Infinite-Series Representations Associated With the Bivariate Rician Distribution and Their Applications

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Abstract—Analytical expressions for the evaluation of the bivariate Rician cumulative distribution function (CDF), the covariance, and the characteristic function (CHF) are not known, despite their usefulness in wireless communications systems analysis. In this letter, motivated by the ability of the Rician model to describe fading in wireless communications, we derive infinite-series representations for the probability density function, the CDF, the covariance, and the CHF of two correlated Rician random variables. It is shown that the presented infinite-series expressions converge rapidly, and can be efficiently used to study several performance criteria for dual-diversity receivers operating over correlated Rician fading channels.

Index Terms—Communications channels, correlated fading, digital communications, fading channels, Rician fading.

I. Introduction

Several statistical models are used in communications systems analysis to describe fading in wireless environments. The most frequently used distributions are Rayleigh, Nakagami-m, Rice, and Weibull. Moreover, several problems in wireless communications theory involve bivariate and, in the general case, multivariate distributions. Examples of such problems can be found in the performance analysis of correlative fading applications with space or frequency diversity, in multichannel reception, or in the transition probabilities in a first (or higher) order Markov chain that models the fading process [1].

Looking at the open up-to-date technical literature, several approaches are presented concerning the multivariate Rayleigh and Nakagami-m distributions. More specifically, Nakagami presented the bivariate Rayleigh and Nakagami-m probability density function (PDF) [2]. Tan and Beaulieu [1] proposed an infinite-series representation for the bivariate Nakagami-m cumulative distribution function (CDF), and Simon and Alouini [3] formulated the bivariate Rayleigh CDF with a single integral with finite limits. Concerning the multivariate Rayleigh and Nakagami-m distributions, Mallik [4] presented analytical results for the multivariate Rayleigh distribution, while Karagiannidis et al. [5], [6] derived expressions for the multivariate Nakagami-m PDF and CDF. Moreover, the covariance of correlated Nakagami-m random variables (RVs) is presented in [7]. Most of these works have been used to study, for example, the effects of correlated fading on the performance

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of wireless communications systems with diversity at the receiver [8]–[12]. However, despite the usefulness of the Rice model, only the bivariate Rician PDF was given in the past as [8, eq. (15)]

$$f_{x_1,x_2}(x_1,x_2) = \frac{(1+K)^2 x_1 x_2}{2\pi\beta^2 (1-u^2)} \exp\left[\frac{-2K}{1+u}\right]$$

$$\times \exp\left[-\frac{(1+K)\left(x_1^2 + x_2^2\right)}{2\left(1-u^2\right)\beta}\right]$$

$$\times \int_0^{2\pi} \exp\left[\frac{u\left(1+K\right) x_1 x_2 \cos\theta}{\left(1-u^2\right)\beta}\right]$$

$$\times I_0\left(\sqrt{\frac{2K\left(x_1^2 + x_2^2 + 2x_1 x_2 \cos\theta\right)}{\beta(1+K)^{-1}(1+u)^2}}\right) d\theta$$
(1)

where $x_1 = |W|$ and $x_2 = |H|$ are correlated Rician RVs, with W, H as nonzero-mean correlated complex Gaussian RVs, u is the correlation coefficient, which is assumed to be real, and is defined as

$$u = \frac{\langle (W^* - \langle W^* \rangle) (H - \langle H \rangle) \rangle}{\sqrt{\langle |W - \langle W \rangle|^2 \rangle} \sqrt{\langle |H - \langle H \rangle|^2 \rangle}}$$
(2)

where

 $\langle \cdot \rangle$ expectation;

 β average power of x_1, x_2 defined as $\beta = \overline{x_1^2}/2 = \overline{x_2^2}/2$;

K Rice factor defined as the ratio of the signal power in the dominant component over the scattered power;

 $I_0(\cdot)$ modified Bessel function of the first kind and zeroth order.

The complicated form of (1) was the reason for the use of numerical integration in [8] and [13] in order to study the error performance of dual switched diversity receivers over correlated Rician flat-fading channels, and of orthogonal frequency-division multiplexing (OFDM) systems over frequency-nonselective fast Rician channels, respectively.

In this letter, motivated by the importance of the Rician model to describe fading in wireless communications systems, we present a novel infinite-series representation for the bivariate Rician PDF. This formula is used to derive similar expressions with rapid convergence for the CDF, the covariance, and the characteristic function (CHF) of two correlated identically distributed (i.d.) Rician RVs. The proposed expressions can be used to study several performance criteria, such as the outage probability and the average error rate for different modulation schemes, of dual-diversity receivers employing selection combining (SC), equal-gain combining (EGC), and

switched-and-stay combining (SSC). However, due to the limited size of this letter, we present, as an indicative example of the mathematical analysis, the evaluation of the outage probability of a dual SC receiver operating over correlated Rician fading channels.

II. INFINITE-SERIES REPRESENTATIONS

A. Probability Density Function

Using the infinite-series representation of $I_0(\cdot)$ [14, eq. (8.445)]

$$I_0(z) = \sum_{k=0}^{\infty} \frac{z^{2k}}{2^{2k} k! \Gamma(k+1)}$$
 (3)

and changing the order of summation and integration, (1) can be written as

$$f_{x_{1},x_{2}}(x_{1},x_{2}) = \sum_{i=0}^{\infty} \frac{(1+K)^{2}}{2\pi\beta^{2}(1-u^{2})} \left[\frac{K(1+K)}{\beta(1+u)^{2}} \right]^{i}$$

$$\times \frac{x_{1}x_{2}}{2^{i}i!\Gamma(i+1)}$$

$$\times \exp\left[\frac{-2K}{1+u} - \frac{(1+K)(x_{1}^{2}+x_{2}^{2})}{2(1-u^{2})\beta} \right]$$

$$\times \int_{0}^{2\pi} \exp\left[\frac{u(1+K)x_{1}x_{2}\cos\theta}{(1-u^{2})\beta} \right]$$

$$\times (x_{1}^{2}+x_{2}^{2}+2x_{1}x_{2}\cos\theta)^{i}d\theta. \tag{4}$$

Expanding the term $(x_1^2 + x_2^2 + 2x_1x_2\cos\theta)^i$ using the multinomial identity results in

$$f_{x_{1},x_{2}}(x_{1},x_{2}) = \sum_{i=0}^{\infty} \frac{(1+K)^{2}}{2\pi\beta^{2}(1-u^{2})} \left[\frac{K(1+K)}{\beta(1+u)^{2}} \right]^{i}$$

$$\times \frac{2^{v_{3}}x_{1}^{2v_{1}+v_{3}+1}x_{2}^{2v_{2}+v_{3}+1}}{2^{i}v_{1}!v_{2}!v_{3}!\Gamma(i+1)}$$

$$\times \exp\left[\frac{-2K}{1+u} - \frac{(1+K)(x_{1}^{2}+x_{2}^{2})}{2(1-u^{2})\beta} \right]$$

$$\times \int_{0}^{2\pi} \exp\left[\frac{u(1+K)x_{1}x_{2}\cos\theta}{(1-u^{2})\beta} \right]$$

$$\times (\cos\theta)^{v_{3}} d\theta \qquad (5)$$

with v_1 , v_2 , v_3 being nonnegative integers. Letting $x = \cos \theta$ and after manipulations, the integral in (5) can be solved with the use of [14, eq. (3.389/1)] resulting in

$$f_{x_{1},x_{2}}(x_{1},x_{2})$$

$$= \sum_{i=0}^{\infty} \mathcal{A} \left[\mathcal{D}_{1} x_{1}^{2v_{1}+v_{3}+1} x_{2}^{2v_{2}+v_{3}+1} \exp \left[-\mathcal{E}(x_{1}^{2}+x_{2}^{2}) \right] \right]$$

$$\times_{1} F_{2} \left(\frac{1+v_{3}}{2}; \frac{1}{2}, \frac{2+v_{3}}{2}; \mathcal{F} x_{1}^{2} x_{2}^{2} \right)$$

$$+ \mathcal{D}_{2} x_{1}^{2v_{1}+v_{3}+1} x_{2}^{2v_{2}+v_{3}+1} \exp \left[-\mathcal{E}(x_{1}^{2}+x_{2}^{2}) \right]$$

$$\times_{1} F_{2} \left(\frac{2+v_{3}}{2}; \frac{3}{2}, \frac{3+v_{3}}{2}; \mathcal{F} x_{1}^{2} x_{2}^{2} \right)$$

$$(6)$$

where

$$\mathcal{A} = \frac{2^{v_3 - i - 2} (1 + K)^{2 + i} K^i (1 + u)^{-2i}}{\beta^{2 + i} (1 - u^2) v_1! v_2! v_3! \Gamma(i + 1)} \exp\left(-\frac{2K}{1 + u}\right)$$

$$\mathcal{D}_1 = 2 (1 + (-1)^{v_3}) \frac{\Gamma\left(\frac{1 + v_3}{2}\right)}{\sqrt{\pi} \Gamma\left(\frac{2 + v_3}{2}\right)}$$

$$\mathcal{D}_2 = \frac{2 (-1 + (-1)^{v_3}) u (1 + K) \Gamma\left(\frac{2 + v_3}{2}\right)}{\sqrt{\pi} \beta (u^2 - 1) \Gamma\left(\frac{3 + v_3}{2}\right)}$$

$$\mathcal{E} = \frac{1 + K}{2\beta (1 - u^2)}$$

$$\mathcal{F} = \frac{(1 + K)^2 u^2}{4\beta^2 (u - 1)^2 (1 + u)^2}.$$

Note that after the truncation of the infinite series in (6) at the value i = I - 1, the remainder R_I converges to zero as I increases (for the proof, see the Appendix).

B. Cumulative Distribution Function

The bivariate Rician CDF is by definition

$$F_{x_1,x_2}(x_1,x_2) = \int_0^{x_1} \int_0^{x_2} f_{x_1,x_2}(y_1,y_2) \, dy_1 dy_2. \tag{7}$$

In order to solve the integrals in (7), we substitute the hypergeometric functions of (6) with their infinite-series representation [14, eq. (9.14/1)], i.e.,

$$_{1}F_{2}(p;q_{1},q_{2};x) = \sum_{h=0}^{\infty} \frac{(p)_{h}x^{h}}{(q_{1})_{h}(q_{2})_{h}h!}$$
 (8)

with $(\cdot)_n$ denoting the Pochhammer symbols [15, eq. (6.1.22)]. This leads to a new formula for the bivariate PDF, shown in (9) at the bottom of the page, where

$$\mathcal{G}_1 = \frac{\left(\frac{1+v_3}{2}\right)_h}{\left(\frac{1}{2}\right)_h \left(\frac{2+v_3}{2}\right)_h h!}, \quad \mathcal{G}_2 = \frac{\left(\frac{2+v_3}{2}\right)_h}{\left(\frac{3}{2}\right)_h \left(\frac{3+v_3}{2}\right)_h h!}.$$

$$f_{x_{1},x_{2}}(x_{1},x_{2}) = \sum_{\substack{i,h=0\\v_{1}+v_{2}+v_{3}=i}}^{\infty} \mathcal{A}\mathcal{F}^{h} \left[\mathcal{D}_{1}\mathcal{G}_{1} \ x_{1}^{2v_{1}+v_{3}+2h+1} x_{2}^{2v_{2}+v_{3}+2h+1} \exp \left[-\mathcal{E} \left(x_{1}^{2} + x_{2}^{2} \right) \right] + \mathcal{D}_{2}\mathcal{G}_{2} x_{1}^{2v_{1}+v_{3}+2h+2} x_{2}^{2v_{2}+v_{3}+2h+2} \exp \left[-\mathcal{E} \left(x_{1}^{2} + x_{2}^{2} \right) \right] \right]$$

$$(9)$$

Substituting (9) into (7) and interchanging the order of summation and integration, integrals of the form

$$\int_{0}^{x} y^{n} \exp\left(-zy^{2}\right) dy \tag{10}$$

are produced. This integral can be solved by making the transformation $t = zy^2$ and using the definition of the "lower" incomplete Gamma function [15, eq. (6.5.2)], i.e.,

$$\gamma(p,x) = \int_0^x y^{p-1} \exp(-y) dy \tag{11}$$

resulting finally in (12), shown at the bottom of the page.

C. Covariance

The covariance of x_1 , x_2 is defined as

$$\operatorname{cov}\langle x_1^{n_1} x_2^{n_2} \rangle = \int_0^\infty \int_0^\infty x_1^{n_1} x_2^{n_2} f_{x_1, x_2} (x_1, x_2) \, dx_1 dx_2.$$
(13)

Similar to the derivation of the bivariate CDF, substituting (9) into (13) and interchanging the order of summation and integration, integrals of the form

$$\int_0^\infty y^n \exp\left(-zy^2\right) dy \tag{14}$$

are produced. The above integrals can be solved using [14, eq. (3.461/3)], and after manipulations, the covariance of x_1 , x_2 is finally given by (15), shown at the bottom of the page.

D. Characteristic Function

The CHF of x_1 , x_2 is, by definition

$$\Phi_{x_1,x_2}(s_1,s_2) = \int_0^\infty \int_0^\infty f_{x_1,x_2}(x_1,x_2) \times \exp(js_1x_1 + js_2x_2) dx_1 dx_2 \quad (16)$$

with $j = \sqrt{-1}$. Substituting (9) into (16), integrals of the form

$$\int_0^\infty y^n \exp\left(-z_1 y^2 - z_2 y\right) dy \tag{17}$$

are produced, which can be solved using [14, eq. (3.462/2)], and $\Phi_{x_1,x_2}(s_1,s_2)$ can be finally written as

$$\Phi_{x_{1},x_{2}}(s_{1},s_{2}) = \sum_{\substack{i,h=0\\v_{1}+v_{2}+v_{3}=i}}^{\infty} \frac{\mathcal{AF}^{h}}{(2\mathcal{E})^{i+2h+2}} \times \exp\left(-\frac{s_{1}^{2}+s_{2}^{2}}{8\mathcal{E}}\right) \times \left[\mathcal{D}_{1}\mathcal{G}_{1}g(z_{1},t_{1})g(z_{2},t_{2}) + \frac{\mathcal{D}_{2}\mathcal{G}_{2}}{2\mathcal{E}} \times g(z_{1}+1,t_{1})g(z_{2}+1,t_{2})\right] (18)$$

where

$$g(z,t) = \Gamma(z) D_{-z}(t)$$

 $z_1 = 2v_1 + v_3 + 2h + 2, \quad t_1 = -\frac{js_1}{\sqrt{2\mathcal{E}}}$
 $z_2 = 2v_2 + v_3 + 2h + 2, \quad t_2 = -\frac{js_2}{\sqrt{2\mathcal{E}}}$

with $D_{-v}(z)$ being the parabolic cylinder function of order v and argument z [15, eq. (9.240)].

III. APPLICATIONS IN DUAL-DIVERSITY RECEIVERS

The proposed infinite-series representations (12), (15), and (18) can be efficiently used to study important performance criteria, such as the outage probability and the average error rate for several modulation schemes, of dual-diversity receivers operating over correlated Rician fading channels.

1) Selection Combining: The formula (12) for the joint Rician CDF can be directly applied for the study of the outage performance of dual SC receivers. Let $\gamma_i = x_i^2 E_s/2N_0$ and $\overline{\gamma}_i = \beta E_s/N_0$, with E_s/N_0 being the symbol energy-to-Gaussian noise spectral density ratio, denote the instantaneous and mean signal-to-noise ratio (SNR) at the

$$F_{x_{1},x_{2}}(x_{1},x_{2}) = \sum_{\substack{i,h=0\\v_{1}+v_{2}+v_{3}=i}}^{\infty} \frac{\mathcal{A}\mathcal{F}^{h}}{4\mathcal{E}^{i+2h+2}} \left[\mathcal{D}_{1}\mathcal{G}_{1}\gamma\left(v_{1} + \frac{v_{3}}{2} + h + 1,\mathcal{E}x_{1}^{2}\right)\gamma\left(v_{2} + \frac{v_{3}}{2} + h + 1,\mathcal{E}x_{2}^{2}\right) + \frac{\mathcal{D}_{2}\mathcal{G}_{2}}{\mathcal{E}}\gamma\left(v_{1} + \frac{v_{3}}{2} + h + \frac{3}{2},\mathcal{E}x_{1}^{2}\right)\gamma\left(v_{2} + \frac{v_{3}}{2} + h + \frac{3}{2},\mathcal{E}x_{2}^{2}\right) \right]$$

$$(12)$$

$$\cos \langle x_1^{n_1} x_2^{n_2} \rangle = \sum_{\substack{i,h=0\\v_1+v_2+v_3=i}}^{\infty} \frac{\mathcal{A}\mathcal{F}^h}{4\mathcal{E}^{i+2h+2+(n_1+n_2/2)}} \left[\mathcal{D}_1 \mathcal{G}_1 \Gamma \left(v_1 + \frac{v_3}{2} + h + \frac{n_1}{2} + 1 \right) \Gamma \left(v_2 + \frac{v_3}{2} + h + \frac{n_2}{2} + 1 \right) + \frac{\mathcal{D}_2 \mathcal{G}_2}{\mathcal{E}} \Gamma \left(v_1 + \frac{v_3}{2} + h + \frac{n_1}{2} + \frac{3}{2} \right) \Gamma \left(v_2 + \frac{v_3}{2} + h + \frac{n_2}{2} + \frac{3}{2} \right) \right]$$
(15)

	K = 0 dB			K = 7 dB		
\underline{u}	$x_1 = 1$ $x_2 = 1$	$ \begin{aligned} x_1 &= 1 \\ x_1 &= 5 \end{aligned} $	$ \begin{aligned} \mathbf{x}_2 &= 5\\ \mathbf{x}_2 &= 5 \end{aligned} $	$x_1 = 1$ $x_2 = 1$	$x_1 = 1$ $x_1 = 5$	$\begin{vmatrix} x_2 = 5 \\ x_2 = 5 \end{vmatrix}$
0.1	6	8	11	14	20	26
0.3	5	7	11	12	18	26
0.6	5	7	17	11	14	29
0.9	10	15	63	23	32	124

TABLE I
NUMBER OF TERMS OF (12) REQUIRED FOR
SIX-SIGNIFICANT-FIGURE ACCURACY

k-branch (k=1,2), respectively, of a dual SC receiver operating over correlated Rician fading channels. The outage probability of the SC output SNR, $P_{\rm out}$, can be evaluated as [9]

$$P_{\text{out}} = F_{\gamma_{\text{out}}}(\gamma_{th}) = F_{\gamma_{1},\gamma_{2}}(\gamma_{th},\gamma_{th})$$

$$= F_{x_{1},x_{2}}\left(\sqrt{2\beta\frac{\gamma_{th}}{\overline{\gamma}}},\sqrt{2\beta\frac{\gamma_{th}}{\overline{\gamma}}}\right). \tag{19}$$

- 2) Equal-Gain Combining: The CHF of two correlated Rician RVs is needed to study the binary phase-shift keying (BPSK) error performance of dual predetection ECG receivers, based on the Gil-Pelaez inversion theorem [11]. Similarly, the joint CHF can be used to find expressions for the CHF of the sum of correlated RVs to study the EGC error performance for several modulation schemes [10], [16]. The covariance of two correlated Rician RVs can be used to evaluate significant performance criteria of the EGC output SNR, such as the average output SNR, the amount of fading (AoF), the higher order moments, and the outage and error performance for a broad range of modulation schemes, using the moments-based approach [7].
- 3) Switched-and-Stay Combining: The infinite series for the joint CDF can be used to study the error performance of switched-and-stay diversity receivers, applying the well-known Beaulieu series approach [8].

IV. NUMERICAL RESULTS

In order to check the accuracy of the proposed mathematical analysis, the results obtained by the infinite series in (12) are compared with the corresponding ones from (7), where the involved integrals are evaluated numerically. Table I gives the results for six-significant-figure accuracy, assuming, without loss of generality, that h = i, with β normalized to one and for a range of values for x_1, x_2, K, u . We observe that an increase of K or x_1, x_2 leads to an increase of the required terms that need to be summed to obtain a target accuracy. Furthermore, comparing with the convergence of the infinite-series representation for the bivariate or multivariate Nakagami-m CDF [1], [5], where an increase of the correlation leads to an increase of the required terms needing to be summed; for the bivariate Rician case, only for high correlation values the required terms increase. Moreover, even for the highest correlation situation $(u \rightarrow 1)$, the required terms are much less than the corresponding ones for the Nakagami-m case.

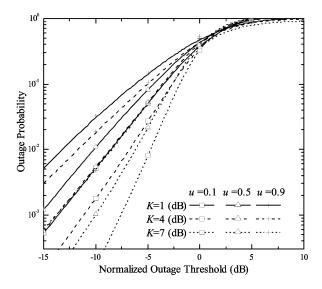


Fig. 1. Outage probability P_{out} versus the normalized outage threshold $\gamma_{th}/\overline{\gamma}$ for a dual SC receiver and for several values of K and u.

Using (19), the outage performance of a dual SC receiver operating in correlated Rician fading channels is plotted in Fig. 1. The obtained results show clearly that the outage performance degrades with an increase of the fading correlation and/or the fading severity.

V. CONCLUSIONS

We derived infinite-series representations for the PDF, the CDF, the covariance, and the CHF of two correlated i.d. Rician RVs. It is shown that the presented infinite-series expressions converge rapidly, and thus, they can efficiently be used to analytically study several performance criteria of dual-diversity receivers when correlated Rician fading is assumed. We investigated, as an indicative example, the outage probability of a dual SC receiver where the numerical results clearly depict the effect of the fading correlation and the fading severity on the receiver's performance.

APPENDIX

CONVERGENCE OF THE INFINITE-SERIES REPRESENTATIONS

After the truncation of the infinite series in (6) at the value i=I-1, the remainder R_I converges to zero as I increases, since the term $\mathcal A$ converges to zero for any combination of v_1 , v_2 , v_3 which satisfies $v_1+v_2+v_3\to\infty$. This can be proved as follows.

The term A can be upper bounded by

$$\mathcal{A} \leq \mathcal{A}' = \frac{2^{-v_1 - v_2 - 2} (1 + K)^{2 + v_1 + v_2 + v_3} K^{v_1 + v_2 + v_3}}{\beta^{2 + v_1 + v_2 + v_3} (1 - u^2) v_1! v_2! v_3! \Gamma(v_3 + 1)} \times (1 + u)^{-2v_1 - 2v_2 - 2v_3} \exp\left(-\frac{2K}{1 + u}\right)$$
(20)

and A' can be written as

$$\mathcal{A}' = \frac{2^{-v_1 - v_2 - 2} (1 + K)^{2 + v_1 + v_2} K^{v_1 + v_2} (1 + u)^{-2v_1 - 2v_2}}{\beta^{2 + v_1 + v_2} (1 - u^2) v_1! v_2!} \times \exp\left(-\frac{2K}{1 + u}\right) \left(\frac{K(K + 1)}{b(1 + u)^2}\right)^{v_3} \frac{1}{v_3!^2}.$$
 (21)

If $v_1=n, v_2=n, v_3\to\infty$ (similar if $v_1\to\infty, v_2=n, v_3=n,$ or $v_1=n, v_2\to\infty, v_3=n$), then

$$\lim_{v_3 \to \infty} \mathcal{A}' = \lim_{v_3 \to \infty} \left(\frac{K(K+1)}{b(1+u)^2} \right)^{v_3} \frac{1}{v_3!^2}.$$
 (22)

The limit in (22) has the form

$$\lim_{t \to \infty} \frac{Z^t}{t!} = 0 \tag{23}$$

with Z being real and t integer. Therefore, from (20)–(23), results

$$\lim_{v_3 \to \infty} \mathcal{A}' = 0 \Rightarrow \lim_{v_3 \to \infty} \mathcal{A} = 0 \Rightarrow \lim_{v_3 \to \infty} R_I = 0.$$
 (24)

The same analysis can be also applied for the infinite-series representations of the CDF, covariance, and CHF to prove their convergence.

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