Cognitive MAC Protocols Using Memory for Distributed Spectrum Sharing Under Limited Spectrum Sensing

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Abstract

The main challenges of cognitive radio include spectrum sensing at the physical (PHY) layer to detect the activity of primary users and spectrum sharing at the medium access control (MAC) layer to coordinate access among coexisting secondary users. In this paper, we consider a cognitive radio network in which a primary user shares a channel with secondary users that cannot distinguish the signals of the primary user from those of a secondary user. We propose a class of distributed cognitive MAC protocols to achieve efficient spectrum sharing among the secondary users while protecting the primary user from potential interference by the secondary users. By using a MAC protocol with one-slot memory, we can obtain high channel utilization by the secondary users while limiting interference to the primary user at a low level. The results of this paper suggest the possibility of utilizing MAC design in cognitive radio networks to overcome limitations in spectrum sensing at the PHY layer as well as to achieve spectrum sharing at the MAC layer.

Index Terms

Cognitive medium access control, cognitive radio networks, protocols with memory, spectrum sensing, spectrum sharing.

I. Introduction

Today's expanding demand for wireless services has necessitated cognitive radio technology in order to overcome the limitations of the conventional static spectrum allocation policy. Cog-

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nitive radio technology enables a more efficient use of limited spectrum resources by allowing unlicensed users (or secondary users) to opportunistically utilize licensed spectral bands. The main challenges of cognitive radio include *spectrum sensing* at the physical (PHY) layer to detect the activity of licensed users (or primary users) and *spectrum sharing* at the medium access control (MAC) layer to coordinate access among coexisting secondary users [1]. Spectrum sensing is needed to identify spectrum opportunities or spectrum holes, while spectrum sharing helps secondary users achieve an efficient and fair use of identified spectrum opportunities.

In this paper, we study a MAC protocol design problem for a cognitive radio network in which a primary user shares a spectral band (or a channel) with multiple secondary users. One of the main assumptions of our model is that the secondary users have limited spectrum sensing capability at the PHY layer in the sense that they are unable to distinguish between the activities (i.e., spectrum access) of the primary user and a secondary user. In other words, the secondary users can sense whether the channel is idle or busy, but when the channel is sensed busy, they do not know whether the channel is accessed by the primary user or not. This assumption contrasts with and is weaker than the prevailing assumption, made in previous work on MAC design for cognitive radio, that sensing at the PHY layer is perfect in that secondary users can always detect the presence of primary users (see, for example, [2],[3]). [4] relaxes the assumption of perfect spectrum sensing and considers sensing errors at the PHY layer. However, [4] requires that the signals of primary users be statistically distinguishable from those of secondary users. On the contrary, our assumption is valid when the signals of primary users are (statistically) indistinguishable from those of secondary users.

Another key assumption we maintain is that explicit coordination messages cannot be communicated between a central controller and a user, or between users. This implies that the primary user cannot broadcast its presence to the secondary users for spectrum sensing and that centralized scheduling schemes such as TDMA cannot be used for spectrum sharing. Again, this assumption contrasts with and is weaker than the assumption made in existing work that requires central controllers or dedicated control channels (see, for example, [2],[3]). As pointed out in [1], in cognitive radio networks, protocols requiring broadcast messages cause a major problem due to the lack of a reliable control channel as a channel has to be vacated whenever a primary user returns to the channel.

Our protocol design for the secondary users is based on MAC protocols with memory, which are formally presented in [5]. Under a protocol with memory, users adjust their transmission parameters depending on the local histories of their own transmission actions and feedback information. Hence, protocols with memory can be implemented in a distributed way without explicit message passing for any given sensing ability of users. Moreover, by exploiting information embedded in local histories, protocols with memory enable a secondary user to “change its transmitter parameters based on interaction with the environment in which it operates,” as demanded by the definition of cognitive radio [6].

In [5], we have focused on the problem of achieving coordinated access among symmetric users by using a protocol with memory. In a cognitive radio network, where a primary user exists, another kind of coordination is needed to ensure that the secondary users do not interfere with the primary user. In this paper, we show that a class of protocols with one-slot memory can achieve high channel utilization by the secondary users while protecting the primary user at a desired level. We also show that the system performance can be improved by utilizing longer memory. The results of this paper suggest that a carefully designed MAC protocol can be used in place of an algorithm for primary user detection at the PHY layer. The main contribution of this paper is to illustrate the possibility of utilizing MAC design to overcome limitations in spectrum sensing at the PHY layer as well as to achieve spectrum sharing at the MAC layer.

In recent years, there have been burgeoning research efforts involving cognitive radio networks. Due to space limitations, we review only a few of them, focusing on the most related work, and refer the interested reader to [1] for a comprehensive survey. [2] examines gains from spectrum agility in terms of spectrum utilization. Our model corresponds to the non-agile case of [2] as secondary users in our model stay in the same channel for the considered horizon of time. This is because our model is not equipped with ideal control devices as assumed in [2]. [3] uses a mechanism design approach to determine the allocation of spectrum opportunities to selfish secondary users. [4] analyzes the decision of secondary users to sense and access channels using a partially observable Markov decision process framework. [7] evaluates performance under two spectrum access schemes using different sensing, back-off, and transmission mechanisms. [8] develops a sensing-period optimization mechanism and an optimal channel-sequencing algorithm for efficient discovery of spectrum opportunities. [9] models the interactions between secondary users as a non-cooperative game and derives the price of anarchy. A survey on MAC protocols for cognitive radio networks is presented in [10]–[12].

The rest of this paper is organized as follows. In Section II, we describe our system model. In Section III, we formulate MAC protocols, performance metrics, and a protocol design problem. In Section IV, we explain how to compute the performance metrics for a given protocol, using Markov chains. In Section V, we solve the protocol design problem numerically. In Section VI, we discuss how the proposed protocols can be enhanced by utilizing longer memory. In Section VII, we conclude this paper.

II. System Model

We consider a licensed channel in a slotted Aloha-type network, as in [5] and [13], with a single primary user and N secondary users. We assume that N is fixed over time. Time is divided into slots of equal length, and the primary and secondary users maintain synchronized time slots. A user can attempt to transmit a packet or wait in a slot in which it has a packet to transmit. Due to interference, only one user can transmit successfully in a slot, and simultaneous transmission by more than one user results in a collision.

The traffic of the primary user arrives following a stochastic process. We assume that an arrival of traffic generates multiple packets, the average number of which is denoted by T_{pac}, and that the average time interval (measured in slots) between two consecutive arrivals of traffic, denoted by T_{int}, is larger than T_{pac}.[1] In each slot, the primary user has either a packet to transmit or none depending on traffic arrivals and transmission results. The state of the primary user, denoted by y_p, is said to be *on* if the primary user has a packet to transmit and *off* otherwise. A similar *on*-*off* model for the primary user can be found in [2] and [8].[2]

Each secondary user always has packets to transmit. After a user makes a transmission attempt, it learns whether the transmission is successful or not using an acknowledgement (ACK) response. The secondary users have the sensing ability to find out whether the channel is accessed or not while they wait. However, when the channel is sensed busy, they do not obtain information

[1] A scenario that fits into our assumptions is one where the primary user has bursty traffic.

[2] Under perfect sensing assumed in [2] and [8], the duration of *on* and *off* periods is independent of the existence of the secondary users because the secondary users can be required to back off when they sense the activity of the primary user. On the contrary, under limited sensing in our model, an *on* period becomes longer while an *off* period becomes shorter as the secondary users create more collisions with the primary user. This fact is taken into account in the objective of the protocol design problem formulated in Section III.

about whether the primary user accessed the channel or not. This assumption limits the ability of the secondary users to detect the presence of the primary user. Using the information from ACK responses and sensing, a secondary user can classify a slot into the four states, *idle*, *busy*, *success*, and *failure*, as in [14]. The state of secondary user i, denoted by y_i, is *idle* if no user transmits, *busy* if secondary user i does not transmit but at least one other user transmits, *success* if secondary user i transmits and succeeds, and *failure* if secondary user i transmits but fails.

III. Protocol Description and Problem Formulation

A. Protocol Description

1) Protocol for the Primary User: The decision rule for the primary user is to transmit whenever it has a packet to transmit. Note that the primary user does not need to modify its decision rule for coexistence with the secondary users, which is consistent with the requirements of cognitive radio networks.

2) Protocol for the Secondary Users: The decision rule for the secondary users is prescribed by a protocol with one-slot memory [5]. A protocol with one-slot memory specifies a transmission probability for each possible state of the previous slot, and thus it can be formally represented by a function $f : \mathcal{Y}_s \to [0, 1]$, where $\mathcal{Y}_s$ is the set of the states of a secondary user, i.e., $\mathcal{Y}_s = \{\textit{idle}, \textit{busy}, \textit{success}, \textit{failure}\}$. A secondary user whose state is $y \in \mathcal{Y}_s$ in the previous slot transmits with probability $f(y)$ in the current slot. We provide two definitions about the properties of a protocol with one-slot memory.

Definition 1: A protocol f with one-slot memory is *non-intrusive* if $f(\textit{busy}) = 0$.

When the secondary users follow a non-intrusive protocol, they wait in a slot following a busy slot. Thus, a non-intrusive protocol allows the primary user not to be interrupted by the secondary users once it has a successful transmission.

Definition 2: A protocol f with one-slot memory has the *fairness level* $\theta \in (0, 1]$ if the average number of consecutive successes by a secondary user while the primary user does not transmit is $1/\theta$, or

$$1 - f(\textit{success})(1 - f(\textit{busy}))^{N-1} = \theta. \tag{1}$$

Suppose that there is no transmission by the primary user. Once a secondary user succeeds, it has a successful transmission in the next slot with probability $f(\textit{success})(1 - f(\textit{busy}))^{N-1}$, and thus the average number of consecutive successes is given by $1/[1 - f(\textit{success})(1 - f(\textit{busy}))^{N-1}]$. As the fairness level is smaller, a secondary user keeps using the channel for a longer period once it succeeds, which makes other secondary users wait longer until they have a successful transmission. In [13], a protocol with fairness level θ is said to be M-short-term fair if $1/\theta \leq M$.

B. Performance Metrics

1) Collision Probability of the Primary User: In overlay spectrum sharing, it is important to protect the primary user from interruption by the secondary users. We measure interference experienced by the primary user by the collision probability of the primary user, defined as

$$P_c = \frac{\text{No. of collisions experienced by PU}}{\text{No. of transmission attempts by PU}},$$

where PU represents "primary user." That is, the collision probability of the primary user is the probability that it experiences a collision when it attempts to transmit a packet.

2) Channel Utilization of the Secondary Users: We measure the utilization of spectrum opportunities by the success probability of the secondary users, defined as

$$P_s = \frac{\text{No. of successes by SUs}}{\text{No. of slots in which PU is } \textit{off}},$$

where SU represents "secondary user." In other words, the success probability of the secondary users is the probability that a secondary user has a successful transmission when the primary user has no packet to transmit. The channel utilization (or throughput) of the secondary users is defined as the proportion of time slots in which a secondary user has a successful transmission, i.e.,

$$C_s = \frac{\text{No. of successes by SUs}}{\text{No. of slots}}.$$

3) Channel Utilization of the System: The channel utilization of the system is defined as the proportion of time slots in which a successful transmission occurs, i.e.,

$$C = \frac{\text{No. of successes}}{\text{No. of slots}}.$$

4) Computation of the Performance Metrics and Performance Bounds: We define an *on* period and an *off* period as a period in which the state of the primary user is *on* and *off*, respectively, between two consecutive arrivals of traffic. Let $T_{\textit{on}}$ and $T_{\textit{off}}$ be the average length (measured in slots) of an *on* period and an *off* period, respectively. Then the average time interval between two consecutive arrivals of traffic can be decomposed as $T_{int} = T_{\textit{on}} + T_{\textit{off}}$. Let T_{col} be the average number of collisions that the primary user experiences while transmitting packets generated by an arrival of traffic. We assume that $T_{col} < T_{int} - T_{pac}$ to assure the stability of the system. Since the primary user transmits whenever it has a packet to transmit, it has either a successful transmission or a collision when its state is *on*. Hence, an *on* period can be decomposed into slots in which the primary user succeeds and those in which it collides, i.e., $T_{\textit{on}} = T_{pac} + T_{col}$. Let T_s and $T_{\textit{ns}}$ be the average numbers of slots in which one and none, respectively, of the secondary users has a successful transmission between two consecutive arrivals of traffic. Given the protocol for the primary user and our contention model, a secondary user can have a successful transmission only when the state of the primary user is *off*. Thus, we can decompose an *off* period into slots in which a secondary user succeeds and those in which no secondary user succeeds, i.e., $T_{\textit{off}} = T_s + T_{\textit{ns}}$. Note that $T_{\textit{on}}$, $T_{\textit{off}}$, T_{col}, T_s, and $T_{\textit{ns}}$ are determined by the protocol and the traffic arrival process whereas T_{pac} and T_{int} are determined entirely by the traffic arrival process.

We explain how we (approximately) compute the performance metrics defined in this section. The collision probability of the primary user can be computed as $P_c = T_{col}/T_{\textit{on}}$ since the primary user transmits whenever its state is *on*. Also, the success probability of the secondary users can be computed as $P_s = T_s/T_{\textit{off}}$. The channel utilization of the primary user is given by $C_p = T_{pac}/T_{int}$, while that of the secondary users is $C_s = T_s/T_{int}$. The channel utilization of the system can be computed as $C = C_p + C_s = (T_{pac} + T_s)/T_{int}$.

When perfect control devices are available to broadcast the presence of the primary user and to schedule access by the secondary users as in [2], we can obtain $T_{col} = 0$ and $T_s = T_{\textit{off}}$. Thus, with control devices, we can achieve the maximum values of the performance metrics $\overline{C}_p = T_{pac}/T_{int}$, $\overline{C}_s = (T_{int} - T_{pac})/T_{int}$, and $\overline{C} = 1$. Note that the channel utilization of the primary user is not affected by the absence of control devices (as long as $T_{col} < T_{int} - T_{pac}$) although the primary user may experience increased delay as T_{col} becomes large due to contention between the primary user and the secondary users. The value of C_s becomes smaller as contention among the secondary users increases. The ratio of C to $\overline{C}$ can be used as a measure of inefficiency due to the absence of control devices.

C. Protocol Design Problem

We formulate a problem solved by the protocol designer to determine a protocol. We assume that the protocol designer considers only non-intrusive protocols with one-slot memory. Non-intrusiveness is a desirable property in that it prevents the secondary users from interrupting the primary user once the primary user obtains a successful transmission. We focus on protocols with one-slot memory because they are simple to design and implement. We also assume that the protocol designer has the most preferred fairness level $\theta \in (0, 1]$. Then non-intrusiveness together with fairness level θ implies that $f(\textit{success}) = 1 - \theta$ by (1), and the remaining elements of a protocol to be specified are transmission probabilities following an idle state and a failure state, denoted by $q = f(\textit{idle})$ and $r = f(\textit{failure})$, respectively. For simplicity, we call hereafter a non-intrusive protocol with one-slot memory having fairness level θ a θ-fair non-intrusive protocol.

The protocol designer aims to maximize the channel utilization of the system while keeping the collision probability of the primary user below a certain threshold level specified as $\eta \in (0, 1)$. The protection level η can be considered as a requirement imposed by the primary user or by spectrum regulators. The protocol design problem can be formally expressed as

$$\max_{f \in \mathcal{F}} C \text{ subject to } P_c \leq \eta,$$

where $\mathcal{F}$ is the set of all θ-fair non-intrusive protocols. Since T_{pac} and T_{int} are independent of the prescribed protocol, the protocol design problem can be rewritten as

$$\max_{(q,r) \in [0,1]^2} C_s = P_s \frac{T_{int} - T_{pac} - T_{col}}{T_{int}} \text{ subject to } T_{col} \leq \gamma, \tag{2}$$

where $\gamma = (\eta/(1-\eta))T_{pac}$ is the threshold level for T_{col}, derived from the relationship $P_c = T_{col}/(T_{pac} + T_{col})$ and the requirement $P_c \leq \eta$. Note that T_{col} appears both in the objective function and in the constraint. The protocol designer prefers small T_{col} for two reasons. Smaller T_{col} implies less interference to the primary user and at the same time longer *off* periods that the secondary users can utilize. In Section IV we explain how to compute P_s and T_{col} analytically given a θ-fair non-intrusive protocol, while in Section V we investigate the solution to the protocol design problem using numerical illustrations.

IV. Analytical Results

A. Derivation of the Success Probability of the Secondary Users

We first study the operation of the system in an *off* period, in which the primary user is inactive. To analyze performance in an *off* period, we construct a Markov chain whose state space is $\{0, 1, \ldots, N\}$, where state k represents transmission outcomes in which exactly k secondary users transmit. The transition probability from state k to state k' in an *off* period, denoted $P_{\textit{off}}(k'|k)$, under a θ-fair non-intrusive protocol is given by

$$P_{\textit{off}}(k'|0) = \binom{N}{k'} q^{k'}(1-q)^{N-k'} \quad \text{for } k' = 0, \ldots, N, \tag{3}$$

$$P_{\textit{off}}(k'|1) = \begin{cases} \theta & \text{for } k' = 0 \\ 1-\theta & \text{for } k' = 1 \\ 0 & \text{for } k' = 2, \ldots, N, \end{cases}$$

$$P_{\textit{off}}(k'|k) = \begin{cases} \binom{k}{k'} r^{k'}(1-r)^{k-k'} & \text{for } k' = 0, \ldots, k \\ 0 & \text{for } k' = k+1, \ldots, N \end{cases} \text{, for } k = 2, \ldots, N. \tag{4}$$

The transition matrix of the Markov chain can be written in the form of

$$\mathbf{P}_{\textit{off}} = \begin{array}{cc} & \begin{array}{cccccc} 0 & 2 & \cdots & N-1 & N & 1 \end{array} \\ \begin{array}{c} 0 \\ 2 \\ \vdots \\ N-1 \\ N \\ 1 \end{array} & \left(\begin{array}{ccccc|c} * & * & \cdots & * & * & * \\ * & * & \cdots & 0 & 0 & * \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ * & * & \cdots & * & 0 & * \\ * & * & \cdots & * & * & * \\ \hline \theta & 0 & \cdots & 0 & 0 & 1-\theta \end{array}\right) \end{array},$$

where the entries marked with an asterisk can be found in (3) and (4).

Consider a slot t in which the state of the primary user has changed from *on* to *off*, i.e., $y_p^{t-1} = on$ and $y_p^t = \textit{off}$, where y_p^τ is the state of the primary user in slot τ. Since such a transition can occur only if the primary user transmitted a packet successfully in slot $t-1$, it must be the case that $y_i^{t-1} = busy$ for every secondary user i, where y_i^τ is the state of secondary user i in slot τ. By non-intrusiveness, no secondary user transmits in slot t, and thus an *off* period always begins with an idle slot (state 0). Starting from an idle slot, the secondary users contend with each other until a secondary user obtains a success, i.e., state 1 is reached. When a secondary user obtains a success, it transmits with probability $1-\theta$ in the next slot while all the other secondary users wait. A period of consecutive successes by a secondary user ends with an idle slot, when the successful user waits. In short, an *off* period can be considered as the alternation of a contention period and a success period, which is continued until traffic arrives to the primary user. A success period consists of slots with consecutive successes by a secondary user, whereas a contention period begins with an idle slot and lasts until a secondary user succeeds. Since all the secondary users transmit with the same transmission probability following an idle slot, they have an equal chance of becoming a successful user for the following success period at the point when a contention period starts.

Let $\tilde{T}_s$ and $\tilde{T}_{ns}$ be the average duration (measured in slots) of a success period and a contention period, respectively. $\tilde{T}_s$ is determined by the fairness level θ, where the relationship is given by $\tilde{T}_s = 1/\theta$. Let $\mathbf{Q}_{off}$ be the N-by-N matrix in the upper-left corner of $\mathbf{P}_{off}$. Suppose that $0 < q, r < 1$ so that all the entries of $\mathbf{P}_{off}$ marked with an asterisk are nonzero. Then $(\mathbf{I}-\mathbf{Q}_{off})^{-1}$ exists and is called the fundamental matrix for $\mathbf{P}_{off}$, when state 1 is absorbing (i.e., $\theta = 0$) [16]. The average number of slots in state $k \neq 1$ starting from state 0 (an idle slot) is given by the $(1,k)$-entry of $(\mathbf{I}-\mathbf{Q}_{off})^{-1}$. Hence, the average number of slots to hit state 1 (a success slot) for the first time starting from an idle slot is given by the first entry of $(\mathbf{I}-\mathbf{Q}_{off})^{-1}\mathbf{e}$, where $\mathbf{e}$ is a column vector of length N all of whose entries are 1. Hence, we obtain $\tilde{T}_{ns} = [(\mathbf{I}-\mathbf{Q}_{off})^{-1}\mathbf{e}]_1$, where $[\mathbf{v}]_k$ denotes the k-th entry of vector $\mathbf{v}$. Note that $\tilde{T}_{ns}$ is independent of θ. That is, the average duration of a contention period is not affected by the average duration of a success period. The success probability of the secondary users can be computed by

$$P_s = \frac{\tilde{T}_s}{\tilde{T}_{ns}+\tilde{T}_s} = \frac{1}{\theta[(\mathbf{I}-\mathbf{Q}_{off})^{-1}\mathbf{e}]_1 + 1}, \tag{5}$$

for $(q,r) \in (0,1)^2$.

An alternative method to compute the success probability of the secondary users is to use a stationary distribution. Since $\theta \in (0,1]$, all states communicate with each other under the transition matrix $\mathbf{P}_{off}$ for all $(q,r) \in (0,1)^2$. Hence, the Markov chain is irreducible, and there exists a unique stationary distribution $\mathbf{w}_{off}$, which satisfies

$$\mathbf{w}_{off} = \mathbf{w}_{off}\mathbf{P}_{off} \text{ and } \mathbf{w}_{off}\mathbf{e} = 1. \tag{6}$$

Let $w_{off}(k)$ be the entry of $\mathbf{w}_{off}$ corresponding to state k, for $k = 0, 1, \ldots, N$. Then $w_{off}(k)$ gives the probability of state k during an *off* period. In particular, the success probability of the secondary users is given by $w_{\textit{off}}(1)$. Since contention and success periods alternate from the beginning of an *off* period, the stationary distribution yields the probabilities of states for any duration of an *off* period (assuming that $T_{\textit{off}}$ is sufficiently larger than $\tilde{T}_{\textit{ns}} + \tilde{T}_s$), not just the limiting probabilities as an *off* period lasts infinitely long. By manipulating (6), we can derive that $w_{\textit{off}}(1) = P_s$, whose expression is given in (5).

B. Derivation of the Collision Probability of the Primary User

We next study the operation of the system in an *on* period, in which the primary user always transmits. To analyze performance in an *on* period, we construct another Markov chain with the same state space $\{0, 1, \ldots, N\}$ as before. Again, state k corresponds to transmission outcomes in which exactly k secondary users transmit. The transition probability from state k to state k' in an *on* period, denoted $P_{\textit{on}}(k'|k)$, under a θ-fair non-intrusive protocol is given by

$$P_{\textit{on}}(k'|k) \quad = \quad \begin{cases} \binom{k}{k'} r^{k'}(1-r)^{k-k'} & \text{for } k' = 0, \ldots, k \\ 0 & \text{for } k' = k+1, \ldots, N \end{cases} \text{, for } k = 0, \ldots, N. \tag{7}$$

The transition matrix of the Markov chain can be written in the form of

$$\mathbf{P}_{\textit{on}} = \begin{array}{cc} & \begin{array}{cccccc} 1 & 2 & \cdots & N{-}1 & N & 0 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ \vdots \\ N{-}1 \\ N \\ 0 \end{array} & \left(\begin{array}{ccccc|c} * & 0 & \cdots & 0 & 0 & * \\ * & * & \cdots & 0 & 0 & * \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ * & * & \cdots & * & 0 & * \\ * & * & \cdots & * & * & * \\ \hline 0 & 0 & \cdots & 0 & 0 & 1 \end{array} \right) \end{array},$$

where the entries marked with an asterisk can be found in (7). Note that state 0, which corresponds to a success by the primary user, is absorbing because once the primary user has a successful transmission, its transmissions in the following slots are not interrupted by the secondary users. Hence, collisions in an *on* period occur only before the primary user obtains a successful transmission. Also, the average number of collisions experienced by the primary user in an *on* period, T_{col}, is independent of the length of traffic, T_{pac}. Let $\mathbf{Q}_{\textit{on}}$ be the N-by-N matrix in the upper-left corner of $\mathbf{P}_{\textit{on}}$. For $r \in [0, 1)$, the matrix $\mathbf{I} - \mathbf{Q}_{\textit{on}}$ is invertible, and the average number of slots until the first success by the primary user starting from state k is given by the k-th entry of $(\mathbf{I} - \mathbf{Q}_{\textit{on}})^{-1}\mathbf{e}$, for $k = 1, \ldots, N$.

Consider a slot t in which the state of the primary user has changed from *off* to *on*, i.e., $y_p^{t-1} = \textit{off}$ and $y_p^t = on$. Then an *on* period begins from slot t. The number of collisions that the primary user expect to experience in the *on* period depends on the transmission outcome in slot $t-1$, the last slot of the preceding *off* period. Suppose that there was a collision among $k \geq 2$ secondary users in slot $t-1$. Then the Markov chain starts from state k in slot $t-1$. Since the *on* period starts in slot t, the number of collisions in the *on* period does not include the collision in slot $t-1$. Hence, the average number of collisions until the first success in an *on* period when the preceding *off* period ended with k transmissions is given by

$$d(k) = [(\mathbf{I} - \mathbf{Q}_{\textit{on}})^{-1}\mathbf{e}]_k - 1,$$

for $k = 2, \ldots, N$.

Suppose that there was a success in slot $t-1$. Then the successful secondary user transmits with probability $1-\theta$ while all the other secondary users wait in slot t. Thus, with probability θ, the primary user succeeds in slot t, and with probability $1-\theta$, state 1 occurs in slot t, from which it takes $[(\mathbf{I} - \mathbf{Q}_{\textit{on}})^{-1}\mathbf{e}]_1$ collisions on average to reach a success by the primary user. Therefore, the average number of collisions until the first success in an *on* period when the preceding *off* period ended with a success is given by

$$d(1) = \theta \cdot 0 + (1-\theta)[(\mathbf{I} - \mathbf{Q}_{\textit{on}})^{-1}\mathbf{e}]_1 = (1-\theta)[(\mathbf{I} - \mathbf{Q}_{\textit{on}})^{-1}\mathbf{e}]_1. \tag{8}$$

Suppose that slot $t-1$ was idle. Then with probability $\binom{N}{k} q^k (1-q)^{N-k}$, slot t contains transmission by k secondary users, for $k = 0, \ldots, N$. With probability $(1-q)^N$ the primary user experiences no collision while with probability $\binom{N}{k} q^k (1-q)^{N-k}$ the *on* period begins with state k, for $k = 1, \ldots, N$. Therefore, the expected number of collisions until the first success in an *on* period when the preceding *off* period ended with an idle slot is given by

$$\begin{aligned} d(0) &= (1-q)^N \cdot 0 + \sum_{k=1}^{N} \binom{N}{k} q^k (1-q)^{N-k} [(\mathbf{I} - \mathbf{Q}_{\textit{on}})^{-1}\mathbf{e}]_k \\ &= \sum_{k=1}^{N} \binom{N}{k} q^k (1-q)^{N-k} [(\mathbf{I} - \mathbf{Q}_{\textit{on}})^{-1}\mathbf{e}]_k. \end{aligned}$$

The probability that the last slot of an *off* period has k transmissions is given by $w_{\textit{off}}(k)$, for $k = 0, 1, \ldots, N$. Hence, the average number of collisions that the primary user experiences before its first success in an *on* period is given by

$$T_{col} = \sum_{k=0}^{N} w_{\mathit{off}}(k) d(k).$$

Once the primary user succeeds in an *on* period, it has successful transmissions until it finishes transmitting all the packets it has, from which point an *off* period begins. Using the relationship $P_c = T_{col}/(T_{pac} + T_{col})$, we can compute the collision probability of the primary user. The operation of the system under a θ-fair non-intrusive protocol is summarized in Fig. 1.

V. Numerical Results

A. Graphical Illustration of the Protocol Design Problem

Based on the results in Section IV, we can show that, for a given fairness level $\theta \in (0, 1]$, P_s and T_{col} are continuous functions of (q, r) on the interior of $[0, 1]^2$. In order to guarantee the existence of a solution, in this section we consider the protocol design problem on a restricted domain,

$$\max_{(q,r)\in[\epsilon,1-\epsilon]^2} C_s = P_s \frac{T_{int} - T_{pac} - T_{col}}{T_{int}} \text{ subject to } T_{col} \leq \gamma, \tag{9}$$

for a small $\epsilon > 0$. Throughout this section, we set $\epsilon = 10^{-4}$. We say that a protocol is optimal if it solves (9). An optimal protocol gives an approximate, if not exact, solution to (2).

In Fig. 2, we show the dependence of the performance metrics, P_s, T_{col}, and C_s, on the protocol (q, r). To obtain the results, we consider a network with $N = 10$, $T_{int} = 100$, and $T_{pac} = 50$, and set $\theta = 0.1$. The maximum value of C_s is thus 0.5, while $\tilde{T}_s = 10$. Fig. 2(a) plots the contour curves of P_s. The success probability of the secondary users P_s is maximized at $q = 0.11$ and $r = 0.48$, and the maximum value of P_s is 0.804, which corresponds to the minimum value of $\tilde{T}_{ns}$ as 2.44. The value of (q, r) that maximizes P_s can be justified as follows. Following an idle slot in an *off* period, every secondary user transmits with probability q, and thus the probability of success is maximized when $q = 1/N$ [15]. During an *off* period, a collision cannot follow a success, and following an idle slot, a collision involving two transmissions is most likely among all kinds of collisions when $q \approx 1/N$. Since non-colliding users do not transmit following a collision under a non-intrusive protocol, the probability of success between two contending users is maximized when $r = 1/2$. r is chosen slightly smaller than $1/2$ because collisions involving more than two transmissions occur with small probability.

Fig. 2(b) plots the contour curves of T_{col}. As q and r are large, secondary users transmit aggressively in a contention period, intensifying interference to the primary user when it starts transmitting. Thus, T_{col} is increasing in both q and r. The set of (q, r) that satisfies the constraint $T_{col} \leq \gamma$ can be represented by the region below the contour curve of T_{col} at level γ. For example, the shaded area in Fig. 2(b) represents the constraint set corresponding to $T_{col} \leq 1$. Since $P_c = T_{col}/(T_{pac} + T_{col})$, P_c is monotonically increasing in T_{col}, and thus the contour curves of P_c have the same shape as those of T_{col}.

Fig. 2(c) plots the contour curves of C_s. Let $(q^*, r^*) = \arg\max_{(q,r)\in[\epsilon,1-\epsilon]^2} C_s$. That is, (q^*, r^*) represents the θ-fair non-intrusive protocol that maximizes the channel utilization of the secondary users when no constraint is imposed on the collision probability of the primary user. Note that the channel utilization of the secondary users can be expressed as $C_s = P_s \times P_{\mathit{off}}$, where P_{off} is the proportion of "*off*" slots, i.e., $P_{\mathit{off}} = T_{\mathit{off}}/T_{int} = (T_{int} - T_{pac} - T_{col})/T_{int}$. Hence, in order to maximize C_s, we need to take into account both P_s and P_{off}. To maximize P_s, (q, r) needs to be chosen at $(0.11, 0.48)$. Since P_{off} is decreasing in T_{col}, maximizing P_{off} requires (q, r) to be (ϵ, ϵ), at which T_{col} is minimized. In Fig. 2(c), it is shown that this conflict is resolved by choosing (q, r) somewhere in between. The protocol that maximizes the channel utilization of the secondary users is given by $(q^*, r^*) = (0.10, 0.37)$, while the maximum value of C_s is 0.390.

Fig. 3 shows the contour curves of C_s and T_{col} in the same graph to illustrate the protocol design problem (9). The protocol design problem is to find the largest value of C_s on the region of (q, r) that satisfies $T_{col} \leq \gamma$. Let γ^* be the value of T_{col} at (q^*, r^*). With the parameter specification to obtain Fig. 3, we have $\gamma^* = 1.376$. We say that a constraint is binding if its removal results in a strict improvement in the objective value and non-binding otherwise. Then the constraint in (9) is binding if $\gamma < \gamma^*$ and non-binding if $\gamma \geq \gamma^*$. For example, if $\gamma = 1$, the constraint is binding and the optimal protocol is given by the point on the contour curve of T_{col} at level 1, marked with '$+$' in Fig. 3, where a contour curve of T_{col} and that of C_s are tangent to each other. In contrast, if $\gamma = 2$, the constraint is non-binding and the optimal protocol is given by the solution to the unconstrained problem, $(q^*, r^*) = (0.10, 0.37)$, marked with '$\times$' in Fig. 3.

Fig. 4 shows the solutions to the protocol design problem for γ between 0.1 and 2. Fig. 4(a) plots optimal protocols, denoted by (q^o, r^o), as γ varies while Fig. 4(b) shows the values of

T_{col} and C_s at the optimal protocols. We can divide the range of γ into three regions: $(0, 0.8]$, $(0.8, 1.38)$, and $[1.38, \infty)$. For $\gamma \leq 0.8$, the optimal protocol occurs at the corner with $r^o = \epsilon$. As γ decreases in this region, q^o decreases to ϵ while r^o stays at ϵ, which makes C_s decrease to 0. Smaller γ means that transmissions by the primary user are less interfered, and this can be achieved by inhibiting transmissions by the secondary users. For $\gamma \in (0.8, 1.38)$, the solution to the protocol design problem is interior while the constraint $T_{col} \leq \gamma$ is still binding. The trade-off between T_{col} and C_s is less severe in this region than in $(0, 0.8]$. Reducing γ from 1.38 to 0.8 results in a slight decrease in C_s from 0.39 to 0.37. For $\gamma \geq 1.38$, the constraint $T_{col} \leq \gamma$ is non-binding, and thus (q^o, r^o) remains at $(q^*, r^*) = (0.10, 0.37)$ while C_s remains at its unconstrained maximum level, 0.39. The rate of change in the maximum value of C_s with respect to γ suggests that keeping T_{col} below 0.8 induces a large cost in terms of the reduced channel utilization, maintaining T_{col} between 0.8 and 1.38 only a minor cost, and tolerating T_{col} larger than 1.38 no cost. In other words, when the optimal solution to the protocol design problem is interior, the optimal dual variable on the constraint $T_{col} \leq \gamma$ is close to zero or is zero.

B. Varying the Number of Secondary Users

We study how the solution to the protocol design problem changes as the number of secondary users varies between 3 and 50. We fix other parameters of the model as before. We first solve the protocol design problem with a non-binding constraint, assuming that γ is sufficiently large. Fig. 5(a) shows optimal protocols (q^*, r^*) when the constraint is non-binding. As N increases from 3 to 50, q^* decreases from 0.33 to 0.02 while r^* increases from 0.36 to 0.37. Fig. 5(b) plots the values of T_{col} and C_s at (q^*, r^*). As N increases from 3 to 50, T_{col} increases from 1.36 to 1.38 while C_s decreases from 0.40 to 0.39. The results show that when the constraint is non-binding, the degree of contention increases with the number of the secondary users but only slightly, as the values of T_{col} and C_s are almost constant as N varies. Almost constant T_{col} implies that, even without a constraint on T_{col}, interruption to the primary user can be kept below a certain level. This is because under optimal protocols the primary user is likely to contend with at most two secondary users when it starts transmitting, regardless of the total number of the secondary users. Also, almost constant C_s implies that optimal protocols are capable of resolving contention among the secondary users efficiently even if there are many secondary users sharing the channel. The values of T_{col} at (q^*, r^*) can be interpreted as the minimum values of γ that make the constraint of the protocol design problem non-binding.

Now we set $\gamma = 1$ so that the constraint is binding for all N between 3 and 50. Fig. 5(a) shows optimal protocols (q^o, r^o) when the constraint is given by $T_{col} \leq 1$. As N increases from 3 to 50, q^o decreases from 0.30 to 0.02 while r^o increases from 0.16 to 0.17. Imposing the constraint limits the values of q and r, but it impacts r more than q, i.e., $q^o \approx q^*$ and $r^o < r^*$ for given N, due to the shape of the contour curves of C_s as illustrated in Fig. 3. Fig. 5(b) plots the values of T_{col} and C_s at (q^o, r^o). As N increases from 3 to 50, T_{col} stays at 1, confirming that the constraint $T_{col} \leq 1$ is binding, while C_s decreases from 0.39 to 0.38. Again, C_s is almost constant with respect to N even when a constraint is imposed on T_{col}. We can see that requiring $T_{col} \leq 1$ decreases the maximum values of C_s only slightly because the constraint with $\gamma = 1$ is mild so that the optimal protocols remain interior. If we impose a sufficiently strong constraint, i.e., choose a small γ, then we have the optimal protocol at the corner, $q^o < q^*$ and $r^o = \epsilon$, and C_s is reduced significantly, as suggested in Fig. 4.

C. Varying the Fairness Level

We investigate the impact of the fairness level on optimal protocols and their performance. We first consider sufficiently large γ so that the constraint is non-binding. Fig. 6(a) shows optimal protocols (q^*, r^*) as θ varies from 0.01 to 0.99 when the constraint is non-binding. As θ increases, q^* stabilizes around 0.10 quickly whereas r^* keeps increasing but at a diminishing rate. As θ is larger, contention periods occur more frequently during an *off* period, and thus it becomes more important to resolve contention among the secondary users quickly by having $r \approx 1/2$ when maximizing C_s. Fig. 6(b) plots the values of T_{col} and C_s at (q^*, r^*). As θ increases, T_{col} increases, reaches a peak at $\theta = 0.1$, and then decreases, whereas C_s decreases monotonically. The negative relationship between C_s and θ can be interpreted as a trade-off between channel utilization and short-term fairness.

Since T_{col} at (q^*, r^*) ranges between 1.00 and 1.37, we set $\gamma = 0.8$ to analyze the protocol design problem with a binding constraint. Fig. 6(a) shows optimal protocols (q^o, r^o) with $\gamma = 0.8$ while Fig. 6(b) plots the values of T_{col} and C_s at (q^o, r^o), as θ varies from 0.01 to 0.99. Note that the optimal protocols are at the corner with $r^o = \epsilon$ for $\theta \leq 0.09$. Imposing the constraint $T_{col} \leq 0.8$ limits the values of q and r. The differences between q^* and q^o and between r^* and

r^o are larger for smaller θ in the region $[0.1, 1]$ because requiring $T_{col} \leq 0.8$ imposes a stronger constraint for smaller θ, which can be seen by comparing the values of T_{col} with binding and non-binding constraints in that region. The impact of the constraint on C_s is marginal as long as the optimal protocols are interior.

D. Estimated Number of Secondary Users

Suppose that the protocol designer solves the protocol design problem for each possible N and prescribes the obtained protocols for the secondary users as a function of N. If the secondary users know the exact number of secondary users sharing the channel, an optimal protocol can be implemented. Here we consider a scenario where the secondary users choose an optimal protocol based on their (possibly incorrect) estimates of the number of secondary users. For simplicity, we assume that all the secondary users have the same estimate. We consider $N = 10$ and the estimated number of secondary users, denoted by $\hat{N}$, between 5 and 15. In Fig. 7, we plot the values of T_{col} and C_s when the N secondary users follow the optimal protocol computed assuming $\hat{N}$ secondary users. As before, we consider the two cases of non-binding and binding constraints, with $\gamma = 1$ for the binding constraint. In both cases, optimal q decreases with the estimated number of secondary users while optimal r is almost constant, as shown in Fig. 5(a). The overall interference level from the secondary users reduces as $\hat{N}$ increases, and thus T_{col} decreases with $\hat{N}$. C_s is not affected much by $\hat{N}$, reaching a peak when $\hat{N} = N$. This result suggests that channel utilization is robust to errors in the estimation of the number of secondary users. Note that, in the case of the binding constraint, the constraint is violated slightly when an underestimation occurs, i.e., $\hat{N} < N$. In order to offset this effect, the protocol designer can choose an estimation procedure that is biased toward overestimation, or specify a smaller γ than the required threshold.

VI. Enhancement Using Longer Memory

We have adopted protocols with one-slot memory for their simplicity. Protocols with one-slot memory not only are easy to design and implement but also allow us to use Markov chains to study performance. However, as illustrated in [5], it is possible to obtain performance improvement by utilizing longer memory. In this section, we explain how longer memory can help reduce the average number of collisions and bound the maximum number of collisions experienced by the primary user in an *on* period. Let p_i^τ be the transmission probability of secondary user i in slot τ. A protocol with B-slot memory that enhances a θ-fair non-intrusive protocol f can be expressed as follows:

(P1) If $y_i^{t-2} = success$ and $y_i^{t-1} = failure$, then $p_i^t = 0$.

(P2) If $y_i^{t-B} = \cdots = y_i^{t-1} = failure$, then $p_i^t = 0$.

(P3) Otherwise, $p_i^t = f(y_i^{t-1})$.

A. Reducing the Average Number of Collisions Experienced by the Primary User

(P1) requires that a secondary user that experiences a collision following a success back off. Note that a collision following a success cannot occur in an *off* period by non-intrusiveness, and thus (P1) does not affect performance in an *off* period. The only possible occasion in which a collision follows a success is when the primary user starts transmitting. Therefore, if a secondary user experiences a collision following a success, it can infer than an *on* period has started. According to a θ-fair non-intrusive protocol, a secondary user transmits with probability r after a collision, which yields $d(1) = (1-\theta)[(\mathbf{I} - \mathbf{Q}_{on})^{-1}\mathbf{e}]_1$ in (8). By imposing (P1), we can reduce the value to $d(1) = 1 - \theta$, which in turn reduces the value of T_{col}. For example, with $N = 10$, $T_{int} = 100$, $T_{pac} = 50$, $\theta = 0.1$, and $(q, r) = (q^*, r^*) = (0.10, 0.37)$, (P1) reduces $d(1)$ from 1.426 to 0.9 and T_{col} from 1.376 to 0.954.

B. Bounding the Maximum Number of Collisions Experienced by the Primary User

In the range of parameter values considered in Section V, the average number of collisions experienced by the primary user in an *on* period is reasonably small, not exceeding 1.5 slots, even without a constraint imposed on it. However, as colliding secondary users transmit with probability $r > 0$, the realized number of collisions in an *on* period can be arbitrarily large with positive probability. That is, the worst-case number of collisions in an *on* period is unbounded under a θ-fair non-intrusive protocol. We can bound the maximum number of collisions in an *on* period by imposing (P2), which requires a secondary user that experiences B consecutive collisions to back off. Since non-colliding secondary users wait after a collision, colliding secondary users must have the same number of consecutive collisions in any slot. Thus, secondary users experiencing B consecutive collisions back off simultaneously, yielding a slot that can be utilized by the primary user. Therefore, the primary user cannot experience more than B collisions in an *on* period. When B is chosen moderately large, B consecutive collisions rarely occur in an *off* period, and thus (P2) has a negligible impact on the success probability of the secondary users P_s while it reduces T_{col}. (P2) can be considered as a safety device to limit the number of collisions that the primary user can experience during an *on* period.

VII. Conclusion

In this paper, we have considered a scenario in which a primary user shares a channel with secondary users that cannot distinguish the signals of the primary user from those of a secondary user. We have shown that a class of distributed MAC protocols can coordinate access among the secondary users while restricting interference to the primary user, thereby overcoming the limited sensing ability of the secondary users at the PHY layer. The basic ideas underlying the proposed protocols can be exploited in different settings. For example, in a random access network with CSMA/CA, protocols with memory can be used to adjust the back-off parameters of secondary users based on their own transmission results and obtained channel information. Also, we can provide quality-of-service differentiation to secondary users by specifying different protocol parameters across secondary users. The fairness level for a secondary user determines the average number of its consecutive successes, while the transmission probabilities following an idle or a collision slot determine the probability that a secondary user is chosen as the successful user for the next success period in a contention period. Finally, the enhanced protocols with longer memory suggest the potential of observed patterns in history as a substitute for explicit information passing. As users make decisions based on history under a protocol with memory, users can adjust their behavior to the network environment or the states of other users by extracting information from history.

References

[1] I. F. Akyildiz, W.-Y. Lee, M. C. Vuran, and S. Mohanty, “NeXt generation/dynamic spectrum access/cognitive radio wireless networks: A survey,” *Comput. Networks*, vol. 50, no. 13, pp. 2127–2159, Sep. 2006.

[2] C.-T. Chou, S. Shankar N, H. Kim, and K.G. Shin, “What and how much to gain by spectrum agility?,” *IEEE J. Sel. Areas Commun.*, vol. 25, no. 3, pp. 576–588, Apr. 2007.

[3] A. R. Fattahi, F. Fu, M. van der Schaar, and F. Paganini, “Mechanism-based resource allocation for multimedia transmission over spectrum agile wireless networks,” *IEEE J. Sel. Areas Commun.*, vol. 25, no. 3, pp. 601–612, Apr. 2007.

[4] Q. Zhao, L. Tong, A. Swami, and Y. Chen, “Decentralized cognitive MAC for opportunistic spectrum access in ad hoc networks: A POMDP framework,” *IEEE J. Sel. Areas Commun.*, vol. 25, no. 3, pp. 589–600, Apr. 2007.

[5] J. Park and M. van der Schaar, “Medium access control protocols with memory,” *IEEE/ACM Trans. Netw.*, to be published.

[6] FCC, ET Docket No 03-222 Notice of proposed rule making and order, Dec. 2003.

[7] S. Huang, X. Liu, and Z. Ding, “Opportunistic spectrum access in cognitive radio networks,” in *Proc. INFOCOM*, 2008, pp. 1427–1435.

[8] H. Kim and K. G. Shin, “Efficient discovery of spectrum opportunities with MAC-layer sensing in cognitive radio networks,” *IEEE Trans. Mobile Computing*, vol. 7, no. 5, pp. 533–545, May 2008.

[9] L. Law, J. Huang, M. Liu and S. Li, “Price of anarchy for cognitive MAC games,” in *Proc. GLOBECOM*, 2009.

[10] H. Wang, H. Qin, and L. Zhu, “A survey on MAC protocols for opportunistic spectrum access in cognitive radio networks,” in *2008 Int. Conf. Comput. Sci. Software Eng.*, pp. 214–218.

[11] T. V. Krishna and A. Dasa, “A survey on MAC protocols in OSA networks,” *Comput. Networks*, vol. 53, no. 9, pp. 1377–1394, Jun. 2009.

[12] C. Cormio and K. R. Chowdhury, “A survey on MAC protocols for cognitive radio networks,” *Ad Hoc Networks*, vol. 7, no. 7, pp. 1315–1329, Sep. 2009.

[13] R. T. Ma, V. Misra, and D. Rubenstein, “An analysis of generalized slotted-Aloha protocols,” *IEEE/ACM Trans. Netw.*, vol. 17, no. 3, pp. 936–949, Jun. 2009.

[14] A. H. Mohsenian-Rad, J. Huang, M. Chiang, and V. W. S. Wong, “Utility-optimal random access without message passing,” *IEEE Trans. Wireless Commun.*, vol. 8, no. 3, pp. 1073–1079, Mar. 2009.

[15] J. L. Massey and P. Mathys, “The collision channel without feedback,” *IEEE Trans. Inf. Theory*, vol. 31, no. 2, pp. 192–204, Mar. 1985.

[16] C. M. Grinstead and J. L. Snell, *Introduction to Probability*. Providence, RI: AMS, 1997.

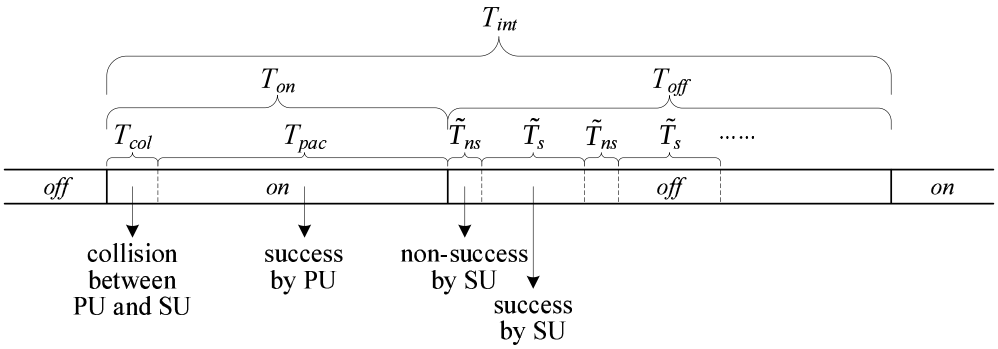
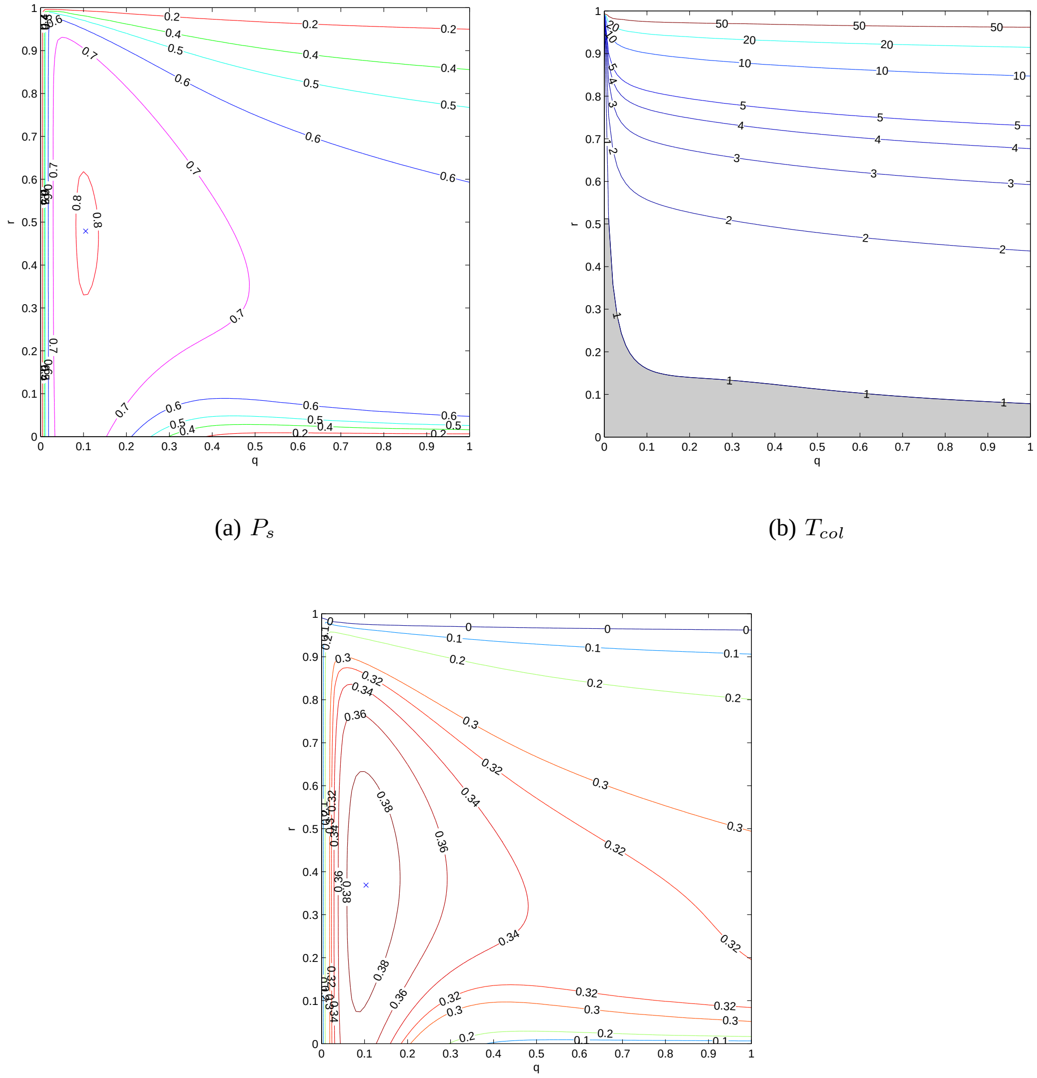


Fig. 1. Operation of the system under a θ-fair non-intrusive protocol.



(a) P_s

(b) T_{col}

(c) C_s

Fig. 2. Contour curves of P_s, T_{col}, and C_s as functions of (q, r) when $N = 10$, $T_{int} = 100$, $T_{pac} = 50$, and $\theta = 0.1$.

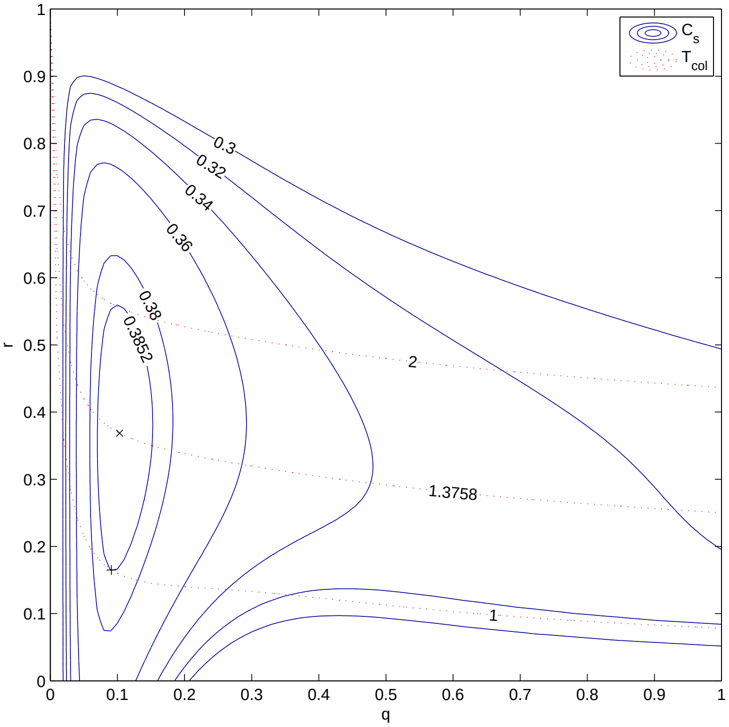
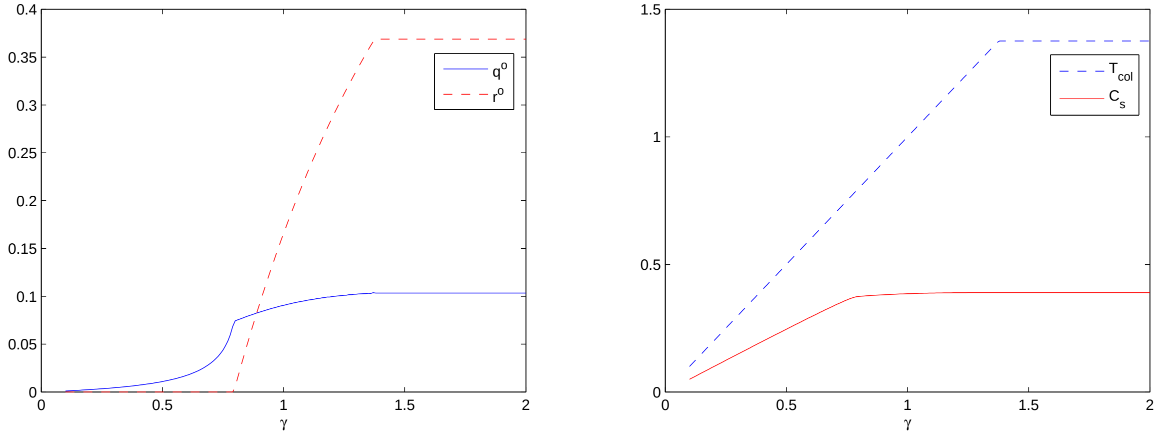


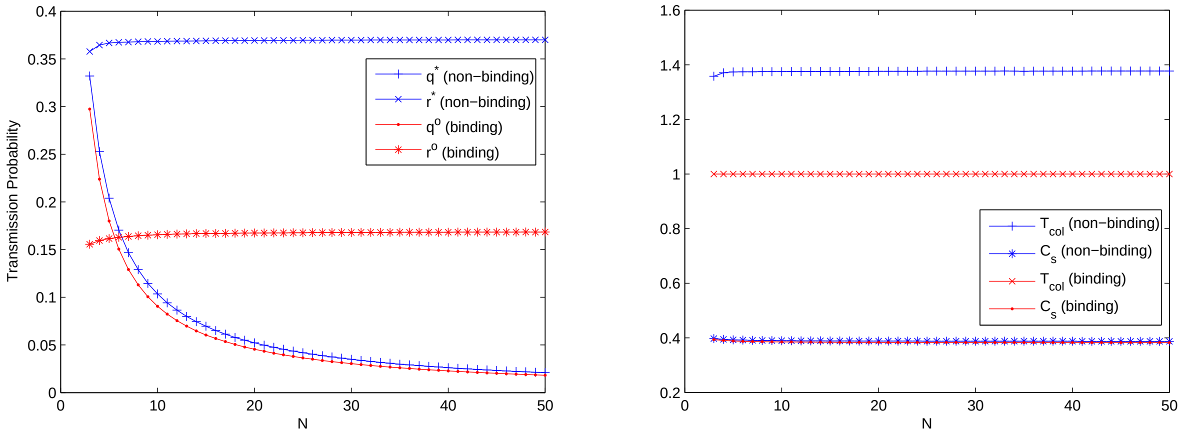
Fig. 3. Illustration of optimal protocols.



(a)

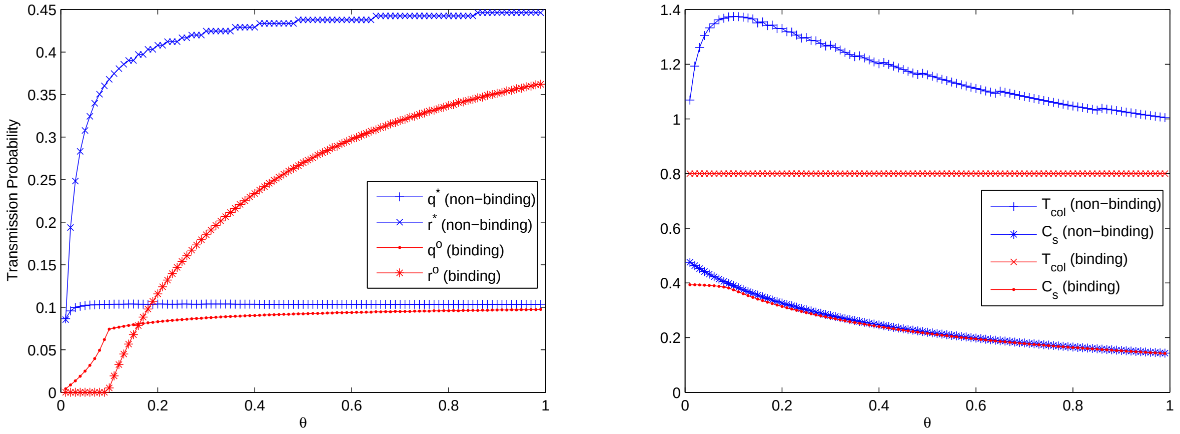
(b)

Fig. 4. Solution to the protocol design problem for γ between 0.1 and 2 when $N = 10$, $T_{int} = 100$, $T_{pac} = 50$, and $\theta = 0.1$: (a) optimal protocols, and (b) the values of T_{col} and C_s at the optimal protocols.



(a) (b)

Fig. 5. Solution to the protocol design problem for N between 3 and 50 when $T_{int} = 100$, $T_{pac} = 50$, and $\theta = 0.1$: (a) optimal protocols, and (b) the values of T_{col} and C_s at the optimal protocols.



(a) (b)

Fig. 6. Solution to the protocol design problem for θ between 0.01 and 0.99 when $N = 10$, $T_{int} = 100$, and $T_{pac} = 50$: (a) optimal protocols, and (b) the values of T_{col} and C_s at the optimal protocols.

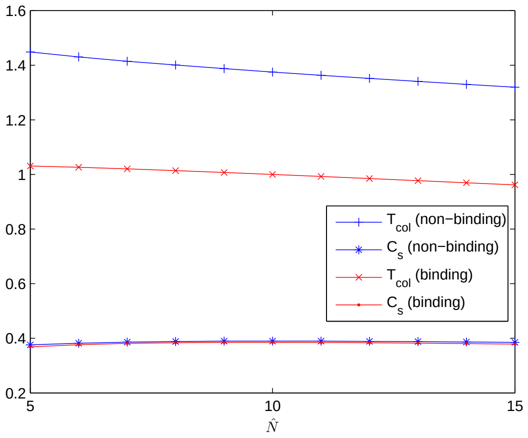


Fig. 7. Values of T_{col} and C_s for $\hat{N}$ between 5 and 15 when $N = 10$, $T_{int} = 100$, $T_{pac} = 50$, and $\theta = 0.1$.