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TEST GENERATION FOR MICROPROCESSORS

by

Satish Mukund Thatte

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BY

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THESIS

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## 20. ABSTRACT (continued)

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## TEST GENERATION FOR MICROPROCESSORS

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Department of Electrical Engineering  
University of Illinois at Urbana-Champaign, 1979

The goal of this thesis is to develop test generation procedures for testing microprocessors in a user environment. Classical fault detection methods based on the gate and flip-flop level or on the state diagram level description of microprocessors are not suitable for test generation. The problem is further compounded by availability of a large variety of microprocessors. They differ widely in their organization, instruction repertoire, addressing modes, data storage and manipulation facilities, etc. In this thesis, a general graph-theoretic model for microprocessors is developed at the register transfer level. Any microprocessor can be easily modeled using information only about the instruction set and the functions performed by it. This information is easily available in the user's manual. A fault model is developed on a functional level quite independent of the implementation details. The effects of faults in the fault model are investigated at the level of the graph-theoretic model. Test generation procedures are proposed which take the microprocessor organization and the instruction set as parameters and generate tests to detect all the faults in the fault model. The complexity of the test sequences measured in terms of the number of instructions is given. Our effort in generating tests for a real microprocessor and evaluating their fault coverage is described.

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## 1. INTRODUCTION

### 1.1. Description of the Problem

Microprocessors constitute a very high percentage of today's large scale integrated (LSI) circuits. The number of microprocessor-based digital systems is expanding rapidly. This has given rise to an acute need for sound theoretical tools to develop efficient, thorough and cost-effective test programs to detect faults in microprocessors at all levels: at the component level during fabrication and before encapsulation, at the chip level before incorporating the microprocessor into a system, and at the system level in the field. These levels have their own testing requirements and constraints on test development and application.

Manufacturers of microprocessors are interested in testing various components and devices on the microprocessor chip during its fabrication for DC parametric behavior (such as power consumption, noise sensitivity, fanin and fanout capability, etc.) as well as dynamic timing problems, etc. Both manufacturers and users are interested in testing microprocessors at the chip level for its correct functional operation at the rated speed. Finally, users and system designers are interested in ensuring that the microprocessor in the system (as well as the rest of the system) is functioning correctly. Classical fault detection methods such as the D-algorithm [RBSc67] used for the chip and system level testing are employed to detect logical faults defined at a low level such as a line stuck-at-one or stuck-at-zero [CMMe70] and [BrFr76]. These faults are associated with lines interconnecting

gates and flip-flops. For microprocessors which contain thousands of gates, flip-flops and interconnections, classical methods must consider a very large number of faults making test generation extremely complicated.

Our approach associates faults with various functions of the microprocessor (defined at a suitably higher level), such as the register decoding function, instruction decoding and control function, data storage function, etc. We give some examples of faults in microprocessors which we are interested in detecting.

Example 1.1: When the instruction "Load register  $R_1$ " is executed, register  $R_2$  is loaded instead of register  $R_1$ . This may happen due to a faulty register decoding function. The instruction "Interrupt enable" correctly enables the interrupt, but at the same time the accumulator is cleared. This can be attributed to a fault in the instruction decoding and control function. The instruction "Add the contents of register  $R_1$  to the contents of the accumulator," is not correctly executed for a few operands due to faults in the arithmetic and logic (ALU) unit. We associate these faults with the data manipulation function. A register may fail to store certain data patterns. This fault is associated with the data storage function. □

Another important reason motivating our approach of considering faults at a functional level is the constraint imposed on testing microprocessors in a user environment: the test programs need to be generated without knowing the implementation details of the chip at the gate and flip-flop level. The only source of information which is readily

available is the typical user's manual detailing the instruction set and describing the architecture of the microprocessor. Using this information it is easier to define the functional behavior of a microprocessor and associate faults with the functions as illustrated in Example 1.1.

In this thesis we are concerned with formulating a sound theoretical foundation for test program generation for testing microprocessors in a user environment, particularly at the chip level. We are interested only in generating deterministic tests to detect permanent faults which give rise to faulty functional behavior as described in Example 1.1. We will not discuss testing issues related with dynamic timing problems, faulty DC parametric behavior or manufacturing or design processes. For these aspects readers are referred to [TEST75]. Of course, the "solution" that proposes the execution of each instruction for all possible operands and in every possible sequence for testing microprocessors is really not a solution. It only serves the purpose of dramatically pointing out how difficult the problem really is.

We assume that the external tester monitors all the input and output pins of the microprocessor. In particular, the status pins and the data and address buses of the microprocessor are continually checked. Testing is stopped on the detection of any fault, (may or may not be in real time) since we are not interested in fault location on a chip. The tester and the external memory which contains the instructions executed by the microprocessor are assumed to be fault free. Various sophisticated testers which are commercially available do satisfy the requirements mentioned above. In this thesis we will not discuss the design and implementation or operation of a tester. For information on testers

readers are referred to [Hust74] and [Ande76].

Sophisticated testers available for testing microprocessor chips cannot be conveniently used for testing microprocessors incorporated in a system in the field. Recognizing this difficulty various schemes such as self testing [Ball79], [LiDo79] and transition counting [Haye76] have been proposed. A notable instrument suitable for field testing and diagnosis is the signature analyzer available from the Hewlett-Packard Company [HPJ077]. These techniques are aimed at the ability to test systems in the field without requiring a sophisticated tester; however, their fault detection capability principally hinges on how thorough the test programs are, again emphasizing the need for good test generation procedures. Though the test generation procedures proposed in this thesis assume the presence of a sophisticated tester, we believe that these procedures can be used, with relatively easy modifications, for generating tests suitable for field testing. However, more research is required in this area.

## 1.2. Thesis Outline

The goal of the thesis is to develop test generation procedures for testing microprocessors. These procedures should treat the microprocessor organization and instruction set as parameters. This is necessary in view of the fact that today's microprocessors differ widely in their organization, instruction repertoire, addressing modes, data storage and manipulation facilities, etc.

In Figure 1.1 the thesis outline is schematically illustrated. In the beginning of Chapter 2 we survey various models and methods of

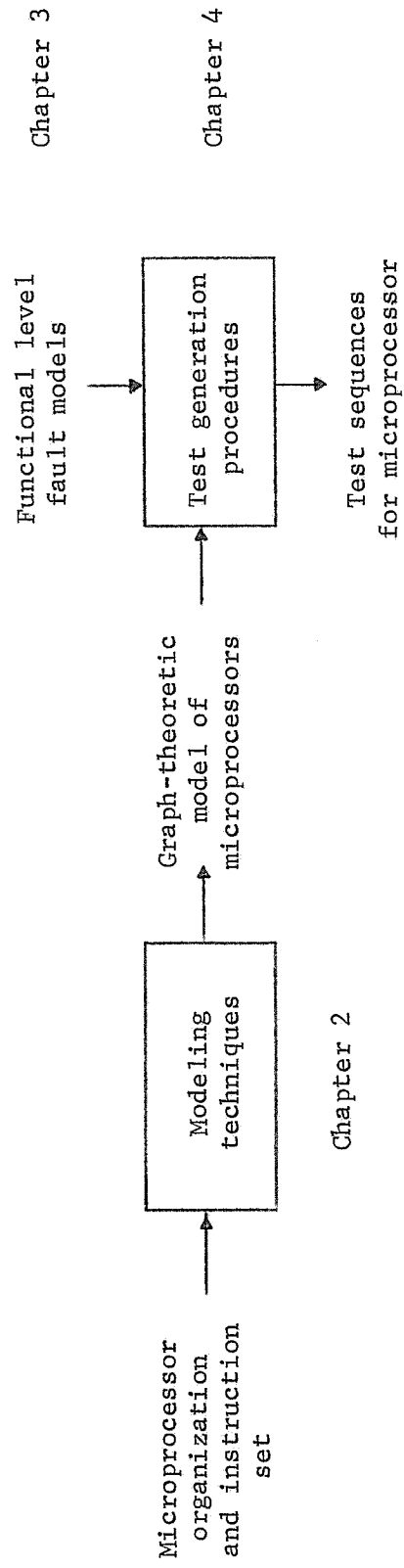


Figure 1.1. Illustrating the thesis outline.

test generation published in literature. Since none of them is suitable for testing microprocessors, we first develop a general graph-theoretic model for microprocessors at the register transfer level. Any microprocessor can be easily modeled on the proposed lines using information only about the instruction set and the functions performed by it.

(This information is easily available in a typical user's manual.) This allows us to treat the microprocessor organization and the instruction set as parameters of the test generation procedures. We will illustrate how to generate the graph-theoretic model for a small example microprocessor.

Functional level fault models capable of describing faulty behavior at a higher level are presented in Chapter 3. These models are quite independent of implementation details of a microprocessor. We will investigate the effects of these faults on the graph-theoretic model of a microprocessor. In Chapter 4 we will present test generation procedures to detect faults in the fault models and prove their fault coverage. The generation of the test sequence will be illustrated for the example microprocessor. The generated test sequences comprise valid machine instructions which are assembled to produce test patterns. This may be contrasted with the classical methods which may generate bit vectors that do not correspond to any instruction.

Chapter 5 discusses the complexity of the test sequences measured in terms of the number of instructions present in these sequences. Chapter 6 reports on the feasibility of our approach. We will describe our effort in generating tests for a real microprocessor. The results were quite encouraging. Finally, in Chapter 7, we summarize the thesis and suggest topics for future research.

## 2. A MODEL FOR MICROPROCESSORS

Any rigorous exercise of generating tests for fault detection in a digital system should consist of three activities:

1. Constructing a model at a suitable level for describing the behavior of the digital system.
2. Developing a fault model to define the scope of allowable faults in the system. A good fault model is usually found as a result of a trade-off between the need to account for most of the faults commonly observed in the system and the need to be able to keep the complexity of test generation low, and the length of tests short. The nature of the fault model is usually influenced by the model used to describe the system as illustrated in Section 2.1 below.
3. Generating tests to detect all the faults in the fault model.

Microprocessor testing practised in industry seems to be based on ad hoc techniques such as "testing" each instruction for many operands, "exercising" various modules in the microprocessor (such as the ALU, shifter, registers, indexing hardware), or running an application program. A typical example based on such ad hoc techniques is [ChMc76]. A good tutorial survey of testing methods and tools used in industry can be found in [FeeW78]. Other sources of information describing testing strategies practised in industry are the digests of the annual Semiconductor Test Symposiums sponsored by the IEEE Computer Society [TEST75]. These techniques are not based on a general model for microprocessors. Moreover, they do not consider any specific fault model. Therefore, the technique followed for testing one microprocessor may be difficult to extend to

other microprocessors having different architectures. It is also very difficult to know what faults can or cannot be detected using these techniques.

We now briefly review various models used in the literature for describing digital systems. We will comment on their applicability for modeling microprocessors for the purpose of test generation, particularly in a user environment.

## 2.1. Review of Previous Models

At the lowest level of the modeling spectrum, models are based on the gate and flip-flop level description of a digital system in order to describe its logic behavior. Most of the classical work on fault diagnosis uses these models. At the highest level of the spectrum, models are based on the so-called "black box" description of the system; truth tables are used to describe a combinational circuit and state tables are employed for describing a sequential circuit. As described below, both of these extremes are unsuitable for modeling microprocessors for the purpose of generating tests for them.

### 2.1.1. Gate and Flip-Flop Level Model

The system is described by a logic diagram consisting of gates and flip-flops. Thus gates and flip-flops are recognized as primitive elements. This model usually supports low-level fault models such as a line stuck-at-one or stuck-at-zero model, which associates faults with lines interconnecting gates and flip-flops [CMMe70] and [BrFr76]. These models were used to test and diagnose digital computers designed with discrete components and with a knowledge of the detailed logic description



[Mann66]. For an excellent annotated bibliography on this topic, readers are referred to [Scol72].

These models are not very useful for generating tests for LSI circuits such as microprocessors which contain a very large number of gates, flip-flops, and interconnections and which therefore require an enormous amount of computation to generate comprehensive test sets. In, addition, the required gate and flip-flop level description is usually not available to microprocessor test designers working in a user environment.

#### 2.1.2. State Diagram Model

This model is based on the state diagram description of the system, giving its output and the next state for any input and present state. The system is viewed as a black box and all the implementation details are ignored. Several test methods have been proposed [Koha70] based on automata identification experiments. Though this model supports a very general fault model, the length of the test sequence generated grows exponentially with the number of inputs and states. This restricts the use of the method only to toy systems having a very small number of inputs and states and rules out its applicability to microprocessors.

#### 2.1.3. Module Level Model

This model views a digital system as a network of interconnected modules such as the ALU, register file, multiplexers, demultiplexers, shifters, control unit, etc. Thus the primitive elements are these higher level modules instead of gates and flip-flops.

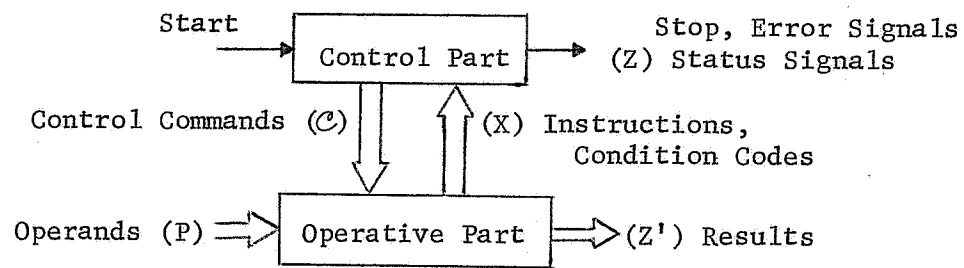
The problem is to generate tests for the entire system using the available tests for individual modules. This problem as stated above

is an extremely difficult and unsolved problem [Powe69] and [BaKi76], though in [BaKi76] methods are given to generate tests for purely combinational systems with some hardware modifications. This model is also not very promising at its present state of research for generating tests for microprocessors because microprocessors also contain sequential logic and no hardware modification is possible in an existing microprocessor chip.

#### 2.1.4. The Robach-Saucier Model

In view of the difficulties pointed out in the previous sections, Robach and Saucier [RoSa75] and [RoSa78] proposed the following model for generating tests for control units of digital systems. Every system can be decomposed into two subsystems, the control and operative parts, as shown in Figure 2.1(a). The control part is characterized by a representation matrix  $M$  as shown in Figure 2.1(b). It has  $n$  rows corresponding to the set of elementary commands  $\mathcal{C} = \{c_1, c_2, \dots, c_n\}$ , and  $m$  columns corresponding to the set of control states  $\{Q_1, Q_2, \dots, Q_m\}$ , such that  $m_{ij} = 1$ , if the state  $Q_j$  activates the command  $c_i$ , and  $m_{ij} = 0$  otherwise. The operative part can be considered to be made up of a set of independent functional units. The set of commands  $\mathcal{C}$  is sent to one or more functional units.

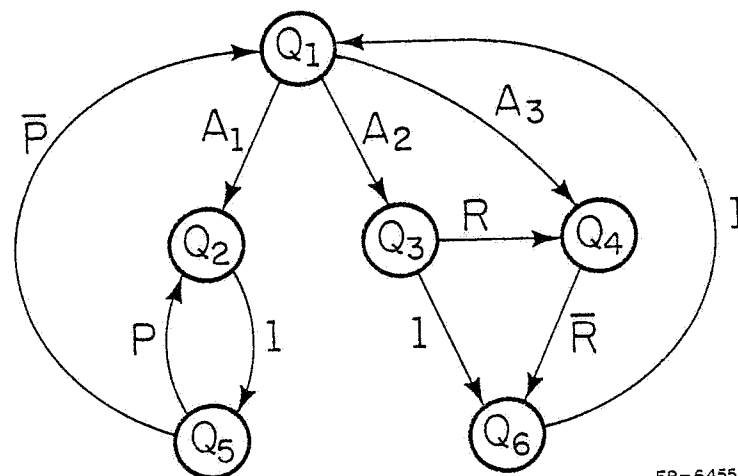
The diagnosis of the control part is performed through the operative part which is assumed fault free. The system is capable of performing a set of "algorithms." For these algorithms, the functioning of the control part is represented by a flow-chart as shown in Figure 2.1(c) where the nodes are the different control states of the considered



(a). A general model for a system.

	$Q_1$	$Q_2$	$Q_3$	.	.	.	$Q_m$
$c_1$	0	1	0	.	.	.	0
$c_2$	0	0	0	.	.	.	0
$c_3$	1	0	0	.	.	.	1
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
$c_n$	0	0	0	.	.	.	1

(b). Representation matrix M.



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(c). Control representation for three algorithms  $A_1$ ,  $A_2$ , and  $A_3$ 

Figure 2.1. The Robach-Saucier Model.

algorithms, and the edges represent the different possible transitions between these states; the edges are labeled with the transition conditions (predicates).

A test of the control unit consists of complete identification of the states (distinguishability of every state from all other states, and checking the commands generated by each state through each functional unit), and verification of sequences. The fault model allows commands to be abnormally activated or abnormally inactive. Thus this model considers basic control commands and control states as primitive elements rather than gates and flip-flops generating these commands.

This approach runs into two problems when applied for generating tests for microprocessors. First, the required information about the details of control states, basic control commands emitted during a state, and flow-charts for algorithms (i.e., each instruction of the microprocessor) may not be available to a test designer working in a user environment. This problem perhaps could be alleviated by formulating the control states and commands at a higher level. Even then, the method faces a second problem; as shown in Figure 2.1(a), it is assumed that the operands (denoted by P) required for the functional units are directly available, and the results (denoted by Z') produced by the functional units are directly observable (possibly with some time delay). On the other hand, in the case of microprocessors a sequence of instructions needs to be executed, in general, to provide proper operands to a functional unit (or to store data in a register) and to read out the result of an operation performed by it (or to read out the contents of a register). For example, consider an "Add" instruction which adds the contents of an accumulator and a

scratch-pad register and stores the result in the accumulator. A "Load accumulator" instruction is needed to load an operand in the accumulator, while two instructions may be required to store an operand in the scratch-pad register: the first to load a general purpose register and the second to transfer the contents from the register to the scratch-pad register. Similarly the accumulator can be read out only by executing the "Store accumulator" instruction.

Though the Robach-Saucier approach is a step in the right direction for testing certain digital systems where the assumptions made in their model are valid, it appears that the limited observability and controllability of internal registers and logic of microprocessors pose a very difficult problem in extending the approach to microprocessor testing.

#### 2.1.5. The Thatte-Abraham Model

A methodology for test generation based on a model for a restricted but "typical" microprocessor organization and instruction set was proposed by Thatte and Abraham [ThAb78]. The model considers an organization for the data processing section of microprocessors shown in Figure 2.2 and allows only limited but commonly observed types of instructions, such as instructions performing information transfers between the main memory and level 1 registers, instructions performing various ALU operations, instructions performing information transfers among level 1 registers, between level 1 and 2 registers, and among level 2 registers.

The fault model takes into account faults associated with registers, ALU, buses, and control section such as incorrect decoding of instructions, missing and extraneous control commands, etc. A drawback

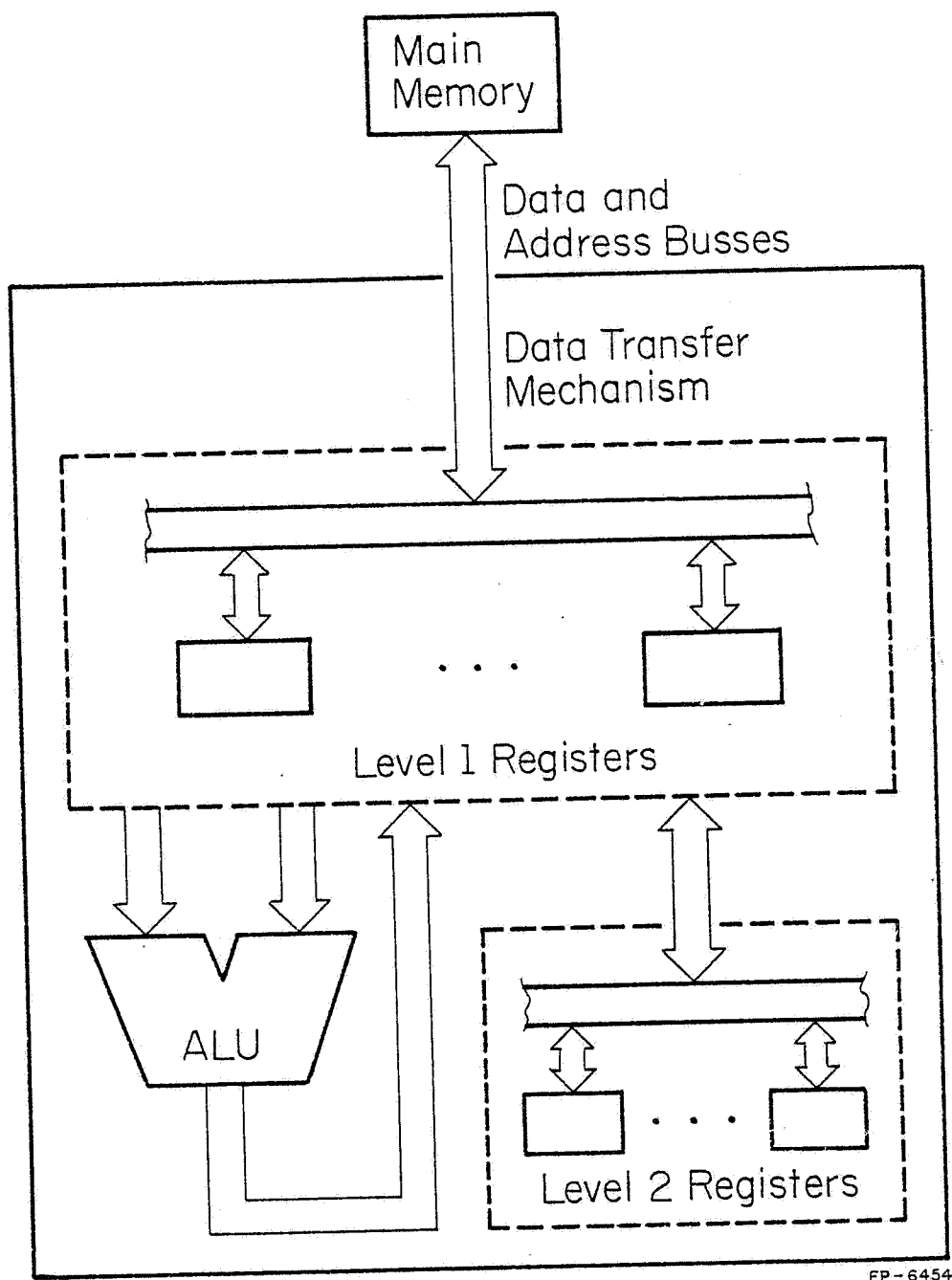


Figure 2.2. The Thatte-Abraham model.

of this model is that it cannot generate tests for different microprocessor organizations and instruction sets, i.e., it fails to treat the microprocessor architecture as a parameter of test generation.

## 2.2. A New Model for Microprocessors

In the light of the discussion in Sections 2.1.1 through 2.1.5, we summarize various requirements for a model suitable for generating tests for microprocessors.

1. The model should be based on a functional description defined at a suitably higher level such as the register transfer level. We will define the model in terms of data flow that occurs between various registers and the main memory during the execution of an instruction. This allows us to describe the functional behavior of a microprocessor by using information about the instruction set and functions performed by it. Since this information is readily available in a typical user's manual, this model is quite suitable, especially in a user environment.

2. The model should be able to treat the microprocessor organization and instruction set as parameters of the test generation procedure, so that for a given microprocessor architecture it would be possible to generate tests. This feature is very desirable as today's microprocessor differ widely in their organization and instruction set, addressing modes, etc. This trend is bound to continue with newly emerging and powerful microprocessors.

3. The model should be able to support a fault model describing faults in various functional primitives such as the data transfer function, the data storage function, the instruction decoding and control function, etc.,

allowing us to describe faulty behavior without knowing the details of their implementation.

For modeling purposes we partition the instruction repertoire of the microprocessor into three classes. This classification scheme is very similar to that proposed by Flynn [Flynn74].

1. Transfer class (denoted by class T):

Instructions of this class perform data transfer between the main memory and a register (on the microprocessor chip), between an I/O device and a register, between registers, and between the main memory locations. Examples are "Load accumulator," "Transfer register  $R_1$  to register  $R_2$ ," I/O instructions, etc.

2. Manipulation class (denoted by class M):

Instructions of this class manipulate the data stored in the main memory or registers by performing operations like "Shift," "Add," "Or," "Decrement," "Compare" instruction, etc.

3. Branch class (denoted by class B):

Class B consists of all those instructions which do not belong to class T or M, e.g., "Conditional and unconditional branches," "Jump to subroutine" and "Return from subroutine (i.e., instructions associated with program sequencing)," "Interrupt enable and disable," "No operation" instruction, etc.

The proposed model is graph-theoretic in nature. Before defining the model formally, we motivate it by means of an example. We represent registers of the microprocessor by labeled nodes and instructions by

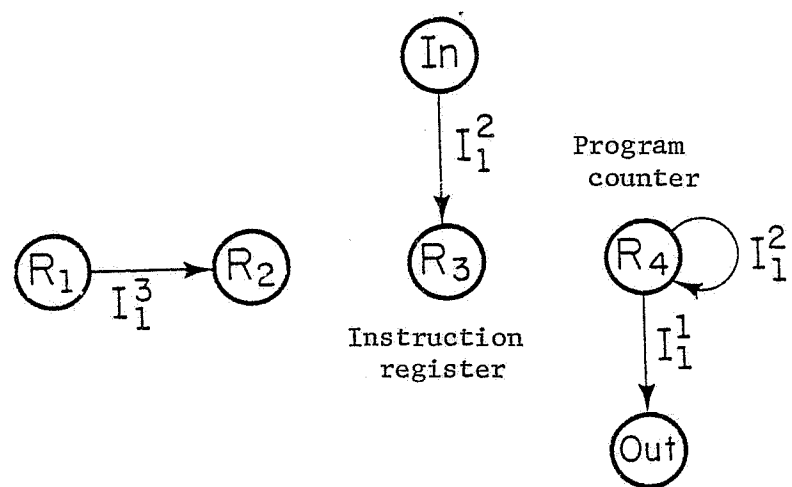


directed labeled edges where edges represent data\* flow during the fetching and execution of instructions.

Example 2.1: Consider a simple register transfer instruction  $I_1$  transferring the contents of register  $R_1$  to register  $R_2$ . The data flow involved during the fetching and execution of  $I_1$  can be represented in a graph as shown in Figure 2.3. Nodes  $R_3$  and  $R_4$  represent the instruction register and the program counter, respectively. The edge from node  $R_4$  to OUT represents the transfer of address of a main memory location containing instruction  $I_1$  from the program counter to the address register of the main memory. This edge is labeled  $I_1^1$ . The edge from node IN to  $R_3$  represents the transfer of instruction  $I_1$  from the main memory to the instruction register. This edge is labeled  $I_1^2$ . While the instruction transfer is taking place, the program counter is incremented. The self loop around node  $R_4$  represents the function of incrementing the program counter. This loop is also labeled  $I_1^2$ . The edge from node  $R_1$  to  $R_2$  indicates the transfer of data stored in register  $R_1$  to register  $R_2$ , i.e., the intended function of the instruction. This edge is labeled  $I_1^3$ . Notice that the data flow represented by the edge labeled  $I_1^1$  takes place before that represented by the edges labeled  $I_1^2$  which, in turn, takes place before that represented by the edge labeled  $I_1^3$ , and so on. Thus the superscripts on the edge labels indicate a precedence relation in time. □

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\* We use data as a generic term referring to the information as well as its address.



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Figure 2.3. Representation of instruction  $I_1$ .

Example 2.1 points out some improvements that can lead to much more concise and succinct representation of an instruction: we may represent that data flow which is involved only during the execution of the instruction and not during its fetching. Since the data flow involved during the fetching of every instruction is the same, no information is gained by representing it for each instruction. On the other hand the data flow involved during the execution of an instruction really characterizes the function performed by the instruction. Thus only the edge from node  $R_1$  to  $R_2$  in Figure 2.3 can be used to represent instruction  $I_1$ .

We now formalize the model. A microprocessor is modeled by a system graph (S-graph). Let  $\mathcal{R} = \{R_1, R_2, R_3, \dots\}$  denote the set of registers in the microprocessor. Set  $\mathcal{R}$  includes the so-called general purpose registers, accumulators, scratch-pad registers, on-chip last-in first-out stack, and the program counter. It also includes index registers, address buffer register, stack pointer, etc., i.e., the registers used in various addressing modes. Included also is the so-called processor status word containing various processor status bits. Each register  $R_i$  is represented by a node (labeled  $R_i$ ) of the S-graph. In addition to the nodes representing registers, we incorporate two more nodes, named "IN" and "OUT" in the S-graph, representing the world external to the microprocessor, i.e., the main memory and I/O devices.

Let  $\mathcal{I} = \{I_1, I_2, I_3, \dots\}$  denote the set of instructions. The execution of instruction  $I_j$  causes data flow among a set of registers, and

between the main memory (or an I/O device) and registers in some sequence. The data may or may not be manipulated during the flow. We can represent the data flow during the execution of any instruction  $I_j$  as follows:

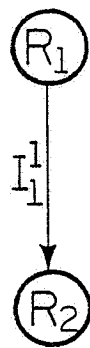
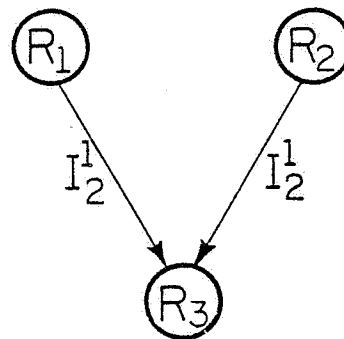
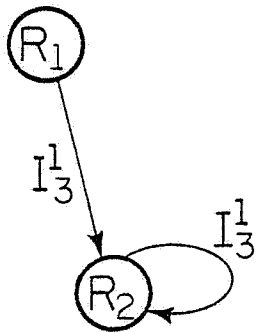
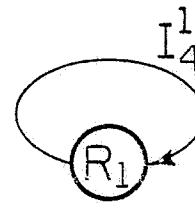
1. There exists a labeled directed edge from node  $R_p$  to node  $R_q$ , if data flow occurs from register  $R_p$  to register  $R_q$  (with or without manipulation) during the execution of  $I_j$ .
2. There exists a labeled directed edge from node IN to node  $R_j$ , if data flow occurs from the main memory or an I/O device to register  $R_j$  (with or without manipulation) during the execution of  $I_j$ .
3. There exists a labeled directed edge from node  $R_j$  to node OUT, if data flow occurs from register  $R_j$  to the main memory (or its address register) or an I/O device (with or without manipulation) during the execution of  $I_j$ .

If more than one edge is required to represent the data flow during the execution of an instruction, the flow may occur in a specific sequence. The exact sequence may not be known to test designers working in a user environment because the sequence depends on the details of implementation of the microprocessor hardware. However, it is possible to deduce the precedence relation in time between the components of the data flow solely on the basis of logical data dependence, independent of the details of implementation. We indicate the precedence relation by means of the labels assigned to directed edges as explained below.

Among the set of edges representing the data flow during the execution of instruction  $I_j$ , two edges are labeled  $I_j^p$  and  $I_j^q$ , where  $p < q$

and  $p$  and  $q$  are smallest such positive integers, if and only if the data flow represented by the edge labeled  $I_j^p$  must take place before that represented by the edge labeled  $I_j^q$  in order to preserve the underlying logical data dependence. Two edges are assigned the same label  $I_j^p$ , if and only if the corresponding data flows can occur simultaneously given the necessary number of resources such as buses and functional units, i.e., the required hardware parallelism exists. In the presence of some limitation on hardware resources, the data flow represented by these two edges may occur in either of the two possible sequences depending on the details of implementation. If only one edge is required to represent the data flow during the execution of instruction  $I_j$ , it is assigned a label  $I_j^1$ . It must be stated that this elaborate notation is used only for clarity in illustrating the data flow sequence and is not really necessary to generate tests.

Example 2.2: Figure 2.4(a) represents a "Transfer" instruction  $I_1$  that transfers the contents of register  $R_1$  to register  $R_2$ . Figure 2.4(b) depicts an "Add" instruction  $I_2$  which adds the contents of registers  $R_1$  and  $R_2$  and stores the result in  $R_3$ . Note that both edges are labeled  $I_2^1$ . If two separate buses are available to route the contents of registers  $R_1$  and  $R_2$  to the ALU simultaneously, the data flow represented by these two edges can take place in parallel. If only one bus is available, the contents of either  $R_1$  or  $R_2$  are transferred to the ALU first and stored in its latch, followed by the transfer of the contents of the other, and then the addition takes place. The actual implementation determines which register is selected first for data transfer.

(a).  $I_1$ - Transfer instruction(b).  $I_2$ - Add instruction(c).  $I_3$ - Or instruction(d).  $I_4$ - Rotate left instruction

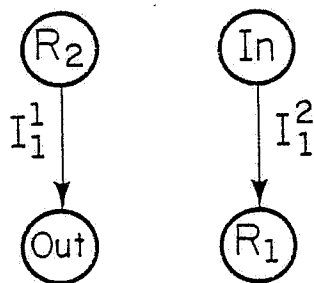
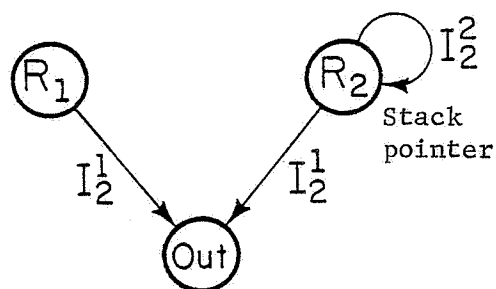
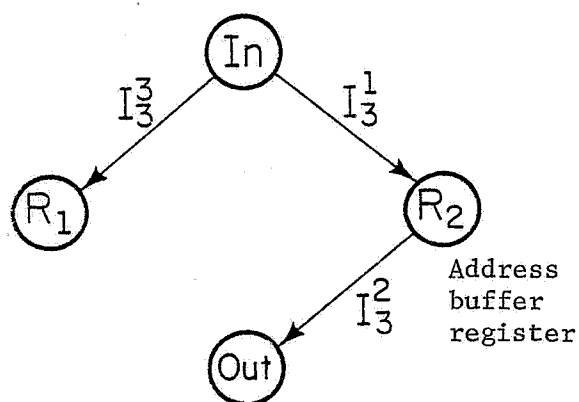
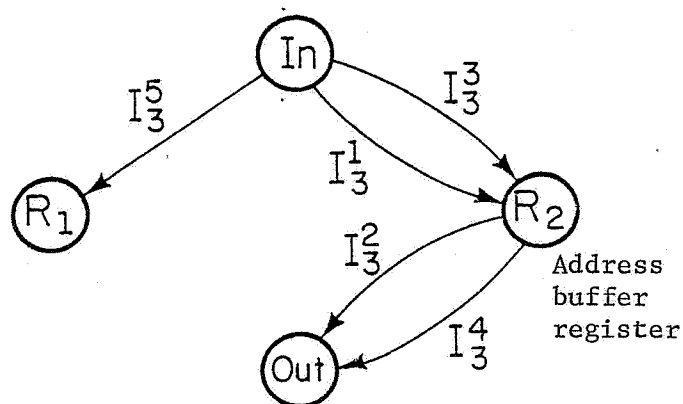
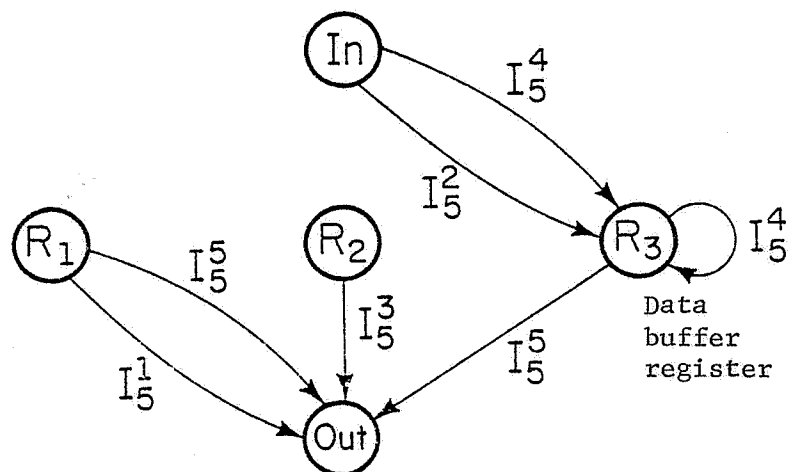
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Figure 2.4. Representation of instructions.

Figure 2.4(c) shows an "Or" instruction  $I_3$  which forms the logical OR of the contents of registers  $R_1$  and  $R_2$  and stores the result in  $R_2$ . As explained above, both edges are labeled  $I_3^1$ . Figure 2.4(d) shows a "Rotate left" instruction which rotates the contents of register  $R_1$  left by 1 bit. □

We now explain how to represent instructions which use addressing modes by these graph-theoretic techniques. A variety of addressing modes is usually available for instructions for fetching operands from the main memory and storing results into the main memory. Various examples of addressing modes are direct, indirect, immediate, indexed, relative, stack, etc. [GsMc75]. Each addressing mode is characterized by a sequence of data transfers between registers and the main memory.

Example 2.3: Figure 2.5(a) represents the "Load register  $R_1$ " instruction,  $I_1$ , using the so-called implied or implicit addressing [GsMc75] where the data to be loaded is contained in the memory location next to the one storing instruction  $I_1$ , i.e., the address of the operand is derived by implication. The edge from node  $R_2$  to OUT represents the transfer of the address of the operand from the program counter ( $R_2$ ), (which is incremented by 1 by this time and points to the word next to the one storing instruction  $I_1$ ), to the main memory address register, and the edge from IN to  $R_1$  represents the data transfer from the main memory to register  $R_1$ . Figure 2.5(b) represents the "Stack push" instruction  $I_2$  which pushes the contents of  $R_1$  into the memory location (top of the last-in first-out (LIFO) stack maintained in the main memory) pointed to by the stack-pointer  $R_2$  and then increments the stack pointer. The edge from node  $R_1$  to node OUT in Figure 2.5(b) represents the transfer of data

Program  
counter(a).  $I_1$ - Load register  $R_1$  instruction using implied addressing(b).  $I_2$ - Stack push instruction(c).  $I_3$ - Load register  $R_1$  instruction using direct addressing(d).  $I_4$ - Load register  $R_1$  instruction using indirect addressing(e).  $I_5$ - Add  $(R_1), (R_2)$  instruction

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Figure 2.5. Representation of addressing modes.



(to be pushed on the stack) from  $R_1$  to the LIFO stack. The  $R_2$ -OUT edge represents the transfer of the address of the top of the stack from  $R_2$  (stack pointer) to the address register of the main memory. Both these edges are labeled  $I_2^1$ . The self loop around the node  $R_2$  represents the stack pointer incrementing function. It is labeled  $I_2^2$  because the stack pointer must be incremented only after the data is pushed so that it points to the location on the stack where next data can be pushed.

Figure 2.5(c) shows how to represent a register load instruction  $I_3$  using direct addressing [GSMc75]. As shown in Figure 2.5(c) the address of the location storing the operand is fetched from the address field of instruction  $I_3$  into the address buffer register  $R_2$  (represented by the IN- $R_2$  edge labeled  $I_3^1$ ). This address is then sent from  $R_2$  to the address register of the main memory (represented by the  $R_2$ -OUT edge labeled  $I_3^2$ ) and the operand is fetched from the main memory and loaded into  $R_1$  (represented by the IN- $R_1$  edge labeled  $I_3^3$ ). The register load instruction  $I_4$  using indirect addressing mode can be represented by incorporating two more edges in Figure 2.5(c) (one more edge from node  $R_2$  to OUT and one more edge from node IN to  $R_2$ ) as shown in Figure 2.5(d).

Figure 2.5(e) shows the representation of a complicated instruction  $I_5$  "Add ( $R_1$ ), ( $R_2$ )," where the contents of registers  $R_1$  and  $R_2$  denote addresses of operands. The first operand for this instruction is fetched from the main memory location pointed to by register  $R_1$  and stored in the data buffer register  $R_3$  (accounting for the  $R_1$ -OUT edge labeled  $I_5^1$  and the IN- $R_3$  edge labeled  $I_5^2$ ). The second operand is fetched from the main memory location pointed to by register  $R_2$  and it is added to the contents of register  $R_3$  (accounting for the  $R_2$ -OUT edge labeled  $I_5^3$ , the other edge

from IN to  $R_3$  labeled  $I_5^4$  and the self loop around  $R_3$ ). Finally the result of the addition (ready in  $R_3$ ) is stored in the main memory location pointed to by register  $R_1$  (accounting for the other edge from  $R_1$  to OUT labeled  $I_5^5$  and the  $R_3$ -OUT edge).  $\square$

We now illustrate the representation of instructions of class B.

Example 2.4: Figure 2.6(a) represents a "Jump" instruction  $I_1$ . The edge from the node IN to  $R_1$  (representing the program counter) represents the transfer of the jump address from the main memory to the program counter. The  $R_1$ -OUT edge represents the transfer of the jump address from the program counter to the address register of the main memory achieving the jump in the program sequencing.

Strictly speaking, the  $R_1$ -OUT edge (indicating the jump in the program sequencing) ~~represents the transfer of the address of a main memory~~ location for fetching a new instruction. Therefore the  $R_1$ -OUT edge really represents a data flow involved during the fetching of the new instruction. We take the flexible viewpoint that the data flow denoted by  $R_1$ -OUT edge could also be considered involved during the execution of the instructions of class B affecting the regular program sequencing. This viewpoint also provides a directed edge from the node representing the program counter to the OUT node, avoiding the awkward situation in which the OUT node would not be reachable from the node representing the program counter.

Figure 2.6(b) represents a "Jump to subroutine" instruction  $I_2$  where the return address for the subroutine is stored in a local register called the subroutine register (denoted by  $R_2$ ). The program counter is represented by node  $R_1$ . The IN- $R_1$  edge represents the transfer of the jump address from the main memory to the program counter, and the  $R_1$ - $R_2$

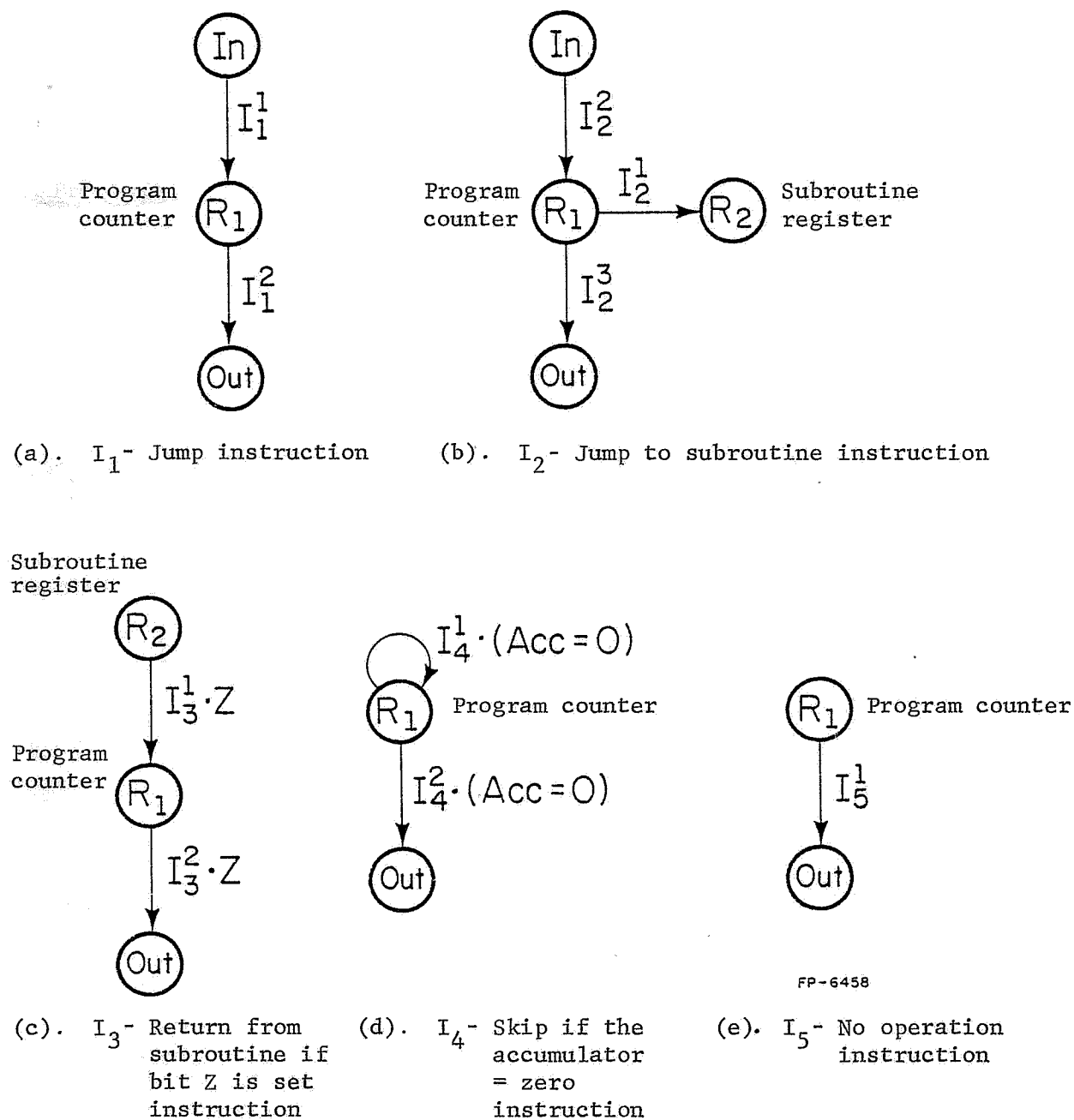


Figure 2.6. Representation of instructions of class B.

edge represents the transfer of the contents of the program counter to the subroutine register, i.e., saving the return address. The return address must be saved before the jump address is transferred from the main memory to the program counter. The  $R_1$ -OUT edge represents the jump in the program sequencing. The labels of edges  $I_2^1$ ,  $I_2^2$ ,  $I_2^3$  indicate the precedence relations in the data flow.

Instructions causing only conditional changes in the program sequencing can be suitably represented by tagging instruction labels with the appropriate condition code (predicate). For example, Figure 2.6(c) represents a "Return from subroutine if bit Z is set" instruction  $I_3$ . If  $Z = 1$ , the contents of the subroutine register  $R_2$ , are transferred to the program counter  $R_1$ . The  $R_1$ -OUT edge represents the jump (conditional) in the program flow.

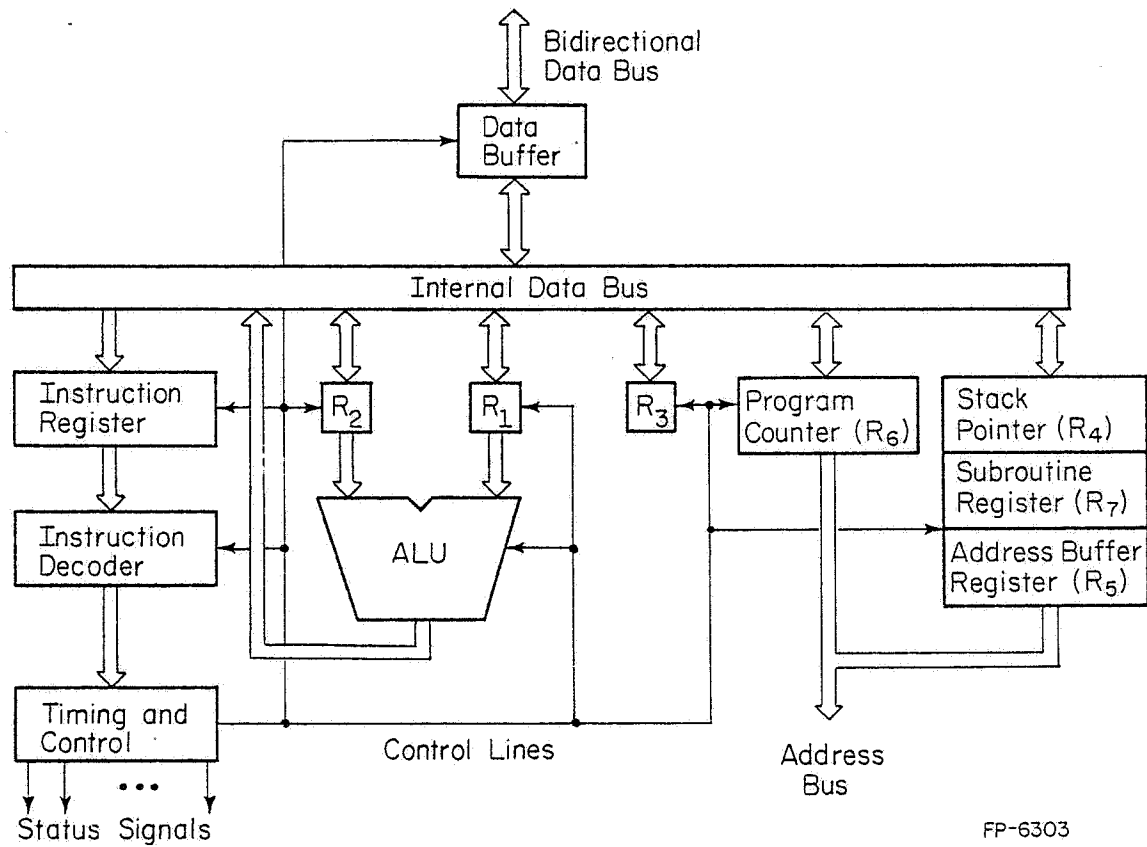
Figure 2.6(d) shows a "Skip if the accumulator equals zero" instruction  $I_4$ . The predicate can be denoted by "ACC = 0." The node  $R_1$  represents the program counter. The self loop around node  $R_1$  denotes the conditional skip, i.e., the program counter is incremented if the accumulator equals zero. Figure 2.6(e) shows a "No operation" instruction  $I_5$ . The  $R_1$ -OUT edge represents the transfer of the contents of the program counter to the address register of the main memory to fetch the next instruction in the regular program sequencing.

Those instructions of class B which do not change the processor status word but only change the logic level on some status pins such as "Interrupt enable" instruction are not represented in the S-graph. □

### 2.3. An Example Microprocessor

We now illustrate the generation of the S-graph for a small hypothetical microprocessor. This example will also be used in Chapter 3 to demonstrate the effects of faults on the S-graph, and in Chapter 4 to illustrate various test generation procedures. Figure 2.7 shows the block diagram of this microprocessor. It has an accumulator ( $R_1$ ), a general purpose register ( $R_2$ ), a scratch-pad register ( $R_3$ ), a program counter ( $R_6$ ) and a subroutine register ( $R_7$ ) to save the return address of subroutines, allowing a single level of subroutine nesting. A stack pointer ( $R_4$ ) is provided which points to the top of a LIFO stack maintained in main memory. An address buffer register ( $R_5$ ) is provided to store the address of operands. The ALU is capable of performing ADD, logical AND, SHIFT and COMPLEMENT operations. The instruction repertoire contains 21 instructions which are listed in Table 2.1. Though all the architectural features of the example microprocessor may not be present in any real microprocessor, they have been carefully chosen to illustrate some of the subtle points involved in test generation. It may be noticed that instructions  $I_1, I_2, I_3, I_5, I_6, I_7, I_8, I_{15}, I_{16}, I_{17}, I_{18}$  and  $I_{19}$  belong to class T, instructions  $I_4, I_{11}, I_{12}$  and  $I_{13}$  constitute class M, while class B contains instructions  $I_9, I_{10}, I_{14}, I_{20}$  and  $I_{21}$ .

Example 2.5: The S-graph for the microprocessor shown in Figure 2.7 is drawn in Figure 2.8. The self loops around  $R_4$ , labeled  $I_{16}^2$  and  $I_{18}^1$ , represent the stack pointer incrementing and decrementing functions, respectively. The self loop around  $R_6$ , labeled  $I_{10}^1 \cdot (R_1 = 0)$ , represents the program counter incrementing function during the "Skip" instruction  $I_{10}$ , if the condition "register  $R_1 = 0$ " is satisfied. All the other edges of Figure 2.8 are self-explanatory. □

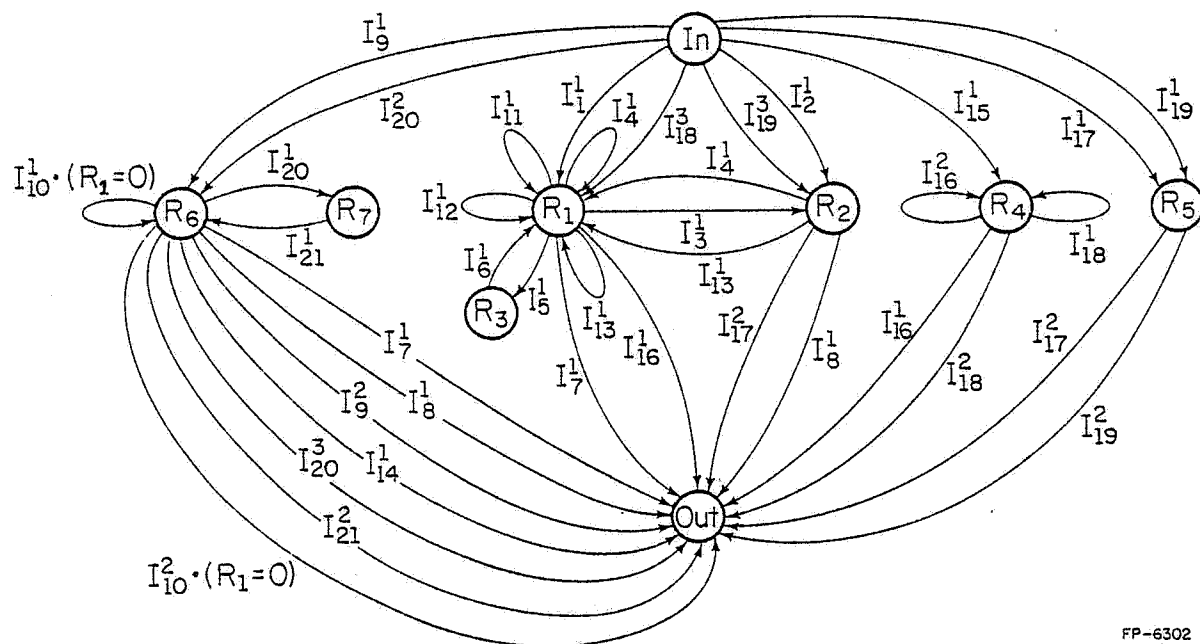


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Figure 2.7. The block diagram of an example microprocessor.

Table 2.1. The instruction repertoire of the example microprocessor.

- I<sub>1</sub> - Load register R<sub>1</sub> from the main memory using immediate addressing.
- I<sub>2</sub> - Load register R<sub>2</sub> from the main memory using immediate addressing.
- I<sub>3</sub> - Transfer the contents of register R<sub>1</sub> to register R<sub>2</sub>.
- I<sub>4</sub> - Add the contents of registers R<sub>1</sub> and R<sub>2</sub> and store the results  
in register R<sub>1</sub>.
- I<sub>5</sub> - Transfer the contents of register R<sub>1</sub> to register R<sub>3</sub>.
- I<sub>6</sub> - Transfer the contents of register R<sub>3</sub> to register R<sub>1</sub>.
- I<sub>7</sub> - Store register R<sub>1</sub> into the main memory using implied addressing.
- I<sub>8</sub> - Store register R<sub>2</sub> into the main memory using implied addressing.
- I<sub>9</sub> - Jump instruction.
- I<sub>10</sub> - Skip if the contents of register R<sub>1</sub> are zero.
- I<sub>11</sub> - Left shift register R<sub>1</sub> by one bit.
- I<sub>12</sub> - Complement (bit-wise) the contents of register R<sub>1</sub>.
- I<sub>13</sub> - Logical AND the contents of registers R<sub>1</sub> and R<sub>2</sub> and store the  
result in register R<sub>1</sub>.
- I<sub>14</sub> - No operation instruction.
- I<sub>15</sub> - Load the stack pointer (R<sub>4</sub>) from the main memory using immediate  
addressing.
- I<sub>16</sub> - PUSH register R<sub>1</sub> on the LIFO stack maintained in the main memory.
- I<sub>17</sub> - Store register R<sub>2</sub> into the main memory using direct addressing.
- I<sub>18</sub> - POP the top of the LIFO stack and store it in R<sub>1</sub>.
- I<sub>19</sub> - Load register R<sub>2</sub> from the main memory using direct addressing.
- I<sub>20</sub> - Jump to subroutine (return address is saved in the subroutine  
register R<sub>7</sub>).
- I<sub>21</sub> - Return from subroutine.



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Figure 2.8. S-graph of the example microprocessor.



## 2.4. Definitions and Notation

Some registers of the microprocessor can be written (loaded with required data) or read out (i.e., its contents can be stored in the main memory or sent to an I/O device) by executing an explicit instruction. Examples of such registers are accumulator and general-purpose registers. On the other hand, some registers cannot be written or read out by executing any explicit instruction. For example, the address buffer register of Figure 2.5(c) can be written as well as "read out" (on the address bus) only implicitly during the execution of instruction  $I_3$ . Similarly, the stack pointer ( $R_2$ ) of Figure 2.5(b) can be read out implicitly on the address bus during the execution of instruction  $I_2$ . Note that in Figure 2.8 also, the stack pointer ( $R_4$ ) can be read out only implicitly during the execution of instructions  $I_{16}$  or  $I_{18}$ , though it is possible to write it explicitly by executing instruction  $I_{15}$ . The data buffer register ( $R_3$ ) of Figure 2.5(e) can be written or read out only implicitly. The subroutine register ( $R_7$ ) in Figure 2.8 can be read out only implicitly by executing the "Return from subroutine" instruction  $I_{21}$ . Finally, the program counter can be written only implicitly during the execution of an instruction of class B which alters the normal program sequencing.

We assume that any register can be written (implicitly or explicitly) as well as read out (implicitly or explicitly) using a sequence of instructions of class T or using an instruction of class B. This assumption can be easily justified for current microprocessors [Cush77]. In terms of the S-graph, there exists a path from the IN node to every node (representing a register) consisting of edges representing instructions of

class T or class B. Similarly there exists a path from every node to the OUT node consisting of edges representing instructions of class T or class B.

Transfer mechanisms such as buses are used to transfer data between registers, functional units, main memory, and I/O devices during the execution of an instruction. Since a test designer working in a user environment may not know the details of implementation of the transfer mechanisms, or how they are shared or time-multiplexed among different data transfers, we "map" a physical transfer mechanism used during the execution of an instruction onto a set of logical entities called transfer paths. We illustrate how to perform this mapping by means of Example 2.6 below. The set of transfer paths associated with instruction  $I_j$  is denoted by  $T(I_j)$ . The motivation for presenting the notion of transfer paths is to be able to develop a fault model for the data transfer function independent of the actual implementation details of the transfer mechanisms.

Example 2.6: With reference to instruction  $I_4$  in Figure 2.8,  $T(I_4)$  contains three transfer paths, two paths for transferring the contents of  $R_1$  and  $R_2$  to the ALU and one path for transferring the output of the ALU to  $R_1$ .  $T(I_6)$  contains only one transfer path for transferring the contents of  $R_3$  to  $R_1$ , while  $T(I_{19})$  contains three transfer paths, one for transferring data (which is actually the address of an operand) from the main memory to  $R_5$ , one for transferring the address from  $R_5$  to the address register of the main memory, and the third one to transfer data from the main memory to  $R_2$ .  $T(I_{21})$  contains two transfer paths, one for transferring the contents of the subroutine register ( $R_7$ ) to the program

counter, and the second one for transferring the contents of the program counter to the address register of the main memory.  $\square$

The set of source registers for an instruction  $I_j$  is defined to be that set of registers which provide the operands for instruction  $I_j$  during its execution. This set is denoted by  $S(I_j)$ . Similarly, the set of destination registers for an instruction  $I_j$  is defined to be that set of registers which are changed by instruction  $I_j$  during its execution. This set is denoted by  $D(I_j)$ . Extending this notation further,  $S(I_1, I_2, \dots, I_n) = S(I_1) \cup S(I_2) \cup \dots \cup S(I_n)$ .  $D(I_1, I_2, \dots, I_n)$  can be defined analogously.  $|S(I_j)|$  and  $|D(I_j)|$  denote the cardinality of the corresponding sets.

Example 2.7: In the S-graph of Figure 2.8,  $S(I_7) = \{R_1\}$ ,  $S(I_4) = \{R_1, R_2\}$ ,  $S(I_6) = \{R_3\}$ ,  $S(I_2) = \{IN\}$ ,  $S(I_{21}) = \{R_7\}$ , etc. Similarly  $D(I_7) = \{OUT\}$ ,  $D(I_4) = D(I_{11}) = D(I_{12}) = \{R_1\}$ ,  $D(I_{16}) = \{R_4, OUT\}$ ,  $D(I_{17}) = \{R_5, OUT\}$ ,  $D(I_{20}) = \{R_6, R_7, OUT\}$ .  $|S(I_4)| = 2$ ,  $|D(I_{20})| = 3$ .  $\square$

The set of directed edges denoting an instruction  $I_j$  in the S-graph is called its edge set and is denoted by  $E(I_j)$ .  $READ(R_i)$  denotes the shortest sequence of instructions of class T or class B that is necessary to read out register  $R_i$  (implicitly or explicitly). Similarly  $WRITE(R_i)$  denotes the shortest sequence of instructions of class T or class B that is necessary to write register  $R_i$  (implicitly or explicitly).  $|E(I_j)|$ ,  $|READ(R_i)|$ , and  $|WRITE(R_i)|$  denote the cardinality of the corresponding set or sequences.

Example 2.8: For the S-graph shown in Figure 2.8,  $READ(R_1) = \langle I_7 \rangle$ ,  $READ(R_3) = \langle I_6, I_7 \rangle$ ,  $READ(R_5) = \langle I_{17} \rangle$ ,  $READ(R_7) = \langle I_{21} \rangle$ , etc.  $WRITE(R_1) = \langle I_1 \rangle$ ,  $WRITE(R_5) = \langle I_{17} \rangle$ ,  $WRITE(R_3) = \langle I_1, I_5 \rangle$ ,  $WRITE(R_7) = \langle I_9, I_{20} \rangle$ . Thus,  $|READ(R_3)| = 2$ ,  $|WRITE(R_7)| = 2$ .  $\square$

We allow  $|D(I_j)| > 1$  only if instruction  $I_j$  involves data transfer between the main memory (or an I/O device) and registers of the microprocessor during its execution. Thus  $|D(I_j)| = 1$  for all those instructions which do not involve data transfer between the main memory and some registers during their execution. We need not consider  $|D(I_j)| > 1$  in the case of these instructions, because the results of instructions of class M or T are usually not stored in more than one register. This does not mean that  $|D(I_j)| > 1$  for every instruction  $I_j$  which causes data transfer to take place between the main memory and registers during its execution.

Thus we have constructed a model based on the data flow involved during the execution of an instruction satisfying the first requirement given in Section 2.2. The S-graph depends on the instruction repertoire and the functions performed by it, i.e., the S-graph reflects the architecture of the microprocessor. As will be described in Chapter 4, this feature makes it possible to consider the instruction set and organization as parameters of the test generation procedures. This satisfies the second requirement given in Section 2.2. The third requirement is related to the development of a fault model defined at functional level. This is the topic of Chapter 3.

## 2.5. Study of Data Transfers Among Registers

In this section we develop a framework to study how the contents of registers in the microprocessor change when a sequence of instructions of class T (called by the generic name T sequence) is executed. Specific occurrences of the T sequence are denoted by symbols  $\sigma$ ,  $\sigma_1$ ,  $\sigma_2$ , etc. A T sequence  $\sigma$  is specified by listing its component instructions, i.e.,  $\sigma = \langle I_{j_1}, I_{j_2}, \dots, I_{j_n} \rangle$ ;  $I_{j_1}$  is executed first, followed by  $I_{j_2}$ , and so on.

We denote this by  $I_{j_1} < I_{j_2} < \dots < I_{j_n}$ . An instruction may occur more than once in a T sequence. Since we are considering data transfers among registers only, we concentrate only on those instructions of class T which transfer data among registers, and not between the main memory and registers. Results derived in this section will be used in Section 4.3.3 for generating tests to detect faults in the instruction decoding and control function, and for proving their fault coverage.

Definition 2.1: Register  $R_i$  is 1-step transferrable to register  $R_j$  under a T sequence  $\sigma = \langle I_{j_1}, I_{j_2}, \dots, I_{j_n} \rangle$ , if the contents of  $R_i$  before the execution of the sequence become the final contents of  $R_j$  at the end of the execution of the sequence. Such a register  $R_i$  is denoted as  $R_j^1(\sigma)$ . □

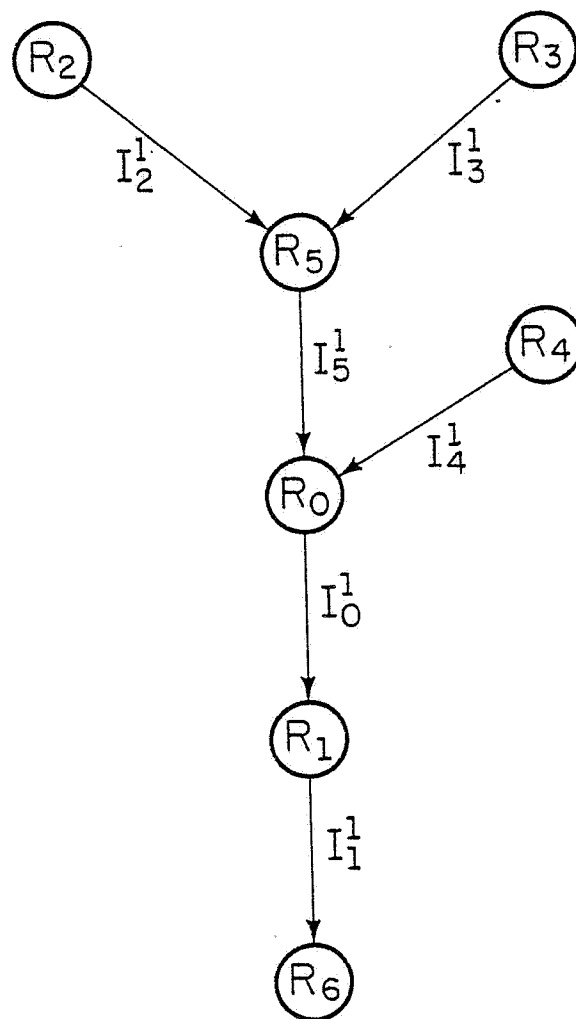
Lemma 2.1: Given a T sequence  $\sigma = \langle I_{j_1}, I_{j_2}, \dots, I_{j_n} \rangle$  and register  $R_j$ , there exists one and only one register  $R_j^1(\sigma)$ .

Proof: Follows immediately from Definition 2.1. □

Example 2.9: Consider a hypothetical S-graph shown in Figure 2.9. Consider the T sequence  $\sigma = \langle I_0, I_1, I_2, I_3, I_4, I_5 \rangle$ . We have  $R_6^1(\sigma) = R_0$ , and  $R_0^1(\sigma) = R_3$ . □

If the T sequence  $\sigma$ , in the example above, is executed one more time the contents of  $R_3$  (before the first execution of the T sequence) would become the final contents of  $R_6$ . This observation motivates the next definition.

Definition 2.2: Register  $R_i$  is K-step transferrable to register  $R_j$  under a T sequence  $\sigma = \langle I_{j_1}, I_{j_2}, \dots, I_{j_n} \rangle$ , if the contents of  $R_i$  before the first execution of the sequence become the final contents of  $R_j$  at the end of the  $K^{\text{th}}$  execution of the sequence, where K is the



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Figure 2.9. An example illustrating Definitions 2.1 and 2.2.

smallest such integer. Such a register  $R_i$  is denoted as  $R_j^K(\sigma)$ . □

Example 2.10: For the S-graph shown in Figure 2.9,

$R_6^2(\sigma) = R_3$  under the T sequence  $\sigma = \langle I_0, I_1, I_2, I_3, I_4, I_5 \rangle$ . □

If we denote the T-sequence formed by concatenating two T sequences  $\sigma_1$  and  $\sigma_2$  as  $\sigma_1 \cdot \sigma_2$ , then in the context of Example 2.10,  $R_6^1(\sigma \cdot \sigma) = R_6^2(\sigma) = R_3$ . Therefore Definition 2.2 may appear rather contrived and artificial because  $R_j^K(\sigma) = R_j^1(\sigma \cdot \sigma \cdot \dots \cdot \sigma)$ , where the T sequence  $\sigma \cdot \sigma \cdot \dots \cdot \sigma$  is formed by concatenating K copies of T sequence  $\sigma$ . However, as mentioned earlier, results derived in this section will be used in Section 4.3.3 where test generation procedures are given. Some of these test generation procedures involve loops containing a T sequence. When the loop is to be executed K times, it is easier and more natural to consider the T sequence being executed K times rather than a long sequence formed by concatenating K copies of the T sequence being executed once.

M denotes the smallest integer such that if any register is K-step transferrable to a given register  $R_j$  under the T sequence  $\sigma = \langle I_{j_1}, I_{j_2}, \dots, I_{j_n} \rangle$ , then  $K \leq M$ .

Lemma 2.2: Given a T sequence  $\sigma = \langle I_{j_1}, I_{j_2}, \dots, I_{j_n} \rangle$  and a register  $R_j$ , there exists one and only one register  $R_j^K(\sigma)$ , where  $K \leq M$ .

Proof: Follows from Definition 2.2. □

We are interested in finding the relation between M and n - the number of instructions in  $\sigma = \langle I_{j_1}, I_{j_2}, \dots, I_{j_n} \rangle$ . (Recall that  $I_{j_1} < I_{j_2} < \dots < I_{j_n}$ .) In order to do this we first show how to

find  $R_j^1(\sigma)$  under a given T sequence  $\sigma$ . Consider a set of instructions  $A = \{I_{j_k} \mid I_{j_k} \in \sigma \text{ and } D(I_{j_k}) = \{R_j\}\}$ . If set A is found to be empty, then, of course,  $R_j^1(\sigma) = R_j$  because the execution of the T sequence  $\sigma$  does not change the contents of  $R_j$ . Moreover,  $M = 1$ . On the other hand, if set A is found to be nonempty, then choose  $I_{j_p} \in A$  such that there exists no other  $I_{j_q} \in A$  with  $I_{j_p} < I_{j_q}$ , i.e., choose  $I_{j_p}$  which is executed latest in  $\sigma$  but which still belongs to set A. Designate the instruction so chosen as  $I_{\ell_1}$ . Note that  $I_{\ell_1}$  is a unique instruction, and it is the last instruction in  $\sigma$  which changes the contents of  $R_j$ .

Consider a set of instructions  $B_1 = \{I_{j_k} \mid I_{j_k} \in \sigma, D(I_{j_k}) = S(I_{\ell_1}), \text{ and } I_{j_k} < I_{\ell_1}\}$ . If set  $B_1$  is found to be empty, then  $R_j^1(\sigma) = S(I_{\ell_1})$  because when  $I_{\ell_1}$  is executed the contents of  $S(I_{\ell_1})$  are the same as they were before the first instruction in  $\sigma$  was executed. (Note that  $I_{\ell_1}$  transfers the contents of  $S(I_{\ell_1})$  to  $R_j$ , and no instruction that occurs after  $I_{\ell_1}$  in  $\sigma$  can change the contents of  $R_j$ .) On the other hand, if set  $B_1$  is found to be nonempty, then choose  $I_{j_p} \in B_1$  such that there exists no other  $I_{j_q} \in B_1$  with  $I_{j_p} < I_{j_q}$ , i.e., choose  $I_{j_p}$  which is executed latest in  $\sigma$  but which still belongs to set  $B_1$ . Designate the instruction so chosen as  $I_{\ell_2}$ . Note that  $I_{\ell_2}$  is a unique instruction, and  $I_{\ell_2} < I_{\ell_1}$ .

Now consider a set of instructions  $B_2 = \{I_{j_k} \mid I_{j_k} \in \sigma, D(I_{j_k}) = S(I_{\ell_2}), \text{ and } I_{j_k} < I_{\ell_2}\}$ . If set  $B_2$  is found to be empty, then  $R_j^1(\sigma) = S(I_{\ell_2})$ . This is explained as follows: When  $I_{\ell_2}$  is executed the contents of  $S(I_{\ell_2})$  are the same as they were before the first instruction in  $\sigma$  was executed.  $I_{\ell_2}$  transfers the contents of  $S(I_{\ell_2})$  to  $S(I_{\ell_1})$  and the contents of  $S(I_{\ell_1})$  do not change between the executions of  $I_{\ell_2}$  and  $I_{\ell_1}$ .  $I_{\ell_1}$  transfers the contents of  $S(I_{\ell_1})$  to  $R_j$ , and no instruction that occurs after  $I_{\ell_1}$  in



$\sigma$  can change the contents of  $R_j$ . Thus the contents of  $S(I_{\ell_2})$  before the execution of  $\sigma$  become the final contents of  $R_j$  at the end of the execution of  $\sigma$ . On the other hand, if set  $B_2$  is found to be nonempty, then choose  $I_{j_p} \in B_2$  such that there exists no other  $I_{j_q} \in B_2$ , with  $I_{j_p} < I_{j_q}$ . Designate the instruction so chosen as  $I_{\ell_3}$ . Note that  $I_{\ell_3}$  is a unique instruction, and  $I_{\ell_3} < I_{\ell_2}$ .

This process of selecting sets  $A, B_1, B_2, \dots$ , and instructions  $I_{\ell_1}, I_{\ell_2}, I_{\ell_3}, \dots$  can be continued until set  $B_i$  is found to be empty. Since  $I_{\ell_i} < I_{\ell_{i-1}} < \dots < I_{\ell_2} < I_{\ell_1}$ , and there are  $n$  instruction in  $\sigma$ , the process must terminate in at most  $n$  steps, i.e., when set  $B_i = \{I_{j_k} \mid I_{j_k} \in \sigma, D(I_{j_k}) = S(I_{\ell_i}), \text{ and } I_{j_k} < I_{\ell_i}\}$  is found empty, the process terminates and we get  $R_j^1(\sigma) = S(I_{\ell_i})$ . Of course,  $S(I_{\ell_i})$  could be the same as  $R_j$ , in which case  $M = 1$ .

We call the sequence of instructions  $\langle I_{\ell_i}, I_{\ell_{i-1}}, \dots, I_{\ell_1} \rangle$  the characteristic sequence associated with the transfer of the contents of  $R_j^1(\sigma)$  to  $R_j$ , and denote it by  $\sigma_1$ . Since each instruction in  $\sigma_1$  is a unique instruction, the characteristic sequence  $\sigma_1$  is also a unique sequence. Note that  $\sigma_1$  is a subsequence of  $\sigma$ . Let the initial contents of  $R_j^1(\sigma)$  (i.e., the contents before the first instruction in  $\sigma$  is executed) be denoted by  $d_1$ . During the execution of  $\sigma$ , instructions in  $\sigma_1 = \langle I_{\ell_i}, I_{\ell_{i-1}}, \dots, I_{\ell_1} \rangle$  form a "chain" of instructions transferring the initial contents of  $R_j^1(\sigma)$  to  $R_j$ , i.e.,  $I_{\ell_i}$  transfers data  $d_1$  from  $R_j^1(\sigma)$  to  $S(I_{\ell_{i-1}})$ ,  $I_{\ell_{i-1}}$  transfers data  $d_1$  from  $S(I_{\ell_{i-1}})$  to  $S(I_{\ell_{i-2}})$ , ..., finally  $I_{\ell_1}$  transfers data  $d_1$  from  $S(I_{\ell_1})$  to  $R_j$ . Concisely we may say that during the execution of  $\sigma$ , each instruction in  $\sigma_1$  transfers data  $d_1$  where  $d_1$  represents the initial contents of  $R_j^1(\sigma)$ .

Example 2.11: Consider a hypothetical S-graph shown in Figure 2.10. Consider the T sequence  $\sigma = \langle I_8, I_{14}, I_1, I_{11}, I_2, I_6, I_9, I_3, I_{13}, I_4, I_{13}, I_{10}, I_{12}, I_5 \rangle$ . Note that instruction  $I_{13}$  is executed twice in the sequence. Of course, in the S-graph it is represented only once using the edge from node  $R_4$  to node  $R_3$ . In this example

$$\begin{aligned}
 A &= \{I_{11}, I_{12}\} & ; & \quad I_{\ell_1} = I_{12}; S(I_{\ell_1}) = \{R_2\}. \\
 B_1 &= \{I_8, I_{10}\} & ; & \quad I_{\ell_2} = I_{10}; S(I_{\ell_2}) = \{R_3\}. \\
 B_2 &= \{I_6, I_9, I_{13}\} & ; & \quad I_{\ell_3} = I_{13}; S(I_{\ell_3}) = \{R_4\}. \\
 B_3 &= \{I_3, I_4\} & ; & \quad I_{\ell_4} = I_4; S(I_{\ell_4}) = \{R_3\}. \\
 B_4 &= \{I_6, I_9, I_{13}\} & ; & \quad I_{\ell_5} = I_{13}; S(I_{\ell_5}) = \{R_4\}. \\
 B_5 &= \{I_3\} & ; & \quad I_{\ell_6} = I_3; S(I_{\ell_6}) = \{R_6\}. \\
 B_6 &= \{I_2, I_{14}\} & ; & \quad I_{\ell_7} = I_2; S(I_{\ell_7}) = \{R_7\}. \\
 B_7 &= \{I_1\} & ; & \quad I_{\ell_8} = I_1; S(I_{\ell_8}) = \{R_4\}. \\
 B_8 &= \{\emptyset\}.
 \end{aligned}$$

Hence  $R_1^1(\sigma) = S(I_{\ell_8}) = R_4$ . Note that  $I_{\ell_3}$  refers to the second occurrence of  $I_{13}$  in  $\sigma$ , while  $I_{\ell_5}$  refers to its first occurrence. The characteristic sequence  $\sigma_1 = \langle I_1, I_2, I_3, I_{13}, I_4, I_{13}, I_{10}, I_{12} \rangle$  which is a subsequence of  $\sigma$ . □

Now we show how to find  $R_j^K(\sigma)$  under the T sequence  $\sigma$  (for  $R$  (for  $K \leq M$ ). If a register which is 1-step transferrable to  $R_j^1(\sigma)$  under the T sequence  $\sigma$  does not belong to the set  $\{R_j, R_j^1(\sigma)\}$ , it must be

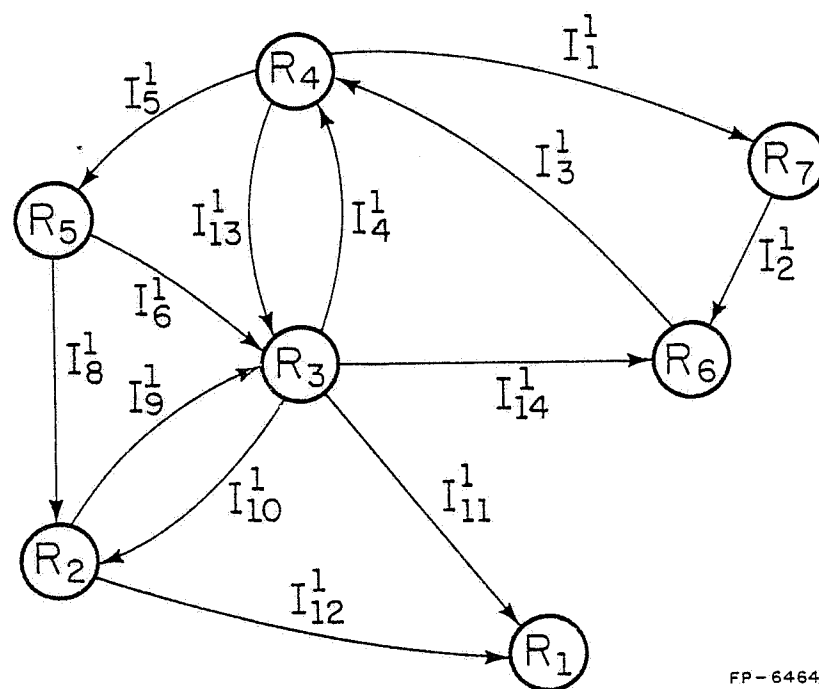


Figure 2.10. S-graph for Example 2.11.

2-step transferrable to  $R_j$  and is designated as  $R_j^2(\sigma)$ . On the other hand, if it belongs to the set  $\{R_j, R_j^1(\sigma)\}$  it is 1-step transferrable to  $R_j$ , allowing us to conclude that  $M = 1$ . Extending this argument, if a register which is 1-step transferrable to  $R_j^K(\sigma)$  under the  $T$  sequence  $\sigma$  does not belong to the set  $\{R_j, R_j^1(\sigma), R_j^2(\sigma), \dots, R_j^K(\sigma)\}$ , it must be  $(K+1)$ -step transferrable to  $R_j$  and is designated as  $R_j^{K+1}(\sigma)$ . On the other hand, if it belongs to the set  $\{R_j, R_j^1(\sigma), R_j^2(\sigma), R_j^3(\sigma), \dots, R_j^K(\sigma)\}$  it is  $p$ -step transferrable to  $R_j$ , where  $1 \leq p \leq K$ . (Refer to Definition 2.2.) In this case we can immediately conclude that  $M = K$ .

We denote the characteristic sequence associated with the transfer of the contents of  $R_j^i(\sigma)$  to  $R_j^{i-1}(\sigma)$  by  $\sigma_i$ , for  $2 \leq i \leq M$ . Concisely we may say that during the execution of  $\sigma$ , each instruction in  $\sigma_i$  transfers data  $d_i$  where  $d_i$  represents the initial contents of  $R_j^i(\sigma)$ , for  $2 \leq i \leq M$ . This discussion leads to the following lemma.

Lemma 2.3:  $R_j^i(\sigma)$  is 1-step transferrable to  $R_j^{i-1}(\sigma)$ , for  $2 \leq i \leq M$ . Some register in the set  $\{R_j, R_j^1(\sigma), R_j^2(\sigma), \dots, R_j^M(\sigma)\}$  is 1-step transferrable to  $R_j^M(\sigma)$ . □

Definition 2.3: Let  $\sigma_i = \langle I_{j_a}, I_{j_b}, I_{j_c}, \dots \rangle$  and  $\sigma_j = \langle I_{j_p}, I_{j_q}, I_{j_r}, \dots \rangle$  be two subsequence of a  $T$  sequence  $\sigma = \langle I_{j_1}, I_{j_2}, \dots, I_{j_n} \rangle$ .  $\sigma_i$  and  $\sigma_j$  are defined to be disjoint if the sets  $\{a, b, c, \dots\}$  and  $\{p, q, r, \dots\}$  are disjoint, i.e.,  $\{a, b, c, \dots\} \cap \{p, q, r, \dots\} = \emptyset$ . If  $\{a, b, c, \dots\} \cap \{p, q, r, \dots\} \neq \emptyset$ ,  $\sigma_i$  and  $\sigma_j$  are not disjoint. □

Definition 2.4: Let  $\sigma_i, \sigma_j, \sigma_k, \dots$  be subsequences of a  $T$  sequence  $\sigma$ . Subsequences in the set  $\{\sigma_i, \sigma_j, \sigma_k, \dots\}$  are defined to be mutually disjoint if each pair of subsequences is disjoint. They are

not mutually disjoint if any pair of subsequences is not disjoint.  $\square$

Lemma 2.4: A T sequence  $\sigma = \langle I_{j_1}, I_{j_2}, \dots, I_{j_n} \rangle$  can be partitioned into a set containing at most  $n$  mutually disjoint subsequences.  $\square$

Lemma 2.5: Subsequences in the set  $\{\sigma_1, \sigma_2, \dots, \sigma_M\}$  are mutually disjoint, where  $\sigma_1$  denotes the characteristic sequence associated with the transfer of  $R_j^1(\sigma)$  to  $R_j$ , and  $\sigma_i$  denotes the characteristic sequence associated with the transfer of  $R_j^i(\sigma)$  to  $R_j^{i-1}(\sigma)$ , for  $2 \leq i \leq M$ .

Proof: During the execution of  $\sigma$ , each instruction in  $\sigma_i$  transfer data  $d_i$  where  $d_i$  represents the initial contents of  $R_j^i(\sigma)$  for  $1 \leq i \leq M$ . Let us assume the contrary, i.e., subsequences in the set  $\{\sigma_1, \sigma_2, \dots, \sigma_M\}$  are not mutually disjoint. Therefore at least one pair of subsequences, say  $\sigma_i$  and  $\sigma_k$ , must not be disjoint. In this case, at the end of execution of  $\sigma$ , either the contents of  $R_j^{i-1}(\sigma)$  are different from  $d_i$ , or the contents of  $R_j^{k-1}(\sigma)$  are different from  $d_k$ . This contradicts the assumption that  $\sigma_i$  is the characteristic sequence associated with the transfer of  $R_j^i(\sigma)$  to  $R_j^{i-1}(\sigma)$ , and  $\sigma_k$  is the characteristic sequence associated with the transfer of  $R_j^k(\sigma)$  to  $R_j^{k-1}(\sigma)$ . Therefore subsequences in the set  $\{\sigma_1, \sigma_2, \dots, \sigma_M\}$  must be mutually disjoint.  $\square$

Theorem 2.1:  $M \leq n$ , where  $n$  = the number of instructions in the T sequence  $\sigma$ .

Proof: Follows immediately from Lemmas 2.4 and 2.5.  $\square$

Corollary 2.1: Let the initial contents of registers  $R_j^1(\sigma), R_j^2(\sigma), \dots, R_j^M(\sigma)$  be  $d_1, d_2, \dots, d_M$ , respectively. Then at the end of  $K^{\text{th}}$  execution ( $1 \leq K \leq M$ ) of the T sequence  $\sigma$ , register  $R_j$  will

contain  $d_K$ . At the end of  $i^{\text{th}}$  execution ( $i > M$ ) of the T sequence  $\sigma$ ,  $R_j$  will contain some data belonging to the set  $\{d_1, d_2, \dots, d_M\}$ .

Proof: Follows from Lemma 2.3. □

Corollary 2.2: If register  $R_j$  contains the same data  $d$  at the end of each of  $i$  executions of the T sequence  $\sigma = \langle I_{j_1}, I_{j_2}, \dots, I_{j_n} \rangle$ , for  $1 \leq i \leq n$ , then at the end of the  $(n+p)^{\text{th}}$  execution of the T sequence  $\sigma$ , for  $p \geq 1$ ,  $R_j$  will contain the same data  $d$ .

Proof: Follows from Corollary 2.1, and  $M \leq n$ . □

Corollary 2.3: If register  $R_j$  contains the same data  $d$  at the end of each of  $i$  executions of the T sequence  $\sigma$  containing at most  $K-1$  instructions, for  $1 \leq i \leq K-1$ , then at the end of  $K^{\text{th}}$  execution of the T sequence  $\sigma$ ,  $R_j$  will contain the same data  $d$ .

Proof: Follows immediately from Corollary 2.2. □

### 3. FUNCTIONAL LEVEL FAULT MODELS FOR MICROPROCESSORS

In this chapter we present fault models for various functions in a microprocessor in accordance with the third requirement cited in Section 2.2. We develop fault models which are quite independent of the implementation details of the microprocessor. We categorize various functions in a microprocessor into the register decoding function, instruction decoding and control function, data storage function, data transfer function, and data manipulation function. We will present a fault model for each of these functions at a higher level. We will, however, point out the underlying fault mechanisms in order to clarify the reasons for choosing the particular models. We will also describe the effects of these faults at the level of the graph-theoretic model for a microprocessor presented in Chapter 2.

#### 3.1. Fault Model for the Register Decoding Function

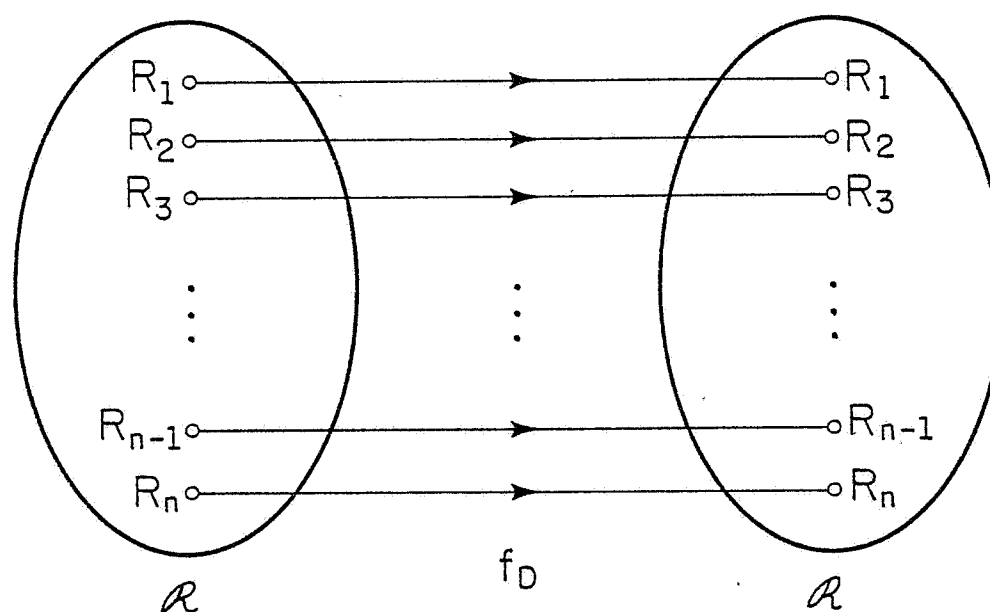
Registers on a microprocessor chip are typically realized as small random-access memories (RAM) [INTE75]. They could also be realized as separate registers interconnected by a network of multiplexers, demultiplexers and buses. Various instructions use registers to fetch operands or address of operands and to store results of operations. Register decoding refers to the task of decoding the "address" of a register which may be stored as a specific bit pattern in the instructions involving that register or which may be generated by the control unit during the execution of the instructions. We want to develop a fault model for this decoding function independent of the realization of the decoding mechanism.

The register decoding function can be modeled as a mapping  $f_D$  from  $\mathcal{R}$  to  $\mathcal{R} \cup \{\emptyset\}$ , where  $\emptyset$  denotes a null or nonexistent register. Let  $f_D(R_i) \subseteq \mathcal{R} \cup \{\emptyset\}$  denote the set of registers which is the image of  $R_i$  under the mapping  $f_D$ . If there is no fault in the register decoding function we get  $f_D(R_i) = \{R_i\}$ , for every  $R_i \in \mathcal{R}$ . Under a fault, if  $f_D(R_i) = \{\emptyset\}$ , whenever register  $R_i$  is to be accessed (while executing any instruction which involves  $R_i$ ), no register is accessed. Obviously if  $\emptyset \in f_D(R_i)$  then  $f_D(R_i) = \{\emptyset\}$  because  $f_D(R_i) = \{R_j, R_k, \dots, \emptyset\}$  is meaningless. If  $f_D(R_i) \neq \{\emptyset\}$  then whenever  $R_i$  is accessed, all the registers in the set  $f_D(R_i)$  are accessed. By this we mean, whenever  $R_i$  is to be written with data  $d$ , all the registers in  $f_D(R_i)$  would be written with data  $d$ , and whenever the contents of  $R_i$  are to be retrieved or used, the contents formed by the bit-wise OR or AND function (depending on technology) over the registers of the set  $f_D(R_i)$  will be retrieved. Under this fault we allow  $f_D(R_i) \neq \{R_i\}$ , for every  $R_i \in \mathcal{R}$ .

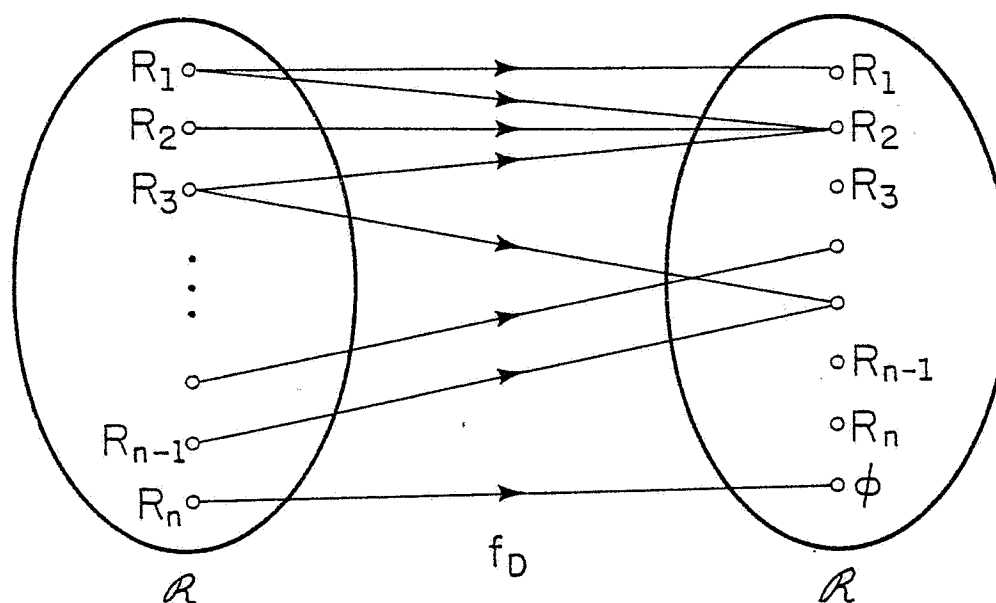
This situation can be best illustrated by means of a pictorial representation shown in Figure 3.1. In Figure 3.1(a) the mapping  $f_D$  is shown under the condition that there is no fault in the register decoding function. Under this condition,  $f_D$  is a one-to-one correspondence from  $\mathcal{R}$  to  $\mathcal{R}$ . When there is a fault in the register decoding function  $f_D$  becomes, in general, a many-to-many correspondence from  $\mathcal{R}$  to  $\mathcal{R} \cup \{\emptyset\}$ . This is illustrated in Figure 3.1(b).

We now briefly comment on the fault mechanisms responsible for faults in the register decoding function. Consider a subset of registers  $\mathcal{R}' \subseteq \mathcal{R}$  which is realized as a random-access memory on the microprocessor chip. Due to faults in the address decoder in this memory some registers





- (a).  $f_D$  is a one-to-one correspondence from  $\mathcal{R}$  to  $\mathcal{R}$  if there is no fault in the register decoding function.



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- (b).  $f_D$  is a many-to-many correspondence from  $\mathcal{R}$  to  $\mathcal{R} \cup \{\emptyset\}$  if there are faults in the register decoding function.

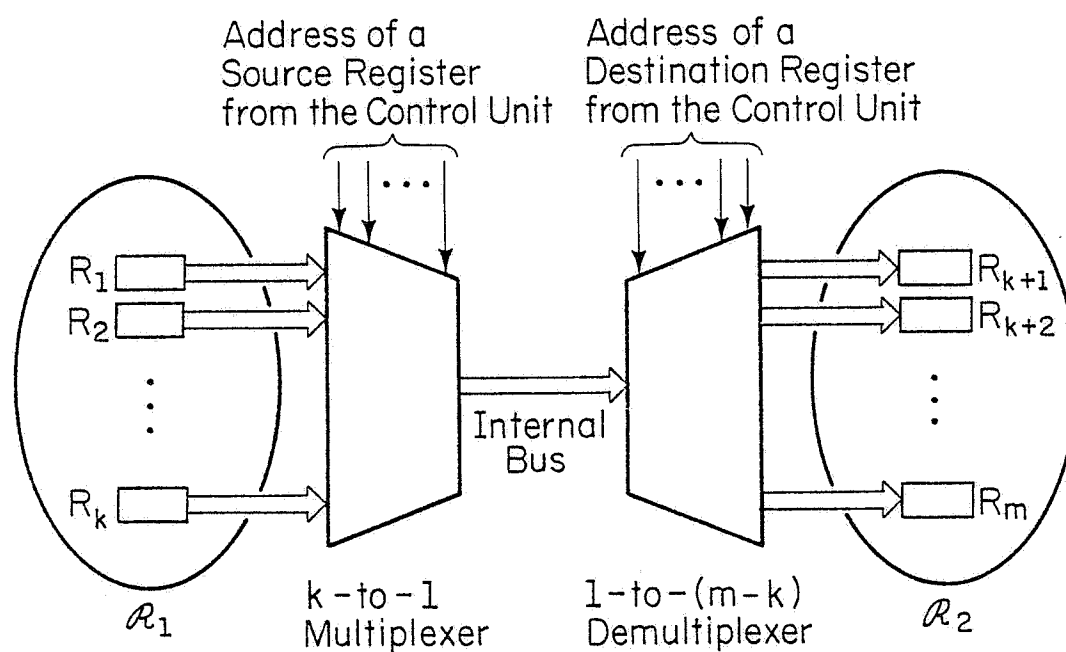
Figure 3.1. Representation of the mapping  $f_D$ .

in  $\mathcal{R}'$  could be decoded as some other registers in  $\mathcal{R}'$  [NTAb78]. This can be adequately modeled by a many-to-many correspondence from  $\mathcal{R}'$  to  $\mathcal{R}' \cup \{\emptyset\}$ . Of course, under this fault a register in  $\mathcal{R}'$  cannot be decoded as some other register not in  $\mathcal{R}'$ .

In order to rule out the possibility of a register  $R_1$  being decoded as another register  $R_2$  under a fault, we must know whether  $R_1$  and  $R_2$  belong to different random-access memories realized on the chip, or we must know of the existence of some mechanism (realizing  $R_1$  and  $R_2$ ) under which this fault cannot be present, i.e., we need to know the implementation details. Our desire is to make the fault model as independent as possible of the actual implementation. Therefore we allow  $f_D$  to be a many-to-many correspondence from  $\mathcal{R}$  to  $\mathcal{R} \cup \{\emptyset\}$  under a fault. We are thus considering the "worst case" behavior under the register decoding faults.

Alternatively registers could be realized as separate registers and interconnected with a network of multiplexers, demultiplexers and buses. A typical situation is shown in Figure 3.2, where it is desired to choose one register from a subset of registers  $\mathcal{R}_1 = \{R_1, R_2, \dots, R_k\} \subset \mathcal{R}$  for transferring its contents to a register to be chosen from another subset of registers  $\mathcal{R}_2 = \{R_{k+1}, R_{k+2}, \dots, R_m\} \subset \mathcal{R}$ . The task is accomplished by using a  $k$ -to-1 multiplexer for choosing a source register from  $\mathcal{R}_1$ , and a 1-to- $(m-k)$  demultiplexer for selecting a destination register from  $\mathcal{R}_2$ . The multiplexer and the demultiplexer receive the addresses of the registers to be selected from the control unit.

Due to faults in these units the wrong registers may be chosen, or more than one registers could be chosen. Under some fault in the control unit, an incorrect register address could be sent to the multiplexer



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Figure 3.2. A multiplexer-demultiplexer mechanism for data transfer.

or demultiplexer resulting into the selection of a wrong register. All these faults can be adequately modeled by a many-to-many correspondence from  $\mathcal{R}_1$  to  $\mathcal{R}_1 \cup \{\emptyset\}$ , and another many-to-many correspondence from  $\mathcal{R}_2$  to  $\mathcal{R}_2 \cup \{\emptyset\}$ . In this particular implementation, a register in  $\mathcal{R}_1$  may not be decoded as a register in  $\mathcal{R}_2$ , and vice-versa. We avoid all these implementation dependent details by allowing  $f_D$  to be a many-to-many correspondence from  $\mathcal{R}$  to  $\mathcal{R} \cup \{\emptyset\}$ .

At this point one may wonder how a register used to store address of operands such as  $R_5$  and  $R_7$  (in Figure 2.8), could be decoded as a register used to store operands, such as  $R_1$  and  $R_2$ , particularly in the light of the fact that the widths of registers used to store addresses usually differ from those used to store data. This is quite likely in the following situation.

All registers are realized as a RAM array on the microprocessor chip. Each word of the RAM is 16 bits in width and can be used as a single register for storing addresses which are 16 bits in width. A single word can also be used as a pair of registers for storing data which is 8 bits in width. This is the way registers are implemented on the INTEL 8080 microprocessor [INTE75].

We now extend the notation developed in Chapter 2.  $f_D(D(I_j))$  denotes the set of registers formed by making the union of the image sets of registers in  $D(I_j)$  under the mapping  $f_D$ .  $f_D(S(I_j))$  can be analogously defined. Extending this notation further,  $f_D(D(I_1, I_2, \dots, I_n)) = f_D(D(I_1)) \cup f_D(D(I_2)) \cup \dots \cup f_D(D(I_n))$ . The set  $f_D(S(I_1, I_2, \dots, I_n))$  can be defined similarly.

We now illustrate the effects of faults in the register decoding function at the level of the S-graph by means of the following example.

Example 3.1: In terms of the S-graph of Figure 2.8,

$\mathcal{R} = \{R_1, R_2, R_3, R_4, R_5, R_6, R_7\}$ . If  $f_D(R_5) = \{R_1\}$ , register  $R_1$  will play the role of the address buffer register  $R_5$  during the execution of instructions  $I_{17}$  and  $I_{19}$ . Thus under  $I_{17}$ ,  $R_2$  will be read out correctly, but the contents of register  $R_1$  will be changed instead of that of  $R_5$ . If  $f_D(R_1) = \{R_2\}$ , when  $I_1$  is executed  $R_2$  will be written instead of  $R_1$ . Moreover,  $I_7$  will read out  $R_2$  instead of  $R_1$ . If  $f_D(R_2) = \{R_1, R_3\}$ , then  $I_8$  will read out  $R_1 * R_3$ , where  $*$  denotes the bit-wise AND or OR function over registers  $R_1$  and  $R_3$  depending on technology. Similarly when  $I_2$  is executed, both  $R_1$  and  $R_3$  will be written instead of  $R_2$ .

If  $f_D(R_3) = \{\emptyset\}$ , then  $I_5$  will not change the contents of any register, and  $I_6$  will transfer a ONE<sup>†</sup> or a ZERO<sup>†</sup> to  $R_1$  depending on technology, instead of the contents of  $R_3$ . If  $f_D(R_7) = \{\emptyset\}$ , the "Jump to subroutine" instruction  $I_{20}$  will correctly execute the jump in the program sequencing, but will not save the return address into  $R_7$ . The fault will show up when the "Return from subroutine" instruction  $I_{21}$  is executed, because the program sequence will return to the main memory location whose address is ONE or ZERO depending on technology, as a ONE or a ZERO will be loaded into the program counter ( $R_6$ ) instead of the contents of  $R_7$ .

---

<sup>†</sup> ONE denotes a binary vector with each of its bits set to logic 1 and having its width equal to that of a register, i.e.,  $\text{ONE} = (11\dots 1)$ ; similarly ZERO stands for  $(00\dots 0)$ .

If  $f_D(R_5) = \{R_7\}$ , instruction  $I_{20}$  will be executed correctly, i.e., the jump to subroutine will occur and the return address will be correctly saved in  $R_7$ , but now the correct execution of  $I_{21}$  will depend on whether  $I_{17}$  or  $I_{19}$  were executed within the subroutine. If they were executed in the subroutine, due to the fault  $f_D(R_5) = \{R_7\}$ , the subroutine register  $R_7$  will be changed instead of the address buffer register  $R_5$ , resulting into the loss of the return address saved in  $R_7$ . Thus  $I_{21}$  will cause the program to branch to some location that equals the address of operand used in the last instruction  $I_{17}$  or  $I_{19}$  executed within the subroutine. □

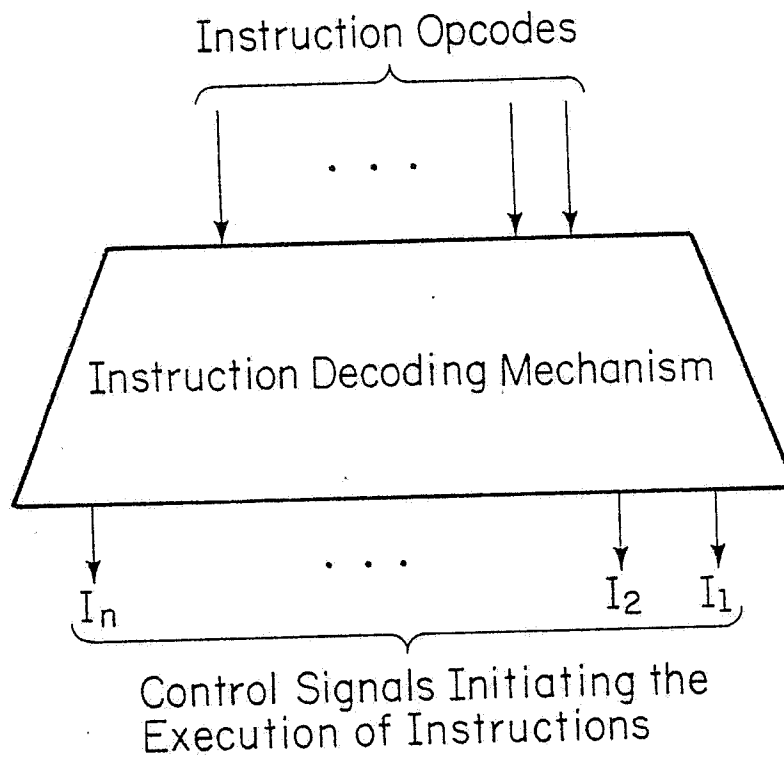
### 3.2. Fault Model for the Instruction Decoding and Control Function

The instruction decoding mechanism is shown as a block diagram in Figure 3.3. Basically it is a decoder whose inputs are the instruction opcodes and whose outputs correspond to the control signals that initiate the execution of instructions. For each valid opcode, one and only one output of the decoder is activated initiating the execution of one and only one instruction.

Under a fault in the instruction decoding and control function, the faulty behavior of the microprocessor can be specified as follows.

When instruction  $I_j$  is executed any one of the following can happen:

1. Instead of instruction  $I_j$  some other instruction  $I_k$  is executed. This fault is denoted by  $f(I_j/I_k)$ .
2. In addition to instruction  $I_j$ , some other instruction  $I_k$  is also activated. This fault is denoted by  $f(I_j/I_j + I_k)$ .
3. No instruction is executed. This fault is denoted by  $f(I_j/\emptyset)$ .



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Figure 3.3. Block diagram of the instruction decoding mechanism.

This fault model is strongly motivated by the fact summarized in the following theorem.

Theorem 3.1: If a decoder is realized without any reconvergent fanout then under a single stuck-at fault its behavior can be formulated independent of its implementation details as follows: for a given valid input to the decoder, instead of, or in addition to the expected output some other output is activated, or no output is activated.

Proof: See the Appendix. □

The assumption of no reconvergent fanout in the instruction decoding mechanism is quite reasonable as it has  $n$  inputs and as many as  $2^n$  outputs. We would like to allow the faulty behavior stated above for each instruction of the microprocessor. However, it makes the test generation procedures extremely complicated. Therefore we impose two constraints (given below as 4 and 5) on the decoder behavior under faults in the instruction decoding and control function.

4. If faults  $f(I_j/I_k)$  or  $f(I_j/I_j+I_k)$  are present then instruction  $I_k$  will be correctly executed.

5. If faults  $f(I_j/I_k)$ ,  $f(I_j/I_j+I_k)$  or  $f(I_j/\phi)$  are present then faults  $f(I_q/I_j)$  or  $f(I_q/I_q+I_j)$  cannot be present.

The behavior of a decoder under a single stuck-at fault does not violate these constraints. This will also be proved in the Appendix. Any number of instructions could be faulty subject to the set of specifications 1 through 5. As an example, under the fault model, faults  $f(I_1/I_2)$ ,  $f(I_3/I_3+I_2)$ ,  $f(I_4/I_2)$  can exist simultaneously, so can  $f(I_1/I_2)$ ,  $f(I_3/\phi)$ ,  $f(I_4/I_6)$ ,  $f(I_5/I_5+I_6)$ . Thus, this fault model can account for all single stuck-at faults in the instruction decoding mechanism.



In practice, some faults in the instruction decoding and control function such as  $f(I_j/I_k)$  or  $f(I_j/I_j+I_k)$  may be readily detected if a different number of machine cycles are needed to execute instructions  $I_j$  and  $I_k$ , or different status signals are emitted during their execution [ThAb78].

We now illustrate the effects of faults in the instruction decoding function at the level of the S-graph by means of the following example.

Example 3.2: In terms of the S-graph of Figure 2.8, under fault  $f(I_2/\emptyset)$ , register  $R_2$  will not be written, i.e., its contents remain unchanged. If  $f(I_4/I_6)$  is present, then the contents of  $R_3$  will be transferred to  $R_1$  instead of the sum of  $R_1$  and  $R_2$ . Under  $f(I_7/I_7+I_8)$ , the contents of  $R_1 * R_2$  will be read out, where as before,  $*$  indicates the bit-wise logical OR or AND function depending on technology. If  $f(I_9/I_9+I_3)$  is present, the "Jump" instruction  $I_9$  will be executed correctly, but at the same time the contents of  $R_1$  will also be transferred to  $R_2$ .

Note that the faults in the instruction decoding and control function cannot be treated as faults in the register decoding function. For example,  $f(I_3/I_5)$  cannot be treated as  $R_2$  being decoded as  $R_3$  if  $I_8$  is executed correctly. Under  $f(I_{10}/I_{21})$ , instead of the "Skip if the contents of  $R_1$  are zero" instruction the "Return from subroutine" instruction is executed. Under  $f(I_6/I_6+I_{13})$ ,  $(RESULT1) * (RESULT2)$  will be transferred to  $R_1$ , where  $RESULT1$  and  $RESULT2$  are the results produced by  $I_6$  and  $I_{13}$ , respectively, and  $*$  indicates the bit-wise logical OR or AND function between  $RESULT1$  and  $RESULT2$ . □

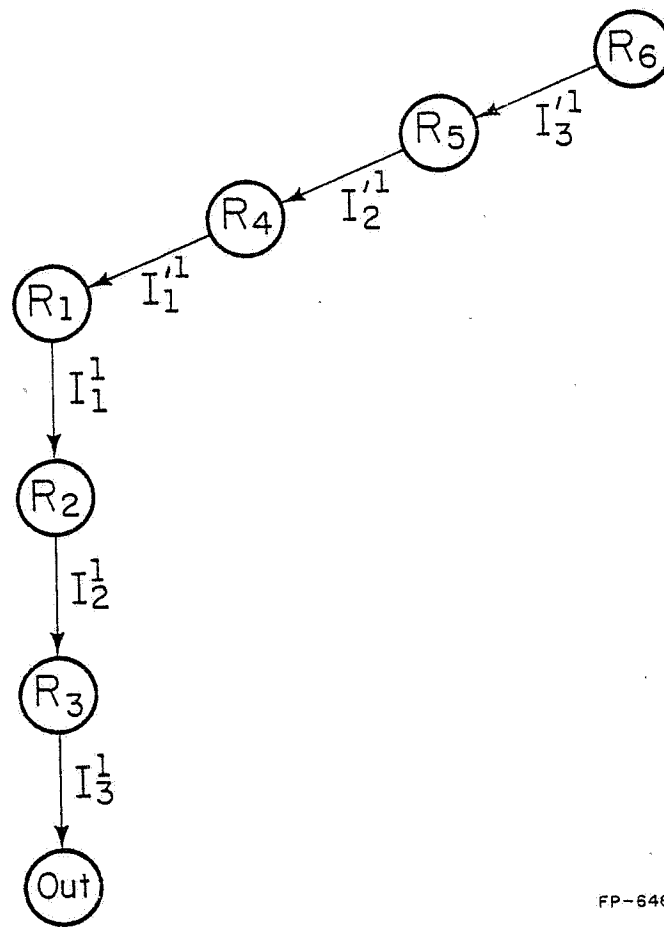
Example 3.3: This example is constructed to show how the result produced by executing a program in the presence of a multiple fault in the instruction decoding and control function differs from the one produced by executing the same program under the fault free condition. A part of the S-graph of a hypothetical microprocessor is shown in Figure 3.4(a).  $I_1, I_2, I_3, I'_1, I'_2, I'_3$  are instructions of class T. Note that instruction  $I_3$  reads out register  $R_3$ . We investigate how the simultaneous existence of three faults  $f(I_1/I_1+I'_1)$ ,  $f(I_2/I_2+I'_2)$ , and  $f(I_3/I_3+I'_3)$  affects the result produced by the microprocessor when it executes the program given in Figure 3.4(b). Only three instructions in the loop are shown. The loop control is given in terms of a high level language construct (FOR loop) only for conciseness and ease of understanding.

Assume that the initial contents of registers  $R_1, R_4, R_5, R_6$  are ONE, ONE, ONE, and ZERO, respectively. Thus under the fault free condition, at the end of each of the  $n$  iterations of the loop, a ONE is read out, independent of the value of  $n$ .

Under the presence of the multiple fault described above, the program would correctly read out a ONE at the end of each of the first three iterations of the loop, but would read out a ZERO instead of a ONE at the end of each iteration after that. Therefore the program would not detect the fault if  $n \leq 3$ . □

### 3.3. Fault Model for the Data Storage Function

In this section a fault model for the data storage function is presented which accounts for the faults in various registers. We allow any cell of a register to be stuck at 0 or 1, and this fault can occur



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(a). A part of the S-graph of a hypothetical microprocessor.

```

FOR K ← 1 TO n DO
  BEGIN
    .
    .
    .
    I1
    I2
    I3
    .
    .
    .
  END

```

(b). The program considered in Example 3.3.

Figure 3.4. Illustrating Example 3.3.

with any number of cells of any number of registers. We now illustrate the effects of faults in the data storage function at the level of the S-graph by means of the following example.

Example 3.4: In terms of the S-graph of Figure 2.8, suppose the first and third bit of register  $R_1$  are stuck at 1 and 0, respectively. Then, it would not be possible to store any data vector whose first and third bits are 0 and 1, respectively, in register  $R_1$ , by executing any instruction whose destination register is  $R_1$ . If the second bit of register  $R_7$  (subroutine register) is stuck at 0, then it would not be possible to execute the "Return from subroutine" instruction  $I_{21}$  successfully if the return address has its second bit equal to 1. Thus under a fault in the data storage function some instructions may not be correctly executed for certain data and address patterns. □

### 3.4. Fault Model for the Data Transfer Function

In this section a fault model for the data transfer function is presented which accounts for faults in various transfer paths, i.e., buses. Under a fault in the data transfer function for any instruction  $I_j$

1. a line in a transfer path in set  $T(I_j)$  can be stuck at 0 or 1,
2. two lines of a transfer path in set  $T(I_j)$  can be coupled, i.e., they fail to carry different logic values. This can happen due to metallization shorts or capacitive couplings [ThAb78].

We allow any number of transfer paths associated with any number of instructions to be faulty in this manner. This fault model is very general and is also independent of implementation details of transfer paths. Even though physical transfer mechanisms may be shared between

transfer paths in practice, by allowing each transfer path to be faulty, we are making the fault model independent of implementation details.

We now illustrate the effects of faults in the data transfer function at the level of the S-graph by means of the following example.

Example 3.5: Suppose the transfer path carrying data from the node  $R_2$  to the OUT node (i.e., the main memory) in instruction  $I_8$  has its second line stuck at 0, then instruction  $I_8$  cannot be executed successfully if the data pattern stored in register  $R_2$  has its second bit equal to 1. Suppose the transfer path used to carry the result from the ALU to register  $R_1$  in instruction  $I_4$  have its first and second line coupled such that the resulting logic value present on these two lines really is a logical OR or AND function (depending on technology) of the logic values that would have been present on these lines, were there no coupling.

Under this fault any ALU result whose first and second bits differ in the logic values will not be successfully transferred to  $R_1$ ; if the coupling results in a logical OR function, the first and second lines of the transfer path will both carry a logic 1 when they are supposed to carry a 1 and a 0, or a 0 and a 1. Similarly if the coupling results in a logical AND function, the first and second lines of the transfer path will both carry a logic 0 when they are supposed to carry a 1 and a 0, or a 0 and a 1. Similar faults could be present with the transfer paths used to carry addresses. Thus under a fault in the data transfer function some instructions cannot be correctly executed for certain data and address patterns. □

### 3.5. Fault Model for the Data Manipulation Function

The data manipulation function refers to various functional units such as the ALU, interrupt handling hardware, hardware used for incrementing (or decrementing) the program counter, stack pointer or index register, hardware used for computing the address of operands in various addressing modes such as indexed and relative, etc.

A microprocessor is not a network of arbitrary interconnections of these functional units. Therefore we need not really worry about the problems involved in testing a digital system comprised of a network of such functional units as mentioned in Section 2.1.3. In fact, recalling the discussion of Section 2.4 any register of a microprocessor can be read or written (explicitly or implicitly) using a sequence of instructions of class T, or using an instruction of class B, i.e., the operands required for an instruction of class M can be stored in the necessary registers (or are available in the main memory) and the result produced by it can be read out from the register where it is stored by using instructions which do not belong to class M.

We do not propose any specific fault model, per se, for the data manipulation function because of the wide variety in existing designs for the ALU and other functional units such as increment or shift logic. We will assume that complete test sets can be derived for the functional units for any given fault model. The operands necessary to execute tests for a given functional unit can be stored in the proper registers by executing instructions of class T or B only, and they do not require the use of any other functional unit. Similarly the results of these tests can be read out by using instructions of class T or B only.

We allow any number of functional units to be faulty at one time.

### 3.6. Fault Model for Microprocessors

We now propose the fault model for microprocessors based on the fault models proposed in Sections 3.1 through 3.5. At any given time we allow the presence of any number of faults but only in one function described above (in Sections 3.1 through 3.5). Note that we are allowing a very general model for microprocessors (as described in Chapter 2). In addition, if we allow multiple faults in different functions, the problem becomes extremely complex. In [ThAb78], a restricted model for microprocessor was considered, (refer to Section 2.5) allowing multiple faults in different functions. In that case the problem turned out to be very complex but of manageable proportions.

#### 4. TEST GENERATION PROCEDURES

In this chapter we present test generation procedures to generate tests for detecting faults covered by the fault models presented in Sections 3.1 through 3.5. The first step in developing test generation procedures is to assign labels to the nodes and edges of the S-graph under consideration by using the labeling algorithm given in Section 4.1. Test generation procedures for detecting faults in the register decoding function, instruction decoding and control function, data transfer and storage function, and data manipulation function are given in Sections 4.2, 4.3, 4.4 and 4.5, respectively. The fault coverage of the tests is also proved.

##### 4.1. Algorithm 4.1: The Labeling Algorithm

This algorithm assigns integer labels to nodes and edges. The label assigned to a node representing register  $R_i$  is denoted by  $\ell(R_i)$ , and the label assigned to the edge set  $E(I_j)$  representing instruction  $I_j$  is denoted by  $\ell(I_j)$ .

Step 1: Assign a label 0 to the OUT node.

Step 2:  $K \leftarrow 0$ ;

WHILE a node remains unlabeled DO

BEGIN

Assign a label  $K+1$  to all unlabeled nodes representing registers whose contents can be transferred (explicitly or implicitly) to any register whose node is labeled  $K$  by executing a single instruction of class T or B;

$K \leftarrow K + 1$

END



Step 3: Assign a label  $l$  to each edge in the set  $E(I_j)$  where  $I_j$  is an instruction that reads out a register (explicitly or implicitly) during its execution.

Step 4: If  $I_j$  is an instruction whose edge set has not been labeled in step 3 then assign a label  $(K+1)$  to each edge in the set  $E(I_j)$ , where  $l(D(I_j)) = K$ . □

Thus the labeling algorithm first assigns an integer label to each node of the S-graph. This label (which is equal to  $|\text{READ}(R_i)|$  as will be shown in Lemma 4.1) indicates the shortest "distance" of that node to the OUT node, i.e., the minimum number of instructions of class T or B that need to be executed to read out (explicitly or implicitly) the contents of the register being represented by that node. After assigning labels to the nodes of the S-graph, the labeling algorithm assigns labels to the edges representing instructions. In step 4, each edge in the set  $E(I_j)$  representing instruction  $I_j$  is assigned a label  $K + 1$ , if the destination register of  $I_j$  was assigned a label  $K$  in step 1 or 2. Note that in step 4, the edge sets of only those instructions are labeled which do not cause data transfers from registers of the microprocessor to the main memory or an I/O device during their execution. For such an instruction  $I_j$ ,  $|D(I_j)| = 1$ . (Recall the discussion in Section 2.4.)

On the other hand, the destination set  $D(I_j)$  of an instruction  $I_j$  that reads out (explicitly or implicitly) a register during its execution may contain more than one register. Since the nodes representing these registers may have different labels, step 4 cannot be applied in this

case. In this case we use step 3 to assign a label 1 to each edge in the set  $E(I_j)$ . The choice of the label may be explained by the fact that in this case the OUT node has to be a member of the set  $D(I_j)$ , and step 1 assigns a label 0 to the OUT node.

For concise description, we will use the phrase "register  $R_i$  with label  $K$ ," if the node representing register  $R_i$  is labeled  $K$ . Similarly we will use the phrase "instruction  $I_j$  with label  $K$ ," if the edge set  $E(I_j)$  representing instruction  $I_j$  is labeled  $K$ . The phrase "Execute  $READ(R_i)$ " means execute instructions in the  $READ(R_i)$  sequence; the phrase "Execute  $WRITE(R_i)$ " can be interpreted in a similar fashion.

Lemma 4.1: a) If  $\ell(R_i) = K$ ,  $|READ(R_i)| = K$ . b) If  $\ell(I_j) = 1$ ,  $I_j$  reads out (explicitly or implicitly) a register with label 1.

Proof: a) Nodes are labeled in step 2 of the labeling algorithm. A node is labeled 1 if the register represented by it can be read out (explicitly or implicitly) by executing a single instruction of class T or B. A node is labeled 2 if the contents of the register represented by it can be transferred to a register whose node is labeled 1 by executing a single instruction of class T or B, and the former register (whose node is labeled 2) cannot be read out by executing a single instruction of class T or B. Thus a register  $R_i$  whose node is labeled 2 can be read out by executing a sequence of instructions of class T or B containing two instructions and no shorter sequence exists to read it out. Therefore  $|READ(R_i)| = 2$ . (Recall the definition of  $READ(R_i)$  in Section 2.4.) Extending the argument in this fashion it can be easily proved that a register  $R_i$  whose node is labeled  $K$  can be read out by executing a sequence of instructions of class T or B containing  $K$  instructions and

no shorter sequence exists to read it out. Therefore  $|\text{READ}(R_i)| = K$ .

(b) If  $\ell(I_j) = 1$ ,  $I_j$  must have been labeled in step 3 of the labeling algorithm, and it reads out a register (explicitly or implicitly) during its execution. Therefore this register must have been labeled 1 in step 2. □

We now comment on the significance of the labels assigned by the labeling algorithm. For each register  $R_i$ ,  $\ell(R_i)$  indicates the minimum number of instructions of class T or B needed to read out  $R_i$ . Therefore  $\ell(R_i)$  can be thought of as an "observability index" for register  $R_i$ .  $\ell(I_j)$  has a similar connotation. If  $\ell(I_j) \geq 2$ ,  $\ell(I_j) - 1$  indicates the minimum number of instructions of class T or B needed to read out register  $D(I_j)$ . If  $\ell(I_j) = 1$ , instruction  $I_j$  reads out some register with label 1; thus the effects of execution of instruction  $I_j$  are directly observable at the external pins of the microprocessor if  $\ell(I_j) = 1$ .

Various test generation procedures to be presented in the following sections of this chapter generate tests in such a way that the knowledge gained from the correct execution of tests used to check the decoding of registers and instructions with lower labels is utilized in generating tests to check the decoding of registers and instructions with higher labels. Thus the fact that a register with a given label can be correctly "observed" is used to generate suitable tests for correctly observing registers with higher labels. Recall that a register with a lower label implies that it has better observability than the one with a higher label.

These test generation procedures may generate instructions with higher labels to set up proper operands in various registers while "checking out" instructions with lower labels. Since, as described above,

the instructions with higher labels are not checked out yet, they may not be able to set up the required operands successfully. This imposes the basic requirement on the test generation procedure: the tests must be able to check for proper execution of every instruction using other potentially faulty instructions; otherwise certain faults may mask each other and never be detected. This point will be illustrated by means of examples in Sections 4.2 and 4.3.

Since each instruction is checked for its proper execution using other potentially faulty instructions, it is not necessary to devise some labeling scheme that assigns labels to registers indicating their "distance" from the IN node which can signify their "controllability index." The test generation procedures take into consideration the presence of faulty instructions (which may fail to store required operands in registers, i.e., fail to control the registers correctly) to be used in checking out other instructions.

Recall (Example 2.4) that those instructions of class B which only change the logic level on some status pins of the microprocessor (e.g. "Interrupt enable") are not represented in the S-graph. Therefore, they are not labeled by the labeling algorithm. We assign  $\ell(I_j) = 1$  for every instruction  $I_j$  of class B which is not represented in the S-graph. This assignment can be justified as follows: the effects of these instructions of class B are directly observable at the external pins of the microprocessor. Since the instructions which read out registers are labeled 1 by the labeling algorithm, it makes sense to assign label 1 to these instructions of class B.

Lemma 4.2: All instructions of class B are assigned label 1.

Proof: Those instructions of class B which are represented in the S-graph implicitly read out the program counter or a register containing the address of an instruction. These instructions are assigned label 1 in step 3 of the labeling algorithm. All the other instructions of class B (which are not represented in the S-graph) are also assigned label 1 as explained above.  $\square$

Recall that instructions with label 1 have the highest observability. Instructions of class B enjoy the highest observability.

Example 4.1: The labeling algorithm will assign the following labels to the nodes and edges of the S-graph in Figure 2.8.

Step 1:  $\ell(\text{OUT}) = 0$ .

Step 2:  $\ell(R_1) = \ell(R_2) = \ell(R_4) = \ell(R_5) = \ell(R_6) = \ell(R_7) = 1$ ,  $\ell(R_3) = 2$ .

Step 3:  $\ell(I_7) = \ell(I_8) = \ell(I_9) = \ell(I_{10}) = \ell(I_{14}) = \ell(I_{16}) = \ell(I_{17}) = \ell(I_{18}) = \ell(I_{19}) = \ell(I_{20}) = \ell(I_{21}) = 1$ .

Step 4:  $\ell(I_1) = \ell(I_2) = \ell(I_3) = \ell(I_4) = \ell(I_6) = \ell(I_{11}) = \ell(I_{12}) = \ell(I_{13}) = \ell(I_{15}) = 2$ ,  $\ell(I_5) = 3$ .

The contents of the program counter ( $R_6$ ) are read out (implicitly) on the address bus during the fetching of every instruction, therefore  $\ell(R_6) = 1$ . The contents of the subroutine register ( $R_7$ ) are implicitly read out on the address bus (by routing the contents through the program counter), hence  $\ell(R_7) = 1$ . Note that  $\ell(I_1) = \ell(I_2) = 2$ , because  $I_1$  and  $I_2$  both use immediate addressing (Refer to Table 2.1.), and  $|D(I_1)| = |D(I_2)| = 1$ . All other labels are self explanatory.  $\square$

#### 4.2. Test Generation Procedure for Detecting Faults in the Register Decoding Function

The tests generated using the procedure guarantee that the register decoding function denoted by the mapping  $f_D$  is a one-to-one correspondence from  $\mathcal{R}$  to  $\mathcal{R}$ .

This procedure uses two data structures, a queue of registers and a set of registers which are named  $Q$  and  $A$  respectively. The queue  $Q$  is initialized with all the registers such that a register  $R_i$  lies ahead of another register  $R_j$  in the queue, if and only if,  $\ell(R_i) \leq \ell(R_j)$ . The set  $A$  is initialized to be empty. In each iteration of the test generation procedure, set  $A$  is progressively augmented by removing the register lying in the front of the queue  $Q$  and including it in set  $A$ ; now the register which was second in the queue before the augmentation of set  $A$  lies in the front of the queue, i.e., the queue is updated. The tests generated so far will assure that at any given stage, registers in set  $A$  have disjoint image sets under mapping  $f_D$ . The procedure terminates when set  $A$  contains all the registers that were initially in the queue and the queue gets empty. At this stage, the generated tests will guarantee that all the registers have disjoint image sets under mapping  $f_D$ , establishing that  $f_D$  is a one-to-one correspondence. Recalling the discussion in Section 4.1, the procedure utilizes the knowledge gained from the correct execution of tests used to check decoding of registers with lower labels to generate tests to check decoding of registers with higher labels.

ONE and ZERO will be frequently used as operands or addresses of operands in various test procedures. This choice is arbitrary. We could have used (1010...10) and its bit-wise complement (0101...01) as operands or addresses of operands instead.

Procedure 4.1 given below generates tests for detecting faults in the register decoding function. Note that the test instructions are generated only in steps 3(a), (b), and (c); other steps perform bookkeeping tasks.

Procedure 4.1: Procedure to generate tests for detecting faults in the register decoding function.

Step 1: Initialize the queue Q with all the registers such that register  $R_i$  lies ahead of register  $R_j$ , if and only if,  $\ell(R_i) \leq \ell(R_j)$ ; Initialize the set A as empty.

Step 2:  $A \leftarrow$  register at the front of Q; Update Q.

Step 3: REPEAT

- a) Generate instructions to write each register  $R_i$  of set A with data ONE, and the register at the front of Q (if there is one) with data ZERO. (The instructions of the corresponding WRITE ( $R_i$ ) sequences need to be generated.)
- b) Generate instructions to read out each register  $R_i$  of set A, such that register  $R_i$  will be read before register  $R_j$ , if and only if,  $\ell(R_i) \leq \ell(R_j)$ . (The instructions of the corresponding READ ( $R_i$ ) sequences need to be generated.)
- c) Generate instructions to read out the register  $R_j$  at the front of Q (if there is one). (The instructions of the READ ( $R_j$ ) sequence need to be generated.)
- d)  $A \leftarrow A \cup \{\text{Register at the front of Q}\}$ .
- e) Update Q.

UNTIL Q = empty.

Step 4: Repeat steps 1, 2, and 3 with complementary data. □

Procedure 4.1 describes the test generation procedure at a higher level. There are many subtle points involved in the execution details of this procedure, particularly if some registers can be written or read out only implicitly. These points can be best illustrated by giving an example. We show how to apply this algorithm to generate the tests for the S-graph of Figure 2.8 in the following example accompanied with the explanatory comments.

Example 4.2: Generation of tests for detecting faults in the register decoding function for the S-graph of Figure 2.8.

Step 1:  $Q \leftarrow R_1 R_2 R_4 R_5 R_6 R_7 R_3$ ;  $A \leftarrow \emptyset$

Step 2:  $A \leftarrow \{R_1\}$ ;  $Q \leftarrow R_2 R_4 R_5 R_6 R_7 R_3$

Step 3:

Iteration 1

- a)  $I_1$  with operand ONE;  $I_2$  with operand ZERO;
- b)  $I_7$ ; /Expected output data = ONE/
- c)  $I_8$ ; /Expected output data = ZERO/
- d)  $A \leftarrow \{R_1, R_2\}$
- e)  $Q \leftarrow R_4 R_5 R_6 R_7 R_3$

Iteration 2

- a)  $I_1$  with operand ONE;  $I_2$  with operand ONE;  
 $I_{15}$  with operand ZERO; /stack pointer ( $R_4$ ) is written with a ZERO/
- b)  $I_7$ ;  $I_8$ ; / $R_1$  and  $R_2$  are read out; expected output data = ONE/
- c)  $I_{16}$ ; /stack pointer is implicitly "read out" on the address bus; expected output "data" = ZERO/



$$d) A \leftarrow \{R_1, R_2, R_4\}$$

$$e) Q \leftarrow R_5 R_6 R_7 R_3$$

### Iteration 3

- a)  $I_1$  with operand ONE;  $I_2$  with operand ONE;  
 $I_{15}$  with operand ONE;  $/R_1, R_2$ , stack pointer ( $R_4$ ) are  
 written with data ONE/  
 $I_{17}$  with address of the operand ZERO;  $/R_5$  is written  
 implicitly with "data" ZERO/
- b)  $I_7; I_8; I_{16}$ ;  $/R_1$  and  $R_2$  are explicitly read out while  
 the stack pointer ( $R_4$ ) is implicitly read out/
- c)  $I_{17}$  with the address of operand ZERO;  $/R_{15}$  is implicitly  
 read out on the address bus with expected output  
 "data" = ZERO/
- d)  $A \leftarrow \{R_1, R_2, R_4, R_5\}$
- e)  $Q \leftarrow R_6, R_7, R_3$

### Iteration 4

- a)  $I_1$  with operand ONE;  $I_2$  with operand ONE;  
 $I_{15}$  with operand ONE;  $/R_1, R_2$ , stack pointer ( $R_4$ ) are  
 written with data ONE/  
 $I_{17}$  with the address of operand ONE;  $/R_5$  is written  
 implicitly with "data" ONE/
- $I_9$  with jump address = LOC 1; /program counter is  
 written implicitly with data = LOC 1. LOC 1 is chosen  
 different from ONE/

- b,c) LOC 1:  $I_7$ ;  $/I_7$  is stored in location LOC 1. When this instruction is fetched, the program counter is implicitly read out on the address bus; expected output "data" = LOC 1  $\neq$  ONE.  $R_1$  is explicitly read out as  $I_7$  is executed/  
 $I_8$ ;  $I_{16}$ ;  $I_{17}$  with the address of operand ONE;  $/R_2$  is explicitly read out.  $R_4$  and  $R_5$  are implicitly read out/  
 d)  $A \leftarrow \{R_1, R_2, R_4, R_5, R_6\}$   
 e)  $Q \leftarrow R_7, R_3$

#### Iteration 5

- a)  $I_1$  with operand ONE;  $I_2$  with operand ONE;  
 $I_{15}$  with operand ONE;  $/R_1, R_2$ , stack pointer ( $R_4$ ) are written with data ONE/  
 $I_{17}$  with the address of operand ONE;  $/R_5$  is written implicitly/with "data" ONE/  
 $I_9$  with jump address = LOC 2;  $/$ program counter is written implicitly with data = LOC 2/  
 LOC 2:  $I_{20}$  with jump address = LOC 3;  $/$ Note that  $I_{20}$  is the "Jump to subroutine" instruction, hence program counter (now containing LOC 2 + 1) is saved in the subroutine register ( $R_7$ ), and a new jump address = LOC 3 is loaded into the program counter. Thus  $R_6$  and  $R_7$  are written implicitly with "data" LOC 3 and LOC 2 + 1, respectively. Choose LOC 3 different from LOC 2 + 1 and also different from ONE/

b,c) LOC 3:  $I_7$ ;  $I_7$  is stored in LOC 3. When  $I_7$  is fetched, the program counter is read out implicitly on the address bus; expected output "data" = LOC 3. Thus,  $I_7$  is the first instruction in the subroutine.

$I_7$  explicitly reads out  $R_1$ /

$I_8$ ;  $I_{16}$ ;  $I_{17}$  with the address of operand ONE;  $R_2$  is explicitly read out.  $R_4$  and  $R_5$  are implicitly read out/

$I_{21}$ ;  $I_{21}$  is the "Return from subroutine" instruction.

The contents of the subroutine register ( $R_7$ ) are transferred to the program counter. The next instruction will be fetched from the location LOC 2 + 1, as LOC 2 + 1 is the return address for the subroutine. When this new instruction is fetched, the subroutine register will be effectively read out through the program counter/

d)  $A \leftarrow \{R_1, R_2, R_4, R_5, R_6, R_7\}$

e)  $Q \leftarrow R_3$

#### Iteration 6

a)  $I_1$  with operand ZERO;  $I_1$  is stored in location LOC 2 + 1. When it is fetched  $R_7$  is implicitly read out as explained before/

$I_5$ ;  $R_3$  is written with data ZERO/

$I_1$  with operand ONE;  $I_2$  with operand ONE; are

$I_{15}$  with operand ONE;  $I_{17}$  with the address of operand ONE;

$R_1$ ,  $R_2$ ,  $R_4$ , and  $R_5$  are written with "data" ONE/

$I_{19}$  with the address of operand ONE/

$I_9$  with jump address = LOC 4; /the program counter implicitly written with LOC 4. Choose LOC 4 different from ZERO/

LOC 4:  $I_{20}$  with jump address = LOC 5; /the subroutine register ( $R_7$ ) is written implicitly with data LOC 4 + 1.  
Choose LOC 4 + 1 different from ZERO. When  $I_{20}$  is fetched the program counter is implicitly read out; expected output data = LOC 4/

b) LOC 5:  $I_7; I_8; I_{16}; I_{17}$  with the address of operand ONE; / $R_1$  and  $R_2$  are explicitly read out;  $R_4$  and  $R_5$  are implicitly read out/

$I_{21}$ ; /causes the subroutine register ( $R_7$ ) to be implicitly read out through the program counter when the next instruction will be fetched from location LOC 4 + 1 /

c) LOC 4 + 1:  $I_6; I_7$ ; / $R_3$  is read out using READ ( $R_3$ ) =  $\langle I_6, I_7 \rangle$  sequence; expected output data = ZERO. When  $I_6$  is fetched  $R_7$  is implicitly read out as explained above/

d)  $A \leftarrow \{R_1, R_2, R_4, R_5, R_6, R_7, R_3\}$

e)  $Q \leftarrow \text{empty}$

Step 4: Repeat steps 1, 2, 3 using complementary data. □

The generated test sequence will be  $I_1, I_2, I_7, I_8, I_1, I_2, I_{15}, \dots, I_7, I_8, I_{16}, I_{17}, I_{21}, I_6, I_7$ . Thus the "writing" process involved in step 3(a), and the "reading" process involved in step 3(b) and (c), do involve implicit writing and reading of registers. The whole procedure requires careful selection of "data", i.e., both the operands and address of operands. The jump addresses in the "Jump", "Jump to

subroutine", and "Return from subroutine" instructions must be carefully chosen to avoid reexecuting the already executed tests or overwriting the instructions.

We now present a lemma describing the behavior of a micro-processor under faulty register decoding. This will then be used to prove that the tests generated using Procedure 4.1 will detect any detectable fault in the register decoding function.

Lemma 4.3: Let  $\text{READ}(R_i) = \langle I_{k_1}, I_{k_2}, \dots, I_{k_m} \rangle$  and  $\text{WRITE}(R_i) = \langle I_{p_1}, I_{p_2}, \dots, I_{p_n} \rangle$ .

a) When  $R_i$  is written with data  $d$  by executing the instructions in the  $\text{WRITE}(R_i)$  sequence, all registers in the set  $f_D(D(\text{WRITE}(R_i))) = f_D(D(I_{p_1}, I_{p_2}, \dots, I_{p_n}))$  are written with data  $d$ , unless (1).  $f_D(D(I_{p_j})) = \{\emptyset\}$  for some  $I_{p_j} \in \text{WRITE}(R_i)$ ,  $1 \leq j \leq n-1$ , in which case  $R_i$  is written with either a ONE or a ZERO depending on technology, or (2).  $f_D(D(I_{p_n})) = \{\emptyset\}$ , in which case the contents of  $R_i$  remain unchanged.

b) When  $R_i$  is read out by executing the instructions in the  $\text{READ}(R_i)$  sequence, during the process of reading out  $R_i$  all registers in the set  $f_D(D(I_{k_1}, I_{k_2}, \dots, I_{k_m}))$  are written with data  $d$  and data  $d$  is read out, if  $R_i$  contains data  $d$ , unless  $f_D(S(I_{k_j})) = \{\emptyset\}$ , for some  $I_{k_j} \in \text{READ}(R_i)$ , in which case a ONE or a ZERO will be read out, depending on technology.

Proof: The lemma follows immediately from the faulty behavior of the register decoding function  $f_D$ . □

Example 4.3: With reference to the S-graph of Figure 2.8,  $\text{READ}(R_3) = \langle I_6, I_7 \rangle$  and  $\text{WRITE}(R_3) = \langle I_1, I_5 \rangle$ . When  $R_3$  is written with data  $d$  by executing  $I_1$  with data  $d$ , and  $I_5$ , all registers in the set

$f_D(D(\text{WRITE}(R_3))) = f_D(D(I_1, I_5)) = f_D(D(I_1)) \cup f_D(D(I_5)) = f_D(R_1) \cup f_D(R_3)$   
 are written with data  $d$ , unless  $f_D(R_1) = \{\emptyset\}$ , in which case  $R_3$  is written  
 with either a ONE or a ZERO, or  $f_D(R_3) = \{\emptyset\}$ , in which case the contents of  
 $R_3$  remain unchanged. When  $R_3$  is read out by executing  $I_6$  and  $I_7$ , all registers  
 in the set  $f_D(R_1)$  are written with data  $d$  and data  $d$  is read out, unless  $f_D(R_1)$   
 $= \{\emptyset\}$  or  $f_D(R_3) = \{\emptyset\}$ , in which case either a ONE or a ZERO is read out.  $\square$

**Theorem 4.1:** The test sequence generated by using Procedure 4.1 is capable of detecting any detectable fault in the fault model for the register decoding function.

**Proof:** The proof is by induction. At the beginning of step 3(a) of Procedure 4.1, let set  $A = \{R_{i_1}, R_{i_2}, \dots, R_{i_k}\}$ . Let the induction hypothesis be  $f_D(R_{i_1}) \cap f_D(R_{i_2}) \cap \dots \cap f_D(R_{i_k}) = \{\emptyset\}$ , and  $f_D(R_{i_j}) \neq \{\emptyset\}$ , for each  $R_{i_j} \in A$ . At the end of step 3(d) of Procedure 4.1, i.e., when set  $A$  is augmented, let set  $A = \{R_{i_1}, R_{i_2}, \dots, R_{i_k}, R_{i_{k+1}}\}$ . We will prove that  $f_D(R_{i_1}) \cap f_D(R_{i_2}) \cap \dots \cap f_D(R_{i_k}) \cap f_D(R_{i_{k+1}}) = \{\emptyset\}$ , and  $f_D(R_{i_{k+1}}) \neq \{\emptyset\}$ .

In step 3(a), registers of set  $A$  are written with data ONE (ZERO), and register  $R_{i_{k+1}}$ , which is at the front of the queue, is written with data ZERO (ONE), by executing\* the instructions in the corresponding WRITE sequences. If  $f_D(R_{i_{k+1}}) = \{\emptyset\}$ , the fault will be readily detected when appropriate instructions are executed to read out  $R_{i_{k+1}}$  in step 3(c), as it will fail to produce either a ONE or a ZERO following Lemma 4.3. Assume that for some register  $R_{i_\ell}$  of set  $A$ ,  $f_D(R_{i_\ell}) \cap f_D(R_{i_{k+1}}) \neq \{\emptyset\}$ . If  $R_{i_\ell}$  is written after  $R_{i_{k+1}}$  in step 3(a), the fault will be detected when

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\* Though strictly speaking Procedure 4.1 is a test generation procedure in the proof we are assuming that the tests are executed.

$R_{i_{k+1}}$  is read out in step 3(c), since according to Lemma 4.3, all the registers in the set  $f_D(R_{i_\ell})$ , and hence in the set  $f_D(R_{i_\ell}) \cap f_D(R_{i_{k+1}})$  will be written with ONE (ZERO). Since  $f_D(R_{i_\ell}) \cap f_D(R_{i_{k+1}}) \subseteq f_D(R_{i_{k+1}})$ , when  $R_{i_{k+1}}$  is read out in step 3(c), it will either produce a ONE instead of a ZERO, or a ZERO instead of a ONE. Note that since  $\ell(R_{i_\ell}) \leq \ell(R_{i_{k+1}})$ , the process of reading out of  $R_{i_\ell}$  will not require routing of  $R_{i_\ell}$  through  $R_{i_{k+1}}$ . Similar arguments apply when  $R_{i_\ell}$  is written before  $R_{i_{k+1}}$  in step 3(a). In this case the fault will be detected when  $R_{i_\ell}$  is read out in step 3(c).

The basis of induction, i.e., when  $A = \{R_{i_1}\}$ ,  $f_D(R_{i_1}) \neq \{\emptyset\}$ , and when  $A = \{R_{i_1}, R_{i_2}\}$ ,  $f_D(R_{i_1}) \cap f_D(R_{i_2}) = \{\emptyset\}$  can be readily proved following the same arguments used so far. Using these arguments, it is guaranteed that  $\bigcap_{i=1}^n f_D(R_{i_1}) = \{\emptyset\}$ , where  $\mathcal{R} = \{R_1, R_2, \dots, R_n\}$ . Since all the registers have disjoint image sets under mapping  $f_D$ ,  $f_D$  cannot be a many-to-one correspondence. Moreover, since  $f_D(R_{i_1}) \neq \{\emptyset\}$  for each  $R_{i_1} \in \mathcal{R}$ ,  $f_D$  cannot be a one-to-many correspondence from  $\mathcal{R}$  to  $\mathcal{R}$ . Therefore  $f_D$  is guaranteed to be a one-to-one correspondence from  $\mathcal{R}$  to  $\mathcal{R}$ . It follows immediately that the register decoding function (denoted by  $f_D$ ) is free of any detectable fault. Note that for some registers, even if  $f_D(R_{i_1}) \neq \{R_{i_1}\}$ ,  $f_D$  could still be a one-to-one correspondence. For example, we may have  $f_D(R_{i_1}) = \{R_{j_1}\}$  and  $f_D(R_{j_1}) = \{R_{i_1}\}$ . In such a case, the fault is an undetectable fault. □

#### 4.3. Test Generation Procedures for Detecting Faults in the Instruction Decoding and Control Function

In this section we present the test generation procedures to detect faults  $f(I_j/\phi)$ ,  $f(I_j/I_k)$ , and  $f(I_j/I_j+I_k)$  for each ordered pair of instructions  $I_j$  and  $I_k$ . We divide the overall task of test generation into three subtasks depending on which class (T, M, or B) instructions  $I_j$  and  $I_k$  belong to. Following this, we first give the order in which the tests are applied, and then describe the details of test generation.

The overall task of detecting faults in the instruction decoding and control function can be divided into three subtasks.

subtask 1: Test for fault  $f(I_j/\phi)$ ,  $f(I_j/I_k)$ , and  $f(I_j/I_j+I_k)$ ,  
where  $I_j \in \text{class T}$ , and  $I_k \in \text{class } (T \cup M)$ .

subtask 2: Test for faults  $f(I_j/\phi)$ ,  $f(I_j/I_k)$ , and  $f(I_j/I_j+I_k)$ ,  
where  $I_j \in \text{class M}$ , and  $I_k \in \text{class } (T \cup M)$ .

subtask 3: Test for faults  $f(I_j/\phi)$ ,  $f(I_j/I_k)$ , and  $f(I_j/I_j+I_k)$ ,  
where  $I_j \in \text{class B}$ , and  $I_k \in \text{class } (T \cup M \cup B)$ ;  
or  $I_j \in \text{class } (T \cup M)$  and  $I_k \in \text{class B}$ .

The basic philosophy behind this task division is to employ a systematic approach that tackles a complex problem by dividing it into logically distinct and smaller subproblems.

##### 4.3.1. Order of Test Application

Before presenting the details of test generation, we first describe the order in which the tests are applied. The tests for each subtask described above are to be executed by the microprocessor under test in the order given below.



First we concentrate on instructions with label 1, i.e.,  $\ell(I_j) = \ell(I_k) = 1$ , and apply tests to detect faults  $f(I_j/\phi)$ ,  $f(I_j/I_k)$ , and  $f(I_j/I_j+I_k)$ . Then we apply tests to detect faults  $f(I_j/I_j+I_k)$ , where  $\ell(I_j) = 1$  and  $\ell(I_k) = 2$ . This is followed by tests to detect faults  $f(I_j/I_j+I_k)$ , where  $2 \leq \ell(I_j) \leq K_{\max}$  and  $\ell(I_k) = 1$ . ( $K_{\max}$  indicates the maximum value of the labels of edges representing instructions in the S-graph.) Thus we check that all instructions with label 1 are decoded correctly, no instruction with label 1 causes additional execution of an instruction with label 2, and no instruction with a label greater than 1 causes additional execution of an instruction with label 1. This procedure can be easily generalized and is given in a precise algorithmic form below.

Algorithm 4.2: Algorithm to determine the order of test application for detecting  $f(I_j/\phi)$ ,  $f(I_j/I_k)$ , and  $f(I_j/I_j+I_k)$ .

FOR  $K \leftarrow 1$  TO  $K_{\max}$  DO

BEGIN

Step 1: Apply tests to detect faults  $f(I_j/\phi)$ ,  $f(I_j/I_k)$ , and  $f(I_j/I_j+I_k)$ , where  $\ell(I_j) = \ell(I_k) = K$ .

Step 2: Apply tests to detect faults  $f(I_j/I_j+I_k)$ , where  $1 \leq \ell(I_j) \leq K$ ,  $\ell(I_k) = K + 1$ , and  $K < K_{\max}$ .

Step 3: Apply tests to detect faults  $f(I_j/I_j+I_k)$ , where  $K + 1 \leq \ell(I_j) \leq K_{\max}$ ,  $\ell(I_k) = K$ .

END

Strictly speaking, this order need not be followed during the actual application of tests, but it plays a very crucial role in

proving that the tests detect faults in the instruction decoding and control function. Therefore we assume that the tests are applied in the order given by Algorithm 4.2.

Note that in steps 2 and 3 it is not necessary to check for faults  $f(I_j/\phi)$  or  $f(I_j/I_k)$  because these faults are detected by tests involved in step 1. We now present an example to illustrate the three steps of the algorithm.

For concise representation we introduce some notation at this stage. Let  $I_A, I_B$ , etc., denote sets of instructions. Then  $f(I_A/I_B)$  would denote a set of faults given by  $\{f(I_i/I_j) \mid I_i \in I_A \text{ and } I_j \in I_B\}$ . For example, let  $I_A = \{I_1, I_2\}$  and  $I_B = \{I_3, I_4\}$ , then  $f(I_A/I_B) = \{f(I_1/I_3), f(I_1/I_4), f(I_2/I_3), f(I_2/I_4)\}$ . Similarly  $f(I_A/I_{A+B})$  and  $f(I_A/\phi)$  denote the corresponding sets of faults. Needless to say, we do not incorporate  $f(I_i/I_i)$  or  $f(I_i/I_{i+I_i})$  in the sets  $f(I_A/I_B)$  or  $f(I_A/I_{A+B})$ .  $\ell(I_A)$  denotes the set of labels of instructions in set  $I_A$ .

Example 4.4: This example illustrates the steps of Algorithm 4.2 in the case of subtask 1 for the S-graph of Figure 2.8.

Iteration 1:

Step 1: Apply tests for faults  $f(I_A/\phi)$ ,  $f(I_A/I_B)$ , and  $f(I_A/I_{A+B})$

where  $I_A = I_B = \{I_7, I_8, I_{16}, I_{17}, I_{18}, I_{19}\}$ , and  $\ell(I_A) = \ell(I_B) = \{1\}$ .

Step 2: Apply tests for faults  $f(I_A/I_{A+B})$  where  $I_A = \{I_7, I_8, I_{16}, I_{17}, I_{18}, I_{19}\}$ ,  $I_B = \{I_1, I_2, I_3, I_4, I_6, I_{11}, I_{12}, I_{13}, I_{15}\}$ , and  $\ell(I_A) = \{1\}$ ,  $\ell(I_B) = \{2\}$ .

Step 3: Apply tests for faults  $f(I_A/I_{A+B})$  where  $I_A = \{I_1, I_2, I_3, I_5, I_6, I_{15}\}$ ,  $I_B = \{I_7, I_8, I_{16}, I_{17}, I_{18}, I_{19}\}$ , and

$$\ell(I_A) = \{2, 3\}, \ell(I_B) = \{1\}.$$

Iteration 2:

Step 1: Apply tests for faults  $f(I_A/\phi)$ ,  $f(I_A/I_B)$ , and  $f(I_A/I_A+I_B)$  where  $I_A = \{I_1, I_2, I_3, I_6, I_{15}\}$ ,  $I_B = \{I_1, I_2, I_3, I_4, I_6, I_{11}, I_{12}, I_{13}, I_{15}\}$ , and  $\ell(I_A) = \ell(I_B) = 2$ .

Step 2: Apply tests for faults  $f(I_A/I_A+I_B)$  where  $I_A = \{I_7, I_8, I_{16}, I_{17}, I_{18}, I_{19}, I_1, I_2, I_3, I_6, I_{15}\}$ ,  $I_B = \{I_5\}$ , and  $\ell(I_A) = \{1, 2\}$ ,  $\ell(I_B) = \{3\}$ .

Step 3: Apply tests for faults  $f(I_A/I_A+I_B)$  where  $I_A = \{I_5\}$ ,  $I_B = \{I_1, I_2, I_3, I_4, I_6, I_{11}, I_{12}, I_{13}, I_{15}\}$ , and  $\ell(I_A) = \{3\}$ ,  $\ell(I_B) = \{2\}$ .

Iteration 3:

Step 1: Apply tests for fault  $f(I_5/\phi)$ . (Note that  $\ell(I_5) = \{3\}$ .)  $\square$

The next job is to develop the details of test generation for detecting faults  $f(I_j/\phi)$ ,  $f(I_j/I_k)$ , and  $f(I_j/I_j+I_k)$ . These details depend very heavily on the labels of instructions and their source and destination registers. Therefore we partition the job into various cases depending on these labels and present the test generation procedure for each case.

At this stage we make an assumption about the labels of edges representing instructions. If  $I_j \in \text{class M}$  then  $\ell(I_j) \leq 2$ , i.e., if  $\ell(I_j) > 2$  then  $I_j \in \text{class T}$ . Recall from Lemma 4.2 that all instructions of class B are labeled 1. Thus the destination of an instruction of class M can only be a main memory location or a register with label 1. This assumption can be easily justified for available microprocessors,

since the result of an instruction of class M is usually stored in a main memory location or an accumulator [Cush77]; on the other hand, the destination register of an instruction of class T can have any label. Without this assumption the details of the test generation procedures become extremely complicated.

#### 4.3.2. Test Generation for $f(I_j/\phi)$

The details of test generation depend principally on  $\ell(I_j)$ . We consider three cases, namely, case A(1), case A(2), and case A(3), depending on  $\ell(I_j)$ . The suffix A is used to denote that a case belongs to the details of test generation for  $f(I_j/\phi)$ . These cases are divided into subcases which are listed in Table 4.1. For each subcase, the table gives which test generation procedure is applicable and which theorem proves the fault coverage.

##### 4.3.2.1. Test Generation for $f(I_j/\phi)$ When $\ell(I_j) = 1$

This case is referred to as case A(1) and is divided into subcases two subcases.

Case A(1.1):  $OUT \in D(I_j)$ , i.e.,  $I_j$  is expected to read out a register with label 1 (according to Lemma 4.1).

Case A(1.2):  $I_j$  is an instruction of class B not represented in the S-graph, i.e.,  $I_j$  only changes the logic level on some status pins.

In either subcase the fault detection is easy since  $I_j$  has the highest observability as signified by  $\ell(I_j) = 1$ .

We give the test generation procedures below. It is straightforward to derive the tests in terms of the assembly language instructions of the microprocessor to be tested from the procedures to follow. For

Table 4.1. Cases for the details of test generation for detecting  $f(I_j/\phi)$ .

Case and its description		Test generation procedure	Theorem proving the fault coverage
<u>Case A(1)</u>  $\ell(I_j) = 1$	<u>Case A(1.1)</u>  $OUT \in D(I_j)$	Procedure 4.2	*
	<u>Case A(1.2)</u>  $I_j \in \text{class B, and it is not represented in the S-graph.}$	Procedure 4.3	*
<u>Case A(2)</u>	$\ell(I_j) = 2$	Procedure 4.4	Theorem 4.2
<u>Case A(3)</u>  $\ell(I_j) = K \geq 3$	<u>Case A(3.1)</u>  $S(I_j)$ is not the destination register of any instruction belonging to the $READ(D(I_j))$ sequence.	Procedure 4.4	Theorem 4.3
	<u>Case A(3.2)</u>  $S(I_j)$ is the destination register of an instruction belonging to the $READ(D(I_j))$ sequence.	Procedure 4.5	Theorem 4.4

\* Fault coverage follows immediately from the description of the procedure.

conciseness we say that the microprocessor executes these procedures instead of saying that the microprocessor executes the tests generated by these procedures. For the same reason we treat these procedures as if they are test execution procedures instead of test generation procedures. We denote various operands for instructions by OPERAND 1, OPERAND 2, etc., and various results stored in registers (as a consequence of instruction execution) by RESULT 1, RESULT 2, etc.

Procedure 4.2:

This procedure is applicable for case A(1.1). It generates tests to detect fault  $f(I_j/\phi)$  when  $\ell(I_j) = 1$  and  $OUT \in D(I_j)$ .

Step 1: Store proper operand(s) in  $S(I_j)$  such that when  $I_j$  is executed, the expected output "data" is different from the quiescent logic value on the data (or address) bus.

Step 2: Execute  $I_j$ . □

Procedure 4.3:

This procedure is applicable for case A(1.2). It generates tests to detect fault  $f(I_j/\phi)$  when  $\ell(I_j) = 1$  and  $I_j$  belongs to class B but it is not represented in the S graph.

Step 1: Execute the proper instruction to set up the logic value on a status pin to  $x$  ( $x \in \{0, 1\}$ ) if the instruction  $I_j$  under consideration, when executed, sets up the logic value on that status pin to  $\bar{x}$ .

Step 2: Execute  $I_j$ . □

#### 4.3.2.2. Test Generation for $f(I_j/\phi)$ When $\ell(I_j) = 2$

This case is referred to as case A(2). In this case  $|\text{READ}(D(I_j))| = 1$ . Let  $\text{READ}(D(I_j)) = \langle I_k \rangle$ . Of course,  $\ell(I_k) = 1$ , and by definition of the READ sequence  $I_k \in (T \cup B)$  and  $\text{OUT} \in D(I_k)$ .

#### Procedure 4.4:

This procedure is applicable for case A(2). It generates tests to detect fault  $f(I_j/\phi)$  when  $\ell(I_j) = 2$ .

Step 1: Store OPERAND 1 in  $D(I_j)$  and proper operand(s) in  $S(I_j)$  such that when  $I_j$  is executed it produces RESULT 1 in  $D(I_j)$ , and  $\text{RESULT } 1 \neq \text{OPERAND } 1$ .

Step 2: Read out  $D(I_j)$  by executing  $\text{READ}(D(I_j))$ .  
/Expected output data = OPERAND 1/

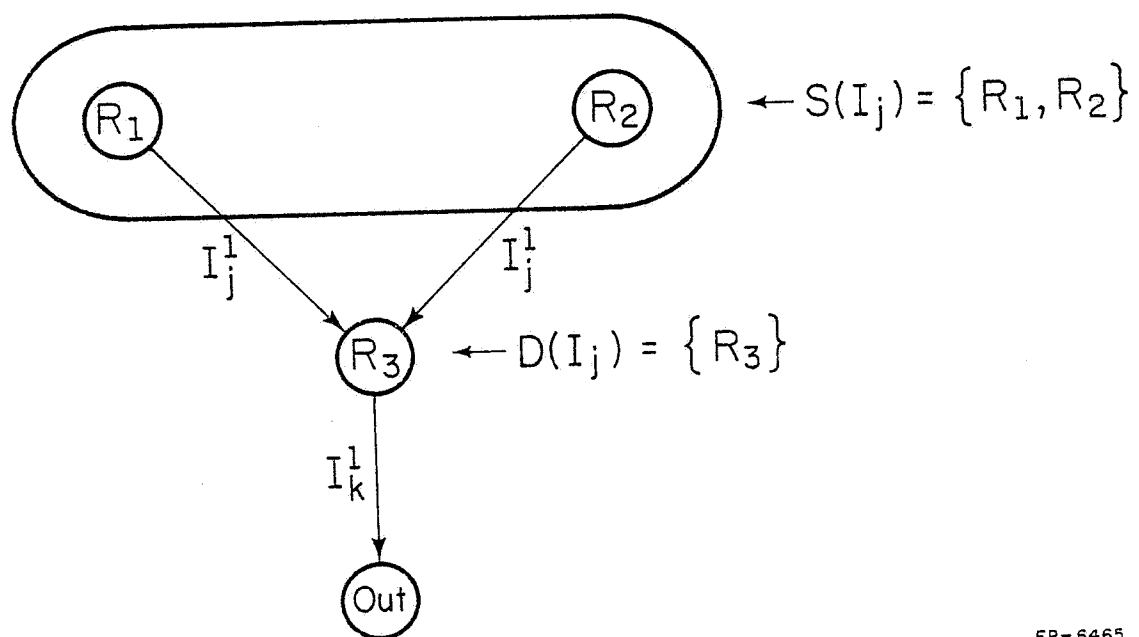
Step 3: Execute  $I_j$ .

Step 4: Read out  $D(I_j)$  by executing  $\text{READ}(D(I_j))$ .  
/Expected output data = RESULT 1/

□

Example 4.5: This example (depicted in Figure 4.1) illustrates Procedure 4.4.  $I_j$  is an "Add" instruction which adds the contents of registers  $R_1$  and  $R_2$  and stores the result in  $R_3$ .  $\text{READ}(D(I_j)) = \langle I_k \rangle$ ,  $\ell(I_k) = 1$  and  $I_k \in \text{class } T$ . OPERAND 1 can be chosen to be ONE, and RESULT 1 to be ZERO, requiring that operand ZERO be stored in both  $R_1$  and  $R_2$ .  $I_k$  is executed to make sure that  $R_3$  does store OPERAND 1. This is followed by execution of  $I_j$  and  $I_k$ .

□



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Figure 4.1. Example illustrating Procedure 4.4.



Theorem 4.2: Procedure 4.4 detects  $f(I_j/\phi)$  in case A(2).

Proof: Note that  $\ell(I_k) = 1$  where  $\text{READ}(D(I_j)) = \langle I_k \rangle$ .

(Refer to Figure 4.1.) Since the tests are applied in the order specified by Algorithm 4.2, the microprocessor under test executes this procedure after executing the tests required to detect  $f(I_p/\phi)$ ,  $f(I_p/I_q)$ , and  $f(I_p/I_p + I_q)$  where  $\ell(I_p) = \ell(I_q) = 1$ , and  $f(I_v/I_v + I_w)$  where  $\ell(I_v) = 1$  and  $\ell(I_w) = 2$ . Therefore, when the microprocessor under test executes this procedure it has been already checked that the  $\text{READ}(D(I_j)) = \langle I_k \rangle$  sequence can correctly read out  $D(I_j)$  and its execution does not cause additional execution of any instruction with label 2; in particular the contents of  $D(I_j)$  are not changed after the execution of  $\text{READ}(D(I_j))$ . Therefore step 2 ensures that  $D(I_j)$  stores OPERAND 1. This step is necessary because due to faults involved in the instructions used to write data in  $D(I_j)$ , RESULT 1 may be stored in  $D(I_j)$  instead of OPERAND 1. If step 2 is not executed, the fault  $f(I_j/\phi)$  will be masked.

In step 3,  $I_j$  is executed which is expected to produce RESULT 1 in  $D(I_j)$ . If  $f(I_j/\phi)$  is present, the contents of  $D(I_j)$  will not change. Consequently when  $D(I_j)$  is read out in step 4 the fault will be detected.

#### 4.3.2.3. Test Generation for $f(I_j/\phi)$ When $\ell(I_j) = K \geq 3$

This case is referred to as case A(3). According to our assumption in Section 4.3.1,  $I_j \in$  class T. This case is divided into two subcases.

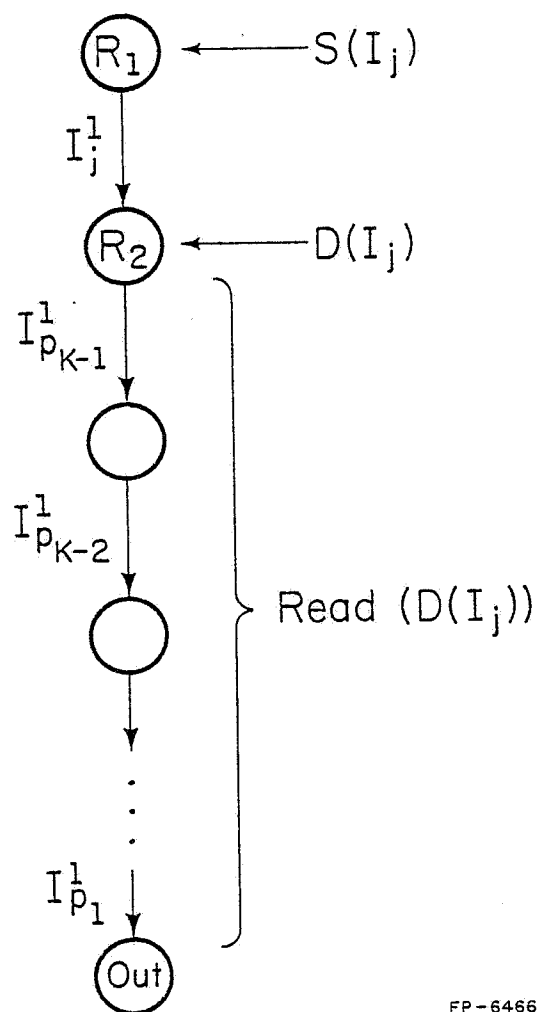
Case A(3.1):  $S(I_j)$  is not the destination register of any instruction belonging to the  $READ(D(I_j))$  sequence. Figure 4.2 illustrates this case,  $\ell(I_j) = K \geq 3$ ,  $\ell(D(I_j)) = K-1$ ,  $\ell(S(I_j)) = K$ , and  $READ(D(I_j)) = \langle I_{p_{K-1}}, I_{p_{K-2}}, \dots, I_{p_1} \rangle$ . Thus it is possible to read out  $D(I_j)$  without routing the contents of  $D(I_j)$  through  $S(I_j)$ . In this case we follow Procedure 4.4. (The same procedure which is used in case A(2).)

Theorem 4.3: Procedure 4.4 detects  $f(I_j/\phi)$  in case A(3.1).

Proof: The proof follows the same arguments given for the proof of Theorem 4.2, except for one change. The microprocessor under test executes this procedure after executing the tests required to detect  $f(I_p/\phi)$ ,  $f(I_p/I_q)$ ,  $f(I_p/I_p + I_q)$  where  $1 \leq \ell(I_p), \ell(I_q) \leq K-1$ , and  $f(I_v/I_v + I_w)$  where  $1 \leq \ell(I_v) \leq K-1$  and  $\ell(I_w) = K$ . Note that in this case  $READ(D(I_j)) = \langle I_{p_{K-1}}, I_{p_{K-2}}, \dots, I_{p_1} \rangle$  as illustrated in Figure 4.2. Since  $\ell(D(I_j)) = K-1$ ,  $\ell(I_{p_i}) = i$ , for  $1 \leq i \leq K-1$ .

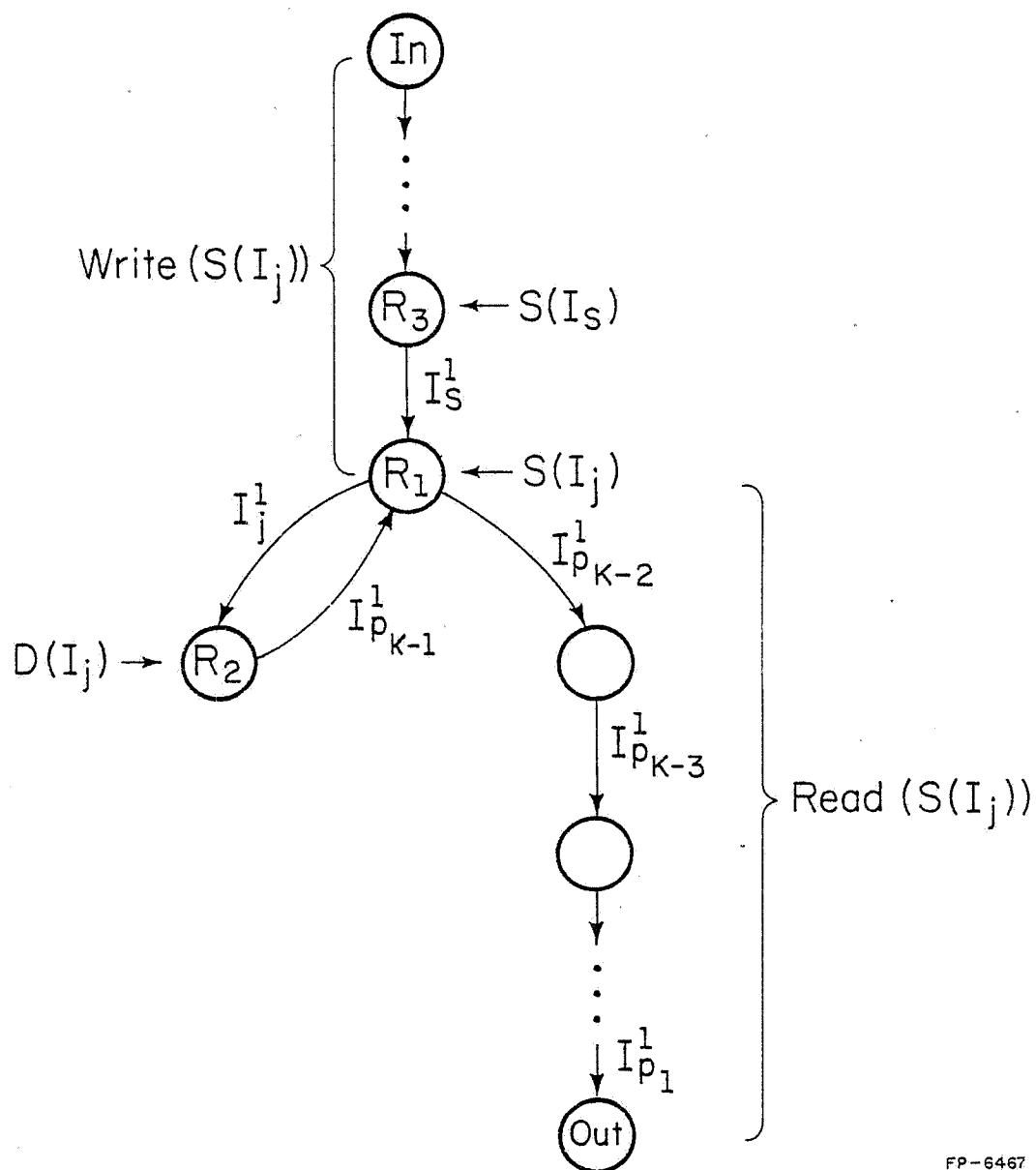
Therefore, when the microprocessor under test executes this procedure it has been already checked that the execution of any instruction  $I_{p_i} \in READ(D(I_j))$  does not give rise to the additional execution of any instruction with label  $K$ .  $D(I_j)$  can be correctly read out by executing  $READ(D(I_j))$  in step 2 of this procedure, and after the execution of  $READ(D(I_j))$  the contents of  $D(I_j)$  are not changed (note that exactly  $\ell(D(I_j)) = K-1$  and  $\ell(I_j) = K$ ). All the remaining arguments are exactly the same as given in the proof of Theorem 4.2.  $\square$

Case A(3.2):  $S(I_j)$  is the destination register of an instruction belonging to the  $READ(D(I_j))$  sequence. Figure 4.3 illustrates this case;  $\ell(I_j) = K \geq 3$ ,  $\ell(D(I_j)) = K-1$ ,  $\ell(S(I_j)) = K-2$ .



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Figure 4.2. Illustrating case A(3.1) in Section 4.3.2.



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Figure 4.3. Illustrating case A(3.2) in Section 4.3.2.

$\text{READ}(D(I_j)) = \langle I_{P_{K-1}}, I_{P_{K-2}}, \dots, I_{P_1} \rangle$  and  $\text{READ}(S(I_j)) = \langle I_{P_{K-1}}, I_{P_{K-2}}, \dots, I_{P_1} \rangle$  where  $\ell(I_{P_i}) = i$  for  $1 \leq i \leq K-1$ . Let  $I_s \in \text{WRITE}(S(I_j))$  and  $D(I_s) = S(I_j)$ . Therefore,  $\ell(I_s) = K-1$ , and  $\ell(S(I_s)) = K-1$ .

Procedure 4.5:

This procedure is applicable for case A(3.2). It generates tests to detect fault  $f(I_j/\phi)$  when  $\ell(I_j) = K \geq 3$ , and  $S(I_j)$  is the destination register of an instruction belonging to the  $\text{READ}(D(I_j))$  sequence.

Step 1: Store OPERAND 1 in  $D(I_j)$  by executing  $\text{WRITE}(D(I_j))$ .

Step 2: Read out  $D(I_j)$  by executing  $\text{READ}(D(I_j))$ .

/Expected output data = OPERAND 1/

Step 3: Store OPERAND 2  $\neq$  OPERAND 1 in  $S(I_s)$  by executing  $\text{WRITE}(S(I_s))$ .

Step 4: Read out  $D(I_j)$  by executing  $\text{READ}(D(I_j))$ .

/Expected output data = OPERAND 1/

Step 5: Execute  $I_s$  and  $I_j$ . /Refer to Figure 4.3/

Step 6: Read out  $D(I_j)$  by executing  $\text{READ}(D(I_j))$ .

/Expected output data = OPERAND 2  $\neq$  OPERAND 1/

□

Theorem 4.4: Procedure 4.5 detects  $f(I_j/\phi)$  in case A(3.2).

Proof:  $1 \leq \ell(I_{P_i}) = i \leq K-1$ , for each instruction  $I_{P_i}$  in the  $\text{READ}(D(I_j))$  sequence. When the microprocessor under test executes this procedure it has already executed the tests required to detect  $f(I_p/\phi)$ ,  $f(I_p/I_q)$  and  $f(I_p/I_p + I_q)$  where  $1 \leq \ell(I_p)$ ,  $\ell(I_q) \leq K-1$ , and  $f(I_v/I_v + I_w)$  where  $1 \leq \ell(I_v) \leq K-1$  and  $\ell(I_w) = K$ .

Therefore, when the microprocessor under test executes this procedure it has been already checked that the  $\text{READ}(D(I_j))$  sequence can correctly read out  $D(I_j)$  and the execution of any instruction in this sequence will not give rise to additional execution of any instruction with label  $K$ ; in particular the contents of  $D(I_j)$  and  $S(I_s)$  remain unchanged. (Note that  $\ell(D(I_j)) = \ell(S(I_s)) = K-1$ .) Moreover, the execution of  $I_s$  does not give rise to additional execution of any instruction with label  $K$ , since  $\ell(I_s) = K-1$ ; thus in particular the contents of  $D(I_j)$  remain unchanged after the execution of  $I_s$ .

In step 2,  $D(I_j)$  is correctly read out by executing  $\text{READ}(D(I_j))$  and it continues to store the operand stored in it, i.e., OPERAND 1, after this "read out" process. In step 3,  $S(I_s)$  is written with OPERAND 2 by executing  $\text{WRITE}(S(I_s))$ . Since  $\ell(S(I_s)) = \ell(D(I_j)) = K-1$ , it is possible to write data in  $S(I_s)$  without routing it through  $D(I_j)$  during the execution of  $\text{WRITE}(S(I_s))$ . Step 4 ensures that  $D(I_j)$  continues to store OPERAND 1 after step 3. Moreover,  $D(I_j)$  and  $S(I_s)$  continue to store OPERAND 1 and OPERAND 2, respectively, after the "read out" process in step 4.

Since  $\ell(I_s) = K-1$ , the execution of  $I_s$  in step 5 will correctly transfer the contents of  $S(I_s)$  to  $S(I_j)$ , i.e.,  $S(I_j)$  now contains OPERAND 2  $\neq$  OPERAND 1. Also the contents of  $D(I_j)$  remain unchanged, since  $\ell(I_s) = K-1$  and  $\ell(D(I_j)) = K-1$ . After this  $I_j$  is executed which is expected to change the contents of  $D(I_j)$  to OPERAND 2. If  $f(I_j/\phi)$  exists,  $I_j$  will fail to do so and the fault will be detected when  $D(I_j)$  is read out in step 6. □

#### 4.3.3. Test Generation for $f(I_j/I_k)$

The details of test generation depend principally on  $\ell(I_j)$  and  $\ell(I_k)$ . We consider three cases, namely, case B(1), case B(2), and case B(3) depending on  $\ell(I_j)$ . The suffix B is used to denote that a case belongs to the details of test generation for  $f(I_j/I_k)$ . These cases are divided into subcases which are listed in Table 4.2. For each case, the table gives which test generation procedure is applicable and which theorem proves the fault coverage.

##### 4.3.3.1. Test Generation for $f(I_j/I_k)$ when $\ell(I_j) = 1$

This case is referred to as case B(1) and is divided into two subcases depending on  $\ell(I_k)$ .

Case B(1.1):  $\ell(I_k) \geq 2$ . Since  $\ell(I_j) = 1$ , the results of the execution of  $I_j$  are directly observable while those of  $I_k$  are not. Hence the behavior of the microprocessor as observed at its external pins under the fault  $f(I_j/I_k)$  is the same as it would be under the fault  $f(I_j/\emptyset)$ . Therefore Procedures 4.2 and 4.3 given in Section 4.3.2.1 should be followed in this case.

Case B(1.2):  $\ell(I_j) = \ell(I_k) = 1$ . Many of the faults in this case would be readily detected because  $I_j$  and  $I_k$  have the highest observability. For example,  $I_j$  and  $I_k$  may read and write data into the main memory during different machine cycles of the corresponding instruction cycles; or during the execution of  $I_j$  and  $I_k$  a different sequence of status signals may be emitted on the status pins. Therefore we will explicitly discuss the detection of those faults which cannot be easily detected in this fashion. This case can be further divided into two subcases, namely, case B(1.2.1) and case B(1.2.2) as described below.

Table 4.2. Cases for the details of test generation for detecting  $f(I_j/I_k)$ 

Case and its description			Test generation procedures	Theorems proving the fault coverage
<u>Case B(1)</u>	<u>Case B(1.1)</u>	$\ell(I_k) \geq 2$	Procedures 4.2 and 4.3	*
$\ell(I_j) = 1$	<u>Case B(1.2)</u>	<u>Case B(1.2.1)</u> During the execution of $I_j$ and $I_k$ the results of the operations performed are read out.	Procedure 4.6	Theorem 4.5
	$\ell(I_k) = 1$	<u>Case B(1.2.2)</u> During the execution of $I_j$ and $I_k$ the results of the operations performed are stored in registers, and not read out.	Procedure 4.7	Theorem 4.6
		<u>Case B(1.2.2.1)</u> Results produced by $I_j$ and $I_k$ are stored in the same register.	Procedure 4.8	Theorem 4.7
		<u>Case B(1.2.2.2)</u> Results produced by $I_j$ and $I_k$ are stored in different registers.		
<u>Case B(2)</u>	<u>Case B(2.1)</u>	$D(I_j) \neq D(I_k)$	Procedure 4.4	Theorem <sup>†</sup> 4.2
$\ell(I_j) = 2$	<u>Case B(2.2)</u>	$D(I_j) = D(I_k)$	Procedure 4.17	Theorem 4.16
<u>Case B(3)</u>	<u>Case B(3.1)</u>	$D(I_j) \neq D(I_k)$	Procedures 4.4 and 4.5	Theorems <sup>†</sup> 4.3 and 4.4
$\ell(I_j) = K \geq 3$	<u>Case B(3.2)</u>	<u>Case B(3.2.1)</u> $\ell(S(I_k)) < K$	Procedure 4.9	Theorem 4.8
	$D(I_j) = D(I_k)$	<u>Case B(3.2.2)</u> $\ell(S(I_k)) = K$	Procedure 4.10	Theorem 4.9

\* Fault coverage follows immediately from the description of the procedure.

† Fault coverage can be proved by following arguments similar to those in the proof of the theorem (marked with †).



The tests generated by Procedure 4.6 for case B(1.2.1) are to be applied before those generated by Procedures 4.7 and 4.8 for case B(1.2.2).

Case B(1.2.1): During the execution of instructions  $I_j$  and  $I_k$  the results of the operations performed are "read out". The operation performed could be as simple as a data transfer from a register to the main memory. Examples of this case are given below.

Example 4.6: Instruction  $I_j$  is "Store the contents of register  $R_1$  into the main memory using direct addressing," and instruction  $I_k$  is "Store the contents of register  $R_2$  into the main memory using direct addressing."

Now consider another example. Instruction  $I_j$  is "Add the contents of the accumulator and the contents stored at the top of a LIFO stack (maintained in the main memory) and store the result at the top of the stack," and instruction  $I_k$  is "Subtract the contents of the accumulator from the contents stored at the top of the LIFO stack and store the result at the top of the stack." □

Procedure 4.6:

This procedure is applicable for case B(1.2.1). It generates tests to detect fault  $f(I_j/I_k)$  when  $\ell(I_j) = \ell(I_k) = 1$  and during the execution of  $I_j$  and  $I_k$  the results of the operations are read out.

Step 1: Store proper operands in  $S(I_j)$  and  $S(I_k)$  such that when  $I_j$  is executed RESULT 1 is read out and when  $I_k$  is executed RESULT 2 is read out, and  $\text{RESULT } 1 \neq \text{RESULT } 2$ .

Step 2: Execute  $I_j$ . /Expected output data = RESULT 1/

Step 3: Execute  $I_k$ . /Expected output data = RESULT 2/ □

Theorem 4.5: Procedure 4.6 detects  $f(I_j/I_k)$  in case B(1.2.1).

Proof: If the proper operands are really stored in  $S(I_k)$  in step 1 so that RESULT2 is read out when  $I_k$  is executed, the fault will be detected in step 2 itself, as RESULT2 will be read out (instead of expected output data = RESULT1).

On the other hand, if the required operands are not stored in  $S(I_k)$  due to faults involved in the instructions used to write data in  $S(I_k)$ , the fault may not be detected in step 2; the execution of  $I_k$  may read out RESULT1 as the wrong operands are stored in  $S(I_k)$ . But in this case the fault will be detected in step 3 as the execution of  $I_k$  will produce RESULT1. Note that if  $f(I_j/I_k)$  is present we assume that  $I_k$  is correctly executed. (Recall the fault model in Section 3.2.)  $\square$

Case B(1.2.2): During the execution of instructions  $I_j$  and  $I_k$  the results of the operations performed are stored in registers. (The results are not read out as in case B(1.2.1).) The operation performed could be as simple as a data transfer from a main memory location to a register. In this situation the instruction belongs to class T. If the operation involves some data manipulation, the instruction belongs to class M.

Note that  $\ell(I_j) = \ell(I_k) = 1$ . If the instruction  $I_j$  or  $I_k$  belongs to class M, at least one of the operands for  $I_j$  or  $I_k$  must be stored in the main memory. (If all the operands for  $I_j$  and  $I_k$  are available in registers we would have  $\ell(I_j), \ell(I_k) \geq 2$ . Refer to step 4 of the labeling algorithm given in Section 4.1.) The address of the operand stored in the main memory is computed and then transferred from a register holding

it to the address register of the main memory. Thus the register storing the address of the operand is implicitly read out during the execution of  $I_j$  and  $I_k$ . This is precisely the reason to assign label 1 to  $I_j$  and  $I_k$ . (Refer to step 3 of the labeling algorithm.) As mentioned earlier, the results of operations performed under  $I_j$  and  $I_k$  are stored in registers. If the instruction belongs to class M, the result produced is stored in a register with label 1. This is consistent with the assumption (regarding the label of an instruction of class M) made in Section 4.3.1.

If the instruction belongs to class T the data transferred from the main memory may be stored in a register with label greater than 1. In this situation fault detection may or may not be easy as illustrated in the following example.

Example 4.7: Let instruction  $I_j$  be "Load the contents of the memory location pointed to by register  $R_1$  in register  $R_2$ ," and let instruction  $I_k$  be "Load the contents of the memory location pointed to by register  $R_3$  in register  $R_2$ ." Both instructions store the result of their operation (which is a simple data transfer) in register  $R_2$ . Even if  $l(R_2) \geq 2$ , fault  $f(I_j/I_k)$  can be easily detected by choosing different addresses (pointers) in  $R_1$  and  $R_3$ . If  $f(I_j/I_k)$  is present, the address stored in  $R_3$  will be (implicitly) read out on the address bus instead of the address stored in  $R_1$  when  $I_j$  is executed, and the fault will be detected. Thus this case is really not different from case B(1.2.1).

We now consider another example where the fault detection is not so easy. Let instruction  $I_j$  be "Load the contents of the memory location pointed to by register  $R_1$  in register  $R_2$ ," and let instruction  $I_k$  be "Load the contents of the memory location pointed to by register  $R_1$  in

register  $R_3$ ." In this case  $f(I_j/I_k)$  can be detected by storing different data in  $R_2$  and  $R_3$  and then reading out these registers by executing  $\text{READ}(R_2)$  and  $\text{READ}(R_3)$ . If  $\ell(R_2)$  or  $\ell(R_3)$  is greater than 1, (i.e.,  $\text{READ}(R_2)$  or  $\text{READ}(R_3)$  contain instructions with label greater than 1) it is not guaranteed that  $\text{READ}(R_2)$  and  $\text{READ}(R_3)$  sequences can correctly read out  $R_2$  and  $R_3$  because the microprocessor under test has not yet executed tests to detect  $f(I_p/I_q)$ ,  $f(I_p/I_p + I_q)$  where  $\ell(I_p), \ell(I_q) \geq 2$ . In this case we treat  $I_j$  and  $I_k$  as if they have label  $\ell(R_2) + 1$  and  $\ell(R_3) + 1$ , respectively, as far as the test generation for faults  $f(I_j/I_k)$ ,  $f(I_j/I_j + I_k)$ ,  $f(I_k/I_j)$ , or  $f(I_k/I_k + I_j)$  is concerned.  $\square$

Case B(1.2.2) being considered applies only to those instructions which store their result in a register with label 1. Depending on whether the results of  $I_j$  and  $I_k$  are stored in the same or different registers we divide this case further into two subcases.

Case B(1.2.2.1): The results produced by executing instructions  $I_j$  and  $I_k$  are stored in the same register. Let this register be designated as  $R_p$ ; furthermore, let  $\text{READ}(R_p) = \langle I_p \rangle$ . Note that  $\ell(I_p) = 1$ . An example of this case is given below.

Example 4.8: Instruction  $I_j$  is "Add the contents of the accumulator and the contents of the memory location (next to the one storing the opcode of instruction  $I_j$ ) and store the result in the accumulator," and instruction  $I_k$  is "Subtract the contents of the accumulator from the contents of the memory location and store the result in the accumulator."  $\square$

Procedure 4.7:

This procedure is applicable for case B(1.2.2.1). It generates tests to detect fault  $f(I_j/I_k)$  when  $\ell(I_j) = \ell(I_k) = 1$ , and during

the execution of  $I_j$  and  $I_k$  the results of the operations are stored in the same register  $R_p$ .

Step 1: Store proper operands in  $S(I_j)$  and  $S(I_k)$  such that when  $I_j$  is executed RESULT 1 is produced and when  $I_k$  is executed RESULT 2 is produced, and  $\text{RESULT 1} \neq \text{RESULT 2}$ .

Step 2: If  $R_p \in S(I_j) \cup S(I_k)$  then read out register  $R_p$  by executing  $\text{READ}(R_p) = \langle I_p \rangle$ . /To make sure that  $R_p$  contains proper operand to be stored in step 1. The other member of  $S(I_j)$  or  $S(I_k)$  is a location in the main memory./

Step 3: If  $R_p \in S(I_j) \cup S(I_k)$  then repeat step 2.

Step 4: Execute  $I_j$ .

Step 5: Read out register  $R_p$  by executing  $\text{READ}(R_p) = \langle I_p \rangle$ .

/Expected output data = RESULT 1/

□

Theorem 4.6: Procedure 4.7 detects  $f(I_j/I_k)$  in case B(1.2.2.1).

Proof: If the register  $R_p$  is a member of  $S(I_j)$  or  $S(I_k)$  it must be ensured that it contains the proper operand to be stored in step 1, otherwise  $I_k$  could produce RESULT 1 instead of RESULT 2 and fault masking would occur.

The instruction  $I_p$  involved in step 2 will correctly read out  $R_p$  because  $\ell(I_p) = 1$ , and the microprocessor under test would have already executed the tests to detect  $f(I_p/I_q)$  where  $\ell(I_q) = 1$  (generated by Procedure 4.6 for case B(1.2.1)); however, if the fault  $f(I_p/I_{p'})$  is present, where  $\ell(I_{p'}) = 2$  and  $D(I_{p'}) = \{R_p\}$ , the contents of  $R_p$  may change after it is read out by executing  $I_p$ . If this happens it will be detected in step 3. On the other hand if the contents of  $R_p$  do not change

after the first execution of  $I_p$  (in step 2), they will not change after the second execution of  $I_p$  (in step 3) either.

In step 4  $I_j$  is executed. If  $f(I_j/I_k)$  is present, RESULT 2 will be produced in  $R_p$  in step 4, and it will be detected when  $R_p$  is read out in step 5. □

Case B(1.2.2.2): The results produced by executing instructions  $I_j$  and  $I_k$  are stored in different registers. Let the result produced by  $I_j$  be stored in register  $R_p$ . Furthermore, let  $READ(R_p) = \langle I_p \rangle$ , and  $\ell(I_p) = 1$ . An example of this case is given below.

Example 4.9: Instruction  $I_j$  is "Load register  $R_1$  from the main memory location (next to the one storing the opcode of instruction  $I_j$ )," and instruction  $I_k$  is "Load register  $R_2$  from the main memory location." □

Procedure 4.8:

This procedure is applicable for case B(1.2.2.2). It generates tests to detect fault  $f(I_j/I_k)$  when  $\ell(I_j) = \ell(I_k) = 1$ , and during the execution of  $I_j$  and  $I_k$  the results of the operations are stored in different registers.

Step 1: Store OPERAND 1 in register  $R_p$  and proper operands in  $S(I_j)$  such that when  $I_j$  is executed RESULT 1 is produced in  $R_p$ , and  $RESULT\ 1 \neq OPERAND\ 1$ .

Step 2: Read out register  $R_p$  by executing  $READ(R_p) = \langle I_p \rangle$ .  
/Expected output data = OPERAND 1/

Step 3: Repeat step 2.

Step 4: Execute  $I_j$ .

Step 5: Read out register  $R_p$  by executing  $READ(R_p) = \langle I_p \rangle$ .

/Expected output data = RESULT 1  $\neq$  OPERAND 1/

□

Theorem 4.7: Procedure 4.8 detects  $f(I_j/I_k)$  in case B(1.2.2.2).

The proof of this theorem follows the proof of Theorem 4.6 closely and will not be repeated here.

#### 4.3.3.2. Test Generation for $f(I_j/I_k)$ when $\ell(I_j) = 2$

This case is referred to as case B(2) and is divided into two subcases depending on whether or not  $D(I_j) = D(I_k)$ .

Case B(2.1):  $D(I_j) \neq D(I_k)$ . In this case under the fault  $f(I_j/I_k)$ , the contents of  $D(I_j)$  remain unchanged as they would remain under the fault  $f(I_j/\phi)$ . Hence, the procedure for this case is the same as Procedure 4.4. Furthermore, using arguments similar to those in the proof of Theorem 4.2, it can be proved that Procedure 4.4 detects  $f(I_j/I_k)$  in this case.

Case B(2.2):  $D(I_j) = D(I_k)$ . ~~This case is not treated here because all the details considered for case C(3.1) in Section 4.3.4.3 apply to this case.~~ Procedure 4.17 used to detect  $f(I_j/I_j + I_k)$  in case C(3.1) (refer to Table 4.3) can detect  $f(I_j/I_k)$  in case B(2.2) discussed here. This will be pointed out in the proof of Theorem 4.16.

#### 4.3.3.3. Test Generation for $f(I_j/I_k)$ When $\ell(I_j) = K \geq 3$

This case is referred to as case B(3). Note that  $I_j$  belongs to class T according to our assumption in Section 4.3.1. This case can be further divided into two subcases depending on whether or not  $D(I_j) = D(I_k)$ .

Case B(3.1):  $D(I_j) \neq D(I_k)$ . In this case under the fault  $f(I_j/I_k)$ , the contents of  $D(I_j)$  remain unchanged as under the fault  $f(I_j/\phi)$ . Hence, the procedures for this case will be the same as Procedures 4.4 and 4.5. Note that Procedure 4.4 is applied in case A(3.1) and Procedure 4.5 is applied in case A(3.2). Using arguments similar to those in the proofs of Theorems 4.3 and 4.4, it can be proved that Procedures 4.4 and 4.5 detect  $f(I_j/I_k)$  in this case.

Case B(3.2):  $D(I_j) = D(I_k)$ . Note that  $\ell(I_j) = \ell(I_k) = K$ , and  $\ell(D(I_j)) = \ell(D(I_k)) = K-1$ . Therefore no instruction in the READ ( $D(I_j)$ ) sequence can have a label greater than  $K-1$ . This case is illustrated in Figure 4.4. Note that  $\text{READ}(D(I_j)) = \langle I_{p_{K-1}}, I_{p_{K-2}}, \dots, I_{p_1} \rangle$  where  $\ell(I_{p_i}) = i$ , for  $1 \leq i \leq K-1$ . Depending on  $\ell(S(I_k))$  the case can be further divided into two subcases. Case B(3.2.1) applies when  $\ell(S(I_k)) < K$ , and case B(3.2.2) applies when  $\ell(S(I_k)) = K$ . Note that  $\ell(S(I_k)) \neq K$ , since  $\ell(I_k) = K$  and  $I_k$  belongs to class T. (Also refer to the labeling algorithm given in Section 4.1.)

Case B(3.2.1):  $\ell(S(I_k)) < K$ . In this case the instruction  $I_k$  does not belong to the READ ( $S(I_k)$ ) sequence. (This will be proved in the proof of Theorem 4.8.)

#### Procedure 4.9:

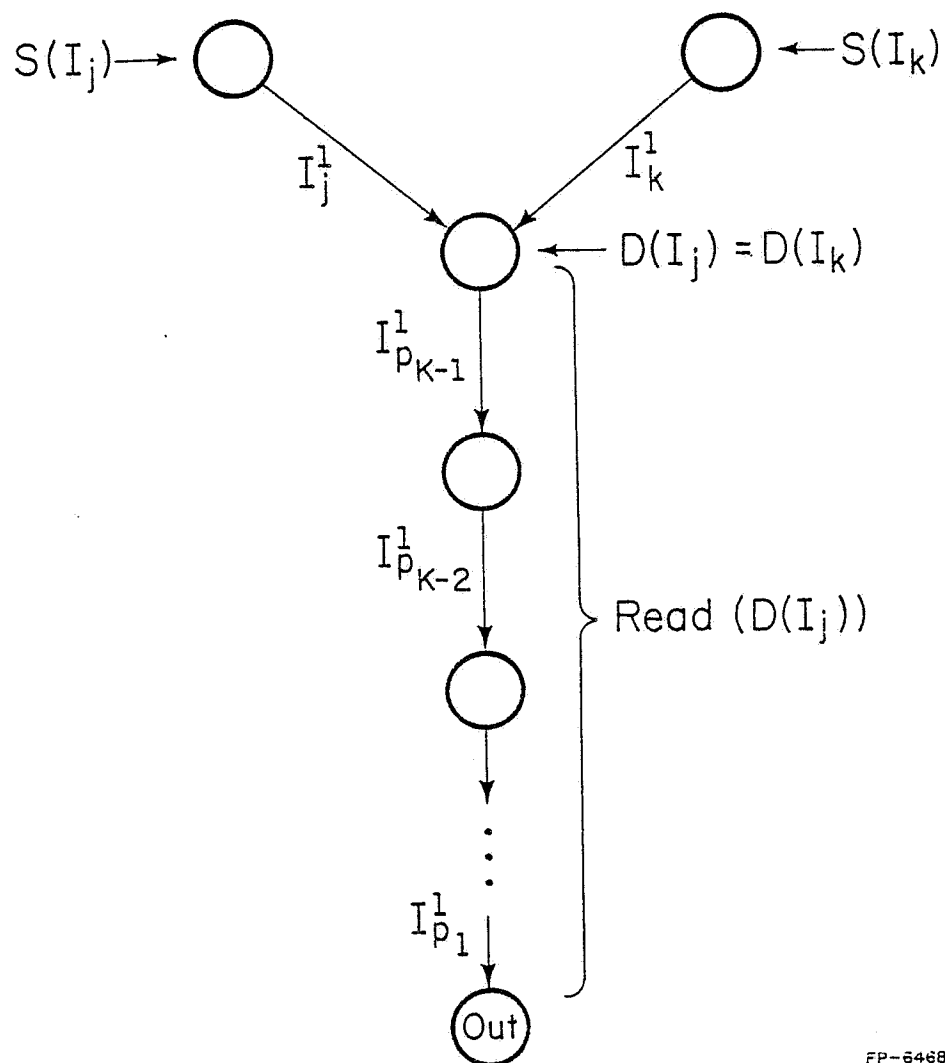
This procedure is applicable for case B(3.2.1). It generates tests to detect fault  $f(I_j/I_k)$  when  $\ell(I_j) = \ell(I_k) = K \geq 3$ ,  $D(I_j) = D(I_k)$ , and  $\ell(S(I_k)) < K$ .

Step 1: Store OPERAND 1 in  $S(I_j)$  and OPERAND 2 in  $S(I_k)$  such that

OPERAND 1  $\neq$  OPERAND 2.

Step 2: Execute  $I_j$ .





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Figure 4.4. Illustrating case B(3.2) in Section 4.3.3.3.

Step 3: Read out  $S(I_k)$  by executing  $READ(S(I_k))$ .

/Expected output data = OPERAND 2/

Step 4: Read out  $D(I_j)$  by executing  $READ(D(I_j))$ .

/Expected output data = OPERAND 1/

□

Theorem 4.8: Procedure 4.9 detects  $f(I_j/I_k)$  in case B(3.2.1).

Proof: Note that  $\ell(I_k) = K$ . Since  $\ell(S(I_k)) \leq K$ , no instruction in the  $READ(S(I_k))$  sequence can have a label greater than  $K-1$ , implying that  $I_k \notin READ(S(I_k))$ . Therefore  $READ(S(I_k))$  reads out  $S(I_k)$  without routing it through  $D(I_j) = D(I_k)$ .

When the microprocessor under test executes this procedure, it has already executed the tests required to detect  $f(I_p/\phi)$ ,  $f(I_p/I_q)$ , and  $f(I_p/I_p + I_q)$  where  $1 \leq \ell(I_p)$ ,  $\ell(I_q) \leq K-1$ , and  $f(I_v/I_v + I_w)$  where  $1 \leq \ell(I_v) \leq K-1$  and  $\ell(I_w) = K$ . Therefore, when the microprocessor under test executes this procedure it has already been checked that the  $READ(D(I_j))$  and  $READ(S(I_k))$  sequences correctly read out  $D(I_j)$  and  $S(I_k)$ , respectively. (Recall that no instruction in the  $READ(D(I_j))$  or  $READ(S(I_k))$  sequences can have a label greater than  $K-1$ .) Moreover, after the execution of these sequences the contents of  $D(I_j)$  and  $S(I_k)$  do not alter.

In step 2,  $I_j$  is executed and it is expected to produce OPERAND 1 in  $D(I_j)$ . If  $f(I_j/I_k)$  is present, OPERAND 2 will be produced in  $D(I_j)$  instead of OPERAND 1, provided  $S(I_k)$  really contained OPERAND 2 when  $I_j$  is executed. In this situation the fault will be detected in step 4. (Note that in step 3,  $READ(S(I_k))$  reads out  $S(I_k)$  without routing it through  $D(I_j) = D(I_k)$ .) On the other hand, due to faults involved in the instructions used to write data in  $S(I_k)$ , OPERAND 2 may not be stored

in  $S(I_k)$ ; in this case the fault will be detected in step 3 itself.  $\square$

Case B(3.2.2):  $\ell(S(I_k)) = K$ . In this case  $I_k \in \text{READ}(S(I_k))$ .

Procedure 4.9 used for case B(3.2.1) cannot be used in this case

because in step 3,  $\text{READ}(S(I_k))$  will read out  $S(I_k)$  by routing it through  $D(I_j) = D(I_k)$ , destroying its contents.

Procedure 4.10:

This procedure is applicable for case B(3.2.2). It generates tests to detect fault  $f(I_j/I_k)$  when  $\ell(I_j) = \ell(I_k) = K \geq 3$ ,  $D(I_j) = D(I_k)$ , and  $\ell(S(I_k)) = K$ .

Step 1: Store OPERAND 1 in  $S(I_j)$  and OPERAND 2 in  $S(I_k)$  such that  
OPERAND 1  $\neq$  OPERAND 2.

Step 2: For  $i \leftarrow 1$  TO  $K$  DO

BEGIN

Execute  $I_j$ ;

Read out  $D(I_j)$  by executing  $\text{READ}(D(I_j))$ .

/Expected output data = OPERAND 1/

END

Step 3: Execute  $I_k$ .

Step 4: Read out  $D(I_k)$  by executing  $\text{READ}(D(I_k)) = \text{READ}(D(I_j))$ .

/Expected output data = OPERAND 2/  $\square$

Theorem 4.9: Procedure 4.10 detects  $f(I_j/I_k)$  in case B(3.2.2).

Proof: In step 2 of Procedure 4.10, register  $D(I_j)$  is read out by executing  $\text{READ}(D(I_j)) = \langle I_{p_{K-1}}, I_{p_{K-2}}, \dots, I_{p_1} \rangle$ . (Refer to Figure 4.4.) Recall that  $\ell(I_j) = \ell(I_k) = K$ , and  $\ell(I_{p_i}) = i$ , for  $1 \leq i \leq K-1$ . When the microprocessor under test executes this procedure,

it has already been checked that  $f(I_p/\phi)$ ,  $f(I_p/I_q)$  and  $f(I_p/I_p+I_q)$  do not exist, where  $1 \leq \ell(I_p)$ ,  $\ell(I_q) \leq K-1$ . Therefore,  $D(I_j)$  can be correctly read out by executing  $\text{READ}(D(I_j))$ .

If OPERAND 2 is really stored in  $S(I_k)$  in step 1, the fault  $f(I_j/I_k)$  will be detected when  $D(I_j)$  is read out during the first iteration of step 2, since OPERAND 2 will be read out instead of OPERAND 1. On the other hand, due to the faults involved in instructions used to write data in  $S(I_k)$ , OPERAND 1 may have been stored in  $S(I_k)$ . In this situation the fault will not be detected during the first iteration of step 2. We now prove that it is necessary and sufficient to repeat the loop in step 2  $K$  times, where  $|\text{READ}(D(I_j))| = K-1$ .

Sufficiency: So far the microprocessor under test has not executed the tests to detect  $f(I_v/I_v+I_w)$ , where  $1 \leq \ell(I_v) \leq K-1$  and  $K+1 \leq \ell(I_w) \leq K_{\max}$ . If a fault  $f(I_{p_i}/I_{p_i}+I_{p'_i})$  is present, where  $I_{p_i} \in \text{READ}(D(I_j))$  and  $K+1 \leq \ell(I_{p'_i}) \leq K_{\max}$ , it has yet not been detected. Consider a sequence of instructions  $\langle I_{p_{K-1}'}, I_{p_{K-2}'}, \dots, I_{p_1}' \rangle$  where  $I_{p_i}'$  belongs to this sequence if and only if the fault  $f(I_{p_i}/I_{p_i}+I_{p'_i})$  is present, for  $1 \leq i \leq K-1$ . Since  $K+1 \leq \ell(I_{p'_i}) \leq K_{\max}$  and  $K \geq 3$ ,  $I_{p_i}'$  belongs to class T according to our assumption in Section 4.3.1.

Therefore, we designate the sequence  $\langle I_{p_{K-1}'}, I_{p_{K-2}'}, \dots, I_{p_1}' \rangle$  as the T sequence  $\sigma'$ . Note that there are at most  $K-1$  instructions in  $\sigma'$ . Thus when the  $\text{READ}(D(I_j))$  sequence is executed in step 2, the T sequence  $\sigma'$  is also executed in addition. Consider the register which is 1-step transferrable to  $S(I_k)$  under the T sequence  $\sigma'$ . The contents of this register before the execution of  $\text{READ}(D(I_j))$  in the first iteration of step 2 will become the final contents of  $S(I_k)$  at the end of the first

iteration of step 2. This may alter the contents of  $S(I_k)$ .

During the first  $K-1$  iterations of step 2  $READ(D(I_j))$  is executed  $K-1$  times, and so is the T sequence  $\sigma'$ . If register  $S(I_k)$  contains OPERAND 1 at the end of each of  $i$  iterations of step 2, (i.e., at the end of each of  $i$  executions of the T sequence  $\sigma'$ , for  $1 \leq i \leq K-1$ ), then by Corollary 2.3, at the end of the  $K^{th}$  execution of the T sequence  $\sigma'$ ,  $S(I_k)$  will contain OPERAND 1. Recall that there are at most  $K-1$  instructions in the T sequence  $\sigma'$ . If this is so, the fault will be detected in step 4 as  $OPERAND\ 1 \neq OPERAND\ 2$  will be read out.

On the other hand, if at the end of any of the first  $K-1$  iterations of step 2,  $S(I_k)$  contains data different from OPERAND 1, the fault will be detected in the next iteration of step 2 when  $D(I_j)$  is read out by executing  $READ(D(I_j))$ , as the output data will be different from OPERAND 1.

Necessity: Let the loop in step 2 be repeated only  $p$  times, where  $1 \leq p \leq K-1$ . (Recall that  $K \geq 3$ .) Consider the register which is  $p$ -step transferrable to  $S(I_k)$  under the T sequence  $\sigma'$ . Since there can be as many as  $K-1$  instructions in  $\sigma'$ , such a register can exist (from Theorem 2.1). If this register contains OPERAND 2, and  $S(I_k)$  and each register which is  $j$ -step transferrable to  $S(I_k)$  ( $1 \leq j \leq p-1$ ) under the T sequence  $\sigma'$  contains OPERAND 1 at the beginning of the first iteration of the loop, then the contents of  $S(I_k)$  will be equal to OPERAND 1 at the end of each of  $i$  iterations of the loop, for  $1 \leq i \leq p-1$ , and they will become OPERAND 2 at the end of the  $p^{th}$  iteration of the loop. (Refer to Corollary 2.1.)

Therefore, OPERAND 1 will be read out during each of  $p$  iterations of the loop in step 2, and OPERAND 2 will be read out in step 4, as expected. Thus the fault may remain undetected if the loop in step 2 is repeated less than  $K$  times.  $\square$

#### 4.3.4. Test Generation for $f(I_j/I_j+I_k)$

The details of test generation depend on  $\ell(I_j)$  and  $\ell(I_k)$ . We consider six cases, namely, case C(1) through case C(6). The suffix C is used to denote that a case belongs to the details of test generation for  $f(I_j/I_j+I_k)$ . These cases are divided into subcases which are listed in Table 4.3. For each case, the table gives which test generation procedure is applicable and which theorem proves the fault coverage.

##### 4.3.4.1. Test Generation for $f(I_j/I_j+I_k)$ When $\ell(I_j) = \ell(I_k) = 1$

This case is referred to as case C(1). As in the case B(1.2), many faults in this case would be readily detected because  $I_j$  and  $I_k$  have the highest observability. This case can be divided into two subcases, namely case C(1.1) and case C(1.2), exactly in the same way case B(1.2) was divided into case B(1.2.1) and case B(1.2.2). (Refer to Section 4.3.3.1.) The tests generated by Procedure 4.11 for case C(1.1) are to be applied before those generated by procedures for case C(1.2).

Case C(1.1): During the execution of  $I_j$  and  $I_k$  the results of the operation performed are "read out".

##### Procedure 4.11:

This procedure is applicable for case C(1.1). It generates tests to detect  $f(I_j/I_j+I_k)$  when  $\ell(I_j) = \ell(I_k) = 1$  and during the execution of  $I_j$  and  $I_k$  the results of the operations are read out.

Table 4.3. Cases for the details of test generation for detecting  $f(I_j/I_j + I_k)$ .

Case and its description		Test generation Procedures	Theorems proving the fault coverage
<u>Case C(1)</u>  $\ell(I_j) = \ell(I_k) = 1$	<u>Case C(1.1)</u> During the execution $I_j$ and $I_k$ the results of the operations performed are read out.	Procedure 4.11	Theorem 4.10
	<u>Case C(1.2)</u> During the execution of $I_j$ and $I_k$ the results of the operations performed are stored in registers, and not read out.	Procedure 4.7 (with two modifications)	Theorems <sup>†</sup> 4.6 and 4.10
	<u>Case C(1.2.1)</u> Results produced by $I_j$ and $I_k$ are stored in the same register. <u>Case C(1.2.2)</u> Results produced by $I_j$ and $I_k$ are stored in different registers.	Procedure 4.12	Theorem 4.11
<u>Case C(2)</u>  $\ell(I_j) = 1,$ $\ell(I_k) = 2$	<u>Case C(2.1)</u> $D(I_k) \subseteq S(I_j)$ , and the result of the operation performed under $I_j$ is read out during its execution.	Procedure 4.13	Theorem 4.12
	<u>Case C(2.2)</u> $D(I_k) \not\subseteq S(I_j)$ , and the result of the operation performed under $I_j$ is read out during its execution.	Procedure 4.14	Theorem 4.13
	<u>Case C(2.3)</u> Results produced by $I_j$ and $I_k$ are stored in the same register.	Procedure 4.15	Theorem 4.14
<u>Case C(3)</u>  $\ell(I_j) = \ell(I_k) = 2$	<u>Case C(2.4)</u> Results produced by $I_j$ and $I_k$ are stored in different registers.	Procedure 4.16	Theorem 4.15
	<u>Case C(3.1)</u> $D(I_j) = D(I_k)$	Procedure 4.17	Theorem 4.16
	<u>Case C(3.2)</u> $D(I_j) \neq D(I_k)$	Procedure 4.18	Theorem 4.17

<sup>†</sup> Fault coverage can be proved by following arguments similar to those in the proof of the theorem (marked with †).

Table 4.3 (continued). Cases for the details of test generation for detecting  $f(I_j/I_j + I_k)$ .

Case and its description		Test generation procedure	Theorems proving the fault coverage
Case C(4)	Case C(4.1) $D(I_j) = D(I_k)$		
$\ell(I_j) = \ell(I_k)$ $= K \geq 3$	Case C(4.1.1) $\ell(S(I_k)) < K$	Procedure 4.19	Theorem 4.18
	Case C(4.1.2) $\ell(S(I_k)) = K$	Procedure 4.20	Theorem 4.19
	Case C(4.2.1) $D(I_j) \neq D(I_k)$ $\ell(S(I_k)) < K$	Procedure 4.21	Theorem 4.20
	Case C(4.2.2) $\ell(S(I_k)) = K$	Procedure 4.22	Theorem 4.21
Case C(5) $1 \leq \ell(I_j) \leq K$ , $\ell(I_k) = K+1$ , and $K \geq 2$	Case C(5.1) $\ell(S(I_k)) \leq K-1$	Procedure 4.21	Theorem <sup>†</sup> 4.20
	Case C(5.2) $\ell(S(I_k)) = K$ , or $K+1$	Procedure 4.22	Theorem <sup>†</sup> 4.9
Case C(6) $K+1 \leq \ell(I_j) \leq K_{\max}$ , and $\ell(I_k) = K$	Case C(6.1) $2 \leq \ell(I_j) \leq K_{\max}$ , and $\ell(I_k) = 1$	Readily detected by executing $I_j$	-
	Case C(6.2) $3 \leq \ell(I_j) \leq K_{\max}$ , and $\ell(I_k) = 2$	Procedure 4.23	Theorems <sup>†</sup> 4.10 through 4.17
	Case C(6.3) $K+1 \leq \ell(I_j) \leq K_{\max}$ , and $\ell(I_k) = K \geq 3$	Procedure 4.24	Theorems <sup>†</sup> 4.10 through 4.17

<sup>†</sup>Fault coverage can be proved by following arguments similar to those in the proof of the theorem (marked with †).



Step 1: If possible, store proper operands in  $S(I_j)$  and  $S(I_k)$  such that when  $I_j$  is executed RESULT 1 is read out and when  $I_k$  is executed RESULT 2 is read out, and  $(\text{RESULT } 1) \vee (\text{RESULT } 2) \neq (\text{RESULT } 1)$ .  
 $\vee$  denotes the bit-wise logical OR function/

Step 2: Execute  $I_j$ . /Expected output data = RESULT 1/

Step 3: Execute  $I_k$ . /Expected output data = RESULT 2//

Step 4: If possible, store proper operands in  $S(I_j)$  and  $S(I_k)$  such that when  $I_j$  is executed RESULT 3 is read out and when  $I_k$  is executed RESULT 4 is read out, and  $(\text{RESULT } 3) \wedge (\text{RESULT } 4) \neq (\text{RESULT } 3)$ .  
 $\wedge$  denotes the bit-wise logical AND function/

Step 5: Execute  $I_j$ . /Expected output data = RESULT 3/

Step 6: Execute  $I_k$ . /Expected output data = RESULT 4/ □

The underlined phrase "if possible" in steps 1 and 4 may come as a surprise. However, it may not be possible to satisfy conditions given in both the steps due to the nature of the operations performed under the instructions  $I_j$  and  $I_k$ . The following example illustrates the point.

Example 4.10: Let instruction  $I_j$  be "Store the contents of register  $R_1$  into the main memory location (next to the one containing the opcode of instruction  $I_j$ )," and instruction  $I_k$  be "Perform the logical AND of the contents of registers  $R_1$  and  $R_2$  and store the result into the main memory location." The requirement in step 4 of Procedure 4.11 can be satisfied by storing a ONE in  $R_1$  and a ZERO in  $R_2$  so that under  $I_j$  a ONE is read out and under  $I_k$  a ZERO is read out, and  $(\text{ONE} \wedge \text{ZERO}) = \text{ZERO} \neq \text{ONE}$ .

On the other hand, no operands would satisfy the requirement in step 1. This can be easily proved. Let OPERAND 1 and OPERAND 2 be stored in  $R_1$  and  $R_2$  respectively.  $I_j$  will read out RESULT 1 = OPERAND 1 and  $I_k$  will read out RESULT 2 = (OPERAND 1)  $\wedge$  (OPERAND 2). If  $f(I_j/I_j + I_k)$  is present, then as illustrated in Example 3.2, (RESULT 1) \* (RESULT 2) would be read out, where \* denotes the bit-wise logical AND or OR function depending on technology. In this situation, if \* is the OR function,  $I_j$  would read out (RESULT 1)  $\vee$  (RESULT 2) = (OPERAND 1)  $\vee$  ((OPERAND 1)  $\wedge$  (OPERAND 2)) = OPERAND 1 = RESULT 1, as expected. Thus, the fault  $f(I_j/I_j + I_k)$  is an undetectable fault.  $\square$

Theorem 4.10: Procedure 4.11 detects all detectable  $f(I_j/I_j + I_k)$  faults in case C(1.1).

Proof: If the proper operands as required by the condition in step 1 are really stored in  $S(I_k)$ , and  $f(I_j/I_j + I_k)$  causes the actual result read out under instruction  $I_j$  to be the bit-wise logical OR combination of RESULT 1 and RESULT 2, the fault will be detected in step 2 as (RESULT 1)  $\vee$  (RESULT 2)  $\neq$  RESULT 1 will be read out. On the other hand, under faults involved in the instructions used to store data in  $S(I_k)$ , wrong operands may have been stored in  $S(I_k)$  in step 1. In this situation the fault will be detected not in step 2 but in step 3 when  $I_k$  is executed. (Recall the fault model in Section 3.2; if  $f(I_j/I_j + I_k)$  exists,  $I_k$  is correctly executed.) Steps 4, 5, and 6 detect the fault if  $f(I_j/I_j + I_k)$  causes the actual result read out under instruction  $I_j$  to be the bit-wise logical AND combination of RESULT 1 and RESULT 2.  $\square$

Case C(1.2): During the execution of instructions  $I_j$  and  $I_k$  the results of the operations performed are stored in registers. (The results are not read out as in case C(1.1).) This case is identical to case B(1.2.2), and all the points illustrated in Example 4.7 do apply here too. Depending on whether the results of  $I_j$  and  $I_k$  are stored in the same or different registers we divided this case into two subcases, namely, case C(1.2.1) and case C(1.2.2), exactly in the same way case B(1.2.1) was divided into case B(1.2.1.1) and case B(1.2.1.2).

Case C(1.2.1): The results produced by executing instructions  $I_j$  and  $I_k$  are stored in the same register. Let this register be designated as  $R_p$ ; furthermore let  $\text{READ}(R_p) = \langle I_p \rangle$ . Note that  $\ell(I_p) = 1$ . (Refer to Example 4.8.) In this case we follow Procedure 4.7 given for case B(1.2.2.1), except for two modifications:

1. In step 1 of Procedure 4.7, RESULT 1 and RESULT 2 should satisfy the condition  $(\text{RESULT 1}) \vee (\text{RESULT 2}) \neq \text{RESULT 1}$ .
2. Step 1 through step 5 of Procedure 4.7 are repeated, and RESULT 1 and RESULT 2 should satisfy the condition  $(\text{RESULT 1}) \wedge (\text{RESULT 2}) \neq \text{RESULT 1}$ .

It can be easily proved that the procedure detects  $f(I_j/I_j + I_k)$  in this case by following the similar arguments used in the proofs of Theorems 4.6 and 4.10.

Case C(1.2.2): The results produced by executing instructions  $I_j$  and  $I_k$  are stored in different registers. Let the result produced by  $I_k$  be stored in register  $R_p$ . Furthermore, let  $\text{READ}(R_p) = \langle I_p \rangle$ . Note that  $\ell(I_p) = 1$ .

Pro

Procedure 4.12:

This procedure is applicable for case C(1.2.2). It generates tests to detect fault  $f(I_j/I_j + I_k)$  when  $\ell(I_j) = \ell(I_k) = 1$ , and during the execution of  $I_j$  and  $I_k$  the results of the operations are stored in different registers.

Step 1: Store OPERAND 1 in register  $R_p$  and proper operands in  $S(I_k)$  such that when  $I_k$  is executed, RESULT1 is stored in  $R_p$  and  $RESULT1 \neq \text{OPERAND 1}$ .

Step 2: Read out register  $R_p$  by executing  $READ(R_p) = \langle I_p \rangle$ .

/Expected output data = OPERAND 1 /

Step 3: Repeat step 2.

Step 4: Execute  $I_j$ .

Step 5: Read out register  $R_p$  by executing  $READ(R_p) = \langle I_p \rangle$ .

/Expected output data = OPERAND 1  $\neq$  RESULT 1/ □

Theorem 4.11: Procedure 4.12 detects  $f(I_j/I_j + I_k)$  in case C(1.2.2).

The proof of this theorem follows closely that of Theorem 4.6 and will not be repeated here. □

4.3.4.2. Test Generation for  $f(I_j/I_j + I_k)$  When  $\ell(I_j) = 1$  and  $\ell(I_k) = 2$  and  $\ell(I_j) = 2$  and  $\ell(I_k) = 2$

This case is referred to as case C(2). We divide this case into various subcases, namely, case C(2.1) through case C(2.4), depending on whether  $D(I_k) \subseteq S(I_j)$  and whether during the execution of  $I_j$  the result of the operation performed under  $I_j$  is read out. Tests generated for case C(2.1) and case C(2.2) are to be applied before those for case C(2.3) and case C(2.4).

Case C(2.1):  $D(I_k) \subseteq S(I_j)$ , and the result of the operation performed under  $I_j$  is read out during its execution. An example of this case is given below.

Example 4.11: Instruction  $I_j$  is "Push the contents of the accumulator on the top of a LIFO stack maintained in the main memory," and instruction  $I_k$  is "Transfer the contents of register  $R_1$  to the accumulator." In general,  $I_j$  may perform some operation on its operands and then read out the result. □

Procedure 4.13:

This procedure is applicable for case C(2.1). It generates tests to detect  $f(I_j/I_j+I_k)$  when  $\ell(I_j) = 1$ ,  $\ell(I_k) = 2$ ,  $D(I_k) \subseteq S(I_j)$ , and the result of the operation performed under  $I_j$  is read out during its execution.

Step 1: Store proper operands in  $S(I_j)$  such that when  $I_j$  is executed RESULT 1 is read out. Store proper operands in  $S(I_k)$  such that when  $I_k$  is executed, it changes the contents of  $D(I_k)$  so that if  $I_j$  is executed after  $I_k$ , RESULT 2  $\neq$  RESULT 1 will be read out.

Step 2: Execute  $I_j$ . /Expected output data = RESULT 1 /

Step 3: Repeat step 2. /Expected output data = RESULT 1  $\neq$  RESULT 2 /

Step 4: Execute  $I_k$ .

Step 5: Execute  $I_j$ . /Expected output data = RESULT 2 / □

Theorem 4.12: Procedure 4.13 detects  $f(I_j/I_j+I_k)$  in case C(2.1).

Proof: If the proper operands as required in step 1 are really stored in  $S(I_k)$  the fault will be detected in step 3, because RESULT 2 will be read out instead of RESULT 1. If proper operands are not stored in  $S(I_k)$  due to the faults in instructions used to write data in  $S(I_k)$ , the

fault may not be detected in step 3, but it will be detected in step 5 because the output data will then be different from RESULT 2.  $\square$

Case C(2.2):  $D(I_k) \neq S(I_j)$  and the result of the operation performed under  $I_j$  is read out during its execution.

Procedure 4.14:

This procedure is applicable for case C(2.2). It generates tests to detect fault  $f(I_j/I_j+I_k)$  when  $\ell(I_j) = 1$ ,  $\ell(I_k) = 2$ ,  $D(I_k) \neq S(I_j)$ , and the result of the operation performed under  $I_j$  is read out during its execution.

Step 1: Store proper operands in  $S(I_k)$  and OPERAND 1 in  $D(I_k)$  such that when  $I_k$  is executed it produces RESULT 1  $\neq$  OPERAND 1 in  $D(I_k)$ .

Step 2: Execute  $I_j$ .

Step 3: Read out  $D(I_k)$  by executing READ ( $D(I_k)$ ).

/Expected output data = OPERAND 1 /

Step 4: If  $\ell(S(I_k)) \geq 2$  then repeat steps 2 and 3 else go to step 5.

Step 5: Execute  $I_k$ .

Step 6: Read out  $D(I_k)$  by executing READ ( $D(I_k)$ ).

/Expected output data = RESULT 1  $\neq$  OPERAND 1 /  $\square$

Theorem 4.13: Procedure 4.14 detects  $f(I_j/I_j+I_k)$  in case C(2.2).

Proof: If the proper operands as required in step 1 are really stored in  $S(I_k)$  the fault  $f(I_j/I_j+I_k)$  will be detected in step 3 because RESULT 1  $\neq$  OPERAND 1 will be read out. On the other hand, if proper operands are not stored in  $S(I_k)$  because of faults in the instructions used to store data in  $S(I_k)$ , OPERAND 1 may be produced in  $D(I_k)$  when  $I_k$  is executed. In this situation step 3 will not detect the fault  $f(I_j/I_j+I_k)$ . The necessity for step 4 is now explained.

At this stage the microprocessor under test has not executed the tests to detect  $f(I_p/I_p+I_q)$  where  $\ell(I_p) = 1$  and  $\ell(I_q) \geq 3$ . If  $f(I_p/I_p+I_q)$  is present and  $D(I_q) \subseteq S(I_k)$  (requiring  $\ell(S(I_k)) \geq 2$ ), then the execution of  $READ(D(I_k))$  in step 3 may cause a change in the contents of  $S(I_k)$  due to the additional execution of  $I_q$ . This will be detected in step 4 as the output data will be different from OPERAND 1. On the other hand, if the contents of  $S(I_k)$  remain unchanged after step 3 they will remain unchanged after step 4 also. In this situation the fault will be detected in step 6 as the output data will be different from RESULT 1. □

Case C(2.3): The results produced during the execution of  $I_j$  and  $I_k$  are stored in the same register. (The result of the operation performed under  $I_j$  is not read out, as in case C(2.1) or case C(2.2).) Let this register be designated as  $R_p$ ; furthermore let  $READ(R_p) = \langle I_p \rangle$ . Note that  $\ell(I_p) = 1$ . (This is consistent with our discussion in Example 4.7.)

Procedure 4.15:

This procedure is applicable for case C(2.3). It generates tests to detect fault  $f(I_j/I_j+I_k)$  when  $\ell(I_j) = 1$ ,  $\ell(I_k) = 2$ , and the results produced during the execution of  $I_j$  and  $I_k$  are stored in the same register designated as  $R_p$ .

Step 1: If possible, store proper operands in  $S(I_j)$  and  $S(I_k)$  so that when  $I_j$  is executed RESULT 1 is produced in  $R_p$  and when  $I_k$  is executed RESULT 2 is produced in  $R_p$ , and  $(RESULT 1) \vee (RESULT 2) \neq (RESULT 1)$ .

Step 2: Execute  $I_j$ .

Step 3: Read out  $R_p$  by executing  $READ(R_p) = \langle I_p \rangle$ .

/Expected output data = RESULT 1 /

Step 4: If  $\ell(S(I_k)) \geq 2$  then repeat steps 2 and 3 else go to step 5.

Step 5: Execute  $I_k$ .

Step 6: Read out  $R_p$  by executing  $READ(R_p) = \langle I_p \rangle$ .

/Expected output data = RESULT 2 /

Step 7: Repeat step 1 with the change that  $(RESULT 1) \wedge (RESULT 2) \neq (RESULT 1)$ .

Step 8: Repeat steps 2 through 6. □

Theorem 4.14: Procedure 4.15 detects  $f(I_j/I_j+I_k)$  in case C(2.3). □

The proof of this theorem follows closely those of Theorems 4.10 and 4.13, and hence is not given.

Case C(2.4): The results produced during the execution of  $I_j$  and  $I_k$  are stored in different registers.

Procedures 4.16:

This procedure is applicable for case C(2.4). It generates tests to detect fault  $f(I_j/I_j+I_k)$  when  $\ell(I_j) = 1$ ,  $\ell(I_k) = 2$ , and the results produced during the execution of  $I_j$  and  $I_k$  are stored in different registers.

Step 1: If possible, store proper operands in  $S(I_k)$  and OPERAND 1 in  $D(I_k)$  so that when  $I_k$  is executed RESULT 1 is produced in  $D(I_k)$  and  $RESULT 1 \neq OPERAND 1$ .

Step 2: Execute  $I_j$ .



Step 3: Read out  $D(I_k)$  by executing  $READ(D(I_k))$ .

/Expected output data = OPERAND 1 /

Step 4: If  $\ell(S(I_k)) \geq 2$  then repeat steps 2 and 3 else go to step 5.

Step 5: Execute  $I_k$ .

Step 6: Read out  $D(I_k)$  by executing  $READ(D(I_k))$ .

/Expected output data = RESULT 1/

□

Theorem 4.15: Procedure 4.16 detects  $f(I_j/I_j+I_k)$  in case C(2.4).

□

The proof of this theorem follows the arguments given in the proof of Theorem 4.13, and hence is not given.

#### 4.3.4.3. Test Generation for $f(I_j/I_j+I_k)$ When $\ell(I_j) = \ell(I_k) = 2$

This case is referred to as case C(3) and is divided into two subcases, namely, case C(3.1) and case C(3.2) depending on whether or not  $D(I_j) = D(I_k)$ .

Case C(3.1):  $D(I_j) = D(I_k)$ . The basic requirement of test generation in this case is to store proper operands in  $S(I_j)$  and  $S(I_k)$  such that if  $I_j$  is executed it produces an  $x \in \{0,1\}$  in some bit (say the  $p^{\text{th}}$  bit) of  $D(I_j)$ , and if  $I_k$  is executed it produces  $\bar{x}$  in the same bit of  $D(I_j)$ . When  $I_j$  is executed  $x * \bar{x}$  will be produced in the  $p^{\text{th}}$  bit of  $D(I_j)$ , if  $f(I_j/I_j+I_k)$  is present. (\* denotes the logic AND or OR function.) Note that  $x * \bar{x} \neq x$ , for  $x = 1$  and  $*$  = AND, and for  $x = 0$  and  $*$  = OR. Thus, if  $D(I_j)$  is read out after executing  $I_j$  the fault will be detected.

If  $D(I_j) \not\subseteq S(I_j)$  and  $D(I_j) \not\subseteq S(I_k)$ , no specific data need to be stored in  $D(I_j)$  in order to satisfy this requirement. On the other hand if  $D(I_j) \subseteq S(I_j)$  or  $D(I_j) \subseteq S(I_k)$ , in order to produce an  $x$  in the  $p^{\text{th}}$

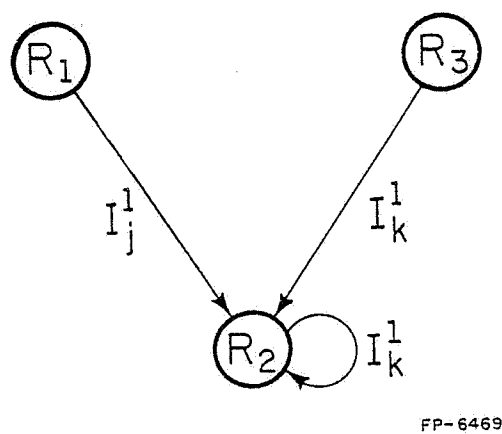
bit of  $D(I_j)$  when  $I_j$  is executed, and to produce  $\bar{x}$  in the  $p^{\text{th}}$  bit  $D(I_j)$  when  $I_k$  is executed, it may be required that some specific logic value must be stored in the  $p^{\text{th}}$  bit of  $D(I_j)$ . If  $x$  must be stored in the  $p^{\text{th}}$  bit of  $D(I_j)$ , the situation is referred to as "situation A"; if  $\bar{x}$  must be stored in the  $p^{\text{th}}$  bit of  $D(I_j)$ , the situation is referred to as "situation B." The following example should clarify these two situations.

Example 4.12: Suppose instruction  $I_j$  is "Transfer the contents of register  $R_1$  to register  $R_2$ ," and instruction  $I_k$  is "Perform the logical AND operation on the contents of register  $R_2$  and  $R_3$ , and store the result in  $R_2$ ." Instructions  $I_j$  and  $I_k$  are represented in a graph in Figure 4.5. We must store  $x = 0$  in the  $p^{\text{th}}$  bit of  $R_1$ , and  $\bar{x} = 1$  in the  $p^{\text{th}}$  bit of both  $R_2$  and  $R_3$ , so that if  $I_j$  is executed  $x = 0$  is produced in the  $p^{\text{th}}$  bit of  $R_2$ , and if  $I_k$  is executed  $\bar{x} = 1$  is produced in the  $p^{\text{th}}$  bit of  $R_2$ . Thus, this is an example of what we have referred to as situation B. An example of situation A can be easily obtained by simply renaming instruction  $I_j$  as  $I_k$ , and vice-versa, and letting  $x = 1$ .  $\square$

We now present two subprocedures to be used in these two different situations. Followed by this we will present Procedure 4.17 (which is a test generation procedure for case C(3.1)) which calls Subprocedure A (as a subroutine) when situation A is present and calls Subprocedure B when situation B is present.

#### Subprocedure A:

This procedure is called as a subroutine by Procedure 4.17 when situation A is present.



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Figure 4.5. Illustrating Example 4.12.

Step 1: Execute  $I_j$ .

Step 2: Read out  $D(I_j)$  by executing READ ( $D(I_j)$ ).

/The  $p^{\text{th}}$  bit of the output data =  $x$ /

Step 3: If  $\ell(S(I_k)) \geq 2$  then repeat steps 1 and 2 else go to step 4.

Step 4: Execute  $I_k$ .

Step 5: Read out  $D(I_j)$  by executing READ ( $D(I_j)$ ).

/The  $p^{\text{th}}$  bit of the output data =  $\bar{x}$ /

□

#### Subprocedure B:

This procedure is called as a subroutine by Procedure 4.17

when situation B is present.

Step 1: Execute  $I_k$ .

Step 2: Read out  $D(I_k)$  by executing READ ( $D(I_k)$ ).

/The  $p^{\text{th}}$  bit of the output data =  $\bar{x}$ /

Step 3: If  $\ell(S(I_k)) \geq 2$  then repeat steps 1 and 2 else go to step 4.

Step 4: Execute  $I_j$ .

Step 5: Read out  $D(I_j)$  by executing READ ( $D(I_j)$ ).

/The  $p^{\text{th}}$  bit of the output data =  $x$ /

□

#### Procedure 4.17:

This procedure is applicable for case C(3.1) and

case B(2.2). It generates tests to detect fault  $f(I_j/I_j+I_k)$  when  $\ell(I_j) = \ell(I_k) = 2$  and  $D(I_j) = D(I_k)$ , and fault  $f(I_j/I_k)$  when  $\ell(I_j) = \ell(I_k) = 2$  and  $D(I_j) \neq D(I_k)$ .

FOR  $i \leftarrow 1$  TO 2 DO

BEGIN

Step 1: IF  $i = 1$  THEN  $x \leftarrow 0$  ELSE  $x \leftarrow 1$ .

Step 2: If possible, store proper operands in  $S(I_j)$  and  $S(I_k)$  such that if  $I_j$  is executed it produces  $x$  in some bit (say the  $p^{\text{th}}$  bit) of  $D(I_j)$ , and if  $I_k$  is executed it produces  $\bar{x}$  in the  $p^{\text{th}}$  bit of  $D(I_j)$ .

Step 3: If step 2 requires  $x$  to be stored in the  $p^{\text{th}}$  bit of  $D(I_j)$  then execute Subprocedure A else execute Subprocedure B.

END.

Note that if  $D(I_j) \not\subseteq S(I_j)$  and  $D(I_j) \not\subseteq S(I_k)$ , then as mentioned earlier no specific data need to be stored in  $D(I_j)$  in step 2 of Procedure 4.17. Therefore strictly speaking either of the subprocedures could be called in step 3 when  $D(I_j) \not\subseteq S(I_j)$  and  $D(I_j) \not\subseteq S(I_k)$ .

Theorem 4.16: Procedure 4.17 detects  $f(I_j/I_j + I_k)$  in case C(3.1) as well as  $f(I_j/I_k)$  in case B(2.2). *given in Section 4.3.3.2.*

Proof: In step 2, proper operands are chosen in  $S(I_j)$  and  $S(I_k)$  to produce  $x$  in some bit (say the  $p^{\text{th}}$  bit) of  $D(I_j)$ , if  $I_j$  is executed, and to produce  $\bar{x}$  in the same bit of  $D(I_j)$ , if  $I_k$  is executed. If  $f(I_j/I_j + I_k)$  is present,  $x * \bar{x}$  will be produced in the  $p^{\text{th}}$  bit of  $D(I_j)$ , when  $I_j$  is executed. If  $f(I_j/I_k)$  is present  $\bar{x}$  will be produced in the  $p^{\text{th}}$  bit of  $D(I_j)$ . The first iteration of the FOR loop is for detecting the fault if  $*$  denotes the OR function, and the second iteration of the loop is for detecting the fault if  $*$  denotes the AND function.

Without loss of generality we assume that  $*$  denotes the OR function and situation A exists. Therefore in step 3, Subprocedure A is executed as a subroutine. If proper operands are really stored in  $S(I_k)$  in step 2 of Procedure 4.17, the fault will be detected in step 2 of

Subprocedure A when  $D(I_j)$  is read out. On the other hand, proper operands may not be stored in  $S(I_k)$  due to faults involved in instructions used to write data in  $S(I_k)$ . In this case the fault will not be detected in step 2 of Subprocedure A. The microprocessor under test has not yet executed tests to detect  $f(I_p/I_p + I_q)$  where  $\ell(I_p) = 1$  and  $\ell(I_q) \geq 3$ . Therefore the execution of  $\text{READ}(D(I_j))$  in step 2 of the subprocedure may cause additional execution of an instruction  $I_q$  such that  $\ell(I_q) \geq 3$  and  $D(I_q) \subseteq S(I_k)$ . If this changes the contents of  $S(I_k)$  the fault may be detected in step 3 of the subprocedure, otherwise it will be detected in step 5. (Refer to the details of the proof of Theorem 4.13.)  $\square$

Case C(3.2):  $D(I_j) \neq D(I_k)$ . In this case we follow the procedure below.

Procedure 4.18:

This procedure is applicable for case C(3.2). It generates tests to detect fault  $f(I_j/I_j + I_k)$  when  $\ell(I_j) = \ell(I_k) = 2$  and  $D(I_j) \neq D(I_k)$ .

Step 1: Store proper operands in  $S(I_k)$  and OPERAND 1 in  $D(I_k)$  such that when  $I_k$  is executed it produces RESULT 1 in  $D(I_k)$ , and  $\text{RESULT } 1 \neq \text{OPERAND } 1$ . If  $\ell(S(I_k)) = 1$ , read out  $S(I_k)$  by executing  $\text{READ}(S(I_k))$ .

/To make sure that  $S(I_k)$  contains expected operands/

Step 2: Execute  $I_j$ .

Step 3: Read out  $D(I_k)$  by executing  $\text{READ}(D(I_k))$ .

/Expected output data = OPERAND 1  $\neq$  RESULT 1/

Step 4: If  $\ell(S(I_k)) \geq 2$  then repeat steps 2 and 3 else go to step 5.

Step 5: Execute  $I_k$ .

Step 6: Read out  $D(I_k)$  by executing  $READ(D(I_k))$ .

/Expected output data = RESULT 1/

□

Theorem 4.17: Procedure 4.18 detects  $f(I_j/I_j+I_k)$  in case C(3.2).

Proof: We will give only the sketch of the proof since the basic ideas are essentially the same as used in the proofs of Theorem 4.10 through 4.17. If proper operands are really stored in  $S(I_k)$  in step 1 the fault will be detected in step 3, since RESULT 1 will be read out. On the other hand, if proper operands are not stored in  $S(I_k)$  due to faults in instructions used to write data in  $S(I_k)$ , the fault will be detected either in step 1 (if  $\ell(S(I_k)) = 1$ ) when  $S(I_k)$  is read out, or in step 4 or 6. □

#### 4.3.4.4. Test Generation for $f(I_j/I_j+I_k)$ When $\ell(I_j) = \ell(I_k) = K \geq 3$

This case is referred to as case C(4). Note that according to the assumption in Section 4.3.1, instructions  $I_j$  and  $I_k$  belong to class T. When the microprocessor executes the tests to detect faults  $f(I_j/I_j+I_k)$ , where  $\ell(I_j) = \ell(I_k) = K \geq 3$ , it has already executed the tests to detect faults  $f(I_p/I_p+I_q)$  where  $1 \leq \ell(I_p) \leq K-1$  and  $\ell(I_q) \leq 2$ ; in particular it has been already checked that the execution of any instruction of label  $\leq K-1$  will not give rise to the additional execution of any instruction of class M. (Recall our assumption in Section 4.3.1 that if  $I_j$  belongs to class M,  $\ell(I_j) \leq 2$ .) We divide this case into two subcases depending on whether or not  $D(I_j) = D(I_k)$ .

Case C(4.1):  $D(I_j) = D(I_k)$ . Since  $\ell(I_j) = \ell(I_k) = K$ ,  $\ell(D(I_j)) = K-1$ . This case is illustrated in Figure 4.4, where depending on  $\ell(S(I_k))$  this case can be further divided into two subcases.

Case C(4.1.1) refers to  $\ell(S(I_k)) < K$ , and case C(4.1.2) refers to  $\ell(S(I_k)) = K$ . Note that  $\ell(S(I_k)) \neq K$ , since  $\ell(I_k) = K$  and  $I_k \in$  class T. (Refer to the labeling algorithm given in Section 4.1.)

Case C(4.1.1):  $\ell(S(I_k)) < K$ . In this case we follow the procedure below.

Procedure 4.19:

This procedure is applicable for case C(4.1.1). It generates tests to detect fault  $f(I_j/I_j+I_k)$  when  $\ell(I_j) = \ell(I_k) = K \geq 3$ ,  $D(I_j) = D(I_k)$ , and  $\ell(S(I_k)) < K$ . This procedure is essentially the same as Procedure 4.9 executed twice, with the following modifications: During the first execution of the procedure, in step 1 the condition to be satisfied by OPERAND 1 and OPERAND 2 is given by  $(\text{OPERAND 1}) \vee (\text{OPERAND 2}) \neq \text{OPERAND 1}$ , and during the second execution of the procedure the condition to be satisfied is given by  $(\text{OPERAND 1}) \wedge (\text{OPERAND 2}) \neq \text{OPERAND 1}$ .

Theorem 4.18: Procedure 4.19 detects  $f(I_j/I_j+I_k)$  in case C(4.1.1).

Proof: The proof of this theorem parallels closely that of Theorem 4.8. We will, therefore, stress only those points where they differ. In step 2 of Procedure 4.19,  $I_j$  is executed and it is expected to produce OPERAND 1 in  $D(I_j)$ . If  $f(I_j/I_j+I_k)$  exists,  $(\text{OPERAND 1}) * (\text{OPERAND 2})$  would be produced instead, where as before  $*$  denotes the logical AND or OR function, provided  $S(I_k)$  really contained OPERAND 2 when  $I_j$  is executed. In this case the fault will be detected in step 4. On the other hand,



due to the faults involved in the instructions used to write data in  $S(I_k)$ , OPERAND 2 may not be stored in  $S(I_k)$ ; in this case the fault will be detected in step 3 itself.  $\square$

Case C(4.1.2):  $\ell(S(I_k)) = K$ . In this case we follow Procedure 4.10 twice, with the same modifications given for case C(4.1.1). We refer to this modified Procedure 4.10 as Procedure 4.20.

Theorem 4.19: Procedure 4.20 detects  $f(I_j/I_j + I_k)$  in case C(4.1.2).  $\square$

The proof of this theorem follows closely those of Theorem 4.9 and 4.18, and hence is not repeated here.

Case C(4.2):  $D(I_j) \neq D(I_k)$ . Depending on  $\ell(S(I_k))$  this case can be further divided into two subcases. Case C(4.2.1) refers to  $\ell(S(I_k)) < K$ , and case C(4.2.2) refers to  $\ell(S(I_k)) = K$ . Note that  $\ell(S(I_k)) \neq K$ , since  $\ell(I_k) = K$  and  $I_k$  belongs to class T.

Case C(4.2.1):  $\ell(S(I_k)) < K$ . In this case we use the procedure below.

Procedure 4.21:

This procedure is applicable for case C(4.2.1). It generates tests to detect fault  $f(I_j/I_j + I_k)$  when  $\ell(I_j) = \ell(I_k) = K \geq 3$ ,  $D(I_j) \neq D(I_k)$ , and  $\ell(S(I_k)) < K$ .

Step 1: Store OPERAND 1 in  $S(I_k)$  and OPERAND 2 in  $D(I_k)$ , such that  
 OPERAND 1  $\neq$  OPERAND 2.

Step 2: Read out  $S(I_k)$  by executing READ ( $S(I_k)$ ).

/Expected output data = OPERAND/1/

Step 3: Execute  $I_j$ .

Step 4: Read out  $D(I_k)$  by executing  $READ(D(I_k))$ .

/Expected output data = OPERAND 2 /

□

Theorem 4.20: Procedure 4.21 detects  $f(I_j/I_j+I_k)$  in case C(4.2.1).

Proof: Since  $\ell(S(I_k)) < K$ , no instruction in the  $READ(S(I_k))$  sequence can have label greater than  $K-1$ . Also  $\ell(D(I_k)) = K-1$ , hence  $READ(S(I_k))$  reads out  $S(I_k)$  without routing its contents through  $D(I_k)$ . When the microprocessor under test executes this procedure, it has already executed the tests to detect  $f(I_p/\phi)$ ,  $f(I_p/I_q)$ , and  $f(I_p/I_p+I_q)$  where  $1 \leq \ell(I_p)$ ,  $\ell(I_q) \leq K-1$ , and  $f(I_v/I_v+I_w)$  where  $1 \leq \ell(I_v) \leq K-1$  and  $\ell(I_w) = K$ . Therefore, in step 2 of this procedure,  $S(I_k)$  is correctly read out to make sure that it stores OPERAND 1, and it continues to store OPERAND 1 after  $READ(S(I_k))$  is executed. In step 3,  $I_j$  is executed. If  $f(I_j/I_j+I_k)$  is present, OPERAND 1 will be stored in  $D(I_k)$  and the fault will be detected in step 4. □

Case C(4.2.2):  $\ell(S(I_k)) = K$ . In this case we follow the procedure below.

Procedure 4.22:

This procedure is applicable for case C(4.2.2). It generates tests to detect fault  $f(I_j/I_j+I_k)$  when  $\ell(I_j) = \ell(I_k) = K \geq 3$ ,  $D(I_j) \neq D(I_k)$ , and  $\ell(S(I_k)) = K$ .

Step 1: Store OPERAND 1 in  $S(I_k)$  and OPERAND 2 in  $D(I_k)$ , such that OPERAND 1  $\neq$  OPERAND 2.

Step 2: FOR  $i \leftarrow 1$  TO  $K$  DO

BEGIN

Execute  $I_j$ ;

Read out  $D(I_k)$  by executing READ ( $S(I_k)$ ).

/Expected output data = OPERAND 2/

END

Step 3: Execute  $I_k$ .

Step 4: Read out  $D(I_k)$  by executing READ ( $D(I_k)$ ).

/Expected output data = OPERAND 1/

□

Theorem 4.21: Procedure 4.22 detects  $f(I_j/I_j+I_k)$  in case C(4.2.2). □

The proof of this theorem follows very closely that of Theorem 4.9 and is not repeated here.

4.3.4.5. Test Generation for  $f(I_j/I_j+I_k)$  When  $1 \leq \ell(I_j) \leq K$ ,  $\ell(I_k) = K+1$ , and  $K \geq 2$ .

This case is referred to as case C(5). Note that  $I_k$  belongs to class T because  $\ell(I_k) = K+1$  and  $K \geq 2$ . Since  $\ell(I_j) \leq K$  and  $\ell(I_k) = K+1$ ,  $\ell(D(I_j)) \leq K-1$  and  $\ell(D(I_k)) = K$ ; hence  $D(I_j) \neq D(I_k)$ . We divide this case into two subcases depending on whether the value of  $\ell(S(I_k))$  is less than  $K$  or is equal to  $K$  or  $K+1$ . Note that it cannot be greater than  $K+1$  because  $\ell(I_k) = K+1$  and  $I_k$  belongs to class T.

Case C(5.1):  $\ell(S(I_k)) \leq K-1$ . We follow Procedure 4.21 in this case. Following the arguments similar to those given in the proof of Theorem 4.20, it can be proved that Procedure 4.21 also detects  $f(I_j/I_j+I_k)$  in case C(5.1).

Case C(5.2):  $\ell(S(I_k)) = K$  or  $K+1$ . We follow Procedure 4.22 in this case. Following the arguments similar to those in the proof of Theorem 4.9, it can be proved that Procedure 4.22 also detects  $f(I_j/I_j+I_k)$  in case C(5.2).

4.3.4.6. Test Generation for  $f(I_j/I_j+I_k)$  When  $K+1 \leq \ell(I_j) \leq K_{\max}$ , and  $\ell(I_k) = K$ .

This case is referred to as case C(6) and is divided into three subcases depending on the value of  $K$ .

Case C(6.1):  $2 \leq \ell(I_j) \leq K_{\max}$ , and  $\ell(I_k) = 1$ . In this case the fault  $f(I_j/I_j+I_k)$  will be readily detected due to the highest observability of  $I_k$ . For example, during the execution of  $I_k$ , data is transmitted between the microprocessor and the main memory or an I/O device, or the logic values on certain status pins are changed (e.g., during the execution of the "Interrupt enable" instruction). Such is not the case during the execution of  $I_j$ , therefore  $f(I_j/I_j+I_k)$  will be readily detected when  $I_j$  is executed.

Case C(6.2):  $3 \leq \ell(I_j) \leq K_{\max}$ , and  $\ell(I_k) = 2$ . The following procedure is followed in this case.

Procedure 4.23:

This procedure is applicable for case C(6.2). It generates tests to detect fault  $f(I_j/I_j+I_k)$  when  $3 \leq \ell(I_j) \leq K_{\max}$ , and  $\ell(I_k) = 2$ .

Step 1: Store proper operands in  $S(I_k)$  and OPERAND 1 in  $D(I_k)$  such that when  $I_k$  is executed RESULT 1 is produced in  $D(I_k)$ , and OPERAND 1  $\neq$  RESULT 1. If  $D(I_j) \subseteq S(I_k)$  then store a proper operand in  $S(I_j)$  such that when  $I_j$  is executed the contents of  $D(I_j)$  will not change. /To ensure that after executing  $I_j$ ,

the contents of  $S(I_k)$  remain unchanged/

Step 2: Execute  $I_j$ .

Step 3: Read out  $D(I_k)$  by executing  $\text{READ}(D(I_k))$ .

/Expected output data = OPERAND 1/

Step 4: If  $\ell(S(I_k)) \geq 3$  then repeat steps 2 and 3 else go to step 5.

Step 5: Execute  $I_k$ .

Step 6: Read out  $D(I_k)$  by executing  $\text{READ}(D(I_k))$ .

/Expected output data = RESULT 1//

□

Theorem 4.22: Procedure 4.23 detects  $f(I_j/I_j + I_k)$  in case C(6.2).

The proof of this theorem is not given as it follows the same arguments given in the proofs of Theorems 4.10 through 4.17.

Case C(6.3):  $K+1 \leq \ell(I_j) \leq K_{\max}$ ,  $\ell(I_k) = K \geq 3$ . Therefore according to the assumption in Section 4.3.1, instructions  $I_j$  and  $I_k$  belong to class T. In this case the procedure given below is followed.

Procedure 4.24:

This procedure is applicable for case C(6.3). It generates tests to detect fault  $f(I_j/I_j + I_k)$  when  $K+1 \leq \ell(I_k) \leq K_{\max}$ ,  $\ell(I_k) = K$ , and  $K \geq 3$ .

Step 1: Store OPERAND 1 in  $S(I_k)$  and OPERAND 2 in  $D(I_k)$  such that  $\text{OPERAND 1} \neq \text{OPERAND 2}$ . If  $D(I_j) = S(I_k)$  then store OPERAND 1 in  $S(I_j)$ . /To ensure that after executing  $I_j$ , the contents of  $S(I_k)$  remain unchanged/

Step 2: Execute  $I_j$ .

Step 3: Read out  $D(I_k)$  by executing  $READ(D(I_k))$ .

/Expected output data = OPERAND 2.  $S(I_k)$  would continue to store its data after executing  $READ(D(I_k))$ , since

$$l(S(I_k)) \leq K/$$

Step 4: Execute  $I_k$ .

Step 5: Read out  $D(I_k)$  by executing  $READ(D(I_k))$

/Expected output data = OPERAND 1/ □

Theorem 4.23: Procedure 4.24 detects  $f(I_j/I_j+I_k)$  in case C(6.3). □

The proof of this theorem is not given as it follows the same arguments used in the proofs of Theorems 4.10 through 4.17.

#### 4.4. Test Generation Procedures for Detecting Faults in the Data Transfer Function and the Data Storage Function

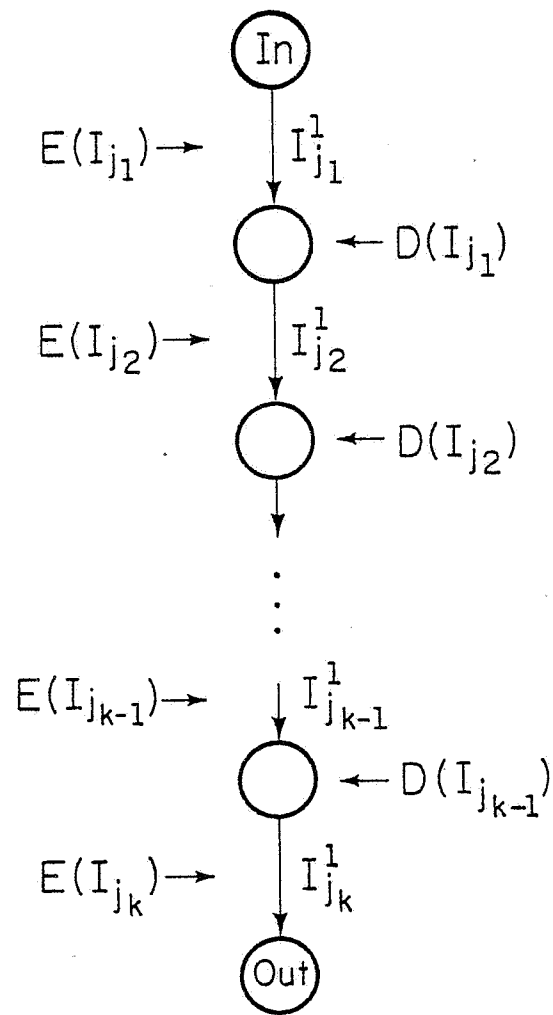
We motivate the discussion by means of an example. Let  $I_{j_1}, I_{j_2}, \dots, I_{j_k}$  be a sequence of instructions of class T such that  $E(I_{j_1}), E(I_{j_2}), \dots, E(I_{j_k})$  form a directed path from the IN node to the OUT node in the corresponding S-graph. Let the transfer paths in sets  $T(I_{j_1}), T(I_{j_2}), \dots, T(I_{j_k})$  each be  $w$  lines in width. Figure 4.6 illustrates the notation. We propose Procedure 4.25 to detect any fault in  $T(I_{j_1}), T(I_{j_2}), \dots, T(I_{j_k})$ , and in registers  $D(I_{j_1}), D(I_{j_2}), \dots, D(I_{j_{k-1}})$ . The fault models for the data storage function and data transfer function are given in Section 3.3 and 3.4, respectively.

##### Procedure 4.25:

$I_{j_1}$  with data:  $111\dots 1$  ;    /Write  $D(I_{j_1})$  with data  $111\dots 1/$   

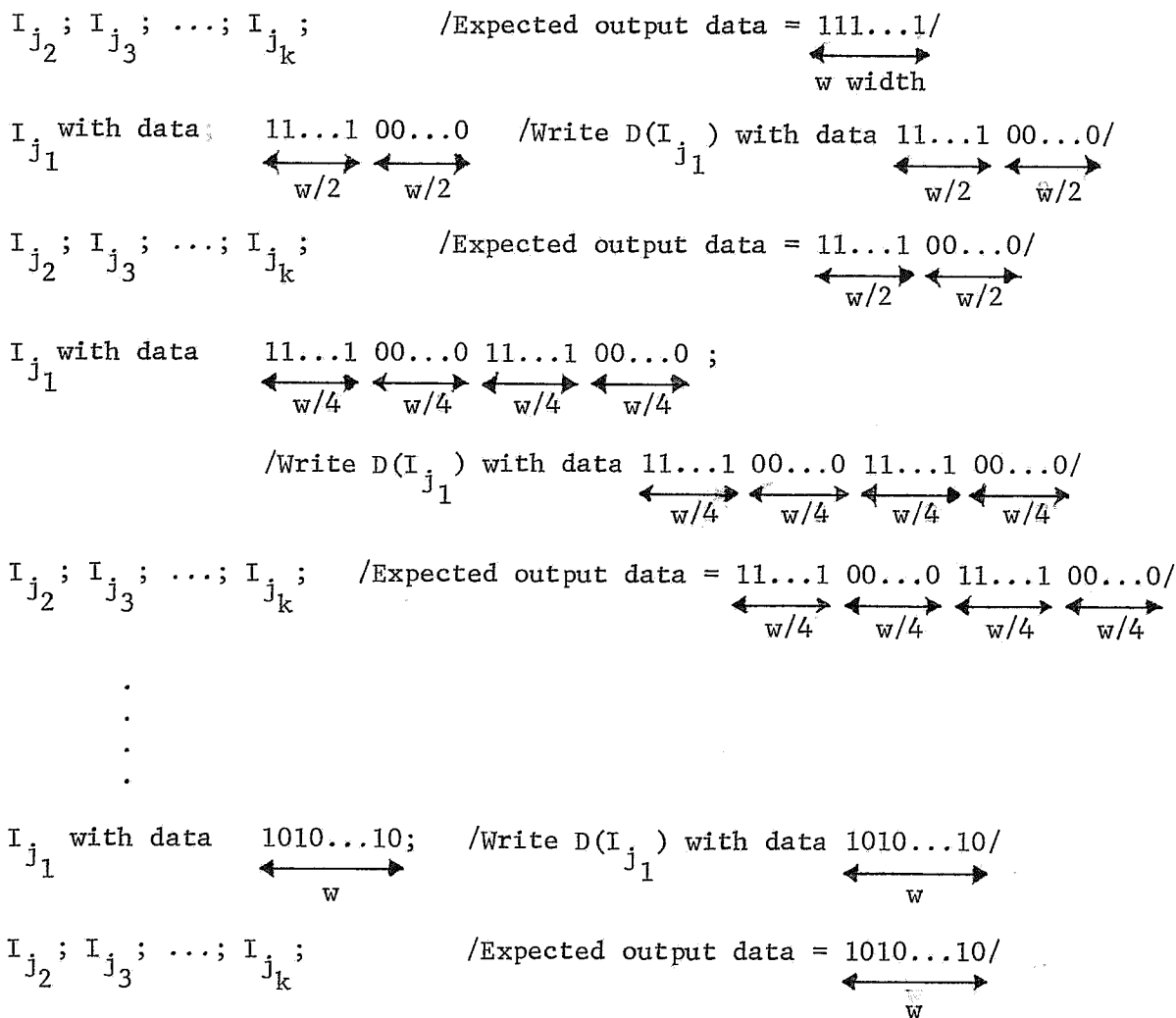
 $\longleftrightarrow$   
 $w$  width
 

 $\longleftrightarrow$   
 $w$  width



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Figure 4.6. Illustrating the notation used in Procedure 4.25.



Repeat the instructions above with complementary data. □

Theorem 4.24: Procedure 4.25 detects

- 1) a line in any transfer path in the set  $T(I_{j_1}) \cup T(I_{j_2}) \cup \dots \cup T(I_{j_k})$  stuck at 0 or 1.
- 2) two or more lines in any transfer path in the set  $T(I_{j_1}) \cup T(I_{j_2}) \cup \dots \cup T(I_{j_k})$  coupled.
- 3) a cell of any register in the set  $D(I_{j_1}) \cup D(I_{j_2}) \cup \dots \cup D(I_{j_{k-1}})$  stuck at 0 or 1.



Proof: If a fault is described by 1 or 3 above, it will be detected either after the execution of sequence  $I_{j_2}, I_{j_3}, \dots, I_{j_k}$  when the expected output data = 111...1, or after the execution of the sequence when the expected output data = 000...0.

Procedure 4.25 also detects any fault described by 2, because at some stage of the procedure any given two lines of a transfer path are required to carry different logic values i.e.,  $x$  and  $\bar{x}$ ,  $x \in \{0,1\}$ , respectively. If these two lines are coupled they will fail to carry different logic values, and the fault will be detected after the subsequent execution of the sequence  $I_{j_2}, I_{j_3}, \dots, I_{j_k}$ .  $\square$

We define the set of tests in Procedure 4.25 "transfer test set" and the associated data being routed on the corresponding transfer paths "transfer test data". Consider the transfer paths associated with the instructions of class T. Concentrate on the subgraph of the S-graph that represents instructions of class T only. We call this subgraph the T-subgraph. Let  $P_1$  be a directed path from the IN node to the OUT node in the T-subgraph. All the instructions which are represented by edges constituting path  $P_1$  are said to be covered by path  $P_1$ . Let  $\{P_1, P_2, \dots, P_n\}$  be a set of directed paths from the IN node to the OUT node of the T-subgraph such that this set collectively covers all the instructions of class T. It is clear from Theorem 4.24 that if the transfer test data is routed from the IN node to the OUT node using the transfer test sets consisting of instructions covered by each path in set  $\{P_1, P_2, \dots, P_n\}$  any fault in the transfer paths associated with the instructions of class T, or any fault in the data storage function will be detected (since the transfer test data is "routed" on every edge of the T-subgraph, every node of the graph

is also visited).

We need to test the transfer paths associated with the instructions of class M. Let " $R_i \circ R_j \rightarrow R_k$ " denote a typical instruction of class M which performs an operation denoted by " $\circ$ " on the contents of register  $R_i$  and  $R_j$ , and stores the result in register  $R_k$ . We need to choose a set of proper operands in  $R_i$  and  $R_j$  such that when the instruction " $R_i \circ R_j \rightarrow R_k$ " is executed it generates and stores data corresponding to the transfer test set in  $R_k$ . We then need to route these data from  $R_k$  to the OUT node by executing READ ( $R_k$ ). This test ensures that the transfer path from the data manipulation logic (e.g. the ALU used in executing instruction " $R_i \circ R_j \rightarrow R_k$ ") to register  $R_k$  is fault free. Therefore any result generated by this instruction can be faithfully transferred to  $R_k$ . Consider instruction  $I_4$  in the S-graph of Figure 2.8. We execute Procedure 4.26 to test the transfer path from the ALU to  $R_1$ . (It is assumed that the microprocessor is an 8-bit processor.)

Procedure 4.26:

$I_1$  with data 1111 1111 ;

$I_2$  with data 0000 0000 ;

$I_4$  ;

$I_7$  ;

Repeat the tests above with data (1111 0000, 0000 0000), (1100 1100, 0000 0000), (1010 1010, 0000 0000), (0000 0000 0000 0000), (0000 1111, 0000 0000), (0011 0011, 0000 0000), (0101 0101, 0000 0000).  $\square$

We also need to check that the transfer paths connecting  $R_i$  and  $R_j$  to the ALU in the " $R_i \circ R_j \rightarrow R_k$ " instruction are fault free. This

ensures that any pair of operands can be applied to the ALU in the " $R_i \circ R_j \rightarrow R_k$ " instruction. For this we need to check that any line in the transfer paths from  $R_i$  and  $R_j$  to the ALU can be set to 0 or 1 independent of the logic values on any other line in these transfer paths. Consider instruction  $I_4$  in Figure 2.8. We want to test the transfer paths connecting  $R_1$  and  $R_2$  to the ALU. We execute Procedure 4.27.

Procedure 4.27:

```

I1 with data 0000 0001 ; /R1 stores 0000 0001/
I2 with data 0000 0000 ; /R2 stores 0000 0000/
I4 ;
I7 ; /Expected output data 0000 0001/

I1 with data 0000 0010 ; /R1 stores 0000 0010/
I2 with data 0000 0000 ; /R2 stores 0000 0000/
I4 ;
I7 ; /Expected output data 0000 0010/

.
.
.
.
.

I1 with data 1000 0000 ; /R1 stores 1000 0000/
I2 with data 0000 0000 ; /R2 stores 0000 0000/
I4 ;
I7 ; /Expected output data 1000 0000/

I1 with data 0000 0000 ; /R1 stores 0000 0000/
I2 with data 0000 0001 ; /R2 stores 0000 0001/
I4 ;
I7 ; /Expected output data 0000 0001/

```

$I_1$  with data 0000 0000 ; / $R_1$  stores 0000 0000/  
 $I_2$  with data 0000 0010 ; / $R_2$  stores 0000 0010/  
 $I_4$  ;  
 $I_7$  ; /Expected output data 0000 0010/

.  
 .  
 .  
 .

$I_1$  with data 0000 0000 ; / $R_1$  stores 0000 0000/  
 $I_2$  with data 1000 0000 ; / $R_2$  stores 1000 0000/  
 $I_4$  ;  
 $I_7$  ; /Expected output data 1000 0000/

Repeat the test above with complementary data. □

We need to execute tests similar to those given in Procedures 4.26 and 4.27 for every instruction of class M. Finally we must test the transfer paths associated with the instructions of class B. This is accomplished by choosing the set of jump or branch addresses such that they correspond to the transfer test set for jump, branch, return from subroutine, etc., instructions. For example, in order to test the transfer path associated with the jump instruction (instruction  $I_9$ ) in Figure 2.8 we need to execute Procedure 4.28. It is assumed that the width of the address bus is 16.

Procedure 4.28:

$I_9$  with jump address 0000 0000 0000 0000 ;  
 $I_9$  with jump address 0000 0000 1111 1111 ;  
 $I_9$  with jump address 0000 1111 0000 1111 ;  
 $I_9$  with jump address 0011 0011 0011 0011 ;  
 $I_9$  with jump address 0101 0101 0101 0101 ;

Repeat the test above with the complementary set of jump addresses. □

#### 4.5. Test Generation Procedure for Detecting Faults in the Data Manipulation Function

As described in Section 3.5 we assume that complete test sets are available for detecting faults (for some specified fault model) in the ALU and other functional units such as a shifter, logic used to increment the program counter, or the interrupt handling logic. The operands specified by such test sets can be provided to a functional unit using, in general, a sequence of instructions of class T. Similarly, the result produced by a functional unit can be read out using a sequence of instructions of class T.

If the logic level description of functional units is available, test sets can be generated for them using classical fault detection algorithms based on the stuck-at fault model. On the other hand, if we do not know the logic level details of the ALU, but know, for example, that it is realized using an iterative logic array we can generate test sets for it as explained in [Dias76]. Another approach would be to generate test sets for functional testing of the ALU, shift, increment, compare logic, etc., using binary decision diagrams [Aker78].

Even though some faults associated with the instruction decoding and control function look like faults in the data manipulation function, and vice versa, the set of faults in one function is not a subset or superset of the set of faults in the other function. For example, under a fault in the instruction decoding function an "Add" instruction may be decoded as a "Subtract" instruction. This fault cannot be distinguished from a fault in the ALU; however, if the test procedure

for detecting faults in the instruction decoding and control function (presented in Section 4.3) is executed correctly, it cannot guarantee the absence of faults in the ALU. In order to detect a fault in the ALU or any other functional units we need to execute the corresponding test sets. Similarly if the test procedure for detecting faults in the data manipulation function is executed correctly, it does not guarantee the absence of faults in the instruction decoding and control function. For example, whenever an "Add" instruction is executed, it may additionally activate an instruction that complements a certain register not involved in the "Add" instruction. Such a fault can be detected by the test procedure used to detect faults in the instruction decoding and control function and not by the test procedure used to detect faults in the data manipulation function.

## 5. COMPLEXITY OF THE TEST SEQUENCES

We now determine the complexity of the test sequences generated by various procedures given in Chapter 4. The complexity is measured in terms of the number of instructions generated as a function of  $n_R$  - the number of registers in set  $\mathcal{R}$ , or  $n_I$  - the number of instructions in the instruction repertoire. This will help in exploring the relationship between the architecture of a microprocessor and the complexity of the test sequences.

Theorem 5.1: The worst case complexity of the test sequence generated by Procedure 4.1 is  $\mathcal{O}(n_R^3)$ , where  $|\mathcal{R}| = n_R$ .

Proof: Let us consider the number of instructions that are generated in steps 3(a), (b), and (c) of Procedure 4.1 when there are  $K$  registers in set  $A$ , i.e.,  $|A| = K$ . As a result of the labeling algorithm,  $\max(\ell(R_i)) = K' \leq K$  for every register  $R_i$  of set  $A$ . Also, every number in the integer set  $\{1, 2, \dots, K'\}$  is assigned as a label to at least one register in set  $A$ . Therefore, in the worst case, during step 3(b) we need to generate  $\Sigma(1 + 2 + \dots + K)$  instructions which would read out the registers of set  $A$ , where  $|A| = K$ . Since set  $A$  is augmented only by one register during each iteration of step 3, in the worst case,  $\sum_{k=1}^{n_R-1} \Sigma(1 + 2 + \dots + K)$  instructions need to be performed for reading out registers of set  $A$ . Similarly, in all iterations of step 3(c), a total of  $\sum_{K=1}^{n_R-1} (K + 1)$  instructions need to be generated to read out the register at the front of the queue.

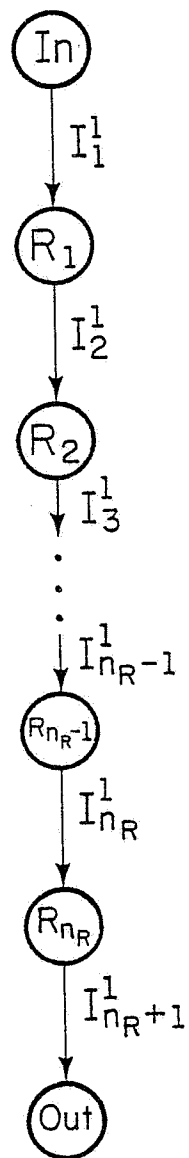
Since there are only  $n_R$  registers, the register that is "farthest away" from the IN node can be written by executing at most  $n_R$  instructions of class T. Moreover, if this register needs  $n_R$  instructions

to be written into, the "next farthest away" register will require  $(n_R - 1)$  instructions, and so on. Therefore when  $|A| = K$ , in the worst case,  $n_R - (K - 1) + n_R - (K - 2) + (n_R - 1) + n_R$  instructions are required to write the registers of a set A during step 3(a). During all iterations of step 3(a), a total of  $\sum_{K=1}^{n_R-1} n_R - (K - 1) + n_R - (K - 2) + \dots + n_R$  instructions are needed to write the registers of set A. Similarly in all iterations of step 3(a), a total of  $\sum_{K=1}^{n_R-1} (n_R - K)$  instructions are needed to write the register at the front of the queue, in the worst case. When all the terms involved in series are summed up, a total of  $n_R^3 + 2n_R^2 - n_R - 2$  instructions are generated. Hence the worst case complexity of the test sequence is  $O(n_R^3)$ .  $\square$

It is instructive to illustrate the worst case example which is shown by a partial S-graph in Figure 5.1. Note that  $O(n_R^3)$  complexity is only for the worst case. For  $n_R = 7$ , 432 instructions are generated in the worst case. However, for the example microprocessor (where  $n_R = 7$ ) only  $2 \times 53 = 106$  instructions are generated as shown in Example 4.2. The exact number of instructions depends on the distribution of integer labels of the nodes. In fact, if all nodes have label 1 (i.e., if the architecture contains only the so-called accumulators and general purpose registers which can be directly loaded into and stored from the main memory), the complexity of the test sequence would be  $O(n_R^2)$ . On the other hand, if the architecture allows scratch-pad registers and on-chip LIFO stacks, for example, (giving rise to nodes with labels greater than 1 in the S-graph), the complexity of the test sequence approaches  $O(n_R^3)$ .

It is very difficult to find a closed form solution for the worst case complexity of the test sequence generated by procedures in





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Figure 5.1. The worst-case example requiring  $\mathcal{O}(n_R^3)$  instructions to be generated by Procedure 4.1.

Section 4.3. We will therefore concentrate on the dominating term in the worst case complexity calculation. This dominating term can be attributed to the loop in step 2 of Procedures 4.10, 4.20 and 4.22. Let  $n_i$  be the number of instructions whose edge sets have been labeled  $i$  in the S-graph. As denoted earlier, let  $K_{\max}$  be the maximum value of labels associated with the edge sets. Thus  $\sum_{i=1}^{K_{\max}} n_i = n$ . The dominating term accounts for  $\sum_{i=1}^{K_{\max}} n_i (i^i)$  instructions generated by step 2 of Procedures 4.10, 4.20 and 4.22.

If the architecture allows instructions which are represented by edge sets with labels much greater than 1, the length of the test sequence could become very large in the worst case, since the complexity grows exponentially (note the  $i^i$  factor in the expression above). This is because instructions represented by edge sets with large labels impart very poor observability to the architecture, i.e., a large number of instructions need to be executed to read out internal registers; this is reflected in the increased length of the test sequences generated by procedures in Section 4.3. However, the expression above is applicable only in the worst case; in many typical architectures  $K_{\max} \leq 3$ , deemphasizing the effect of the dominating term. In fact, if  $K_{\max} \leq 2$  (i.e., the instruction repertoire contains instructions that store their result in the main memory or the accumulators and general-purpose registers), Procedures 4.10, 4.20 and 4.22 will not be required at all. In the case of such architectures, the complexity of the test sequences generated by the procedures given in Section 4.3 can be approximated to  $O(n_I^2)$  because there are  $O(n_I^2)$  faults (in the instruction decoding and control function) and none of them will require Procedure 4.10, 4.20 or 4.22 to generate tests for it. Note that no other procedure has a loop similar to that in step 2 of Procedures 4.10, 4.20

and 4.22. Therefore the complexity of test sequences generated by these procedures can be approximated to  $O(n_I^2)$ .

The length of the test sequence generated by various procedures given in Section 4.4 depends on the widths of data and address buses, the nature of operation "o" performed by instructions " $R_i \circ R_j \rightarrow R_k$ " of class M, and the distribution of integer labels associated with edge sets, i.e.,  $n_{I_i}$  and  $K_{\max}$ . If there are many instructions with higher labels (i.e., large  $n_{I_i}$  for larger i), the length of the test sequences required to detect a fault in the data transfer and data storage functions increases.

For today's microprocessors  $n_R$  typically ranges from 4 to 32, while  $n_I$  ranges from 30 to 200. Note that the complexity of the test sequences for detecting faults in the instruction decoding and control function is at least  $O(n_I^2)$ , while the complexity of the test sequences for detecting faults in the register decoding function is between  $O(n_R^2)$  and  $O(n_R^3)$ . Therefore the test sequences used to detect faults in the instruction decoding and control function constitute a dominant portion of the test sequences for a microprocessor.

## 6. A CASE STUDY

Test sequences were generated for a real microprocessor by applying the test generation procedures developed in the thesis. The goal of the study was two-fold. First, we wanted to generate the test sequences to gain insight into problems involved in using the test generation procedures. We believe that this is an essential first step towards automating the test generation procedures which will operate on a given S-graph. Secondly, we wanted to evaluate the fault coverage of the test sequences for stuck-at faults for a real microprocessor.

A microprocessor from the Hewlett-Packard Company was used. The HP microprocessor is a single chip, n-channel MOS, 8 bit parallel, control oriented processor. All instructions and data are transferred in and out of the microprocessor with an 8 bit bidirectional data bus. Program addresses are transferred out on an 11 bit address bus. There can be up to 15 I/O ports. The normal program may be interrupted by use of the interrupt request control line. The interrupt scheme is fully vectored with 256 possible vectors. The processor can control external circuits and check their status through the use of 7 bidirectional control lines.

The microprocessor contains one 8 bit accumulator, one control logic unit, one 1 bit extend register, sixteen 8 bit registers, one 8 bit magnitude comparator, one 11 bit program counter, one 11 bit subroutine stack register, and one 11 bit interrupt stack register. The instruction set has 187 instructions that includes instructions transferring data between the memory and the accumulator, between the memory and (sixteen 8 bit) registers, between the accumulator and registers, between the accumulator and I/O devices, instructions performing bit manipulations and magnitude comparisons, instructions performing conditional and unconditional jumps in the program sequencing, etc.

We adopted the following strategy in applying the test sequences. Since, as shown in Chapter 5, the test sequences for the instruction decoding and control function form the dominant portion of the test sequences for the microprocessor, we first applied the test sequences for the register decoding function, the data transfer and the data storage function, and the data manipulation function. The length of these sequences was approximately 1 K instructions. This was followed by application of the test sequences for the instruction decoding and control function. The length of these sequences was approximately 8 K instructions. (Recall that there are 187 instructions in the instruction repertoire.) The test sequences were generated by using only the information about the instruction set and organization of the microprocessor.

In order to determine the fault coverage of the test sequences for stuck-at faults, a detailed gate and subnetwork model of the microprocessor (obtained from Hewlett-Packard) was used on the TESTAID III fault simulator. Approximately 2200 single stuck-at faults were simulated. The test sequences generated were run in segments (since the simulator could not handle all the tests at one time) and the fault coverage of each segment was noted. The test sequences for the register decoding, data storage, data transfer, and data manipulation functions were able to detect about 90% of all single stuck-at faults. About 6% of the faults gave rise to simultaneous execution of multiple instructions as described by the fault model for the instruction decoding and control function. Many of these faults were subtle and difficult to detect and very elaborate test sequences were required (accounting for 8 K instructions). For example, when executing the instruction "Skip if the  $n^{\text{th}}$  bit of the accumulator is 0" (with  $n$  between 0 and 7), under a particular single-at fault, the above

instruction will be executed correctly, but at the same time, the contents of the accumulator are also stored in the  $n^{\text{th}}$  register. Some examples of  $f(I_j/I_j+I_k)$  faults found in the case study are given in Table 6.1. (the table lists instructions  $I_j$  and  $I_k$ .)

The remaining faults (about 4%) were associated with the power-up and initialization logic, or were undetectable because of redundancies in the logic, or required invalid opcodes to detect them. Thus for this particular microprocessor the fault coverage was excellent.

The test generation effort was quite straightforward and we believe that it can be automated without much difficulty. The overall results of the case study were quite promising and we are convinced that our approach is a viable and effective one for generating tests for microprocessors.

Table 6.1. Instructions  $I_j$  and  $I_k$  for which fault  $f(I_j/I_j+I_k)$  exists.

Instruction $I_j$	Instruction $I_k$
Clear the extend bit.	Transfer the contents of the accumulator to register $R_5$ .
Return from interrupt and enable interrupt.	Transfer the contents of the accumulator to register $R_1$ .
Clear the third control flag.	Disable interrupt.
Skip if the first control flag is zero.	Enable interrupt.
Clear the first bit of the accumulator.	Clear the zeroth bit of the accumulator.

## 7. CONCLUDING REMARKS

### 7.1. Summary of Thesis

The purpose of this research has been to develop test generation procedures for testing microprocessors that would treat the microprocessor organization and instruction set as parameters. The test generation effort is assumed to be in a user environment where the gate and flip-flop level details of the microprocessor are not known. The procedures will generate tests which can be assembled into valid machine instructions. The microprocessor under test executes these instructions which are stored in the memory of an external tester which continually monitors all the input and output pins of the microprocessor. A fault is detected when the data on any output pin is different from the expected data.

In Chapter 2, the instruction repertoire of the microprocessor was divided into three classes (T, M, and B). Then a graph-theoretic model for microprocessor (called the S-graph) was developed. Each register is represented by a node in the S-graph and data flow involved during the execution of an instruction is represented by a set of directed edges. The motivation behind this approach was to be able to construct a model for the microprocessor for test generation purposes using only the information available in the typical user's manual. This is because the gate and flip-flop level information needed to construct a model at the logic level is not only unavailable, but classical test generation methods which go hand in hand with the logic level model will be very complicated and expensive due to the very large number of gates and flip-flops on the microprocessor chip.

Functional level fault models describing faulty behavior in the register decoding function, instruction decoding and control function, data



transfer function, and data manipulation function were presented in Chapter 3. Various underlying fault mechanisms responsible for functional level faults were pointed out. The fault models are quite independent of the details of implementation. The effects of faults on the behavior of the micro-processor were described at the level of the S-graph.

In Chapter 4, test generation procedures were given to detect faults in the fault models. The first step in test generation is to assign integer labels to the nodes and edges of the S-graph by using the labeling algorithm given in Section 4.1. The label assigned to a node indicates the shortest "distance" of that node to the OUT node (in terms of the instructions of class T or B); the label assigned to the edge set representing an instruction is directly derived from the label assigned to its destination register.

Test generation procedures presented in the subsequent sections of the chapter take full advantage of the information obtained from these labels; tests are generated in such a way that the knowledge gained from the correct execution of tests used for checking the decoding of registers and instructions with lower labels is utilized in generating tests for checking the decoding of registers and instructions with higher labels.

In Chapter 5, the complexity of test sequences generated by the test generation procedures in Chapter 4 was studied. The complexity is measured in terms of the number of instructions generated as a function of  $n_R$  - the number of instructions in the instruction repertoire. The worst case complexity of the test sequence for the register decoding function was shown to be  $O(n_R^3)$ ; however, if all registers have label 1 (indicating the highest observability) the complexity would be  $O(n_R^2)$ .

It was shown that if the instructions have labels less than or equal to two, the complexity of test sequences for the instruction decoding and control function is  $O(n_I^2)$ . If the architecture allows instructions with labels greater than two, the complexity increases very rapidly. Since  $n_R$  typically ranges from 4 to 32, while  $n_I$  ranges from 30 to 200, the test sequences for the instruction decoding and control function constitute a dominant part of the test sequences of a microprocessor.

In Chapter 6 we have described our effort regarding the development of test sequences based on the test generation procedures in Chapter 4 for a real 8-bit microprocessor from the Hewlett-Packard Company. Approximately 2200 single stuck-at faults were simulated. About 96% of these faults were detected by these test sequences. The remaining faults were associated with the power-up and initialization logic, or were undetectable because of redundancies in the logic or they required invalid opcodes for their detection. The results of our study were quite promising.

Thus to summarize the thesis, our approach allows us to treat the organization and instruction set of microprocessors as parameters of the test generation procedures. The information needed to construct the S-graph is easily available in the user's manual. We believe our approach is a viable and effective one towards generating test sequences for microprocessors.

## 7.2. Suggested Future Research

The S-graph of the microprocessor is capable of modeling most of the architectural features observed with current microprocessors. However, it cannot model some of the features observed in the new, powerful 16-bit microprocessors. For example, instructions exchanging data among two

register files cannot be adequately modeled. In order to understand the effects of these architectural features on test generation, further research needs to be done to model these architectural features using the S-graph or some other similar technique.

Fault model for the instruction decoding and control function considers only "gross" faults  $f(I_j/\phi)$ ,  $f(I_j/I_k)$ , and  $f(I_j/I_j+I_k)$ . Our case study regarding the test generation for the Hewlett-Packard microprocessor and the subsequent evaluation of the fault coverage showed that this fault model was adequate to account for all single stuck-at faults in the instruction decoding and control function of this particular microprocessor. We do not know how adequate the fault model would be for other microprocessors, particularly the new 16-bit microprocessors. (Some of them have an on-chip microprogrammed control unit.) Further research needs to be directed towards evaluating the necessity of modeling other faults such as the ones that give rise to partial execution of an instruction, or a change in the sequence of data flow involved in an instruction. Furthermore, if the evaluation study points to the necessity of the improved fault models, the next problem will be to describe the effects of these faults at the level of the S-graph and then develop test generation procedures to detect these faults.

Another important area of future research is to study the applicability of the test sequences generated by the procedures of Chapter 4 in a testing environment where the sophisticated tester required by our approach cannot be used. For example, in field testing, the external tester must be very simple and most of the testing tasks (such as comparing the output result on a bus with the expected result) must themselves be

carried out by the microprocessor under test. Future research must be directed towards investigating the modifications to the proposed test generation procedures to make them suitable for field testing, or for the so-called "self-testing" operations. Self-testing involves some hardcore, i.e., that part of hardware which must be assumed to be fault free. Therefore identification of the hardcore and its testing by an external tester are two major problems that need to be solved for any self-testing scheme.

Finally, future research needs to be directed towards the challenging problem of design for testability. Architectural features which enhance testability should be investigated. Allowing registers and instructions with as low labels as possible (imparting high observability) is obviously a step in the right direction. Such a solution might degrade the performance of the microprocessor in yet unknown way. The underlying testability-performance trade-offs should also be investigated.

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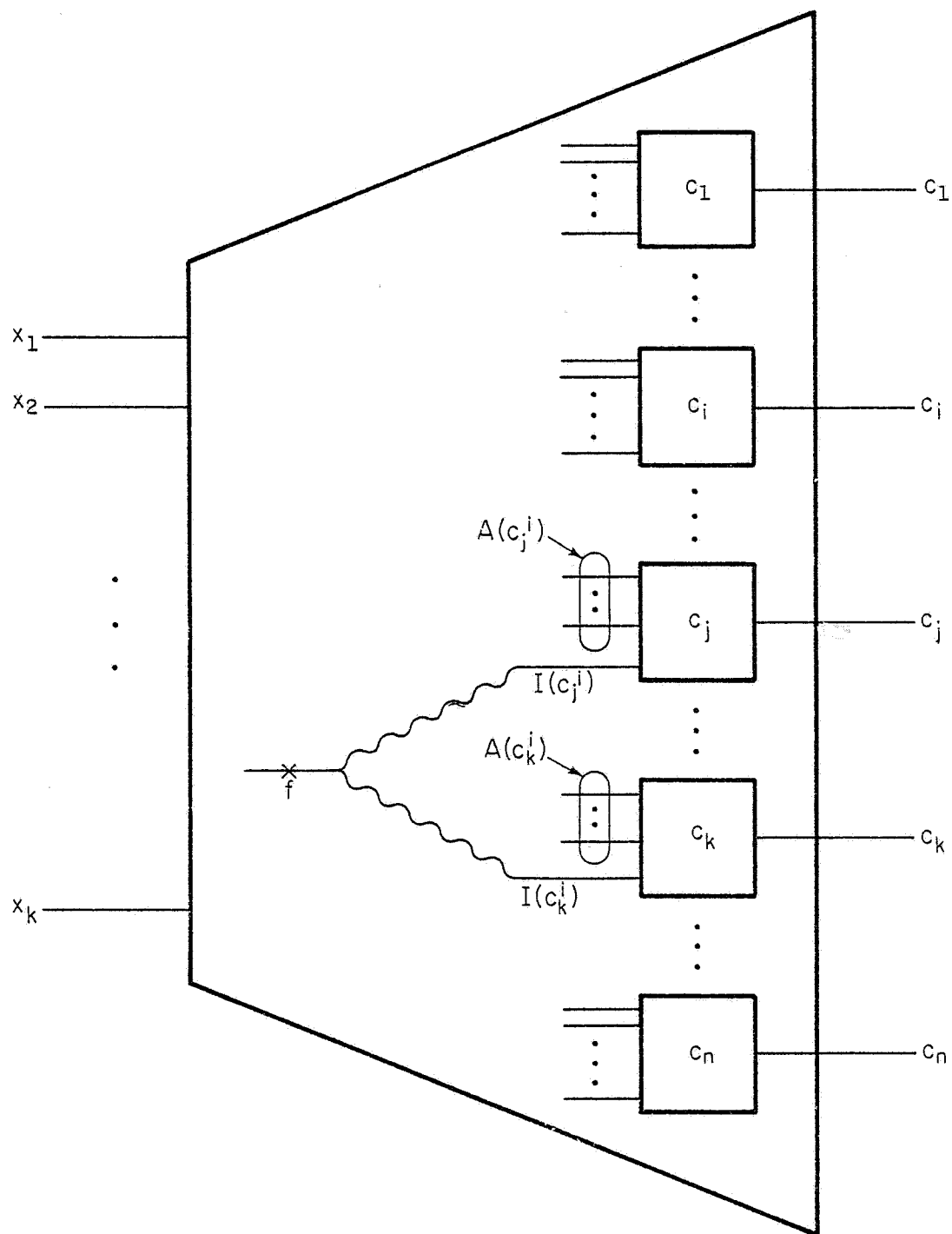
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## APPENDIX

We consider the behavior of a decoder (for a given valid input) under a single stuck-at fault. The decoder is assumed to be realized without any reconvergent fanout. This assumption is quite reasonable as a decoder has  $n$  inputs and as many as  $2^n$  outputs. No other restriction is imposed on its implementation.

Figure A.1 shows a schematic diagram of a decoder which has  $k$  primary inputs labeled  $x_1, x_2, \dots, x_k$ , and  $n$  primary outputs labeled  $c_1, c_2, \dots, c_n$ , where  $n \leq 2^k$ . The set of valid input vectors is a subset of the set of all possible input vectors. Therefore, the set of valid input vectors contains  $n \leq 2^k$  input vectors. The set of valid input vectors which activate output  $c_i$  is denoted by  $X(c_i)$ . Since for any given valid input vector one and only one output is activated  $|X(c_i)| = 1$  and  $X(c_i) \cap X(c_j) = \emptyset$  if and only if  $c_i \neq c_j$ .

Figure A.1 also shows the last level of gates just before the primary outputs. In order to maintain complete generality, each gate is shown as a module and not as a specific gate (such as AND, OR, NOR or NAND). These gates are labeled with the corresponding outputs.  $c_i$  becomes active if and only if all inputs to gate  $c_i$  are active. (For AND and NAND gates, logic 1 is an active input; while for OR and NOR gates, logic 0 is an active input.) If the output of gate  $c_i$  is active, the output of gate  $c_j$  must be inactive ( $c_i \neq c_j$ ). Thus when  $c_i$  is active, at least one input of gate  $c_j$  must remain inactive to ensure that  $c_j$  is inactive. We can partition the inputs of gate  $c_j$  into two sets,  $A(c_j^i)$  and  $I(c_j^i)$ , where  $A(c_j^i)$  is the set of inputs of gate  $c_j$  which are active



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Figure A.1. Schematic diagram of a decoder illustrating the notation used in the proof of Theorem 3.1.



when output  $c_i$  is active, and  $I(c_j^i)$  is the set of inputs of gate  $c_j$  which are inactive when output  $c_i$  is active. Note that  $I(c_j^i) \neq \emptyset$ . We now prove Theorem 3.1 which is restated below for easy reference.

Theorem 3.1: If a decoder is realized without any reconvergent fanout then under a single stuck-at fault its behavior can be formulated independent of its implementation detail as follows: for a given valid input to the decoder, instead of, or in addition to the expected output some other output is activated, or no output is activated.

Proof: We prove the theorem by contradiction. Assume that under a single stuck-at fault, the input vector  $X(c_i)$  activates outputs  $c_j$  and  $c_k$ , in addition to, or instead of  $c_i$ . Therefore under the fault, both  $I(c_j^i)$  and  $I(c_k^i)$  become active in addition to  $A(c_j^i)$  and  $A(c_k^i)$ . Since there is only a single fault, the inputs in  $I(c_j^i)$  and  $I(c_k^i)$  can be traced back to a line where the fault occurs. This line is denoted by  $f$  in Figure A.1. (This could be a primary input line.) Since the decoder does not have any reconvergent fanout,  $|I(c_j^i)| = 1$  and  $|I(c_k^i)| = 1$ ; moreover, the primary inputs which can be traced back from the inputs in  $A(c_j^i)$  are different from those which can be traced back from line  $f$ . Similarly, the primary inputs which can be traced back from the inputs in  $A(c_k^i)$  are different from those which can be traced back from line  $f$ . Thus, if there is no fault in the decoder, the logic value on line  $f$  can be controlled by changing the logic values only on those primary inputs which can be traced back from line  $f$ , without changing the logic values on inputs in  $A(c_j^i)$  and  $A(c_k^i)$ .

We now consider the following "thought experiment" under the fault free condition.

1. Apply  $X(c_i)$ . This will also activate the inputs in  $A(c_j^i)$  and  $A(c_k^i)$ .  $X(c_i)$  does not activate line  $f$ .

2. If necessary, change the logic value on those primary inputs which can be traced back from line  $f$  in order to activate line  $f$ , without making the inputs in  $A(c_j^i)$  or  $A(c_k^i)$  inactive.

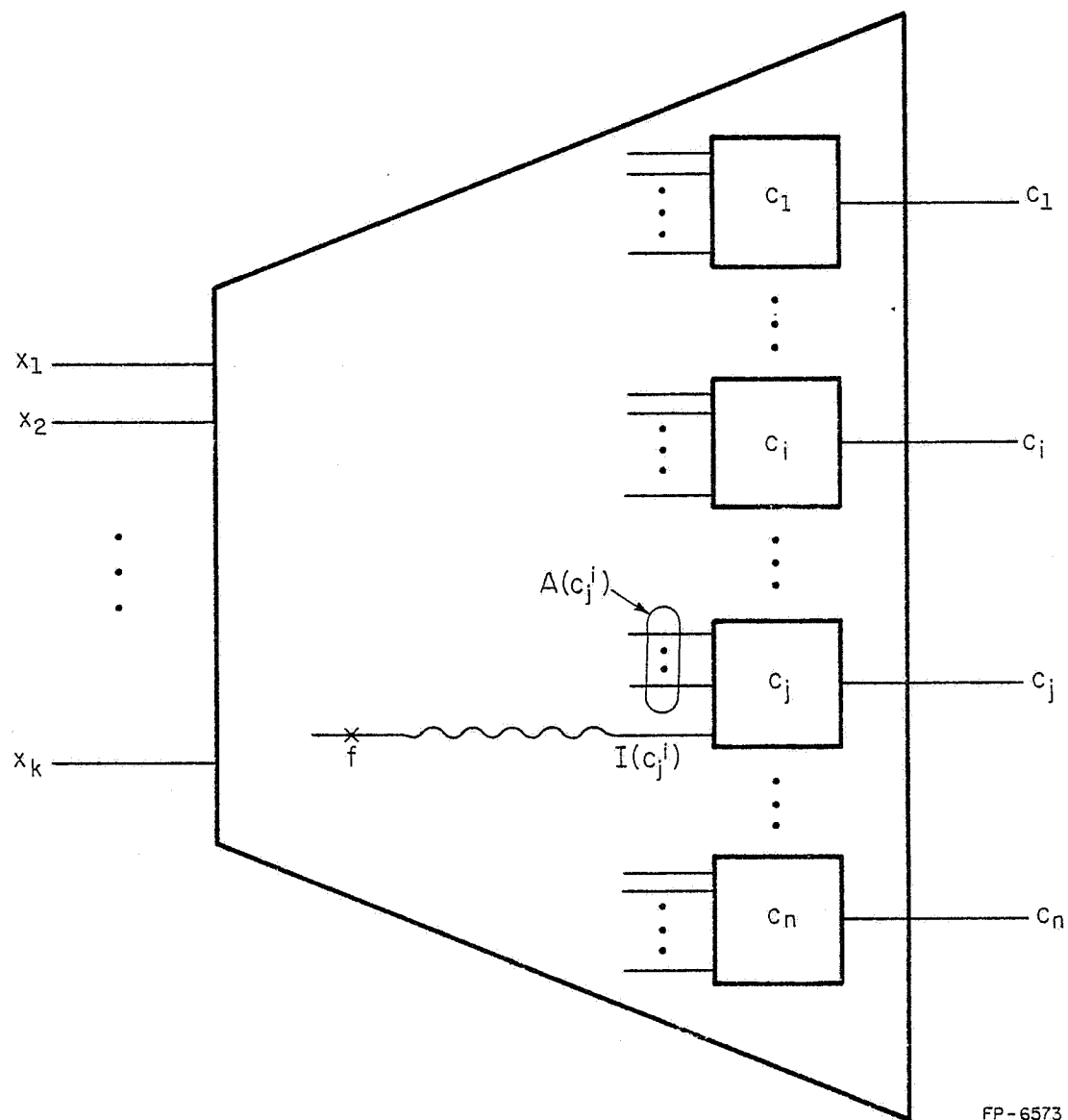
This will make both  $I(c_j^i)$  and  $I(c_k^i)$  active which means that both  $c_j$  and  $c_k$  will become active. Thus even though there is no fault in the decoder, some valid input vector activates both  $c_j$  and  $c_k$ , which is impossible. □

Theorem A.1: If a decoder is realized without any reconvergent fanout then under a single stuck-at fault if  $X(c_i)$  activates  $c_j$  instead of, or in addition to  $c_i$ ,  $X(c_j)$  will activate only  $c_j$ .

Proof: We prove this theorem also by contradiction.

1) First assume that under a single stuck-at fault  $X(c_i)$  activates  $c_j$ , instead of, or in addition to  $c_i$ , and  $X(c_j)$  does not activate any output. Therefore the inputs in  $I(c_j^i)$  can be traced back to a line where the fault occurs. This line is denoted by  $f$  in Figure A.2. Since the decoder does not have any reconvergent fanout,  $|I(c_j^i)| = 1$ , and no input in  $A(c_j^i)$  can be traced back to line  $f$ . When  $X(c_j)$  is applied no output is activated; in particular  $c_j$  is not activated. This can happen only if some input in  $A(c_j^i)$  can be traced back to another fault which keeps that input permanently inactive; but this would violate the assumption of a single stuck-at fault.

2) Now assume that under a single stuck-at fault  $X(c_i)$  activates  $c_j$ , instead of, or in addition to  $c_i$ , and  $X(c_j)$  activates  $c_k$ , instead of, or in addition to  $c_j$ . Therefore, the inputs in  $I(c_j^i)$  and  $I(c_k^j)$  can be



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Figure A.2. Schematic diagram of a decoder illustrating the notation used in the first part of the proof of Theorem A.1.

traced back to a line where the fault occurs. This line is denoted by  $f$  in Figure A.3. Since the decoder does not have any reconvergent fanout,  $|I(c_j^i)| = 1$  and  $|I(c_k^j)| = 1$ ; moreover, the primary inputs which can be traced back from the inputs in  $A(c_j^i)$  are different from those which can be traced back from line  $f$ . Similarly, the primary inputs which can be traced back from the inputs in  $A(c_k^j)$  are different from those which can be traced back from line  $f$ . Thus the logic value on line  $f$  can be controlled by changing the logic values only on those primary inputs which can be traced back from line  $f$ , without changing the logic values on inputs in  $A(c_j^i)$  and  $A(c_k^j)$ .

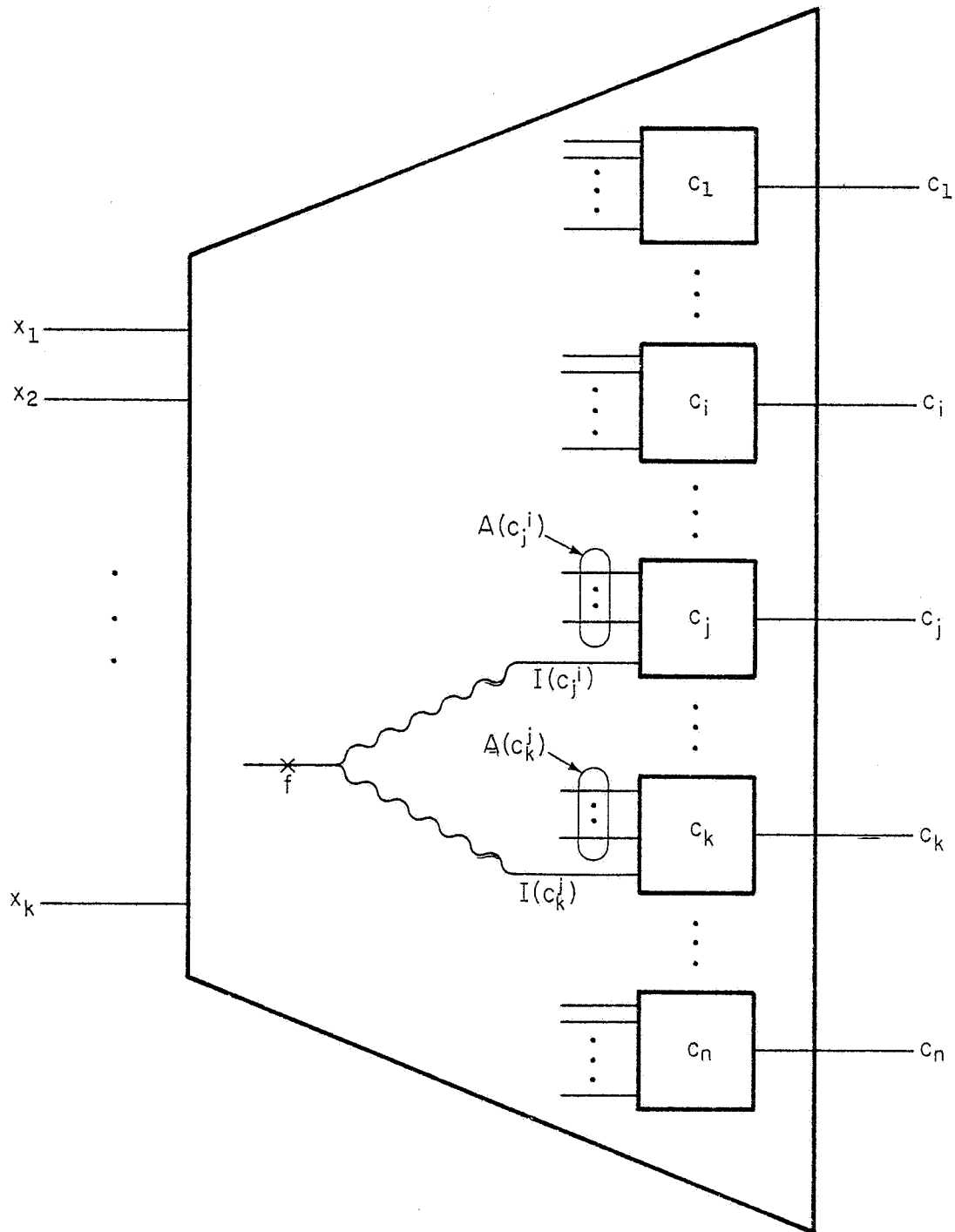
We now consider the following "thought experiment" when there is a fault on line  $f$  as shown in Figure A.3.

1. Apply  $X(c_j)$ . This will also activate the inputs in  $A(c_j^i)$ . Due to the fault on line  $f$ , the input in  $I(c_j^i)$  also becomes active, consequently activating  $c_j$ . At this time some input(s) in  $A(c_k^j)$  must be inactive because  $c_k$  is not active.

2. Change the logic value on those primary inputs which can be traced back from line  $f$  in order to activate line  $f$  (for this  $X(c_j)$  needs to be applied), without changing the logic values in  $A(c_j^i)$  and  $A(c_k^j)$ , i.e., the inputs in  $A(c_j^i)$  are active and some input(s) in  $A(c_k^j)$  are inactive.

Thus we get in a situation where  $X(c_j)$  does not activate  $c_k$  even though fault on line  $f$  exists, contradicting our assumption.  $\square$

Corollary A.1: If a decoder is realized without any reconvergent fanout then under a single stuck-at fault if  $X(c_j)$  does not activate any output, or activates  $c_k$ , instead of, or in addition to  $c_j$ , no  $X(c_q)$  will activate  $c_j$ , instead of, or in addition to  $c_q$ , for  $c_q \neq c_j$ .



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Figure A.3. Schematic diagram of a decoder illustrating the notation used in the second part of the proof of Theorem A.1.

Proof: Follows directly from Theorem A.1.

□

Constraints 4 and 5 given in Section 3.2 are consistent with Theorem A.1 and Corollary A.1.

## VITA

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