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Quadratic Backward Propagation of Variance for Nonlinear Statistical Circuit Modeling

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Abstract—Accurate statistical modeling and simulation are keys to ensure that integrated circuits (ICs) meet the specifications over the stochastic variations that are inherent in IC manufacturing technologies. Backward propagation of variance (BPV) is a general technique for statistical modeling of semiconductor devices. However, the BPV approach assumes that statistical fluctuations are not large so that variations in device electrical performances can be modeled as linear functions of process parameters. With technology scaling, device performance variability over manufacturing variations becomes nonlinear. In this paper, we extend the BPV technique to take into account these nonlinearities. We present the theory behind the technique and apply it to specific examples. We also investigate the effectiveness of several possible solution algorithms.

Index Terms—Backward propagation of variance (BPV), nonlinear devices, semiconductor device modeling, statistical modeling.

I. INTRODUCTION

Integrated circuit (IC) design in the presence of manufacturing process variations requires statistical simulation [1], [2] and, therefore, statistical device models [3]. The models should work with all types of statistical analyses: process corner simulation, distributional (Monte Carlo or MC) simulation, and mismatch simulation.

A number of different approaches for statistical modeling have been proposed. The simplest method is to introduce variations directly into Simulation Program with Integrated Circuit Emphasis (SPICE) model parameters; however, this leads to inaccurate results as it ignores correlations between SPICE model parameters.

Extreme case data, either from split manufacturing lots or from physical simulations of manufacturing extremes, can be used to generate models [4]. However, this does not give distributional models. Standard numerical techniques like principle components analysis [5]–[7] and response surface modeling [8]–[10] can be used to generate statistical models. However, these require significant effort to extract model parameter sets from many statistical device samples and can neither be updated efficiently if a process changes, as it does during development, nor be used for prediction for new generation technologies.

Given a process and a geometry level model, statistical modeling can be done by directly measuring the statistics of the process level model parameters p and then basing the statistical SPICE models on these variations [11], [12]. Although this provides both corner and distributional models, it is not always possible to directly measure all required process parameters. More importantly, this does not take into account the fact that the same variation in similar parameters for different models will give different variations in the modeled electrical performances e [3]. The goal of modeling is to accurately represent

Manuscript received April 18, 2008; revised November 7, 2008 and March 11, 2009. Current version published August 19, 2009. This paper was recommended by Associate Editor D. Sylvester.

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Digital Object Identifier 10.1109/TCAD.2009.2023194

the device electrical performance variation, not the model parameter variation.

A general technique for statistical modeling of semiconductor devices, called backward propagation of variance (BPV), was introduced in [3], [13], and [14]. This technique is based on the models defined as functions of the fundamental (independent) process parameters \mathbf{p} (used for statistical simulation for IC design) and on the statistical device data collected in manufacturing. The method is termed BPV because it takes measurements in variances of important electrical quantities and then calculates the variances in the process parameters necessary to fit the measured data. This gives models that are guaranteed to match the manufacturing extremes and distributions of e (including mismatch). In addition, the models can easily be updated when the process changes or new generation technologies are introduced. However, BPV assumes that statistical variations are not large so that linearization of $e(\mathbf{p})$ is accurate. For general measures of circuit performance, this is not always valid [15], and even for common measures of device performances, this assumption is becoming less valid because variability increases as device dimensions continue to decrease.

In this paper, we extend the BPV technique to account for the nonlinearities in $e(\mathbf{p})$. We denote this technique “quadratic BPV” (QBPV) as it is based on the quadratic (forward) propagation of variance approach introduced in [16]. We present the theory behind the technique and its application to specific examples. We also investigate the effectiveness of several possible solution algorithms.

II. PROPAGATION OF VARIANCE

Consider an electrical performance $e_i = e_i(\mathbf{p})$, where $\mathbf{p} = (p_1, p_2, \dots, p_N)$ is the vector of N process parameters. The second-degree Taylor series expansion of e_i in the neighborhood of $\bar{\mathbf{p}} = (\bar{p}_1, \bar{p}_2, \dots, \bar{p}_N)$ gives

$$e_i(\mathbf{p}) \approx e_i(\bar{\mathbf{p}}) + \sum_{j=1}^N (p_j - \bar{p}_j) \left(\sum_{k=1}^N s_{i,jk} + s_{i,jk} (p_k - \bar{p}_k) \right) \quad (1)$$

where

$$s_{i,j} = \left. \frac{\partial e_i(\mathbf{p})}{\partial p_j} \right|_{\mathbf{p}=\bar{\mathbf{p}}} \quad s_{i,jk} = \left. \frac{1}{2} \frac{\partial^2 e_i(\mathbf{p})}{\partial p_j \partial p_k} \right|_{\mathbf{p}=\bar{\mathbf{p}}}.$$

If p_j is statistically independent and normally distributed with $p_j \sim \mathcal{N}(\bar{p}_j, \sigma_j^2)$, then it follows that [16], [17]

$$\mu_{e_i} = \mathbb{E}\{e_i\} \approx e_i(\bar{\mathbf{p}}) + \sum_{j=1}^N s_{i,jj} \sigma_j^2 \quad (2)$$

$$\sigma_{e_i}^2 = \text{var}\{e_i\} \approx \sum_{j=1}^N s_{i,j}^2 \sigma_j^2 + 2 \sum_{j,k=1}^N s_{i,jk}^2 \sigma_j^2 \sigma_k^2. \quad (3)$$

For $s_{i,jk} = 0$, this gives the standard linear form

$$\mu_{e_i} = \mathbb{E}\{e_i\} \approx e_i(\bar{\mathbf{p}}) \quad \sigma_{e_i}^2 = \text{var}\{e_i\} \approx \sum_{j=1}^N s_{i,j}^2 \sigma_j^2. \quad (4)$$

The standard linear form (4) is sufficiently accurate if e_i is only weakly nonlinear in the neighborhood of $\bar{\mathbf{p}}$. For electrical performances that are not well approximated by the linear functions of the process parameters, a better approximation uses the quadratic forms (2) and (3). In contrast to the linear approximation, $\mathbb{E}\{e_i\}$ is no longer equal to $e_i(\bar{\mathbf{p}})$; therefore, e_i is no longer normally distributed.

The measure of the degree of asymmetry (deviation from normality) of the probability distribution function of e_i is the skewness [18]

$$\gamma_{e_i} = \mu_{e_i^3} / \mu_{e_i^2}^{3/2} \quad (5)$$

where $\mu_{e_i^n} = \mathbb{E}\{(e_i - \mathbb{E}\{e_i\})^n\}$ denotes the n th-order central moments of the distribution of e_i . After some algebraic manipulation

$$\gamma_{e_i} \approx \frac{1}{\sigma_{e_i}^3} \sum_{j,k=1}^N s_{i,jk} \sigma_j^2 \sigma_k^2 \left(6 s_{i,j} s_{i,k} + 8 \sum_{l=1}^N s_{i,kl} s_{i,lj} \sigma_l^2 \right) \quad (6)$$

from which it is apparent that, for a linear response ($s_{i,jk} = 0$), the skewness of e_i is zero, as expected.

III. QBPV

The statistical variation in a particular process parameter p_j can be modeled as

$$p_j = \bar{p}_j + \delta p_j \quad (7)$$

where \bar{p}_j is the nominal (mean) value of the parameter and δp_j is its variation. We assume that all process parameter variations are normally distributed and statistically independent. (A parameter that is distributed lognormally, i.e., the nonideal base-emitter saturation current for bipolar transistors I_{ben} , can be modeled by introducing a normally distributed statistical parameter and multiplying I_{ben} by the exponential of that statistical parameter. Parameters that are partially correlated, for example, channel length variation for NMOS and PMOS devices, can be modeled by using appropriate combinations of independent parameters [19]. We have yet to encounter any statistical device modeling problem that, with some thought, cannot be formulated based on independent normally distributed process parameters.)

Let $\mathbf{e} = (e_1, e_2, \dots, e_M)$ be a vector of the device electrical performances that can be measured in IC manufacturing lines. Statistical variations in \mathbf{p} directly affect the variations in \mathbf{e} . Rather than calculating the variances of \mathbf{e} from the variances of \mathbf{p} using (4), for statistical modeling purposes, the procedure can be reversed to characterize the variances of \mathbf{p} from the measured variances of \mathbf{e} . If σ_{e_i} is known, then $s_{i,j} = \partial e_i / \partial p_j$ can be calculated about the nominal value $\bar{\mathbf{p}}$ from the underlying SPICE models. The variance equations of (4) then give a set of linear equations that can be solved for the unknowns σ_j . This procedure is termed BPV [3]. (The nominal values of \mathbf{p} , which give the nominal values of \mathbf{e} , are determined using nonlinear least squares optimization. The sensitivities are computed from the $\pm 3\sigma$ variations in each p_j ; therefore, the solution for σ_j is done iteratively until the sensitivities and the standard deviations are self-consistent.)

Note that not all process parameters p_j need to be characterized statistically using BPV [3]. For example, for MOSFETs, it can be best to directly measure the distribution of the oxide thickness from the capacitance data. Therefore, statistical characterization can be a mix of forward and backward propagations of variance. If the variances of n of the process parameters p_j , $j \in \mathcal{F} = \{j_1, j_2, \dots, j_n\}$, are known, then (4) becomes

$$\sigma_{e_i}^2 - \sum_{j \in \mathcal{F}} s_{i,j}^2 \sigma_j^2 = \sum_{j \in \mathcal{B}} s_{i,j}^2 \sigma_j^2, \quad i = 1, \dots, M \quad (8)$$

where $\mathcal{B} = \{j = 1, 2, \dots, N : j \notin \mathcal{F}\}$ is the set of indexes of the BPV process parameters. These equations can be solved as long as $M \geq N - n$.

For situations where approximation (4) is not sufficiently accurate, the statistical modeling problem then becomes: How can the nominal values and variances of p_j be calculated so that the nominal values,

variances, and skewness of e_i are modeled accurately? We propose a new approach, i.e., the QBPV, for this task.

If n process parameters are Forward Propagation of Variance (FPV) parameters, then, from (2)

$$\mu_{e_i} - e_i(\bar{p}) - \sum_{j \in \mathcal{F}} s_{i,jj} \sigma_j^2 = \sum_{j \in \mathcal{B}} s_{i,jj} \sigma_j^2. \quad (9)$$

The mean value of $\delta e_i = e_i - e_i(\bar{p})$ is not zero since a nonlinear function of expected values is not equal to the expected value of the nonlinear function. Therefore, the processes of finding the mean values and the variances of the process parameters cannot be anymore decoupled, as is the case for the BPV procedure. The first M of the QBPV system of equations is, therefore, (9).

Accounting for the FPV parameters, from (3), one obtains

$$\begin{aligned} \sigma_{e_i}^2 - \sum_{j \in \mathcal{F}} (s_{i,j}^2 \sigma_j^2 + 2s_{i,jj}^2 \sigma_j^4) \\ = \sum_{j \in \mathcal{B}} \left(s_{i,j}^2 + 4 \sum_{k \in \mathcal{F}} s_{i,jk}^2 \sigma_k^2 \right) \sigma_j^2 + 2 \sum_{j,k \in \mathcal{B}} s_{i,jk}^2 \sigma_j^2 \sigma_k^2. \end{aligned} \quad (10)$$

These provide additional M equations that must be solved as part of the QBPV procedure.

The final set of M equations for the QBPV process targets the fitting of the skewness of the electrical performances. From (6)

$$\begin{aligned} \gamma_{e_i} \sigma_{e_i}^3 - \sum_{j \in \mathcal{F}} (6s_{i,j}^2 s_{i,jj} \sigma_j^4 + 8s_{i,jj}^3 \sigma_j^6) \\ = 12 \sum_{j \in \mathcal{B}} \sum_{k \in \mathcal{F}} (s_{i,j} s_{i,k} + 2s_{i,jk} s_{i,kk} \sigma_k^2) s_{i,jk} \sigma_j^2 \sigma_k^2 \\ + 6 \sum_{j,k \in \mathcal{B}} \left(s_{i,j} s_{i,k} + 4 \sum_{m \in \mathcal{F}} s_{i,jm} s_{i,km} \sigma_m^2 \right) s_{i,jk} \sigma_j^2 \sigma_k^2 \\ + 8 \sum_{j,k,l \in \mathcal{B}} s_{i,jk} s_{i,kl} s_{i,lj} \sigma_j^2 \sigma_k^2 \sigma_l^2. \end{aligned} \quad (11)$$

The QBPV equations (9)–(11) give $3M$ equations to solve for the $2(N - n)$ unknowns, i.e., the means and the variances of the BPV process parameters p_j , $j \in \mathcal{B}$. The defined modeling targets are the means, the variances, and the skewnesses of each of the M electrical performances.

IV. NUMERICAL RESULTS

A. Three Methods to Numerically Solve the QBPV Equations

We have a system of $3M$ equations (for the mean, variance, and skewness of each device performance) with at most $2N$ unknowns (the unknown means and variances of the process parameters). The first M equations (9) have \bar{p}_j coupled with σ_j as the unknowns. The remaining $2M$ equations (10) and (11) are the nonlinear functions of σ_j . These equations all involve sensitivity coefficients that depend on the underlying SPICE models and, more importantly, on the unknown values \bar{p}_j (as for a nonlinear function, the sensitivity $s_{i,j} = \partial e_i / \partial p_j$ depends on the point \bar{p} at which it is evaluated). Therefore, in contrast to the (linear) BPV process, where the means \bar{p}_j can be determined as one step and the sensitivity and variance computations can be done as independent and subsequent steps, for QBPV, the means and the sensitivities need to be calculated self-consistently with, and coupled to, the solution for σ_j . We have investigated three methods to solve the QBPV equations (see Fig. 1).

Method 1: Obtain initial estimates for the means and variances of the process parameters, for example, from linear BPV. From these

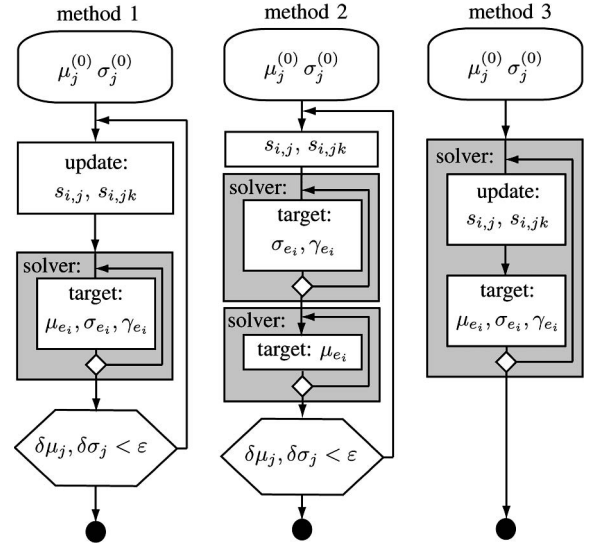


Fig. 1. Three methods to numerically solve the QBPV equations.

values, compute the (first and second order) sensitivity coefficients $s_{i,j}$ and $s_{i,jk}$. Based on these sensitivities, use a nonlinear solver to compute the means and the variances of p_j that best satisfy (9)–(11). Iterate the sensitivity calculations and subsequent computations of the means and variances of p_j until they are, within some convergence criterion, self-consistent.

Method 2: Obtain initial estimates for the means and variances of the process parameters, for example, from linear BPV. From these values, compute the (first and second order) sensitivity coefficients $s_{i,j}$ and $s_{i,jk}$. Based on these sensitivities, use a nonlinear solver to compute the variances of p_j that best satisfy (10) and (11). Use these variances in (9) to update values for the process parameter means, and, then, recalculate the sensitivities. Iterate the sequential variance and mean calculations until they converge.

Method 3: Obtain initial estimates for the means and variances of the process parameters, for example, from linear BPV. Use a nonlinear (iterative) solver to self-consistently solve the system of equations (9)–(11). Here, “self-consistent” means that, as the means and the standard deviations of the process parameters change at each step of the iterative solution process, the calculations of the sensitivity coefficients, which are based on the perturbations of $\pm 3\sigma$ of each p_j , are updated at each iteration. Although this takes more computation time per iteration than methods 1 and 2, for the sensitivity calculations, it obviates the need for the outer iteration loop of methods 1 and 2 and could improve the overall convergence.

For both methods 1 and 2, damping is used for the outer loop iterations on both the means and the variances. Among the software packages for solving systems of nonlinear equations and nonlinear least square problems, TENSOLVE [20], which is based on the tensor method, is proven to be robust and efficient [20], [21] and is used here.

B. Evaluation of Convergence Properties

To evaluate the methods, we first investigate a known analytical nonlinear problem with $N = 3$ process parameters p_j and $M = 3$ device performances e_i related by the following system of equations:

$$e_i = \sum_{j=1}^N a_{ij}^2 p_j^2.$$

This nonlinear example is quadratic; therefore, the QBPV procedure should give an exact solution. It can be shown that the e_i for this

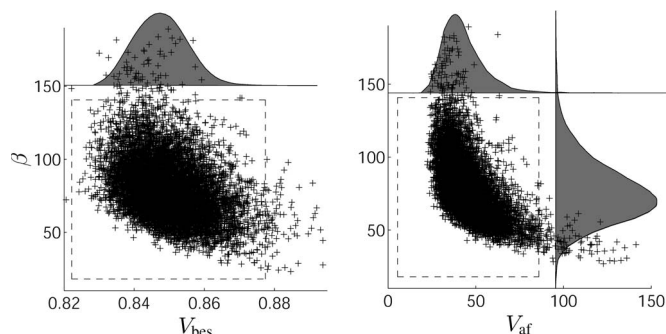


Fig. 2. Scatter plots of the BJT device performances from the MC simulations. The dashed lines are the $\pm 3\sigma$ limits; the shaded bell curves are the sample distributions.

parameters had to be skewed as part of the statistical model generation to center the model on the target fab data.)

Comparison of the measured values with the MC simulations shows that the QBPV results are in better agreement with the measurements than the BPV results. This is expected, because the $e(p)$ mappings are fairly nonlinear for BJTs, and the variations are reasonably large. That this problem is nonlinear was confirmed from the skewnesses computed from the MC simulation ($\gamma_\beta = 1.2$, $\gamma_{V_{bes}} = 0.7$, and $\gamma_{V_{af}} = 4.2$) and is visually apparent in the scatter plots of the MC results (see Fig. 2).

V. CONCLUSION

BPV is a common technique to characterize process parameter variability. This method is effective if the random process parameter deviations are normally distributed and statistically independent and if the dependence of the device electrical performances on the process parameters is nearly linear. With technology scaling and increased variability of process parameters, the assumption of linearity is no longer always reasonable. In this paper, we have introduced QBPV that takes into account the nonlinear dependences of the device electrical performances on the process parameters. We have investigated several different techniques for solving the QBPV equations and compared them in terms of accuracy and computational efficiency. QBPV has been applied to a set of nonlinear problems and provides a significant improvement in the accuracy compared with the (linear) BPV procedure. The method is important in practice as it helps in the reduction of parametric yield loss and overdesign in high-variability and nonlinear IC processes.

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