# Cognitive Beamforming Made Practical: Effective Interference Channel and Learning-Throughput Tradeoff

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#### Abstract

This paper studies the transmission strategy for cognitive radio (CR) under spectrum sharing with primary radio (PR). It is assumed that the CR transmitter is equipped with multi-antennas, whereby transmit precoding and power control are jointly deployed to balance between the interference avoidance at the PR terminals and the throughput maximization of the CR link. This operation is generally known as *cognitive beamforming* (CB). Unlike prior study on CB that assumes perfect knowledge on the channels over which the CR transmitter interferes with the PR terminals, this paper removes such assumption and proposes a *practical* CB scheme by utilizing a new idea of *effective interference channel*, which can be efficiently learned/estimated at the CR transmitter from the received PR signals. Interestingly, it is shown that the CB scheme based upon the effective interference channel can be superior over that utilizing the exact channel knowledge, when the PR terminals are equipped with multi-antennas but only communicate over a subspace of the available spatial dimensions. Furthermore, this paper presents algorithms for the CR to estimate the effective interference channel over a finite learning time. Due to the channel estimation errors, the proposed CB scheme results in leakage interferences at the PR terminals, which in turn limits the maximum transmit power of the CR. This interesting phenomenon creates a general *learning-throughput tradeoff* for the CR, pertinent to the amount of time allocation between channel learning and data transmission. This paper characterizes this tradeoff by studying the optimal learning time to maximize the CR throughput, given the fixed total learning and transmission time, the CR's own transmit power constraint, and the maximum tolerable leakage interference power level at the PR terminals.

#### **Index Terms**

Cognitive beamforming, cognitive radio, multiple-input multiple-output (MIMO) systems, spectrum sharing.

#### I. INTRODUCTION

Cognitive radio (CR), since the name was coined by Mitola in his seminal work [1], has drawn intensive attentions from both academic and industrial communities. Generally speaking, there are three operational models for the CR known in the literature, namely, *Interweave*, *Overlay*, and *Underlay* (see, e.g., [2] and references therein). Interweave method is also known as *opportunistic spectrum access* (OSA), originally outlined in [1] and later introduced by DARPA, whereby the CR transmits over the spectrum allocated to an existing primary radio (PR) only when the PR transmission is detected to be

off, while Overlay and Underlay methods allow the CR to transmit concurrently with the PR. Overlay method is based upon the "cognitive relay" idea [3], [4]. For this method, the CR transmitter is assumed to know perfectly all the channels in the coexisting PR and CR network as well as the PR's messages prior to transmission. Thereby, the CR is able to transmit messages to its own receiver and at the same time compensate for the resultant interference to the PR by relaying the PR's messages to the PR receiver. On the other hand, Underlay method only requires the channel gain knowledge from the CR transmitter to PR receiver, with which the CR transmits regardless of the PR's on/off status provided that the resultant interference power level at the PR receiver is kept below some predefined threshold, also known as the *interference-temperature* constraint [5], [6]. In general, Interweave and Underlay

methods are more favorable over Overlay method from an implementation viewpoint.

In wireless environment, due to the randomness and variation of wireless channels, dynamic resource allocation (DRA) for the CR becomes crucial, whereby the transmit power level, bit-rate, bandwidth, and/or antenna beam of the CR are dynamically changed based upon the channel state information (CSI) in the coexisting PR and CR network (see, e.g., [7]-[14]). In this paper, we are particularly interested in the case where the CR transmitter is equipped with multi-antennas so that it can deploy joint transmit precoding and power control to effectively balance between avoiding the interferences at the PR terminals and maximizing the throughput of the CR link. This operation is in general known as cognitive beamforming (CB). In [14], both optimal and suboptimal CB schemes were presented to maximize the CR channel capacity under the CR's transmit-power constraint and a set of interferencepower constraints for PRs, under the assumption that the CR transmitter knows perfectly the channels over which it interferes with the PR terminals. In contrast, in this paper we propose a *practical* CB scheme, which does not require any prior knowledge on the CR to PR channels. Instead, the proposed scheme exploits the time-division-duplex (TDD) operation mode of the PR link as well as the property of channel reciprocity, and designs the CB based upon a so-called *effective interference channel*, which can be efficiently learned/estimated at the CR transmitter via periodically listening to the PR transmissions. Thereby, the proposed scheme eliminates the training/feedback overhead for the PR to convey the exact interference channel knowledge to the CR and, thus, makes the CB towards being more practically implementable.

Furthermore, the proposed CB scheme utilizing the effective interference channel creates a new operational model for the CR, which is different from the conventional models. We thus name this new model as *opportunistic spatial sharing* (OSS). On the one hand, OSS, like Underlay, is more spectral efficient for the CR transmission than the conventional Interweave method since it allows the CR to transmit concurrently with the PR via transmit beamforming. On the other hand, OSS also improves over Underlay by exploiting the additional PR transmission characteristics (e.g., on/off status, degree of

freedom) learned from the observed effective interference channel for opportunistic transmission, thereby further boosting the CR's transmission spectral efficiency. Therefore, OSS is a superior operational model for the CR over both Underlay and Interweave methods.

The main results of this paper constitute two parts summarized as follows:

- In the first part, we consider the ideal case where the CR's estimation on the effective interference channel is *perfect* or noiseless. In this case, we provide the conditions under which the effective interference channel is sufficient for the proposed CB scheme to result in no adverse effect on the PR transmissions, or in other words, the PRs transmit as if there is no concurrent CR transmission. In addition, we show that when the PR terminals are equipped with multi-antennas but only communicate over a subspace of the available spatial dimensions, the CB scheme based upon the effective interference channel achieve a capacity gain for the CR over that utilizing the exact interference channel knowledge, thanks to the OSS that exploits the additional PR transmission characteristics learned from the effective interference channel.
- In the second part, we consider the more practical case with *imperfect* effective interference channel estimation due to finite learning time. We propose a *two-phase* transmission protocol for the CR to support the practical CB: the first phase is for the CR to estimate the effective interference channel, while the second phase is for the CR to transmit with the CB designed from the estimated channel. We present two practical algorithms for the CR to estimate the effective interference channel, under different assumptions on the availability of the noise power knowledge at the estimator. Furthermore, we show that due to imperfect channel learning, there exists a general *learning-throughput tradeoff* associated with the proposed scheme, pertinent to the amount of time allocation between channel learning time for the effective interference channel to maximize the CR throughput, and derive the solution of this problem by applying convex optimization techniques.

The rest of this paper is organized as follows. Section II presents the system model. Section III describes the effective interference channel concept. Section IV studies the CB based upon the effective interference channel by assuming perfect channel learning. Section V considers the case of imperfect channel learning, presents practical estimation algorithms, and characterizes the learning-throughput tradeoff for the CR. Section VI presents the simulation results. Finally, Section VII concludes the paper.

*Notation*: Scalar is denoted by lower-case letter, e.g., x, and bold-face lower-case letter is used for vector, e.g., x, and bold-face upper-case letter for matrix, e.g., X. Tr(S), |S|,  $S^{-1}$ ,  $S^{\dagger}$ , and  $S^{1/2}$  denote the trace, determinant, inverse, pseudo inverse, and square root ( $S = S^{1/2}(S^{1/2})^H$ ) of a square matrix S, respectively, and  $Diag(S_1, \ldots, S_M)$  denotes a block-diagonal square matrix with  $S_1, \ldots, S_M$  as the diagonal square matrices.  $S \succeq 0$  means that S is a positive semi-definite matrix. For any general matrix

 $M, M^T$  and  $M^H$  denote the transpose and the conjugate transpose of M, respectively, Rank(M) denotes the rank of M, and  $\lambda_{max}(M)$  and  $\lambda_{min}(M)$  denote the maximum and minimum eigenvalues of M, respectively. I and 0 denote the identity matrix and the zero matrix, respectively. ||x|| denotes the Euclidean norm of a complex vector x.  $\mathbb{C}^{x \times y}$  denotes the space of  $x \times y$  matrices with complex entries. The distribution of a circular symmetric complex Gaussian (CSCG) vector with mean x and covariance matrix  $\Sigma$  is denoted by  $\mathcal{CN}(x, \Sigma)$ , and  $\sim$  means "distributed as".  $\mathbb{E}[\cdot]$  denotes the statistical expectation.  $Prob\{\cdot\}$  denotes the probability. max(x, y) and min(x, y) denote, respectively, the maximum and the minimum between two real numbers x and y. For a real number a,  $(a)^+ \triangleq max(0, a)$ .

## II. SYSTEM MODEL

The system of interest is shown in Fig. 1, where a CR link consisting of the CR transmitter (CR-Tx) and CR receiver (CR-Rx) coexists with a PR link consisting of two terminals denoted by PR<sub>1</sub> and PR<sub>2</sub>, respectively. The number of antennas equipped at CR-Tx, CR-Rx, PR<sub>1</sub>, and PR<sub>2</sub> are denoted as  $M_t$ ,  $M_r$ ,  $M_1$ , and  $M_2$ , respectively. It is assumed that  $M_t > 1$ , while  $M_r$ ,  $M_1$ , and  $M_2$  can be any positive integers. For the PR link, it is assumed that PR<sub>1</sub> and PR<sub>2</sub> operate in a TDD mode over a single narrow-band flat-fading channel. Furthermore, reciprocity is assumed for the channels between PR<sub>1</sub> and PR<sub>2</sub>, i.e., if the channel from PR<sub>1</sub> to PR<sub>2</sub> is denoted by  $F \in \mathbb{C}^{M_2 \times M_1}$ , then the channel from PR<sub>2</sub> to PR<sub>1</sub> becomes  $F^{H,1}$  Without loss of generality (W.I.o.g.), the transmit precoding matrix for PR<sub>j</sub>, j = 1, 2, is denoted by  $A_j \in \mathbb{C}^{M_j \times d_j}$ , with  $d_j$ ,  $1 \le d_j \le M_j$ , denoting the corresponding number of transmitted data streams. The transmit covariance matrix for PR<sub>j</sub> is then defined as  $S_j \triangleq A_j A_j^H$ . We assume that  $A_j$  is a full-rank matrix and thus Rank( $S_j$ ) =  $d_j$ . Furthermore, define  $B_1 \in \mathbb{C}^{d_2 \times M_1}$  as the decoding matrix at PR<sub>1</sub> and  $B_2 \in \mathbb{C}^{d_1 \times M_2}$  for PR<sub>1</sub>. Both  $B_j$ 's are also assumed to be full-rank.

In addition, it is assumed that PR<sub>1</sub> and PR<sub>2</sub> are both oblivious to the existence of the CR, while the CR is aware of the PRs and protects the PR transmissions by limiting the resultant interference power levels at both PR<sub>j</sub>'s to be below some prescribed threshold. Let  $\boldsymbol{H} \in \mathbb{C}^{M_r \times M_t}$  denote the CR channel, and  $\boldsymbol{G}_j \in \mathbb{C}^{M_j \times M_t}$  denote the interference channel from CR-Tx to PR<sub>j</sub>, j = 1, 2. Let the transmit precoding matrix of CR-Tx be denoted by a full-rank matrix  $\boldsymbol{A}_{CR} \in \mathbb{C}^{M_t \times d_{CR}}$ , where  $d_{CR} \leq M_t$ , and  $d_{CR} = \text{Rank}(\boldsymbol{S}_{CR})$ , with  $\boldsymbol{S}_{CR}$  denoting the transmit covariance matrix of CR-Tx, i.e.,  $\boldsymbol{S}_{CR} \triangleq \boldsymbol{A}_{CR}\boldsymbol{A}_{CR}^H$ . Notice that we are not concerned with the channels from PRs to CR-Rx since any interference signals from PRs over these channels can simply be treated as additional noise at CR-Rx.

In [14], the optimal design of  $S_{CR}$  has been studied by assuming that the CR has perfect knowledge on H,  $G_1$ , and  $G_2$  at CR-Tx. In this paper, we remove the assumption of any prior knowledge on  $G_1$  and  $G_2$  for the CB deign, as motivated by the following practical considerations. Since CR and

<sup>&</sup>lt;sup>1</sup>The results of this paper hold similarly for the case where  $F^{T}$  instead of  $F^{H}$  is used to represent the reverse channel of F.

PR usually belong to different legitimate systems, it is unlikely that PR will use dedicated resources such as training or feedback to make  $G_1$  and  $G_2$  known to CR-Tx. Consequently, it seems that the only possible way for CR-Tx to learn some knowledge on these channels is by listening to the PR transmissions over a certain period and assuming the channel reciprocities between CR-Tx and PR<sub>j</sub>'s. However, there are several issues related with this approach highlighted as follows:

- What CR-Tx can possibly estimate is indeed the "effective" channels,  $G_j^H A_j$ , from PR<sub>j</sub>, j = 1, 2, instead of the actual interference channels,  $G_j$ 's.
- The conventional CB scheme in [14] requires that the channels  $G_1$  and  $G_2$  are separately estimated. As such, CR-Tx needs to synchronize with the PR TDD transmissions, which requires the knowledge on the exact time instants for each transmit direction between PR<sub>1</sub> and PR<sub>2</sub>.
- If CR-Tx designs  $S_{CR}$  based on the estimated channels,  $G_j^H A_j$ 's, or even the actual channels,  $G_j$ 's, it is unclear whether the effect of the resulted interferences at  $PR_j$ 's can be properly controlled because the transmitted signals from CR-Tx experience the equivalent channel,  $B_jG_j$ , to  $PR_j$ , which is in general different from  $A_i^H G_j$  or  $G_j$ .

Therefore, to make the CB truly implementable in practice, the above issues need to be carefully addressed. One solution that is able to effectively resolve these issues will be shown later in this paper, which utilizes a new concept named *effective interference channel*.

# III. EFFECTIVE INTERFERENCE CHANNEL

Suppose that prior to data transmission, CR-Tx first listens to the frequency band of interest for the PR transmissions over N symbol periods. The received baseband signals can be represented as

$$\boldsymbol{y}(n) = \boldsymbol{G}_{j}^{H} \boldsymbol{A}_{j} \boldsymbol{t}_{j}(n) + \boldsymbol{z}(n), \ n = 1, \dots, N$$
(1)

where j = 1 if  $n \in \mathcal{N}_1$ , and j = 2 if  $n \in \mathcal{N}_2$ , with  $\mathcal{N}_1, \mathcal{N}_2 \subseteq \{1, \ldots, N\}$  denoting the time instants when PR<sub>1</sub> transmits to PR<sub>2</sub> and PR<sub>2</sub> transmits to PR<sub>1</sub>, respectively, and  $\mathcal{N}_1 \cap \mathcal{N}_2 = \emptyset$  due to the assumed TDD mode;  $t_j(n)$ 's are the encoded signals (prior to power control and precoding) for the corresponding PR<sub>j</sub>, and solely for the convenience of later analysis, it is assumed that  $t_j(n)$ 's are independent over n's and  $\mathbb{E}[t_j(n)(t_j(n))^H] = I_{d_j \times d_j}, j = 1, 2; z(n)$ 's are the additive noises assumed to be independent CSCG random vectors with zero-mean elements and the covariance matrix denoted by  $\rho_0 I_{M_t \times M_t}$ . Denote the cardinality of the set  $\mathcal{N}_j$  as  $|\mathcal{N}_j|$ . It is reasonable to assume that PR<sub>j</sub> will transmit, with a constant probability  $\alpha_j < 1$ , during a certain time period. Mathematically, we may use  $\mathbb{E}\left[\frac{|\mathcal{N}_j|}{N}\right] = \alpha_j$  or  $\mathbb{E}\left[\frac{|\mathcal{N}_j|}{N}\right] = \alpha_j$ . Note that  $\alpha_1 + \alpha_2 \leq 1$ .

Define  $s_j(n)$  as  $q_j(n)t_j(n)$ , where  $q_j(n) = 1$ , if  $n \in \mathcal{N}_j$  and  $q_j(n) = 0$  otherwise. Obviously,  $q_j(n)$ 's are random variables with  $\mathbb{E}[q_j(n)] = \alpha_j$ . Meanwhile,  $q_1(n)$  and  $q_2(n)$  are related by  $q_1(n)q_2(n) = 0$ .

Then, we have  $\mathbb{E}\{s_j(n)(s_j(n))^H\} = \alpha_j I$ , j = 1, 2, but  $\mathbb{E}\{s_1(n)(s_2(n))^H\} = 0$ . The signal model in (1) can then be equivalently rewritten as

$$\boldsymbol{y}(n) = \boldsymbol{\mathcal{A}}\boldsymbol{s}(n) + \boldsymbol{z}(n), \ n = 1, \dots, N$$
(2)

where  $\mathcal{A} = [\mathbf{G}_1^H \mathbf{A}_1, \mathbf{G}_2^H \mathbf{A}_2]$  and  $\mathbf{s}(n) = [(\mathbf{s}_1(n))^T, (\mathbf{s}_2(n))^T]^T$ . The covariance matrix of the received signals at CR-Tx is then defined as

$$\boldsymbol{Q}_{\boldsymbol{y}} = \mathbb{E}\{\boldsymbol{y}(n)(\boldsymbol{y}(n))^{H}\} = \boldsymbol{Q}_{s} + \rho_{0}\boldsymbol{I}$$
(3)

where

$$\boldsymbol{Q}_{s} \triangleq \alpha_{1} \boldsymbol{G}_{1}^{H} \boldsymbol{S}_{1} \boldsymbol{G}_{1} + \alpha_{2} \boldsymbol{G}_{2}^{H} \boldsymbol{S}_{2} \boldsymbol{G}_{2}$$

$$\tag{4}$$

denotes the covariance matrix due to only the signals from  $PR_j$ 's.

Practically, only the sample covariance matrix can be obtained at CR-Rx, which is expressed as

$$\hat{\boldsymbol{Q}}_{y} = \frac{1}{N} \sum_{n=1}^{N} \boldsymbol{y}(n) (\boldsymbol{y}(n))^{H}.$$
(5)

From law of large number (LLN), it is easy to verify that  $\hat{Q}_y \to Q_s + \rho_0 I$  with probability one as  $N \to \infty$ , while for finite values of N,  $Q_s$  can only be estimated from  $\hat{Q}_y$ .<sup>2</sup> Denote  $\hat{Q}_s$  as the estimated value of  $Q_s$ . Note that  $\hat{Q}_s$  is a covariance matrix and hence  $\hat{Q}_s \succeq 0$  and  $(\hat{Q}_s)^H = \hat{Q}_s$ . Thus, we define the aggregate "effective" channel from  $PR_i$ 's to CR-Tx as

$$\boldsymbol{G}_{\text{eff}}^{H} = (\hat{\boldsymbol{Q}}_{s})^{1/2} \tag{6}$$

while because of channel reciprocity, we define the *effective interference channel* from CR-Tx to  $PR_j$ 's as  $G_{\text{eff}}$ . In the following parts of this paper, practical CB schemes based on this effective interference channel instead of the actual CR to PR channels will be studied. First, we will consider the ideal case where the estimation of  $G_{\text{eff}}$  is perfect, i.e.,  $\hat{Q}_s = Q_s$  in (6), in Section IV. Then, we will consider the more practical case where  $G_{\text{eff}}$  is not perfectly estimated due to finite N in Section V.

#### IV. COGNITIVE BEAMFORMING WITH PERFECT CHANNEL LEARNING

In this section, we design the CR precoding matrix,  $A_{CR}$ , which contains the information of transmit precoding and power allocation at CR-Tx, based on the effective interference channel  $G_{eff}$  with perfect learning, i.e.,  $\hat{Q}_s = Q_s$  in (6). Note that introduction of the effective interference channel resolves the first two items of issues on implementing the CB raised in Section II: for the first issue, by its definition, the effective interference channel is known to be obtained from the effective channels from PRs to CR-Tx; for the second issue, since learning the effective interference channel does not attempt to separate

<sup>&</sup>lt;sup>2</sup>Discussions on algorithms for such estimation are postponed to Section V.

the channels from  $PR_1$  and  $PR_2$ , sophisticated synchronization with each transmit direction between  $PR_1$  and  $PR_2$  is no longer required. However, the third issue on the effect of the resultant interferences on the PR transmissions is yet unaddressed for the CB designed from the effective interference channel. In this section, we will address this issue in a detailed manner.

First, we present the following proposition:

<u>Proposition</u> 4.1: Under the assumption of perfect channel learning, if the conditions  $A_j^H G_j \supseteq B_j G_j$ , j = 1, 2 hold,<sup>3</sup> then the CB designed utilizing the effective interference channel  $G_{\text{eff}}$  and satisfying the constraint  $G_{\text{eff}} A_{\text{CR}} = 0$  will cause no adverse effect on the PR transmissions, regardless of the actual interference channels  $G_1$  and  $G_2$ .

The conditions in the above proposition can also be explained as  $\text{Span}(A_j^H G_j) \supseteq \text{Span}(B_j G_j)$ , j = 1, 2, where Span(X) denotes the subspace spanned by the rows of X. Intuitively speaking, these conditions hold when the transmitted signal space of  $\text{PR}_j$  after propagating through the PR to CR channel  $G_j^H$ , i.e.,  $G_j^H A_j$ , if reversed (conjugated and transposed), will subsume the equivalent received signal space from CR at  $\text{PR}_j$ ,  $B_jG_j$ , as a subspace, for j = 1, 2. Note that  $A_j^H G_j$  and  $B_jG_j$  may not have the same column size, and  $A_j^H$  and  $B_j$  may differ from each other for any j = 1, 2. Therefore, the validity of such conditions needs to be examined. Before we proceed to the proof of Proposition 4.1, we first present two well-known examples of practical multi-antenna transmission schemes for the PR, for both of which the conditions  $A_j^H G_j \supseteq B_j G_j$ , j = 1, 2 are usually satisfied.<sup>4</sup>

<u>Example</u> 4.1: Spatial Multiplexing: When the PR channel CSI is unknown at transmitter but known at receiver, spatial multiplexing mode is usually adopted to assign equal power levels and rate values to each of transmit antennas (e.g., the V-BLAST scheme [15]). In this case, the transmit covariance matrix at PR<sub>j</sub>, j = 1, 2, becomes  $S_j = \frac{P_j}{M_j} I_{M_j \times M_j}$ , with  $P_j$  denoting the total transmit power of PR<sub>j</sub>. Thus,  $d_j = M_j$ , and  $A_j$ 's are both scaled identity matrices. It then follows that  $A_j^H G_j \supseteq B_j G_j$  regardless of  $B_j$  or the receiver structure.

<u>Example</u> 4.2: Eigenmode Transmission: In the case where the PR CSI is known at both transmitter and receiver, which is usually a valid assumption for the TDD mode, eigenmode transmission mode is usually used to decompose the MIMO channel into parallel scalar channels [15]. In this case,  $S_1$  and  $S_2$  are designed based on the singular-value-decomposition (SVD) of F and  $F^H$ , respectively. Let the SVD of F be  $U_F \Sigma_F V_F^H$ . It then follows that  $A_1 = V_{F(1)} \Lambda_1$ ,  $B_1 = V_{F(2)}^H$ ,  $A_2 = U_{F(2)} \Lambda_2^{1/2}$ , and  $B_2 = U_{F(1)}^H$ , where  $\Lambda_j = \text{Diag}(\lambda_{j,1}, \ldots, \lambda_{j,d_j})$  and  $V_{F(j)}$  ( $U_{F(j)}$ ) denotes the first  $d_j$  columns in  $V_F$  ( $U_F$ ). Note that  $d_j \leq \min(M_1, M_2)$ . If it is true that  $d_1 = d_2$ , then it follows that  $\text{Span}(A_j^H G) = \text{Span}(B_j G)$  and

<sup>4</sup>Note that there exist cases where the conditions in Proposition 4.1 are not satisfied in practice. In such cases, the proposed CB scheme will cause certain performance loss of the PR transmissions, but the resulted interference power levels are in general reduced by the CB.

 $<sup>{}^{3}</sup>X \supseteq Y$  means that for two given matrices X and Y, if Xe = 0 for any arbitrary vector e, then Ye = 0 must hold.

thus  $A_j^H G_j \supseteq B_j G_j$ . Note that a special case here is the "beamforming mode" [15] with  $d_1 = d_2 = 1$ . Next, we present the proof of Proposition 4.1:

*Proof:* First, under the assumption of perfect channel learning,  $G_{\text{eff}} \sqsupseteq A_j^H G_j$  is true for j = 1, 2. This can be shown as follows:  $G_{\text{eff}} e = 0 \stackrel{(a)}{\Rightarrow} (\hat{Q}_s^{1/2})^H e = 0 \stackrel{(b)}{\Rightarrow} (Q_s^{1/2})^H e = 0 \Rightarrow e^H Q_s e = 0 \stackrel{(c)}{\Rightarrow} \|A_j^H G_j e\|^2 = 0, j = 1, 2 \Rightarrow A_j^H G_j e = 0, j = 1, 2$ , where (a) is from (6), (b) is due to  $\hat{Q}_s = Q_s$  under the assumption of perfect channel learning, and (c) is from (4). Since for arbitrary matrices, X, Y, and  $Z, X \sqsupseteq Y$  and  $Y \sqsupseteq Z$  imply that  $X \sqsupseteq Z$ , from  $G_{\text{eff}} \sqsupset A_j^H G_j$  (shown above) and  $A_j^H G_j \sqsupseteq B_j G_j$  (given in Proposition 4.1) it follows that  $G_{\text{eff}} \sqsupset B_j G_j$ , j = 1, 2. Therefore, if the constraint  $G_{\text{eff}} A_{\text{CR}} = 0$  is satisfied by the CB, we have  $B_j G_j A_{\text{CR}} = 0, j = 1, 2$ , i.e., the received interference from the CR,  $G_j A_{\text{CR}}$ , lies in the null space of the receiver decoding matrix,  $B_j$ , at PR<sub>j</sub>, and thus has no effect on the PR transmission.

From Proposition 4.1, it is known that if the given conditions are satisfied, then it is sufficient for us to design  $A_{CR}$  subject to the constraint  $G_{eff}A_{CR} = 0$ . Let the eigenvalue decomposition (EVD) of  $Q_s$ be represented as  $Q_s = V\Sigma V^H$ , where  $V \in \mathbb{C}^{M_t \times d_{eff}}$  and  $\Sigma$  is a positive  $d_{eff} \times d_{eff}$  diagonal matrix, with  $d_{eff} = \text{Rank}(Q_s)$ . Due to independence of  $G_j$ 's, j = 1, 2, it follows that  $d_{eff} = \min(d_1 + d_2, M_t)$ . From (6),  $G_{eff}^H$  can then be represented as  $G_{eff}^H = V\Sigma^{1/2}$ . Define the projection matrix based on V as  $P_G \triangleq I - VV^H = UU^H$ , where  $U \in \mathbb{C}^{M_t \times (M_t - d_{eff})}$  satisfies  $V^H U = 0$ . We are now ready to present the general form of  $A_{CR}$  under the constraint  $G_{eff}A_{CR} = 0$  as

$$\boldsymbol{A}_{\mathrm{CR}} = \boldsymbol{U}\boldsymbol{C}_{\mathrm{CR}}^{1/2} \tag{7}$$

where  $C_{CR}^{1/2} \in \mathbb{C}^{(M_t - d_{eff}) \times d_{CR}}$  with  $d_{CR}$  denoting the number of transmitted data streams for the CR, and  $C_{CR} \in \mathbb{C}^{(M_t - d_{eff}) \times (M_t - d_{eff})}$  satisfies that  $C_{CR} \succeq 0$  and  $Tr(C_{CR}) = Tr(S_{CR}) \leq P_{CR}$ , with  $P_{CR}$ denoting the transmit power of CR-Tx. From (7), it follows that designing  $A_{CR}$  becomes identical to designing  $C_{CR}$  over an equivalent CR channel, denoted by HU, subject to a transmit-power constraint  $Tr(C_{CR}) \leq P_{CR}$ . This observation simplifies significantly the design for the remaining part of  $A_{CR}$ , i.e.,  $C_{CR}$ , since a great deal of work in the literature (see, e.g., [15] and references therein) has studied this precoder design problem for the point-to-point channel with multi-antenna transmitter.

At last, we demonstrate an interesting property for the proposed CB scheme in (7) as follows. If the conditions given in Proposition 4.1 are satisfied and furthermore  $PR_1$  and/or  $PR_2$  have multi-antennas but transmit only over a subspace of the available spatial dimensions, i.e.,  $d_j < \min(M_1, M_2), j = 1, 2$ , the proposed scheme in (7) that utilizes the effective interference channel,  $G_{\text{eff}}$ , can be superior over the conventional CB scheme, the so-called projected-channel SVD (P-SVD) in [14] based on the actual interference channels,  $G_1$  and  $G_2$ , in terms of the achievable degree of freedom (DoF) for the CR transmission. At a first glance, this result is some contra-intuitive since  $G_{\text{eff}}$  contains only partial

information on  $G_j$ 's. The key observation here is that  $G_{\text{eff}}$  contains information on  $A_j^H G_j$ , which also exhibits side information on  $B_j G_j$  via the condition,  $A_j^H G_j \supseteq B_j G_j$ , given in Proposition 4.1, while  $B_j G_j$ 's are assumed to be unknown in [14]. Specifically, for the proposed scheme, the DoF is given as  $d_{\text{CR}}$ , which can be shown to be upper-bounded by  $d_{\text{CR}} \leq \min(M_t - d_{\text{eff}}, M_r) = \min((M_t - d_1 - d_2)^+, M_r)$ . In contrast, for a fair comparison with the proposed scheme, the P-SVD scheme in [14] with perfect knowledge on  $G_1$  and  $G_2$  needs to project onto the null space of  $G_1$  and  $G_2$  (assumed to be independent of each other) so as to completely remove the interferences at both PRs, thus resulting in the DoF to be at most  $\min((M_t - M_1 - M_2)^+, M_r)$ . Therefore, the proposed scheme can have strictly positive DoF even when  $M_1 + M_2 \geq M_t$ , provided that  $d_1 + d_2 < M_t$ , i.e., the total number of antennas of PR<sub>j</sub>'s is no smaller than  $M_t$ , while the total number of transmitted data streams over both transmit directions between PR<sub>1</sub> and PR<sub>2</sub> is smaller than  $M_t$ , while the P-SVD scheme has zero DoF in this case since  $M_t \leq M_1 + M_2$ . In general, since  $d_j \leq \min(M_1, M_2), j = 1, 2$ , it follows that  $(d_1 + d_2) \leq (M_1 + M_2)$  and thus the DoF gain of the proposed scheme over the P-SVD scheme, i.e.,  $\min((M_t - d_1 - d_2)^+, M_r) - \min((M_t - M_1 - M_2)^+, M_r)$ , is always non-negative.

<u>Example</u> 4.3: The capacity gain of the proposed scheme in (7) over the P-SVD scheme in [14], as above discussed, is shown in Fig. 2 for a PR link with  $M_1 = M_2 = 2$ ,  $d_1 = d_2 = 1$  (i.e., beamforming mode corresponding to the largest singular value of F in Example 4.2), and a CR link with  $M_t = 5$ and  $M_r = 3$ . All the channels involved are assumed to have the standard Rayleigh-fading distribution, i.e., each element of the channel matrix is independent CSCG random variable  $\sim C\mathcal{N}(0, 1)$ . W.l.o.g., it is assumed that the interference due to PR transmissions at CR-Rx is included in the additive noise, which is assumed to be  $\sim C\mathcal{N}(0, \rho_1 I)$ . The signal-to-noise ratio (SNR) in this case is thus defined as  $P_{CR}/\rho_1$ . The DoF can be visually seen in the figure to be proportional to the asymptotic ratio between the capacity value over the log-SNR value as SNR goes to infinity [15]. It is observed that the DoF for the proposed scheme is approximately three times of that for the P-SVD scheme in this case, since  $\min((M_t - d_1 - d_2)^+, M_r)/\min((M_t - M_1 - M_2)^+, M_r) = 3/1 = 3$ .

#### V. COGNITIVE BEAMFORMING WITH IMPERFECT CHANNEL LEARNING

In the previous section, CB is designed under the assumption that the effective interference channel,  $G_{\text{eff}}$ , is perfectly estimated at CR-Tx. In this section, we will study the effect of imperfect estimation of  $G_{\text{eff}}$  due to finite sample size N on the performance of the proposed CB. Consider the following *two-phase* transmission protocol for the CR to support the practical CB operation as shown in Fig. 3. Each block transmission of CR with duration T is divided into two consecutive sub-blocks. During the first sub-block of duration  $\tau$ ,  $G_{\text{eff}}$  is estimated; during the second sub-block of duration  $T - \tau$ , CR transmits using the CB derived from the estimated  $G_{\text{eff}}$ . Note that T needs to be chosen such that, on the one

hand, sufficiently small compared with the channel coherence time to maintain constant channels during each block, and on the other hand, as large as possible compared to the inverse of the channel bandwidth to make T constitute a large number of symbols, in order to reduce the percentage of symbol periods for channel learning. In this paper, it is assumed that T is preselected and is thus fixed. For a given T, intuitively, a larger value of  $\tau$  is desirable from the perspective of estimating  $G_{\text{eff}}$ , while a smaller  $\tau$  is favorable in terms of the achievable CR throughput that is proportional to  $(T - \tau)/T$ . Consequently, we will show in this section a general *learning-throughput tradeoff* for the proposed scheme,<sup>5</sup> pertinent to the effect of the value  $\tau$  on the CR throughput. First, we present practical algorithms for estimating  $G_{\text{eff}}$ in Section V-A. Next, we derive the so-called "effective leakage interference power" at PR terminals due to the CB designed from the estimated  $G_{\text{eff}}$  in Section V-B. At last, we study the optimization problem to determine the optimal value of  $\tau$  to maximize the CR throughput in Section V-C, under fixed T,  $P_{\text{CR}}$ , and the constraint on the maximum leakage power level at the PRs.

# A. Estimation of $G_{\text{eff}}$

From (6), it is known that  $G_{\text{eff}}$  depends solely on  $\hat{Q}_s$ , the estimated value of the received PR signal covariance matrix  $Q_s$  defined in (4). Thus, in this subsection, we present algorithms to obtain  $\hat{Q}_s$  from the received sample covariance matrix  $\hat{Q}_y$  given in (5). Denote the EVD of  $\hat{Q}_y$  as

$$\hat{\boldsymbol{Q}}_{y} = \hat{\boldsymbol{T}}_{y} \hat{\boldsymbol{\Lambda}}_{y} \hat{\boldsymbol{T}}_{y}^{H} \tag{8}$$

where  $\hat{\Lambda}_y = \text{Diag}(\hat{\lambda}_1, \hat{\lambda}_2, \dots, \hat{\lambda}_{M_t})$  is a  $M_t \times M_t$  positive diagonal matrix whose diagonal elements are the eigenvalues of  $\hat{Q}_y$ . W.l.o.g., we assume that  $\hat{\lambda}_i$ 's,  $i = 1, \dots, M_t$ , are arranged in a decreasing order. We obtain  $\hat{Q}_s$  from  $\hat{Q}_y$  based on the maximum likelihood (ML) criterion, for the following two cases:

1) Known noise power  $\rho_0$ : In the case where the noise power,  $\rho_0$ , is assumed to be known at CR-Tx prior to channel learning, it follows from [17] that the ML estimate of  $Q_s$  is obtained as

$$\hat{\boldsymbol{Q}}_{s} = \hat{\boldsymbol{T}}_{y} \text{Diag} \left( (\hat{\lambda}_{1} - \rho_{0})^{+}, \dots, (\hat{\lambda}_{M_{t}} - \rho_{0})^{+} \right) \hat{\boldsymbol{T}}_{y}^{H}.$$
(9)

The rank of  $\hat{\boldsymbol{Q}}_s$ , or the estimated value of  $d_{\text{eff}}$ , denoted as  $\hat{d}_{\text{eff}}$ , can be found as the largest integer such that  $\hat{\lambda}_{\hat{d}_{\text{eff}}} > \rho_0$ . Therefore, the first  $\hat{d}_{\text{eff}}$  columns of  $\hat{\boldsymbol{T}}_y$  give the estimate of  $\boldsymbol{V}$ , denoted by  $\hat{\boldsymbol{V}}$ , and the last  $M_t - \hat{d}_{\text{eff}}$  columns of  $\hat{\boldsymbol{T}}_y$  are deemed as the estimate of  $\boldsymbol{U}$ , denoted by  $\hat{\boldsymbol{U}}$ . Note that  $\hat{\boldsymbol{U}}$  will replace the true value  $\boldsymbol{U}$  for the proposed CB design in (7).

<sup>5</sup>Note that the learning-throughput tradeoff includes the sensing-throughput tradeoff studied in [16] as a special case since channel sensing of the CR to detect the PR transmission can be considered as a hard version of channel learning considered in this paper.

2) Unknown noise power  $\rho_0$ : In this case,  $\rho_0$  is unknown to CR-Tx and has to be estimated along with  $\hat{Q}_s$ . The ML estimate of  $\rho_0$  can first be obtained as [18]

$$\hat{\rho}_{0} = \frac{1}{M_{t} - \hat{d}_{\text{eff}}} \sum_{i=\hat{d}_{\text{eff}}+1}^{M_{t}} \hat{\lambda}_{i}$$
(10)

where  $\hat{d}_{\text{eff}}$  is the ML estimate of  $d_{\text{eff}}$ . Specifically,  $\hat{d}_{\text{eff}}$  can be obtained as [18]

$$\hat{d}_{\text{eff}} = \arg\max_{k} \left(M_t - k\right) N \log\left(\frac{\prod_{i=k+1}^{M_t} \hat{\lambda}_i^{1/(M_t - k)}}{\frac{1}{M_t - k} \sum_{i=k+1}^{M_t} \hat{\lambda}_i}\right) = \arg\max_{k} \left(M_t - k\right) N \log\left(\frac{\text{GM}(k)}{\text{AM}(k)}\right)$$
(11)

where GM(k) and AM(k) denote the geometric mean and the arithmetic mean of the last  $M_t - k$  eigenvalues of  $\hat{Q}_y$ , respectively. To make this estimation unbiased, we conventionally adopt the so-called minimum description length (MDL) estimator expressed as [18]

$$\hat{d}_{\text{eff}} = \arg\min_{k} \ (M_t - k)N \log\left(\frac{\text{AM}(k)}{\text{GM}(k)}\right) + \frac{1}{2}k(2M_t - k)\log N \tag{12}$$

where the second term on the right-hand side (RHS) is a bias correction term. The ML estimates of V and  $\hat{U}$ , denoted by  $\hat{V}$  and  $\hat{U}$ , are then obtained from the first  $\hat{d}_{\text{eff}}$  and the last  $M_t - \hat{d}_{\text{eff}}$  columns of  $\hat{T}_y$ , respectively.

After knowing  $\hat{\rho}_0$ ,  $\hat{d}_{\text{eff}}$ ,  $\hat{V}$ , and  $\hat{U}$ , the ML estimate of  $Q_s$  is obtained as

$$\hat{\boldsymbol{Q}}_{s} = \hat{\boldsymbol{V}} \text{Diag} \left( \hat{\lambda}_{1} - \hat{\rho}_{0}, \dots, \hat{\lambda}_{\hat{d}_{\text{eff}}} - \hat{\rho}_{0} \right) \hat{\boldsymbol{V}}^{H}.$$
(13)

From (9) and (13), it is observed that these two estimators have a similar structure while they differ in the noise power term adopted and the way to estimate the rank of  $Q_s$ ,  $d_{\text{eff}}$ .

## B. Effective Leakage Interference Power

Due to imperfect channel estimation, the CB in (7) based on  $\hat{U}$  cannot perfectly remove the effective interference at PR<sub>j</sub>'s. In this subsection, the effect of the channel estimation errors on the resultant leakage interference power levels at PR<sub>j</sub>'s will be analytically quantified so as to assist the later studies. Define the rank over-estimation probability  $p_o(k) = \text{Prob}(\hat{d}_{\text{eff}} - d_{\text{eff}} = k | \hat{d}_{\text{eff}}), k = 1, \dots, \hat{d}_{\text{eff}}$ , and the rank under-estimation probability  $p_u(k) = \text{Prob}(d_{\text{eff}} - \hat{d}_{\text{eff}} = k | \hat{d}_{\text{eff}}), k = 1, \dots, M_t - \hat{d}_{\text{eff}}$ , conditioned on the observation  $\hat{d}_{\text{eff}}$ . If the over-estimation of  $d_{\text{eff}}$  is encountered, the upper bound on the number of data streams from CR-Tx,  $d_{\text{CR}}$ , may be affected. However, as long as  $(M_t - \hat{d}_{\text{eff}}) \ge M_r$ ,  $d_{\text{CR}}$  is more tightly bounded by  $M_r$  and the over-estimation of  $d_{\text{eff}}$  does not cause any problem. On the other hand, the under-estimation of  $d_{\text{eff}}$  will bring a severe issue, since some columns in  $\hat{U}$  may actually come from the PR signal subspace spanned by V. In this case, the interferences at PRs will be tremendously increased, which is similar to the scenario in the conventional Interleave-based CR system when a misdetection of PR transmission occurs. In practice, a threshold  $\xi$  should be properly set, and the last  $M_t - (\hat{d}_{\text{eff}} + k_0)$  columns in  $\hat{T}_y$  are chosen as  $\hat{U}$  only if  $p_o(k_0) \ge \xi$ .

Detailed study on  $p_o(k)$ ,  $p_u(k)$ , and  $\xi$  is deemed as a separate topic of this paper and will not be further addressed here. In this paper, for simplicity we will assume that the rank of  $Q_s$  or  $d_{\text{eff}}$  is correctly estimated. We will then focus our study on the effect of finite N on the distortion of the estimated eigenspace  $\hat{U}$ . From (7), the transmitted signal at CR-Tx is expressed as

$$\boldsymbol{s}_{\mathrm{CR}}(n) = \boldsymbol{A}_{\mathrm{CR}} \boldsymbol{t}_{\mathrm{CR}}(n) = \hat{\boldsymbol{U}} \boldsymbol{C}_{\mathrm{CR}}^{1/2} \boldsymbol{t}_{\mathrm{CR}}(n), \quad n > N$$
(14)

where  $s_{CR}(n)$  is the precoded version of the data vector  $t_{CR}(n)$ . Note that  $\mathbb{E}[t_{CR}(n)(t_{CR}(n))^H] = I$ and  $S_{CR} = \mathbb{E}[s_{CR}(n)(s_{CR}(n))^H]$ . The average leakage interference power at  $PR_j$ , j = 1, 2, due to the CR transmission is then expressed as

$$I_j = \mathbb{E}[\|\boldsymbol{B}_j \boldsymbol{G}_j \boldsymbol{s}_{CR}(n)\|^2].$$
(15)

Next,  $I_j$  is normalized by the respective processed (multiplied by  $B_j$ ) noise power to unify the discussions for  $PR_j$ 's. W.l.o.g., it is assumed that the additive noise power at  $PR_j$  is equal to  $\rho_0$ , the same as that at CR-Tx, and thus the processed noise power becomes  $\rho_0 Tr(B_j B_j^H)$ . Define

$$\bar{I}_j \triangleq \frac{I_j}{\rho_0 \operatorname{Tr}(\boldsymbol{B}_j \boldsymbol{B}_j^H)}.$$
(16)

 $\bar{I}_j$  is named as "effective leakage interference power" at PR<sub>j</sub> since it measures the power of interference normalized by that of noise after they are both processed by the receiver decoding matrix,  $B_j$ .

<u>Lemma</u> 5.1: The upper bounds on  $\overline{I}_j$ , j = 1, 2, are given as

$$\bar{I}_{j} \leq \frac{\operatorname{Tr}(\boldsymbol{C}_{CR})}{\alpha_{j}N} \frac{\lambda_{\max}(\boldsymbol{G}_{j}\boldsymbol{G}_{j}^{H})}{\lambda_{\min}(\boldsymbol{A}_{j}^{H}\boldsymbol{G}_{j}\boldsymbol{G}_{j}^{H}\boldsymbol{A}_{j})}.$$
(17)

Proof: Please refer to Appendix I.

From Lemma 5.1, it follows that the upper bound on  $\bar{I}_j$  is proportional to the CR transmit power  $P_{CR}$  or  $Tr(C_{CR})$ , but inversely proportional to  $\alpha_j$ , N, and the PR<sub>j</sub>'s average transmit power  $P_j$  (through  $A_j$ ). Some nice properties on the resulted leakage interference power by the proposed CB scheme based on the effective interference channel are listed as follows:

- *Ī<sub>j</sub>* is upper-bounded by a finite value provided that α<sub>j</sub> > 0.<sup>6</sup> Note that λ<sub>min</sub>(**A**<sup>H</sup><sub>j</sub>**G**<sub>j</sub>**G**<sup>H</sup><sub>j</sub>**A**<sub>j</sub>) > 0 if M<sub>t</sub> > d<sub>j</sub> and thus **A**<sup>H</sup><sub>j</sub>**G**<sub>j</sub> is a full-rank and fat matrix.
- $\bar{I}_j$  can be easily shown to be invariant to any scalar multiplication over  $G_j$ . Thus, the CR protects PR<sub>j</sub> regardless of its distance-dependent signal attenuation to CR-Tx.

<sup>6</sup>Note that the derived upper bound on  $\bar{I}_j$  is practically useful for  $\alpha_j$  to be a non-negligible positive number, since in the extreme case of  $\alpha_j = 0$ , PR<sub>j</sub> switches off its transmission over the whole learning period and as a result the CR is unable to listen anything from PR<sub>j</sub>.  Since for fixed N and P<sub>CR</sub> the upper bound on I
<sub>j</sub> is inversely proportional to α<sub>j</sub> and P<sub>j</sub>, PR<sub>j</sub> gets better protected if it transmits more frequently and/or with more power. This property is useful for the CR system to design a *fair* rule for distributing interferences among the coexisting PRs.

<u>Example</u> 5.1: In Figs. 4 (a) and 4 (b), numerical results on  $\bar{I}_j$ 's given in (16) as well as theoretical results on the upper bounds on  $\bar{I}_j$ 's given in (17) are compared for PR SNR being 15 dB and 0 dB, respectively. Note that  $P_1 = P_2 = P$  in this example and PR SNR is defined as  $P/\rho_0$ . For the PR, it is assumed that  $M_1 = M_2 = 1$ ,  $\alpha_1 = 0.3$ , and  $\alpha_2 = 0.6$ , while for the CR,  $M_t = 4$ ,  $P_{CR} = 100$ , and  $C_{CR}$  is designed based upon eigenmode transmission. 2000 random channel realizations are considered where the standard Rayleigh fading distribution is adopted. To clearly see the effect of N, we take the inverses of  $\bar{I}_j$ 's or their upper bounds for the vertical axis of each figure. It is observed that at high-SNR region, the theoretical and numerical results match well, and the interference powers are inversely linearly proportional to N. However, at low-SNR region, there exists big mismatch between the two results. This is reasonable since the first order approximation of (29) in Appendix I is inaccurate at low-SNR region. Nonetheless, the good news is that the inverse of interference power is observed to be still linearly proportional to N from the numerical results.

## C. Optimal Learning Time

At last, we study the leaning-throughput tradeoff for CR by determining the optimal learning time  $\tau$  for a given T to maximize the CR throughput, subject to both the interference-power constraints at PR terminals as well as the transmit-power constraint of the CR. It is assumed that the CR channel H is known at both CR-Tx and CR-Rx. From (7) with U replaced by  $\hat{U}$ , the maximum CR throughput is defined as

$$\frac{T-\tau}{T}\log\left|\boldsymbol{I}+\boldsymbol{H}\hat{\boldsymbol{U}}\boldsymbol{C}_{CR}\hat{\boldsymbol{U}}^{H}\boldsymbol{H}^{H}/\rho_{1}\right|$$
(18)

where the term  $(T - \tau)/T$  accounts for the throughput loss due to channel learning.

If peak transmit power constraint for the CR is adopted, we have  $\text{Tr}(C_{CR}) \leq P_{CR}$ , while if average transmit power constraint is adopted, we may allocate the total power for each block to the second phase transmission, resulting in  $\text{Tr}(C_{CR}) \leq \frac{T}{T-\tau}P_{CR}$ . Let  $\Gamma$  denote the prescribed effective interference-power constraint for  $\bar{I}_j$ 's defined in (16). Note that N is related with  $\tau$  by  $N = \tau/T_s$ , where  $T_s$  is the symbol period. From Lemma 5.1, it follows that it is sufficient for  $C_{CR}$  to satisfy the following inequality to ensure the given interference-power constraint,  $\Gamma$ :

$$\operatorname{Tr}(\boldsymbol{C}_{CR}) \le \gamma_j \tau, \ j = 1,2 \tag{19}$$

where

$$\gamma_j = \frac{\zeta_j \alpha_j \Gamma}{T_s} \frac{\lambda_{\min}(\boldsymbol{A}_j^H \boldsymbol{G}_j \boldsymbol{G}_j^H \boldsymbol{A}_j)}{\lambda_{\max}(\boldsymbol{G}_j \boldsymbol{G}_j^H)}$$
(20)

and  $\zeta_j$ ,  $\zeta_j \leq 1$ , is an additional margin that accounts for any analytical error (e.g., at low-SNR region in Example 5.1). In practice, the choice of  $\gamma_j$ 's in (20) depends on the calibration process at CR-Tx, based on prior knowledge of  $\zeta_j$ 's,  $\Gamma$ , and  $T_s$ , as well as the observed average signal power from PRs.

Let  $\gamma = \min(\gamma_1, \gamma_2)$ . Then, the interference-power constraints in (19) become equivalent to  $\operatorname{Tr}(C_{CR}) \leq \gamma \tau$ . The maximization of CR throughput is thus expressed as

(P1): 
$$\max_{\tau, \boldsymbol{C}_{CR}} \quad \frac{T - \tau}{T} \log \left| \boldsymbol{I} + \boldsymbol{H} \hat{\boldsymbol{U}} \boldsymbol{C}_{CR} \hat{\boldsymbol{U}}^{H} \boldsymbol{H}^{H} / \rho_{1} \right|$$
  
s.t. 
$$\operatorname{Tr}(\boldsymbol{C}_{CR}) \leq J, \quad \boldsymbol{C}_{CR} \succeq \boldsymbol{0}, \quad 0 \leq \tau < T$$

where  $J = \min(P_{CR}, \gamma \tau)$  for the case of peak transmit power constraint, while  $J = \min\left(\frac{T}{T-\tau}P_{CR}, \gamma \tau\right)$  for the case of average transmit power constraint.

For Problem (P1), it is noted that  $\hat{U}$  is related with  $\tau$ , which makes the maximization over  $\tau$  complicated. However, it can be verified that the matrix norm of  $\Delta U$  decreases in the order of  $O(1/\sqrt{\tau})$ , as compared to the norm of U. Therefore, the overall term  $\hat{U} = U + \Delta U$  in the objective function is dominated by U, and changes slowly with  $\tau$  when  $\tau$  is sufficiently large. Thus, we assume that the effect of  $\tau$  on  $\hat{U}$  is ignored in subsequent analysis, and will verify this assumption by simulations.

Let the EVD of  $\hat{\boldsymbol{U}}^{H}\boldsymbol{H}^{H}\boldsymbol{H}\hat{\boldsymbol{U}}$  be  $\boldsymbol{U}_{h}\boldsymbol{\Sigma}_{h}\boldsymbol{U}_{h}^{H}$ , where  $\boldsymbol{U}_{h}$  is a  $(M_{t}-d_{\text{eff}}) \times (M_{t}-d_{\text{eff}})$  unitary matrix and  $\boldsymbol{\Sigma}_{h} = \text{Diag}(\sigma_{h,1}^{2}, \ldots, \sigma_{h,M_{t}-d_{\text{eff}}}^{2})$ . W.l.o.g., we assume that  $\sigma_{h,i}^{2}$ 's are arranged in a descending order. Note that if  $(M_{t}-d_{\text{eff}}) > M_{r}$ , then  $\sigma_{h,i}$ 's,  $i = M_{r} + 1, \ldots, M_{t} - d_{\text{eff}}$ , all have zero values. Define  $\boldsymbol{X}$ as  $\boldsymbol{U}_{h}^{H}\boldsymbol{C}_{\text{CR}}\boldsymbol{U}_{h}$ . Problem (P1) is then converted to

$$\begin{array}{ll} (\mathrm{P2}): & \max_{\tau, \boldsymbol{X}} & \frac{T - \tau}{T} \log |\boldsymbol{I} + \boldsymbol{X} \boldsymbol{\Sigma}_h / \rho_1| \\ & \text{s.t.} & \mathrm{Tr}(\boldsymbol{X}) \leq J, \quad \boldsymbol{X} \succcurlyeq \boldsymbol{0}, \quad 0 \leq \tau < T \end{array}$$

where the optimal  $C_{CR}$  can be later recovered as  $U_h X U_h^H$ . By the standard approach like in [21, Chapter 10.5], it can be shown that the optimal X is a diagonal matrix  $X = \text{Diag}(x_1, \ldots, x_{M_t-d_{\text{eff}}})$  and  $x_i$ 's,  $i = 1, \ldots, M_t - d_{\text{eff}}$ , are obtained from

$$(P3): \max_{\tau,\{x_i\}} \frac{T-\tau}{T} \sum_{i=1}^{M_t-d_{\text{eff}}} \log\left(1 + \frac{\sigma_{h,i}^2 x_i}{\rho_1}\right)$$
  
s.t. 
$$\sum_{i=1}^{M_t-d_{\text{eff}}} x_i \leq J, \quad x_i \geq 0, \quad 0 \leq \tau < T.$$

Next, we will study Problem (P3) with peak and average transmit power constraint, respectively.

1) Peak CR power constraint: In this case, if  $P_{CR} > \gamma T$ , then J is always equal to  $\gamma \tau$ . Therefore, we consider the more general case when  $P_{CR} \leq \gamma T$ . The remaining discussion will then be divided into the following two parts for  $P_{CR}/\gamma < \tau < T$  and  $0 \leq \tau \leq P_{CR}/\gamma$ , respectively.

If  $P_{\rm CR}/\gamma < \tau < T$ , then  $J = P_{\rm CR}$  and the optimization in Problem (P3) over  $\tau$  and  $x_i$ 's can be separated. The optimization over  $x_i$ 's directly follows the conventional water-filling (WF) algorithm [21]. For the ease of later discussion, we define

$$f(z) = \max_{\{x_i\}} \sum_{i=1}^{M_t - d_{\text{eff}}} \log\left(1 + \frac{\sigma_{h,i}^2 x_i}{\rho_1}\right)$$
  
s.t. 
$$\sum_{i=1}^{M_t - d_{\text{eff}}} x_i \le z, \quad x_i \ge 0.$$
 (21)

The WF solution of the above optimization problem is then given as  $x_i = (\frac{1}{\mu} - \frac{\rho_1}{\sigma_{h,i}^2})^+$ , where  $\frac{1}{\mu}$  is the water level that should satisfy

$$\sum_{i=1}^{M_t - d_{\text{eff}}} \left( \frac{1}{\mu} - \frac{\rho_1}{\sigma_{h,i}^2} \right)^+ = z.$$
(22)

Denote  $q_k = \frac{k\rho_1}{\sigma_{h,k+1}^2} - \sum_{i=1}^k \frac{\rho_1}{\sigma_{h,i}^2}$ , for  $k = 0, \dots, M_t - d_{\text{eff}}$ . Obviously,  $q_0 = 0$ , and  $q_{M_t-d_{\text{eff}}} = +\infty$  since  $\sigma_{h,M_t-d_{\text{eff}}+1}^2$  is set to be zero. Then, we can express f(z) as

$$f(z) = \sum_{i=1}^{k} \log\left(\frac{\sigma_{h,i}^2}{k\rho_1}\left(z + \sum_{i=1}^{k}\frac{\rho_1}{\sigma_{h,i}^2}\right)\right), \quad z \in [q_{k-1}, q_k].$$

$$(23)$$

Note that k is the number of dimensions assigned with positive  $x_i$ 's. The objective function of Problem (P3) in this case can then be explicitly written as

$$g_1(\tau) \triangleq \frac{T - \tau}{T} f(P_{\rm CR}). \tag{24}$$

Since  $\frac{T-\tau}{T}$  is a decreasing function of  $\tau$ , the optimal  $\tau$  to maximize  $g_1(\tau)$  over  $P_{CR}/\gamma < \tau \leq T$  is simply  $P_{CR}/\gamma$ .

Next, consider  $0 \le \tau \le P_{\rm CR}/\gamma$ . In this case,  $J = \gamma \tau$ , and Problem (P3) becomes

$$\max_{0 \le \tau \le P_{\rm CR}/\gamma} \quad g_2(\tau) \triangleq \frac{T - \tau}{T} f(\gamma \tau).$$
(25)

In order to study the function  $g_2(\tau)$ , some properties of the function f(z) are given below.

<u>Lemma</u> 5.2: f(z) is a continuously increasing, differentiable, and concave function of z.

*Proof:* Please refer to Appendix II.

With Lemma 5.2, it can be easily verified that  $g_2(\tau)$  is also a continuous, differentiable, and concave function of  $\tau$ . Thus, the optimal value of  $\tau$ , denoted as  $\tau_2^*$ , to maximize  $g_2(\tau)$  can be easily obtained by, e.g., the Newton method [22].

To summarize the above two cases, the optimal solution of  $\tau$  for Problem (P3) in the case of peak transmit power constraint can be obtained as

$$\tau^* = \begin{cases} \tau_2^*, & \tau_2^* < P_{\rm CR}/\gamma \\ P_{CR}/\gamma, & \text{otherwise.} \end{cases}$$
(26)

The above solution is illustrated in Fig. 5. The optimal value of (P3) then becomes  $g_2(\tau_2^*)$  if  $\tau_2^* < P_{\rm CR}/\gamma$ , and  $g_1(P_{CR}/\gamma)$  otherwise.

2) Average CR power constraint: In this case, J in Problem (P3) takes the value of  $T/(T-\tau)P_{CR}$  if  $T/(T-\tau)P_{CR} < \gamma\tau$ , and  $\gamma\tau$  otherwise. It can be verified that  $T/(T-\tau)P_{CR} < \gamma\tau$  for some  $\tau$  in [0,T) only when  $P_{CR}/\gamma < T/4$ . In other words, if  $P_{CR}/\gamma \ge T/4$ , J always takes the value  $\gamma\tau$  regardless of  $\tau$ . Thus, the objective function of (P3) is always given as  $g_2(\tau)$ , and the optimal solution of  $\tau$  is  $\tau_2^*$ .

Therefore, we consider the more general case of  $P_{CR}/\gamma < T/4$  here. In this case, it can be shown that the equation  $T/(T-\tau)P_{CR} = \gamma\tau$  always has two positive roots of  $\tau$ , denoted as  $\tau_l$  and  $\tau_u$ , respectively, and  $0 \le \tau_l < \tau_u < T$ . If  $0 \le \tau \le \tau_l$  or  $\tau_u \le \tau < T$ , J takes the value of  $\gamma\tau$ , and then the maximum value of (P3) is obtained by the  $\tau$  that maximizes  $g_2(\tau)$  over this interval of  $\tau$ . Otherwise, the maximum value occurs when  $\tau$  is given as

$$\arg \max_{\tau,\tau_l < \tau < \tau_u} g_3(\tau) \triangleq \frac{T - \tau}{T} f\left(\frac{T}{T - \tau} P_{\rm CR}\right).$$
(27)

It can be shown that  $g_3(\tau)$  is a continuously decreasing function of  $\tau$ , for  $\tau \in [0, T)$ . Thus, the optimal value of  $\tau$  to maximize  $g_3(\tau)$  over this interval of  $\tau$  is simply  $\tau_l$ .

To summarize the above discussions, we obtain the optimal solution of  $\tau$  for Problem (P3) in the case of average transmit power constraint as

$$\tau^* = \begin{cases} \tau_2^*, & \tau_2^* < \tau_l \\ \tau_l, & \text{otherwise.} \end{cases}$$
(28)

The above solution is illustrated in Fig. 6. The optimal value of (P3) then becomes  $g_2(\tau_2^*)$  if  $\tau_2^* < \tau_l$ , and  $g_3(\tau_l)$  otherwise.

#### VI. SIMULATION RESULTS

In the section, we present the simulation results to demonstrate the performance of the proposed CB scheme under imperfect channel learning. The system parameters are taken as  $M_t = 6$ ,  $M_r = 3$ ,  $M_1 = 4$ , and  $M_2 = 2$ . Eigenmode transmission is considered for the PR with  $d_1 = d_2 = 2$ . The channels F,  $G_1$ ,  $G_2$ , and H are randomly generated from the standard Rayleigh fading distribution, and are then fixed in all the examples. The parameters  $\tau$  and T are normalized by the symbol period  $T_s$ . T is set as 1000, and the lowest value of  $\tau$  is set as 10 in all the examples. The CR capacity is measured in nats/complex dimension (dim.). The peak transmit-power constraint for the CR is assumed.

We first fix  $P_{\text{CR}}$  at CR-Tx as 100 and show the variations of the CR throughput as a function of  $\tau$ . Both theoretical results obtained in Section V-C where  $\hat{U}$  is not considered as a function of  $\tau$  and is replaced by the true value U, and numerical results where  $\hat{U}$  changes with  $\tau$  are shown in Fig. 7. The values of  $\gamma$  are taken as 0.2 and 0.6, respectively. From Fig. 7, the first observation is that the numerical and theoretical results almost merge with each other, which supports our previous assumption of ignoring  $\hat{U}$  to be a function of  $\tau$  during the optimization process. We also observe that the CR throughput for  $\gamma = 0.2$  and that for  $\gamma = 0.6$  start to merge when  $\tau$  is sufficiently large due to the fact that  $g_1(\tau)$  defined in (24) does not change with  $\gamma$ . However, the maximum CR throughput is observed to increase with  $\gamma$  because when the PRs can tolerate more leakage interference powers, the optimal learning time is reduced and the CR transmit power becomes less restricted, which in turn enhance the CR throughput.

We then display the maximum CR throughput versus  $P_{\rm CR}$ , or equivalently, the CR SNR, in Fig. 8 for different values of  $\gamma$ . Only the theoretical results are shown here. The first observation is that there exist thresholds on CR SNR, beyond which the maximum throughput cannot be improved for a given  $\gamma$ . This is because that when  $P_{\rm CR}$  is too large, the dominant constraint for throughput maximization becomes the interference-power constraint instead of transmit-power constraint. When this occurs, the intersection point  $P_{\rm CR}/\gamma$  in Fig. 5 moves towards T. Thus, the optimal value of  $\tau$  and the corresponding maximum throughput are determined from  $g_2(\tau)$  in (25), which is not related with  $P_{\rm CR}$ . Meanwhile, when  $\gamma$  increases, it is observed that the maximum throughput also increases, similarly like in Fig. 7.

Our last example shows the change of the optimal  $\tau$  as a function of  $P_{\text{CR}}$  or the CR SNR in Fig. 9, where only the theoretical results are shown. From Fig. 5, we know that when  $P_{\text{CR}}$  decreases, the intersection point moves towards zero. Thus, the curves of the optimal learning time for different  $\gamma$ 's all merge to the presumed minimum value for  $\tau$ ,  $\tau = 10$ , at low-SNR region. On the other side, the optimal values of  $\tau$  stop increasing at high-SNR region for a given  $\gamma$ , similarly as explained for Fig. 8. Moreover, the optimal  $\tau$  is observed to increase with the decreasing of  $\gamma$ .

#### VII. CONCLUDING REMARKS

Cognitive beamforming (CB) is a promising technology to enable high-rate CR transmission and yet provide effective interference avoidance at the coexisting PRs. The main challenge for implementing CB in practice is how to obtain the channel knowledge from CR transmitter to PRs. In this paper, we propose a new solution to this problem by utilizing the idea of effective interference channel, which can be efficiently learned at CR transmitter via blind/semiblind estimation over the received PR signals. Based on the effective interference channel, we then design a practical CB scheme to eliminate the effect of the resulted interferences on the PR transmissions. Furthermore, we show that with finite sample size for channel learning, there exists an optimal learning time to maximize the CR throughput.

The developed results in this paper can be easily extended to the case of multiple PR links. This is so because the proposed CB scheme is based on the effective interference channel that measures the space spanned by all the coexisting PR signals as a whole, and thus it works regardless of these PR signals coming from a single PR link or multiple PR links.

#### APPENDIX I

#### PROOF OF LEMMA 5.1

Define S = [s(1), ..., s(N)] and  $Y_s = AS$  where s(n)'s, n = 1, ..., N, and A are given in Section III. From [19, Appendix I], we know that the first order perturbation<sup>7</sup> to U due to the finite number of samples N and the additive noise  $Z \triangleq [z(1), ..., z(N)]$  can be approximated by

$$\Delta \boldsymbol{U} \triangleq \hat{\boldsymbol{U}} - \boldsymbol{U} \approx -(\boldsymbol{Y}_s^H)^{\dagger} \boldsymbol{Z}^H \boldsymbol{U}.$$
<sup>(29)</sup>

Since the discussions on  $\bar{I}_1$  and  $\bar{I}_2$  are similar, in the following we restrict our study on  $\bar{I}_1$ . From the conditions given in Proposition 4.1, we know that there exists a constant matrix  $W_1 \in \mathbb{C}^{d_2 \times d_1}$ , such that  $B_1 G_1 = W_1 A_1^H G_1$ . The average interference power,  $I_1$  defined in (15), is then re-expressed as

$$I_{1} \stackrel{(a)}{=} \mathbb{E}[\operatorname{Tr}(\boldsymbol{B}_{1}\boldsymbol{G}_{1}\hat{\boldsymbol{U}}\boldsymbol{C}_{CR}\hat{\boldsymbol{U}}^{H}\boldsymbol{G}_{1}^{H}\boldsymbol{B}_{1}^{H})]$$

$$\stackrel{(b)}{=} \mathbb{E}[\operatorname{Tr}(\boldsymbol{B}_{1}\boldsymbol{G}_{1}\Delta\boldsymbol{U}\boldsymbol{C}_{CR}\Delta\boldsymbol{U}^{H}\boldsymbol{G}_{1}^{H}\boldsymbol{B}_{1}^{H})]$$

$$\stackrel{(c)}{=} \mathbb{E}[\operatorname{Tr}(\boldsymbol{B}_{1}\boldsymbol{G}_{1}(\boldsymbol{Y}_{s}^{H})^{\dagger}\boldsymbol{Z}^{H}\boldsymbol{U}\boldsymbol{C}_{CR}\boldsymbol{U}^{H}\boldsymbol{Z}\boldsymbol{Y}_{s}^{\dagger}\boldsymbol{G}_{1}^{H}\boldsymbol{B}_{1}^{H})]$$

$$\stackrel{(d)}{=} \rho_{0}\operatorname{Tr}(\boldsymbol{C}_{CR})\mathbb{E}[\operatorname{Tr}(\boldsymbol{B}_{1}\boldsymbol{G}_{1}(\boldsymbol{Y}_{s}^{H})^{\dagger}\boldsymbol{Y}_{s}^{\dagger}\boldsymbol{G}_{1}^{H}\boldsymbol{B}_{1}^{H})]$$

$$\stackrel{(e)}{=} \rho_{0}\operatorname{Tr}(\boldsymbol{C}_{CR})\mathbb{E}[\operatorname{Tr}(\boldsymbol{W}_{1}\boldsymbol{A}_{1}^{H}\boldsymbol{G}_{1}(\boldsymbol{A}^{H})^{\dagger}(\boldsymbol{S}\boldsymbol{S}^{H})^{-1}\boldsymbol{A}^{\dagger}\boldsymbol{G}_{1}^{H}\boldsymbol{A}_{1}\boldsymbol{W}_{1}^{H})]$$

$$\stackrel{(f)}{\approx} \rho_{0}\operatorname{Tr}(\boldsymbol{C}_{CR})\operatorname{Tr}\left(\boldsymbol{W}_{1}[\boldsymbol{I},\boldsymbol{0}]\begin{bmatrix}\frac{1}{|\mathcal{N}_{1}|}\boldsymbol{I} & \boldsymbol{0}\\ \boldsymbol{0} & \frac{1}{|\mathcal{N}_{2}|}\boldsymbol{I}\end{bmatrix}\begin{bmatrix}\boldsymbol{I}\\\boldsymbol{0}\end{bmatrix}\boldsymbol{W}_{1}^{H}\right)$$

$$= \frac{\rho_{0}}{\alpha_{1}N}\operatorname{Tr}(\boldsymbol{C}_{CR})\operatorname{Tr}(\boldsymbol{W}_{1}\boldsymbol{W}_{1}^{H}) \qquad (30)$$

where (a) is via substituting (14) into (15) and using the independence of  $\hat{U}$  and  $t_{CR}(n)$ ; (b) is due to  $B_1G_1U = 0$ ; (c) is due to (29); (d) is due to independence of  $Y_s$  and Z and  $\mathbb{E}[Z^H X Z] = \rho_0 \operatorname{Tr}(X)I$  for any constant matrix X; (e) is due to the definitions of  $W_1$  and  $Y_s$ ; and (f) is approximately true since N is usually a large number.

From [20], we have

$$\operatorname{Tr}(\boldsymbol{W}_{1}\boldsymbol{A}_{1}^{H}\boldsymbol{G}_{1}\boldsymbol{G}_{1}^{H}\boldsymbol{A}_{1}\boldsymbol{W}_{1}^{H}) \geq \lambda_{\min}(\boldsymbol{A}_{1}^{H}\boldsymbol{G}_{1}\boldsymbol{G}_{1}^{H}\boldsymbol{A}_{1})\operatorname{Tr}(\boldsymbol{W}_{1}\boldsymbol{W}_{1}^{H})$$
(31)

$$\operatorname{Tr}(\boldsymbol{B}_{1}\boldsymbol{G}_{1}\boldsymbol{G}_{1}^{H}\boldsymbol{B}_{1}^{H}) \leq \lambda_{\max}(\boldsymbol{G}_{1}\boldsymbol{G}_{1}^{H})\operatorname{Tr}(\boldsymbol{B}_{1}\boldsymbol{B}_{1}^{H}).$$
(32)

By noting  $\boldsymbol{B}_1\boldsymbol{G}_1 = \boldsymbol{W}_1\boldsymbol{A}_1^H\boldsymbol{G}_1$ , from (31) and (32) it follows that

$$\operatorname{Tr}(\boldsymbol{W}_{1}\boldsymbol{W}_{1}^{H}) \leq \frac{\lambda_{\max}(\boldsymbol{G}_{1}\boldsymbol{G}_{1}^{H})\operatorname{Tr}(\boldsymbol{B}_{1}\boldsymbol{B}_{1}^{H})}{\lambda_{\min}(\boldsymbol{A}_{1}^{H}\boldsymbol{G}_{1}\boldsymbol{G}_{1}^{H}\boldsymbol{A}_{1})}.$$
(33)

Using (16), (30), and (33), the upper bound on  $\overline{I}_1$  given in (17) is obtained.

<sup>&</sup>lt;sup>7</sup>Note that the first order approximation is more valid at high-SNR region.

#### APPENDIX II

#### PROOF OF LEMMA 5.2

First, it is easily known that f(z) is an increasing function of z. Next, we prove the continuity, differentiability, and concavity of f(z), respectively.

### A. Continuity

From (23), it is known that in each section  $[q_{k-1}, q_k]$ , f(z) is obviously continuous. For boundary points of each section, we have

$$\lim_{z \to q_k^-} f(z) = \sum_{i=1}^k \log\left(\frac{\sigma_{h,i}^2}{\sigma_{h,k+1}^2}\right) = \lim_{z \to q_k^+} f(z), \quad k = 1, \dots, M_t - d_{\text{eff}} - 1.$$
(34)

Thus, f(z) is continuous at all the points.

# B. Differentiability

From (23), it is known that in each section  $[q_{k-1}, q_k]$ , f(z) is differentiable. For boundary points of each section, it can be verified that

$$\lim_{z \to q_k^-} \dot{f}(z) = \frac{\sigma_{h,k+1}^2}{\rho_1} = \lim_{z \to q_k^+} \dot{f}(z), \quad k = 1, \dots, M_t - d_{\text{eff}} - 1.$$
(35)

Therefore, f(z) is differentiable at all the points.

# C. Concavity

For a given z, f(z) is obtained by solving the optimization problem in (21), which can be easily verified to be a convex optimization problem [22]. Thus, the duality gap for this optimization problem is zero and f(z) can be equivalently obtained as the optimal value of the following min-max optimization problem:

$$f(z) = \min_{\mu:\mu \ge 0} \max_{\{x_i\}:x_i \ge 0} \sum_{i=1}^{M_t - d_{\text{eff}}} \log\left(1 + \frac{\sigma_{h,i}^2 x_i}{\rho_1}\right) - \mu\left(\sum_{i=1}^{M_t - d_{\text{eff}}} x_i - z\right)$$
(36)

$$= \min_{\mu:\mu \ge 0} \sum_{i=1}^{M_t - d_{\text{eff}}} \left( \log\left(\frac{\sigma_{h,i}^2}{\rho_1 \mu}\right) \right)^+ - \sum_{i=1}^{M_t - d_{\text{eff}}} \left( 1 - \frac{\rho_1 \mu}{\sigma_{h,i}^2} \right)^+ + \mu z$$
(37)

$$=\sum_{i=1}^{M_t-d_{\text{eff}}} \left( \log\left(\frac{\sigma_{h,i}^2}{\rho_1 \mu^{(z)}}\right) \right)^+ - \sum_{i=1}^{M_t-d_{\text{eff}}} \left(1 - \frac{\rho_1 \mu^{(z)}}{\sigma_{h,i}^2}\right)^+ + \mu^{(z)} z$$
(38)

where  $\mu^{(z)} \ge 0$  is the optimal dual variable for a given z. In fact, it can be shown that  $1/\mu^{(z)}$  is just the water level given in (22) corresponding to the total power z.

Denote  $\omega$  as any constant in [0, 1]. Let  $\mu^{(z_1)}$ ,  $\mu^{(z_2)}$ , and  $\mu^{(z_3)}$  be the optimal  $\mu$  for  $f(z_1)$ ,  $f(z_2)$ , and  $f(z_3)$ ,  $z_3 = \omega z_1 + (1 - \omega)z_2$ , respectively. For j = 1, 2, we have

$$f(z_j) = \sum_{i=1}^{M_t - d_{\text{eff}}} \left( \log\left(\frac{\sigma_{h,i}^2}{\rho_1 \mu^{(z_j)}}\right) \right)^+ - \sum_{i=1}^{M_t - d_{\text{eff}}} \left(1 - \frac{\rho_1 \mu^{(z_j)}}{\sigma_{h,i}^2}\right)^+ + \mu^{(z_j)} z_j \tag{39}$$

$$\leq \sum_{i=1}^{M_t - d_{\text{eff}}} \left( \log \left( \frac{\sigma_{h,i}^2}{\rho_1 \mu^{(z_3)}} \right) \right)^+ - \sum_{i=1}^{M_t - d_{\text{eff}}} \left( 1 - \frac{\rho_1 \mu^{(z_3)}}{\sigma_{h,i}^2} \right)^+ + \mu^{(z_3)} z_j \tag{40}$$

where the inequality is due to that  $\mu^{(z_3)}$  is not the optimal dual solution for j = 1, 2. Therefore,

$$\omega f(z_1) + (1-\omega)f(z_2) \le \sum_{i=1}^{M_t - d_{\text{eff}}} \left( \log\left(\frac{\sigma_{h,i}^2}{\rho_1 \mu^{(z_3)}}\right) \right)^+ - \sum_{i=1}^{M_t - d_{\text{eff}}} \left( 1 - \frac{\rho_1 \mu^{(z_3)}}{\sigma_{h,i}^2} \right)^+ + \mu^{(z_3)} z_3 \tag{41}$$

$$=f(z_3) \tag{42}$$

$$= f(\omega z_1 + (1 - \omega) z_2).$$
(43)

Thus, f(z) is a concave function of z.

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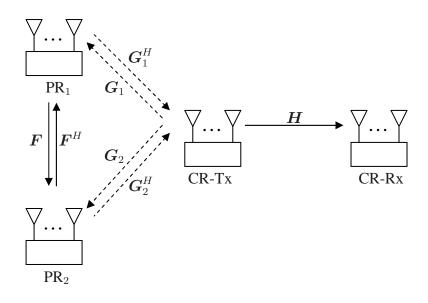


Fig. 1. Spectrum sharing between a CR link and a PR link.

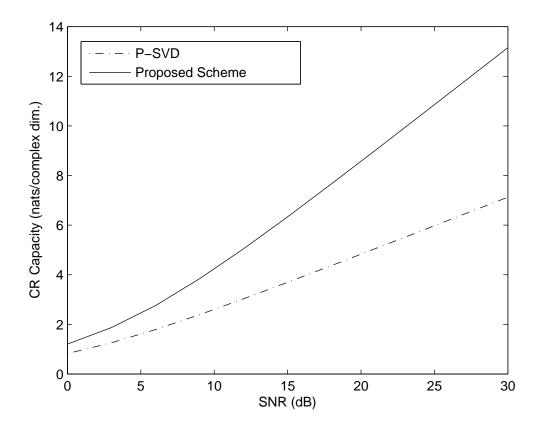


Fig. 2. CR capacity comparison for the proposed CB scheme and the P-SVD scheme in [14].

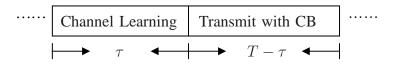
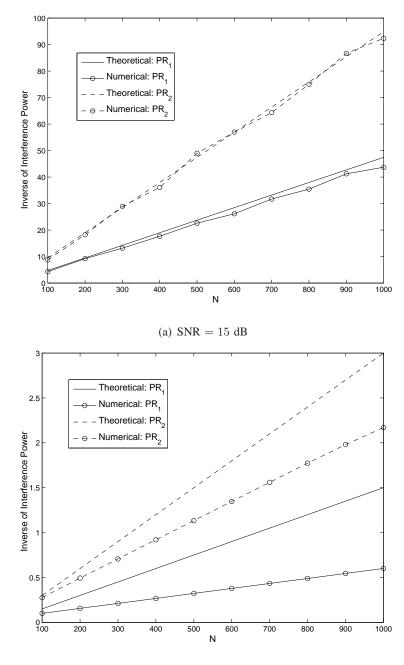


Fig. 3. Proposed two-phase transmission protocol for CR.



(b) SNR = 0 dB

Fig. 4. Leakage interference power levels at  $PR_1$  and  $PR_2$  for different PR SNRs.

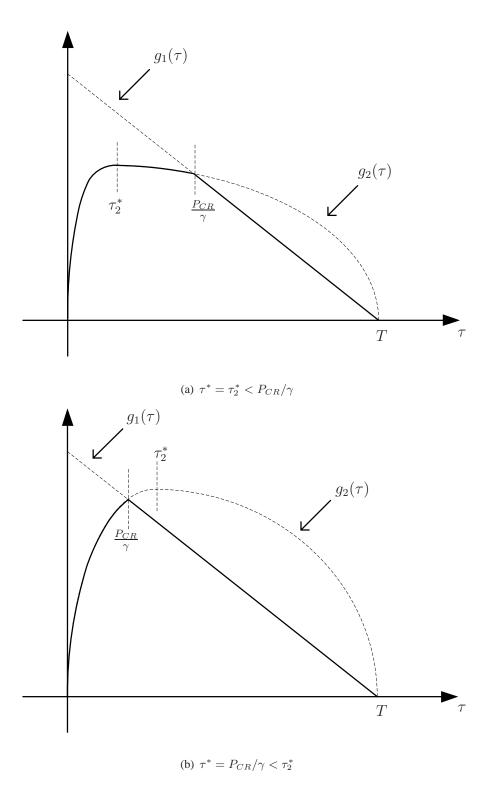


Fig. 5. Illustration of the optimal learning time  $\tau^*$  for the case of peak CR transmit-power constraint.

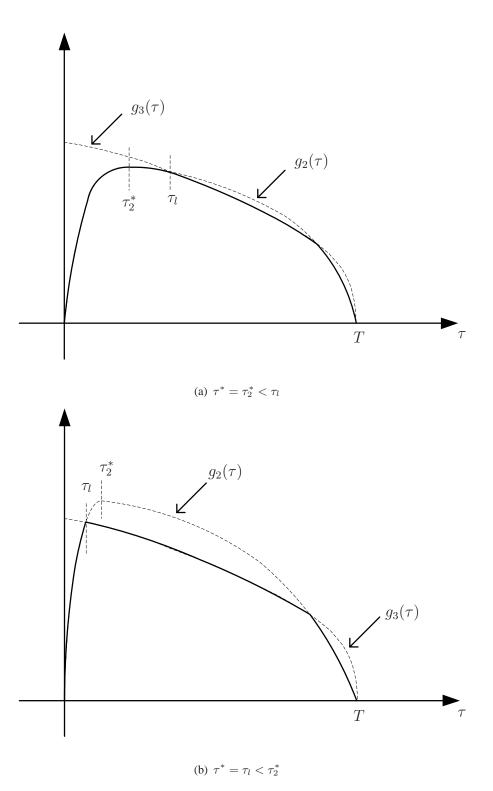


Fig. 6. Illustration of the optimal learning time  $\tau^*$  for the case of average CR transmit-power constraint.

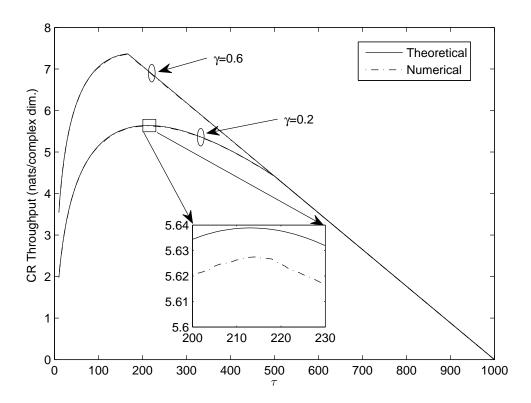


Fig. 7. CR throughput versus learning time.

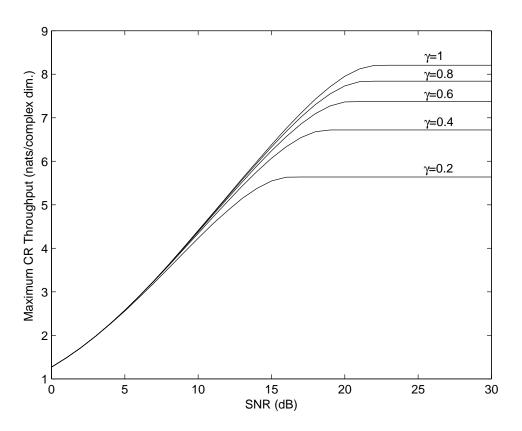


Fig. 8. Maximum CR throughput versus CR SNR.

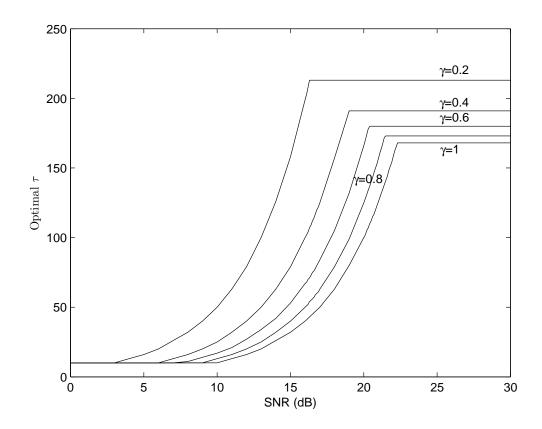


Fig. 9. Optimal learning time versus CR SNR.