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The Capacity of Wireless Ad Hoc Networks With Multi-Packet Reception

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Abstract

We compute the throughput capacity of random dense wireless ad hoc networks for multi-pair unicast traffic in which nodes are endowed with multi-packet reception (MPR) capabilities. We show that $\Theta\left(\frac{(R(n))^{(1-\frac{2}{\alpha})}}{n^{1/\alpha}}\right)$ and $\Theta\left(R(n)\right)$ bits per second constitute tight bounds for the throughput capacity under the physical and protocol model assumptions, respectively, where n is the total number of nodes in the network, $\alpha > 2$ is the path-loss parameter in the physcial model, and $R(n)$ is the MPR receiver range. In so doing, we close the gap between the lower and upper bounds of throughput capacity in the physical model. Compared to the capacity of point-to-point communication reported by Gupta and Kumar [1], MPR increases the order capacity of random wireless ad hoc networks under both protocol and physical models by at least $\Theta(\log n)$ and $\Theta\left((\log n)^{\frac{\alpha-2}{2\alpha}}\right)$, respectively. We address the cost incurred in increasing the transport capacity of wireless ad hoc networks over what can be attained when sources and destinations communicate over multi-hop paths under the physical model assumption. We define the *energy efficiency* $\eta(n)$ as the bit-meters of information transferred in the network for each unit energy, and compute such energy efficiency for different techniques. We show that a lower energy efficiency is attained in order to achieve higher transport capacity.

I. Introduction

The work by Gupta and Kumar [1] demonstrated that wireless ad hoc networks do not scale well for the case of multi-pair unicasts when nodes are able to encode and decode at most one packet at a time. This has motivated the study of different approaches to "embrace interference" in order to increase the

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capacity of wireless ad hoc networks. Embracing interference consists of increasing the concurrency with which the channel is accessed.

One approach to embracing interference consists of allowing a receiver node to decode correctly multiple packets transmitted concurrently from different nodes, which we call multi-packet reception (MPR) [2]. In practice, MPR can be achieved with a variety of techniques, including multiuser detection (MUD) [3], directional antennas [4], [5] or multiple input multiple output (MIMO) techniques. A complementary approach to embracing interference consists of increasing the amount of information sent per transmission. Network coding (NC) [6] was introduced and shown to achieve the optimal capacity for single-source multicast in directed graphs corresponding to wired networks in which nodes are connected by point-to-point links. Since then, many attempts have been made to apply NC to wireless ad hoc networks, and Liu et al. [7] have shown that NC cannot increase the order capacity of wireless ad hoc networks for multi-pair unicast traffic. However, recent work [8]–[12] has shown promising results on the advantage of NC in wireless ad hoc networks subject to multicast traffic. An interesting aspect of these works is that nodes are also assumed to have multi-packet transmission (MPT) and MPR capabilities in addition to using NC for multicasting. Recently, Katti et al. [8] and Zhang et al. [9] proposed analog network coding (ANC) and physical-layer network coding (PNC) respectively, as ways to embrace interference. Interestingly, a careful review of ANC and PNC reveals that they consist of the integration of NC with a form of MPR, in that receivers must be allowed to decode successfully concurrent transmissions from multiple senders by taking advantage of the modulation scheme used at the physical layer (e.g., MSK modulation in ANC [8]). Furthermore, the prior work, which we summarize in Section II, has not addressed the contribution that MPR can make on the scaling laws of wireless ad hoc networks.

This paper focuses on multi-pair unicast traffic in wireless ad hoc networks when nodes are endowed with MPR. Section III describes the network model we use to obtain upper and lower bounds on the throughput capacity of wireless networks with MPR. Section IV presents the derivation of these bounds, which constitutes the first contribution of this paper. We show that $\Theta\left(R(n)\right)$ and $\Theta\left(\frac{(R(n))^{(1-2/\alpha)}}{n^{1/\alpha}}\right)$ bits per second constitute tight bounds for the throughput capacity per node in random wireless ad hoc networks for protocol and physical models respectively, where $R(n)$ is the MPR receiver range and α is the channel path loss parameter. We have been able to close the gap in the physcial model and achieve higher capacity than the bound in [1] and [13]. When $R(n) = \Theta\left(\sqrt{\frac{\log n}{n}}\right)$, the throughput capacity is tight bounded by

$\Theta\left(\sqrt{\frac{\log n}{n}}\right)$ and $\Theta\left(\frac{(\log n)^{\frac{1}{2}-\frac{1}{\alpha}}}{\sqrt{n}}\right)$ for protocol and physical models respectively. This is a gain of $\Theta\left(\log n\right)$ and $\Theta\left((\log n)^{\frac{\alpha-2}{2\alpha}}\right)$ compared to the bound in [1]. The assumptions we use to obtain these results are similar to those made by Gupta and Kumar [1], except that each node is equipped with MPR.

Several schemes have been proposed in the recent past [1], [13], [14] that achieve different capacities for multi-pair unicast under the physical model. Intuitively, there must be a price paid in any scheme aimed at increasing the capacity of wireless networks, including MPR of course. This price is the energy required to transport information across the network. Section V presents our second contribution, which is to compare all these existing techniques in terms of energy efficiency. To do so, we introduce a new parameter to quantify how many bit-meters of information are transferred across the network per each unit energy. We call this metric *energy efficiency*, computed by normalizing the transport capacity by the total transmitted power and measured in units of bit-meters per joule. We compute the energy efficiency of many existing techniques [1], [13], [14] and compare them to the energy efficiency of using MPR. We show that MPR provides a tradeoff between throughput capacity, node decoding complexity, and energy efficiency in random wireless ad hoc networks. We also show that achieving higher throughput capacity leads to a lower energy efficiency in all techniques, including MPR. Note that the transport capacity for random wireless ad hoc networks is defined in bits per second, while for arbitrary wireless ad hoc networks, the transport capacity is measured in bit-meters per second [1]. The focus of this paper is only on random wireless ad hoc networks. Section VI discusses several possible implications of this study.

II. Related Work

Since the landmark work by Gupta and Kumar [1] on the scalability of wireless networks, considerable attention has been devoted to improving or analyzing their results, and we only mention a very small fraction of these works due to space limitations. Grossglauser and Tse [15] demonstrated that a non-vanishing capacity can be attained at the price of long delivery latencies by taking advantage of long-term storage in mobile nodes. The throughput capacity can also be increased by using multiple channels [16] or sender-receiver cooperation [17]. Recently, Ozgur et al. [14] demonstrated that the capacity of random wireless ad hoc network scales linearly with n by allowing nodes to cooperate intelligently using distributed MIMO communications.

Under the physical model assumption, Gupta and Kumar [1] showed that the throughput capacity of random wireless ad hoc networks has lower and upper bounds of $\Theta(\sqrt{1/n \log n})$ and $\Theta(\sqrt{1/n})$, respectively. Franceschetti et al. [13] closed the gap between these two bounds and obtained a tight bound of $\Theta(\sqrt{1/n})$ using percolation theory. In this approach, all communications are simple point-to-point without any cooperation between senders and receivers.

We note that previous work [18] has suggested the concept of bits per joule capacity to evaluate how much information can be transmitted with each unit energy. Our definition of energy efficiency is an extension of this prior work for transport capacity in wireless ad hoc networks.

III. Network Model

We consider a dense wireless ad hoc network with n nodes distributed uniformly in a square of unit area. Hence, in our model, as n goes to infinity, the density of the network also goes to infinity. Our capacity analysis is based on the extension of protocol and physical models for dense networks introduced by Gupta and Kumar [1].

According to the Gupta-Kumar protocol model for point-to-point communications, a common transmission range $R(n)$ for all nodes is defined. Node X_i can successfully transmit to node X_j if for any node $X_k, k \neq i$, that transmits at the same time as X_i, then $|X_i - X_j| \leq R(n)$ and $|X_k - X_j| \geq (1 + \Delta)R(n)$, where X_i, X_j and X_k are the cartesian position in the unit square network for these nodes. We need to define the protocol model for MPR.

Definition 3.1: Protocol Model with Multi-packet Reception: In wireless ad hoc networks with MPR, the protocol model assumption allows MPR capability at nodes as long as they are within a radius of $R(n)$ from the receiver and all other transmitting nodes have a distance larger than $(1 + \Delta)R(n)$. The difference is that we allow the receiver node to receive multiple packets from different nodes within its disk of radius $R(n)$ simultaneously in MPR scheme.

Note that the communication range in Gupta and Kumar's model is a random variable, while $R(n)$ in MPR is a predefined value that depends on the complexity of the nodes. We assume that nodes cannot transmit and receive at the same time, which is equivalent to half duplex communication [1]. The data rate for each transmitter-receiver pair is a constant value of W bits/second and does not depend on n. Given that W does not change the order capacity of the network, we normalize its value to one. The MPR protocol model is shown in Fig. 1. Note that in the protocol model, the communication range $R(n)$

has a minimum value to gaurantee the connectivity in the network, i.e., $R(n) \geq \Theta\left(\sqrt{\frac{\log n}{n}}\right)$.

In the physical model of dense random wireless ad hoc networks in [1], a successful communication occurs if signal to interference and noise ratio (*SINR*) of the pair of transmitter i and receiver j satisfies

$$\textit{SINR}_{i\rightarrow j} = \frac{Pg_{ij}}{BN_0 + \sum_{k\neq i,k=1}^{n} Pg_{kj}} \geq \beta, \tag{1}$$

where P is the transmit power of a node, g_{ij} is the channel attenuation factor between nodes i and j, and BN_0 is the total noise power. The channel attenuation factors g_{ij} and g_{kj} are only functions of the distance under the simple path loss propagation model, i.e., $g_{ij} = |X_i - X_j|^{-\alpha}$.

However, in the physical model definition for MPR, each receiving node has a receiver range such that all the nodes transmitting within this range will be decoded by the receiver. Consequently, the definition of physical model should incorporate this fact in order to better represent this new many-to-one communication scheme. The following proposition states the decoding procedure for MPR. Note that, with MPR, we can either decode the received signal for multiple transmitters jointly using maximum likelihood decoding or decode transmitters sequentially as long as the *SINR* condition is satisfied. Definition 3.3 below describes the condition that satisfies the minimum required *SINR*.

Proposition 3.2: The transmitter-receiver pair with maximum *SINR* is the nearest set of transmitters, after decoding and subtracting this group from the received signal, the set with the next highest *SINR* is the second nearest group of transmitters, and this continues; i.e., receivers decode the information from the nearest transmitters to farthest ones whose positions are the maximum distance inside of communication range.

Because the channel propagation model is based on the path-loss parameter, it is clear from (1) that the node (or group of nodes) with the closest distance to the receiver has the highest *SINR*. After decoding this (their) packet(s) and subtracting it (them) from the received data, it is obvious that the next packet(s) with highest *SINR* is (are) from the second closest node(s) to the receiver node and this procedure can continue. At a given time t, the decoding procedure for any receiver j in the MPR scheme is sequential, i.e., a receiver decodes the information from the highest *SINR* to the lowest *SINR* for MPR using SIC.

Essentially, this proposition states that each group of transmissions from some transmitters can be decoded if and only if the previous group of transmissions from transmitters that are closer to the receiver node was decoded first by the receiver node. The last decoded node occurs at the edge of the circle whose radius is $R(n)$.

Definition 3.3: Physical Model with Multi-packet Reception: In the physical model of dense random wireless ad hoc networks [1], the transmissions from all of the transmitters centered around a receiver j with a distance smaller or equal to $R(n)$ occur successfully if the *SINR* of the transmitter $Z(R(n))$ at the edge of this receiver circle satisfies

$$\mathit{SINR}_{Z(R(n))\to j} = \frac{Pg_{Z(R(n))j}}{BN_0 + \sum_{k\notin A_{Z(R(n))}} Pg_{kj}} \geq \beta, \tag{2}$$

where $g_{Z(R(n))j}$ is the channel attenuation factor between nodes $Z(R(n))$ and j and BN_0 is the total noise power. $A_{Z(R(n))} = \pi R^2(n)$ is the receiver communication range (circle) centered around the receiver j.

Any transmission outside the receiver range is considered interference while all the transmissions inside receiver range will be decoded jointly or separately. For this reason, we denote the interference inside area $A_{Z(R(n))}$ as constructive interference, because it consists of transmissions that will be eventually decoded, while all the transmissions from nodes outside of area A are called destructive interference and are not decoded. Note that in the physical model for the MPR scheme, the receiver range $R(n)$ defines the area where the receiver is capable of decoding, which contrasts with point-to-point communication [1], for which the transmission range $r(n)$ defines the possible area where the receiver can decode, given that only one transmission is successful at a receiver. Given that any transmitter that is closer to the receiver has a smaller channel attenuation compared to the edges of the circle, it is easy to show that the *SINR* of these transmitter nodes are always greater than the value in (2) if these nodes are decoded jointly or separately depending on the distribution of these nodes around the receiver node j.

Definition 3.4: Feasible throughput capacity of unicast: A throughput of $\lambda(n)$ bits per second for each node is feasible if we can define a scheduling transmission scheme that allows each node in the network to transmit $\lambda(n)$ bits per second on average to its destination.

Definition 3.5: Order of throughput capacity: $\lambda(n)$ is said to be of order $\Theta(f(n))$ bits per second if there exist deterministic positive constants c and c' such that

$$\begin{cases} \lim_{n\to\infty} \text{Prob } (\lambda(n) = cf(n) \text{ is feasible}) = 1 \\ \lim_{n\to\infty} \text{Prob } (\lambda(n) = c'f(n) \text{ is feasible}) < 1. \end{cases} \tag{3}$$

The distribution of nodes in random networks is uniform. Therefore, if there are n nodes in a unit square, then the density of nodes equals n. Hence, if $|S|$ denotes the area of space region S, the expected number of the nodes, $E(N_S)$, in this area is given by $E(N_S) = n|S|$. Let N_j be a random variable defining the number of nodes in S_j . Then, for the family of variables N_j, we have the following standard results known as the Chernoff bound [19]:

Lemma 3.6: Chernoff bound: For any $0 < \delta < 1$, we have

$$P[|N_j - n|S_j|| > \delta n|S_j|] < e^{-\theta n|S_j|}, \tag{4}$$

where θ is a function of δ.

Therefore, for any $\theta > 0$, there exist constants such that deviations from the mean by more than these constants occur with probability approaching zero as $n \to \infty$. An event occurs with high probability (w.h.p.) if its probability tends to one as $n \to \infty$. It follows that, w.h.p., we can get a very sharp concentration on the number of nodes in an area, so we can find the achievable lower bound w.h.p., provided that the upper bound (mean) is given.

IV. Capacity with MPR

We compute the capacity of wireless ad hoc networks for both protocol and physical models. To accomplish this, we first present some definitions and preliminary results from our earlier work [20].

The per-node throughput capacity of the network is defined as the number of bits per second that every node can transmit w.h.p. to its destination. Note that throughput capacity is equivalent to transport capacity in this paper. Transport capacity is defined in units of bits per second in random networks and bit-meters per second in arbitrary networks as originally described in [1]. In random networks, source-destination distance is on average the same order for all pairs and therefore, the distance is simply a normalization factor. Since this paper only discusses random networks, we use bits per second unit for transport capacity consistent with the definition in [1].

A cut Γ is a partition of the vertices (i.e. nodes in the wireless networks) of a graph into two sets. The cut capacity is defined to be the sum of bandwidth of all the edges crossing the cut that can transmit simultaneously. Min-cut is a cut whose capacity is the minimum value among the capacity of all cuts. For the wireless networks, we use the concept of *sparsity cut*, as defined by Liu et al. [7], instead of min-cut, to take into account the differences between wired and wireless links. The cut length l_Γ is defined as the length of the cut line segment. For the square region illustrated in Fig. 2, the middle line induces a sparsity cut Γ. Because nodes are uniformly deployed in a random network, such a sparsity cut captures the traffic bottleneck of these random networks on average [7]. The sparsity-cut capacity is upper bounded by deriving the maximum number of simultaneous transmissions across the cut.

Let $R(n)$ be the radius of the receiver range A, i.e., $A = \pi R^2(n)$. Given that we assume omni-antenna broadcasting, this is the radius that distinguishes the decode-able transmitter nodes from the interference.

We assume that each disk with radius $R(n)$ centered at any receiver is disjoint from the other disks centered at the other receivers. It will be shown later that this assumption is necessary in order to guarantee that the physical model condition, $SINR \geq \beta$, is satisfied.

A. Upper Bound for Protocol Model

We first derive the sparsity cut for a random wireless ad hoc netework under the protocol model.

Lemma 4.1: The asymptotic throughput capacity of a sparsity cut Γ for a unit square region has an upper bound of $c_1 l_\Gamma n R(n)$, where, $c_1 = \pi/2(2+\Delta)$.

Proof: The cut capacity is upper bounded by the maximum number of simultaneous transmissions across the cut. We observe from Fig. 2 that all the nodes located in the shaded area S_{xy} can send their packets to the receiver node located at (x, y). These nodes lie in the left side of the cut Γ within an area called S_{xy} and the assumption is that all these nodes are sending packets to the right side of the cut Γ.

For a node at location (x, y), any node in the disk of radius $R(n)$ can transmit information to this receiver simultaneously and the node can successfully decode those packets. In order to obtain an upper bound, we only need to consider edges that cross the cut. Let us first consider all possible nodes in the S_{xy} region that can transmit to the receiver node. Because nodes are uniformly distributed, the average number of transmitters located in S_{xy} is $n \times S_{xy}$. The number of nodes that are able to transmit at the same time from left to right is upper bounded as a function of S_{xy}.

The area of S_{xy} is

$$S_{xy} \quad = \quad \frac{1}{2}R^2(n)(\theta - \sin\theta). \tag{5}$$

This area is maximized when $\theta = \pi$,

$$\max_{0\leq\theta\leq\pi}[S_{xy}] = \frac{1}{2}\pi R^2(n). \tag{6}$$

The total number of nodes that can send packets across the cut is upper bounded as

$$\frac{l_\Gamma}{(2+\Delta)R(n)}\frac{1}{2}\pi R^2(n)n = c_1 l_\Gamma n R(n), \tag{7}$$

where $c_1 = \pi/2(2+\Delta)$. ■

Corollary 4.2: For any arbitrary shape unit area random network, if the minimum cut length l_Γ is not a function of n, then the sparsity cut capacity has an upper bound of $\Theta(nR(n))$.

Proof: Regardless of the shape of the unit area region, there exists a sparsity cut for each orientation of the cut line. This sparsity cut capacity depends only on the minimum cut length l_Γ. If l_Γ is not a function of n, then the capacity is always upper bounded as $\Theta(nR(n))$. ■

Theorem 4.3: The per source-destination throughput of *MPR* scheme in a 2-D random network is upper bounded by $\Theta(R(n))$.

Proof: For a sparsity cut Γ in the middle, on average, we have that w.h.p. there are $\Theta(n)$ pairs of source-destination nodes that need to cross Γ in one direction, i.e., $n_{\Gamma_{1,2}} = n_{\Gamma_{2,1}} = \Theta(n)$ w.h.p.. Combining this result with Corollary 4.2, we can easily prove this theorem. ■

B. Lower Bound for Protocol Model

We now prove that, when n nodes are distributed uniformly over a unit square area, we have simultaneously at least $\frac{l_\Gamma}{(2+\Delta)R(n)}$ circular regions in Fig. 2, each one contains $\Theta(nR^2(n))$ nodes w.h.p. The objective is to find the achievable lower bound using Chernoff bound such that the distribution of the number of edges across the cut is sharply concentrated around its mean, and hence in a randomly chosen network, the actual number of edges crossing the sparsity cut is indeed $\Theta(nR(n))$ w.h.p..

Theorem 4.4: Each A_j with circular shape contains $\Theta(nR^2(n))$ nodes w.h.p. and uniformly for all values of $j, 1 \le j \le \frac{l_\Gamma}{(2+\Delta)R(n)}$.

This theorem can be expressed as

$$\lim_{n\to\infty} P\left[\bigcap_{j=1}^{l_\Gamma/(2+\Delta)R(n)} |N_j - E(N_j)| < \delta E(N_j)\right] = 1, \tag{8}$$

where δ is a positive small value arbitrarily close to zero.

Proof: From Chernoff bound lemma 3.6 and (4), for any given $0 < \delta < 1$, we can find $\theta > 0$ such that

$$P\left[|N_j - E(N_j)| > \delta E(N_j)\right] < e^{-\theta E(N_j)} = e^{-\theta n|A_j|} \tag{9}$$

Thus, we can conclude that the probability that the value of the random variable N_j deviates by an arbitrarily small constant value from the mean tends to zero as $n \to \infty$. This is a key step in showing that when all the events $\bigcap_{j=1}^{l_\Gamma/(2+\Delta)R(n)} |N_j - E(N_j)| < \delta E(N_j)$ occur simultaneously, then all N_j's converge uniformly to their expected values. Utilizing the union bound, we arrive at

$$P\left[\bigcap_{j=1}^{l_\Gamma/(2+\Delta)R(n)} |N_j - E(N_j)| < \delta E(N_j)\right] = 1 - P\left[\bigcup_{j=1}^{l_\Gamma/(2+\Delta)R(n)} |N_j - E(N_j)| > \delta E(N_j)\right]$$
$$\geq 1 - \sum_{j=1}^{l_\Gamma/(2+\Delta)R(n)} P\left[|N_j - E(N_j)| > \delta E(N_j)\right] > 1 - \frac{l_\Gamma}{(2+\Delta)R(n)} e^{-\theta E(N_j)} = 1 - \frac{l_\Gamma}{(2+\Delta)R(n)} e^{-\frac{\theta\pi n R^2(n)}{2}} \quad (10)$$

The last term is derived from the fact that $E(N_j) = \frac{\pi}{2} n R^2(n)$. In order to guarantee connectivity, we need $R(n) \geq \Theta(\sqrt{\frac{\log n}{n}})$ [1]. Therefore as $n \to \infty$, then $\frac{e^{-\frac{\theta\pi n R^2(n)}{2}}}{R(n)} \to 0$. ■

This theorem demonstrates that w.h.p., we can achieve the lower bound.

Corollary 4.5: The per source-destination throughput of *MPR* scheme for a 2-D random network has a lower bound of $\Theta(R(n))$ w.h.p..

Proof: It is proved in Theorem 4.4, there are $\frac{l_\Gamma}{(2+\Delta)R(n)}$ different circles of radius $R(n)$ each of them having $\Theta(nR^2(n))$ nodes w.h.p. Therefore, per source-destination is the multiplications of these two values divided by the total number of nodes which proves the corollary. ■

C. Upper Bound for Physical Model

Lemma 4.6: The asymptotic throughput capacity of a sparsity cut Γ for a unit square region has an upper bound of $\frac{\pi l_\Gamma n}{2} \frac{R^2(n)}{D(n)}$, where, $R(n)$ and $D(n)$ are receiver range and division range of MPR respectively as illustrated in Fig. 3.

Proof: The cut capacity is upper bounded by the maximum number of simultaneous transmissions across the cut. Based on the results from section IV-A and the total number of nodes in each area S_{xy}, we can compute the total information capacity C_j for one receiver j at the right side of the cut as

$$C_j = \frac{1}{2}\pi n R^2(n). \quad (11)$$

The constraint to guarantee that Eq. (11) is true for all of the nodes inside the circle of radius $R(n)$, is to satisfy *SINR*$_{i \in S_{xy}} \geq \beta$. For this reason, the circles whose nodes are transmitting concurrently must be away from each other far enough ($D(n) \geq 2R(n)$) as shown in Fig. 3. Therefore, the total throughput capacity $C(n)$ across the sparsity cut is

$$C(n) \leq \left(\left\lfloor \frac{l_\Gamma}{D(n)} \right\rfloor + 1 \right) C_j < \frac{\pi n R^2(n)(l_\Gamma + D(n))}{2D(n)}. \tag{12}$$

Note that $D(n)$ and $R(n)$ are decreasing functions of n, and $\lim\left(l_\Gamma + D(n)\right) = l_\Gamma$ asymptotically because $\lim D(n) = 0$ as $n \to \infty$. This proves the lemma. ■

Lemma 4.7: The per source-destination throughput of MPR scheme in a 2-D random network is upper bounded by $O\left(\frac{R^2(n)}{D(n)}\right)$.

Proof: From lemma 4.6, there are $l_\Gamma/D(n)$ different circles of radius $R(n)$ each of them having $\Theta(nR^2(n))$ nodes on average. Therefore, the average per node throughput capacity can be derived as

$$\lambda(n) = \frac{C(n)}{n} = O\left(\frac{R^2(n)}{D(n)}\right). \tag{13}$$

■

To derive an upper bound for the throughput capacity, we need to obtain a minimum $D(n)$, such that it guarantees $\textit{SINR}_{Z(R(n))} \geq \beta$. From Proposition 3.2, the decoding sequence of transmissions is from nearest nodes to farthest nodes, i.e., the information of the next transmitter in the communication range can be decoded if and only if the previous one is decoded successfully and then it is subtracted from the received data. So if the *SINR* of the outmost node can be decoded, then all of the nodes inside that circle can be decoded separately or at least jointly. Based on this assumption, we only need to compute the *SINR* of the farthest nodes $Z(R(n))$ (i.e., at the conjunction edge of the communication circle) to make sure $\textit{SINR}_{Z(R(n))} \geq \beta$. Therefore, the upper bound capacity exists if maximizing this capacity is equivalent of maximizing the following function.

$$\max_{\textit{SINR}_{Z(R(n))} \geq \beta} \lambda(n) = \max_{\textit{SINR}_{Z(R(n))} \geq \beta} \frac{R^2(n)}{D(n)} \tag{14}$$

Note that the throughput capacity is maximized by minimizing $D(n)$, while if this value is too small, then Eq. (2) will not be satisfied. Our aim is to find the optimum value for $D(n)$ such that both conditions are satisfied. The following theorem establishes the optimum value that will satisfy Eq. (2).

Theorem 4.8: The per source-destination throughput of MPR scheme in a 2-D random network is upper bounded by $O\left(\frac{(R(n))^{(1-2/\alpha)}}{n^{1/\alpha}}\right)$.

Proof: In order to compute the upper bound, we derive the *SINR* for the node that is in a circle close to the edge of the network. For this receiver node, the Euclidean distances of interfering nodes are at $(iD(n)+R(n))$ if we assume all interfering nodes are at the farthest distance from the receiver node. Then the *SINR* of the transmitter node that is located at the circumference of the communication circle is given by

$$\textit{SINR}_{Z(R(n))} \leq \frac{P/R^{\alpha}(n)}{\frac{\pi}{2}nR^2(n)\sum_{i=1}^{l_\Gamma/D(n)}\frac{P}{(iD(n)+R(n))^{\alpha}}} \leq \left(\frac{D(n)}{R(n)}\right)^{\alpha}\frac{1}{\frac{\pi}{2}nR^2(n)\sum_{i=1}^{l_\Gamma/D(n)}\frac{1}{(i+\frac{1}{2})^{\alpha}}}.$$

The second inequality above stems from the fact that $\frac{R(n)}{D(n)} \leq \frac{1}{2}$. Note that $l_\Gamma/D(n)$ approaches infinity when $n \to \infty$; therefore, the summation $\sum_{i=1}^{l_\Gamma/D(n)}\frac{1}{(i+\frac{1}{2})^{\alpha}}$ converges to a bounded value. This means that there are constant values c_3 and c_4 such that

$$c_3 \leq \sum_{i=1}^{l_\Gamma/D(n)}\frac{1}{(i+\frac{1}{2})^{\alpha}} \leq \sum_{i=1}^{l_\Gamma/D(n)}\frac{1}{(i)^{\alpha}} \leq c_4. \tag{15}$$

Combining (15) and (1), the *SINR* constraint can be revised as

$$\beta \leq \textit{SINR}_{Z(R(n))} \leq \left(\frac{D(n)}{R(n)}\right)^{\alpha}\frac{2}{\pi c_3 nR^2(n)}. \tag{16}$$

Then the relationship between $R(n)$ and $D(n)$ can be expressed as

$$D(n) \geq \left(\frac{c_3\beta\pi}{2}\right)^{\frac{1}{\alpha}} n^{\frac{1}{\alpha}}(R(n))^{(1+2/\alpha)}. \tag{17}$$

From Eqs. (13) and (17), the upper bound of the throughput capacity is computed as

$$\lambda(n) = O\left(\frac{R^2(n)}{D(n)}\right) = O\left(\frac{(R(n))^{(1-2/\alpha)}}{n^{1/\alpha}}\right). \tag{18}$$

■

The above upper bound is derived based on the assumption that the *SINR* for the nodes that are located on the circumference of communication circle A of radius $R(n)$ satisfy the physical model, i.e., $\textit{SINR}_{Z(R(n))} \geq \beta$. Note that successive decoding of nodes in A starts with nodes with the highest *SINR* or equivalently with the closest distance to the receiver j. Let's define $U_i \in A$ as a subset of the set A that contains a group of nodes in the communication circle with the closest distance to the receiver j that will be decoded jointly in the next step. Then it is easy to show that $\textit{SINR}_{U_i} \geq \textit{SINR}_{Z(R(n))} \geq \beta$. We will show that this upper bound capacity is also an achievable lower bound.

D. Lower Bound for Physical Model

Given the upper bound derived in the previous section, the Chernoff Bound is used to prove the achievable lower bound w.h.p.. We prove that, when n nodes are distributed uniformly over a square area, we have simultaneously at least $\frac{l_\Gamma}{D(n)}$ circular regions (see fig. 2), each one containing $\Theta(nR^2(n))$ nodes w.h.p.. The objective is to find the achievable lower bound using the Chernoff bound, such that the distribution of the number of edges across the cut is sharply concentrated around its mean, and hence in a randomly chosen network, the actual number of edges crossing the sparsity cut is indeed $\Theta\left(\frac{(R(n))^{(1-2/\alpha)}}{n^{(1/\alpha)}}\right)$ w.h.p..

Theorem 4.9: Each area A_j with circular shape of radius $R(n)$ contains $\Theta(nR^2(n))$ nodes w.h.p. and uniformly for all values of $j, 1 \leq j \leq \frac{l_\Gamma}{D(n)}$ under the condition that $R(n) \geq \Theta\left(\sqrt{\frac{\log n}{n}}\right)$. Equivalently, this can be expressed as

$$\lim_{n\to\infty} P\left[\bigcap_{j=1}^{l_\Gamma/D(n)} |N_j - E(N_j)| < \delta E(N_j)\right] = 1, \tag{19}$$

where δ is a positive arbitrarily small value close to zero.

Proof: From Equation (4), for any given $0 < \delta < 1$, there exists a $\theta > 0$ such that

$$P\left[N_j - |E(N_j)| > \delta E(N_j)\right] < e^{-\theta E(N_j)} = e^{-\theta n |A_j|}. \tag{20}$$

Thus, we can conclude that the probability that the value of the random variable N_j deviates by an arbitrarily small constant value from the mean tends to zero as $n \to \infty$. This is a key step in showing that when all the events $\bigcap_{j=1}^{l_\Gamma/D(n)} |N_j - E(N_j)| < \delta E(N_j)$ occur simultaneously, then all N_js converge uniformly to their expected values. Utilizing the same technique as in IV-B, we obtain

$$P\left[\bigcap_{j=1}^{l_\Gamma/D(n)} |N_j - E(N_j)| < \delta E(N_j)\right] \geq 1 - \sum_{j=1}^{l_\Gamma/D(n)} P\left[|N_j - E(N_j)| > \delta E(N_j)\right] > 1 - \frac{l_\Gamma}{D(n)} e^{-\theta E(N_j)}.$$

Because $E(N_j) = \frac{\pi}{2} n R^2(n)$, the final result is

$$\lim_{n\to\infty} P\left[\bigcap_{j=1}^{l_\Gamma/D(n)} |N_j - E(N_j)| < \delta E(N_j)\right] \geq 1 - \frac{l_\Gamma}{D(n)} e^{-\frac{\theta \pi n R^2(n)}{2}} \geq 1 - \frac{l_\Gamma}{2R(n)} e^{-\frac{\theta \pi n R^2(n)}{2}}. \tag{21}$$

If $R(n) \geq \sqrt{\frac{c_5 \log n}{n}} = \Theta\left(\sqrt{\frac{\log n}{n}}\right)$ and as $n \to \infty$, then $\frac{e^{-\frac{\theta \pi n R^2(n)}{2}}}{R(n)} \to 0$, when $\theta > 1/\pi c_5$. Here, the key constraint of $R(n)$ is given as

$$R(n) \geq \Theta\left(\sqrt{\frac{\log n}{n}}\right). \tag{22}$$

■

Eq. (22) is equivalent to the connectivity condition in the protocol model [1], [20]. It is interesting to note that we did not really use connectivity criterion in the physical model, however, it turns out that the minimum distance for the receiver range in MPR model is equivalent to the connectivity constraint in protocol model for random networks.

The above theorem demonstrates that w.h.p., there are indeed $\Theta(nR^2(n))$ nodes in each communication region with the constraint in (22). The achievable capacity is only feasible when the receiver range of each node in MPR scheme is at least equal to the connectivity criterion of transmission range in point-to-point communication [1]. Combining the result of Eq. (18) in Theorem 4.8 and (22) in Theorem 4.9, we can state the following theorem for the lower bound of throughput capacity, which implies the lower bound order capacity achieves the upper bound in physical model.

Theorem 4.10: The per source-destination throughput capacity of MPR scheme in a 2-D static wireless ad hoc network is lower bounded by $\Omega\left(\frac{(R(n))^{(1-2/\alpha)}}{n^{1/\alpha}}\right)$ provided that $R(n) \geq \Theta\left(\sqrt{\frac{\log n}{n}}\right)$, which means the tight bound is at least $\Theta\left(\frac{(\log n)^{\frac{1}{2}-\frac{1}{\alpha}}}{\sqrt{n}}\right)$ for $\alpha > 2$.

Proof: We first prove that Eq. (18) is an achievable bound and then by applying the minimum receiver range constraint in Eq. (22), we derive the lower bound for this theorem.

To derive the achievable lower bound, we design a scheme for separating decode-able transmitter nodes inside the communication circle and interference, such that $\mathit{SINR}_{Z(R(n))} \geq \beta_1$. Similar to the derivations in Eq. (15) and using Fig. 3, it is clear that the *SINR* is minimized when the largest value for interference is considered. This value is achieved when we compute the interference for a receiver node in the middle of the network and use the closest possible distance to the receiver node[1]. This lower bound can be written as

$$\mathit{SINR}_{Z(R(n))} \geq \frac{\frac{P}{R^{\alpha}(n)}}{BN_0 + \frac{\pi}{2}nR^2(n)\sum_{i=1}^{l_\Gamma/2D(n)} \frac{2P}{(iD(n)-R(n))^{\alpha}}}. \tag{23}$$

Assume that $D(n)$ satisfies the condition in Eq. (17). If we use the constraint for $R(n)$ in (22), we arrive at

$$\frac{D(n)}{R(n)} \geq \left(\frac{c_3\beta\pi}{2}\right)^{\frac{1}{\alpha}} n^{\frac{1}{\alpha}}(R(n))^{2/\alpha} \geq \Theta\left((\log n)^{\frac{1}{\alpha}}\right), \tag{24}$$

[1]Note that the difference between maximum and minimum value of interference is a constant value

which illustrates that $R(n)$ can be ignored compared with $D(n)$ for large values of n, i.e., $n \to \infty$. We now evaluate the asymptotic behavior of (23) when $n \to \infty$. Combining Eqs. (24) and (23), $\mathit{SINR}_{Z(R(n))}$ can be lower bounded by

$$\begin{aligned}
\lim_{n\to\infty} \mathit{SINR}_{Z(R(n))} &\geq \left(\frac{D(n)}{R(n)}\right)^{\alpha} \frac{1}{\pi n R^2(n) \sum_{i=1}^{l_\Gamma/D(n)} \frac{1}{i^\alpha}} \\
&\geq \left(\frac{D(n)}{R(n)}\right)^{\alpha} \frac{1}{\pi c_4 n R^2(n)} \geq \frac{c_3}{2c_4}\beta = \beta_1.
\end{aligned}$$

This inequality is derived using Eqs. (17) and (15), together with the fact that the second term in the denominator of *SINR* goes to infinity when $n \to \infty$ and, therefore, we can drop the first term related to the noise. Using the same arguments introduced for the computation of the upper bound, we can show that a non-zero value for $\mathit{SINR}_{Z(R(n))}$ can be achieved which implies that the throughput capacity can be achieved asymptotically. ■

The above theorem demonstrates that a gain of at least $\Theta\left((\log n)^{\frac{\alpha-2}{2\alpha}}\right)$ can be achieved compared with the results by Gupta and Kumar [1] and Franceschetti et al. [13]. Combining Theorems 4.8 and 4.10, we arrive at our next major contribution of this paper.

Theorem 4.11: The per source-destination throughput capacity of MPR scheme in a 2-D static wireless ad hoc network is tight bounded as $\Theta\left(\frac{(R(n))^{(1-2/\alpha)}}{n^{1/\alpha}}\right)$. The minimum receiver range is lower bounded as $R(n) \geq \Theta\left(\sqrt{\frac{\log n}{n}}\right)$, which implies a lower tight bound of $\Theta\left(\frac{(\log n)^{\frac{1}{2}-\frac{1}{\alpha}}}{\sqrt{n}}\right)$.

Note that this result shows that we can close the gap in the physical model similar to the results derived by Franceschetti et al. [13] but achieving higher throughput capacity with MPR.

V. Energy Efficiency

Many wireless sensor and ad hoc networks are energy and power limited systems and it is natural to ask what the price of achieving higher capacities in wireless ad hoc networks is.

The transport capacity was originally defined in [1] based on bits per second for random networks and bit-meters per second for arbitrary networks. The definition of bits-per Joule was defined in [18] that takes into consideration the energy consumption that is required to transmit information bits.

To incorporate the effect of energy consumption for communication in wireless networks, we define *bit-meters per Joule*, or simply energy efficiency. This new metric is a measure for evaluating the energy efficiency of the transport capacity in wireless sensor and ad hoc networks. This definition is general and it becomes equal to the bits per Joule definition of [18] in the special case of random networks. The formal definition is as follows.

Definition 5.1: [Energy Efficiency: bit-meters per joule] In wireless ad hoc networks with limited energy, the energy efficiency is

$$\eta(n) = \frac{\lambda(n)}{P(n)}, \tag{25}$$

where $\lambda(n)$ is the transport capacity of the network and $P(n)$ is the total minimum average power required to achieve $\lambda(n)$ for each source-destination pair in the network.

With this definition of efficiency, we compute the relationship between transport capacity and the energy efficiency for the various approaches defined to increase the transport capacity of wireless ad hoc networks, including our own.

A. Energy Efficiency in Approach by Gupta and Kumar [1]

It is easy to show that [21] the minimum transmit power P for each hop to guarantee that the *SINR* $\geq \beta$ and the total average power to transmit this information are

$$\min(P) = \Theta(s_n^\alpha) = \Theta\left(\left(\frac{\log n}{n}\right)^{\frac{\alpha}{2}}\right),$$

$$P(n) = \min(P) \times \text{total number of hops} = \Theta\left(\left(\frac{\log n}{n}\right)^{\frac{\alpha}{2}-\frac{1}{2}}\right). \tag{26}$$

where $s_n = \Theta\left(\sqrt{\frac{\log n}{n}}\right)$. The energy efficiency for this scheme can be computed by dividing the transport capacity by the total average power required to achieve this capacity. This renders

$$\eta(n) = \Theta\left(\frac{n^{\frac{\alpha}{2}-1}}{(\log n)^{\frac{\alpha}{2}}}\right). \tag{27}$$

B. Energy Efficiency in Approach by Franceschetti et al. [13]

The communication in the approach by Franceschetti et al. [13] is based on dividing the transfer of packets into four phases. In the first phase, the source transmits a packet to a relay inside a path that is called ”highway path.” The distance between the source and highway path is considered a long range communication and is proportional to $\Theta(\frac{\log n}{\sqrt{n}})$. Inside the highway path in phases two and three, multiple hop communication occurs horizontally and vertically respectively. The communication range is of short range and proportional to $\Theta(\frac{1}{\sqrt{n}})$. Communication in phase four is similar to phase one and it is between relay and destination.

Assume that $P_h(n)$ is the transmit power at the highway path in phases two and three. Following the definition in [13], the interference from the other cells can be expressed as

$$I(d,n) \leq P_h(n)\left(s_n(d+1)\right)^{-\alpha} c_6. \tag{28}$$

where c_6 is a constant value. The signal power at the receiver is lower bounded as

$$S(d,n) \geq P_h(n)\left(s_n\sqrt{2}(d+1)\right)^{-\alpha}. \tag{29}$$

Using the above results, the *SINR* is derived as

$$\textit{SINR} = \frac{S(d,n)}{BN_0 + I(d,n)} \geq \frac{P_h(n)\left(\sqrt{2}\right)^{-\alpha}}{BN_0(s_n(d+1))^{\alpha} + P_h(n)c_6}. \tag{30}$$

In the limit, the minimum required power to guarantee that the *SINR* satisfies the physical model when $n \to \infty$ is $\min(P_h(n)) = \Theta((s_n(d+1))^{\alpha}) = \Theta\left((n)^{-\alpha/2}\right)$.

For the long-range communications in the first and fourth phase, there is no interference. Therefore, the *SINR* can be expressed as

$$\textit{SINR} = \frac{P_u\left(\frac{\log n}{\sqrt{n}}\right)^{-\alpha}}{BN_0}. \tag{31}$$

The minimum required power for this case to guarantee the physical model condition is given by

$$\min(P_u) = \Theta\left(\left(\frac{(\log n)^2}{n}\right)^{\frac{\alpha}{2}}\right).$$

Using the definition of energy efficiency, we can compute its value for this case as

$$\eta(n) = \lambda(n)/P(n) = \frac{\lambda(n)}{2\min(P_u) + \sqrt{n}\min(P_h)} = \Theta\left(n^{\frac{\alpha}{2}-1}\right).$$

C. Energy Efficiency in Approach by Ozgur et al. [14]

Ozgur et al. [14] proposed a hierarchical cooperation method to achieve linear scalability with virtual MIMO techniques. In this method, the communication range is of order $\Theta(1)$ and, therefore, $P_{\min} = \Theta(1)$, $\lambda(n) = \Theta(1)$, and $\eta(n) = \Theta(1)$. Note again that we do not consider the signaling overhead required for cooperation in this approach. We only consider the throughput capacity in the physical layer.

D. Energy Efficiency with MPR

In this paper, we demonstrated that MPR closes the gap between the upper and lower bounds of the capacity of wireless ad hoc networks by achieving higher transport capacity. However, it is important to find out the energy efficiency of this approach. From the derivation of transport capacity for MPR in Eq. (23), the *SINR* is given by

$$\textit{SINR} \geq \frac{P(R(n))^{-\alpha}}{BN_0 + \frac{\pi}{2}nR^2(n)\sum_{i=1}^{l_\Gamma/2D(n)} 2P(iD(n)-R(n))^{-\alpha}} \tag{32}$$

The physical model constraint is guaranteed for *SINR* asymptotically when the minimum transmit power $P_{\text{MPR}}(n)$ is

$$\min(P_{\text{MPR}}(n)) = \Theta\left(R^\alpha(n)\right) = \left(\frac{\log n}{n}\right)^{\frac{\alpha}{2}}. \tag{33}$$

Eq. (33) is derived using Eqs. (22) and (24) when $n \to \infty$.

The relationship between $\lambda(n)$ and $P_{\text{MPR}}(n)$ can be computed from Theorem 4.11 as

$$\lambda(n) = n^{-1/\alpha}\left(P_{\text{MPR}}(n)\right)^{\frac{\alpha-2}{\alpha^2}}. \tag{34}$$

Because the communication range in MPR is equal to $R(n)$, the total minimum transmit power from source to destination is equal to $\frac{P_{\text{MPR}}(n)}{R(n)}$.

The energy efficiency of MPR scheme is given by

$$\eta(n) = \frac{\lambda(n)R(n)}{P_{\text{MPR}}(n)} = \lambda(n)(R(n))^{1-\alpha} = n^{-\frac{\alpha-1}{\alpha-2}}\lambda(n)^{\frac{-(\alpha-1)^2-1}{\alpha-2}}.$$

VI. Discussion

The reason for the significant increase in capacity with MPR is that, unlike point-to-point communication in which nodes compete to access the channel, MPR embraces (strong) interference by utilizing higher decoding complexity for all nodes. As we have pointed out, recent work on network coding [8], [9] implicitly assumes some form of MPR. These results clearly demonstrate that embracing interference is crucial to improve the performance of wireless ad hoc networks, and that MPR constitutes an important component of that.

Another interesting observation is the fact that increasing the receiver range $R(n)$ increases the throughput capacity. This is in sharp contrast with point-to-point communication in which increasing the communication range actually decreases the throughput capacity and it is again due to the fact that MPR embraces the interference.

Figure 4 shows the tradeoff between the total minimum transmit power and the transport capacity. From this figure, it is clear that the total transmit power for the network must be increased in order to increase the per source-destination transport capacity in random wireless ad hoc networks.

Figure 5 shows that, by increasing the transport capacity in wireless ad hoc networks, the energy efficiency of all the schemes we analyzed decreases. Many wireless ad hoc networks are limited in total available energy or power for each node. Therefore, increasing the transport capacity may not be feasible if the required power to do so is not available. This result also shows that the transport capacity should not be the only metric used in evaluating and comparing the merits of different schemes. The energy efficiency of these schemes is also very important. Based on different values for $R(n)$, different transport capacities can be attained. In general, MPR allows to have tradeoff between receiver complexity and transport capacity.

There are certain issues that we did not discuss in this paper. Our analysis does not include the energy required for increased decoding complexity, which is necessary for MPR. Our analysis also does not include the additional required overhead related to cooperation among nodes. Such topics are the subject of future studies.

VII. Conclusion

This paper shows that the use of MPR can close the gap for the transport (throughput) capacity in random wireless ad hoc networks under the physical model, while achieving much higher capacity gain than that of [13]. The tight bounds are $\Theta(R(n))$ and $\Theta\left(\frac{(R(n))^{1-2/\alpha}}{(n^{(1/\alpha)})}\right)$ where $R(n)$ is the receiver range in MPR model for protocol and physical models respectively.

We introduced a new definition related to energy efficiency based on bit-meters per Joule metric. Our results show that increasing the transport capacity by means of MPR or any of the other techniques proposed to date [1], [13], [14] results in a reduction of energy efficiency in the network. Accordingly, there is a tradeoff to be made between increasing transport capacity and decreasing energy efficiency. Determining what is the optimum tradeoff between capacity and energy efficiency is an open problem.

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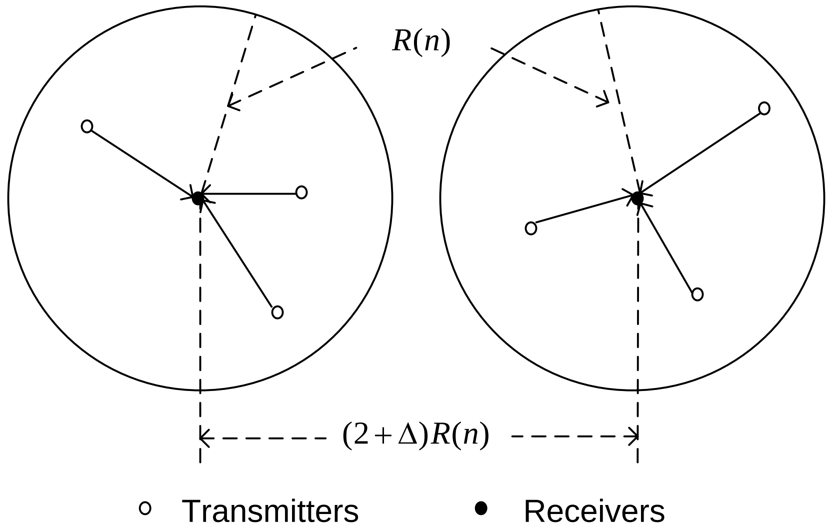

Fig. 1. MPR protocol model

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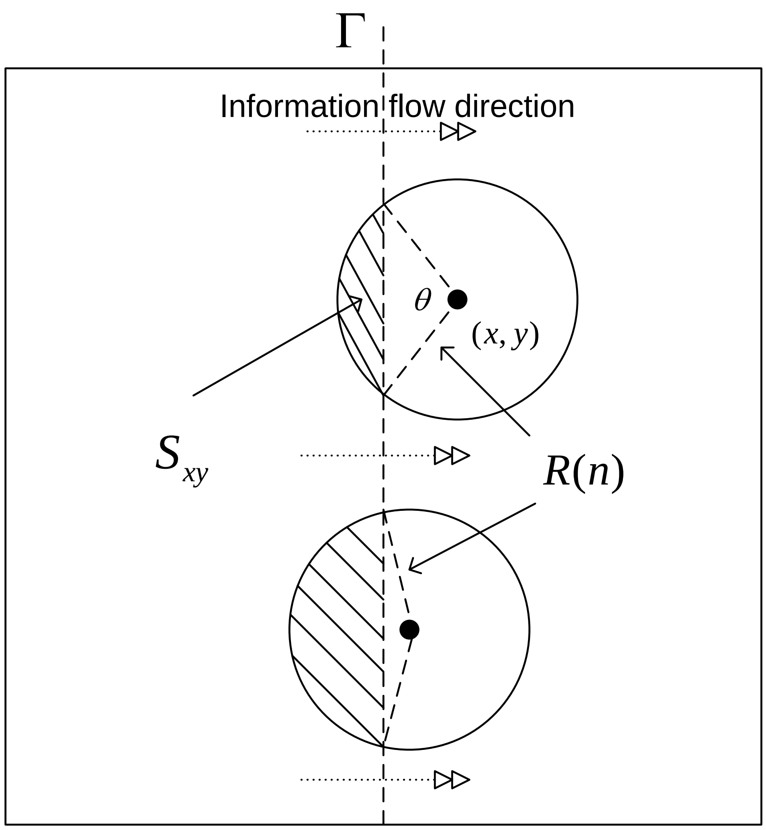

Fig. 2. For a receiver at location (x, y), all the nodes in the shaded region S_{xy} can send a message successfully and simultaneously.

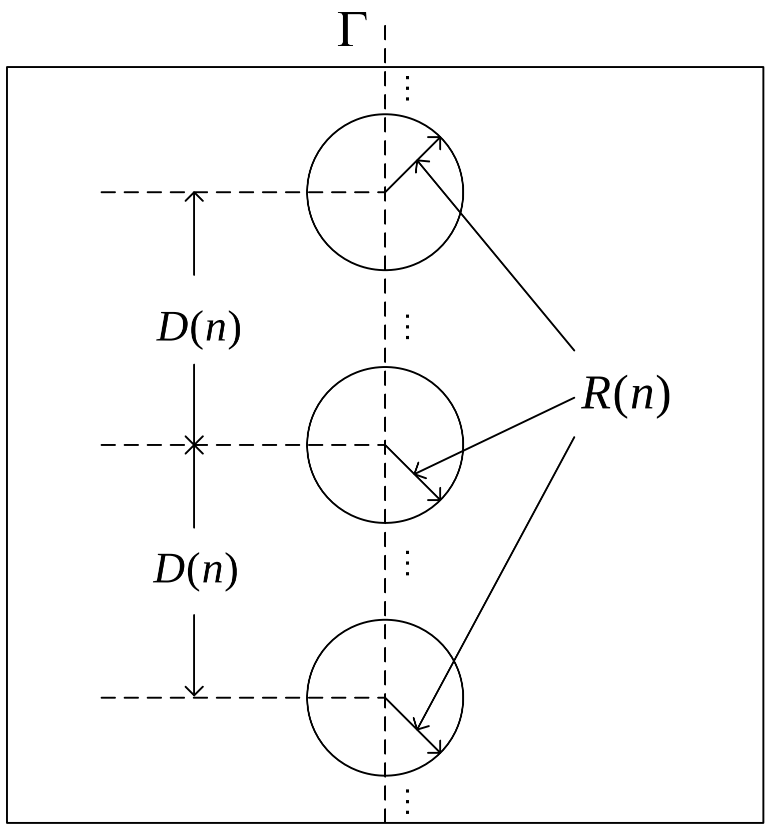

Fig. 3. Upper bound design of the network

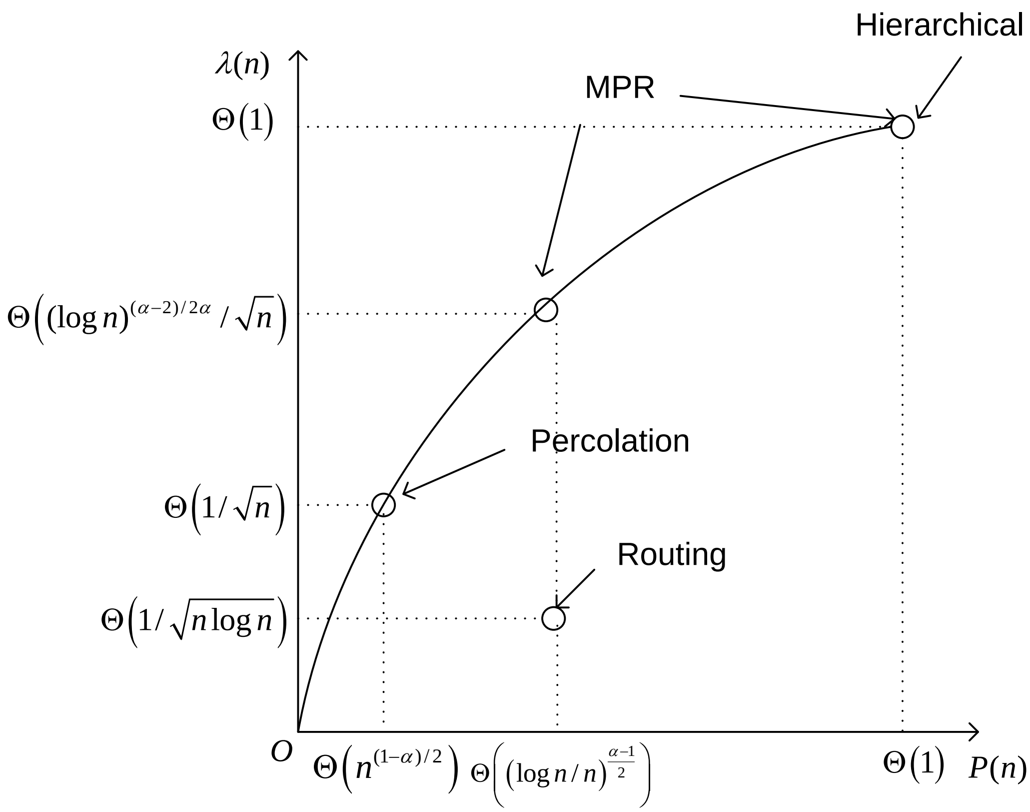

Fig. 4. Power and Capacity tradeoff

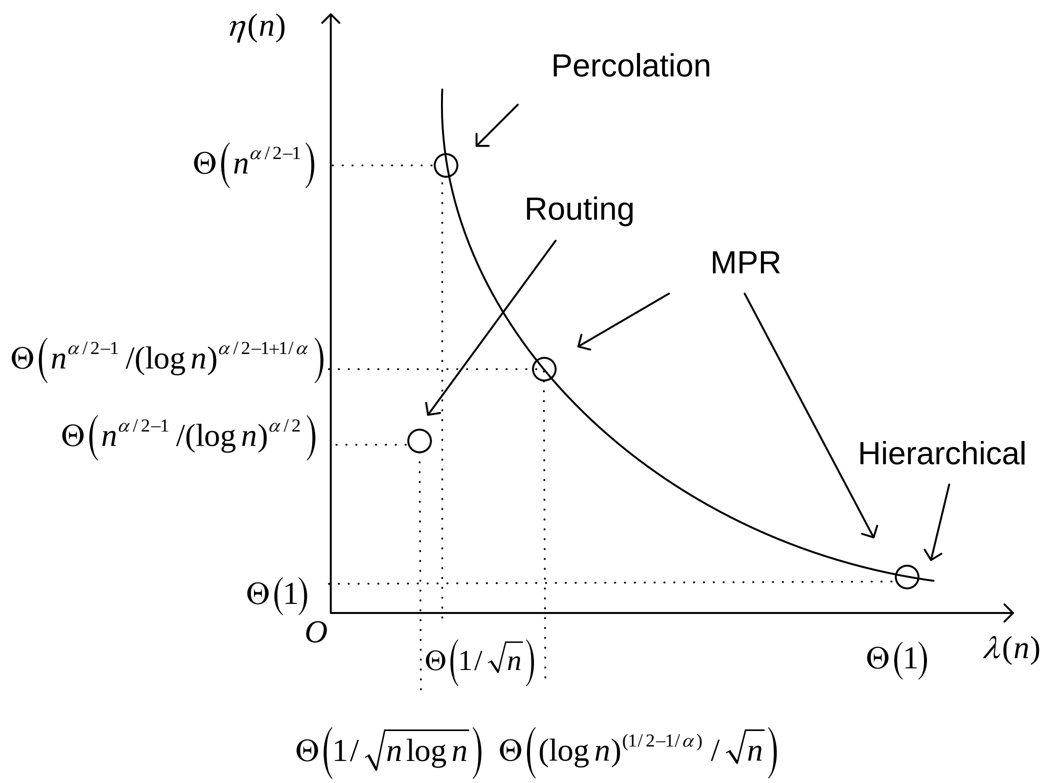

Fig. 5. Capacity and energy efficiency tradeoff