

# Iterative Spectrum Shaping with Opportunistic Multiuser Detection

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## Abstract

This paper studies a new decentralized resource allocation strategy, named *iterative spectrum shaping* (ISS), for the multi-carrier-based multiuser communication system, where two coexisting users independently and sequentially update transmit power allocations over parallel subcarriers to maximize their individual transmit rates. Unlike the conventional iterative water-filling (IWF) algorithm that applies the single-user detection (SD) at each user's receiver by treating the interference from the other user as additional noise, the proposed ISS algorithm applies multiuser detection techniques to decode both the desired user's and interference user's messages if it is feasible, thus termed as *opportunistic multiuser detection* (OMD). Two encoding methods are considered for ISS: One is *carrier independent encoding* where independent codewords are modulated by different subcarriers for which different decoding methods can be applied; the other is *carrier joint encoding* where a single codeword is modulated by all the subcarriers for which a single decoder is applied. For each encoding method, this paper presents the associated optimal user power and rate allocation strategy at each iteration of transmit adaptation. It is shown that under many circumstances the proposed ISS algorithm employing OMD is able to achieve substantial throughput gains over the conventional IWF algorithm employing SD for decentralized spectrum sharing. Applications of ISS in cognitive radio communication systems are also discussed.

## Index Terms

Spectrum sharing, interference channel, multi-carrier systems, decentralized resource allocation, multiuser detection, iterative water-filling, cognitive radio.

## I. INTRODUCTION

This paper is concerned with spectrum sharing in a multiuser communication system based on multi-carrier modulation techniques such as discrete multitone (DMT) for wired-line communication and orthogonal frequency division multiplexing (OFDM) for wireless communication. It is assumed that neither the users' transmitters nor their receivers are collocated and as a result there is no centralized control over the users' transmissions. In addition, all users are assumed to transmit over the same frequency band and thus possibly interfere with each other. The above scenario exists in many wire-line/wireless

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broadband communication systems in practice, e.g., the DMT-based digital subscriber line (DSL) network, and the OFDM-based wireless *ad hoc* network.

The system of interest is in nature a competitive environment due to the lack of cooperation among the users. Therefore, decentralized strategies for allocation of users' transmit resources such as powers, bit rates, bandwidths, and/or antenna beams become crucial to the achievable system throughput. Consequently, a great deal of valuable scholarly work has been done in the literature on this study. For the conventional narrow-band spectrum sharing over single-antenna slow-fading channels, distributed transmit power control has been studied in, e.g., [1]–[4], for minimizing the sum power consumption to meet with each individual user's quality-of-service (QoS) requirement. Following the similar problem formulation, decentralized joint power control and beamforming have been studied in, e.g., [5]–[7] for the case of multi-antenna transceivers. In [8], a decentralized power allocation strategy so-called *iterative water-filling* (IWF) was proposed for a 2-user DSL system, where each of the two users independently and sequentially updates transmit power levels over different subcarriers so as to maximize individual transmit rate, subject to the coexisting user's interference treated as additional background noise at the receiver. Because of its practical advantages for implementation, the IWF algorithm has been thoroughly investigated in the subsequent literature. For example, in [9], [10], IWF has been studied for spectrum sharing scenarios with more than two users. In [11], [12], conditions on the convergence of IWF have been rigorously characterized. Motivated by IWF, semi-centralized and centralized power allocation schemes for multiuser spectrum sharing have also been studied in [13]–[15] and [16]–[18], respectively, all based on the primal-dual Lagrange duality approach.

The existing works on decentralized/centralized resource allocation schemes for multiuser spectrum sharing [1]–[18] have mostly assumed the *single-user detection* (SD) at the receiver by treating the interference from the other coexisting users as additional noise, mainly because of implementation ease of the proposed schemes. During the past decade, multiuser detection techniques (see, e.g., [19] and references therein) have been thoroughly studied in the literature, and proved under many circumstances to be able to provide substantial performance gains such as rate improvement and decoding error reduction over the conventional SD. This fact motivates this paper to make an attempt to combine the well-known IWF with multiuser detection such that at each iteration of user transmit adaptation, the corresponding

user is able to decode both the desired message and some/all of the interference users' messages – thereby reducing the overall interference at the receiver – if such decoding is feasible, thus termed as *opportunistic multiuser detection* (OMD). The resultant new decentralized resource allocation algorithm is named *iterative spectrum sharing* (ISS). Note that the proposed ISS maintains the main advantage of IWF to be a purely decentralized algorithm, while it improves over IWF via replacing the SD by the more advanced OMD. With OMD, the transmission of the updating user at each iteration subject to concurrent transmissions of the other coexisting users can be generally modeled by the Gaussian multiple-access channel (MAC) [20], whereas there is a key difference pointed out as follows. Unlike the conventional MAC, the coexisting users considered in this paper are non-cooperative in allocating transmit rates/powers over subcarriers due to the lack of centralized control over their transmissions. As a result, whether OMD should be applied and over which subset of users it should be applied depend on the instantaneous channel gains as well as the interference users' power and rate allocations. Note that the OMD in the context of this paper is analogous to the “successive group decoder (SGD)” in the fading MAC with unknown channel state information (CSI) at the user transmitters (see, e.g., [21] and references therein). The main contributions of this paper are summarized as follows:

- This paper considers two encoding methods for the proposed ISS. One is *carrier joint encoding* (CJE) where a single codeword is modulated by all the subcarriers and is decoded at the receiver by a single decoder. The other encoding method is designed to maximally exploit the advantage of OMD, named *carrier independent encoding* (CIE), where independent codewords are modulated by different subcarriers and thus allow for variable rate assignments and adaptive decoding methods. For both encoding methods, this paper derives the optimal user power allocation strategies to maximize individual transmit rate at each iteration. The derived power allocation schemes are shown to be non-trivial extensions of the standard “water-filling” (WF) power control [20] for IWF.
- This paper investigates the converged user power spectrums by the proposed ISS, and compares them to those by IWF for various system setups. Such comparison reveals some important insights on why ISS is able to outperform IWF in terms of the achievable system throughput for decentralized spectrum sharing.

The rest of this paper is organized as follows. Section II presents the system model of multi-carrier-

based multiuser spectrum sharing. Section III provides the problem formulations to determine the optimal user power allocation policies for the proposed ISS with CIE and CJE. Section IV presents the solutions to the formulated problems. Section V provides the simulation results to demonstrate the performance gains of ISS over IWF. Finally, Section VI concludes the paper.

## II. SYSTEM MODEL

Consider a typical spectrum sharing scenario where  $K$  users transmit independent messages to their corresponding receivers simultaneously over the same frequency band. For the purpose of exposition, in this paper it is assumed that  $K = 2$ , while the general case of  $K > 2$  is to be studied in the future work. Both the users are assumed to adopt a multi-carrier (DMT/OFDM) -based transmission and have the same symbol period and cyclic prefix (CP) period that is assumed to be larger than the maximal signal multipath spread of the two users. The total bandwidth for spectrum sharing is equally divided into  $N$  orthogonal sub-channels. For the time being, it is assumed that perfect time and frequency synchronization with reference to a common clock system have been established for both the users prior to their data transmission. In addition, it is assumed that the difference between the propagation delays from the two user transmitters to either one of their receivers is much smaller than the CP period and, thus, such delay differences can be safely accommodated within the CP period. Consider a block-based transmission for the two users with each block consisting of  $L$  DMT/OFDM symbols, while  $L$  is usually a large number to guarantee sufficient coding protection within each block transmission. For typical wireless applications, it is also assumed that the block duration is sufficiently small as compared to the coherence time of any channel between the users. Thus, all the channels involved in this paper can be assumed to be block fading (BF), i.e., they are constant during each block transmission but can vary from block to block. Based on the standard DMT/OFDM modulation and demodulation, the discrete-time baseband signals for the system of interest are given by

$$\begin{aligned} y_{1,n} &= \tilde{h}_{11,n}x_{1,n} + \tilde{h}_{21,n}x_{2,n} + z_{1,n} \\ y_{2,n} &= \tilde{h}_{22,n}x_{2,n} + \tilde{h}_{12,n}x_{1,n} + z_{2,n} \end{aligned} \quad (1)$$

where  $n = 1, \dots, N$  is the subcarrier index;  $x_{i,n}$  and  $y_{i,n}$  are the transmitted signal and received signal at subcarrier  $n$ , respectively, for user  $i = 1, 2$ ;  $\tilde{h}_{11,n}$  and  $\tilde{h}_{22,n}$  are the ‘‘direct’’ channel complex coefficients

for user 1 and 2, respectively, at subcarrier  $n$ , while  $\tilde{h}_{21,n}$  and  $\tilde{h}_{12,n}$  are the “interference” channel complex coefficients from user 2 to 1, and from user 1 to 2, respectively, at subcarrier  $n$ ; and  $z_{i,n}$  is the receiver noise at subcarrier  $n$  for user  $i = 1, 2$ . Note that both the block and symbol indexes are dropped in (1) for conciseness. Without loss of generality, it is assumed that  $\{z_{i,n}\}, \forall i, n$  are independent circularly symmetric complex Gaussian (CSCG) random variables (RVs) each having zero mean and unit variance. It is also assumed that  $x_{i,n}$ ’s are independent RVs each with zero mean and respective variance  $p_{i,n}$ , while  $p_{i,n}$  denotes the transmit power allocated to subcarrier  $n$  of user  $i$ . Let  $P_1$  and  $P_2$  denote the average transmit power constraint for user 1 and 2, respectively. It thus holds that  $\frac{1}{N} \sum_{n=1}^N p_{i,n} \leq P_i, i = 1, 2$ .

Two encoding methods are considered at each user transmitter. One is *carrier independent encoding* (CIE), where each subcarrier is assigned an independent codebook and from each codebook a codeword is chosen to be modulated into  $L$  consecutive DMT/OFDM symbols at the corresponding subcarrier in each block. At the receiver,  $N$  independent decoders are used to decode the corresponding messages from different subcarriers. Let  $r_{i,n}$  denote the rate of the codebook assigned to user  $i$  at subcarrier  $n$ . The average transmit rate of user  $i$  then becomes  $R_i^{\text{CIE}} = \frac{1}{N} \sum_{n=1}^N r_{i,n}$ . The other encoding method is *carrier joint encoding* (CJE), where a single codebook is used for each block transmission and only one codeword is chosen from this codebook and is modulated into all  $N$  subcarriers of  $L$  DMT/OFDM symbols. At the receiver, a single decoder is used to decode the message from all the subcarriers. Let  $R_i^{\text{CJE}}$  denote the rate of this single codebook for user  $i$ . Comparing CIE and CJE, it is easily seen that CIE requires more encoding and decoding complexities over CJE, due to the use of independent codebooks over different subcarriers. In addition, for the same finite value of  $L$ , the effective codeword length for CIE is reduced by a factor  $1/N$  as compared to that for CJE, thus resulting in inferior error-correcting capabilities. Therefore, the existing multi-carrier-based transmission systems in practice have all chosen to use CJE instead of CIE. Nevertheless, it is worth noticing that CIE provides more flexibility over CJE in adaptive rate assignments and decoding methods over subcarriers, which, as will be shown later in this paper, can be a beneficial factor for the proposed ISS under certain circumstances.

The system model considered in this paper is known as the *2-user parallel Gaussian interference channel*, for which characterization of the capacity region is in general still an unsolved problem (see, e.g., [22] and references therein). Nevertheless, achievable rates of this channel have been thoroughly

studied in the literature based on different assumptions on the level of cooperations between the users for encoding and decoding as well as power and rate allocations over the subcarriers. In this work, we constrain our study on this channel by making the following major assumptions:

- Each of the two users only has the knowledge on its own channel as well as the channel from the other user's transmitter to its receiver.
- Each of the two users *independently* and *sequentially* updates its transmit power allocations over different subcarriers to maximize individual transmit rate.
- Each of the two users is able to obtain the knowledge on transmit rates/rate (for CIE/CJE) of the other user over subcarriers; and both the users employ the same type of encoding method (CIE or CJE) and the same set of codebooks. Thereby, at one user's receiver, it is possible to apply multiuser detection (MD) to decode both the desired user's message and the interference user's message.

Note that in the above assumptions, the first two are due to practical considerations and are same as those made by the conventional IWF proposed in [8],<sup>1</sup> while the third assumption is a new one and is not present in IWF where only the single-user detection (SD) is applied. The decentralized resource allocation scheme motivated by IWF while employing the more advanced MD is named *iterative spectrum sharing* (ISS) in this paper.

### III. PROBLEM FORMULATION

In this section, problem formulations are provided for the users to determine their transmit power and rate allocations over different subcarriers at each iteration of transmit adaptation. Both encoding methods, namely, CIE and CJE, are considered. For brevity, only user 1's transmit adaptation is addressed here, while the developed results also apply to user 2.

Consider first CIE. At a particular iteration for user 1 to update its transmission, since user 2's transmit powers  $\{p_{2,n}\}$  and rates  $\{r_{2,n}\}$  over different subcarriers are fixed values, the maximum transmit rate of user 1 at subcarrier  $n$  with an arbitrary allocated transmit power  $p_{1,n}$  can be expressed as<sup>2</sup>

<sup>1</sup>More precisely, in the first assumption on the known interference channel between the users, only the channel gain is to be known for SD of IWF while both the channel gain and phase information are required for MD of the proposed scheme.

<sup>2</sup>For the purpose of exposition, continuous rate and power values are assumed in this paper. In addition, it is assumed that the optimal Gaussian codebook is employed by the two users. The developed results in this paper are readily extended to the more practical cases with discrete power and rate values and/or non-optimal modulation and coding schemes via, e.g., applying the optimal discrete bit-loading algorithm with the "SNR gap" approximation [23].

$$r_{1,n}(p_{1,n}) = \begin{cases} C(h_{11,n}p_{1,n}) & r_{2,n} \leq C\left(\frac{h_{21,n}p_{2,n}}{1+h_{11,n}p_{1,n}}\right) \\ C(h_{11,n}p_{1,n} + h_{21,n}p_{2,n}) - r_{2,n} & C\left(\frac{h_{21,n}p_{2,n}}{1+h_{11,n}p_{1,n}}\right) < r_{2,n} \leq C(h_{21,n}p_{2,n}) \\ C\left(\frac{h_{11,n}p_{1,n}}{1+h_{21,n}p_{2,n}}\right) & r_{2,n} > C(h_{21,n}p_{2,n}) \end{cases} \quad (2)$$

where  $C(x) \triangleq \log_2(1+x)$  is the capacity function of the AWGN channel [20], while  $h_{11,n} \triangleq |\tilde{h}_{11,n}|^2$  and  $h_{21,n} \triangleq |\tilde{h}_{21,n}|^2$ . The above result is illustrated in the following three cases corresponding to the three expressions of  $r_{1,n}$  in (2) from top to bottom. Note that the following discussions apply to any subcarrier  $n$  of user 1.

- *Strong Interference*: In this case, the received interference signal power from user 2 at user 1's receiver is sufficiently large such that the contained message with rate  $r_{2,n}$  can be first decoded by SD with user 1's signal taken as additional Gaussian noise. After that, by reconstructing the received user 2's signal and subtracting it from  $y_{1,n}$ , user 1's message can be decoded by SD. The above operation is known as *successive decoding* in the MAC [20].
- *Moderate Interference*: In this case, the received signal power from user 2 is not as large as that in the previous case of strong interference and as a result, user 2's message can not be directly decoded by SD. However, it is still feasible for user 1 to apply *joint decoding* [20] to decode both users' messages.<sup>3</sup> In this case, the rate pair of the two users falls on the 45-degree segment of the corresponding MAC capacity region boundary [20].
- *Weak Interference*: In this case, the received signal power from user 2 is too weak to be decoded even without the presence of user 1's signal. As such, user 1's receiver has the only option of treating user 2's signal as the additional Gaussian noise and applying SD to decode directly user 1's message. Note that the above SD is used in the conventional IWF regardless of the received signal power from the interference user (user 2).

From the above discussions, it is known that MD is applied in both cases of strong and moderate interferences, but not in the case of weak interference. Thus, user 1's receiver opportunistically applies MD to the interference user signal if it has a sufficiently large received power to be decoded either successively or jointly with the desired user signal. Therefore, the MD in the context of this paper is called *opportunistic multiuser detection* (OMD).

<sup>3</sup>Note that an alternative decoding method in this case is successive decoding along with "rate splitting" [24] or "time sharing" [20] encoding technique. However, these techniques require certain cooperation between the users and are thus not considered in this paper.

In Fig. 1 (a),  $r_1(p_1)$  in (2) is illustrated. For conciseness, the index  $n$  is dropped here. It is assumed that  $p_2 = 1$ ,  $r_2 = 0.5$ , and  $h_{21} = h_{11} = 1$ . Note that in this case  $r_2 < C(h_{21}p_2)$  and thus OMD instead of SD should be applied. The rate achievable by SD, denoted by  $r_1^{\text{SD}}(p_1) = C(\frac{h_{11}p_1}{1+h_{21}p_2})$  from (2), is also shown for comparison. It is observed that user 1's rate with OMD is improved over that with SD, and  $r_1(p_1)$  is the minimum of the two functions defined as  $f(p_1) \triangleq C(h_{11}p_1)$  and  $h(p_1) \triangleq C(h_{11}p_1 + h_{21}p_2) - r_2$ , which are the rates achievable by successive decoding and joint decoding, respectively. The threshold value of  $p_1$ , denoted by  $p_{th}$ , for which  $r_1(p_1) = f(p_1)$  if  $p_1 \leq p_{th}$  and otherwise  $r_1(p_1) = h(p_1)$ , is obtained from (2) as

$$p_{th} = \frac{1}{h_{11}} \left( \frac{h_{21}p_2}{2^{r_2} - 1} - 1 \right). \quad (3)$$

Note that  $p_{th} \geq 0$  if  $r_2 < C(h_{21}p_2)$ .

With  $r_{1,n}(p_{1,n})$  given in (2) for all  $n$ 's, the problem can be formulated for user 1 to optimize its power and rate allocations over subcarriers to maximize its average rate in the case of CIE. This problem is denoted as (P1) and is expressed as

$$\begin{aligned} \text{(P1)} \quad & \max_{p_{1,n} \geq 0, \forall n} R_1^{\text{CIE}}(\{p_{1,n}\}) := \frac{1}{N} \sum_{n=1}^N r_{1,n}(p_{1,n}) \\ & \text{s. t.} \quad \frac{1}{N} \sum_{n=1}^N p_{1,n} \leq P_1. \end{aligned}$$

After (P1) is solved, from the obtained solution for  $p_{1,n}$  at subcarrier  $n$ , the corresponding transmit rate and decoding method can be obtained from (2). The solution of (P1) is given later in Section IV-A.

Next, the case of CJE is considered. Recall that  $R_2^{\text{CJE}}$  and  $\{p_{2,n}\}$  are user 2's transmit rate value and power allocations over subcarriers, respectively, which are all fixed for user 1's transmit optimization. With joint encoding over all the subcarriers, the maximum transmit rate of user 1 under arbitrary power allocations  $\{p_{1,n}\}$  is expressed as

$$R_1^{\text{CJE}}(\{p_{1,n}\}) = \begin{cases} \mathbb{E}[C(h_{11,n}p_{1,n})] & R_2^{\text{CJE}} \leq \mathbb{E}[C(\frac{h_{21,n}p_{2,n}}{1+h_{11,n}p_{1,n}})] \\ \mathbb{E}[C(h_{11,n}p_{1,n} + h_{21,n}p_{2,n})] - R_2^{\text{CJE}} & \mathbb{E}[C(\frac{h_{21,n}p_{2,n}}{1+h_{11,n}p_{1,n}})] < R_2^{\text{CJE}} \leq \mathbb{E}[C(h_{21,n}p_{2,n})] \\ \mathbb{E}[C(\frac{h_{11,n}p_{1,n}}{1+h_{21,n}p_{2,n}})] & R_2^{\text{CJE}} > \mathbb{E}[C(h_{21,n}p_{2,n})] \end{cases} \quad (4)$$

where for notational brevity,  $\mathbb{E}[\cdot]$  is used to represent the operation  $\frac{1}{N} \sum_{n=1}^N (\cdot)$ . Note that the rate  $R_1^{\text{CJE}}$  here is analogous to the ergodic capacity in wireless fading channels where a sufficient long codeword spans over all possible fading states and the codeword rate is the average of all the instantaneous mutual

information of the channel at different fading states [25]. Similar to CIE, the three rate expressions of  $R_1^{\text{CJE}}$  in (4) are also achievable by successive decoding, joint decoding, and SD, respectively, whereas there is a key difference that only one of these decoding methods is applied over all the subcarriers for CJE, in contrast to the case of CIE, where each subcarrier can be independently assigned one of these decoding methods.<sup>4</sup> Thus, unlike CIE, the user in the case of CJE does not have the flexibility for transmit rate and decoding method adaptations over different subcarriers, while it still can optimize over transmit power allocations and choose the best decoding method to maximize its transmit rate.

The problem for user 1 to optimize its power allocations in the case of CJE is denoted as (P2), and is expressed as

$$\begin{aligned}
 \text{(P2)} \quad & \max_{p_{1,n} \geq 0, \forall n} R_1^{\text{CJE}}(\{p_{1,n}\}) \\
 & \text{s.t.} \quad \frac{1}{N} \sum_{n=1}^N p_{1,n} \leq P_1.
 \end{aligned}$$

After solving the optimal power allocations in (P2), the maximum transmit rate and its achievable decoding method can be obtained from (4). The solution of (P2) is provided later in Section IV-B.

#### IV. OPTIMAL POWER ALLOCATION

In this section, (P1) and (P2) for the case of CIE and CJE, respectively, are solved to obtain the optimal power allocations for user 1 at each iteration of transmit adaptation. It is shown that the obtained power allocation solutions in both cases are non-trivial variations of the standard WF solution [20], which is employed in IWF.

##### A. Carrier Independent Encoding

In this part, (P1) for the case of CIE is studied. The objective function of (P1) is the sum of  $N$  independent functions,  $r_{1,n}(p_{1,n})$ 's, each of which can be easily shown to be a concave function of  $p_{1,n}$ . Therefore, the objective function is concave in  $\{p_{1,n}\}$ . In addition, the constraint of (P1) is a linear function of  $p_{1,n}$ 's. Thus, (P1) is a convex optimization problem, and thus can be solved via convex optimization techniques.

<sup>4</sup>Due to frequency-selective channel variation, it may be possible that at some subcarriers of user 1, the interference channel gains from user 2 are sufficiently large such that if CIE is used, OMD can be applied at these subcarriers to immediately remove the effect of these interferences, while in the case of CJE, whether OMD can be applied depends on the interference channel gains at all the subcarriers.

In (P1), the objective function is separable in  $n$  while the constraint is not. Therefore, the *Lagrange dual decomposition* method, which has been applied in prior works (see, e.g., [13]-[18]), is also proposed here to decouple the constraint in  $n$ , and thereby decomposes (P1) into a set of  $N$  independent subproblems each for a different subcarrier. First, the Lagrangian of (P1) is written as

$$\mathcal{L}(\{p_{1,n}\}, \lambda) = \frac{1}{N} \sum_{n=1}^N r_{1,n}(p_{1,n}) - \lambda \left( \frac{1}{N} \sum_{n=1}^N p_{1,n} - P_1 \right) \quad (5)$$

where  $\lambda$  is the non-negative dual variable associated with the power constraint. Then, the Lagrange dual function of (P1) is defined as

$$g(\lambda) = \max_{p_{1,n} \geq 0, \forall n} \mathcal{L}(\{p_{1,n}\}). \quad (6)$$

The value of the dual function serves as an upper bound on the optimal value of the original (primal) problem, denoted by  $r^*$ , i.e.,  $r^* \leq g(\lambda)$  for any  $\lambda \geq 0$ . The dual problem of (P1) is then defined as  $\min_{\lambda \geq 0} g(\lambda)$ . Let the optimal value of the dual problem be denoted by  $d^*$ , which is achievable by the optimal dual solution  $\lambda^*$ , i.e.,  $d^* = g(\lambda^*)$ . For a convex optimization problem with a strictly feasible point, the Slater's condition [26] is satisfied and thus the duality gap,  $r^* - d^* \leq 0$ , is indeed zero for (P1). This result suggests that (P1) can be equivalently solved by first maximizing its Lagrangian to obtain the dual function for some given dual variable  $\lambda$ , and then solving the dual problem over  $\lambda \geq 0$ .

Consider first the problem for maximizing the Lagrangian to obtain the dual function  $g(\lambda)$  for some given  $\lambda$ . It is interesting to observe that  $g(\lambda)$  can be rewritten as

$$g(\lambda) = \frac{1}{N} \sum_{n=1}^N g_n(\lambda) + \lambda P_1 \quad (7)$$

where

$$g_n(\lambda) = \max_{p_{1,n} \geq 0} r_{1,n}(p_{1,n}) - \lambda p_{1,n} \quad n = 1, \dots, N. \quad (8)$$

By this way,  $g(\lambda)$  can be obtained via solving a set of  $N$  independent subproblems, each for a different subcarrier  $n$ . Note that the maximization problems in (8) at different  $n$ 's all have the same structure and thus can be solved using the same computational routine. For conciseness, the index  $n$  is dropped in (8) and the resultant problem is re-expressed as

$$(P3) \quad \max_{p_1 \geq 0} a(p_1) := r_1(p_1) - \lambda p_1$$

where  $r_1(p_1)$  is given by (2) with the index  $n$  dropped.

Solutions of (P3) for all the subcarriers can then be used to obtain the dual function  $g(\lambda)$  in (6) for any given  $\lambda$ . Then, the dual function needs to be minimized over  $\lambda \geq 0$  in the dual problem to obtain the optimal dual solution  $\lambda^*$  with which the duality gap is zero, i.e., the original problem (P1) is equivalently solved. The standard routine in convex optimization to iteratively update  $\lambda$  toward its optimal solution is via the bisection method [26] based on the subgradient of  $g(\lambda)$ , which can be shown to be  $P_1 - \frac{1}{N} \sum_{n=1}^N p_{1,n}$ . When  $\lambda = \lambda^*$ , the associated optimal solution of (P1), denoted by  $\{p_{1,n}^*\}$ , satisfies  $\frac{1}{N} \sum_{n=1}^N p_{1,n}^* = P_1$ . For brevity, the details of this standard routine are omitted here.

Next, the solution of (P3) is derived for some given  $\lambda$ . Note that since  $r_1(p_1)$  is a concave function of  $p_1$ , so is  $a(p_1)$  and thus (P3) is a convex optimization problem. The following discussions are then made on the solution to (P3):

If  $r_2 \leq C(h_{21}p_2)$ , from (2) it follows that OMD should be applied in this case. Note that  $p_{th}$  given in (3) satisfies  $p_{th} \geq 0$  in this case, and  $a(p_1)$  is the minimum of two functions defined as  $f_\lambda(p_1) \triangleq f(p_1) - \lambda p_1$  and  $h_\lambda(p_1) \triangleq h(p_1) - \lambda p_1$ , where  $f(p_1)$  and  $h(p_1)$  are defined earlier in Section III. Also note that when  $p_1 \leq p_{th}$ ,  $a(p_1) = f_\lambda(p_1)$ ; otherwise,  $a(p_1) = h_\lambda(p_1)$ . The optimal values of  $p_1$  that maximize  $f_\lambda(p_1)$  and  $h_\lambda(p_1)$  can be obtained as the standard WF solutions

$$p_1^{(f)} = \left( \frac{1}{(\ln 2)\lambda} - \frac{1}{h_{11}} \right)^+ \quad (9)$$

with  $(\cdot)^+ \triangleq \max(0, \cdot)$  and

$$p_1^{(h)} = \left( \frac{1}{(\ln 2)\lambda} - \frac{1 + h_{21}p_2}{h_{11}} \right)^+ \quad (10)$$

respectively. Note that  $0 \leq p_1^{(h)} \leq p_1^{(f)}$ . Let  $a^*$  denote the optimal value of (P3), which is achievable by the optimal solution  $p_1^*$ , i.e.,  $a^* = a(p_1^*)$ . Since  $\max_{p_1} \min(f_\lambda(p_1), h_\lambda(p_1)) \leq \min(f_\lambda(p_1^{(f)}), h_\lambda(p_1^{(h)}))$ , it follows that  $a^* \leq f_\lambda(p_1^{(f)})$  and  $a^* \leq h_\lambda(p_1^{(h)})$ . Based on this result,  $p_1^*$  is obtained for the following three cases:

- $p_{th} \geq p_1^{(f)}$ : In this case,  $a(p_1^{(f)}) = f_\lambda(p_1^{(f)})$ , thus it follows that  $a^* \geq f_\lambda(p_1^{(f)})$ . Since it has been shown that  $a^* \leq f_\lambda(p_1^{(f)})$ , it follows that  $a^* = f_\lambda(p_1^{(f)})$  and  $p_1^* = p_1^{(f)}$ , as shown in Fig. 1 (b). Note that successive decoding is optimal in this case.
- $p_{th} \leq p_1^{(h)}$ : Similar to the first case, it can be shown that  $a^* = h_\lambda(p_1^{(h)})$  and thus  $p_1^* = p_1^{(h)}$ , as shown in Fig. 1 (d). Note that joint decoding is optimal in this case.

- $p_1^{(h)} < p_{th} < p_1^{(f)}$ : Since  $p_{th} < p_1^{(f)}$ , it follows that  $f_\lambda(p_1)$  is an increasing function for  $p_1 \leq p_{th}$ . Moreover, since  $a(p_1) = f_\lambda(p_1)$ , for  $p_1 \leq p_{th}$ , it follows that  $f_\lambda(p_{th}) \geq a(p_1)$  for any  $p_1 \leq p_{th}$ . Similarly, it can be shown that  $h_\lambda(p_{th}) \geq a(p_1)$  for any  $p_1 \geq p_{th}$ . Since  $h_\lambda(p_{th}) = f_\lambda(p_{th})$ , it concludes that  $p_1^* = p_{th}$ , as shown in Fig. 1 (c). In this case, either successive decoding or joint decoding achieves the optimum, while this paper adopts the former due to its more implementation ease over the latter.

If  $r_2 > C(h_{21}p_2)$ , SD should be used. Note that  $p_{th} < 0$  in this case. It is easy to show that the optimal solution  $p_1^*$  of (P3) in this case is same as  $p_1^{(h)}$  in (10) obtained earlier. Note that this WF-based power allocation policy is also used in IWF.

By summarizing the above discussions, the following theorem is obtained:

**Theorem 4.1:** The optimal solution of (P1) at subcarrier  $n$ ,  $n = 1, \dots, N$ , is (with the index  $n$  dropped for conciseness)

$$p_1^* = \begin{cases} p_1^{(f)}, & p_{th} \geq p_1^{(f)} \\ p_{th}, & p_1^{(h)} < p_{th} < p_1^{(f)} \\ p_1^{(h)}, & 0 \leq p_{th} \leq p_1^{(h)} \\ p_1^{(h)}, & p_{th} < 0 \end{cases} \quad (11)$$

where  $p_{th}$  is given in (3), while  $p_1^{(f)}$  and  $p_1^{(h)}$  are given in (9) and (10), respectively, with  $\lambda = \lambda^*$ . The corresponding optimal decoding methods at subcarrier  $n$  are (from top to bottom) successive decoding, successive decoding, joint decoding, and SD, respectively.

In Fig. 2, the optimal power allocation  $p_1^*$  in (11) at a particular subcarrier  $n$  is shown for different values of  $\lambda^*$ . Note that  $\lambda^*$  is a decreasing function of user'1 average power constraint  $P_1$ . Only the case of  $r_2 \leq C(h_{21}p_2)$  where OMD should be applied is considered here. Thus,  $p_{th} \geq 0$  and only the first three expressions of  $p_1^*$  in (11) are illustrated in this figure. It is observed that the obtained power allocation is a variation of the standard WF solutions, e.g.,  $p_1^{(f)}$  in (9) and  $p_1^{(h)}$  in (10). There are two fixed noise levels  $w^{(f)} = 1/h_{11}$  and  $w^{(h)} = (1 + h_{21}p_2)/h_{11}$ , corresponding to the power allocations  $p_1^{(f)}$  and  $p_1^{(h)}$ , respectively. The amount of power (water) to be allocated (filled) then depends on the water-level  $1/((\ln 2)\lambda^*)$ . If  $P_1$  is sufficiently large such that  $1/((\ln 2)\lambda^*) \geq w^{(f)}$  and at the same time  $P_1$  is sufficiently small such that  $1/((\ln 2)\lambda^*) \leq w^{(f)} + p_{th}$ , then  $p_1^* = 1/((\ln 2)\lambda^*) - w^{(f)} = p_1^{(f)}$ ; if  $P_1$  is sufficiently large such that  $1/((\ln 2)\lambda^*) > w^{(f)} + p_{th}$ , but not yet large to make  $1/((\ln 2)\lambda^*) \geq w^{(h)} + p_{th}$ , then  $p_1^* = p_{th}$

regardless of  $\lambda^*$  and the resultant noise-plus-power level is below the water-level  $1/((\ln 2)\lambda^*)$ ;<sup>5</sup> if  $P_1$  is sufficiently large such that  $1/((\ln 2)\lambda^*) \geq w^{(f)} + p_{th}$ , then  $p_1^* = 1/((\ln 2)\lambda^*) - w^{(h)} = p_1^{(h)}$ . The above three cases are illustrated by Fig. 2 (a), (b), and (c), respectively.

### B. Carrier Joint Encoding

Next, the problem (P2) for the case of CJE is studied. Similar to the case of CIE, it can be shown that  $R_1^{\text{CJE}}(\{p_{1,n}\})$  in (4) is a concave function of  $\{p_{1,n}\}$  and thus (P2) is a convex optimization problem. Similar to (P1), the Lagrange duality method is applied to solve (P2). Like (P1), the Lagrangian and the dual function for (P2) can be obtained, and it can be shown that (P2) has a zero duality gap. For brevity, these details are skipped here and the min-max form of (P2) is directly given as

$$\min_{\mu \geq 0} \max_{p_{1,n} \geq 0, \forall n} R_1^{\text{CJE}}(\{p_{1,n}\}) - \mu \left( \frac{1}{N} \sum_{n=1}^N p_{1,n} - P_1 \right) \quad (12)$$

with  $\mu$  denoting the non-negative dual variable associated with the transmit power constraint. The optimal dual solution of  $\mu$ , denoted by  $\mu^*$ , in the above minimization problem can be similarly obtained by the bisection method as in (P1). In the following, the maximization problem in (12) over  $\{p_{1,n}\}$  with some fixed  $\mu$  is addressed, which can first be simplified as (by removing the irrelevant constant term)

$$(P4) \quad \max_{p_{1,n} \geq 0, \forall n} b(\{p_{1,n}\}) := R_1^{\text{CJE}}(\{p_{1,n}\}) - \mu \mathbb{E}[p_{1,n}].$$

Similar to (P3), the following two cases are studied for (P4):

If  $R_2^{\text{CJE}} \leq \mathbb{E}[C(h_{21,n}p_{2,n})]$ , it is known from (4) that OMD should be used in this case. Compared with the previously studied case of CIE, the power optimization in the case of CJE is more involved, as explained as follows: From (4), it is easy to show that if  $R_2^{\text{CJE}} \leq \mathbb{E}[C(h_{21,n}p_{2,n})]$ ,  $R_1^{\text{CJE}}(\{p_{1,n}\})$  can be expressed as the minimum of two functions defined as  $f_\mu(\{p_{1,n}\}) \triangleq \mathbb{E}[C(h_{11,n}p_{1,n})] - \mu \mathbb{E}[p_{1,n}]$  and  $h_\mu(\{p_{1,n}\}) \triangleq \mathbb{E}[C(h_{11,n}p_{1,n} + h_{21,n}p_{2,n})] - R_2^{\text{CJE}} - \mu \mathbb{E}[p_{1,n}]$ . Then, let  $f_{\mu,n}(p_{1,n}) \triangleq C(h_{11,n}p_{1,n}) - \mu p_{1,n}$  and  $h_{\mu,n}(p_{1,n}) \triangleq C(h_{11,n}p_{1,n} + h_{21,n}p_{2,n}) - R_2^{\text{CJE}} - \mu p_{1,n}$ ,  $n = 1, \dots, N$ , be the component in  $f_\mu$  and  $h_\mu$  at subcarrier  $n$ , respectively, i.e.,  $f_\mu = \mathbb{E}[f_{\mu,n}]$ ,  $h_\mu = \mathbb{E}[h_{\mu,n}]$ . Since  $\min(f_\mu, h_\mu)$  is not necessarily equal to  $\mathbb{E}[\min(f_{\mu,n}, h_{\mu,n})]$ , it is unclear whether  $R_1^{\text{CJE}}(\{p_{1,n}\})$  is separable in  $n$ , which makes unclear whether the maximization of  $b(\{p_{1,n}\})$  over  $\{p_{1,n}\}$  is solvable directly by the dual decomposition method.

<sup>5</sup>It is noted that in realistic multi-carrier systems, the channel conditions vary from subcarrier to subcarrier and as a result it is unlikely that all the subcarriers will fall into this case and are thus allocated powers  $p_{th}(n)$ 's regardless of  $\lambda^*$  or  $P_1$ .

Let  $b^*$  denote the maximum value of  $b(\{p_{1,n}\})$  achievable by the optimal solution  $\{p_{1,n}^*\}$ . Note that both  $f_\mu(\{p_{1,n}\})$  and  $h_\mu(\{p_{1,n}\})$  are concave functions in  $\{p_{1,n}\}$  and achieve their respective maximum values at

$$p_{1,n}^{(f)} = \left( \frac{1}{(\ln 2)\mu} - \frac{1}{h_{11,n}} \right)^+, \quad n = 1, \dots, N \quad (13)$$

$$p_{1,n}^{(h)} = \left( \frac{1}{(\ln 2)\mu} - \frac{1 + h_{21,n}p_{2,n}}{h_{11,n}} \right)^+, \quad n = 1, \dots, N. \quad (14)$$

Note that  $p_{1,n}^{(f)} \geq p_{1,n}^{(h)}, \forall n$ . Since  $b(\{p_{1,n}\}) = \min(f_\mu(\{p_{1,n}\}), h_\mu(\{p_{1,n}\}))$ , it follows that  $b^* \leq f_\mu(\{p_{1,n}^{(f)}\})$  and  $b^* \leq h_\mu(\{p_{1,n}^{(h)}\})$ . Next, the following cases are discussed on  $\{p_{1,n}^*\}$ :

- $\mathbb{E}[C(\frac{h_{21,n}p_{2,n}}{1+h_{11,n}p_{1,n}^{(f)}})] \geq R_2^{\text{CJE}}$ : In this case,  $b(\{p_{1,n}^{(f)}\}) = f_\mu(\{p_{1,n}^{(f)}\})$ . Since  $b^* \leq f_\mu(\{p_{1,n}^{(f)}\})$ , it follows that  $b^* = f_\mu(\{p_{1,n}^{(f)}\})$  and thus  $p_{1,n}^* = p_{1,n}^{(f)}$ . Note that successive decoding is optimal in this case.
- $\mathbb{E}[C(\frac{h_{21,n}p_{2,n}}{1+h_{11,n}p_{1,n}^{(h)}})] \leq R_2^{\text{CJE}}$ : In this case,  $b(\{p_{1,n}^{(h)}\}) = h_\mu(\{p_{1,n}^{(h)}\})$ . Since  $b^* \leq h_\mu(\{p_{1,n}^{(h)}\})$ , it follows that  $b^* = h_\mu(\{p_{1,n}^{(h)}\})$  and thus  $p_{1,n}^* = p_{1,n}^{(h)}$ . Joint decoding is thus optimal.
- $\mathbb{E}[C(\frac{h_{21,n}p_{2,n}}{1+h_{11,n}p_{1,n}^{(f)}})] < R_2^{\text{CJE}} < \mathbb{E}[C(\frac{h_{21,n}p_{2,n}}{1+h_{11,n}p_{1,n}^{(h)}})]$ : In this case,  $\{p_{1,n}^*\}$  is neither  $\{p_{1,n}^{(f)}\}$  nor  $\{p_{1,n}^{(h)}\}$ . Furthermore, by contradiction it can be shown that  $\mathbb{E}[C(\frac{h_{21,n}p_{2,n}}{1+h_{11,n}p_{1,n}^*})] = R_2^{\text{CJE}}$  must hold in this case.

Thus,  $\{p_{1,n}^*\}$  can be obtained by solving either one of the following two equivalent problems:

$$\begin{aligned} \text{(P5)} \quad & \max_{p_{1,n} \geq 0, \forall n} \mathbb{E}[C(h_{11,n}p_{1,n})] - \mu\mathbb{E}[p_{1,n}] \\ & \text{s.t. } \mathbb{E}[C(\frac{h_{21,n}p_{2,n}}{1+h_{11,n}p_{1,n}})] \geq R_2^{\text{CJE}}. \end{aligned}$$

$$\begin{aligned} \text{(P6)} \quad & \max_{p_{1,n} \geq 0, \forall n} \mathbb{E}[C(h_{11,n}p_{1,n} + h_{21,n}p_{2,n})] - R_2^{\text{CJE}} - \mu\mathbb{E}[p_{1,n}] \\ & \text{s.t. } \mathbb{E}[C(\frac{h_{21,n}p_{2,n}}{1+h_{11,n}p_{1,n}})] \leq R_2^{\text{CJE}}. \end{aligned}$$

Note that the objective functions of (P5) and (P6) are both concave in  $\{p_{1,n}\}$ . However, (P5) is a non-convex optimization problem since its constraint is not necessarily convex due to the fact that  $C(\frac{b}{1+ax})$  is a convex function of  $x$  for  $x \geq 0$  with any positive constants  $a$  and  $b$ , while (P6) is a convex optimization problem since its constraint has the reversed inequality of that in (P5) and is thus a convex constraint. Therefore, without loss of generality, (P6) is considered for this case, while the obtained solution is optimal for both (P5) and (P6). Similar to the third case of (P3), both successive decoding and joint decoding achieve the maximum rate given the optimal power allocations, whereas the former is more preferable than the latter from an implementation viewpoint.

Lemma 4.1: The optimal solution of (P6) is

$$\tilde{p}_{1,n}^{(h)} = \begin{cases} 0, & \frac{1}{(\ln 2)\mu F_n(0)} - \frac{1+h_{21,n}p_{2,n}}{h_{11,n}} \leq 0 \\ x_n^*, & \text{otherwise} \end{cases} \quad (15)$$

for  $n = 1, \dots, N$ , where  $x_n^*$  is the unique positive root of the equation

$$x_n = \frac{1}{(\ln 2)\mu F_n(x_n)} - \frac{1 + h_{21,n}p_{2,n}}{h_{11,n}} \quad (16)$$

while  $F_n(x_n)$  is defined as

$$F_n(x_n) = \frac{1 + h_{11,n}x_n}{1 + h_{11,n}x_n + \nu h_{21,n}p_{2,n}} \quad (17)$$

and  $\nu > 0$  with which the constraint of (P6) is satisfied with equality.

*Proof:* Please see Appendix I. ■

It is observed from (15) and (16) that the optimal solution of (P6) resembles a *biased* version of the standard WF solution  $\{p_{1,n}^{(h)}\}$  given in (14) because the associated water-level is biased by an additional factor  $F_n$ , which itself is a function of the optimal power allocation. It is also observed from (17) that the biasing factor is an increasing function of the allocated power. The algorithm that resolves the biasing factor  $F_n(x_n)$  to obtain the solution of  $x_n$  in (16) is given in Appendix II.

If  $R_2^{\text{CJE}} > \mathbb{E}[C(h_{21,n}p_{2,n})]$ , from (4) it is known that SD should be applied at user 1's receiver in this case and  $R_1^{\text{CJE}}(\{p_{1,n}\}) = \mathbb{E}[C(\frac{h_{11,n}p_{1,n}}{1+h_{21,n}p_{2,n}})]$ , which is separable in  $n$ . Thus,  $b(\{p_{1,n}\})$  is also separable in  $n$  and can be maximized independently over different  $n$ 's. It is not hard to show that the optimal power allocations  $\{p_{1,n}^*\}$  in this case are equal to  $\{p_{1,n}^{(h)}\}$  given in (14). Note that the power allocation policy (14) is same as (10), which is used in IWF. Also note that the achievable rate of IWF is same with CIE or CJE.

Summarizing the discussions on the above two cases, the following theorem is obtained:

Theorem 4.2: The optimal solution of (P2) is

$$p_{1,n}^* = \begin{cases} p_{1,n}^{(f)}, & \mathbb{E}[C(\frac{h_{21,n}p_{2,n}}{1+h_{11,n}p_{1,n}^{(f)}})] \geq R_2^{\text{CJE}} \\ \tilde{p}_{1,n}^{(h)}, & \mathbb{E}[C(\frac{h_{21,n}p_{2,n}}{1+h_{11,n}p_{1,n}^{(f)}})] < R_2^{\text{CJE}} < \mathbb{E}[C(\frac{h_{21,n}p_{2,n}}{1+h_{11,n}p_{1,n}^{(h)}})] \\ p_{1,n}^{(h)}, & \mathbb{E}[C(\frac{h_{21,n}p_{2,n}}{1+h_{11,n}p_{1,n}^{(h)}})] \leq R_2^{\text{CJE}} \leq \mathbb{E}[C(h_{21,n}p_{2,n})] \\ p_{1,n}^{(h)}, & R_2^{\text{CJE}} > \mathbb{E}[C(h_{21,n}p_{2,n})] \end{cases} \quad (18)$$

for  $n = 1, \dots, N$ , where  $p_{1,n}^{(f)}$ ,  $p_{1,n}^{(h)}$ , and  $\tilde{p}_{1,n}^{(h)}$  are given in (13), (14), and (15), respectively, with  $\mu = \mu^*$ . The corresponding optimal decoding methods are (from top to bottom) successive decoding, successive decoding, joint decoding, and SD, respectively.

## V. SIMULATION RESULTS

In this section, the performance of the proposed ISS algorithm with OMD is evaluated and compared to that of the conventional IWF algorithm with SD. It is assumed that the multi-carrier system has the number of subcarriers  $N = 64$  and the CP period is equal to  $1/4$  of the symbol period. All the channels involved in the system, including users' direct channels and interference channels, are assumed to each have 16 independent, equal-power, multipath taps. In addition, a symmetric channel model is assumed where the two users' direct channels have the same average unit power, and the two interference channels between users have the same average power denoted by  $\rho$ , while  $\rho$  may take different values in order to investigate the effect of the interference between the two users on their achievable rates. In total, 1000 independent channel realizations are simulated over which each user's achievable average rate is computed, while the rate loss due to the insertion of CP is ignored. For each channel realization, the multipath taps of the direct/interference channels are generated by independent CSCG RVs with zero mean and equal variance. The ISS/IWF algorithm is then implemented over each channel realization where the two users iteratively update their power allocations until their rates both get converged.

In Fig. 4, the achievable average sum-rate of the two users is shown for different values of the interference channel power gain,  $\rho$ . It is assumed that  $P_1 = P_2 = 100$ . It is observed that the proposed ISS algorithm with either CIE or CJE improves the sum-rate over IWF, thanks to the more superior OMD over SD. It is also observed that the achievable sum-rate of IWF fluctuates over different values of  $\rho$ , while ISS ensures a consistent rate increase with  $\rho$  except the region of very low values of  $\rho$  where OMD is not frequently applied. Interestingly, it is observed that as  $\rho$  increases, ISS with CJE becomes superior over that with CIE in terms of the achievable sum-rate. Since CJE has a lower complexity to implement than CIE, this result provides a useful guidance for practical system design. However, this phenomenon is some counter-intuitive since CIE provides each user more flexibility for rate adaptations over different subcarriers and is thus expected to be more suitable than CJE to exploit the benefit of OMD. A reasonable explanation for this observation can be obtained by looking at a snapshot of the users' converged power

spectrums in this case, as shown in Fig. 5 for  $\rho = 10$ . It is observed that the two users' power spectrums in the case of IWF are close to be orthogonal in frequency, which suggests that "interference avoidance" is probably the expected solution by IWF in this case. In contrast, the power spectrums of the two users in the case of ISS with CIE are observed to be almost overlapped in frequency, as a result of OMD being applied at different subcarriers, while the spectrums in the case of ISS with CJE appear to be in between those of IWF and ISS with CIE. It is thus conjectured that neither completely orthogonal nor overlapped spectrum is the best converged solution for decentralized spectrum sharing, which could probably explain why ISS with CJE performs the best when the interference channel gains are large.

In Fig. 6, the achievable users' individual rates are shown for a special case of the general channel model studied in this paper. In this case, a "cognitive radio" type of newly emerging wireless system is considered, where user 1 is the so-called primary (non-cognitive) user (PU) that is the legitimate user operating in the frequency band of interest, while user 2 is the secondary (cognitive) user (SU) that transmits at the same time over the same spectrum under the constraint that its transmission will not cause the PU's QoS to an unacceptable level. Note that a similar scenario has also been considered in [27]. The PU is non-cognitive since it is oblivious to the existence of the SU and, thus, it applies the conventional IWF algorithm with SD by treating the interference from the SU as additional noise. While for the SU, it is cognitive in the sense that it is aware of the PU and thus transmits with a much lower average power than that of the PU in order to protect the PU. In this simulation, it is assumed that  $P_1 = 100$  and  $P_2 = 1$ . In addition, since the SU is cognitive, it may choose to use the more advanced resource allocation scheme, e.g., ISS with OMD instead of IWF with SD. Two cases are then studied in this simulation: Case I, both user 1 and user 2 employ IWF; Case II, user 1 employs IWF while user 2 employs ISS. Note that in both cases, CJE is assumed for both users since the PU, with no knowledge on the existence of the SU, should use CJE instead of CIE from a practical consideration. In Fig. 6, it is observed that the achievable rate of user 1 (the PU) drops slightly in Case II as compared to Case I when  $\rho$  is sufficiently large, while the achievable rate of user 2 (the SU) improves significantly. For example, at  $\rho = 1$ , user 1's rate drop is only 3% (a negligible rate loss), while user 2's rate improvement is as large as 140% (a dramatic rate increase) by comparing Cases I and II.

The above observations can be explained by looking at a snapshot of both users' converged power

spectrums (normalized by users' respective average powers) at a typical value of  $\rho = 5$  dB, as shown in Fig. 7. It is observed that user 1's spectrum does not change much over the two cases, while user 2's spectrum changes dramatically from a very "peaky" one in Case I to a more spread one in Case II. The SU's rate improvement in Case II over Case I is due to OMD, which removes the effect of the PU's interference and thus the SU can allocate powers based on its own channel condition, while the PU's rate drop in Case II over Case I is due to the "interference diversity" phenomenon [28], namely, the more peaky interference in Case I is more advantageous for minimizing the resultant PU's rate loss as compared to the more spread one in Case II.

## VI. CONCLUDING REMARKS

This paper studies a new decentralized resource allocation scheme, ISS, for multi-carrier-based multiuser spectrum sharing. ISS maintains the main advantages of the well-known IWF algorithm, e.g., being purely distributed and requiring only practical channel knowledge, while it improves over IWF by exploiting OMD at the user receiver. The resultant benefits are twofold: First, OMD improves the user transmit rate at each iteration of resource adaptation as compared to SD; Second, ISS with OMD leads to more balanced converged user power spectrums than IWF with SD.

This paper presents the very initial results on ISS, for which many issues remain unaddressed yet and are worth further investigating. First, it is shown by simulation that for ISS, CJE performs better than CIE with large interference channel gains, while the opposite is true for moderate or small interference channel gains. This observation raises the question on whether there exists an optimal *multi-band encoding* scheme that divides the total bandwidth into multiple sub-bands over which CIE is applied while within each sub-band CJE is applied. Second, simulation results verify that the convergence of ISS, like IWF, is always guaranteed with realistic channel realizations, while characterizing the exact conditions for the convergence of ISS is an important topic for the future study. Last, extending the results of this paper to the cases with more than two users and/or multi-antenna terminals will also be interesting.

## APPENDIX I

### PROOF OF LEMMA 4.1

Since (P6) is a convex optimization problem, the Lagrange dual decomposition method can be applied to solve it, similar to that for (P1). Let  $\nu$  be the dual variable associated with the constraint of (P6). Since

it is already known that for the problem of interest the constraint is satisfied with equality, it follows that  $\nu > 0$  from the Karush-Kuhn-Tucker (KKT) optimality condition [26]. Then, (P6) can be written as the following equivalent min-max optimization problem:

$$\min_{\nu > 0} \max_{p_{1,n} \geq 0, \forall n} \mathbb{E}[C(h_{11,n}p_{1,n} + h_{21,n}p_{2,n})] - R_2^{\text{CJE}} - \mu \mathbb{E}[p_{1,n}] - \nu (\mathbb{E}[C(\frac{h_{21,n}p_{2,n}}{1 + h_{11,n}p_{1,n}})] - R_2^{\text{CJE}}) \quad (19)$$

where the ‘‘min’’ part can be solved by the bisection method similarly like (P1), while the ‘‘max’’ part for some given  $\nu$  can be decomposed into  $N$  subproblems each for a different subcarrier. At subcarrier  $n$ , the associated subproblem is expressed as

$$\max_{p_{1,n} \geq 0} C(h_{11,n}p_{1,n} + h_{21,n}p_{2,n}) - \mu p_{1,n} - \nu C(\frac{h_{21,n}p_{2,n}}{1 + h_{11,n}p_{1,n}}) \quad (20)$$

Let  $\delta_n$  be the non-negative dual variable associated with the constraint  $p_{1,n} \geq 0$ . The KKT optimality conditions for the optimal primal and dual solutions of the above problem, denoted by  $p_{1,n}^*$  and  $\delta_n^*$ , respectively, are then obtained as

$$p_{1,n}^* = \frac{1}{(\ln 2)(\mu - \delta_n)F_n(p_{1,n}^*)} - \frac{1 + h_{21,n}p_{2,n}}{h_{11,n}}, \quad p_{1,n}^* \delta_n^* = 0, \quad p_{1,n}^* \geq 0, \quad \delta_n^* \geq 0$$

where  $F_n(\cdot)$  is given in (17). From the above KKT conditions, by considering the following two cases: (1)  $\delta_n^* > 0, p_{1,n}^* = 0$ ; and (2)  $p_{1,n}^* > 0, \delta_n^* = 0$ , (15) can be correspondingly obtained.

## APPENDIX II

### ALGORITHM TO SOLVE (16)

The algorithm to obtain the unique positive root  $x_n^*$  of the equation (16) is given in this appendix. Define  $G_n(x_n) = 1/((\ln 2)\mu F_n(x_n))$ . Note that  $G_n$  is a decreasing function of  $x_n$  for  $x_n \geq 0$ , and  $G_n(0) \geq \zeta_n \triangleq (1 + h_{21,n}p_{2,n})/h_{11,n}$  from (15), and  $G_n(\infty) = 1/((\ln 2)\mu)$ . As shown in Fig. 3,  $x_n^*$  is then obtained as the intersection between a 45-degree line starting from the point  $(0, \zeta_n)$  and the plot of the function  $G_n(x_n)$  in the region of  $x_n \geq 0$ . Numerically,  $x_n^*$  can be obtained by a simple iterative algorithm based on the bisection search described as follows. Let  $x_n^* \in [0, x_n^{\max}]$ , where  $x_n^{\max}$  is an upper bound on  $x_n^*$ . A proper value of  $x_n^{\max}$  may be  $G_n(0) - \zeta_n$  from Fig. 3. For the first iteration, let  $\hat{x}_n$  be the midpoint of the initial interval for  $x_n^*$ , i.e.,  $\hat{x}_n = \frac{1}{2}x_n^{\max}$ . The value of  $G_n(\hat{x}_n) - \zeta_n$  is then computed, and compared to  $\hat{x}_n$ : if it is larger than  $\hat{x}_n$ , it follows that  $x_n^* > \hat{x}_n$  and thus  $x_n^* \in (\frac{1}{2}x_n^{\max}, x_n^{\max}]$ ; otherwise,  $x_n^* \leq \hat{x}_n$  and  $x_n^* \in [0, \frac{1}{2}x_n^{\max}]$ . Thereby, after the first iteration, the interval for searching  $x_n^*$  is reduced by half. The above process is repeated until  $x_n^*$  is found within any given accuracy.

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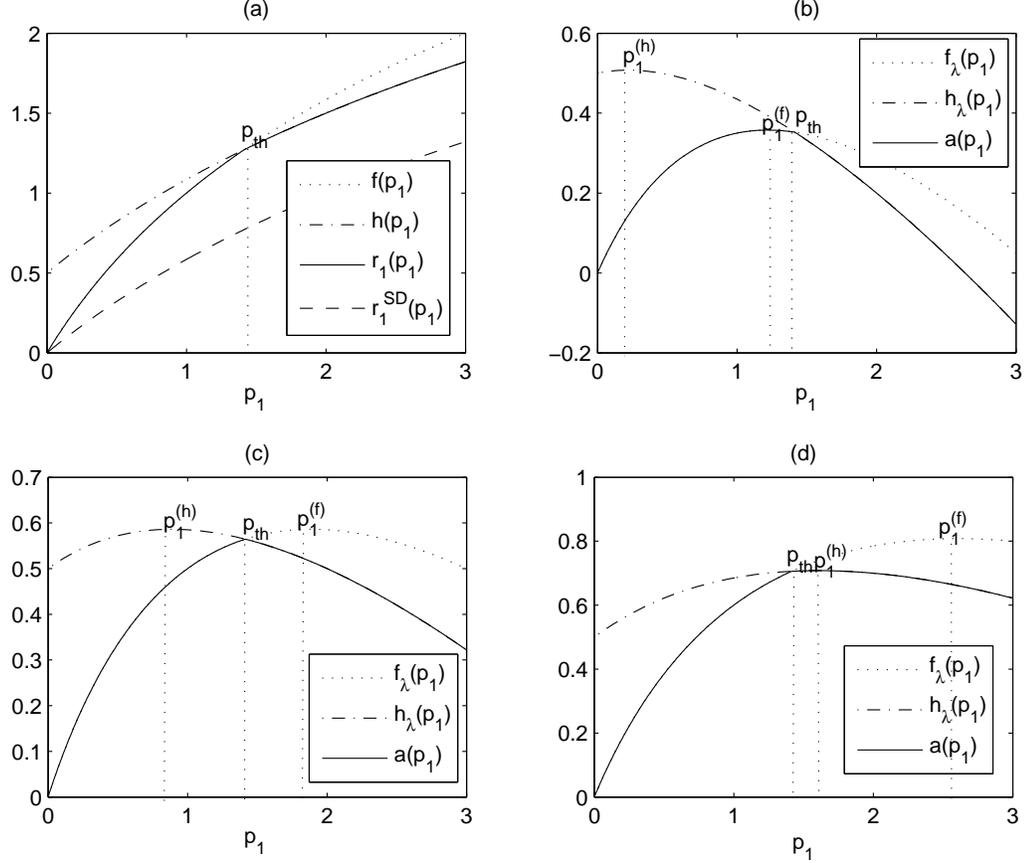


Fig. 1. Illustration of the functions  $r_1(p_1)$  and  $a(p_1) \triangleq r_1(p_1) - \lambda p_1$  in the case of  $r_2 \leq C(h_{21}p_2)$ . Sub-figure (a) illustrates the function  $r_1(p_1)$ ; sub-figures (b), (c), and (d) illustrate the function  $a(p_1) = \min(f_\lambda(p_1), h_\lambda(p_1))$  for  $\lambda = 0.65, 0.5$ , and  $0.4$ , respectively, where the function's maximum value is achieved by  $p_1^* = p_1^{(f)}, p_{th}$ , and  $p_1^{(h)}$ , respectively.

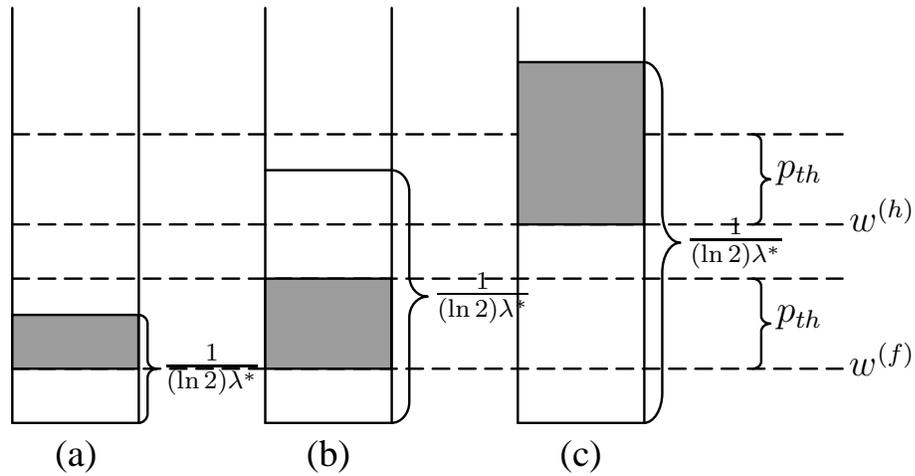


Fig. 2. Illustration of the optimal power allocation (11) in the case of  $r_2 \leq C(h_1h_2)$ : (a)  $w^{(f)} \leq \frac{1}{(\ln 2)\lambda^*} \leq w^{(f)} + p_{th}$ ; (b)  $w^{(f)} + p_{th} < \frac{1}{(\ln 2)\lambda^*} < w^{(h)} + p_{th}$ ; and (c)  $\frac{1}{(\ln 2)\lambda^*} \geq w^{(h)} + p_{th}$ . The height of the grey area in each case is the corresponding allocated power.

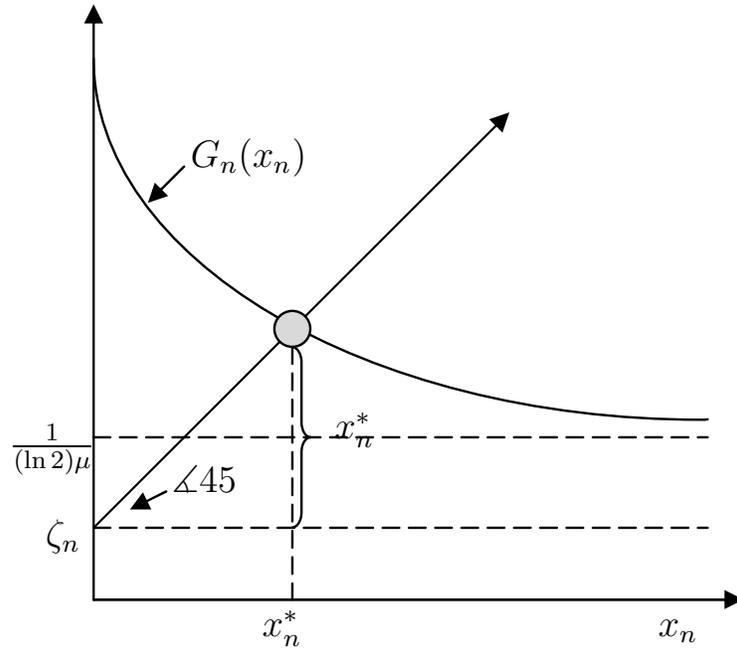


Fig. 3. Illustration of the unique positive root  $x_n^*$  for the equation (16).

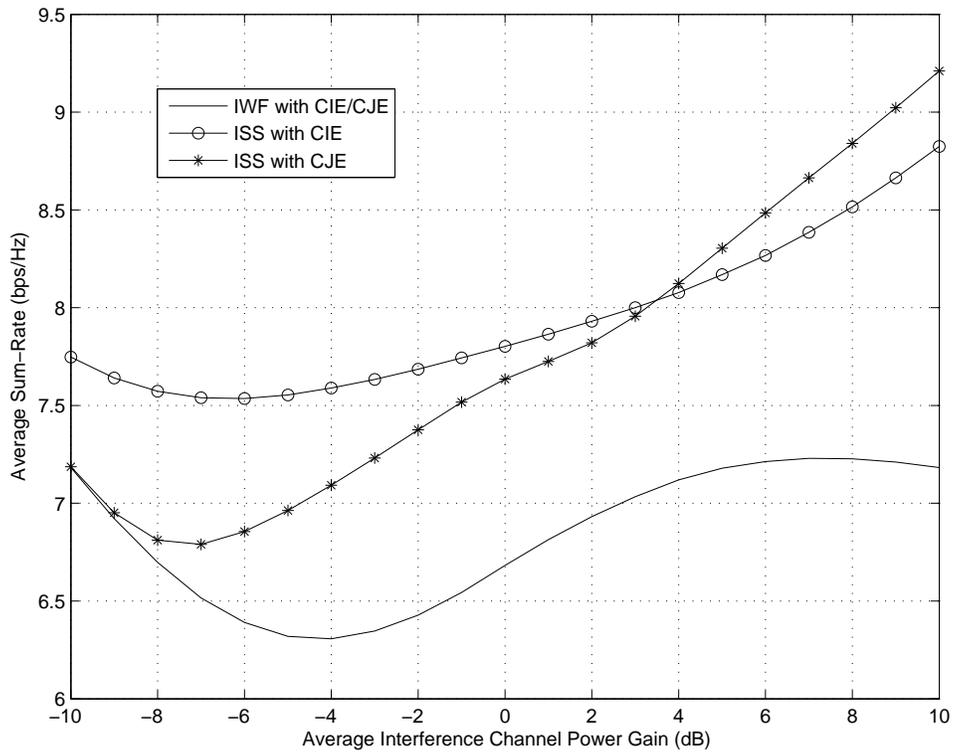


Fig. 4. The achievable sum-rate versus the average interference channel power gain  $\rho$  between the users for  $P_1 = P_2 = 100$ .

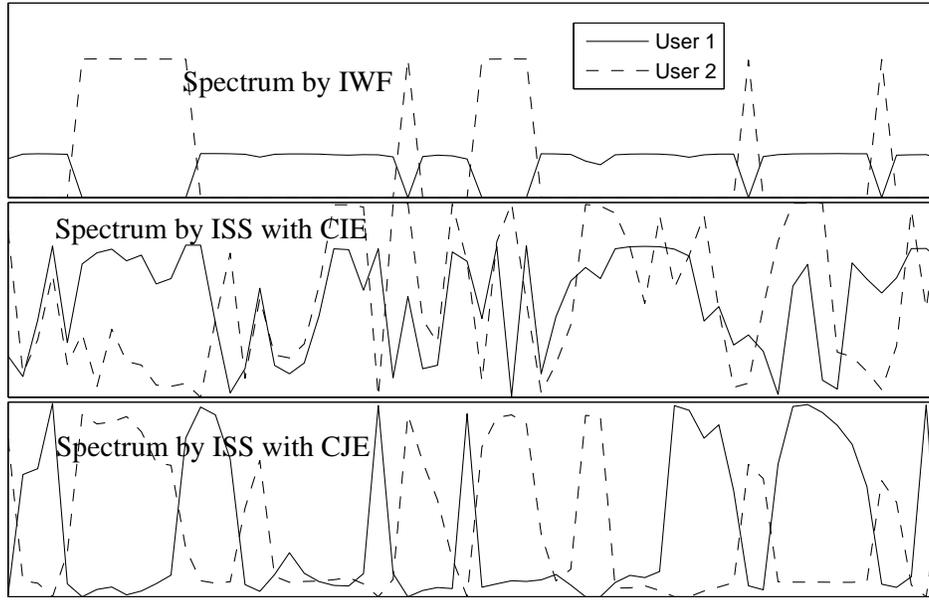


Fig. 5. A snapshot on the converged user power spectrums in the case of  $P_1 = P_2 = 100$ , and  $\rho = 10$ .

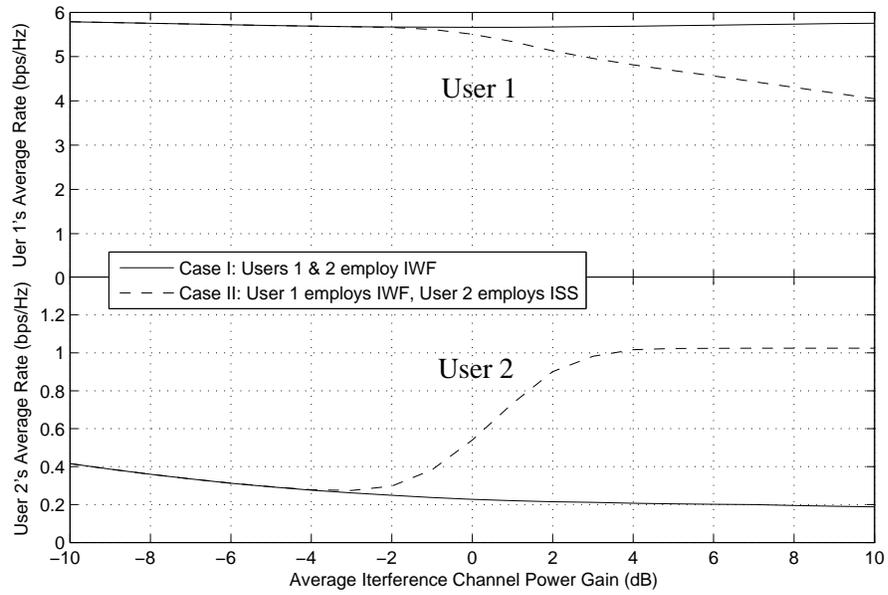


Fig. 6. The achievable user rates versus the average interference channel power gain  $\rho$  for  $P_1 = 100$  and  $P_2 = 1$ .

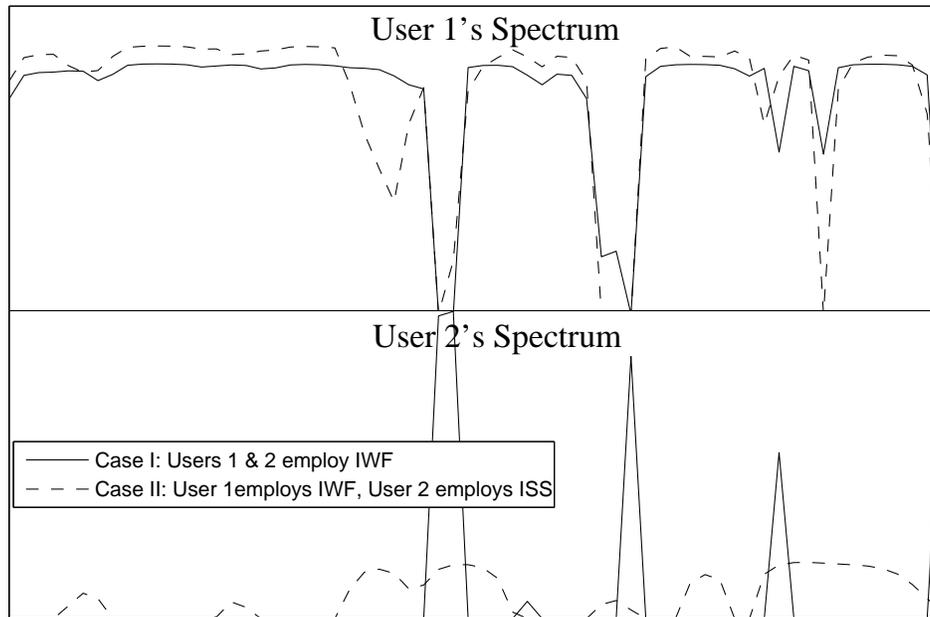


Fig. 7. A snapshot on the converged user power spectrums in the case of  $P_1 = 100$ ,  $P_2 = 1$ , and  $\rho = 5$  dB.