R}/\omega)d\phi/dR$. Hence

$$\epsilon = q' - q = q(\dot{R}/\omega_2) \frac{d}{dR} (\phi_2 - q\phi_1), \text{ or}$$

$$\frac{\epsilon}{q\dot{R}} = \frac{1}{\omega_2} \frac{d}{dR} (\phi_2 - q\phi_1). \quad (2)$$

The latter expression is a measure of the dispersion per unit range rate.

In [2] the distribution of this latter quantity (with \dot{R} in knots) over the range of 50–500 nmi for 15 values of q ($= 1.33$ – 8.00) was determined. The frequencies used in the calculation were 10, 15, 20, 30, 40, and 80 Hz. From the resulting histograms along with a calculation of the variance of each, it was judged

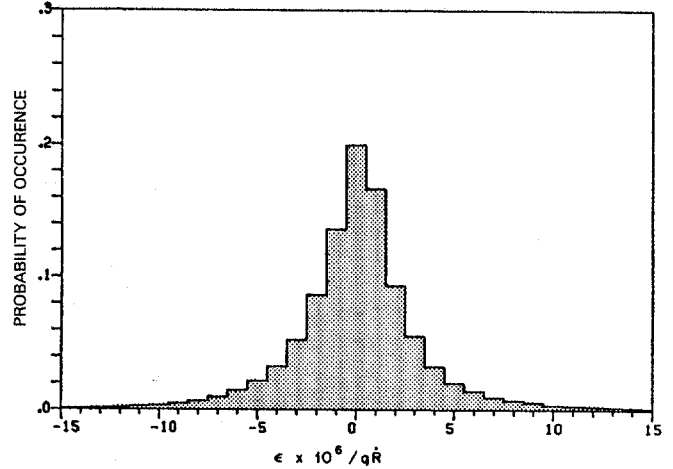


Fig. 3. Histogram depicting the probability density of the spectral dispersion measure ϵ over the range 50–500 nmi. The histogram is an average of 15 histograms developed from the 15 possible ordered frequency pairs selected from (10, 15, 20; 30, 40, and 80 Hz). The range rate \dot{R} is in knots.

that the difference between them was statistically insignificant. In Fig. 3 (Fig. 42 of Ref. 3) is depicted the resulting average histogram of $\epsilon/q\dot{R}$ which is assumed to be independent of q and the signal frequencies used. From (2)

$$\epsilon/q\dot{R} = \frac{1}{\omega_2} \frac{d}{dR} (\phi_2 - q\phi_1) = \frac{1}{\omega_2 \dot{R}} \frac{d}{dt} (\phi_2 - q\phi_1);$$

hence $(d/dt)(\phi_2 - q\phi_1)$ is similarly distributed.

If we assume that over the integration time T the amplitudes of the received signals are constant, then (1) reduces to

$$C(T) = \frac{1}{T} \left| \int dt \exp [i(\phi_2 - q\phi_1)] \right|.$$

If we further assume that $\phi_2 - q\phi_1 = a + bt$ (see discussion below) over the integration time T , then

$$C(T) = \left| \frac{\sin bT/2}{bT/2} \right|,$$

where $b = (d/dt)(\phi_2 - q\phi_1)$. Thus

$$\begin{aligned} E\{C\} &= \sum_n \left| \frac{\sin b_n T/2}{b_n T/2} \right| P(b = b_n) \\ &= \sum_n \left| \frac{\sin n\gamma}{n\gamma} \right| h(n) \end{aligned}$$

where $\gamma = \pi f_2 \dot{R} T \times 10^{-6}$ and $h(n)$ are the histogram values for $-15 \leq n \leq 15$.

This equation can be numerically solved for the product $f_2 \dot{R} T$ for specified values of $E\{C\}$. In Fig. 4 we plot $E\{C\}$ versus $f_2 T$ (Hz·s) for several different range rates \dot{R} (knots). From these curves one can determine the integration time T (s) for an acceptable level of “dispersion degradation” of $E\{C\}$ as a function of the upper frequency f_2 and range rate \dot{R} . In Fig. 5 the integration time T (min) is plotted versus $f_2 \dot{R}$ (Hz·knots) for the $E\{C\} = .9, .7, \text{ and } .5$. From this plot it

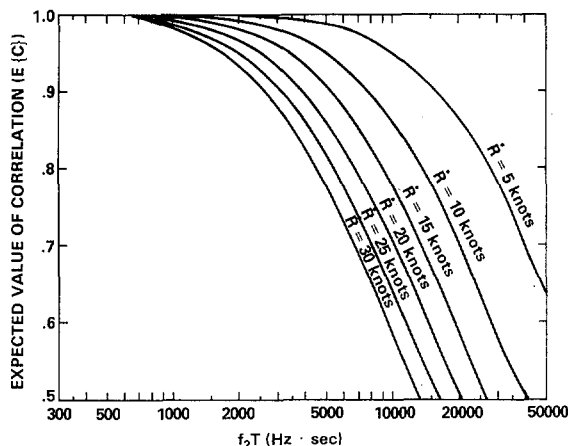


Fig. 4. The expected value of the correlation ($E\{C\}$) is plotted versus f_2T with range rate \dot{R} (knots) as a parameter. The frequency f_2 is the higher of the received pair and T is the integration time in seconds. From this set of curves one can determine the degradation in correlation due to dispersion as a function of the integration time T (assuming \dot{R} and f_2 known). The general applicability of these curves is limited to $\dot{R}T \leq 1$ nmi (see discussion in text).

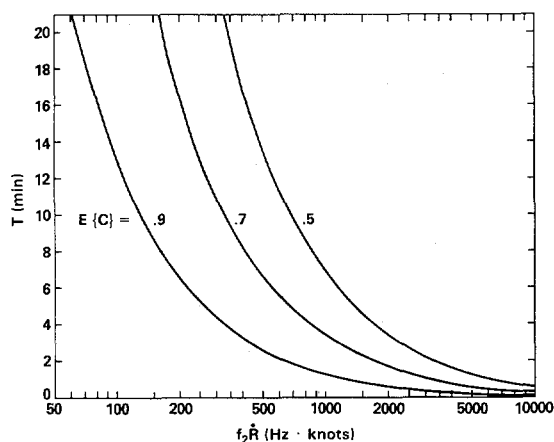


Fig. 5. The integration time T in minutes versus $f_2\dot{R}$ for expected values of the correlation ($E\{C\}$). The higher signal frequency F_2 is in Hz and the range rate \dot{R} between the source and receiver is in knots. To determine the maximum integration time T for a given situation, calculate the expected range of $f_2\dot{R}$ and the amount of degradation, due to dispersion, acceptable in the correlation of the two signals. One then selects the smallest T corresponding to these conditions. The general applicability of these curves is limited to $\dot{R}T \leq 1$ nmi (see discussion in text).

can be seen that for a given set of parameters (\dot{R} and f_2) the effects of dispersion on $E\{C\}$ can be quite significant if the integration time T is not chosen properly.

Both Figs. 4 and 5 were developed on the assumption that the phase $\phi_2 - q\phi_1$ was a linear function of time over the integration interval. An examination of plots of $\phi_2 - q\phi_1$ versus range in [3] reveals that, with the exception of isolated points, the linearity assumption is almost always good for range changes of 1 nmi or less. Hence we restrict the

applicability of the curves in Figs. 4 and 5 to conditions of $\dot{R}T \leq 1$ nmi.

V. SUMMARY AND CONCLUSIONS

We have evaluated the effects of modal dispersion on acoustic CW signals propagated in the deep ocean to long ranges. The model used for this evaluation has been experimentally validated. It was found that the effect of dispersion on the bandwidth of the signal was small, but in the coherent processing of two signals, the degradation of the correlation function is highly significant if one uses an improperly long integration interval. This interval can be less than a minute for some signal conditions. The calculations herein considered a source that transmitted two CW signals with a fixed frequency ratio and reception at a single receiver. However, the effects of dispersion on correlations between signals received on spatially separated receivers (interarray processing) will be similar. These effects will be investigated in future work.

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