

# MIMO Capacity Convergence in Frequency-Selective Channels

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**Abstract**—The dependence of multi-antenna capacity on bandwidth is characterized empirically for narrowband, wideband and ultrawideband indoor channels using spatial and polar arrays. It is shown that both the mean and the outage MIMO capacity increase with bandwidth, whilst the capacity coefficient of variation asymptotically vanishes.

**Index Terms**—Capacity, fading, multiple-input multiple-output (MIMO), ultrawideband (UWB).

## I. INTRODUCTION

IN a rich multipath environment, multiple-input multiple-output (MIMO) techniques can exploit the increased spatial degrees of freedom to substantially boost the information capacity or reliability of a wireless system [1]. Transmission using wideband and ultrawideband (UWB) signals on frequency-selective channels provides a similar effect in the frequency domain, increasing the link robustness and achievable rates [2]. Owing to these advantages, both wideband and MIMO systems have been studied extensively in recent years. The exploitation of MIMO techniques on wideband channels, with application in future gigabit wireless networks, has also attracted considerable attention [3]–[8]. Polarized arrays, which offer a compact alternative to spatial arrays with similar performance, are of particular interest for practical implementation [4], [9].

Given the significant impact of channel bandwidth on fading [10], some research has been undertaken on the influence of bandwidth and frequency-selectivity on MIMO diversity and spatial multiplexing statistics. Using an information theoretic treatment, Telatar and Tse showed that the fading channel capacity approaches the additive white Gaussian noise (AWGN) channel capacity in the wideband limit [8]. Molisch *et al.* used wideband measurements to show that the MIMO capacity distribution tightens in 100 MHz channels compared with narrowband channels [3]. Bölcskei *et al.* demonstrated that the wideband ergodic capacity depends on the channel delay-spread for the MIMO case, as does the capacity variance [5]. Other researchers have analyzed the capacity of Nakagmi-*m* channels, examining the impact of bandwidth with this generalized formulation. Thus Zheng and Kaiser showed that the ergodic capacity increases with the value of *m* [11]. Holter and Øien derived expressions for the amount of fading in correlated MIMO diversity systems [12], while Liu *et al.* presented the error probability expressions [13]. We note here

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that increasing the MIMO array size also leads to capacity convergence, as shown by Smith and Shafi for Gaussian channels [14].

In this letter, we extend our previous measurement-based UWB MIMO analysis [4] to study the impact of bandwidth on ergodic and outage capacity. Another contribution is the analysis of capacity variance of tri-polar MIMO arrays in addition to conventional spatial arrays. The capacity coefficient of variation is presented as a measure of the variation of capacity due to small-scale fading, and its dependence on channel bandwidth and MIMO array size is investigated.

## II. ANALYSIS METHODOLOGY

### A. Channel Model

Consider an  $N_t \times N_r$  MIMO wideband system for which the propagation channel is represented in the frequency domain by  $\mathbf{H} \in \mathcal{C}^{N_r \times N_t \times N_f}$ , where  $N_f$  is the number of discrete frequency components. If  $\mathbf{h}_f$ ,  $f = 1, \dots, N_f$ , is the channel transfer function between the  $t^{\text{th}}$  transmitting antenna and  $r^{\text{th}}$  receiving antenna, then  $\mathbf{H} = [\mathbf{h}_k]_{r,t}$ ,  $t = 1, \dots, N_t$ ,  $r = 1, \dots, N_r$ . We represent the frequency resolution of  $\mathbf{h}_f$  by  $\Delta f = W_{max}/(N_f - 1)$ , where  $W_{max}$  specifies the maximum bandwidth of the frequency-selective channel. In this analysis, MIMO channel power normalization is required to remove pathloss from  $\mathbf{H}$ . If the MIMO subchannels have equal average power, we can normalize the power to remove pathloss effects so that each subchannel has unit mean energy, as in [1], [3]. However, we adopt a more generalized approach that does not assume equi-power subchannels. The normalization factor in our treatment is the root energy of a reference subchannel, i.e.,

$$\eta = \sqrt{\frac{1}{N_f} \sum_{f=1}^{N_f} |h_{r_0, t_0, f}|^2}. \quad (1)$$

Thus  $\mathbf{H}$  is divided by  $\eta$  for pathloss normalization. Note that the two approaches are equivalent when the subchannels are balanced, but (1) is applicable to a wider class of MIMO architectures such as polarized antennas and cooperative multiuser (virtual) arrays. We designate the  $\mathbf{h}_{1,1}$  subchannel as reference, without loss of generality, so that  $r_0 = t_0 = 1$  in (1). In the polar MIMO system,  $\mathbf{h}_{1,1}$  corresponds to the vertical transmit and receive polarization. In this analysis, the center frequency is kept constant and the bandwidth is varied, similar to [10].

Given that  $\mathbf{H}$  is a random fading channel, its statistics can be analyzed over an ensemble,  $\mathcal{X}$ , of  $N_x$  individual realizations. We denote the  $x^{\text{th}}$  MIMO channel realization in  $\mathcal{X}$  by  $\mathbf{H}(x)$ .

### B. Capacity Evaluation

The mutual information capacity of a flat-fading  $N_t \times N_r$  MIMO channel,  $\mathbf{H}_f(x) = [h_f]_{r,t}$ , given perfect channel

information at the receiver, can be evaluated in bps/Hz as [1]

$$C_f(x) = \log_2 \det \left\{ \mathbf{I}_{N_r} + \frac{\rho}{N_t} \mathbf{H}_f^H(x) \mathbf{H}_f(x) \right\}, \quad (2)$$

where  $\rho$  is the average receive SNR and  $\mathbf{I}_{N_r}$  is an  $N_r \times N_r$  identity matrix. As channel information is usually not available at the transmitter, we will assume an isotropic Gaussian input for capacity calculation. We use the outage and ergodic channel capacity as the capacity measures in this letter [1]. Now, for a frequency-selective channel,

$$C_b(x) = \mathcal{E}_{f \in \mathcal{W}_b} \{C_f(x)\}, \quad (3)$$

where  $\mathcal{E}_{f \in \mathcal{W}_b}$  denotes the statistical average over the channel bandwidth, and  $\mathcal{W}_b$  is the set of  $N_b$  frequency components in the channel with bandwidth  $W_b = \Delta f(N_b - 1)$  such that  $1 \leq N_b \leq N_f$  and  $0 \leq W_b \leq W_{max}$ . Note that the capacity in (2) is also sometimes referred to as the spectral efficiency, and the maximum achievable rate is given by  $R_b = W_b C_b$ . In this treatment, we vary the channel bandwidth while keeping its center frequency constant at  $f_c$ . Of interest here is the relative variation in channel capacity with bandwidth. We characterize this behavior using the coefficient of variation, defined as [15]

$$CV_b = \sqrt{AF_b} = \frac{\sqrt{\mathcal{E}_{x \in \mathcal{X}} \{C_b^2(x)\} - \mathcal{E}_{x \in \mathcal{X}}^2 \{C_b(x)\}}}{\mathcal{E}_{x \in \mathcal{X}} \{C_b(x)\}}, \quad (4)$$

where  $AF_b$  is the amount of fading in the capacity of the wide-band channel with bandwidth  $W_b$ . Note that  $AF_b \rightarrow 0$  and  $AF_b \rightarrow 1$  signify nonfading AWGN channels and Rayleigh fading channels, respectively.

### C. Channel Measurements

The analysis in this letter is based on indoor MIMO channel measurements conducted in the FCC-allocated UWB band, i.e. 3.1–10.6 GHz [2]. Linear spatial arrays and orthogonal polar arrays are synthesized using up to three omnidirectional antennas at the transmitter and receiver. The frequency-domain complex amplitudes corresponding to the elements of  $\mathbf{H}(x)$  are measured. Thus we have  $N_t = 3$ ,  $N_r = 3$ ,  $N_f = 1601$ ,  $W_{max} = 7.5$  GHz,  $f_c = 6.85$  GHz, and  $N_x = 1800$ . Further details of the measurement procedure can be found in [4].

## III. RESULTS

We calculate the MIMO channel capacity,  $C_b(x)$ , for a range of  $W_b$  from the measurement data, and estimate the corresponding first-order statistics over  $\mathcal{X}$ . In this analysis, we assume  $\rho = 10$  dB and limit our attention to symmetric i.e.  $N \times N$  MIMO configurations. Fig. 1 shows the cumulative distribution function (CDF) of  $C_b$  for a range of narrowband, wideband and ultrawideband channels, obtained from the LOS spatial MIMO measurements. It is noticed that an increase in the channel bandwidth is accompanied by an increase in the  $q\%$  outage capacity, indicated by the capacity where the value of the CDF is  $q$ , where typical values of  $q$  are 1 or 10 [1]. With a SISO ( $1 \times 1$ ) system, the mean capacity is 2.2 bps/Hz. The 10% outage capacity of a narrowband system ( $W_b = 1$  Hz) is 0.6 bps/Hz, rising to 2 bps/Hz for a full-band UWB system ( $W_b = 7.5$  GHz), while at 1% outage, the capacity increases

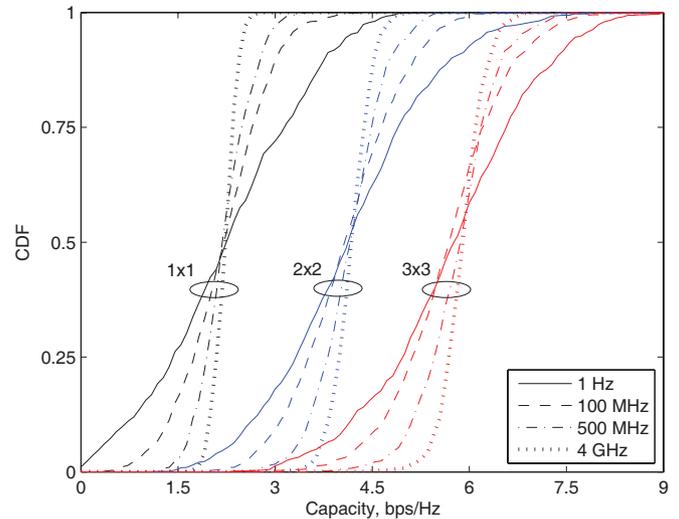


Fig. 1. CDFs of MIMO capacity with the specified bandwidth using spatial arrays in an indoor LOS channel.

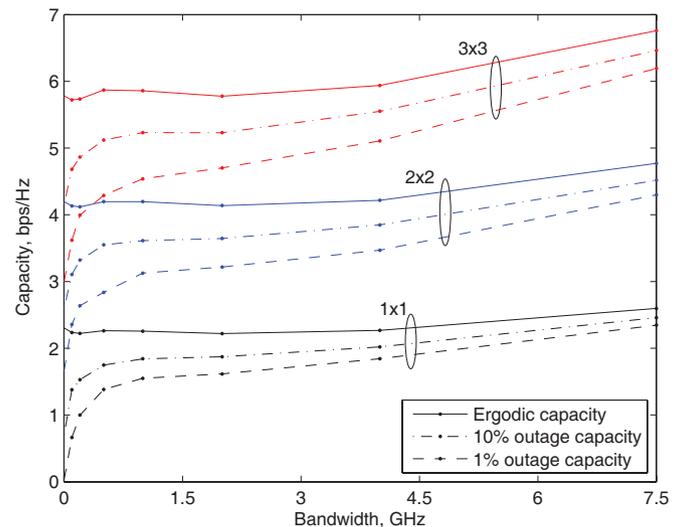


Fig. 2. The ergodic and 1% outage capacity of the ultrawideband MIMO channel in an LOS environment.

from 0 to 1.8 bps/Hz. Thus, as the bandwidth increases, the capacity distribution is more concentrated about the mean. Similar trends are observed in the  $N \times N$  MIMO systems. These results are in agreement with earlier theoretical studies [5] and measurement results comparing narrowband and wideband ( $W_b = 10$  and 100 MHz) channels [3], and extend the analysis to UWB channels. In the polar MIMO system, the capacity does not scale linearly with  $N$  due to unequal branch power ratios [4], but a reduction in the capacity variation as  $W_b \rightarrow W_{max}$  is still observed.

Fig. 2 quantifies this bandwidth dependence of the ergodic and outage capacities of  $N \times N$  spatial MIMO systems. A small increase in the ergodic capacity with bandwidth is in evidence. From the figure, as  $W_b$  increases from 1 Hz to 7.5 GHz in our measured LOS spatial MIMO channel, the ergodic capacity undergoes an increase of 11%, 15% and 18% with  $N = 1, 2, 3$ , respectively. Such behavior was suggested on a theoretical basis for frequency-selective MIMO channels in [5], [11] and was observed empirically in [3]. The outage

capacity, also shown in Fig. 2, undergoes a much more substantial, monotonic increase with bandwidth, approaching the ergodic capacity in the large bandwidth asymptote. Unlike the ergodic capacity, the outage capacity growth with bandwidth is logarithmic, saturating at large  $W_b$ . The capacity convergence with  $W_b$  also appears to depend on the array size, and becomes slower as  $N$  increases. Thus the 10% outage capacity increases by a factor of 3.5, 1.7 and 1.5 for the  $1 \times 1$ ,  $2 \times 2$  and  $3 \times 3$  channels, respectively, when  $W_b$  increases from narrowband to 7.5 GHz UWB. Thus we infer that whilst both ergodic and outage capacity increase with bandwidth, their difference at large  $W_b$  grows with  $N$ . This observation holds true in both spatial and polar arrays, and in LOS and NLOS scenarios. The polar array ergodic capacity, but not outage capacity, increases under LOS availability at UWB bandwidth. This difference does not exist to an appreciable extent with spatial arrays.

We now characterize the variance of capacity in terms of the coefficient of variation in (4), and the results are shown in Fig. 3. From the comparison of LOS and NLOS capacity, the latter has lower  $CV_b \forall W_b$ , and the difference is more pronounced with polar MIMO. We observe an exponential decay in  $CV_b$  with  $W_b$ , which is steeper when  $N$  is large. In general, we observe from Fig. 3 that a given value of  $CV_b$  can be achieved by increasing either  $W_b$  or  $N$ .

With the spatial array, as  $W_b$  increases from 500 MHz to 7.5 GHz, the  $CV_b$  decreases only by about 10%. A spatial MIMO system with small  $N$  has higher  $CV_b$  at small  $W_b$ , and therefore is less stable, but the dependence on  $N$  vanishes as  $W_b \rightarrow \infty$ , or alternatively, the dependence on  $W_b$  vanishes as  $N \rightarrow \infty$ . The  $CV_b$  of capacity for a narrowband SISO channel is 50%, while for  $2 \times 2$  and  $3 \times 3$  spatial systems, it is 30% and 20%, respectively. At  $W_b = 7.5$  GHz, it falls to 4%, signifying the dramatically reduced variability of MIMO capacity in the UWB channel. The reason is that the ergodic capacity increases only slightly with  $W_b$ , while the standard deviation decreases significantly, leading to a smaller  $CV_b$ .

Some differences are observed for polar arrays, as illustrated by Fig. 3(b). At large  $W_b$ , the MIMO  $CV_b$  is larger than that obtained with SISO. The  $CV_b$  also does not vary monotonically with  $N$  due to the effect of unequal branch powers and unequal dependence of energy capture on scattering [9]. Detailed analysis reveals that the polar MIMO ergodic capacity (with  $N = 2, 3$ ) is higher in LOS than in NLOS, but the difference in the LOS and NLOS variance is comparatively larger, leading to the observed behavior of  $CV_b$ . The capacity, however, converges rapidly with  $W_b$  when  $N = 3$ , similar to spatial arrays. Thus for  $3 \times 3$  polar MIMO, at  $W_b = 500$  MHz the  $CV_b$  reaches within 3% of its value at  $W_b = 7.5$  GHz.

Physically, the channel capacity is a monotonically increasing function of SNR, and therefore of the channel energy under constant noise variance. The variation of UWB channel energy due to small-scale fading is considerably lower than in narrowband channels [10], and capacity mimics this behavior.

The analysis in this letter has provided results useful for the design of wideband MIMO systems. By considering the tradeoffs between the achievable capacity, antenna array size and channel bandwidth, as discussed in this letter, the required level of robustness and stability can be achieved while minimizing the system complexity and resource usage.

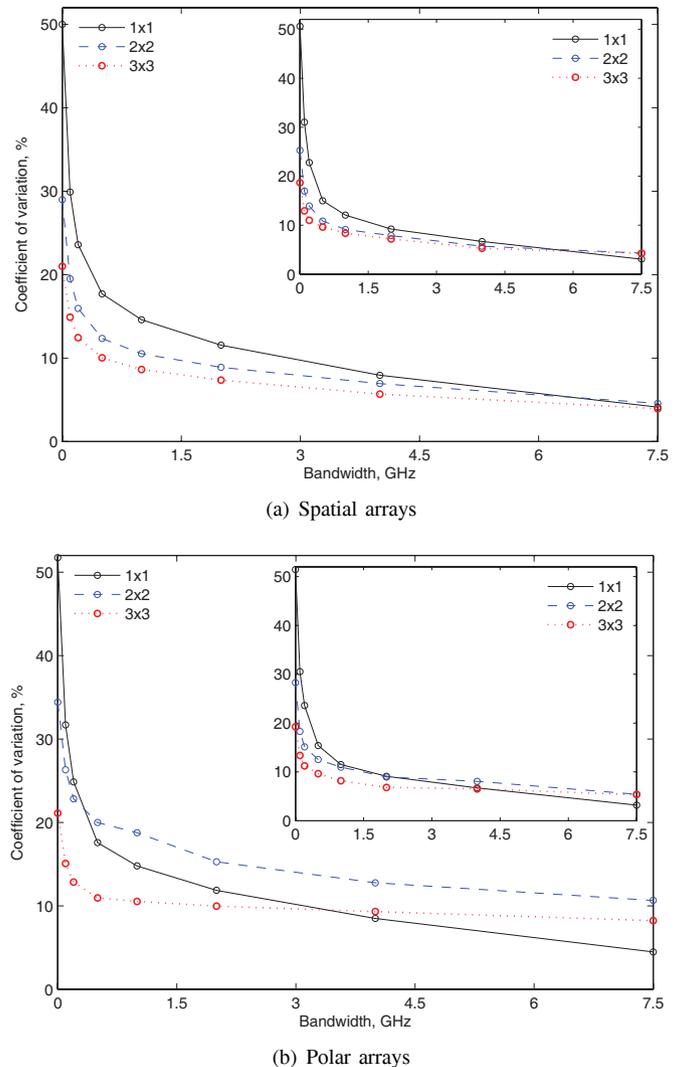


Fig. 3. The coefficient of variation of MIMO capacity estimated from measurements in the LOS channel. The insets show the NLOS results.

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