Various Compressed Sensing Set-Ups Evaluated Against Shannon Sampling Under Constraint of Constant Illumination

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Abstract

Under the constraint of constant illumination, an information criterion is formulated for the Fisher information that compressed sensing measurements in optical and transmission electron microscopy contain about the underlying parameters. Since this approach requires prior knowledge of the signal's support in the sparse basis, we develop a heuristic quantity, the detective quantum efficiency (DQE), that tracks this information criterion well without this knowledge. It is shown that for the investigated choice of sensing matrices, and in the absence of read-out noise, i.e. with only Poisson noise present, compressed sensing does not raise the amount of Fisher information in the recordings above that of Shannon sampling. Furthermore, enabled by the DQE's analytical tractability, the experimental designs are optimized by finding out the optimal fraction of on-pixels as a function of dose and read-out noise. Finally, we introduce a regularization and demonstrate, through simulations and experiment, that it yields recon-

structions attaining minimum mean squared error at experimental settings predicted by the DQE as optimal.

Keywords. Fisher information, Statistical experimental design, Poisson noise, read-out noise, Dose limitation, Detective quantum efficiency, Single-pixel camera, Transmission electron microscopy, ADF-STEM.

1 Introduction

Beam damage to the specimen is one of the most fundamental limits to the data quality in electron microscopy. In biological applications the acceptable dose is often below ten electrons per square ångström, no matter if one images biological macromolecules [1, 2, 3] or records diffraction patterns of protein crystals [4]. Although not as severe, beam sensitivity is an issue in materials science as well and researchers go to great lengths to limit it [5, 6, 7, 8]. Furthermore, with the increased occurrence of soft and hard matter being interfaced into so-called hybrid materials [9] biology's low upper bound for the electron dose now encroaches on the realm of materials physics too. A current trend in scanning transmission electron microscopy (STEM) seeks to limit electron exposure of the specimen

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by invoking compressed sensing [10] (CS) in the recording process [11, 12, 13, 14].

For applications with photon radiation, dose limitation is often a driving factor as well, for example when using potentially hazardous X-rays for computed tomography [15]. Furthermore, limiting the recording time can be a valid goal in itself.

In CS, the signal x is retrieved from the recordings y that have been produced by the sensing matrix A operating on x, i.e. y = Ax. Recovery of x from a surprisingly low number of measurements y is possible if these measurements are incoherent [16] to the signal and a sparsity constraint can be imposed. In general that involves expressing x in a mathematical basis where it is sparse, for instance many photographs are sparse in a wavelet basis. For piecewise linear signals such an explicit decomposition can be omitted and x can be retrieved instead by expressing that its total generalized variation (TGV) must be minimal; this approach is known as TV minimization [17].

In order to make the measurements incoherent to the signal, sensing matrices conventionally have zero-mean independent and identically distributed (iid) random variables for entries. In the analysis of the respective error bounds, noise is often not considered, or assumed additive and/or bounded.

1.1 Contribution of this paper

In this paper it is acknowledged that in many experiments the total dose on the specimen is of greater importance than the number of measurements, and hence the performance of various CS set-ups is assessed under the constraint of constant total illumination. In other words, we investigate how to optimally make use of a given electron or photon budget.

Two fundamentally different and common setups are analyzed: the single-pixel set-up and annular dark field STEM (ADF-STEM). These two instances respectively represent experiments where the illumination is caused by external sources outside of the scientist's control and those where they are in full command of the irradiation, between them covering most practical situations.

The Fisher information that the measurements contain about the underlying parameters is evaluated through the A-optimality information criterion (AOC) [18, 19, 20, 21, 22]. Furthermore, a heuristic quantity, the detective quantum efficiency (DQE), is developed. With the aid of simulations it is shown that the DQE tracks the AOC well. Furthermore, a novel regularization is introduced and through simulations and experiment it is empirically demonstrated that the associated reconstructions attain minimum mean squared error (MSE) at experimental settings predicted by the DQE as optimal.

Having established the DQE's validity, its analytical tractability is used to show that for the investigated sensing matrices and in the absence of read-out noise, i.e. with only Poisson noise present, compressed sensing does not raise the amount of Fisher information in the recordings above that of a Shannon sampled signal. This is reflected in the reconstruction results that yield a comparable MSE when both data sets are treated with the same algorithm to isolate the influence of the recording protocol.

Furthermore, the experimental designs are optimized, i.e. the fraction of on-pixels for best reconstruction quality is given as a function of particle dose, read-out noise and other experimental parameters.

1.2 Relation to previous work

The conventional zero-mean sensing matrices in, for instance, [17, 16] are not physically realizable for the particle-counting experiments investigated in this paper: the single-pixel camera and ADF-STEM. In a pivotal paper, Raginsky [23] et al. have shown that in that context the use of a sensing matrix that preserves non-negativity and flux combined with the non-additive and dose dependent unbounded Poisson noise, has such a large and adverse effect on the derived error bounds that they do not even approach the conventional bounds in the limit of high dose and the associated relatively low noise.

CS has been analyzed from the perspective of Fisher information and the AOC before [24, 25, 26, 27, 28, 29]. These works, however, did not consider sensing matrices that preserve non-negativity and flux or the non-additive and dose dependent unbounded Poisson noise. That doing so yields qualitatively different results is illustrated by the fact that the finding in [27] that "the estimation accuracy [degrades] by at least the down-sample factor" is reproduced in this work when just read-out noise is considered, but not when only Poisson noise is taken into account.

1.3 Organization of the paper

This paper is organized as follows. The image formation of ADF-STEM and the single-pixel camera is treated in Sec. 2; the basics of compressed sensing pertaining to our problem are dealt with in Sec. 3; in Sec. 4 Fisher information and the Aoptimality information criterion are explained and the application of statistical experimental design to the compressed sensing set-ups is developed; the results are reported and discussed in Secs. 5 and 6; and in Sec. 7 the conclusions are drawn.

2 Image formation

In this section the image formation for the singlepixel camera and ADF-STEM is established.

2.1 Single-pixel camera

A way of realizing a single-pixel camera [30] is by projecting an image onto an array of switchable mirrors located in the image plane of an objective lens whose conjugate plane contains a photon detector on the optical axis; see Fig. 1 for a sketch of the setup. Each of the mirrors in the array can be switched on or off, and fractional on-values can be obtained by tuning the pixels' on-time.

The linear image formation model is given as

$$\mathbf{E}(y) = IAx,\tag{1}$$

where the measurement vector y has M elements, the two-dimensional object is written in long-vector format as x and has N elements, the sensing matrix A has dimensions $M \times N$ and $I\mu$ is the average number of photons reflecting off each of the object's pixels, with μ the average of x and E denoting the expectation value. Although not treated explicitly in this work, it is worthwhile noting that I depends on N through $I \propto 1/N$, thus expressing that a finite number of photons must be budgeted over the N pixels.

Two distinct choices for A are investigated in this paper:



Figure 1: (a): In the single-pixel camera the scene is projected onto an array of switchable mirrors, and the reflected light is focused onto the singlepixel detector. (Sketch after [31]) (b): In ADF-STEM an electron probe is scanned over the object, and electrons scattered to higher angles are integrated in an annular detector in the far-field.

PIX_{Di}: each mirror in the array has a chance p to provide the discrete value v of 1 ("on") and a chance 1 - p to provide 0 ("off");

PIX_{Fr}: each mirror has a chance p to provide a fractional value v drawn from a uniform distribution on [0, 1], and a chance 1-p to provide 0.

This approach allows the value for p to be optimized. Since the single-pixel camera is to be evaluated for a constant recording time, the elements of A corresponding to on-pixels are set to v/M and those corresponding to off-pixels to 0, expressing that a constant expected total number of photons $NI\mu$ is reflecting off the object. Note that although the number of photons impinging on the sample is constant, the recorded number of photons scales with p.

The pixel-by-pixel raster scan, or Shannon sampling, is obtained with sensing matrix PIX_{Di} when p is set to 1/N and the matrix A is diagonal with M = N. The subsequent reconstruction of the image under the application of a sparsity constraint then comes down to denoising.

The case of Gaussian distributed values in the sensing matrix is not considered in this paper. It is trivial to adapt the conventional zero-mean binary and uniformly distributed values to the nonnegative distributions considered here: shifting the distributions' mean accomplishes this without introducing distortions to the distributions. A mere translation is not sufficient for the Gaussian distribution however, its non-boundedness implies additional clipping of remaining negative values *and* too-high positive values. The latter is necessary because a finite recording time sets an upper limit on the value of an on-pixel. The analysis needed to separate the influence of such a distortion from the influence of the mere non-negativity falls outside of the scope of this work.

2.2 ADF-STEM

For ADF-STEM the electron beam is condensed onto the specimen and scanned across it. For each beam position an annular detector in the far-field integrates the electrons scattered to its surface; see Fig. 1. Although analyzing the resulting gray values in terms of the specimen's chemical composition [32] is important, it is not the subject of this work and is hence not treated or attempted. Instead the signal x is taken as the expected detector output for a certain beam position.

The image formation model is again given by (1), and these choices of A are investigated:

ADF_{CS}: *A*'s elements are switched on with a probability p and given a value of 1/(pN), off-pixels are set to 0;

ADF_{Sh}: A is $N \times N$ with only the diagonal elements non-zero and equal to M/N.

These two matrices allow comparison under equal dose in the recordings since they yield an expected total dose in y of $MI\mu$, corresponding to an average of $I\mu$ and $IM\mu/N$ per individual measurement for sensing matrices ADF_{CS} and ADF_{Sh}, respectively.

Note that in ADF-STEM the operator has direct control over the delivered dose through the beam intensity, dwell time and beam-blanking at will so that all electrons that scatter from the object and that could in principle be detected, actually are detected and contribute to the signal y; this recording mode is fundamentally different from the singlepixel case.

Sensing matrix ADF_{CS} is analogous to the singlepixel set-up and can be experimentally realized through fast beam deflection as detailed in [12, 13]; matrix ADF_{Sh} represents a classic raster scan, or Shannon scan, with a total intensity matched to the CS case. A special case of ADF_{CS} is obtained by equating p to 1/N and corresponds to sampling the specimen in M randomly selected points. This is known as inpainting [33] and its treatment here is motivated by the interest it currently receives from various research groups [11, 12].

2.3 Noise models

Since both imaging techniques are essentially particle counting experiments (photons or electrons) the measurements y_i follow a Poisson probability density distribution

$$\frac{(IA_i x)^{y_i}}{y_i!} \exp\left(-IA_i x\right) \tag{2}$$

with A_i the i^{th} row of A. This Poisson distribution has mean and variance IA_ix .

In the case the recording device is not perfect it could add read-out noise. To keep the problem analytically tractable the Poisson distribution is approximated with a normal distribution with mean and variance IA_ix , and the read-out noise is modeled as normal additive noise with zero mean and variance c. Since both contributions are independent their variances add, yielding

$$\frac{1}{\sqrt{2\pi \left(IA_i x + c\right)}} \exp\left(\frac{-\left(y_i - IA_i x\right)^2}{2 \left(IA_i x + c\right)}\right) \tag{3}$$

as distribution for y_i .

The approximation of a Poisson distribution as a Gaussian is of great practical value in the theoretical analysis but needs some justification. In practice, the actual noise properties are rarely truly Poisson; the Poisson distribution is itself an approximation to the real system, and this provides some latitude to choose a more mathematically convenient approximate model.

For example, in ADF-STEM, the detection is indirect such that the signal produced by each electron is a random variable, and the aggregated signal yields a probability distribution that scales as Poisson but is not actually Poisson for several reasons. First, the distribution is continuous, not discrete. Second, the constant of proportionality between the signal and the variance is in general not unity. The effect of this calibration factor on the results in this article is nothing more than a trivial rescaling factor. Third, the readout noise also contributes, especially at low signal levels where it can be dominant. Thus, in the very regime where the Poisson distribution is most poorly approximated by a Gaussian, the dominant noise contribution is in fact Gaussian.

Since much of CS literature considers read-out noise only it is investigated in this paper too. The noise is assumed normal and additive with zero mean and variance c, so that y_i is drawn from the distribution

$$\frac{1}{\sqrt{2\pi c}} \exp\left(\frac{-\left(y_i - IA_i x\right)^2}{2c}\right). \tag{4}$$

3 Compressed sensing

In compressed sensing (CS) under-determined or ill-posed problems are alleviated by regularization of the solution. In general a basis is defined in which the solution is expected to be sparse, and regularizing then means finding that solution with minimal ℓ_1 -norm in said basis.

An explicit decomposition can be avoided in the special case of piecewise linear objects x with the aid of Hessian regularization [34, 33]. To this end the total generalized variation (TGV) is defined as,

$$TGV(x) = \sum_{k} |s_k|, \text{ with } s = Hx, \qquad (5)$$

where s_k is the Laplacian in x_k , calculated approximately by the operator H which implements a convolution with the kernel

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$
 (6)

This approach is often used in inpainting [33].

The signal x can then be retrieved by solving the constraint optimization problem

$$\min_{x} \mathrm{TGV}(x) \quad \text{s.t.} \quad IAx = y. \tag{7}$$

However, solving this system with exact compliance to the constraint leads to overfitting and a solution that cannot be considered very sparse anymore. Instead, a constraint is stated that can be obeyed exactly without overfitting and can be complied with by minimizing an augmented Lagrangian [35] through an alternating direction scheme [36, 37, 38]

$$\min_{x} \operatorname{TGV}(x) \quad \text{s.t.} \quad \ln L(y|x) = \operatorname{E}\left(\ln L(y|x)\right). \tag{8}$$

 $\ln L$ is the log-likelihood of the measurements y conditional on the model parameters x. More details and the equivalence to a convex optimization problem are shown in App. A. The encouraging results and close correspondence of this choice of regularization to the behavior predicted by the DQE are presented in this paper as empirical results.¹

Although not needed for solving (8), an explicit transformation

$$x = Gs, \tag{9}$$

is necessary for our analysis, changing the image formation model from (1) into

$$\mathbf{E}(y) = IAGs. \tag{10}$$

Since s is computed through a convolution with kernel (6), the inverse operator G can be obtained by transforming to Fourier space, inverting, zeroing the dc-component and transforming back to real space. As this resets the mean value to 0, s is extended by one element containing the mean, and the appropriate column is appended to G; similarly, a corresponding row is appended to H as well.

4 Statistical experimental design

In this section the Fisher information matrix, the A-optimality criterion (AOC) and the detective quantum efficiency (DQE) of the various measurement schemes are discussed.

4.1 Fisher information matrix

The Fisher information [39, 20] \mathcal{I} quantifies how much information the measurements y contain about the unknown parameters s that model it. In this section and in Sec. 4.2 the signal support is assumed known, i.e. only the non-zero elements of s are retained. The Fisher information is defined as the negative expectation value of the curvature of the measurements' log-likelihood function $\ln L(y|s)$

$$\mathcal{I}_{k,\ell}(s) = -\mathbf{E}\left[\frac{\partial^2 \ln L(y|s)}{\partial s_k \partial s_\ell}\right].$$
 (11)

¹The MATLAB implementation of this algorithm, sparseElnL, is made freely available (https://github.com/ woutervandenbroek/sparseElnL).

If the measurements y_i are drawn from the independent distributions $p_i(y_i|s)$, then

$$L(y|s) = \prod_{i} p_i(y_i|s).$$
(12)

For the problems at hand, the distributions p_i are given by the noise distributions in (2), (3) and (4). Working out (11) while taking (9) into account yields

$$\mathcal{I}_{k,\ell}(s) = I \sum_{i} \frac{[AG]_{i,k} [AG]_{i,\ell}}{[AG]_{i}s}$$
(13)

for the Poisson distribution in (2),

$$\mathcal{I}_{k,\ell}(s) = I \sum_{i} \frac{[AG]_{i,k} [AG]_{i,\ell}}{[AG]_{i}s + c/I} \left(1 + \frac{1}{2I\left([AG]_{i}s + c/I\right)}\right)$$
(14)

for the case in (3) with Poisson and read-out noise, and

$$\mathcal{I}_{k,\ell}(s) = \frac{I^2}{c} \sum_{i} [AG]_{i,k} [AG]_{i,\ell}$$
(15)

for read-out noise only, as described in (4).

In order to calculate $\mathcal{I}(s)$, all zero entries of s are removed, as are the corresponding columns of G and rows of H. It can be seen immediately from (9) that this does not affect the image formation.

4.2 Information criterion

In statistical experimental design experiments are set up so as to maximize a measure of the information the recordings contain about the unknown parameters that model them. This requires compression of the information matrix into a single number, a so-called information criterion, that then is optimized with respect to the experimental settings (*p* in this paper).

For this work, the oft-used A-optimality is chosen. It is the trace of the inverse of the information matrix, divided by the number of unknowns, K,

$$AOC = \operatorname{tr}\left(\mathcal{I}(s)^{-1}\right)/K.$$
(16)

Although various choices are possible, in [40] it is noted that "[A] design that is optimal for a given model using one [...] criteri[on] is usually nearoptimal for the same model with respect to [...] other criteria."

A-optimality has the added advantage that in case of unbiased estimators it serves as the lower bound on the mean squared error. It is possible to attain this instance of the so-called Cramér Rao lower bound [18, 19, 20] in practice; most notably by a maximum likelihood estimation from a sufficient number of measurements [20]. Nevertheless, one must keep in mind that foremost AOC is a statement about the information contained in the measurements, independent of any estimation algorithm. In view of the fact that the regularized estimates produced in CS are generally biased, the interpretation as a lower bound on the mean squared error is only of secondary importance in this context.²

4.3 Detective quantum efficiency

Rigorous as the AOC is, it lacks generality in this context as it must be calculated for a particular sparsifying basis, example object x and realization of A. The latter problem can be countered by averaging over multiple realizations of A, although this exacerbates an already demanding computation. Furthermore, it requires knowledge of the signal support and no analytical relation between the imaging system's settings and the reconstruction quality AOC is provided.

In order to gain insight, the detective quantum efficiencies (DQE) of the various CS setups are introduced. While conventionally the DQE characterizes noise properties of imaging devices [41, 42], it is slightly generalized here in order to characterize the recording set-up as a whole,

$$DQE = \alpha \frac{SNR_{out}^2}{SNR_{in}^2}.$$
 (17)

SNR_{out} and SNR_{in} are the signal-to-noise ratios of the recorded signal and the incoming signal, respectively. The prefactor α reflects the different number of measurements in both cases, hence $\alpha = M/N$ for sensing matrices PIX_{Di}, PIX_{Fr} and ADF_{CS}, and $\alpha = 1$ for sensing matrix ADF_{Sh}. More details are provided in App. B.

²The MATLAB implementation of these calculations, sparseAOC, is made freely available (https://github.com/woutervandenbroek/sparseAOC).

Sensing		Noise model	
matrix	Poisson	Poisson + read-out	read-out
PIX _{Di}	$\frac{1-p}{N}$	$\frac{p(1-p)}{nN+r\gamma}$	$\frac{p(1-p)}{r\alpha}$

 PIX_{Fr}

 ADF_{CS}

ADF_{Sh}

1

Table 1: DQEs for Various Sensing Matrices andNoise Models.

The SNR is defined as the ratio of the standard deviation of the signal to the standard deviation of the noise. For SNR_{out} this is calculated as the recorded signal y, while for SNR_{in} we use the best possible hypothetical reference signal allowed by Poisson noise given the number of available particles. In case of the single-pixel camera this constitutes the signal that would be obtained with an ideal detector in lieu of each of the mirrors in the image plane, while for ADF-STEM it means spreading out the available dose over all pixels in the image; the respective expressions are (44) and (45).

Since the DQE is calculated from y instead of s knowledge of the support does not come into play. Although more heuristic than the AOC, the DQE is more general as s enters its expression through the average μ of x only, and is independent of the particular realization of A and only depends on the variance of A's elements.

The results are summarized in Table 1, where the new variables

$$r = \frac{M}{N}, \ \gamma = \frac{cN}{I\mu} \text{ and } \gamma' = \frac{c}{I\mu},$$
 (18)

have been used. r is the reduction, and γ and γ' the normalized variances of the read-out noise.

Note how the last column of the first two rows of Table 1 reproduces the N/M dependency of the AOC presented in Eq. 21 in [27] for read-out noise without Poisson statistics.

5 Results

(3-p/4)

 $\frac{1-p}{pN}\frac{1}{\gamma'}$

With the aid of simulations a good agreement between AOC and DQE is demonstrated. Furthermore, reconstructions from a variety of simulated single-pixel set-ups and from an ADF-STEM experiment show close correspondence between MSE and DQE. This justifies the use of the DQE to derive optimal experimental settings and to determine when a CS set-up is preferable over a denoised Shannon scan.

5.1 Agreement between DQE and AOC

The agreement between DQE and AOC for singlepixel camera and ADF-STEM is evaluated under the assumption of three different noise models: Poisson noise only, Poisson noise and read-out noise and read-out noise only.

5.1.1 Single-pixel

Two distinct single-pixel set-ups are investigated: PIX_{Di} where the mirrors in the array take on discrete values of either 0 or 1, and PIX_{Fr} where they take a fractional value between 0 and 1.

The test sample x is the 100×100 Ramp-Discs phantom, displayed in Fig. 2, yielding N = 10000. The intensities lie in the interval [0.1, 1] to better mimic realistic experimental conditions; $\mu = 0.53$ and $\sigma = 0.18$. The sparse vector s has approximately N/10 non-zero elements and M is set to double that value, i.e. 2000, to arrive at a reduction r of 20%. The relative read-out noise γ is set to 625 and $I = 7.5 \times 10^5$, ensuring that $p_{\text{opt}} = 0.10$ and that a CS-measurement with sensing matrix PIX_{Di}, p = 0.01 and combined Poisson and readout noise has a SNR²_{out} of 10 as calculated with (46).

The results for Poisson noise only are depicted in Fig. 3. Predictions by DQE^{-1} agree well with the AOC, except for very low p for sensing matrix PIX_{Fr}.

For these simulation settings the optimal value $p_{\rm opt}$ for the case of simultaneous Poisson and readout noise is 0.10 for PIX_{Di} and 0.16 for PIX_{Fr} as given by (21) and (23) respectively. In Fig. 4 it is shown how AOC and DQE⁻¹ coincide for a wide range of p.



Figure 2: (a): The 100×100 Ramp-Discs phantom with gray values in the interval [0.1, 1.0]. (b): The absolute values of the Laplacian.



Figure 4: DQE⁻¹ and AOC vs. the fraction of onpixels for the single-pixel set-up for simultaneous Poisson and read-out noise. The values for p are logarithmically spaced to better sample the region for low p.

The results for read-out noise only are depicted in Fig. 5. For both sensing matrices, PIX_{Di} and PIX_{Fr} , DQE^{-1} and AOC agree well; yielding an optimum at 0.50 and 0.67, respectively.

5.1.2 ADF-STEM

The CS implementation of ADF-STEM, sensing matrix ADF_{CS} , is tested for all three noise models.

Simulations are carried out on the 100×100 Ramp-Discs phantom displayed in Fig. 2. The simulation parameters are identical to those in Sec. 5.3.1, except that γ' has been set to 0.5 and $I = 2.5 \times 10^4$ so that the ADF_{CS}-measurement with p = 0.01 and combined Poisson and read-out noise has a SNR²_{out} of 10.

The values for DQE^{-1} are compared to AOC in Fig. 6. A least absolute differences fit is used to scale both curves to each other, and an excellent agreement can be observed.

5.2 Agreement between DQE and MSE

In this section Monte Carlo simulations and experiments are presented to illustrate that the settings predicted by the DQE as optimal do yield recon-



Figure 3: DQE^{-1} and AOC vs. the fraction of onpixels for the single-pixel set-up for Poisson noise only.



Figure 5: DQE^{-1} and AOC vs. the fraction of onpixels for the single-pixel set-up for just read-out noise.



Figure 6: DQE^{-1} and AOC vs. the fraction of onpixels for sensing matrix ADF_{CS} . All noise models: Pss stands for Poisson noise, rd-t for read-out noise.

structions with minimal mean squared error (MSE) when regularized with the $\ln L = E(\ln L)$ constraint in Sec. 3.

5.2.1 Monte Carlo simulations

Single-pixel and ADF-STEM CS-measurements of the Ramp-Discs phantom were simulated with the same settings as in Sec. 5.1.1. Poisson noise and additive normal noise were added to the recordings as needed. For each value of p ten measurements were simulated with different realizations of sensing matrix and noise. As starting guess

$$\frac{A^T y}{I\langle A \rangle^2 NM} \tag{19}$$

with a slight perturbation was used, where $\langle A \rangle$ denotes the average of all elements of A.

The MSEs were calculated with respect to the original phantoms and their averages plotted as a function of p along with the sample standard deviation. A first order least squares fit was used to match DQE⁻¹ to MSE. That, contrary to Sec. 5.1, a mere scaling does not suffice for a good agreement is an indication of the general biasedness of regularized estimators.

The results for the single-pixel camera with discrete mirror values (PIX_{Di}) in the presence of Poisson noise and read-out noise are presented in Fig. 7. The optimization ran for 50 iterations, with 10 subiterations for step 1; see App. A. The MSEs show the same characteristic optimum for p = 0.10as above. The DQE⁻¹ matches the MSE well and predicts the minimum.

The results for the single-pixel set-up with fractional mirror values (PIX_{Fr}) in the presence of just read-out noise are presented in Fig. 8. The optimization ran for 100 iterations, with 10 subiterations for step 1. The MSEs show the same characteristic optimum at about 0.67 as above. The DQE^{-1} matches the MSE well and can hence be used to predict the optimal experimental settings.

In Fig. 9 results for the single-pixel camera with discrete on-values (PIX_{Di}) in the presence of just Poisson noise are presented. The optimization ran for 50 iterations, with 10 subiterations for step 1. Despite systematic deviations, the DQE^{-1} shows the same characteristic monotonic increase as the MSE and hence predicts the optimal settings well.



Figure 7: DQE^{-1} and MSE vs. the fraction of on-pixels for the single-pixel camera with discrete on-values (PIX_{Di}) with Poisson noise and read-out noise. The values of p are spaced logarithmically to sample the region around the optimum better. Compare to the results in Fig. 4.



Figure 8: DQE^{-1} and MSE vs. the fraction of on-pixels for the single-pixel camera with fractional on-values (PIX_{Fr}) for read-out noise only. Compare to the results in Fig. 5.



Figure 9: DQE^{-1} and MSE vs. the fraction of on-pixels for the single-pixel set-up with discrete on-values (PIX_{Di}) for Poisson noise only. Compare to the results in Fig. 3.

Ten values for p were chosen, equidistantly spaced from 0.01 to 0.90. To avoid purely numerical difficulties, the variable γ was set to 0.05 so that according to (21) the optimal value for p equals 0.001, i.e. ten times smaller than the lowest value encountered. This makes the ratio between the terms cand IAx in constraint (33) of the order of 10^{-4} for p = 0.01; for higher p the ratio is even smaller.

5.2.2 Experimental ADF-STEM

A gold nanorod was imaged in a FEI Titan³ microscope operating in STEM mode at 300 kV. An image x_0 with 256×256 pixels was raster scanned with an intensity $I_0 = 3.9 \times 10^3$. After application of a 3×3 median filter, it served as ground truth to compute the MSE of the reconstructions; see Fig. 10.a. Then, a 256×256 image x_1 was raster scanned with an intensity of $I_1 = 1.1 \times 10^2$; see Fig. 10.b. At a standard deviation \sqrt{c} of 0.32 e^- and 0.43 e^- for x_0 and x_1 , respectively, the read-out noise was small, and this experiment exhibits almost pure Poisson noise.

Ten sensing matrices of type ADF_{CS} were constructed with p = 1/N and M ranging from N/10to N in steps of N/10. This corresponds to an inpainting set-up with a fraction r of M/N pixels available. From x_1 and these ten matrices, ten measurements y_1 were synthesized.



Figure 10: Experimental ADF-STEM data. (a) Ground truth image x_0 . (b) Image x_1 . (c) Reconstruction from M = N/10 measurements, with p = 1/N. (d) Reconstruction from M = N measurements with p = 1/N, i.e. a denoising.

The DQE for this set-up is calculated as laid out in App. B. However, since in this case the measurements are not taken at equal dose, (44) must be used for SNR_{in}^2 instead of (45); resulting in DQE = M/N.

Again constraint (33) was used for the reconstructions. The starting guesses were set according to (19). The reconstructions ran for 200 iterations, with 10 subiterations for step 1. In Figs. 10.c and 10.d results are shown for M = N/10 and M = N. Profiles over the central vertical line of the reconstructions are contrasted with the measurements in Fig. 11 to allow visual assessment of the reconstruction quality .

For calculation of the MSEs, the reconstructions were registered to x_0 and the intensities were matched by a linear least squares fit to x_0 . The resulting MSEs are depicted in Fig. 12 where they are shown to match well with DQE⁻¹ after a linear transformation has been applied to the latter.



Figure 11: Profiles and measurements for the central vertical line of the reconstructions in Fig. 10, for M/N = 0.1 and M/N = 1.



Figure 12: After a linear transformation, the $DQE^{-1}s$ match well to the MSEs of the reconstructions from the experimental ADF-STEM data.

Sensing	Noise model		
matrix	Poisson	Poisson + read-out	read-out
PIX _{Di}	1/N	$\sqrt{r\gamma/N}$	1/2
$\mathrm{PIX}_{\mathrm{Fr}}$	1/N	$\frac{4}{\sqrt{6}}\sqrt{r\gamma/N}$	2/3
$\mathrm{ADF}_{\mathrm{CS}}$	1/N	1/N	1/N
$\mathrm{ADF}_{\mathrm{Sh}}$	n.a.	n.a.	n.a.

Table 2: Optimal Values for p for Various Sensing Matrices and Noise Models. See also (20) and (22).

5.3 Main results: DQE analysis

In this section optimal values p_{opt} for p are derived from the DQE; the results are summarized in Table 2. Furthermore, the DQE's analytical tractability is deployed to investigate if the denoising of a Shannon scan is preferable over a full CS set-up, and under what circumstances this might be the case.

5.3.1 Single-pixel camera

In the presence of just Poisson noise the DQE decreases monotonically with p, thus suggesting that the optimal value is the minimum 1/N, which defines an inpainting set-up. In the case of Poisson noise and read-out noise, the optimal value is obtained by equating the derivative of DQE with respect to p to zero, yielding

$$p_{\text{opt}} = \frac{r\gamma}{N} \left(\sqrt{1 + \frac{N}{r\gamma}} - 1 \right),$$
 (20)

$$\simeq \sqrt{\frac{r\gamma}{N}}, \quad \text{for } \sqrt{\frac{r\gamma}{N}} \ll 1, \qquad (21)$$

for sensing matrix PIX_{Di} . For sensing matrix PIX_{Fr} it holds that,

$$p_{\text{opt}} = 2\frac{r\gamma}{N} \left(\sqrt{1 + \frac{2}{3}\frac{N}{r\gamma}} - 1 \right),$$
 (22)

$$\simeq \frac{4}{\sqrt{6}}\sqrt{\frac{r\gamma}{N}}, \quad \text{for } \sqrt{\frac{r\gamma}{N}} \ll 1.$$
 (23)

For just read-out noise an optimal value of 1/2 is reached for PIX_{Di}, and 2/3 for PIX_{Fr}. See Table 2

The DQE for the Shannon case is derived by setting p = 1/N and r = 1 in the expression for PIX_{Di} with simultaneous Poisson and read-out noise (Table 1, first row, second column), yielding

$$DQE_{Sh} = \frac{1}{N(1+\gamma)}.$$
 (24)

The CS set-up is treated for low read-out noise by evaluating the DQE in the respective optimal values for p (21):

$$DQE_{CS} \simeq \frac{1}{N}, \text{ for } \sqrt{\frac{r\gamma}{N}} \ll 1.$$
 (25)

It is thus shown that in the absence of read-out noise $(\gamma = 0)$, with only Poisson noise present, a CS recording contains the same amount of Fisher information as a Shannon scan.

In the limit of strong read-out noise (large γ) p_{opt} in (20) approaches 1/2, and a lower bound is obtained by filling out p = 1/2 in the expression for the DQE:

$$DQE_{CS}^{lb} = \frac{1}{2N + 4r\gamma}.$$
 (26)

From this it follows that if γ exceeds $(1-4r/N)^{-1} \simeq 1$, a CS reconstruction is preferred since then $DQE_{CS}^{lb} > DQE_{Sh}$. In the limit of strong read-out noise the condition $\gamma > 1$ is met by construction.

Reconstructions from simulations following the settings in Sec. 5.1.1 were carried out to test the dependence of the MSE on the relative read-out noise γ on CS recordings and Shannon scans. The optimization ran for 50 iterations, with 10 subiterations for step 1. For CS, γ was varied from 10^{-1} to 10^5 , and following (20) p was set to p_{opt} with values varying from 0.0014 to 0.45, respectively. For the Shannon scans γ was varied from 10^{-1} to 10^1 .

The results are depicted in Fig. 13. After a linear transformation has been applied to DQE^{-1} , it fits well to MSE. For low noise both approaches yield comparable MSEs, but the much stronger rise for the Shannon scan shows CS is preferable in case of read-out noise. Also, as predicted in (25), MSE for CS is approximately constant for the lower noise levels.

5.3.2 ADF-STEM

The expressions for the DQEs are given in Table 1 for ADF-STEM measurements. Contrary to the



Figure 13: MSE and DQE^{-1} for reconstructions from Shannon (Sh) scans and from CS measurements for the single-pixel set-up.

single-pixel set-up the optimal value for p is independent of the noise model as for all models it holds that,

$$\frac{\partial}{\partial p}$$
DQE $\propto -\frac{1}{p^2}$, (27)

indicating that DQE is a monotonically decreasing function of p. This suggests the optimal value for p is the minimum 1/N (see Table 2) and that an inpainting approach is preferred.

The quality of a CS reconstruction is compared to that of a denoised Shannon scan recorded with the same electron dose. It can be shown that,

if
$$p \ge \frac{1 + \gamma'/r}{N}$$
, then $DQE_{Sh} \ge DQE_{CS}$, (28)

where DQE_{CS} and DQE_{Sh} denote the DQEs for the CS and the Shannon case, defined respectively by sensing matrices ADF_{CS} and ADF_{Sh} for simultaneous Poisson and read-out noise (Table 1, third and fourth row, second column).

In the absence of read-out noise, γ' equals 0 and condition (28) is always met. $DQE_{Sh} \ge DQE_{CS}$ then implies that a Shannon scan contains at least as much Fisher information as a CS measurement. That the respective reconstructions yield approximately the same MSE under equal dose conditions can be inferred from the results in Sec. 5.2.2 for experimental ADF-STEM where read-out noise was negligible. Since these CS measurements were obtained by selecting M pixels at random from a Shannon scan, the total dose for these CS measurements scales directly with M. The results in Fig. 12 show that the MSE is a linear function of $DQE^{-1} = N/M$ and hence of the inverse of the total dose. This implies that for constant dose MSE is constant as well, and hence equal to that of the denoised Shannon scan at M/N = 1.

6 Discussion

Rigorous as the AOC is, it lacks some generality as it yields results for just the test object and the particular realization of the sensing matrix A, and requires knowledge of the support in the sparse basis. In contrast, the DQE, although more heuristic in nature, is an analytical function of the experimental conditions and alleviates some of the AOC's drawbacks: the test object enters through just its mean value, only the variance of the sensing matrix entries is needed and knowledge of the support is not necessary.

A caveat is that for too few on-pixels the Fisher information matrix becomes singular, which means no unbiased estimators are possible [43] and the AOC cannot be computed. This is information that the DQE cannot deliver, and its predictions therefore only hold on the condition of the Fisher information matrix being non-singular.

That regularized estimators are in general biased is reflected in the fact that a linear fit is needed to match DQE^{-1} to the MSE of the reconstructions, instead of the mere scaling that sufficed for matching to the AOC. Nevertheless, the DQE still predicted the optimal experimental settings accurately, thus suggesting that also biased estimators perform best with measurements with maximum Fisher information.

7 Conclusions

In this paper the performance of various CS setups was assessed under the constraint of constant total illumination and compared to that of a denoised Shannon scan with the same dose. The Aoptimality information criterion AOC was chosen as a measure of the amount of Fisher information in the measurements. With the aid of simulations, the heuristic quantity detective quantum efficiency DQE was shown to track the AOC accurately. Also the mean squared error (MSE) of CS reconstructions from experimental and simulated recordings was well tracked by the DQE.

The DQE's analytical tractability was then used to show that for the investigated sensing matrices and in the absence of read-out noise, i.e. with only Poisson noise present, compressed sensing does not raise the amount of Fisher information in the recordings above that of a Shannon sampled signal. As a consequence, reconstruction results yield a comparable MSE when both data sets are treated with the same algorithm. This result holds for both investigated systems, but is particularly surprising for the single-pixel camera, as the Shannon scan makes use of only 1/N of the total dose. This is considered of fundamental importance here as read-out noise can be viewed as a mere engineering problem for particles with sufficiently high energies. This might temper the high expectations for beam damage reduction that were laid out in the TEM community.

The DQE was also used to optimize the experimental designs, i.e. the fraction p of on-pixels for best reconstruction quality as a function of particle dose, read-out noise and other experimental parameters. In the presence of Poisson noise these results, summarized in Table 2, differ markedly from the optimal p = 1/2 often tacitly assumed in literature. When there is only Poisson noise an inpainting setup (p = 1/N) is optimal in all investigated systems. No matter the noise model, p = 1/N is optimal for ADF-STEM. When there is only read-out noise p = 1/2 and p = 2/3 are optimal if the on-values in the single-pixel camera are discrete or fractional, respectively. A combination of Poisson noise and read-out noise yields an optimum between these extremes.

A Expected log-likelihood regularization

The simulation and experimental results have been obtained by solving the following problem by minimizing the associated augmented Lagrangian [35] through an alternating direction scheme [36, 37, 38],

$$\min_{x} \mathrm{TGV}(x) \quad \text{s.t.} \quad \ln L(y|x) = \mathrm{E}\left(\ln L(y|x)\right). \quad (29)$$

 $\ln L$ is the log-likelihood of the measurements y conditional on the model parameters x, and E denotes the expectation value.

A.1 Expected log-likelihood

The likelihood L is defined as

$$L = \prod_{j=1}^{M} p(y_j | IA_j x), \tag{30}$$

with p the probability distribution function of the measurements y as given in (2), (3) or (4). Filling out expression (3), yields

$$\ln L = \sum_{j} -\ln \left(IA_{j}x + c \right) - \frac{\left(IA_{j}x - y_{j} \right)^{2}}{IA_{j}x + c}, \quad (31)$$

omitting constant terms and a factor of 1/2. Taking the expectation value over y gives

$$\operatorname{E}(\ln L) = \sum_{j} -\ln\left(IA_{j}x+c\right) - \frac{\sigma_{y}^{2}}{IA_{j}x+c}.$$
 (32)

The symbol σ_y^2 denotes the variance of the measurements, and equals $IA_jx + c$ for Poisson and read-out noise and c for just read-out noise.

Equating $\ln L$ and $E(\ln L)$ is then equivalent to setting to zero the constraint,

$$c_{\ln L} = \sum_{j=1}^{M} \left(\frac{(IA_j x - y_j)^2}{IA_j x + c} - 1 \right).$$
(33)

The constraint associated with (4) is then

$$\sum_{j=1}^{M} \left(\frac{(IA_j x - y_j)^2}{c} - 1 \right).$$
 (34)

A.2 Augmented Lagrangian, alternating directions

The augmented Lagrangian [35] then becomes,

$$\mathcal{L}_{A} = \sum_{i}^{2N} |s_{i}| - \sum_{i}^{2N} \lambda_{i} (H_{i}x - s_{i}) + \frac{\mu}{2} \sum_{i}^{2N} (H_{i}x - s_{i})^{2} - \nu c_{\ln L} (x) + \frac{\beta}{2} c_{\ln L}^{2} (x).$$
(35)

This problem is then solved with an alternating directions method [36, 37] where in iteration $\ell + 1$:

- 1. \mathcal{L}_{A} is minimized w.r.t. x through a numerical optimization with a Polak-Ribière non-linear conjugate gradient approach [44, 35], with $s^{(\ell)}$, $\lambda^{(\ell)}$ and $\nu^{(\ell)}$ kept constant;
- L_A is minimized analytically w.r.t. s through soft thresholding/shrinkage [45, 37],

$$s^{(\ell+1)} = \max\left(\left|Hx^{(\ell+1)} - \frac{\lambda^{(\ell)}}{\mu}\right| - \frac{1}{\mu}, 0\right) \\ \times \operatorname{sgn}\left(Hx^{(\ell+1)} - \frac{\lambda^{(\ell)}}{\mu}\right),$$
(36)

with $x^{(\ell+1)}$ and $\lambda^{(\ell)}$ kept constant;

3. the multipliers λ and ν are then updated through [35]

$$\lambda^{(\ell+1)} = \lambda^{(\ell)} - \mu \left(H x^{(\ell+1)} - s^{(\ell+1)} \right) (37)$$

$$\nu^{(\ell+1)} = \nu^{(\ell)} - \beta c_{\ln L} \left(x^{(\ell+1)} \right), \qquad (38)$$

with $x^{(\ell+1)}$ and $s^{(\ell+1)}$ kept constant.

In the first iteration μ , ν and all λ_i are set to zero and step 1 is iterated until convergence. In this way an initial estimate for x is obtained that obeys the constraint $\ln L = E(\ln L)$ virtually perfectly. This estimate is then used as the starting point for the subsequent minimization.

During optimization IA_jx can become temporarily negative. To avoid the ensuing numerical difficulties, the respective term in the denominator of (33) is wrapped in the softplus function

$$\epsilon \ln \left(1 + \exp \left(I A_j x/\epsilon \right) \right), \tag{39}$$

which maps positive variables onto themselves and negative variables to zero, with $\epsilon = c/10$ the width of the transition between both regimes.

A.3 Convex optimization

CS reconstruction problems can often be cast into two distinct forms. The first is the convex optimization problem:

$$\arg\min_{x} \left(F(x) = |Rx - b|_{2}^{2} + \lambda |Wx|_{1} \right), \quad (40)$$

for some positive constant λ and sparsifying matrix W; $| |_2$ and $| |_1$ are the ℓ_2 - and ℓ_1 -norm, respectively. The second form is:

$$\arg\min_{x} (|Wx|_1), \text{ s.t. } |Rx - b|_2^2 = C,$$
 (41)

for some positive target value C, chosen for example so as to make the likelihood consistent with the experimental error, such as in (33). We assume C is feasible, i.e. there is a non-empty subset X of potential solutions such that $|Rx - b|_2^2 = C$ for all $x \in X$.

These two forms are equivalent. As shown in [46], the residual $|Rx - b|_2^2$ and the ℓ_1 -norm of the solution, $|Wx|_1$, of (40) define a convex Pareto-curve when plotted against each other. And since this curve is parametrized by λ , λ can be used to tune the solution to obtain $|Rx - b|_2^2 = C$. The result so found is also a solution to the second form (41), for if there were an element of X with lower $|Wx|_1$, the first algorithm could have selected it and reduced F(x).

The constraint (33) is well approximated by a second order expansion in IA_jx around y_j of the individual terms j:

$$c_{\ln L} = \sum_{j=1}^{M} \left(\frac{(IA_j x - y_j)^2}{y_j + c} - 1 \right).$$
(42)

This is correct up to second order in $IA_jx - y_j$, and since the summation averages out the third order error terms, the dominant error is only of fourth order. The translation from the formulation in (41) to the problem at hand is then

$$Rx = \frac{IAx}{\sqrt{y+c}}, \ b = \frac{y}{\sqrt{y+c}}, \ C = M, \text{ and } W = H, (43)$$

where division of vectors is elementwise.

B Derivation of DQEs

In this appendix the detective quantum efficiencies, DQE, as defined in (17) are derived. For SNR_{in}:

$$SNR_{in}^{2} = \frac{(I\sigma)^{2}}{I\mu} = I\frac{\sigma^{2}}{\mu}, \text{ for single-pixel}, (44)$$

$$SNR_{in}^{2} = I\frac{\sigma^{2}}{\mu}\frac{M}{N}, \text{ for ADF-STEM}.$$
(45)

where $I\sigma$ is the signal's standard deviation, $I\mu$ is the average number of photons reflecting off each of the object's pixels, and μ and σ are the mean and standard deviation of x, respectively.

sensing		
matrix	$\langle y angle$	σ_a^2
$\mathrm{PIX}_{\mathrm{Di}}$	$I\mu p\frac{N}{M}$	$\frac{p}{M^2} \left(1 - p \right)$
$\mathrm{PIX}_{\mathrm{Fr}}$	$I\mu p \frac{N}{2M}$	$\frac{p}{M^2}\left(1/3 - p/4\right)$
$\mathrm{ADF}_{\mathrm{CS}}$	$I\mu$	$\frac{1}{N^2} (1/p - 1)$
$\mathrm{ADF}_{\mathrm{Sh}}$	$I\mu \frac{M}{N}$	$\frac{M^2}{N^3}$

Table 3: Average Signals and Variances for Various Sensing Matrices

The SNR of the signal y recorded according to (1) follows,

$$\operatorname{SNR}_{\operatorname{out}}^2 = \frac{\sigma_y^2}{\langle y \rangle + c},$$
 (46)

where σ_y^2 is the variance of y, $\langle y \rangle$ is—as the average of y—the variance of the Poisson noise, and c is the read-out noise variance.

The CS measurement process can be regarded as sampling without replacement from the N members of population x, and the covariance matrix cov_x of x hence has diagonal elements σ^2 and off-diagonal elements $-\sigma^2/(N-1)$. Since the members of y are measured independently but with the same rules for on- and off-pixels, σ_y^2 can be taken as the variance of an arbitrary element k in y,

$$\sigma_y^2 = \left(\frac{\partial y_k}{\partial x}\right)^T \operatorname{cov}_x \frac{\partial y_k}{\partial x},\tag{47}$$

$$= I^{2} \sigma^{2} \sum_{i=1}^{N} a_{ki}^{2} - \frac{I^{2} \sigma^{2}}{N-1} \sum_{i=1}^{N} a_{ki} \sum_{j \neq i}^{N} a_{kj} (48)$$

$$\simeq I^{2} N \sigma^{2} \sigma_{a}^{2}.$$
(49)

with a_{ki} the respective element of sensing matrix A in (1), and σ^2 and σ_a^2 the variances of x and of the elements in A, respectively.

The expressions in Table 1 can be obtained by finding $\langle y \rangle$ and σ_a for all sensing matrices (see Table 3), and setting to zeros the terms c or $\langle y \rangle$ in the denominator in (46) for the cases without read-out noise or only read-out noise, respectively. Furthermore, the change of variables in (18) helps simplifying the results.

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