An analytical model for surface EMG generation in volume conductors with smooth conductivity variations

An analytical model for surface EMG generation in volume conductors with smooth conductivity variations

Luca Mesin[1], Dario Farina[2,*]

Abstract— **A non-space invariant model of volume conductor for surface EMG signal generation is analytically investigated. The volume conductor comprises planar layers representing the muscle and subcutaneous tissues. The muscle tissue is homogeneous and anisotropic while the subcutaneous layer is inhomogeneous and isotropic. The inhomogeneity is modeled as a smooth variation in conductivity along the muscle fiber direction. This may reflect a practical situation of tissues with different conductivity properties in different locations or of transitions between tissues with different properties. The problem is studied with the regular perturbation theory, through a series expansion of the electric potential. This leads to a set of Poisson's problems, for which the source term in an equation and the boundary conditions are determined by the solution of the previous equations. This set of problems can be solved iteratively. The solution is obtained in the two-dimensional Fourier domain, with spatial angular frequencies corresponding to the longitudinal and perpendicular direction with respect to the muscle fibers, in planes parallel to the detection surface. The series expansion is truncated for the practical implementation. Representative simulations are presented. The proposed model constitutes a new approach for surface EMG signal simulation with applications related to the validation of methods for information extraction from this signal.**

Index Terms— **EMG modeling, volume conductor, space-invariance**

I. Introduction

MANY models of volume conductors for the simulation of surface EMG signals have been proposed in the literature [11][20]. These models considered a number of geometries and conductivity tensors. To obtain an analytical solution, the volume conductor should be relatively simple with respect to the actual anatomy. Thus, description of the volume conductor with homogeneous layers have been proposed [1][3][4]. A few attempts to analytically describe tissue inhomogeneities have also been described [14][15][19]. In a recent work [15], we focused on the perturbation effect of small spherical inhomogeneities in the volume conductor on the simulated surface EMG signal. An approximate technique was used to account for the inhomogeneities, which had spherical shape. The approximations introduced imposed limitations on the application of the model, requiring a certain distance between inhomogeneities, which thus can not simulate a distributed change in conductivity. This model was used to evaluate the effect of local inhomogeneities on estimates of conduction velocity from surface EMG [8] and allowed the interpretation of the relatively large variability in conduction velocity estimates when surface EMG signals are detected in different locations over the muscle in experimental tests [7]. The effect of small inhomogeneities on conduction velocity estimates was substantial and dependent on the spatial filter used and inter-channel distance. In another study, we analyzed a model of muscle with two main pinnation angles [14]. In this case, the inhomogeneity is due to the different conductivity tensor in the muscle tissue associated to the two main pinnation directions. Inhomogeneous volume conductors can also result from different fiber curvatures depending on fiber position and fiber shortening [16].

The presence of inhomogeneities along the propagation direction of the intracellular action potential determines changes in the shape of the recorded surface EMG potentials [5]. Thus, surface EMG signals recorded along the fiber direction are not delayed versions of the same prototype shape. The volume conductor is non-space-invariant and the forward problem should be solved for each position of the source. Non-space-invariant volume conductors allow the simulation of important features of surface EMG signals recorded in practice. Thus, they can be used for the validation of methods for information extraction from the surface EMG [5]. Although numerical solutions [5][10][16] may have the advantage of imposing less constraints to the shape of the volume conductor, analytical methods are still desirable for limiting the computational time and providing direct relations between the model parameters and the features of the simulated signals.

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In this study we approached the problem of analytically determining the surface potential distribution in a layered volume conductor in which one layer has conductivity slowly variable along fiber direction. This study and the model of volume conductor with local inhomogeneities previously described [15] provide the means for investigating the effect of both local and distributed tissue inhomogeneities on the surface EMG. The applications of the two models are complementary (sharp vs. slow variations in conductivity).

II. METHODS

2.1 Mathematical problem

The electrical field problems in physiology can be studied, within good approximation, as quasi-static [9]. Thus, from the bio-electrical point of view, the tissues can be described as a volume conductor. In these conditions, the electrical potential solves the Poisson equation:

$$-\nabla\cdot(\underline{\underline{\sigma}}\nabla\phi)=I \tag{1}$$

where ϕ is the potential (V), I the current density source (A/m^3), and $\underline{\underline{\sigma}}$ the conductivity tensor (S/m).

The ideal model considered in this study consists of two planar layers [3] (Figure 1). The muscle is homogeneous and anisotropic, infinite in the *x* and *z* directions, semi-infinite (infinite in the negative direction) in the *y* direction. The fat layer is bounded in the *y* direction, infinite in the *x* and *z* directions. In [3] the fat layer tissue was assumed homogeneous and isotropic. In this study the fat layer is inhomogeneous in the direction of propagation of the intracellular action potentials (*z* direction). The inhomogeneity is introduced by a smooth variation of the conductivity along *z*. Although it will not be treated in this study, the generalization of the problem to the case of a smooth perturbation of conductivity in both spatial directions is straightforward.

The conductivity of the fat layer has the form $\sigma_F=\sigma(1+\varepsilon \mathrm{P}(z))$, where ε (with $|\varepsilon|<1$) is the amplitude of the perturbation term $\mathrm{P}(z)$, which is a function of the direction of propagation *z*, with $\mathrm{P}(z)\le 1, \forall z.$

The Poisson equation in the fat layer is written as:

$$\nabla\cdot\left[\sigma(1+\varepsilon \mathrm{P}(z))\nabla\phi\right]=\sigma(1+\varepsilon \mathrm{P}(z))\left(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}\right)+\varepsilon\sigma \mathrm{P}'(z)\frac{\partial\phi}{\partial z}=0 \tag{2}$$

In the muscle layer, the following equation holds:

$$\nabla\cdot\left[\underline{\underline{\sigma}}\nabla\phi\right]=\sigma_t\left(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}\right)+\sigma_l\frac{\partial^2\phi}{\partial z^2}=-\delta(x-x_0)\delta(y-y_0)\delta(z-z_0) \tag{3}$$

where σ_l, σ_t are the longitudinal and transversal conductivity of the muscle tissue, and the source is an impulsive current. The conditions at the interface between fat and muscle layer are the continuity of the potential and of the current density. To solve the problem, two further conditions are considered, i.e., the Neumann's condition at the surface (air is considered an insulator) and the vanishing of the potential at infinity.

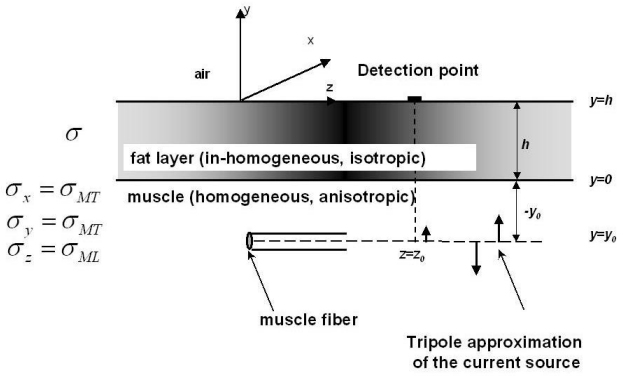

Fig. 1 Planar layered volume conductor model with the notations used in the text. The fat layer has variable conductivity in the direction z of action potential propagation. The source is modeled as a current tripole (impulse amplitudes: I1 = 24.6 A/m2, I2 = -35.4 A/m2, I3 = 10.8 A/m2; distances between poles: a = 2.1 mm, b = 6.9 mm).

2.2 Solution by regular perturbation

The mathematical problem presented above can be solved by the regular perturbation theory [2][21]. The potential is expanded as a power series in the parameter ε, introducing the unknown functions ϕ_n:

$$\phi=\sum_{n=0}^{+\infty}\varepsilon^n\phi_n \tag{4}$$

Since Eq. (2) contains both $\mathrm{P}(z)$ and its derivative $\mathrm{P}'(z)$, convergence can be achieved only for sufficiently smooth functions $\mathrm{P}(z)$ (the issue of convergence will be discussed below). For this reason, local inhomogeneities, as studied in [15], cannot be investigated with the perturbation approach here proposed. Substituting the expression (4) in Eq. (2), the following iterative system of equations is obtained:

$$\begin{cases}\Delta\phi_0=0\\ \Delta\phi_n+\mathrm{P}(z)\Delta\phi_{n-1}+\mathrm{P}'(z)\dfrac{\partial\phi_{n-1}}{\partial z}=\Delta\phi_n+\displaystyle\sum_{j=1}^{n}\left(-\mathrm{P}(z)\right)^{j-1}\mathrm{P}'(z)\frac{\partial\phi_{n-j}}{\partial z}=0\end{cases} \tag{5}$$

Assuming the solution for the muscle tissue in the same form as in Eq. (4) [2]

$$\phi^M=\sum_{n=0}^{+\infty}\varepsilon^n\phi_n^M \tag{6}$$

yields to the following system of equations:

$$\begin{cases}-\sigma_t\dfrac{\partial^2\phi_0^M}{\partial x^2}-\sigma_t\dfrac{\partial^2\phi_0^M}{\partial y^2}-\sigma_l\dfrac{\partial^2\phi_0^M}{\partial z^2}=\delta(x-x_0)\delta(y-y_0)\delta(z-z_0)\\ -\sigma_t\dfrac{\partial^2\phi_n^M}{\partial x^2}-\sigma_t\dfrac{\partial^2\phi_n^M}{\partial y^2}-\sigma_l\dfrac{\partial^2\phi_n^M}{\partial z^2}=0\end{cases} \tag{7}$$

Imposing the continuity of the potential and of the current density at the muscle-fat interface, for each power of ε [2], we obtain

$$\begin{cases} \phi_0 = \phi_0{}^M & \sigma \dfrac{d\phi_0}{dy} = \sigma_t \dfrac{d\phi_0{}^M}{dy} \\ \phi_i = \phi_i{}^M & \sigma \dfrac{d\phi_i}{dy} + \sigma \mathrm{P} \dfrac{d\phi_{i-1}}{dy} = \sigma_t \dfrac{d\phi_i{}^M}{dy} \end{cases}, \tag{8}$$

The two boundary conditions, the Neumann's condition at the surface, and the vanishing of the potential at infinity, are expressed as:

$$\begin{cases} \phi_0{}^M (y \to -\infty) \to 0 & \left.\dfrac{d\phi_0}{dy}\right|_{y=h} = 0 \\ \phi_i{}^M (y \to -\infty) \to 0 & \left.\dfrac{d\phi_i}{dy}\right|_{y=h} = 0 \end{cases} \tag{9}$$

where h is the fat layer thickness, being $y = 0$ the fat-muscle interface; from Eqs. (9), we observe that the conditions on the successive functions ϕ_i do not depend iteratively on the previous ones [contrary to what happens for the interface conditions in Eqs. (8)].

The error in truncating the series expansions in Eqs. (4) and (6) can be expressed in terms of ε. A sufficient condition to achieve convergence is that the norm of the functions ϕ_n does not increase with the order n.

Except for the first, Eqs. (5) are Poisson's equations with the source term which depends on the approximating functions determined by the previous equations. The first equation is a Laplace equation which can be solved analytically. The entire system is then solved iteratively [2][21]. As only a few equations can be solved in practice [which corresponds to a truncation of the series expansion in Eq. (4)], the method yields an approximate solution [2][21]. The solution in the fat layer for the term ϕ_0 is provided in [15]. The other problems associated to Eqs. (5) and (7), with conditions (8) and (9), can be studied in the Fourier domain. By two-dimensional Fourier transformation of the potential in the x and z coordinates, we obtain a system of second order ordinary differential equations in y with parameters k_x and k_z (the spatial angular frequencies) in the fat and muscle domains:

$$\begin{cases} \dfrac{d^2\hat{\phi}_n}{dy^2} - k^2 \hat{\phi}_n = -\sum_{j=1}^{n} \Im\left[(-\mathrm{P})^{j-1} \mathrm{P}'\right] * jk_z \hat{\phi}_{n-j} \\ \dfrac{d^2\hat{\phi}_n{}^M}{dy^2} = k_y{}^2 \hat{\phi}_n{}^M \end{cases} \tag{10}$$

where * indicates the convolution with respect to k_z, $k^2 = k_x{}^2 + k_z{}^2$, and $k_y{}^2 = k_x{}^2 + r_a\, k_z{}^2$, with r_a the ratio between the longitudinal and transversal conductivity of the muscle tissue. The solution of the equation for the muscle tissue is an exponential function which decreases for negative values of y, multiplied by an arbitrary function of k_x and k_z. The solution of the equation in the fat layer is obtained (by linearity) by the summation of the solution of the homogeneous equation with inhomogeneous interface condition (due to the term containing ϕ_{i-1}) and the particular solution of the complete equation, satisfying homogeneous conditions.

The homogeneous solution can be written as:

$$\begin{cases} \hat{\varphi}_i^{hom} = A^i{}_1 e^{ky} + B^i{}_1 e^{-ky} & 0 < y < h \\ \hat{\varphi}_i^{M-hom} = A^i{}_0 e^{k_y y} & y < 0 \end{cases} \tag{11}$$

where $\hat{\varphi}_i^{hom}$ and $\hat{\varphi}_i^{M-hom}$ are the Fourier transforms of the solutions of the homogeneous problem satisfying the in-homogeneous interface condition (due to the term containing ϕ_{i-1}) in the fat and muscle layers, respectively.

Imposing the boundary conditions, we obtain:

$$\begin{cases} kA^i{}_1 e^{kh} - kB^i{}_1 e^{-kh} = 0 \\ A^i{}_1 + B^i{}_1 = A^i{}_0 \\ \sigma k (A^i{}_1 - B^i{}_1) + \sigma \hat{\mathrm{P}} * \left.\dfrac{d\hat{\phi}_{i-1}}{dy}\right|_{y=0} = \sigma_t A^i{}_0 k_y \end{cases} \tag{12}$$

Solving the linear system in Eq. (12), we obtain:

$$\begin{cases} A^i{}_0 = -\dfrac{\sigma \hat{\mathrm{P}} * \left.\dfrac{d\hat{\phi}_{i-1}}{dy}\right|_{y=0} \left(1 + e^{2hk}\right)}{k\sigma\left(1 - e^{2hk}\right) - k_y \sigma_t (1 + e^{2hk})} \\ A^i{}_1 = -\dfrac{\sigma \hat{\mathrm{P}} * \left.\dfrac{d\hat{\phi}_{i-1}}{dy}\right|_{y=0}}{k\sigma\left(1 - e^{2hk}\right) - k_y \sigma_t (e^{2hk} + 1)} \\ B^i{}_1 = -\dfrac{\sigma \hat{\mathrm{P}} * \left.\dfrac{d\hat{\phi}_{i-1}}{dy}\right|_{y=0} e^{2hk}}{k\sigma\left(1 - e^{2hk}\right) - k_y \sigma_t (e^{2hk} + 1)} \end{cases} \tag{13}$$

The particular solution can be expressed as the convolution of the right hand side of Eq. (10) and the Green function G_i, which is solution of the equation:

$$\frac{d^2 G_i}{dy^2} - k^2 G_i = \delta(y - y_0) \tag{14}$$

where $0 < y_0 < h$. The solution can be obtained writing the general solutions in three regions as follows:

$$\begin{cases} G_i = A^i{}_4 e^{ky} + B^i{}_4 e^{-ky} & y_0 < y < h \\ G_i = A^i{}_3 e^{ky} + B^i{}_3 e^{-ky} & 0 < y < y_0 \\ \hat{\phi}_n{}^M = A^i{}_2 e^{k_y y} & y < 0 \end{cases} \tag{15}$$

where G_i is the Green function satisfying Eq. (14) and $\hat{\varphi}_n^M$ is the solution of the Laplace equation in the muscle layer.

The conditions are:

$$\begin{cases} kA^i{}_4 e^{kh} - kB^i{}_4 e^{-kh} = 0 \\ A^i{}_4 e^{ky_0} + B^i{}_4 e^{-ky_0} = A^i{}_3 e^{ky_0} + B^i{}_3 e^{-ky_0} \\ kA^i{}_4 e^{ky_0} - kB^i{}_4 e^{-ky_0} = kA^i{}_3 e^{ky_0} - kB^i{}_3 e^{-ky_0} + 1 \\ A^i{}_3 + B^i{}_3 = A^i{}_2 \\ \sigma k(A^i{}_3 - B^i{}_3) = \sigma_t A^i{}_2 k_y \end{cases} \quad (16)$$

Solving the linear system of Eqs. (16), the following solution is obtained:

$$\begin{cases} A^i{}_2 = \dfrac{\sigma e^{-ky_0}\left(e^{2ky_0} + e^{2hk}\right)}{k\sigma\left(1 - e^{2hk}\right) - k_y\sigma_t(1 + e^{2hk})} \\ A^i{}_3 = \dfrac{e^{-ky_0}(k\sigma + k_y\sigma_t)\left(e^{2ky_0} + e^{2hk}\right)}{2k(k\sigma\left(1 - e^{2hk}\right) - k_y\sigma_t(e^{2hk} + 1))} \\ B^i{}_3 = \dfrac{e^{-ky_0}(k\sigma - k_y\sigma_t)\left(e^{2ky_0} + e^{2hk}\right)}{2k(k\sigma\left(1 - e^{2hk}\right) - k_y\sigma_t(e^{2hk} + 1))} \\ A^i{}_4 = \dfrac{e^{-ky_0}\left(k\sigma(e^{2ky_0} + 1) + k_y\sigma_t\left(e^{2ky_0} - 1\right)\right)}{2k(k\sigma\left(1 - e^{2hk}\right) - k_y\sigma_t(e^{2hk} + 1))} \\ B^i{}_4 = -\dfrac{e^{2hk-ky_0}\left(k\sigma(e^{2y_0k} + 1) + k_y\sigma_t\left(e^{2ky_0} - 1\right)\right)}{2k(k\sigma\left(1 - e^{2hk}\right) - k_y\sigma_t(e^{2hk} + 1))} \end{cases} \quad (17)$$

When $\hat{\varphi}_i^{hom}$ and $G(y,y_0)$ are computed, the surface potential can be obtained from Eq.(4) as a series of the following terms:

$$\hat{\varphi}_i(y=0) = \hat{\varphi}_i^{hom}(y=0) + \int_0^h G(y=0,y_0)F[\hat{\varphi}_{i-1}(y_0),...,\hat{\varphi}_0(y_0)]dy_0 \quad (18)$$

where $F[u]$ indicates the functional on the right hand side of Eq.(9), which is a function of all the determined perturbation functions $\hat{\varphi}_j(y_0)$, $j=0,1,...,i-1$.

2.3 Numerical implementation

The current density source in the muscle was approximated with a current tripole [13] with parameters described in Figure 1. The numerical implementation requires the truncation of the perturbation series in Eqs. (4) and (6). We truncated the series to the first three terms. The six equations to be solved in this case, for the fat and muscle layer, are the following:

$$\begin{cases} \Delta\phi_0 = 0 \\ \Delta\phi_1 + \mathrm{P}(z)\Delta\phi_0 + \mathrm{P}'(z)\dfrac{\partial\phi_0}{\partial z} = \Delta\phi_1 + \mathrm{P}'(z)\dfrac{\partial\phi_0}{\partial z} = 0 \\ \Delta\phi_2 + \mathrm{P}(z)\Delta\phi_1 + \mathrm{P}'(z)\dfrac{\partial\phi_1}{\partial z} = \Delta\phi_2 + \mathrm{P}'(z)\dfrac{\partial\phi_1}{\partial z} - \mathrm{P}(z)\mathrm{P}'(z)\dfrac{\partial\phi_0}{\partial z} = 0 \end{cases} ; \quad (19)$$

$$\begin{cases} -\sigma_t\dfrac{\partial^2\phi_0^M}{\partial x^2} - \sigma_t\dfrac{\partial^2\phi_0^M}{\partial y^2} - \sigma_l\dfrac{\partial^2\phi_0^M}{\partial z^2} = \delta(x-x_0)\delta(y-y_0)\delta(z-z_0) \\ -\sigma_t\dfrac{\partial^2\phi_1^M}{\partial x^2} - \sigma_t\dfrac{\partial^2\phi_1^M}{\partial y^2} - \sigma_l\dfrac{\partial^2\phi_1^M}{\partial z^2} = 0 \\ -\sigma_t\dfrac{\partial^2\phi_2^M}{\partial x^2} - \sigma_t\dfrac{\partial^2\phi_2^M}{\partial y^2} - \sigma_l\dfrac{\partial^2\phi_2^M}{\partial z^2} = 0 \end{cases} . \quad (20)$$

In the spatial frequency domain, Eqs. (19) can be written as:

$$\begin{cases} \dfrac{d^2\hat{\phi}_0}{dy^2} - k^2\hat{\phi}_0 = 0 \\ \dfrac{d^2\hat{\phi}_1}{dy^2} - k^2\hat{\phi}_1 = \Im[\mathrm{P}'] * jk_z\hat{\phi}_0 \\ \dfrac{d^2\hat{\phi}_2}{dy^2} - k^2\hat{\phi}_2 = \Im[\mathrm{P}'] * jk_z\hat{\phi}_1 - \Im[\mathrm{PP}'] * jk_z\hat{\phi}_0 \end{cases} \quad (21)$$

and Eqs. (20) as:

$$\begin{cases} \dfrac{d^2\hat{\phi}_0^M}{dy^2} = k_y{}^2\hat{\phi}_0^M - \delta(y-y_0) \\ \dfrac{d^2\hat{\phi}_1^M}{dy^2} = k_y{}^2\hat{\phi}_1^M \\ \dfrac{d^2\hat{\phi}_2^M}{dy^2} = k_y{}^2\hat{\phi}_2^M \end{cases} \quad (22)$$

Boundary and interface conditions are those provided in Eqs. (8) and (9), with the index i taking values 1 and 2.

The solution $\hat{\phi}(k_x, y, k_z)$ of the previous perturbation problem is obtained analytically, in the Fourier domain for x and z, and in the spatial domain for y. The solution in the spatial domain for the three coordinates is obtained by numerical inverse Fourier transform. The solution $\hat{\phi}(k_x, y, k_z)$ is determined for a discrete set of values of the variables kx and kz. Moreover, the spatial frequencies are bounded to a value corresponding in time domain to 1024 Hz (avoiding aliasing), assuming a conduction velocity of 4 m/s.

III. Results

The concepts previously described have been implemented in Matlab version 6.5 (The Mathworks, Natick, MA). The zero order solution was obtained from [15], sampling the variable y at steps of 0.2 mm. The first order perturbation solution was calculated for equispaced y values (as it enters the second order perturbation equation). The second order perturbation solution was calculated only at the surface of the volume conductor (where the surface EMG signal is recorded).

Figure 2 reports simulated single fiber action potentials (considering the propagation of a tripole current source, neglecting generation and end-of-fiber effects) recorded from the volume conductor described in Figure 1, with two selections of the conductivity of the fat layer. The perturbation function $\mathrm{P}(z)$ was Gaussian in these simulations. The variable conductivity along the muscle fiber direction introduced shape changes in the recorded action potentials. The approximated solution for variable conductivity is consistent with the exact solutions for constant conductivities. Indeed, the amplitude of the monopolar potential detected at the point of maximum conductivity (i.e., at $z = 0$ in Figure 2) is approximately the same as that obtained in the case of a model with homogeneous subcutaneous layer with $\sigma_F = \max_z \sigma(1 + \varepsilon\mathrm{P}(z))$. This property was checked for

many other values of ε and perturbation functions $P(z)$ (results not shown). In a set of simulations (not shown), it was observed that with ε smaller than 0.7 and standard deviation of $P(z)$ larger than 5, a second order perturbation represented the solution with an error smaller than 2 % with respect to higher order perturbations.

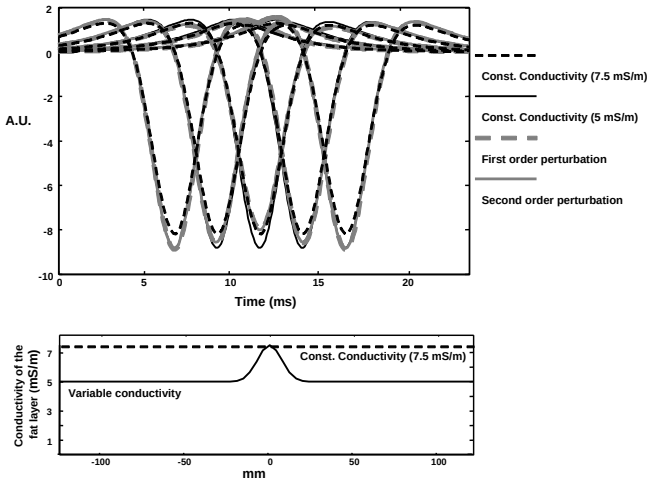

Fig. 2 Monopolar detection by 5 electrodes (inter-electrode distance 10 mm). Conduction velocity 4 m/s; fat layer thickness 4 mm; fiber depth 4 mm; fat conductivities 0.075 S/m (dashed line) and $0.05\left(1+0.5e^{-\frac{z^2}{100}}\right)$ S/m (solid line); muscle longitudinal conductivity 0.5 S/m, transversal conductivity 0.1 S/m. Potentials relative to perturbation free (black, dashed line), zero order (black solid line), first order (grey, dashed line), and second order (grey, solid line) perturbation are shown. A.U.: Arbitrary Units.

Figure 3 shows the surface potential distribution generated by a single muscle fiber and recorded on the surface of the volume conductor with a monopolar system. The second order perturbation term is also shown for different locations of the current tripole along the fiber direction. The spatial filter used for signal detection, fiber depth and thickness of the fat layer affect the perturbation term due to the inhomogeneity, as shown in Figures 4 and 5.

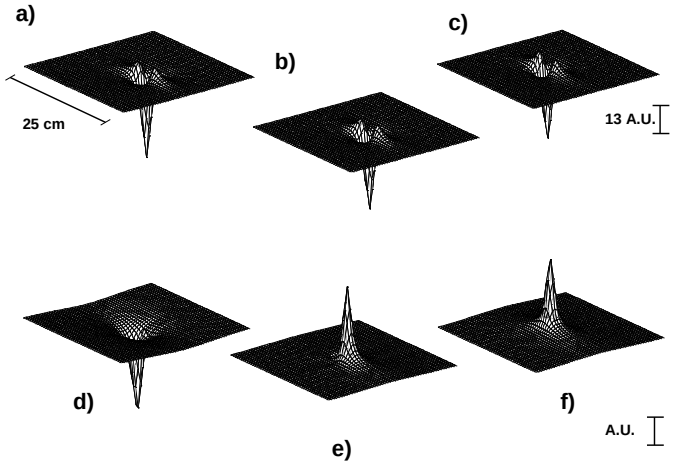

Fig. 3 Monopolar surface potentials (a, b, c) and perturbation term (second order approximation) (d, e, f) for different locations of the propagating source (tripole source as in Figure 1). The central pole of the tripole is at a distance of 16 mm (a, d), 8 mm (b, e), and 0 mm (c, f) from the point in which the conductivity is maximum. Fat layer thickness 4 mm; fiber depth 4 mm; fat conductivity $0.05\left(1+0.5e^{-\frac{z^2}{100}}\right)$ S/m; muscle longitudinal conductivity 0.5 S/m, transversal conductivity 0.1 S/m. A.U.: Arbitrary Units.

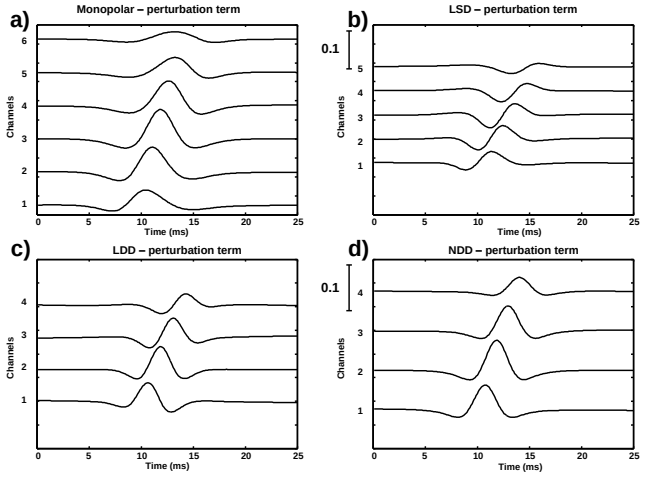

Fig. 4 Perturbation term (second order approximation) in case of monopolar, single differential (LSD), double differential (LDD), and Laplacian (NDD) recording. Interelectrode distance 5 mm; fat layer thickness 4 mm; fiber depth 4 mm; fat conductivity $0.05\left(1+0.5e^{-\frac{z^2}{100}}\right)$ S/m; muscle longitudinal conductivity 0.5 S/m, transversal conductivity 0.1 S/m. The amplitudes of the perturbation signals are normalized with respect to the range of the perturbation free signal.

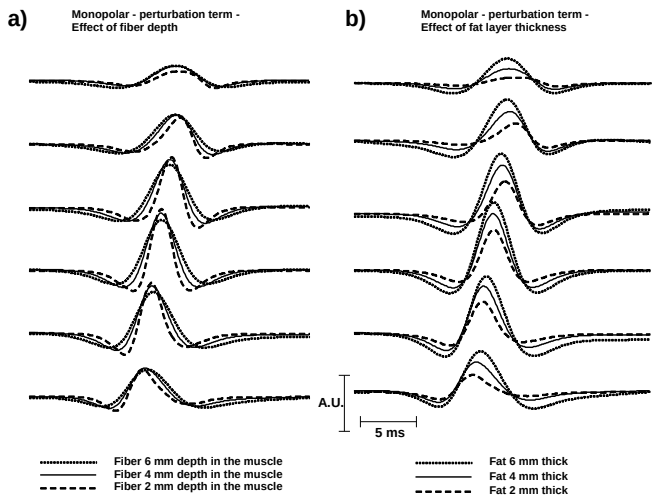

Fig. 5 Effect of a) the muscle fiber depth and b) the fat layer thickness on the perturbation term (monopolar detection; second order approximation). A.U.: Arbitrary Units.

Since the shape of the action potentials is different depending on the location of the recording electrodes, estimates of muscle fiber conduction velocity are influenced by the variable conductivity. Figure 6 shows estimates of conduction velocity from single fiber action potentials simulated with the proposed model by varying the parameter of the generation system (the list of parameters for the simulation conditions is shown in Table 1). Results from monopolar and single differential detection systems are shown. The simulated fibers were infinite in length, thus there were no end-plate or end-of-fiber components and the bias in the estimates is due exclusively to the variation in conductivity.

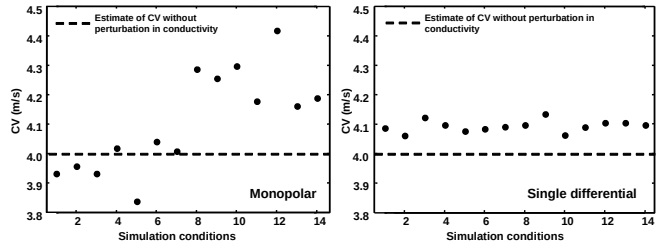

Fig. 6 Estimates of conduction velocity (CV) from two simulated surface EMG signals using the spectral matching approach [12]. The simulation conditions are described in Table 1. The function describing the perturbation on conductivity of the fat layer is $0.05\left(1+\varepsilon e^{-\frac{z^2}{\nu^2}}\right)$, with the values ε and ν reported in Table 1. Single muscle fiber action potentials were simulated. The fibers were infinite in length (no end-plate or end-of-fiber components) in order to show only the effect of the perturbation in conductivity. Signals were detected with two a) monopolar and b) single differential systems. The center of the detection systems corresponded to the maximum of the Gaussian perturbation of conductivity. Inter-electrode and inter-channel distance 5 mm, detection systems aligned along the fiber propagation path. The simulated conduction velocity value was in all cases 4 m/s. The solid line indicates the estimate of conduction velocity without any perturbation in the conductivity of the fat layer. In this case there is no bias since the shape of the action potentials is unchanged.

Table 1 Parameters of the model for the 14 simulation conditions reported in Figure 6.

Anatomy (identification number)	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Fiber Depth (within the muscle) - mm	4	**2**	**6**	4	4	4	4
Fat Thickness - mm	4	4	4	**2**	**6**	4	4
Perturbation Variance	10	10	10	10	10	**20**	10
Perturbation Parameter	0.5	0.5	0.5	0.5	0.5	0.5	**0.25**

Anatomy (identification number)	**8**	**9**	**10**	**11**	**12**	**13**	**14**
Fiber Depth (within the muscle) - mm	4	**2**	**6**	4	4	4	4
Fat Thickness - mm	4	4	4	**2**	**6**	4	4
Perturbation Variance	10	10	10	10	10	**20**	10
Perturbation Parameter	-0.5	-0.5	-0.5	-0.5	-0.5	-0.5	**-0.25**

IV. Discussion and Conclusions

Modeling surface EMG signals provides the means for assessing the limitations of methods for information extraction from the surface EMG [6] and for solving the inverse problem [20]. Many EMG models are currently available but most analytical models assume volume conductors homogeneous in the muscle fiber direction. In this case, the volume conductor is space-invariant in the direction of source propagation, which implies unchanged shape of the recorded surface action potentials along the muscle fiber (excluding end-plate and end-of-fiber components). In this study we propose an analytical method to approximate the solution of the Poisson equation in presence of conductivity slowly varying along the fiber direction. This provides a new tool for simulating surface EMG signals. The effect of a variable conductivity on the action potential shape has been shown in representative simulations (Figures 2-5). It was previously shown that changes in action potential shape due to inhomogeneities affects the estimation of muscle fiber conduction velocity from surface EMG recordings [8]. This has also been confirmed in the present study (Figure 6). Moreover, a non-space-invariant volume conductor allows the analysis of spatial filters for EMG signal detection. The theoretical transfer function of spatial filters is based on the assumption of absence of shape changes during propagation (space-invariant model). With this assumption, the propagation of the potential along the fiber direction and the weighted summation of signals recorded at different detection surfaces is equivalent to a spatial convolution [17][18]. However, in case of inhomogeneous volume conductors, the effect of spatial filters can be significantly different with respect to the ideal condition.

The proposed model of volume conductor assumes a slowly varying conductivity. This may reflect a practical situation of tissues with different conductivity properties in different locations due to glands, small vessels, scars or other structures. Moreover, this model can be used for describing transitions between tissues of different properties. Finally, a change in conductivity along the fiber direction may be considered as an approximation of a change in subcutaneous layer thickness along the muscle, which is relevant in practical condition.

In conclusion, a new model for the generation of surface EMG signals has been proposed. The model describes a layered volume conductor, with one layer presenting variable conductivity along the direction of source propagation. The mathematical problem of the determination of the surface potential distribution has been addressed by the regular perturbation theory, which has never been applied before to surface EMG simulation. The model finds applications in testing algorithms for information extraction from the surface EMG signal.

GRANTS

This work was partially supported by Contract C15097/01/NL/SH of the European Space Agency (ESA) on Microgravity effects on skeletal muscles investigated by surface EMG and mechanomyogram.

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