Codes for Iterative Decoding from Partial Geometries

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Abstract — **This work develops codes suitable for iterative decoding using the sum-product algorithm. We consider regular low-density parity-check (LDPC) codes derived from partial geometries, a large class of combinatorial structures which include several of the previously proposed algebraic constructions for LDPC codes as special cases. We derive bounds on minimum distance and $\mathrm{rank}_2(H)$ for codes from partial geometries, and present constructions and performance results for two classes of partial geometries which have not previously been proposed for use with iterative decoding.**

I. Incidence and designs

A *design* $\mathcal{D} = (\mathcal{P}, \mathcal{B}, \mathcal{I})$ consists of a finite non-empty set $\mathcal{P}$ of points and a finite non-empty set $\mathcal{B}$ of blocks, together with an incidence relation $\mathcal{I} \subseteq \mathcal{P} \times \mathcal{B}$. The *incidence matrix* N of $\mathcal{D}$ is a $|\mathcal{P}| \times |\mathcal{B}|$ matrix indexed by the points and blocks of $\mathcal{D}$, with $N_{i,j} = 1$ if point i is incident with block j. The *adjacency matrix* of $\mathcal{D}$ is a $|\mathcal{P}| \times |\mathcal{P}|$ matrix A, indexed by the points of $\mathcal{D}$ with $A_{i,j} = 1$ if the points i and j are incident with the same block in $\mathcal{D}$. The adjacency matrix is said to be *strongly regular* if the number of other points with which a pair of points are both incident depends only on whether or not the pair of points are themselves incident.

In what follows we consider partial geometries, a class of designs with strongly regular A. For a partial geometry, $\mathrm{pg}(s,t,\alpha)$, each point p is incident with $t+1$ blocks and each block B incident with $s+1$ points, any two blocks have at most one point in common, and for any non-incident point-block pair (p, B) the number of blocks incident with both p and a point in B equals some constant α.

II. Codes from partial geometries

We can take the incidence matrix N of a partial geometry as the parity-check matrix H of an LDPC code C with $|\mathcal{P}|$ parity-checks, length $|\mathcal{B}|$, column weight $s+1$, row weight $t+1$, and girth ≥ 6. Using the properties of strongly regular graphs we can find the eigenvalues of A and then show that NN^T has eigenvalues $(s+1)(t+1)$, $s+t+1-\alpha$, 0 with corresponding multiplicities

$$1, \; \frac{st(s+1)(t+1)}{\alpha(s+t+1-\alpha)}, \; \frac{s(s+1-\alpha)(st+\alpha)}{\alpha(s+t+1-\alpha)}. \qquad (1)$$

Using Tanner's bit- and parity-oriented bounds [3] we obtain the following

Lemma 1 *The minimum distance of a code from* $\mathrm{pg}(s,t,\alpha)$ *satisfies* $d_{\min} \geq \max\{(t+1)(s+1-t+\alpha)/\alpha, 2(s+\alpha)/\alpha\}$.

Designs from two classes of partial geometries, balanced incomplete block designs, BIBDs ($\alpha = s+1$), and generalized quadrangles, GQs ($\alpha = 1$), have been studied previously for use as LDPC codes (see e.g. [2, 4]). The minimum distance bounds from Lemma 1 are weak for the BIBDs, nets ($\alpha = t$) and transversal designs, TDs ($\alpha = s$), and a better bound is provided by Massey's bound ($d_{\min} \geq s+2$ in this case). However, for the proper partial geometry ($1 < \alpha < \min\{s,t\}$) and generalized quadrangle codes the bounds from Lemma 1 significantly improve upon Massey's bound to give a minimum distance bound of up to twice the column weight of H.

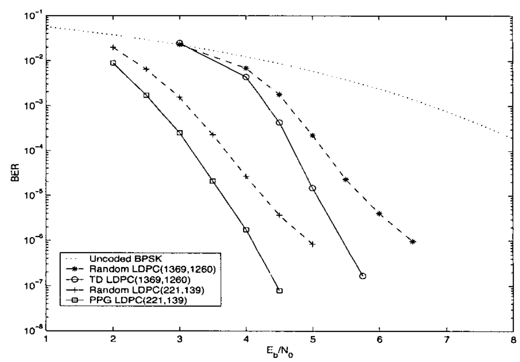

Figure 1: BER vs. E_b/N_0 for LDPC codes in an AWGN channel

The excellent performance of the finite geometry codes [2] has been attributed to the highly redundant parity-check matrices of those codes [2, 4], motivating the search for other designs which give low rank parity-check matrices. A simple upper bound on the 2-rank of a code is the number of non-zero eigenvalues of HH^T which we know from (1):

$$\mathrm{rank}_2(H) \leq \frac{st(s+1)(t+1)}{\alpha(t+s+1-\alpha)} + 1.$$

Further, we use results from [1] to show that this bound is tight (within 1 of the actual rank) for the partial geometry codes with $\mu_2 = s+t+1-\alpha \equiv 1 \bmod 2$. We see that, with the exception of some BIBDs, every partial geometry produces a code with linearly dependent rows in H.

Fig. 1 shows the performance of LDPC codes derived from a $\mathrm{TD}(2, 36, 2)$ and a proper $\mathrm{pg}(12, 12, 9)$ compared with randomly constructed codes of the same rate and length.

References

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