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On the Performance of Cognitive Satellite-Terrestrial Networks

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Abstract—We investigate the performance of a multi-beam cog-² nitive satellite terrestrial network in which a secondary network 3 (mobile terrestrial system) shares resources with a primary satel-4 lite network given that the interference temperature constraint 5 is satisfied. The terrestrial base stations (BSs) and satellite users 6 are modeled as independent homogeneous Poisson point pro-7 cesses. Utilizing tools from stochastic geometry, we study and ⁸ compare the outage performance of three secondary transmis-9 sion schemes: first is the power constraint (PCI) scheme where 10 the transmit power at the terrestrial BS is limited by the interfer-11 ence temperature constraint. In the second scheme, the terrestrial 12 BSs employ directional beamforming to focus the signal intended 13 for the terrestrial user, and in the third, BSs that do not satisfy 14 the interference temperature constraint are thinned out (BTPI). 15 Analytical approximations of all three schemes are derived and ¹⁶ validated through numerical simulations. It is shown that for the 17 least interference to the satellite user, BTPI is the best scheme. 18 However, when thinning is not feasible, PCI scheme is the viable 19 alternative. In addition, the gains of directional beamforming are 20 optimal when the terrestrial system employs massive multiple-21 input-multiple-output transceivers or by the use of millimeter 22 wave links between terrestrial BSs and users.

23 Index Terms—Cognitive radio, interference, multi-beam satel-24 lite, poisson point processes, satellite-terrestrial networks.

I. INTRODUCTION

²⁶ THE KEY goals of future generation wireless communication systems include billions of connected devices,

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data rates in the range of Gbps, lower latencies, increased 28 reliability, improved coverage and environment-friendly, low- 29 cost, and energy-efficient operation. As the existing cellular 30 spectrum approaches its performance limits, there is grow-31 ing interest in and exploration of supplementary resources 32 for meeting these demands [1]. As a result, satellite mobile communication is attracting widespread interest in radio tech-34 nology studies which aim to provide ample coverage with 35 low complexity infrastructure [2]. Multi-beam structure in 36 modern satellite mobile communication has gained massive attention because of the potential to provide a higher coverage area and larger capacity since multiple isolated spot 39 beams can reuse frequency. For example, with a reuse factor 40 of four, hundreds of beams are possible [3]. The frequency 41 reuse in multi-beam satellites gives a trade-off between inter-42 beam interference and available bandwidth as presented in [4]. 43 Precoding techniques have been established to increase communication efficiency [1]. In the context of multi-beam satel-45 lites, precoding techniques are being explored as a means to mitigate inter-beam interference. The work in [5] shows 47 that with the use of linear precoding, spectral efficiency is improved by about fifty percent. Moreover, motivated by the 49 advances in cellular communication to improve spectral effi-50 ciency, hybrid satellite-terrestrial networks have gained interest 51 in research [6], [7]. 52

Cognitive radio is another technology that has attracted 53 considerable research as a means of spectrum management 54 in conventional wireless communication systems because it 55 allows the coexistence of primary and secondary networks 56 using the same resources [8], [9]. A primary network consists 57 of transmitters and receivers with the licence to use a specific 58 frequency band [10] while a secondary network comprises the 59 transmitters and receivers that share resources with the pri-60 mary network. Cognitive radio networks operate three major 61 paradigms: underlay, overlay and interweave [9]. Within the 62 framework of satellite communication, Sharma et al. [11] suggest that the level of interference power can determine which 64 cognitive technique is appropriate. The underlay paradigm, 65 which allows concurrent primary (non-cognitive) and sec-66 ondary (cognitive) transmissions, and is suitable for medium 67 interference regions, is considered in this paper. 68

In addition, the fusion of cognitive radios with hybrid ⁶⁹ satellite-terrestrial networks (cognitive satellite-terrestrial networks, CSTNs) is investigated by many researchers with ⁷¹ the objective of optimizing efficiency and coverage in ⁷² both existing and future wireless communication systems. ⁷³

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⁷⁴ The work in [12] introduced the concept to show the possi-⁷⁵ bility of maximising spectrum utilization for terrestrial ground ⁷⁶ and satellite uplink transmissions. Additional works enhanc-⁷⁷ ing CSTNs include [13]–[16]. Specifically, the work in [13] ⁷⁸ presents methods for utilizing underlay CSTNs, power alloca-⁷⁹ tion is considered in [14] and performance of CSTNs under ⁸⁰ imperfect channel estimations is measured using the metrics of ⁸¹ outage probability and normalised capacity. Lagunas *et al.* [15] ⁸² investigate efficient allocation of more resources such as car-⁸³ rier, power and bandwidth allocations for achieving more gain ⁸⁴ with the CSTNs, and finally, the work in [16] presents a math-⁸⁵ ematical approach to achieve computational efficiency of the ⁸⁶ outage probability of CSTNs.

With the incorporation of base stations (BSs) to satellite communication, terrestrial interference is another key parameter that needs to be characterized for the accurate analysis of the performance of CSTNs. Given the random locations of terrestrial BSs as well as satellite users [17] and motivated by the successes of using stochastic geometry models for interference characterization in cellular cognitive radio networks [18], [19], we employ the probabilistic stochastic geometric tools for characterizing the interference in CSTNs.

To achieve performance gains, numerous studies have sought ways of managing interference. A well known method for this management is directional transmission [20], [21], which focuses a signal to a target direction (unlike the omnidirectional method in which a signal is transmitted in all directions). Directional transmission has the advantage of reducing interference and increasing coverage. In CSTNs, Sharma *et al.* [22] study different beamforming techniques to jointly achieve maximum rate for the secondary user and minimize interference to the satellite users and show that modified linear constrained minimum variance beamformer achieves tor this objective.

108 A. Design Approaches

This paper evaluates the performance of a CSTN where there is concurrent transmission of a primary multi-beam satellite network and a secondary terrestrial mobile network, and where interference to the primary network is not beyond a set limit. We provide a comparative analysis of different methods the for keeping interference generated by the terrestrial network within acceptable limits.

In [13]–[16], all nodes are assumed to be equipped with a single antenna. However, in the proposed CSTN model, the nodes of the secondary (terrestrial) network will be equipped with multiple antennas as well as multiple beams considered for the satellite network. Therefore, unlike the models network where multiple terresscenario with the analysis of a network where multiple terrestrial base stations (BSs) share resources with a multi-beamed satellite to serve the terrestrial user. To the authors' best knowles edge, randomly distributed BS with multiple antennas has not been considered for this network set-up.

¹²⁷ Introducing multiple BSs with multiple antennas at the sec-¹²⁸ ondary network results in a more involved analysis than is ¹²⁹ presented in [13]–[16], because apart from characterizing the strict interference constraints imposed by the satellite network, ¹³⁰ there is an added interference from other terrestrial BSs trying to serve the terrestrial user. In this paper therefore, we ¹³² characterize this added interference by using stochastic geometric tools, and consider its effect on the transmissions in ¹³⁴ both primary and secondary networks. ¹³⁵

The performance of this network is analysed for three differ- 136 ent transmission schemes. In the first, we assume that the BS 137 process of the secondary network is stationary and ergodic 138 so that BS nodes take part in transmission to the terrestrial 139 user only if they satisfy the interference temperature constraint 140 imposed by the satellite. Thus, we design a framework for 141 characterizing the transmission power at the BS to ensure that 142 the interference limit imposed by the primary network is not 143 surpassed, and also characterize the interference by the BSs 144 that do not satisfy the constraint. This scheme is referred to 145 as power constraint to limit interference (PCI). In the second 146 (DBI), we utilize directional transmission at the secondary 147 system to focus the signals intended for the terrestrial user 148 and accordingly restrict interference to acceptable limits. This 149 scheme is based on the interference limit and thus no power 150 restriction is placed on the terrestrial BSs. Finally, because 151 some BSs may not participate in transmission owing to their 152 inability to satisfy this interference temperature constraint, we 153 will consider for the third scheme only the subset of BSs that 154 meet the satellite's requirement. This consideration leads to a 155 marked point process and will be referred to as the BS thinning 156 process to restrict interference (BTPI). It is important to note 157 that the thinning criteria is based on transmit power constraint 158 which will be described in Section II. 159

The performance of these schemes are analysed in terms 160 of outage probability at both satellite and terrestrial users. To 161 gain further insight, we also study the area spectral efficiency 162 of the secondary system in order to investigate the impact 163 of interference temperature on the average number of successful transmitted symbols. The analysis presented here adds 165 valuable insights to recent works on CSTNs. 166

B. Contributions

The main contributions of the paper can be summarized as 168 follows: 169

- We have presented a more general model of CSTN where 170 a multi-beam satellite shares resources with randomly distributed BSs (equipped with multiple antennas) as long 172 as the interference temperature constraint imposed by the 173 satellite system is satisfied. 174
- We have presented analysis of this network under three 175 schemes of limiting interference generated by the sec- 176 ondary system. 177
 - Power constraint to limit interference (PCI): in this 178 method, the only participating BSs are those that satisfy the primary systems requirements. This require- 180 ment is satisfied by restricting the transmit power at 181 the BSs. 182
 - Directional beamforming to control interference 183 (DBI): here, a transmitting BS utilizes directional 184 beamforming to focus the intended signal to the user, 185

thus restricting interference to the primary networkwithin required limits.

- BS thinning process to restrict interference (BTPI):
 the assumption in this method is that not all BSs
 would satisfy the constraint set by the primary net work. These non-satisfying BSs are thinned out so
 that only the subset of BSs that satisfy the constraint
 participate in communication.
- To analyse the performance of this network, we introduce two important metrics: *outage probability* to measure the effect of interference from BSs other than the intended BS on both satellite and terrestrial communication, and *area spectral efficiency* to investigate the impact of interference temperature on spectrum efficiency at the secondary system.
- We also provide a detailed analysis on the effect of channel fading, BS node density and signal-to-interferenceplus-noise ratio (SINR) threshold on a CSTN.
- Via numerical results, we show the effective trade-off 204 between outage probability performance and number of 205 antennas at each BS and terrestrial user. In addition, BTPI 206 is the best scheme of secondary transmission in a CSTN 207 because of its strict adherence to the satellite system's 208 requirements thereby producing least interference to the 209 satellite user of the three schemes. Finally, where thin-210 ning is not feasible, for a conventional terrestrial mobile 211 system, restricting the transmit power at the terrestrial BS 212 (PCI) is the viable option. 213
- Notations: We use upper and lower case to denote cumutis lative distribution functions (CDFs) and probability density functions (PDFs) respectively. \mathbb{R} denotes the real plane, Probability is denoted by \mathcal{P} , expectation by $\mathbb{E}[\cdot]$, and $\exp(\cdot)$ and $e^{(\cdot)}$ are used interchangeably to represent the exponential function, and all other symbols will be explicitly defined wherever used.
- The rest of the paper is organized as follows. Section II describes the system model. The transmission characterization of multi-beam CSTN is presented in Section III. Section IV gives the numerical analysis, followed by the conclusion in Section V.

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II. SYSTEM MODEL

We consider the downlink of a multi-beam CSTN consisting of a satellite whose coverage area is served by *K* spot beams (known as the primary system) and terrestrial BSs sharing resources with the satellite to communicate with a terrestrial user (secondary system) as shown in fig. 1. h_{pp} and h_{cc} represent the direct channel links from the satellite and a given BS to their respective users, while h_{pc} and h_{cp} are the interference links from satellite to terrestrial user and from BS to satellite user respectively.

In the primary system, the satellite transmits to users using K beams. The users are geographically scattered from which a cluster of K beams are formed. Without loss of generality, a single feed per beam is assumed. Thus, each beam is paired with a single user at a given instance. To manage interterter between adjacent beams and reduce the round trip



Fig. 1. An illustration of network set-up.

delays, multiple gateways (GWs) have been proposed to manage clusters of beams so that distributed joint processing can be utilized [23]. However, in this paper we focus on a single gateway (GW) which manages a cluster of *K* beams with an ideal link between satellite and GW. It is assumed that 246 perfect channel state information is obtainable at the GW¹; 247 these assumptions are typical in [3], [17], and [24].² To reduce 248 the expense of backhauling, joint processing is performed at 249 the GW so that each of *K* user's signal is jointly precoded 250 and transmitted across all beams [3]. In addition, zero-forcing 251 (ZF) precoder for interference management between beams is 252 considered.³

In the secondary system, the underlay cognitive paradigm is 254 employed which allows the terrestrial BSs to transmit concurrently with the satellite as long as interference to the primary 256 user is below a certain threshold. 257

A. Network Model

In this section, we illustrate our system model of a downlink ²⁵⁹ multi-beam CSTN consisting of multiple satellite users with ²⁶⁰ terrestrial BSs serving their desired user. The satellite users in ²⁶¹ the network are modelled as points in \mathbb{R}^2 which are distributed ²⁶² uniformly in the beam radius as a homogeneous Poisson point ²⁶³ process (PPP), Φ_U with intensity λ_U as illustrated in Fig. 2. We ²⁶⁴ assume that a cluster of K beams is formed of users geograph-²⁶⁵ ically close together, in other words, the users in a Voronoi ²⁶⁶ cell comprise a cluster resulting in a coverage area that make ²⁶⁷ up a Voronoi tessellation on the plane. Hence, the total num-²⁶⁸ ber of beams, *K*, can be determined with the help of λ_U . The ²⁶⁹ BSs are also modelled as points of a uniform PPP, Φ_{BS} with ²⁷⁰

AQ3

¹It is an assumption in this paper that the gateway contains information about the deployment of BS nodes in the secondary system attempting to share resources with the satellite so that the value of the interference temperature constraint is set according to the number of active nodes.

²Admittedly, obtaining perfect CSI at the GW is difficult since satellite communication systems experience long round trip delays from the GW to users. However, these studies state that reliable CSI is obtainable by the consideration of fixed satellite services. In addition, recent research efforts are considering precoding paradigms to reduce the dependence of effective precoding on accurate CSI, see [4], [25], [26].

³Although, other precoding schemes have been investigated in recent satellite literature, we consider ZF as a simple linear precoder, shown to improve spectral efficiency with a 20–50 % in [3].



Fig. 2. An illustration of the satellite user network under PPP model showing the location of users in a cluster of K beams. The cell boundaries are shown and form a Voronoi tessellation.

²⁷¹ intensity λ_{BS} in \mathbb{R}^2 . It is assumed that the point processes are ²⁷² independent. For the satellite system, transmissions are simul-²⁷³ taneous and use a universal frequency reuse scenario where ²⁷⁴ all users can use the same channel and we consider a typical ²⁷⁵ user receiving information from a multi-beam satellite.

276 B. Satellite System Model

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1) Fading Model: We assume that the forward link contains both the line-of-sight (LOS) component and the scatter component. Hence, consider Ω to be the average receive power of LOS term, b_0 as half of the average power of scattered component, and *m* as the Nakagami fading coefficient by definition. Leveraging the results from [27], the Shadowed-Rician (SR) fading model can be considered to model both the LOS and scatter components. Therefore the probability density function (PDF) can be written as

$$f_{|h|^2}(x) = \left(\frac{2mb_0}{2mb_0 + \Omega}\right)^m \frac{1}{2b_0} \exp\left(-\frac{x}{2b_0}\right)$$
$$\times {}_1F_1\left(m, 1, \frac{\Omega x}{2b_0(2mb_0 + \Omega)}\right)$$

(1)

where $_1F_1$ is the hypergeometric function and the parameters b_0 , m and Ω are connected with the elevation angle θ as illustrated in Fig. 1. We omit the corresponding expressions of parameters b_0 , m and Ω as they are characterized in detail in [27]. Although the SR fading model is widely used in literature, the PDF and cumulative density function (CDF) are approximate the squared SR model with Gamma random variable. Accordingly, the parameters of Gamma random variable are given as [27]

²⁹⁸
$$\alpha_s = \frac{m(2b_0 + \Omega)^2}{4mb_0^2 + 4mb_0\Omega + \Omega^2}, \ \beta_s = \frac{4mb_0^2 + 4mb_0\Omega + \Omega^2}{m(2b_0 + \Omega)}.$$
²⁹⁹ (2)

2) Antenna Gain at Satellite User Terminal: It is worth noticing that the average SINRs are highly dependent on both so2 satellite beam pattern and user position. Therefore, the beam gain can be approximated as [3]

$$G_{ii} = \mathcal{L}_{\max} G_{s,i} G_{r,i} \left(\frac{J_1(x)}{2x} + 36 \frac{J_3(x)}{x^3} \right)^2$$
(3) 30

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where \mathcal{L}_{max} is the free space loss [24],⁴ x = 3052.07123 $\sin(\phi_{ii})/\sin(\phi_{3dB})$, J_1 and J_3 are the first-kind Bessel functions of order 1 and 3. $G_{s,i}$ is the satellite transmit antenna gain for the *i*th beam and $G_{r,i}$ is the satellite user's receive antenna gain. Note that ϕ_{ii} is denoted as the off-axis angle of the *i*th desired beam, and ϕ_{ij} is the off-axis angle from the *i*th desired beam to the center of the *j*th interfering beam. Therefore, G_{ii} can be calculated from (3) with ϕ_{ii} . Similarly, G_{ij} which is the observed antenna gain between the *j*th interfering beam and the *i*th user, is also calculated by (3) in terms of ϕ_{ij} .

C. Terrestrial System Model

1) Fading Model: The impact of small scale fading on ³¹⁷ the transmitted signals of cellular networks is higher than ³¹⁸ satellite systems. The extensive study of cellular networks ³¹⁹ in [29] and [30] show that the Nakagami fading model can ³²⁰ capture a generalised propagation environment. Hence, we ³²¹ consider Nakagami-*m* channel model, and the channel power ³²² is distributed according to ³²³

$$h_i \sim f_{\Gamma}(x; m_i) \triangleq \frac{m_i^{m_i} x^{m_i - 1} e^{-m_i x}}{\Gamma(m_i)}, \qquad (4) \quad {}_{324}$$

where $i \triangleq cc, cp$, and $\Gamma(m_i)$ is the gamma function.

2) Directional Beamforming Model: In order to reduce the ³²⁶ impact of terrestrial interference on the satellite user termi-³²⁷ nals, we employ directional beamforming at BSs [20], [31]. ³²⁸ Accordingly, multiple antenna arrays are deployed at the trans-³²⁹ mitters. It is worth noticing that the receiver, i.e., terrestrial ³³⁰ user is also equipped with directional antennas. We consider ³³¹ static beamforming though sectorized antennas. Hence, we ³³² are directional antennas at transmit and receiver pairs ³³³ are directional antennas with sectorized gain patterns. Let M_{BS} ³³⁴ denote the number of transmit antennas at a BS and M_R denote ³³⁵ receive antennas which could either be a satellite or terrestrial ³³⁶ user. Denoting the in-sector antenna array gain as G_q^M and ³³⁷ the out-of-sector antenna array gain as G_q^m respectively, these ³³⁸ gains are expressed as [32]

$$G_q^{\mathrm{M}} = \frac{\mathrm{M}_{\mathrm{q}}}{1 + \delta_q (\mathrm{M}_{\mathrm{q}} - 1)},$$
 340

$$G_q^{\rm m} = \delta_q \, G_q^{\rm M},\tag{5} \quad {}_{341}$$

where $q \in \{BS, R\}$, δ_q is a factor that measures the ratio of ³⁴² main lobe to side lobe level. We assume adaptive beamform-³⁴³ ing at the BSs such that active transmission link is that where ³⁴⁴ maximum gain can be achieved. Thus, for any intended link, ³⁴⁵ q (i.e., the transmission link between a given BS and the terrestrial user), the beamforming gain, $G_q = G_{BS}^M G_R^M$. The gains ³⁴⁷

⁴We assume the satellite channel is quasi-stationary which implies that the environmental characteristics including the effect of rain attenuation can be neglected. This is levaraging on the results of experimental data from [28] that shows that the environmental attributes of the channel are assumed to be constant within a small area.

³⁴⁸ of links other than the intended link will be denoted as G_t . ³⁴⁹ G_t also depends on the in-sector directivity gains (i.e., G^M) ³⁵⁰ and out-of-sector (i.e., G^m) gains of the antenna beam pattern. ³⁵¹ Accordingly, the effective antenna gain for an interferer seen ³⁵² by the terrestrial user is given by

$$G_{t} = \begin{cases} G_{BS}^{M} G_{R}^{M}, & \mathcal{P}_{MM} = \frac{1}{M_{BS}M_{R}} \\ G_{BS}^{M} G_{R}^{m}, & \mathcal{P}_{Mm} = \frac{(M_{R} - 1)}{M_{BS}M_{R}} \\ G_{BS}^{m} G_{R}^{M}, & \mathcal{P}_{mM} = \frac{(M_{BS} - 1)}{M_{BS}M_{R}} \\ G_{BS}^{m} G_{R}^{m}, & \mathcal{P}_{mm} = \frac{(M_{R} - 1)(M_{BS} - 1)}{M_{BS}M_{R}} \end{cases}$$
(6)

³⁵⁴ where \mathcal{P}_{tk} , with $t, k \in \{M, m\}$ denotes the probability that the ³⁵⁵ antenna gain $G^t G^k$ is seen by the receiver. Here, the effective ³⁵⁶ gain can be considered as a random variable, which can take ³⁵⁷ any of the above-mentioned values.

358 D. Signal Model

1) Satellite Received Signal: The overall channel gain
between the *j*th beam and *i*th user of the satellite can be
given as

$$h_{pp}^{ij} = h_{pp}^{j} G_{ij} (\phi_{ij})^{1/2}, \quad i, j = 1, \dots, K.$$
(7)

³⁶³ Consider P_{si} as the satellite transmit power of *i*th beam, ³⁶⁴ and x_p^i as the transmitted information symbol from beam *i*. ³⁶⁵ The received signal at *i*th beam user can be formulated as

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$$y_i = \sqrt{P_{si}} G_{ii} h_{pp}^i x_p^i + \sum_{j \in \Phi_U, j \neq i} \sqrt{P_{sj}} G_{ij} h_{pp}^i x_p^j + I_{BS} + \omega_i$$

367 (8)

where ω_i is the noise power at beam *i*, P_{sj} is the satellite transmit power of the *j*th beam and I_{BS} is the terrestrial interference given by

$$I_{\rm BS} = \sum_{l \in \Phi_{\rm BS}} \sqrt{P_{\rm ter}} \, G_t \, h_{cp}^l \, x_c^l \, r_{l,i}^{-\alpha}, \tag{9}$$

where P_{ter} , x_c^l are the transmit power and information signal from the l^{th} terrestrial BS, $r_{l,i}$ is the distance from l^{th} BS to the r_{i}^{th} beam of the satellite user, and α is the path loss exponent. *2) Terrestrial Received Signal:* The received signal at the terrestrial user from the l^{th} BS is represented as:

$$y_l = \sqrt{P_{\text{ter}}} G_l r_l^{-\alpha} h_{cc}^l x_c^l + \sum_{m \in \Phi_{\text{BS}}, m \neq l} \sqrt{P_{\text{ter}}} G_l r_m^{-\alpha} h_{cc}^m x_c^m$$

$$+ I_{\text{SAT}} + \omega_l, \qquad (10)$$

379

³⁸⁰ where ω_l is additive white Gaussian noise $\omega_l \sim CN(0, \sigma_l^2)$, ³⁸¹ I_{SAT} is the interference from the satellite given by

$$_{382} I_{\text{SAT}} = \sum_{j \in \Phi_U} \sqrt{P_{sj}} G_{ij} h_{pc}^j x_p^j, \qquad (11)$$

and h_{pc}^{j} is the interference channel from the j^{th} beam of the satellite to terrestrial user.

To ensure a BS does not cause interference to the satellite system beyond the pre-defined threshold, Υ , its transmit power is further constrained by [14]:

$$P_{\text{ter}} = \min\left(\frac{\Upsilon}{|h_{cp}^l|^2}, P_{\text{tot}}\right), \qquad (12)$$

39'

where h_{cp} is the interference channel from the BS to the ³⁸⁹ primary user and P_{tot} is the total available power at the l^{th} BS. ³⁹⁰

E.

In this subsection, we consider the SINR obtained at the 392 terrestrial and satellite users respectively. 393

1) SINR at Terrestrial User: The SINR at the terrestrial $_{394}$ user from the l^{th} BS can be formulated from (10) and given as: $_{395}$

$$\zeta_l = \frac{P_{\text{ter}} G_l |h_{cc}^l|^2 r_l^{-\alpha}}{\sigma_l^2 + I_{\text{BS}} + I_{\text{SAT}}},$$
(13) 396

where h_{cc}^{l} is the fading gain of the channel between l^{th} and ³⁹⁷ the terrestrial user, $I_{\rm BS} = \sum_{m \in \Phi_{\rm BS}, m \neq l} P_m G_t |h_{cc}^m|^2 r_m^{-\alpha}$ is the ³⁹⁸ interference from other BSs in $\Phi_{\rm BS}$, $I_{\rm SAT} = \sum_{j \in \Phi_U} P_{sj} G_{ij} |h_{pc}^j|^2$ ³⁹⁹ represents interferences from each beam of the satellite to terrestrial user, r_l is the distance from the l^{th} BS to the user, σ_l^2 ⁴⁰¹ is the noise power.

SINR at Satellite User: The SINR for the intended link i at 403 the i^{th} user can then be formulated as 404

$$\sigma_{i} \triangleq \frac{P_{si}G_{ii}|h_{pp}^{i}|^{2}}{\sigma_{i}^{2} + \sum_{j \in \Phi_{u}, j \neq i} P_{sj}G_{ij}|h_{pp}^{j}|^{2} + I_{BS}},$$
 (14) 405

where h_{pp}^{i} is the channel fading gain at the i^{th} user, σ_{i}^{2} is the 406 noise power, and h_{pp}^{j} denotes each interference fading gain 407 from other beams to their users, I_{BS} is the interference from 408 the terrestrial system defined in (9).

The second term of the denominator in (14) is zero due to ⁴¹⁰ successful ZF precoding.⁵ Hence, the SINR for the intended ⁴¹¹ link *i* at any particular user considering terrestrial interference ⁴¹² can be re-written as ⁴¹³

$$\hat{\zeta}_{i} \triangleq \frac{P_{si}G_{ii}|h_{pp}^{l}|^{2}}{\sigma_{i}^{2} + \sum_{l \in \Phi_{\rm BS}} P_{\rm ter} G_{l} |h_{cp}^{l}|^{2} r_{l,i}^{-\alpha}},$$
(15) 414

where $r_{l,i}$ is the distance between l^{th} BS and i^{th} satellite user, 415 and α is the path loss exponent. 416

F. Performance Metrics 417

In order to analyse the performance of the system we will 418 use the two fundamental metrics of outage probability and area 419 spectral efficiency. 420

Outage Probability: This is the probability that outage $_{421}$ occurs at either satellite or terrestrial user. Outage occurs when $_{422}$ the received SINR falls below an acceptable threshold, T_t that $_{423}$ is, $_{424}$

$$\mathcal{P}_{\text{out}}(T_t) = \mathcal{P}(\text{SINR} < T_t). \tag{16} \quad 425$$

⁵The ZF precoder is designed using the unconstrained optimization method described in [33] such that the powers of all signals are scaled to correspond with the power increase as a result of precoding. As a result, the transmit power is maintained as the same with the case of no precoding.

Area Spectral Efficiency: This metric is presented to mea-427 sure the utilization of spectrum efficiency of wireless cellular 428 systems. It is defined as the maximum rate per unit bandwidth 429 of a user in a defined coverage area. It can also be described 430 as the average number of successful transmitted bits per unit 431 area and is therefore determined by the outage probability, 432 \mathcal{P}_{out} . Area spectral efficiency, η_{AE} is expressed as [34]

$$\eta_{AE} = \lambda_{BS}(1 - \mathcal{P}_{out})\log_2(1 + T_t), \qquad (17)$$

⁴³⁴ where T_t is the SINR threshold, and λ_{BS} is the BS node ⁴³⁵ density.

436 III. TRANSMISSION CHARACTERISATION 437 IN MULTI-BEAM CSTN

Here, we study the performance of the multi-beam CSTN from the perspective of outage probability and area spectral efficiency. In the context of this system model which permits simultaneous transmission of both satellite and terrestrial BSs tate to their respective users, we consider three practical scenarios. First is the analysis under assumption that all terrestrial BSs tate obey the constraint by using a limited transmit power defined tates in (12), (PCI). Second, we investigate the impact of using date directional beamforming at the secondary system to limit intertates deployed in the secondary system will meet the requirements for transmission, we perform thinning and analyse only the subset of BSs that meet this constraint (BTPI).

Remark 1: The analysis in the paper is done for the outage probability of both satellite and terrestrial systems. However, the area spectral efficiency analysis presented here is done only for the terrestrial system. The main idea behind this consideration is to measure the impact of interference temperties ature constraint imposed by the satellite on spectral utilization efficiency at the terrestrial system.

458 A. PCI: Power Constraint to Limit Interference

In this transmission method, we assume that the terrestrial system is equipped with omnidirectional antennas (i.e., no beamforming is used in transmission). Hence, to manage the interference the terrestrial system causes to the satellite system, the transmission power of terrestrial BSs is limited by the interference constraint imposed by the satellite. We also assume that the terrestrial BSs and users utilize single antennas for transmission. Thus, in the sequel we assess the impact of limited transmit power on the outage performance of the both satellite and terrestrial users. The property of joint random variables is used to quantify the limited transmission power and the interferences from the satellite and terrestrial system as the case requires are characterized by the use of moment transmistions and Laplacian functionals respectively.

⁴⁷³ Outage Probability at the Terrestrial User: At the terrestrial ⁴⁷⁴ user, outage occurs when the SINR falls below the threshold, ⁴⁷⁵ T_t . The outage probability from the l^{th} BS is defined as

$$\mathcal{P}_{\text{out}}(T_t) = \mathcal{P}(\zeta_l < T_t). \tag{18}$$

Thus in the following proposition, we present the outage probability of SINR of the terrestrial user for a predefined treshold, T_t . *Proposition 1:* The outage probability of the received SINR $_{480}$ at the terrestrial user from the l^{th} BS is given at the top of the $_{481}$ next page where $_{482}$

$$\mathbb{E}_{I_{\Phi_{\mathrm{BS}}}}\left[\exp\left(\frac{-A\,k\,r_l^{\alpha}\,T_l\,I_{\Phi_{\mathrm{BS}}}}{P_{\mathrm{tot}}}\right)\right] \tag{20} 483$$

$$= \exp\left(-2\pi \lambda_{\rm BS} \int_{r}^{\infty} \left(1 - \frac{1}{\left(1 + \frac{A k P_m r_l^{\alpha}}{P_{\rm tot} r^{\alpha}}\right)^{m_{cc}}}\right) r \,\mathrm{d}r\right) \quad {}^{484}$$
$$f_{\Gamma}(y) = \frac{m_{cp}^{m_{cp}} y^{m_{cp}-1} e^{-m_{cp}y}}{\Gamma(m_{cp})}, \quad (21) \quad {}^{485}$$

where m_{cp} is the Nakagami fading parameter of the interference channel, $\gamma(.,.)$ is the lower incomplete gamma function, $\Gamma(m_{cp})$ is the gamma function of m_{cp} , and 488

$$\mathbb{E}_{I_{\text{SAT}}}\left[\exp\left(\frac{-A\,k\,r_l^{\alpha}\,T_l\,I_{\text{SAT}}}{P_{\text{tot}}}\right)\right]$$

$$=\exp\left[-2\pi\lambda_U \left(1 - \frac{1}{\left(1 + \frac{A\,k\,T_l\,r_l^{\alpha}\,G_{ij}\,P_{sj}}{\beta_s\,P_{\text{tot}}}\right)}\right)^{\alpha_s}\right]$$
(22) 490

where β_s and α_s are gamma distribution random variable 491 parameters of the satellite. 492

Proof: Refer Appendix A.

B. Special Case: Approximating BS Interference Using Gamma Variable and Negligible Satellite Interference 495

The characterisation of BS interference from Proposition 1, 496 equation (20) is provided in terms of Laplacian and probability 497 generating functionals for which closed forms only exist for 498 order to obtain a more tractable model, we pursue this interference characterisation in terms of their cumulants [35]. Under 501 Rayleigh fading assumption, we approximate the BS interference distribution using the gamma model. In most modern 503 cognitive-satellite networks, the satellite interference to the 504 terrestrial user is not an essential consideration due to it's 505 negligible magnitude compared to the larger values of intra 506 cluster interference power. 507

Under this consideration of, the distribution of the equivalent aggregate of BS interference path gain is given as

$$\bar{I}_{\rm BS} = \sum_{m \in \Phi_{\rm BS}} |h_{cc}^m|^2 r_m^{-\alpha}.$$
 (23) 510

By the use of Campbell's theorem, the characteristic function $_{511}$ of $\bar{I}_{\rm BS}$ is computed as [36] $_{512}$

$$\phi_{\bar{I}_{\rm BS}}(w) = \exp\left(-2\pi\lambda_{\rm BS} \int\limits_{h_{cc}} \int\limits_{\mathbb{R}} \cdot [1 - e^{jwxr_m^{-\alpha}}] \cdot f_{h_{cc}}(x) \,\mathrm{drdx}\right)$$
(24) 514

where $j = \sqrt{-1}$. Using equation (24), we can obtain the corresponding closed forms of the cumulants. Specifically, the n^{th} ⁵¹⁶

$$\mathcal{P}_{\text{out}}(T_{l}) = \frac{\gamma\left(m_{cp}, \frac{\Upsilon m_{cp}}{P_{\text{tot}}}\right)}{\Gamma\left(m_{cp}\right)} \sum_{k=0}^{m_{cc}} \binom{m_{cc}}{k} (-1)^{k} e^{\frac{-Akr_{l}^{\alpha} T_{l} \sigma^{2}}{P_{\text{tot}}}} \mathbb{E}_{I_{\Phi}_{BS}} \left[e^{\frac{-Akr_{l}^{\alpha} T_{l} I_{\Phi}_{BS}}{P_{\text{tot}}}} \right] \mathbb{E}_{I_{SAT}} \left[e^{\frac{-Akr_{l}^{\alpha} T_{l} I_{SAT}}{P_{\text{tot}}}} \right] + \sum_{k=0}^{m_{cc}} \binom{m_{cc}}{k} (-1)^{k} \int_{\frac{\Upsilon}{P_{\text{tot}}}}^{\infty} \mathbb{E}_{I_{\Phi}_{BS}} \left[e^{\frac{-Akr_{l}^{\alpha} T_{l} y I_{\Phi}_{BS}}{\Upsilon}} \right] e^{\frac{-Akr_{l}^{\alpha} T_{l} y I_{\Phi}_{BS}}{\Upsilon}} \mathbb{E}_{I_{SAT}} \left[e^{\frac{-Akr_{l}^{\alpha} T_{l} y I_{SAT}}{\Upsilon}} \right] f_{\Gamma}(y) \, dy$$

$$(19)$$

517 cumulant of $\phi_{\bar{I}_{\rm BS}}(w)$ can be given by

$$\kappa_{\bar{I}_{BS}}(n) = \frac{1}{j^n} \frac{d^n}{dw^n} \frac{\left(\log \phi_{\bar{I}_{BS}}(w)\right)}{1}\Big|_{w=0}$$
(25)

After integration of equation (24) (refer to [36] for detailed 520 derivations), we obtain

$$\kappa_{\bar{I}_{\rm BS}}(n) = \frac{2\pi\lambda_{\rm BS}}{n\,\alpha - 2}\,\mathbb{E}_{h_{cc}}\left(h_{cc}^{2/\alpha}\right).\tag{26}$$

522 To obtain the closed form expressions of $\kappa_{\bar{I}_{\rm RS}}(n)$ under the $_{\rm 523}$ Gamma model, we consider the distribution of ${\it I}_{\rm BS}$ as

$$f_{\tilde{I}_{\text{BS}}}(x;\nu,\theta) = \frac{x^{\nu-1}e^{-\frac{\lambda}{\theta}}}{\theta^{\nu}\Gamma(\nu)},$$
(27)

s25 where the parameters ν and θ are given by

$$\nu = \frac{\kappa_{\tilde{I}_{BS}}(1)}{\kappa_{\tilde{I}_{BS}}(2)} \quad \text{and} \quad \theta = \frac{\kappa_{\tilde{I}_{BS}}(2)}{\kappa_{\tilde{I}_{BS}}(1)}.$$
 (28)

⁵²⁷ with the cumulants $\kappa_{\bar{I}_{\rm RS}}(1)$ and $\kappa_{\bar{I}_{\rm RS}}(2)$ being characterized 528 using equation (26).

The interested reader is referred to [37], to obtain more 529 530 insights on the use of gamma variables.

Accordingly, we obtain the closed form expression of outage 531 ⁵³² probability at the terrestrial user in the following proposition. Proposition 2: The outage probability of the received SINR 533 534 at the terrestrial user from the l^{th} BS is given as

535
$$\mathcal{P}_{\text{out}}(T_t) = \gamma \left(1, \frac{\Upsilon}{P_{\text{tot}}}\right) e^{\frac{-A t_l^{\alpha} T_t \sigma^2}{P_{\text{tot}}}} \left(\frac{A t_l^{\alpha} T_t P_m}{P_{\text{tot}}} + \frac{1}{\theta}\right)^{-\nu}$$
536
$$\times \theta^{-\nu} + e^{\frac{\Upsilon}{P_{\text{tot}}}} - e^{\frac{1+t\sigma^2}{t\theta}} \left(t\frac{\Upsilon}{P_{\text{tot}}} + \frac{1}{\theta}\right)^{-\nu} \theta$$

537

$$\times \left(\frac{t\theta}{1+t\theta\frac{\Upsilon}{P_{\text{tot}}}}\right)^{-\nu} \left(1+t\sigma^{2}\right)^{-1+\nu} \\ \times \Gamma \left[1-\nu, \left(\frac{\Upsilon}{P_{\text{tot}}}+\frac{1}{t\theta}\right)\left(1+t\sigma^{2}\right)\right]$$

so
$$r_{l}$$
 $r_{l}^{\alpha} T_{t} P_{m}$.

Proof: See Appendix B. 540

In order to quantify the impact of restricting the trans-541 542 mit power at terrestrial BS on satellite communication, we 543 consider outage probability at the satellite user.

(29)

Outage Probability at the Satellite User: Here, outage 544 ccurs when the received SINR at the user is less than accept-546 able threshold, T_s . Thus the outage probability is given in the following proposition.

Proposition 3: The outage probability at the *i*th beam of 548 549 the satellite system is given at the top of the next page where $s = \frac{A I \beta_s T_s}{P_{si} G_{ii}}$, $\Gamma(x, y)$, $\gamma(x, y)$, are the upper and lower incomplete gamma functions respectively, and $\Gamma(x)$ is the gamma ⁵⁵¹ function. 552

Proof: See Appendix C.

C. DBI: Directional Beamforming to Control Interference

In this scenario, we investigate limiting the interference 555 of secondary system by employing static directional beam- 556 forming using sectorized antennas to focus the signals for the 557 terrestrial user. Here, the terrestrial system is assumed to be 558 equipped with M_{BS} antennas at the BSs and M_R antennas at 559 the user⁶. We begin with determining the outage probability at 560 the secondary user and then evaluate the impact on the satellite 561 user by measuring its outage probability. This is achieved by 562 using sectorized gain patterns to characterize main lobe and 563 side lobe gains used in transmission. The interference from 564 BSs other than the transmitting BS is quantified with Laplace 565 functionals. 566

The following proposition gives the effect of applying 567 directional beamforming on the terrestrial user's outage 568 performance. 569

Proposition 4: The outage probability at the terrestrial user 570 from the l^{th} BS employing directional beamforming for trans- 571 mission is given as 572

$$\mathcal{P}_{\text{out}}(T_t) = \sum_{k=0}^{m_{cc}} \binom{m_{cc}}{k} (-1)^k$$
⁵⁷³

$$\times \exp\left(\frac{-A k r_l^{\alpha} T_l \sigma^2}{P_{\text{ter}} G_l}\right) \mathbb{E}_{I_{\text{SAT}}}\left[e^{\frac{-A k r_l^{\alpha} T_l I_{\text{SAT}}}{P_{\text{ter}} G_l}}\right] \prod_{t,k \in \{M,m\}} 574$$

$$\times \exp\left[-2\pi \mathcal{P}_{tk}\lambda_{BS} \int_{r}^{\infty} \left(1 - \frac{1}{\left(1 + \frac{A k r_{l}^{\alpha} T_{t} P_{m} G_{l}^{tk}}{P_{ter} G_{l} m_{cc} r_{m}^{d}}}\right)^{m_{cc}}\right) r \, dr\right]. \quad 575$$
(31) 576

Proof: From the proof of Proposition 1, we have

$$\mathcal{P}_{\text{out}}(T_t) = \sum_{k=0}^{m_{cc}} {m_{cc} \choose k} (-1)^k e^{\frac{-A k r_l^{\alpha} T_l \sigma^2}{P_{\text{ter}} G_l}}$$
576

$$\times \mathbb{E}_{I_{\rm BS}} \left[e^{\frac{-A k r_l^{\alpha} T_l I_{\Phi_{\rm BS}}}{P_{\rm tot} G_l}} \right] \mathbb{E}_{I_{\rm SAT}} \left[e^{\frac{-A k r_l^{\alpha} T_l I_{\rm SAT}}{P_{\rm ter} G_l}} \right].$$
(32) 580

⁶This assumption is justified since when employing directional beamforming, the multiple transmit and receive antennas form a transmit beam and a receive beam which is equivalent to communication with a single directional transmit antenna and a single directional receive antenna [38], [39].

553

554

$$\mathcal{P}_{\text{out}}(T_s) \approx \sum_{l=0}^{\alpha_s} {\alpha_s \choose l} (-1)^l \exp\left(-s\sigma_i^2\right) \exp\left[2\pi\lambda_{\text{BS}}\left(\int_r^\infty \frac{m_{cp}\Gamma\left(m_{cp},\frac{\Upsilon m_{cp}}{P_{\text{tot}}}\right) - \Gamma\left(m_{cp}+1\right)}{m_{cp}\Gamma\left(m_{cp}\right)} + \frac{m_{cp}^{m_{cp}-1}}{\Gamma\left(m_{cp}\right)}\left(m_{cp} + P_{\text{tot}}r^{-\alpha}s\right)^{-m_{cp}} \times \left(\Gamma\left(m_{cp}+1\right) - m_{cp}\Gamma\left(m_{cp},\frac{\Upsilon\left(m_{cp}+P_{\text{tot}}r^{-\alpha}s\right)}{P_{\text{tot}}}\right)\right) + \left(1 - e^{-s\Upsilon r^{-\alpha}}\right)\left(\frac{\Upsilon\left(m_{cp},\frac{\Upsilon m_{cp}}{P_{\text{tot}}}\right) - \Gamma\left(m_{cp}\right)}{\Gamma\left(m_{cp}\right)}\right)\right) r dr\right]$$
(30)

However, the terrestrial interference due to other BSs needs 581 582 to be characterized before proceeding. Given that the interfer-583 ence from BSs could be either from main lobe or side lobe 584 as defined in (6), we utilize the notion of marked stochastic ⁵⁸⁵ geometry to characterize the interference as [40]

$$I_{\Phi_{\rm BS}} = I_{\Phi_{\rm BS}}^{MM} + I_{\Phi_{\rm BS}}^{Mm} + I_{\Phi_{\rm BS}}^{mM} + I_{\Phi_{\rm BS}}^{mm}.$$
 (33)

By definition of the Laplace transform, we have 587

$$\mathcal{L}\left\{I_{\Phi_{\mathrm{BS}}}\right\} = \mathcal{L}\left\{I_{\Phi_{\mathrm{BS}}}^{MM}\right\} \mathcal{L}\left\{I_{\Phi_{\mathrm{BS}}}^{Mm}\right\} \mathcal{L}\left\{I_{\Phi_{\mathrm{BS}}}^{mM}\right\} \mathcal{L}\left\{I_{\Phi_{\mathrm{BS}}}^{mm}\right\}.$$
(34)

Starting with the characterisation of $\mathcal{L}\{I_{\Phi_{\text{RS}}}^{MM}\}(s)$, we obtain 589

$$\mathcal{L}\left\{I_{\Phi_{\mathrm{BS}}}^{MM}\right\}(s) = \mathbb{E}\left[\exp\left(-sI_{\Phi_{\mathrm{BS}}}^{MM}\right)\right],$$

$$= \mathbb{E}_{\Phi_{\mathrm{BS}},h_{cc}^{m},G_{t}}\left[\exp\left(-sP_{m}G_{t}^{MM}|h_{cc}^{m}|^{2}r_{m}^{-\alpha}\right)\right],$$

$$= \mathbb{E}_{\Phi_{\mathrm{BS}},G_{t}}\left\{\prod_{m\in\Phi_{\mathrm{BS}}}\left(\frac{1}{1+\frac{sP_{m}G_{t}^{MM}r_{m}^{-\alpha}}{m_{cc}}}\right)^{m_{cc}}\right\},$$

$$= \mathbb{E}_{G_{t}}\left\{\exp\left[-2\pi \mathcal{P}_{MM}\lambda_{\mathrm{BS}}r\left(1-\frac{1}{\left(1+\frac{sP_{m}G_{t}^{MM}}{m_{cc}r_{m}^{\alpha}}\right)^{m_{cc}}}\right)\mathrm{d}r\right]\right\},$$

$$= (35)$$

594

⁵⁹⁵ where \mathcal{P}_{MM} is the probability that $G_t^{MM} = G^M G^M$, $\frac{A k r_l^{\alpha} T_l}{P_{\text{ter}} G_l}$, (a) follows from the use of the moment generating 596 597 function of Gamma random variable with Nakagami fading ⁵⁹⁸ parameter m_{cc} , and (b) follows due to the use of probabil-⁵⁹⁹ ity generating functionals of PPPs. Following similar steps, ⁶⁰⁰ $\mathcal{L}\{I_{\Phi_{BS}}^{Mm}\}, \mathcal{L}\{I_{\Phi_{BS}}^{mm}\}$ can be computed and finally, using ⁶⁰¹ equation (34), the Laplace transform of $I_{\Phi_{BS}}$ is given as

605 where r_m is the distance between the m^{th} BS and the terrestrial 606 user. The characterisation of $\mathcal{L}{I_{SAT}}(s)$ has been outlined in 607 Appendix A and is expressed as

$${}_{608} \quad \mathcal{L}\{I_{\text{SAT}}\}(s) = \exp\left[-2\pi\lambda_U \left(1 - \frac{1}{\left(1 + \frac{s\,G_{ij}\,P_{sj}}{\beta_s}\right)^{\alpha_s}}\right)\right], \quad (37)$$

⁶⁰⁹ where $s = \frac{A k r_l^{\alpha} T_l}{P_{\text{ter}} G_l}$, α_s and β_s are the gamma distribution ⁶¹⁰ parameters of the satellite given in (2).

This proof is concluded by substituting (36) and (37) 611 into (32). 612

Outage Probability at Satellite User: In the following 613 lemma we measure the impact of employing directional beam- 614 forming at the terrestrial BS in terms of outage probability at 615 the satellite user. 616

Lemma 1: The outage probability of at the i^{th} user of the 617 satellite considering directional beamforming at the terrestrial 618 system is given as 619

$$\exp\left[-2\pi P_{tk}\lambda_{BS}\int_{r}\left(1-\frac{Al\beta_{s}T_{s}P_{ter}G_{t}^{k}}{\left(1+\frac{Al\beta_{s}T_{s}P_{ter}G_{t}^{k}}{P_{si}G_{ii}m_{cp}r_{l,i}^{m}}\right)^{m_{cp}}}\right)^{r} dr \right], \quad (28)$$

where $r_{l,i}$ is the distance from the l^{th} BS to the i^{th} satellite 623 624

625

631

Proof: The proof follows from Proposition 4.

Remark 2: It is important to note that with single transmit 626 and receive antennas, directional beamforming cannot be used 627 to manage the interference. Hence, limiting the transmit power 628 of the terrestrial system as in PCI is the method employed. In 629 other words, when $M_{BS} = M_R = 1$, then DBI reduces to PCI. 630

D. BTPI: BS Thinning Process to Restrict Interference

In this subsection, we characterize BSs which do not sat- 632 isfy the interference constraint imposed by primary system. 633 As some of the BSs may not provide sufficient coverage for 634 the terrestrial user, and these BSs may override the interfer- 635 ence temperature constraint set by satellite system and may 636 cause harmful interference to primary users, leading to a dete- 637 rioration of the system's performance. In such conditions, one 638 can make use of a thinning operation on the original PPP 639 of BSs, leading to the well-known Matern Hard-core point 640 process (MHCPP) that has been used to appropriately model 641 networks with guard zones [41]. 642

Additionally, for power constrained terrestrial systems, the 643 characterisation of hardcore models of point processes needs 644 to take into consideration fading and interference constraint. 645 In this regard, thinning with respect to fading is considered. 646 We leverage the results from [41] and [42] and incorporate 647 thinning in the design aspects of our system model. The char- 648 acterization of HCPP models via the Laplace functional and 649 probability generating functionals is quite difficult to analyse 650

$$\mathcal{P}_{\Xi} = \left[\frac{\pi \operatorname{Csc}[(m_{ij} - m_{cp})\pi]}{\Gamma[m_{ij}]\Gamma[m_{cp}]} \left(-m_{ij}^{m_{ij}} \left(\frac{m_{cp} \Upsilon r_c^{\alpha}}{P_t}\right)^{m_{ij}} \Gamma[m_{ij}]_P F_{\Upsilon} \left[\left\{m_{ij}\right\}, \left\{1 + m_{ij}, 1 + m_{ij} - m_{cp}\right\}, m_{ij} m_{cp} \frac{\Upsilon r_c^{\alpha}}{P_t}\right] + m_{ij}^{m_{cp}} \left(\frac{m_{cp} \Upsilon r_c^{\alpha}}{P_t}\right)^{m_{cp}} \Gamma[m_{cp}]_P F_{\Upsilon} \left[\left\{m_{cp}\right\}, \left\{1 + m_{cp}, 1 - m_{ij} + m_{cp}\right\}, m_{ij} m_{cp} \frac{\Upsilon r_c^{\alpha}}{P_t}\right]\right)\right]$$
(42)

and has not been properly done yet. However, the nodes further away from a hard core distance, *d*, can still be modelled as a PPP as shown in [42]. Thus, we take into account such an approximation for analytical tractability, and consider that the distribution of BSs follows a PPP while their density is approximated by that of the density of a modified hard-core PPP, $\bar{\lambda}_{BS}$.

Let Φ_{BS} be the primary point process and $\overline{\Phi}_{BS}$ be the generalised MHCPP. In order to generalise the traditional MHCPP with respect to transmit power with interference constraint, the hard-core distance *d* is replaced with the received power.

Remark 3: A BS node is retained in $\overline{\Phi}_{BS}$ if, and only if, it has the lowest mark in its neighborhood set of BSs, of $N(x_i)$ determined by dynamically changing the random-shaped region defined by instantaneous path gains, which can be looked upon as the communication range.

Lemma 2: Let the number of BSs in communication range be N, the retaining probability of a BS node is $\mathcal{P}_{BS} = \frac{1 - e^{-N\mathcal{P}_{\Xi}}}{N\mathcal{P}_{\Xi}}$. Then the intensity of active number of BSs is given by $\overline{\lambda}_{BS} = \frac{1 - e^{-N\mathcal{P}_{\Xi}}}{N\mathcal{P}_{ES}}$.

Now, in order to find \mathcal{P}_{BS} , we have to compute the neighbourhood success probability \mathcal{P}_{Ξ} . Let x_i represent the location of a BS in Φ_{BS} , i.e., $i \in \Phi_{BS}$. The neighbourhood set of any BS located at x_i is determined by bounding an observation \mathcal{B}_{T_5} region, \mathcal{B}_{x_i} by $\mathcal{B}_{x_i}(r_d)$, where r_d is a sufficiently large distance, such that the probability that any BS located beyond r_d becomes neighbour of the BS at x_i is a very small number, ρ . This probability is expressed as

$$\mathcal{P}\left\{\frac{P_t |h_{ij}|^2}{||x_i - x_j||^{\alpha}} \le \frac{\Upsilon}{|h_{cp}|^2} ||x_i - x_j|| > r_d\right\} \le \varrho, \quad (39)$$

⁶⁸⁰ where P_t is the transmit power of any BS, x_i and x_j represent ⁶⁸¹ the locations of any two BSs in Φ_{BS} , and $||x_i - x_j||$ is the ⁶⁸² distance between two neighbouring BSs.

Then the neighbourhood success probability within the bounded region can be defined as

(40)

$$\mathcal{P}_{\Xi} = \mathcal{P}\Big\{\Psi_{x_i,x_j} \leq \frac{\Upsilon}{|h_{cp}|^2} | x_j \in \mathcal{B}_{x_i}(r_d)\Big\},$$

ň

where $\Psi_{x_i,x_j} = \frac{P_t |h_{ij}|^2}{r_c^{\alpha}}$, and $r_c = ||x_i - x_j||$ is the distance between any two BSs in comparison.

⁶⁸⁸ Following from ratio and product distribution [40], (40) can ⁶⁸⁹ be written as

690

$$\mathcal{P}_{\Xi} = \int_{0}^{\infty} \int_{0}^{\frac{\Upsilon r_c^{\alpha}}{P_t}} f_{|h_{ij}|^2}(x) f_{|h_{cp}|^2}(\frac{y}{x}) \frac{1}{x} \mathrm{d}y \, \mathrm{d}x,$$
$$= \int_{0}^{\infty} f_{|h_{ij}|^2}(x) F_{|h_{cp}|^2}\left(\frac{\Upsilon r_c^{\alpha}}{P_t x}\right) \mathrm{d}x. \tag{41}$$

Using (41), we can derive the generalised MHCPP process 692 of the BSs and their active nodes which satisfy the interference 693 constraint. Therefore, the closed-form expression of the above 694 integral is given at the top of this page, where ${}_{P}F_{\Upsilon}$ is the 695 hypergeometric regularised function, m_{ij} is the Nakagami fading parameter from the distribution of h_{ij} and Csc is cosecant 697 function. 698

From the above analysis, the outage probability at the terrestrial and satellite users can be computed with the updated density, $\bar{\lambda}_{BS}$, by following steps similar to proposition 3 and lemma 1 respectively.⁷

IV. NUMERICAL RESULTS

As previously mentioned, we have analysed three different 704 methods of limiting interference caused by terrestrial commu- 705 nication to the satellite network. In this section, we provide 706 numerical results to validate our system model and present 707 comparison of these three interference limiting schemes. We 708 also verify the accuracy of theoretical results presented in 709 the previous section showcasing the performance metrics of 710 outage probability and area spectral efficiency. The parame-711 ters considered for simulation in this paper are inspired from 712 related studies on CSTNs, satellite and cellular communica- 713 tion [16], [27], [31] and the correctness of the analytical 714 results is verified through Monte Carlo simulations. For the 715 primary satellite network, we consider a K-beam network 716 with an orbit radius of 35786 km where the intensity of 717 satellite users is expressed as $\lambda_U = \frac{K}{\pi R^2}$ where K is any 718 integer that indicates the average number of users/beams 719 being served by the satellite. A few of the parameters 720 with their corresponding values are presented in Table I. 721 All other parameters will be explicitly mentioned wherever 722 used. 723

Figures 3 to 5 illustrate the impact of limiting terrestrial 724 BS transmit power using the imposed interference temperature 725 constraint (PCI). In Fig. 3, we compare the outage probability 726 performance with different values of satellite imposed interference temperature constraint at the terrestrial user. This result 728 is a validation of proposition 1. It can be observed that the 729 simulation results obtained from the numerical evaluation of 730 equation (19) are consistent with the analytical derivations, 731 as shown by the matching of these results. As can be seen, 732 with increasing values of interference temperature constraint, 733 Υ , the outage probability performance is considerably lower. 734 This result is expected as increasing the interference temperature constraint implies that the terrestrial BS can transmit 736

9

⁷It is important to note that although the expressions for outage probability are not presented in closed form, they are not computationally complex and can easily and efficiently be calculated with the use of many computer software programmes including MATLAB and MATHEMATICA.

Notation	Parameter	Values
d_0	Orbit	35786 Km
r_d	Beam radius	50 Km
$G_{s,i}$	Satellite antenna gain	30 dBi
$G_{r,i}$	Satellite terminal gain	15 dBi
3dB	Angle	0.4^{o}
ϕ_{ii}	Off-axis angle of desired user	0.6^{o}
ϕ_{ij}	Off-axis angle of interfering user	0.8^{o}
$\lambda_{ m S}$	Density of users	1e-10
$\lambda_{ m BS}$	Density of BSs	5e-06
G^M	BS antenna gain of main lobe	15 dB
α	Path loss exponent	2.1
P_{ter}	Node transmit power	20 dB
m_{cc}, m_{cp}	Nakagami parameter	1
N_0	Noise power	-174 dB

TABLE I SIMULATION PARAMETERS



Fig. 3. Outage probability as a function of SINR threshold of the secondary network under different satellite interference temperature constraints, Υ and $P_{\text{tot}} = 20$ dB.



Fig. 4. Outage probability as a function of SINR threshold of the secondary network with varying BS node density under different satellite interference temperature constraints, Υ .

737 with more power, which in turn leads to more successful738 communication with the terrestrial user.

After establishing that increased interference temperature constraint has a positive impact on terrestrial communication, we now consider the effect of node density, λ_{BS} , on the outage. Hence, in Fig. 4, we present a plot of outage probability against SINR threshold at the terrestrial user for varying values At of λ_{BS} and Υ . As can be observed, reducing the BS density



Fig. 5. Outage probability at the satellite user as a function of SINR threshold for varying interference temperature constraints, Υ , $P_{\text{tot}} = 20$ dB.



Fig. 6. Outage probability at the terrestrial user as a function of SINR threshold for varying BS node density with varying antenna gain.

leads to a decrease in outage probability. This outcome can be explained by the fact that a higher density of BSs (implying ⁷⁴⁶ more deployed BSs) indicate that there are many more BSs to ⁷⁴⁷ interfere with the intended transmission to the terrestrial user. ⁷⁴⁸ Also, confirming the results from Fig. 3, the outage probability ⁷⁴⁹ is lower for $\Upsilon = 15$ dB in both cases of λ_{BS} when compared ⁷⁵⁰ with values for $\Upsilon = 10$ dB. ⁷⁵¹

In Fig. 5, we analyse the outage probability at the satellite 752 user with respect to restricting the transmit power of the ter- 753 restrial base stations. To provide more insight on the impact 754 of constraint in the CSTN, we compare these results to the 755 case of no interference (non-transmitting terrestrial BSs). It can 756 be seen from the figure that outage probability is appreciably 757 lower with decreasing values of interference temperature con-758 straint. This result is in contrast to the observations of varying 759 constraint at the terrestrial user in Fig. 4, and this outcome 760 implies that lowering the values of interference temperature 761 constraint produces more rigidity in restraining the transmis- 762 sion power of terrestrial BSs, which then results in noticeably 763 lower interference to the satellite user and lesser probablity of 764 outage. In addition, we provide simulation results of the satel-765 lite channel using the SR fading model; as can be observed 766 from the figure, the simulations are closely matched with the 767 simulations using the Gamma random variable approximation 768



Fig. 7. Outage probability at the satellite user as a function of SINR threshold for varying BS node density when terrestrial BS is employing beamforming with M_{BS} =32, $M_r = 16$, $\alpha = 2.5$.

⁷⁶⁹ for the channel. This result is an affirmation of the channel ⁷⁷⁰ approximation we used in our analysis.

Next, we consider the use of directional beamforming for 771 transmission in the terrestrial system. Fig. 6 presents a com-772 parison of outage probability with different BS densities and 773 antenna gains at the terrestrial user. This result verifies propo-774 775 sition 4 as shown by the minimal performance gap between simulation and analytical results. It can be observed that when 776 777 the antenna gain is increased, there is a reduction in outage probability. For example, when $\lambda = 0.000001$, for a spe-778 779 cific threshold of 10 dB, the outage probability is 0.5 when $_{780}$ M_{BS} = M_r = 8 whereas when utilizing 32 antennas at both 781 BS and user, the outage probability reduces to 0.1. This result 782 indicates that directional beamforming has a direct effect on 783 the SINR threshold as an increase in the directional beamform-784 ing gain results in a reduction in the target SINR threshold 785 required for good coverage. It is also evident from the figure 786 that a higher network density yields more outage for a target SINR value. 787

The impact at the satellite user of utilizing directional beamforming for terrestrial transmission and interference mitigation is shown in Fig. 7. It can be identified from the figure that as BS nodal density increases, the probability of outage at the rest satellite user also increases similar to the effect at the terresrest trial user. Also worthy of note, deploying more BSs in the restrial network increases the aggregate interference caused rest to the satellite user.

Next, we present the analysis of thinning out all BSs that 797 do not satisfy the interference temperature constraint imposed 798 by the satellite, as discussed in Section III. After thinning, 799 $\bar{\lambda}_{BS}$ is computed using lemma 2 so that, $\bar{\lambda}_{BS} = \lambda_{BS} \mathcal{P}_{BS}$. 800 Accordingly, in Figures 8 and 9, we present a comparison 801 of outage probability by using all three methods of PCI, DBI 802 and BTPI.

Fig. 8 plots the outage probability as a function of SINR threshold at the terrestrial user. It is evident from the figure that for a fixed interference temperature constraint $\Upsilon = 0$ dB, BTPI has the best performance giving the least outage probability for a given target SINR. What is striking about the performance of DBI is its dependence on the antenna array size. Increasing the



Fig. 8. Comparison of outage probability at the terrestrial user using three methods for $\Upsilon = 0$ dB, $\lambda_{BS} = 0.000009$.



Fig. 9. Comparison of outage probability at both the satellite user and terrestrial user using three methods for $\Upsilon = 10$ dB, $M_{BS} = M_r = 16$.

number of transmit and receive antennas reasonably reduces ⁸⁰⁹ the outage probability, but this comes at a cost. We note that ⁸¹⁰ the gains of employing directional beamforming are optimal ⁸¹¹ when utilizing massive multiple input-multiple output (MIMO) ⁸¹² systems, or employing millimeter wave links at the terrestrial ⁸¹³ system because each of these methods allow for a large array ⁸¹⁴ of antennas. This can be investigated in our future work. ⁸¹⁵

Fig. 9 considers the impact of using all three schemes at 816 both the satellite user and terrestrial user. It is apparent that 817 for a target SINR, BTPI is the best method in both cases 818 to reduce the impact of interference on the satellite system 819 in a multi-beam CSTN as its performance results in fewer 820 outages. This result can be explained by the fact that thin- 821 ning is a strict implementation of the interference temperature 822 constraint imposed by the satellite. DBI gives the worst perfor- 823 mance causing the most interference to satellite transmission 824 and increasing the probability of outage occurrences. We note 825 that using PCI, which restricts transmit power at the terrestrial BS, results in moderate interference to the satellite user, 827 much lower than that produced by directional beamforming. 828 Therefore, for a conventional multi-beam CSTN, where thin- 829 ning is not feasible, PCI is a more viable scheme than DBI 830 but at cost of moderate interference to satellite user. 831



Fig. 10. Area spectral efficiency as a function of SINR threshold for varying interference temperature constraints, Υ .

Finally, in Fig. 10, we illustrate the area spectral efficiency 832 at the terrestrial user with respect to SINR threshold under 833 different values of Υ . It can be seen from the figure that for 834 835 higher values of interference temperature constraint, the area 836 spectral efficiency increases, which implies that the terrestrial can transmit with more power. This outcome is the evi-837 BS 838 dence for reduced outage probability observed at the terrestrial user for increasing values of Υ . It is worthy of mention that 839 840 there is an optimal value of area spectral efficiency as indicated by the shape of the curves in Fig. 10 with the implication 841 842 that increasing the SINR threshold has a diminishing returns 843 effect. Further, when the optimal SINR threshold is deter-⁸⁴⁴ mined, this can be used to determine the optimal BS density which maximises the area spectral efficiency of the terrestrial 845 846 system whilst taking into account the constraint imposed by 847 the satellite system. Determination of these optimal points can 848 be explored in future works.

V. CONCLUSION

The impact of interference in a multi-beam CSTN was 850 investigated. From our analysis, it is clear that successful trans-851 852 mission at both satellite and terrestrial systems depends on 853 network conditions such as BS node density, antenna gain, and interference temperature constraint imposed by the satel-854 855 lite. Accordingly, performance metrics of outage probability 856 and area spectral efficiency were analysed. With simulation 857 results we show the effect of varying the network parame-858 ters such as BS node density and the value of interference ⁸⁵⁹ temperature constraint on the network. After comparing three 860 secondary system transmission schemes (PCI, DBI and BPTI) ⁸⁶¹ aimed at keeping interference to the satellite system within the ⁸⁶² predefined limits, we observed that for a target SINR, BTPI (which strictly adheres to the satellite's requirements) gives the 863 best performance. We also showed that for conventional ter-864 ⁸⁶⁵ restrial mobile networks, DBI performed the worst. However, the performance when utilizing directional beamforming can 867 be improved at the cost of increasing the antenna gain. In practical scenarios, this would mean employing massive 869 MIMO transceivers or millimeter wave links at the terres-870 trial system. In addition, when BS thinning is not feasible, restricting the transmit power at the terrestrial BS by lowering ⁸⁷¹ the value of interference temperature constraint is the viable ⁸⁷² method to obtain reduced outage probability of the satellite ⁸⁷³ communication. ⁸⁷⁴

The terrestrial user experiences outage when its SINR⁸ falls
$$_{877}$$

$$\mathcal{P}_{\text{out}}(T_t) = \mathcal{P}(\text{SINR} < T_t),$$
 879

$$= \mathcal{P}\left(\frac{P_{\text{ter }}|h_{cc}^{l}|^{2}r_{l}^{-\alpha}}{\sigma^{2} + I_{\text{BS}} + I_{\text{SAT}}} < T_{t}\right).$$
(43) 880

Substituting P_{ter} in (43) with the interference temperature ⁸⁸¹ constraint defined in (12) as

$$P_{\text{ter}} = \min\left(\frac{\Upsilon}{|h_{cp}^l|^2}, P_{\text{tot}}\right),\tag{44} \quad \text{$883}$$

And using the property of joint distribution of random variables 884 X and Y from [43], we have: 885

$$\mathcal{P}(\min(X, Y) < t) = \mathcal{P}(X < t, Y < t),$$
886

and

$$\min(X, Y) = \begin{cases} X & \text{if } Y > X, \\ Y & \text{if } Y \le X. \end{cases}$$
(45) 888

887

889

Therefore, (43) becomes

9

$$\mathcal{P}_{\text{out}}(T_t) = \mathcal{P}\left(\frac{P_{\text{tot}} \mid h_{cc}^l \mid^2 r_l^{-\alpha}}{\sigma^2 + I_{\text{BS}} + I_{\text{SAT}}} < T_t, P_{\text{tot}} \le \frac{\Upsilon}{\mid h_{cp}^l \mid^2}\right)$$
⁸⁹⁰

$$+ \mathscr{P}\left(\frac{\frac{1}{|h_{cp}^{l}|^{2}} |h_{cc}^{l}|^{2} r_{l}^{-\alpha}}{\sigma^{2} + I_{\rm BS} + I_{\rm SAT}} < T_{t}, P_{\rm tot} > \frac{\Upsilon}{|h_{cp}^{l}|^{2}}\right).$$
(46) 892

Let $\Gamma = |h_{cp}^l|^2$. The outage probability conditioned on Γ is 893 defined as: 894

⁸In this scenario, since we limit the interference using interference temperature constraint, the beamforming gain, $G_l = 1$ and is omitted for subsequent analysis.

Given that fading of the channel of the l^{th} BS, h_{cc}^{l} fol-898 899 lows the Nakagami fading model described in Section II-C1, 900 we employ the upper bound approximation of gamma dis-⁹⁰¹ tribution with parameter m_{cc} such that: $\mathcal{P}[|h_{cc}^l|^2 < \gamma <$ $_{902} (1 - e^{-A\gamma})^{m_{cc}}$ with $A = m_{cc}(m_{cc}!)^{\frac{-1}{m_{cc}}}$, therefore, starting with $_{903}$ I, the conditional outage probability is expressed as:

$${}_{904} \quad \mathcal{P}_{\text{out}|\Gamma}^{I}(T_t) = \int_{0}^{\frac{\Upsilon}{P_{\text{tot}}}} \mathcal{P}\left[\frac{P_{\text{tot}} \ |h_{cc}^{l}|^2 r_l^{-\alpha}}{\sigma^2 + I_{\text{BS}} + I_{\text{SAT}}} < T_t\right] f_{\Gamma}(\mathbf{y}) \, \mathrm{d}\mathbf{y}, \quad (48)$$

 $_{905}$ where $f_{\Gamma}(y)$ is the density of fading of interference channel 906 given by

907
$$f_{\Gamma}(y; m_{cp}) = \frac{m_{cp}^{m_{cp}} y^{m_{cp}-1} e^{-m_{cp}y}}{\Gamma(m_{cp})},$$
(49)

⁹⁰⁸ where m_{cp} is the Nakagami fading parameter, and $\Gamma(m_{cp})$ is 909 the Gamma function,

$$\begin{aligned} & \mathcal{P}\left[\frac{P_{\text{tot}} |h_{cc}^{l}|^{2} r_{l}^{-\alpha}}{\sigma^{2} + I_{\text{BS}} + I_{\text{SAT}}} < T_{t}\right] \\ & \text{910} \quad \mathcal{P}\left[\frac{P_{\text{tot}} |h_{cc}^{l}|^{2} r_{l}^{-\alpha}}{\sigma^{2} + I_{\text{BS}} + I_{\text{SAT}}} \left[\mathcal{P}\left[|h_{cc}^{l}|^{2} < \frac{T_{t} r_{l}^{\alpha}}{P_{\text{tot}}} \left(\sigma^{2} + I_{\text{BS}} + I_{\text{SAT}}\right)\right]\right], \\ & \text{912} \quad \overset{(a)}{=} \mathbb{E}_{I_{\text{BS}}, I_{\text{SAT}}}\left[\left(1 - e^{-A \frac{r_{l}^{\alpha} T_{l}}{P_{\text{tot}}} \left(\sigma^{2} + I_{\text{BS}} + I_{\text{SAT}}\right)\right)^{m_{cc}}\right], \end{aligned}$$

913
$$\stackrel{(b)}{=} \sum_{k=0}^{m_{cc}} {m_{cc} \choose k} (-1)^k e^{\frac{-Akr_l^{\alpha} T_l \sigma^2}{P_{tot}}} \mathbb{E}_{I_{BS}} \left[e^{-\frac{-Akr_l^{\alpha} T_l I_{BS}}{P_{tot}}} \right]$$

 $\times \mathbb{E}_{I_{\text{SAT}}} e^{-}$ 914

925

915
$$\stackrel{(c)}{=} \sum_{k=0}^{m_{cc}} {m_{cc} \choose k} (-1)^k e^{\frac{-A k t_I^{\alpha} T_I \sigma^2}{P_{tot}}} \prod_{m \in \Phi_{BS}} \mathbb{E}_{I_{BS}} \left[e^{-\frac{-A k t_I^{\alpha} T_I T_{BS}}{P_{tot}}} \right]$$

916
$$\times \prod_{j \in \Phi_U} \mathbb{E}_{I_{\text{SAT}}} \left[e^{-\frac{-AkT_U^T T_I I_{\text{SAT}}}{P_{\text{tot}}}} \right],$$
(50)

917 where (a) follows from the tight gamma approximation previ-918 ously defined, (b) follows from applying binomial expansion, 919 and (c) follows from the product of both satellite and tergeo restrial links such that $I_{\rm BS} = \sum_{m \in \Phi_{\rm BS}, m \neq l} P_m |h_{cc}^m|^2 r_m^{-\alpha}$ and

 $_{921} I_{\text{SAT}} = \sum_{j \in \Phi_U} P_{sj} G_{i,j} |h_{pc}^j|^2$. Now substituting (c) into (48), the 922 solution yields

923
$$\mathcal{P}_{\text{out}|\Gamma}^{I}(T_{t}) = \frac{\gamma\left(m_{cp}, \frac{\Upsilon m_{cp}}{P_{\text{tot}}}\right)}{\Gamma\left(m_{cp}\right)} \sum_{k=0}^{m_{cc}} \binom{m_{cc}}{k} (-1)^{k}$$
924
$$\times e^{\frac{-Ak r_{l}^{\alpha} T_{l} \sigma^{2}}{P_{\text{tot}}}} \mathbb{E}_{I_{\text{BS}}}\left[e^{\frac{-A k r_{l}^{\alpha} T_{l} I_{\text{BS}}}{P_{\text{tot}}}}\right] \mathbb{E}_{I_{\text{SAT}}}\left[e^{\frac{-A k r_{l}^{\alpha} T_{l} I_{\text{SAT}}}{P_{\text{tot}}}}\right].$$
925 (51)

The Laplace transform of terrestrial interference is given as 926

$$\mathbb{E}_{I_{\rm BS}}\left[\exp(-sI_{\rm BS})\right]$$

$$= \mathbb{E}_{I_{\rm BS}} \left[\prod_{m \in \Phi_{\rm BS}} \exp\left(-sP_m X_{cc} r_m^{-\alpha}\right) \right],$$
⁹²⁸

$$= \mathbb{E}_{I_{\rm BS}} \left[\exp\left(-s \sum_{m \in \Phi_{\rm BS}} P_m X_{cc} r_m^{-\alpha} \right) \right], \qquad (52) \quad \text{see}$$

where, $s = \frac{A k r_l^{\alpha} T_t}{P_{\text{tot}}} X_{cc} = |h_{cc}^m|^2$.

=

$$\mathbb{E}_{I_{\Phi_{\mathrm{BS}}}}\left[\exp\left(\frac{-A\,k\,r_l^{\alpha}\,T_t\,I_{\Phi_{\mathrm{BS}}}}{P_{\mathrm{tot}}}\right)\right]$$
932

$$= \exp\left(-2\pi\lambda_{\rm BS}\int_{r}^{\infty} \left(1 - \frac{1}{\left(1 + \frac{A\,k\,P_m\,r_l^{\alpha}}{P_{\rm tot}\,r^{\alpha}}\right)^{m_{cc}}}\right) r\,{\rm d}r\,\right). \tag{53}$$

The expectation of interfering link from the satellite is 935 obtained thus: Let $s = \frac{A k r_l^{\alpha} T_t}{P_{\perp}}$ 936

$$\mathcal{L}\{I_{\text{SAT}}\}(s) = \mathbb{E}\big[\exp(-s I_{\text{SAT}})\big],$$
⁹³⁷

$$= \mathbb{E}_{\Phi_U, X_{pc}} \left[\prod_{i \in \Phi_U} \exp\left(-s P_{sj} G_{ij} X_{pc}\right) \right],$$
938

$$\stackrel{(a)}{=} \mathbb{E}_{\Phi_U} \left\{ \prod_{i \in \Phi_U} \mathbb{E}_{X_{pc}} \left[\exp(-sP_{sj}G_{ij}X_{pc}) \right] \right\},$$
⁹³⁹

$$\stackrel{(b)}{=} \exp\left[-2\pi\lambda_U \left(1 - \left(\frac{1}{1 + \frac{sP_{sj}G_{ij}}{\beta_s}}\right)^{\alpha_s}\right)\right], \quad (54) \quad _{940}$$

where $X_{pc} = |h_{pc}^j|^2$, (a) follows from the assumption of 941 independent fading, (b) follows from the use of Campbell's 942 theorem, moment generating function of Gamma random 943 variable and probability generating functionals of PPPs. 944 945

For the second part of $\mathcal{P}_{out|\Gamma}$ in (47), we obtain:

$$\mathcal{P}_{\text{out}|\Gamma}^{II}(T_t) = \int_{\frac{\Upsilon}{P_{\text{tot}}}}^{\infty} \mathcal{P}\left[\underbrace{\frac{\frac{1}{\Gamma} |h_{cc}^l|^2 r_l^{-\alpha}}{\sigma^2 + I_{\text{BS}} + I_{\text{SAT}}} < T_t}_{III}\right] f_{\Gamma}(y) \, \mathrm{d}y, \qquad _{946}$$

with f_{Γ} defined in (49). We solve *III* by following steps similar ₉₄₈ to those outlined in (50) and obtain 949

$$\mathcal{P}\left[\frac{\frac{\Upsilon}{\Gamma}|h_{cc}^{l}|^{2}r_{l}^{-\alpha}}{\sigma^{2}+I_{\rm BS}+I_{\rm SAT}} < T_{t}\right] = \sum_{k=0}^{m_{cc}} \binom{m_{cc}}{k} (-1)^{k} e^{\frac{-Akr_{l}^{\alpha}T_{t}\Gamma\sigma^{2}}{\Upsilon}}$$
950

$$\times \prod_{m \in \Phi_{\rm BS}} \mathbb{E}_{I_{\rm BS}} \left[e^{-\frac{A k r_l^{\alpha} T_l \Gamma I_{\rm BS}}{P_{\rm tot}}} \right] \prod_{j \in \Phi_U} \mathbb{E}_{I_{\rm SAT}} \left[e^{-\frac{A k r_l^{\alpha} T_l \Gamma I_{\rm SAT}}{P_{\rm tot}}} \right].$$
(56)

 $9r_m$ is subsequently referred to as r.

Now, substituting (56) into (55), we obtain \mathcal{P}_{outly}^{II} given as

954
$$\mathcal{P}_{\text{out}|\Gamma}^{II}(T_{l}) = \int_{\frac{\Upsilon}{P_{\text{tot}}}}^{\infty} \sum_{k=0}^{m_{cc}} {m_{cc} \choose k} (-1)^{k} e^{\frac{-A k r_{l}^{\alpha} T_{l} y \sigma^{2}}{\Upsilon}}$$
955
$$\times \prod_{m \in \Phi_{\text{BS}}} \mathbb{E}_{I_{\text{BS}}} \left[e^{-\frac{-A k r_{l}^{\alpha} T_{l} y I_{\text{BS}}}{P_{\text{tot}}}} \right]$$
956
$$\times \prod_{i \in \Phi_{I}} \mathbb{E}_{I_{\text{SAT}}} \left[e^{-\frac{-A k r_{l}^{\alpha} T_{l} y I_{\text{SAT}}}{P_{\text{tot}}}} \right] \frac{m_{cp}^{m_{cp}} y^{m_{cp}-1} e^{-m_{cp} y}}{\Gamma(m_{cp})} \, \mathrm{dy}. \quad (57)$$

⁹⁶⁷ The expectations of interfering links from the ⁹⁵⁸ other BSs, $\mathbb{E}_{I_{\text{BS}}}[\exp(-\frac{A k r_l^{\alpha} T_i y I_{\text{BS}}}{P_{\text{tot}}})]$ and the satellite, ⁹⁵⁹ $\mathbb{E}_{I_{\text{SAT}}}[\exp(-\frac{A k r_l^{\alpha} T_i y I_{\text{SAT}}}{P_{\text{tot}}})]$ are obtained by following similar ⁹⁶⁰ steps to (53) and (54) respectively. Finally, the proof of outage ⁹⁶¹ probability for the terrestrial user is realised by summation ⁹⁶² of $\mathcal{P}_{\text{out}|y}^{I}$ and $\mathcal{P}_{\text{out}|y}^{II}$ respectively.

963 APPENDIX B 964 PROOF OF PROPOSITION 2

The approximated outage probability for the terrestrial user when $f_{\bar{I}_{BS}}(x; \nu, \theta) = \frac{x^{\nu-1}e^{-\frac{x}{\theta}}}{\theta^{\nu}\Gamma(\nu)}$ and $I_{SAT} = 0$ is given as

967
$$\mathcal{P}_{\text{out}|\Gamma}(T_t) = \underbrace{\int_{0}^{\frac{1}{P_{\text{tot}}}} \mathcal{P}\left[\frac{P_{\text{tot}} |h_{cc}^{l}|^2 r_l^{-\alpha}}{\sigma^2 + I_{\text{BS}}} < T_t\right] f_{\Gamma}(y) \, \mathrm{d}y}_{I} + \underbrace{\int_{0}^{\infty} \mathcal{P}\left[\frac{\Upsilon}{\frac{\Gamma}{P_{\text{tot}}}} |h_{cc}^{l}|^2 r_l^{-\alpha}}{\sigma^2 + I_{\text{BS}}} < T_t\right] f_{\Gamma}(y) \, \mathrm{d}y. \quad (58)$$

⁹⁶⁹ The expectation of the interfering links from other BSs is ⁹⁷⁰ given as

$${}_{971} \quad \mathbb{E}_{I_{\rm BS}}\left[e^{\left(-\frac{A\,k\,r_l^{\alpha}\,T_l\,T_{\rm BS}}{P_{\rm tot}}\right)}\right] = \int_0^{\infty} e^{-\frac{A\,k\,r_l^{\alpha}\,T_lx}{P_{\rm tot}}} \frac{x^{\nu-1}e^{-\frac{x}{\theta}}}{\theta^{\nu}\Gamma(\nu)} \mathrm{d}x, \qquad (59)$$

972 Solving for (59) yields

973
$$\mathbb{E}_{I_{\rm BS}}\left[e^{\left(-\frac{Akr_l^{\alpha}T_tI_{\rm BS}}{P_{\rm tot}}\right)}\right] = \left(\frac{Akr_l^{\alpha}P_mT_t}{P_{\rm tot}} + \frac{1}{\theta}\right)^{-\nu}\theta^{-\nu}.$$
 (60)

⁹⁷⁴ Using the expressions $\mathbb{E}_{I_{\text{BS}}}\left[e^{\left(-\frac{A k r_{\Gamma}^{T} T_{I} I_{\text{BS}}}{P_{\text{tot}}}\right)}\right]$ and $f_{\Gamma}(y) = e^{-y}$

⁹⁷⁵ to solve (58) and following similar steps to Appendix A will ⁹⁷⁶ yield (29).

977 APPENDIX C 978 PROOF OF PROPOSITION 3

Now, the outage probability of SINR distribution using (15) eso can be given as

981
$$\mathbb{P}\left[\frac{P_{si}G_{ii}|h_{pp}^{i}|^{2}}{\sigma^{2}+I_{BS}} < T_{s}\right] = \mathbb{P}\left[h_{pp}^{i}|^{2} < \frac{T_{s}}{P_{si}G_{ii}}\left(\sigma^{2}+I_{BS}\right)\right].$$
982 (61)

Leveraging the tight upper bound of a Gamma random variable of parameters α_s and β_s as $\mathbb{P}[h_{pp}^i|^2 < \gamma < (1 - e^{-A\beta_s\gamma})^{\alpha_s}$ with $A = \alpha_s(\alpha_s!)^{\frac{-1}{\alpha_s}}$, and by applying binomial theorem we set approximate (61) as

$$\mathbb{P}\left[h_{pp}^{i}|^{2} < \frac{T_{s}}{P_{si}G_{ii}}\left(\sigma^{2} + I_{\mathrm{BS}}\right)\right]$$

$$\approx \sum_{l=0}^{\alpha_s} {\alpha_s \choose l} (-1)^l e^{\frac{-A l \beta_s T_s \sigma^2}{P_{si} G_{ii}}} \mathcal{L} \{ I_{\Phi_{\rm BS}} \} (s), \qquad (62) \quad {}_{968}$$

where $s = \frac{A l \beta_s T_s}{P_{si} G_{ii}}$. Next, the terrestrial interference due to 989 BSs is characterized as 990

$$\mathcal{L}\left\{I_{\Phi_{\mathrm{BS}}}\right\}(s) = \mathbb{E}_{I_{\Phi_{\mathrm{BS}}}}\left[\exp\left(-sI_{\Phi_{\mathrm{BS}}}\right)\right],$$
991

$$= \mathbb{E}_{I_{\Phi_{\mathrm{BS}}}} \left[\prod_{l \in \Phi_{\mathrm{BS}}} \exp\left(-s P_{\mathrm{ter}} |h_{cp}|^2 r_l^{-\alpha}\right) \right], \quad (63) \quad _{992}$$

which is gotten by substituting $I_{\Phi_{BS}} = \sum_{l \in \Phi_{BS}} P_{ter} |h_{cp}^i|^2 r_l^{-\alpha}$. 993 Applying Campbell's theorem [40], we obtain 994

$$\mathcal{L}\{I_{\Phi_{\mathrm{BS}}}\}(s) = \exp\left[2\pi\lambda_{\mathrm{BS}}\int_{r}^{\infty} \left(e^{-sP_{\mathrm{ter}}|h_{cp}^{i}|^{2}r^{-\alpha}} - 1\right)r\mathrm{dr}\right].$$
(64) 995

Taking the expectation with respect to $|h_{cp}^i|^2$ and recalling that997 P_{ter} is constrained as in equation (12), we obtain998 $\mathcal{L}{I_{\Phi_{BS}}}(s)$ 999

$$= \exp\left[2\pi\lambda_{\rm BS}\left(\underbrace{\int\limits_{r}^{\infty}\int\limits_{0}^{\frac{\Upsilon}{P_{\rm tot}}}\int\limits_{r}^{(e^{-sP_{\rm tot}yr^{-\alpha}}-1)f_{\Gamma}(y)\,\rm dy\,r\,dr}_{I} + \underbrace{\int\limits_{r}^{\infty}\int\limits_{\frac{\Upsilon}{P_{\rm tot}}}^{\infty}\left(e^{-s\frac{\Upsilon}{\Gamma}yr^{-\alpha}}-1\right)f_{\Gamma}(y)\,\rm dy\,r\,dr}_{II}\right)\right], \quad 1001$$

(65) 1002

1003

where $f_{\Gamma}(y)$ is as defined in (49).

After solving the inner integrals of *I* and *II* with respect to $_{1004}$ *y*, the expectation of the interference from BSs limited by the $_{1005}$ interference temperature constraint is given as $_{1006}$

$$\mathcal{L}\left\{I_{\Phi_{\mathrm{BS}}}\right\}(s) \tag{1007}$$

$$= \exp\left[2\pi\lambda_{\rm BS}\left(\int\limits_{r}^{\infty} \frac{m_{cp}\,\Gamma\left(m_{cp},\frac{\Upsilon m_{cp}}{P_{\rm tot}}\right) - \Gamma\left(m_{cp}+1\right)}{m_{cp}\Gamma\left(m_{cp}\right)}\right]^{1000}$$

$$+ \frac{m_{cp}^{m_{cp}-1}}{\Gamma(m_{cp})} (m_{cp} + P_{tot}r^{-\alpha}s)^{-m_{cp}}$$

$$(- (\gamma(m_{cp} + P_{tot}r^{-\alpha}s)))$$
1009

$$\times \left(\Gamma(m_{cp}+1) - m_{cp} \Gamma(m_{cp}, \frac{\Gamma(m_{cp}+\Gamma_{tot}(x-3))}{P_{tot}}) \right)$$
 1010
$$\sum_{c} \left(\gamma(m_{cp}, \frac{\Upsilon m_{cp}}{P_{tot}}) - \Gamma(m_{cp}) \right)$$

$$+ \int_{r} \left(1 - e^{-s\Upsilon r^{-\alpha}}\right) \left(\frac{\gamma(m_{cp}, \overline{P_{tot}}) - \Gamma(m_{cp})}{\Gamma(m_{cp})}\right) r dr \left[, 1011\right]$$
(66) 1012

¹⁰¹³ where $\Gamma(x, y)$, $\gamma(x, y)$, are the upper and lower incom-¹⁰¹⁴ plete gamma functions respectively, and $\Gamma(x)$ is the gamma ¹⁰¹⁵ function.

¹⁰¹⁶ This proof is concluded by substituting (66) into (62).

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