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A Novel Quantitative Measure of Breast Curvature Based on Catenary

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Abstract

Quantitative, objective measurements of breast curvature computed from clinical photographs could be used to investigate factors that impact reconstruction and facilitate surgical planning. This paper introduces a novel quantitative measure of breast curvature based on catenary. A catenary curve is used to approximate the overall curvature of the breast contour, and the curvature measure is extracted from the catenary curve. The catenary curve was verified by comparing its length, the area enclosed by the curve, and the curvature measure from the catenary curve to those from manual tracings of the breast contour. The evaluation of the proposed analysis employed untreated and postoperative clinical photographs of women who were undergoing tissue expander/implant (TE/Implant) reconstruction. Logistic regression models were developed to distinguish between the curvature of breasts undergoing TE/Implant reconstruction and that of untreated breasts based on the curvature measure and patient variables (age and body mass index). The

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relationships between the curvature measures of untreated breasts and patient variables were also investigated. The catenary curve approximates breast curvature reliably. The curvature measure contains useful information for quantifying the curvature differences between breasts undergoing TE/Implant reconstruction and untreated breasts, and identifying the effect of patient variables on the breast shape.

Index Terms

Breast cancer; breast curvature; breast reconstruction; catenary; digital photographs

I. Introduction

Breast cancer is the most common type of cancer for women in the United States, with approximately 200 000 new cases of invasive breast cancer diagnosed each year [1]. Fortunately, due to early detection techniques and improved cancer treatments, the death rate caused by breast cancer has decreased since 1990 [1]–[3].

Current breast cancer treatment encompasses not only surgical removal of the tumor and medical adjuvant and neo-adjuvant therapies to control the cancer, but also an increasing emphasis on restoration of the breast shape to improve the patient's quality of life. In this respect, the goal of breast reconstruction is to recreate a breast form that is satisfying to the patient, facilitating her psychosocial adjustment to living as a breast cancer survivor.

Recreating natural-appearing breast curvature is an important aspect of breast reconstruction. This process can be very challenging and currently depends largely on the individual surgeon's qualitative, subjective assessment of breast morphology. Quantitative, objective measurements of breast curvature computed from clinical photographs could potentially be used to investigate factors that impact reconstruction and facilitate surgical planning.

Many research groups have proposed methods for quantifying breast morphology to evaluate the aesthetic outcomes of breast cancer treatments [4]–[6]. However, those methods are not sufficient for quantifying the curvature of the breast since they focus on other properties such as ptosis and symmetry of the nipple–areola locations.

Only a few previous studies have investigated methods for quantifying characteristics related to breast curvature. Cardoso and Cardoso [7] developed a breast conservation therapy outcome estimate system by using features extracted from clinical photographs of the patient taken in four different postures. The extracted features of their study include the following asymmetry measures: the difference between the levels of the inferior left and right breast contours, the difference between the lengths of the left and right breast contours, and the nonoverlapping area of the two breasts to capture the asymmetry of the breast contours. In subsequent studies [8]–[10] from the same research group, they introduced an automated algorithm for tracing the breast contour by applying Dijkstra's shortest path algorithm on the gradient image with constraints on the path. They showed the efficacy of their algorithms on 190 patients who underwent breast conservation therapy. Van Limbergen *et al.* [11] used nipple displacement and breast contour retraction as quantitative measurements and correlated those with the subjective, qualitative scoring. The breast contour retraction measure was based on the vertical and horizontal length differences between the breasts. However, all measures developed in these previous studies quantify asymmetry; they do not provide a quantitative description of the curvature of the breast. Moreover, it is important for the measure to help investigators understand the role of physical properties of the breast such as its volume, size, and tissue stiffness, in determining the shape of the breast contour. However, the models used in previous studies [8]–[10] are not able to explain physical properties of the breast contour since they are based on low-level image features, such as the gradient of the image.

In this study, we introduce a novel quantitative measure of the overall curvature of the breast contour that is derived from a physical model of the breast contour. It should be noted that the purpose of the proposed measure is to quantify the overall shape of the breast contour, rather than tracing local contour variations. Our new measure is based on catenary theory [12]. Catenary is the theoretical shape of a flexible chain suspended by two fixed points and it can be used to approximate any string-like object. First, we used a catenary curve to approximate the overall curvature of the breast contour. After that, we extracted the shape parameter, which is our curvature measure of the breast, from the resulting catenary curve. Catenary theory has been adapted to approximate interesting curves in other medical applications, especially in orthodontics [13], [14]. However, our study is the first to apply catenary theory to approximate the overall shape of the breast contour and to quantify the curvature of the breast by using a key parameter of the catenary curve.

In a preliminary study [15], we introduced a semiautomatic method to fit a catenary curve using three manually annotated fiducial points along a breast contour with the limited size of data; however, the approach was sensitive to error in the manual annotation. In this paper, we present a new robust method for quantifying the curvature of the breast with larger dataset. Moreover, to validate our quantification method, we evaluated the catenary curve fitted on the breast contour by comparing clinically meaningful measures extracted from the catenary curve to those from the manual tracing of the breast contour on clinical photographs. The measures include the length of the curve, the enclosed breast area by the contour and a straight line passing through the two end points of the same contour, and the breast curvature measure.

To show that the shape parameter of the catenary is able to meaningfully quantify the breast curvature, we compared the curvature measurements of subjects who were undergoing tissue expander/implant (TE/Implant) reconstruction, especially the ones in the tissue expander (TE) phase, to those of untreated women. The analysis was motivated by previous clinical studies indicating possible shape difference between breasts of TE/Implant reconstructions (either completed or in the process of), especially for unilateral reconstruction cases, and untreated breasts [16]–[19]. However, none of these prior studies reported the shape difference quantitatively. These clinical studies suggest that TE/Implant reconstructed breasts have lower curvature than untreated breasts. We limited our comparison to breasts in the TE phase of TE/Implant reconstruction to untreated breasts since breasts in the TE phase are expected to have clearly lower curvature than untreated breasts. This is because of the high tension introduced by the TE on the overlying breast skin and soft tissue, necessary for creating a loose soft tissue envelope and the potential for appropriate ptosis when the TE is exchanged for a permanent implant [20]. To evaluate this hypothesis, we developed logistic regression models to quantify the differences between the curvature of untreated breasts and that of breasts in the TE phase during TE/Implant reconstruction in terms of the extracted curvature measure, patient age, and patient body mass index (BMI, which is a measure of body fat based on the ratio of weight in kilograms to height in meters). The consideration of these patient variables is key since breast morphology is widely believed to depend upon them [21]–[24]. In addition, we compared the relationship between the curvature measures of untreated breasts and corresponding patient variables as a complementary analysis, to demonstrate the effectiveness of the proposed measure. Our hypotheses are: 1) the breasts of younger patients will have lower breast curvatures than the breasts of older patients; and 2) the breasts of patients with lower BMI will have lower breast curvatures than the breasts of patients with higher BMI. This analysis was motivated by previous clinical studies indicating that age and BMI are factors that impact measures of breast ptosis [21], [24]. As a ptotic breast is one in which the nipple is lower than the inframammary fold, the curvature of a ptotic breast is expected to be high.

II. Materials and Methods

A. Dataset

The study population for this paper consists of women aged 21 or older who underwent or were scheduled for breast reconstruction surgery from January 1, 2004 to June 31, 2011 at The University of Texas MD Anderson Cancer Center. A Nikon Coolpix 8400 (Nikon, NY) was used to obtain anterior posterior (AP) images of 79 patients (136 breasts). Of the 136 breasts, 96 breasts were either healthy or untreated breasts and 40 breasts were in the TE phase of TE/Implant reconstruction. Of 40 TE/implant reconstruction breasts, 8 were undergoing inflation of the TE by 50%–100% of the manufacturers volume of the TE, 15 were undergoing over inflation by 4%–30%, 10 by 35%–70%, 4 by 75%–95%, and 3 by 110%–135%.

B. Breast Contour From the Manual Tracing

To evaluate the proposed method of quantifying the breast curvature using catenary, a manual tracing of the breast contour on the clinical photograph was obtained as the "gold standard" of the breast contour. One nonclinical observer (J.L.) traced the breast contour using a stylus and a tablet computer for 12 randomly selected patients (total 20 breasts). As described in Section II-E1, the length, area enclosed by the curve, and the curvature measure (see Section II-C) from the approximated breast contour were compared to those of the manually traced contour.

C. Quantifying Breast Curvature Using Catenary

To quantify the breast curvature, catenary theory was adopted from differential geometry. A catenary is a perfectly flexible and inextensible string of uniform density supported by two distinct points [12]. Catenary curves have been used to model many objects shaped like hanging strings, such as arches and suspension bridges [25].

The parametric equation of a catenary curve in the image coordinate system, where its origin is located in the upper left with x-axis extending to the right and the y-axis extending downward, has the form

$$\begin{bmatrix} x_s \\ y_s \end{bmatrix} = \begin{bmatrix} \alpha \cdot s + b \\ -\alpha \cosh(s) + c \end{bmatrix} \quad (1)$$

where b and c are the offset of the x-axis and y-axis, respectively, and s is a free parameter for the catenary curve, where its range governs the evolvement of the curve. Moreover, α is the ratio of the tension to the weight applied to each point on the curve. Fig. 1(a) shows how α determines the shape of a catenary, and Fig. 1(b) shows how the range of s affects the evolvement of the catenary curve.

The parameter α is the key factor in determining the shape of a catenary curve. A larger α decreases the curvature of the curve whereas a smaller α increases the curvature of the curve. As previously stated, catenary theory can be easily extended to approximate the contour of a patient's breast. A large α will represent a breast with low curvature and a small α will denote a breast with high curvature. In this respect, we used the parameter α as our curvature measure of the breast in this study.

However, due to the weight of breasts and the shape of a person's chest wall, breast contours are at an angle to the imaginary horizontal line. In order to capture this angle, a rotation parameter θ is introduced. Thus, the final equation for the proposed model for the breast curvature is

$$\begin{bmatrix} \widehat{x_s} \\ \widehat{y_s} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x_s \\ y_s \end{bmatrix}. \tag{2}$$

Fig. 1(c) shows how α and θ affect the appearance of the proposed catenary curve when it approximates the contour of a right breast.

D. Obtaining Breast Curvature Measure

As stated in the previous section, the catenary curve parameter α is the key parameter in determining the shape of the curve, i.e., the curvature of the breast. In order to obtain the parameter α from a given photograph, we need to solve the following equation:

$$\begin{bmatrix} x \\ y \end{bmatrix} = F(s, P), P = [\alpha, \theta, b, c] \tag{3}$$

where F denotes the catenary curve on a breast contour, i.e., (2), and x and y represent the x- and y-coordinates of the breast contour itself.

From multiple points (x- and y-coordinates) along the breast contour, we can solve (3) in the least-squares sense. These points are the M point, where M is the closest point on the inframammary fold to the midline of the torso, the anterior axillary point A, which is the start of the crease between the axilla and the inner arm, and several equally spaced points (L_{multi}, more than 10) between M and the anterior axillary point, which are illustrated in Fig. 2(a). It should be noted that there are existing algorithms for automatically tracing the breast contour [8]–[10]. Such a method could be used as a preprocessing step to identify the required points for the proposed measurement algorithm. However, we did not employ one of the previously published methods for automatically tracing the breast contour because they have been validated only for patients who underwent breast conservation therapy and not for patients who underwent breast reconstruction. Although the existing algorithm may work for identifying the contour of untreated breast at least, it should be validated for reconstructed breast, in order to be used as a preprocessing step. Moreover, as the purpose of this paper is to introduce a new measure for quantifying the overall curvature of the breast contour, how to provide the required points for the proposed measure is not a primary focus of this study. In addition, in our previous study [4], we showed that human observer, both clinical experts and novice observers, could annotate points on the breast contour with low inter- and intraobserver variability. For these reasons, we used manually annotated points for the study.

We used the trust-region-reflective algorithm [26], [27] to solve (3) considering it as a nonlinear data-fitting problem. We first interpolated the aforementioned reference points to provide the algorithm with a denser data sampling to enable a better approximation of the overall shape of the breast contour [see Fig. 2(b)]. Then, we selected the points on the middle one fifth to four fifths of the breast contour, and passed those points to the algorithm since they capture the portion of the breast contour with high curvature [see Fig. 2(c)]. The range of s used in the algorithm must account for the fact that the length of the breast contour varies across women. From our dataset, we found that the contours of extremely small breasts can typically be fit by $s = [-1\ 1]$. The contours of extremely large breasts can typically be fit by $s = [-2.5\ 2.5]$. Therefore, we selected [−1 1] and [−2.5 2.5] as the lower and upper bounds of the range of s, respectively. Then, the bisection algorithm was used to refine the range of s based on the error between the breast contour and the approximated one. In summary, the algorithm solves the following problem in the least-squares sense

$$\min_{P_j} \sum_j \left(F(s_{i,j}, P_j) - \begin{bmatrix} x_i \\ y_i \end{bmatrix} \right)^2 \quad (4)$$

where x_i and y_i are the coordinates of the ith point on the selected breast contour, and $s_{i,j}$ is the catenary point in jth sets of the range of s, corresponding to the location of $[x_i, y_i]^T$. The algorithm selects P_j that minimizes (4) as a solution P.

Once the solution is found, we can obtain the breast curvature measure a of a patient in the given clinical photograph [see Fig. 2(d)]. However, as (4) is an overdetermined problem, the solution cannot guarantee that the resulting catenary curve passes through all of the reference points; moreover, either one of the two end points (either M or A) or both of them are usually missed. To address this problem, we introduce new points to replace any missed end points; the new point, M_{new} and/or A_{new}, is defined as the point where the resulting curve from the solution P and the imaginary line between two end points (M and A) intersect [see Fig. 3].

In order to compare the a values of different individuals, they must be unitless quantities. Therefore, we normalized a by the length of the curve. The normalized version of measure a is $a' = a/l$, where l denotes the length of the approximated breast contour from the one end point (either M_{new} or M) to the other end point (either A_{new} or A).

E. Evaluation Methods

1) Evaluating the Approximated Breast Contour—Our catenary model is derived from a physical interpretation of the approximated breast contour. A detailed tracing of the exact breast contour is not necessary for our analysis. Nevertheless, the value of the proposed measure will be limited if our model is not able to *approximate* the breast contour with high reliability. To address this, we evaluated the proposed method for quantifying the breast curvature by comparing the length, the area enclosed by the curve, and the curvature measure a from the approximated breast contour with those from the breast contour manually annotated as described in Section II-B.

For the area comparison, we define an enclosed region that is surrounded by the breast contour demarcated in the photograph and the straight line passing through the two end points of the contour [see Fig. 2(d)]. In order to compute a for the manually traced breast contour, we construct the catenary curve with the same length and end points as the manually traced breast contour. The parameter a for the manually annotated breast contour is then calculated as follows. The length of the curve from (x_1, y_1) to (x_2, y_2) can be computed as

$$l = \int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx. \quad (5)$$

Assuming, without loss of generality, that $\theta = 0$, we can formulate the length of the catenary curve by substituting (1) in (5) as follows:

$$l=\left[\alpha\sinh\left(\frac{x-b}{\alpha}\right)\right]_{x_1}^{x_2}=\alpha\left(\sinh\left(\frac{x_2-b}{\alpha}\right)-\sinh\left(\frac{x_1-b}{\alpha}\right)\right). \quad (6)$$

If we set one end point as (x_1, y_1), and the other as (x_2, y_2), the vertical displacement between two end points of the catenary is given by

$$y_2-y_1=2\alpha\sinh\left(\frac{(x_1+x_2)/2-b}{\alpha}\right)\sinh\left(\frac{(x_1-x_2)/2}{\alpha}\right). \quad (7)$$

Since we assume $\theta = 0$, the vertical height of the two end points of the catenary curve to be the same, and therefore, we can derive the following relationship:

$$\frac{x_1+x_2}{2}=b. \quad (8)$$

By substituting (8) into (6), we can compute the length of the catenary curve with the ends points of the same vertical height as follows:

$$l=2\alpha\sinh\left(\frac{x_2-x_1}{2\alpha}\right)=2\alpha\sinh\left(\frac{\text{dist}((x_2,y_2),(x_1,y_1))}{2\alpha}\right) \quad (9)$$

where dist(p, q) denotes the Euclidean distance between the points p and q. Equation (9) shows that we can obtain α for the catenary curve, if we know its curve length and the distance between two end points of the curve. The aforementioned equation can be easily extended to the case with $\theta \neq 0$ by applying a 2-D rotation matrix to the original x- and y-coordinates of the catenary curve, where θ can be estimated from the angle between the x-axis and the straight line passing through the two end points of the curve [see Figs. 1(c) and 2(d)]. Therefore, from (9), we can compute the breast curvature measure α from the given length of the annotated breast contour and the distance between two end points.

The intraclass correlation coefficient (ICC) was used to compare the measurements obtained from the approximated contour to those from the manually traced contour. The ICC is a measure of the reproducibility of replicate measures from the same subject [28]. A two-way random effect model was used to evaluate the agreement between the length, area, and curvature measure of the breast contour as approximated by our catenary method and those of the contour as manually traced. The interpretation of the ICC used for this study is as follows: ICC <0.4 indicates poor reproducibility, $0.4 \leq$ ICC <0.75 indicates fair to good reproducibility, and ICC ≥ 0.75 indicates excellent reproducibility [29]. Analyses were performed using the MATLAB v.7.11.0 (R2010b) (The Mathworks, Natick, MA) statistics toolbox v.7.4 (R2010b).

2) Effectiveness of the Breast Curvature Measure—Our hypothesis is that there is a difference in the breast curvature between breasts in the TE phase of a TE/Implant reconstruction and untreated breasts, and the proposed normalized breast curvature measure α' can capture the difference.

However, as described earlier, the breast morphology is widely believed to depend on patient variables such as age and BMI [21]–[23]. Therefore, we developed logistic regression models to investigate the use of α', age, and BMI for quantifying differences in breast morphology among women with different surgical histories and, therefore, to show that α' captures useful information about the breast curvature. The logistic regression models relate the feature vector V to the breast reconstruction outcome O, where they are defined as follows:

$$V_{i,1}=[1,\alpha',\mathrm{age}_i,\mathrm{BMI}_i]^T,\ i=1,2,\ldots,136,$$
$$V_{i,2}=[1,\alpha']^T,\ i=1,2,\ldots,136,$$
$$V_{i,3}=[1,\mathrm{age}_i,\mathrm{BMI}_i]^T,\ i=1,2,\ldots,136,$$
$$O_i=\begin{cases}1, & \text{TE reconstructed}\\ 0, & \text{untreated}\end{cases},\ i=1,2,\ldots,136, \qquad (10)$$

where subscript i denotes the parameters associated with the ith breast in the dataset. If we denote the logistic regression model resulting from the feature vector $V_{i,j}$ as the jth model, we have three models as follows:

$$\ln\left(\frac{p_i}{1-p_i}\right)=\beta_{0,j}\cdot V_{i,j},\quad i=1,2,\ldots,136,\ j=1,2,3 \qquad (11)$$

where $\beta_{0,j}=[\beta_0,\beta_1,\ldots,\beta_j]$ and p_i represents the conditional probability of patients having a breast in the TE phase of a TE/Implant reconstruction based on the given feature vector $V_{i,j}$, i.e., $p_i = \mathrm{P}(O_i \mid V_{i,j})$.

Leave-one-out cross-validation was utilized to evaluate the efficacy of the logistic regression models. We used the area under the receiver operating characteristic curve (AUC) as the efficacy metric of the logistic regression model. We used the nonparametric receiver operating characteristic (ROC) curves with $N = 1000$ bootstrap replicates to compute the AUC and their 95% confidence interval (CI) values for the three logistic regression models. The AUC values of three logistic regression models were compared to show the effectiveness of the full model (α', age, and BMI) over the partial models (α' only and patient variables only).

As a second complementary analysis to demonstrate the utility of the proposed measure α', we divided the untreated breasts into two groups as follows: 1) patients whose age is less than 50 and 2) patients whose age is over 50. We selected the threshold of 50 years since it is the average age at menopause, which brings with it many changes to the woman's body [30]. We conducted a single-tailed two-sample equal variance Student's t-test to compare the average α' of the younger age group and that of the older age group. Similarly, we divided the same untreated breasts using BMI as follows: 1) patients whose BMI is less than 25 and 2) whose BMI is over 25. We selected the BMI of 25 as a threshold since it is the indicator of overweight [31]. We likewise conducted a t-test to compare the α' of the low BMI group and that of the high BMI group. Our hypotheses for these tests are: 1) women with low age have breasts with low curvature and 2) women with low BMI have breasts with low curvature. All analyses were performed using the MATLAB v.7.11.0 (R2010b) (The Mathworks, Natick, MA) statistics toolbox v.7.4 (R2010b).

III. Results

A. Evaluating the Approximated Breast Contour

Table I summarizes the length, the area enclosed by the curve, and the curvature measure α of the approximated breast contour and those of the manually traced breast contour for the dataset described in Section II-B. Fig. 4 depicts the approximated breast contours and the manually traced breast contours for four patients as examples. The ICC values for all measures were above 0.75, which indicates excellent agreement between the parameters extracted from the manually traced breast contour and from the approximated breast contour identified by the proposed method.

In summary, the results of the statistical analyses support the fact that the curve length, the area enclosed by the curve, and the breast curvature measure obtained from the approximated breast contour agree well with those from the manually traced breast contour. These findings are encouraging since they suggest that the approximated breast contour using catenary preserves the clinically meaningful features of the actual breast contour.

B. Effectiveness of the Breast Curvature Measure

The median and the range of parameters extracted from 136 breasts are summarized in Table II. Three logistic regression models were trained and tested on the feature vector V [see Table II] and the reconstruction outcome O by leave-one-out cross-validation. Fig. 5 shows the resulting ROC curves of the logistic regression models and the chance reference line of the random predictive model.

The AUCs of the logistic regression model for all variables (i.e., α', age, and BMI), breast curvature measure α', and patient variables (i.e., age and BMI) are 0.69, 0.71, and 0.52, respectively. Both the logistic regression model using all variables and the model using only the breast curvature measure α' showed better prediction results than the model using only patient variables. Moreover, the performance of the model using only patient variables was similar to that expected from chance alone. In other words, the feature vectors $V_{i,1}$ (α', age, and BMI) and $V_{i,2}$ (α' only) contain useful information for quantifying the differences between breasts in the TE phase of a TE/Implant reconstruction and untreated breasts, while the feature vector $V_{i,3}$ (age and BMI only) has no useful information for quantifying the differences. As expected, the results indicate that patient age and BMI alone have no predictive power for breast curvature changes resulting from surgery. The results also suggest that there may be no advantage in including patient age and BMI in addition to our curvature measure for quantifying the difference between breasts in the TE phase of a TE/Implant reconstruction and untreated breasts. (In contrast, our preliminary study with a much smaller dataset had suggested that patient age and BMI might help the breast curvature measure α' to distinguish the difference in shape between breasts undergoing TE/Implant reconstruction and untreated breasts [15].)

In the logistic regression model for the breast curvature measure α', coefficients associated with parameter α' ranged from 25.8 to 29.1. The values of those coefficients demonstrate that increasing α' is associated with increasing probability that the contour is from a breast in the TE phase during TE/Implant reconstruction, given feature vector $V_{i,2}$. In other words, a breast in the TE phase of a TE/Implant reconstruction will result in a relatively high value of α', which means that the breast contour has low curvature.

These findings are encouraging since they confirm qualitative remarks of previous clinical studies. Specifically, breast reconstruction by tissue expansion typically results in rounded breast mound in appearance and the projection of lower portion of breast mound is limited [32], [33]. Therefore, additional breast revision surgeries including contralateral symmetry treatments may be required to recreate a natural-appearing breast with appropriate ptosis [19], [20]. As expected from this qualitative observation, our quantitative analysis demonstrates that breasts in the TE phase of a TE/Implant reconstruction have a rounded and less curved contour compared to untreated breasts.

Table III shows the Student's *t*-test result for two analyses: age and BMI, respectively. The results indicate that patients with lower BMIs have lower breast curvature (higher α') as compared to patients with higher BMIs. This result supports our hypothesis about the effect

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of BMI on the breast curvature. Although we did not observe statistical significance for the analysis comparing the breast curvatures of younger and older women, the *p*-value is close to the traditional cut-off value 0.05 ($p = 0.0525$), and the *post hoc* statistical power was only 0.46. Hence, the results are suggestive that our curvature measure captures the differences in breast curvature that arise as part of the aging process.

IV. Discussion

The objective of this study was to introduce a novel quantitative measure of the breast curvature that can explain physical properties of the breast contour. We proposed a new application of catenary for approximating the overall shape of the breast contour and a normalized version (a') of the catenary curve parameter a for quantifying the breast curvature.

We introduced a semiautomatic method that approximates the breast contour using the catenary curve and calculates a measure of the breast curvature from that catenary curve, i.e., the catenary shape parameter a. The proposed method uses multiple points on the breast contour, which include the M point and the anterior axillary point A as two end points, and several equally spaced points between those two end points. In order to evaluate the proposed method for quantifying breast curvature, we compared the length, the area enclosed by the curve, and the breast curvature measure of the approximated breast contour and those of the breast contour manually traced on the clinical photograph. The ICC showed that the measures from the approximated breast contour were in agreement with those from the manually annotated breast contour. This finding was also verified with a second statistical method (hypothesis test for equivalence [34], data not shown).

To show that the proposed measure a captures relevant information about the breast curvature, we analyzed the normalized measure a' along with age and BMI for patients who were undergoing or scheduled for a breast reconstruction surgery. We focused on women with breasts in the TE phase of a TE/Implant reconstruction and untreated breasts since the surgical literature provides clear qualitative descriptions of the typical differences in breast curvature between these two groups. A logistic regression model was applied to the curvature measure a', age, and BMI. We observed that there was no advantage of including age and BMI for quantifying the curvature differences of the two groups. In addition, the logistic regression model showed the expected association between the measure a' and TE/Implant reconstruction. We found that breasts in the TE phase have higher values of the measure a', which corresponds to a breast contour with lower curvature relative to that of the untreated breasts. In addition, we showed the utility of the proposed measure through analyses of the curvature for women with different BMI or different ages. We found that women with low BMI have breasts with low curvature as compared to women with high BMI. A similar, albeit not statistically significant, trend was observed when the measure was used to compare the breast curvature of younger and older women. Thus, our analyses show that the proposed measure can be used to quantify the change in breast shape due to demographic and clinical properties such as age and BMI.

The obvious surgical application of the proposed measure a' is in comparing the curvature of pre- and postoperative breast contours to evaluate the reconstruction outcome. Moreover, if one established a reference database of the change of the curvature value for different surgical procedures, surgeons could use it to select an appropriate procedure to achieve their goal. For example, surgeons could use the reference database to select the appropriate shape and size of the breast implant for TE/implant reconstruction. In addition, if a unilateral reconstructed breast shows lower curvature than the opposite breast, then surgeons may be able to perform a mastopexy on the opposite breast to achieve better symmetry, and the choice of the surgical procedure (e.g., the amount of horizontal and vertical skin to be resected) could be made from the aforementioned database.

The application of the proposed measure α' is not limited to the quantification of the breast curvature but can also be used to understand the underlying physical properties of the breast that result from a series of reconstruction procedures. The shape of the catenary is determined by forces applied on each small segment of the catenary, which are the weight applied to the segment and the driven tension due to the weight. Similarly, the shape of the breast contour is also determined by forces in equilibrium, which are the breast weight and the tension applied on the breast soft tissue at the contour. The ratio of the tension to the weight of the breast is the parameter α. Therefore, by tracking α values at different time points of a series of reconstructive procedures, we can estimate the change in the underlying physical relationship between the breast soft tissue and the total breast weight applied on the breast contour. If we consider TE/Implant reconstruction as an example, an increase in the value of α (i.e., decreased curvature) may mean either that there has been an increase in tension on the breast's skin and overlying soft tissue due to the capsular contracture or radiation-induced shrinkage around implants, or a reduction in breast weight due to the deflation of the TE or the implant leaking. Similarly, a decrease in the value of α (i.e., increased curvature) may represent either a decrease in tension on the breast soft tissue or an increase in breast weight due to the inflation of the TE.

The proposed catenary measure can be extended to capture other important curvatures, such as the curvature of sagittal plane, lower and upper breast. Fig. 6 depicts how our catenary measure could be used to measure other curvatures. The breast curvature of sagittal plane can be measured using the points that lie between the transition point (i.e., where breast mound first starts to leave the chest wall), nipple, and inframammary fold in lateral view [see Fig. 6(b)]. Moreover, the curvature of lower breast contour in the lateral view can be measured from the points that lie between the nipple and the lateral terminus of the inframammary fold [see Fig. 6(c)]. In addition, we can measure the lower breast contour in the oblique view from point M to the lateral breast–chest wall junction point [see Fig. 6(d)]. In the AP view, the curvature of the upper pole of the breast can be measured between the lower axillary fat pad point (the point in the upper lateral portion of the breast showing the natural transitions from a convex to a concave shape), the transition point, and the M point [see Fig. 6(a)].

More in-depth analysis of breast curvature can be conducted using 3-D image of patient's torso. The curvature change of the entire breast can be analyzed from: 1) transverse catenary contours; 2) sagittal catenary contours; 3) radial catenary contours (radiating from the real or potential location of the nipple); and 4) catenary contours lying on concentric contours as shown in Fig. 6(e)–(g), respectively. Each contour is about 1 cm or 36° apart from each other.

The proposed curvature measure is applicable to the quantification of the changes in breast shape due to other events such as pregnancy, age, weight gain/loss, breastfeeding, or following breast augmentation, mastopexy, and reduction. For instance, we have already demonstrated that the proposed measure can be used to assess the effects of age and BMI on the shape of breast contour. Similar analysis could be conducted for other events to evaluate their effects on the shape of breast contour. In addition to the above, we can use the proposed measure to quantify the shape of ptotic breast, and use its value to investigate the new relationship between the ptotic breast and the demographic and clinical properties (e.g., pregnancy, age, and weight loss), which have been shown as factors of impacting ptosis [21], [24].

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The proposed method for quantifying the curvature of the breast does not capture local contour variations. Local variations can include naturally existing transitions from a convex to a concave shape, such as is common in the upper lateral portion of the breast [see Fig. 4(d)], as well as local contour defects introduced during treatment and reconstruction. As these characteristics also impact the breast appearance, additional measures will be needed to quantify local contour variations.

It is possible that other mathematical models of the breast contour, such as parabolas or high-order spline models, could likewise statistically capture interesting curvature features. However, the usage of such values is limited only to the quantification of the breast curvature since they have no physical meaning, while our measure can provide physical interpretation of the curvature that might be useful to reconstructive surgeons for surgical planning.

The future work using the new catenary measure includes: 1) quantitative analysis of autologous reconstructed breasts; 2) comparison between TE/Implant reconstructed breast and autologous reconstructed breast; and 3) analysis of the change of the reconstructed breast shape over time. From such future studies, we will be able answer several important questions, including: 1) in the case of unilateral reconstruction, whether autologous reconstruction is superior to TE/implant reconstruction in terms of symmetry and 2) whether the size of the autologous tissue affects the gradual change of lower breast contour over time.

V. Conclusion

This study is the first to quantitatively investigate breast curvature in patients with untreated breasts and breasts in the TE phase of TE/Implant reconstruction using a sophisticated model. Prior work was limited to simple asymmetry measures [7], [11]. Our new curvature measure lays the groundwork for developing systems to support both plastic surgeons and breast cancer survivors in decision making for breast reconstruction. By establishing a reference database of the curvature measure α' for different breast reconstruction procedures, a case-based reasoning system could be developed to predict the likely changes to a woman's breast shape if she were to undergo a given reconstructive procedure. Moreover, because the curvature measure is based on a physical model of the breast and not simply on abstract properties of an image, it could be a valuable component of a surgical planning system in the future.

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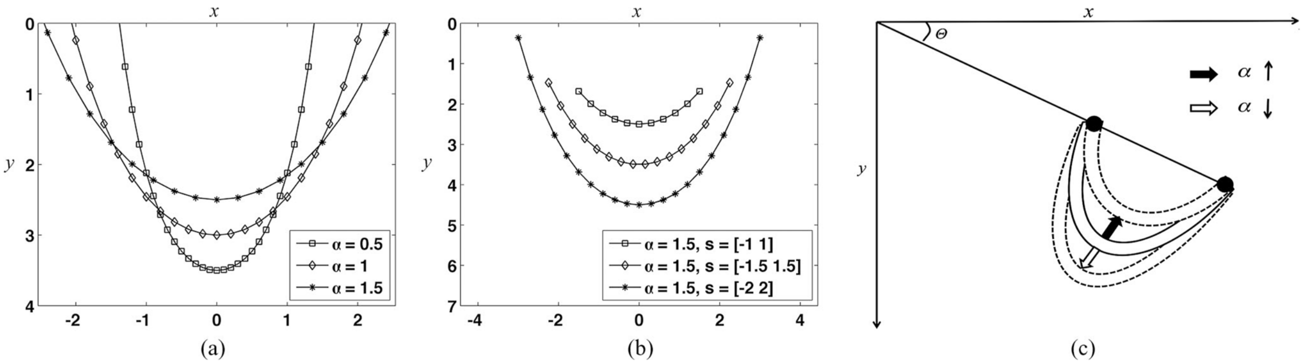

Fig. 1.

Graphs of catenaries for (a) different α values and (b) different ranges of s, and (c) catenary with α and θ to approximate a right breast contour. (a) Smaller α corresponds to a curve with higher curvature whereas a larger α corresponds to a curve with lower curvature. (b) Catenary evolves as the range of s increases. (c) α approximates the breast contour curvature and θ captures the orientation of the breast contour from the x-axis.

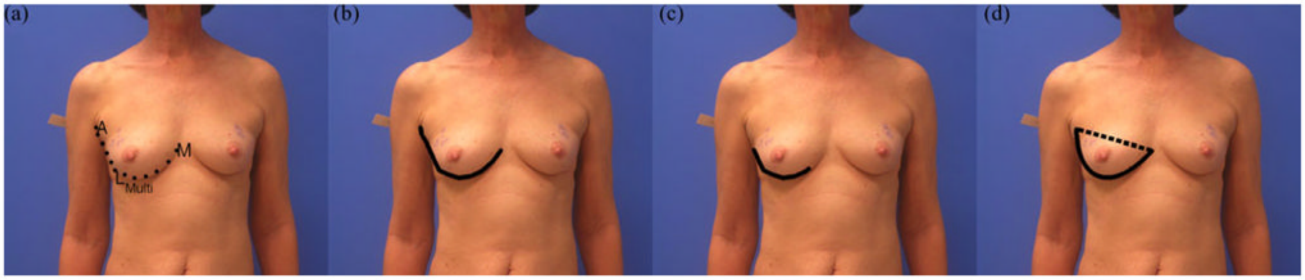

Fig. 2.

Procedures to approximate the right breast contour using the proposed method. (a) These points are the M point (where M is the closest point on the inframammary fold to the midline of the torso), the anterior axillary point (A, the start of the crease between the axilla and the inner arm), and several equally spaced points (L_{multi}, more than 10) between the midline and the anterior axillary point. (b) Interpolated points from the reference points (M, A, L_{multi}). (c) Selected points on the middle one fifth to four fifths of the interpolated points on the breast contour that were used by the proposed algorithm. (d) Final rotated catenary curve is fitted on the subject's right breast. The bounded region by the final catenary curve and the dashed line were used for calculating the area enclosed by the curve. Such an area obtained from the proposed method was compared to that of the manually traced breast contour.

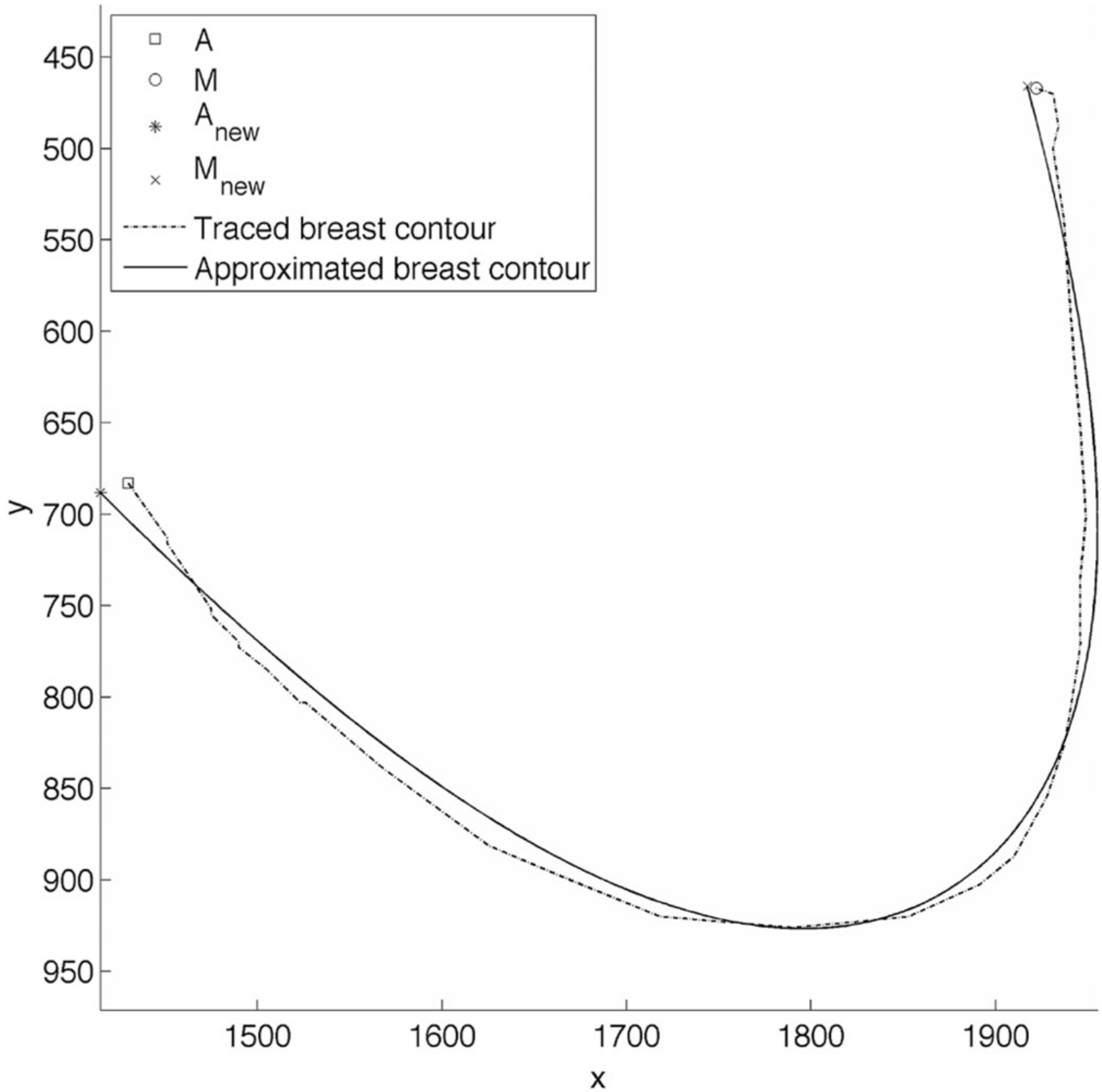

Fig. 3.

Finding the catenary fitted on the given breast contour using the proposed method is equivalent to solving an overdetermined problem. Therefore, the solution *P* cannot guarantee that the resulting catenary curve passes through all of the reference points, and either one of the two end points (either M or A) or both of them are usually missed. We introduced a replacement point for the missed one (M_{new} and/or A_{new}) as the point where the resulting curve from the solution *P* and the imaginary line between the two end points (M and A) intersect, to compute the length of the approximated breast contour.

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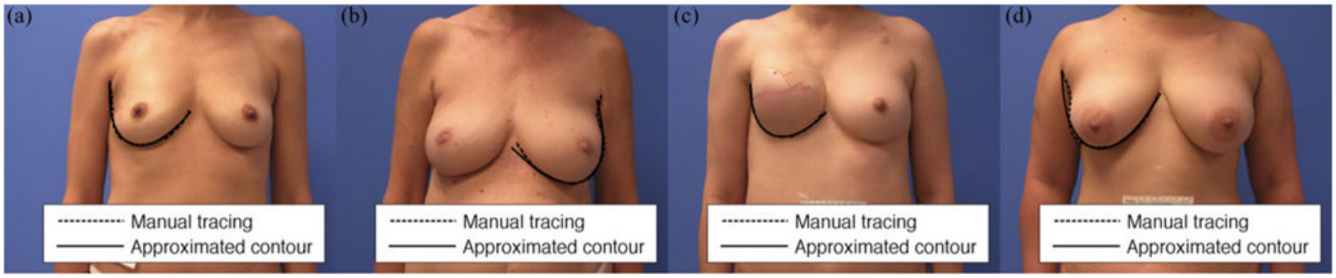

Fig. 4.
Approximated breast contours by the proposed method and the manual tracing of the actual breast contour for four patients' breasts. A total of three untreated breasts (a, b, and d) and one breast in the TE phase of TE/Implant reconstruction (c) are shown. One image (d) shows an example of the normal local contour in the upper lateral portion of the breast.

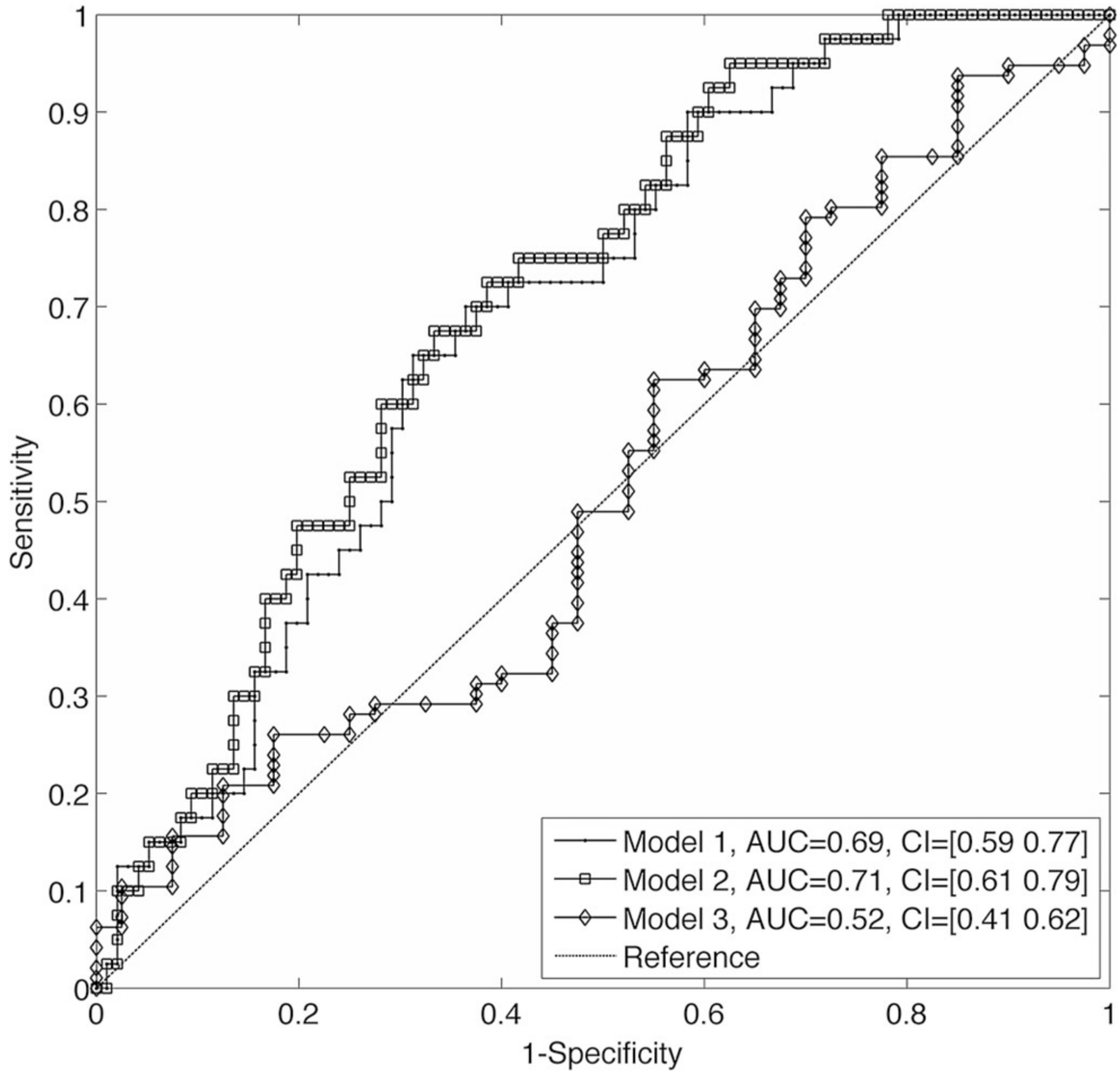

Fig. 5.

ROC curve of the resulting logistic regression models. The AUCs of the logistic regression model and their 95% CI for all variables (Model 1|α', age, and BMI), breast curvature measure α' (Model 2|α'), and patient variables (Model 3|age, and BMI) are 0.69 (CI: 0.59–0.77), 0.71 (CI: 0.61–0.79), and 0.52 (CI: 0.41–0.62), respectively. Models 1 and 2 effectively quantify the differences between the TE/Implant reconstructed breasts and the untreated breasts.

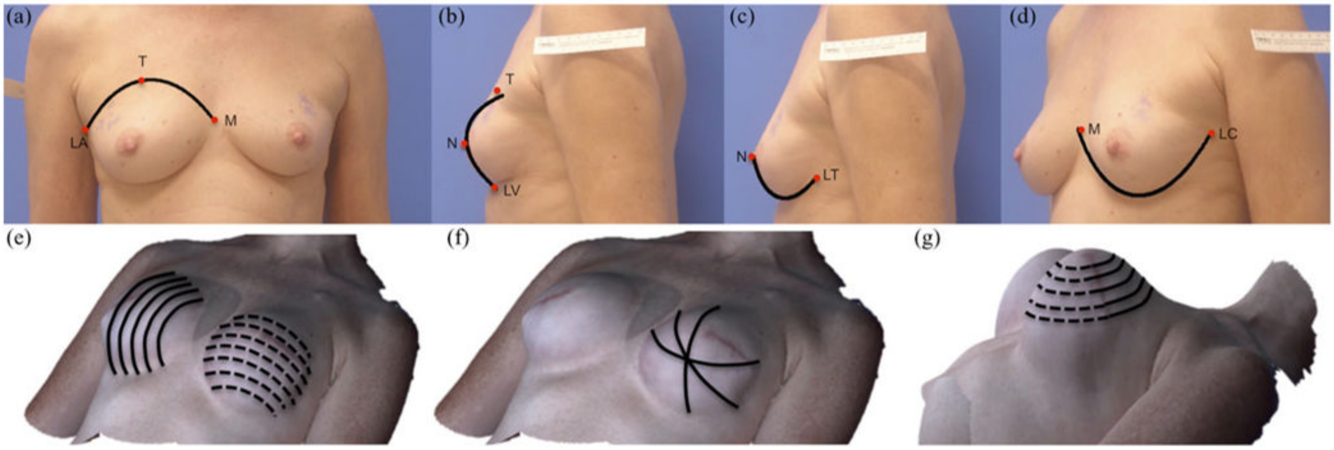

Fig. 6.

Breast curvatures to which the proposed measure is applicable. (a) Curvature of upper breast which can be used to measure the upper pole fullness. The curvature can be measured from multiple points between the lower axillary fat pad point (LA), transition point (T, where breast mound first starts to leave the chest wall), and M point. (b) Breast curvature of sagittal plane that can be measured from multiple points between T, nipple (N), and lowest visible point of the breast (LV). (c) Curvature of lower breast contour in lateral view that can be measured from multiple points between N and lateral terminus of inframammary fold (LT). (d) Curvature of lower breast contour in oblique view that can be measured from multiple points between M and the lateral breast–chest wall junction point (LC). (e) Sagittal catenary contours (solid lines) and the transverse catenary contours (dashed lines). (f) Radial catenary contours (radiating from the real or potential location of the nipple). (g) Upper (solid lines) and lower (dashed lines) catenary contours lying on concnetric contours of the breast. Contours in (e)–(g) can capture the curvature changes of the breast.

TABLE I

Statistical Analysis Results and Summary Statistics for the Measures

	Measures	**Mean *(pixels)***	**Standard Deviation *(pixels)***	***ICC***
Manual Tracing	α	163	22	N/A
	Area	178,918	52,574	
	Length	1,116	173	
Proposed	α	170	26	0.82
	Area	180,556	55,567	0.99
	Length	1,088	171	0.98

This table summarizes the length, the area enclosed by the curve, and the curvature measure α of the approximated breast contour and those of the manually traced breast contour. The intraclass correlation coefficient values for all measures were above 0.75, which indicates excellent agreement between the manually traced breast contour and the approximated breast contour from the proposed method.

TABLE II

Summary Statistics of the Feature Vector V Extracted from the Data set

	Median	**Min**	**Max**
$\alpha_i^{\prime}$ *Untreated*	0.13	0.07	0.22
$\alpha_i^{\prime}$ *TE/Implant*	0.16	0.11	0.22
BMI Untreated	26	18	46
BMI TE/Implant	24.14	19	35
Age Untreated	46	27	66
Age TE/Implant	43	34	65

TABLE III

Student's *t*-Test Result on Untreated Breast Groups

	Age		***BMI***	
	<50 (N=58)	**≥ 50 (N=38)**	**<25 (N=47)**	**≥ 25 (N=49)**
a' mean [min max]	0.14 [0.09 0.21]	0.13 [0.07 0.22]	0.15 [0.07 0.22]	0.12 [0.07 0.2]
p-value	0.0525		<0.0001	