

Energy Efficient Transmission over Space Shift Keying Modulated MIMO Channels

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Abstract—Energy-efficient communication using a class of spatial modulation (SM) that encodes the source information entirely in the antenna indices is considered in this paper. The energy-efficient modulation design is formulated as a convex optimization problem, where minimum achievable average symbol power consumption is derived with rate, performance, and hardware constraints. The theoretical result bounds any modulation scheme of this class, and encompasses the existing space shift keying (SSK), generalized SSK (GSSK), and Hamming code-aided SSK (HSSK) schemes as special cases. The theoretical optimum is achieved by the proposed practical energy-efficient HSSK (EE-HSSK) scheme that incorporates a novel use of the Hamming code and Huffman code techniques in the alphabet and bit-mapping designs. Experimental studies demonstrate that EE-HSSK significantly outperforms existing schemes in achieving near-optimal energy efficiency. An analytical exposition of key properties of the existing GSSK (including SSK) modulation that motivates a fundamental consideration for the proposed energy-efficient modulation design is also provided.

Index Terms—Energy efficiency, green communications, multiple-input multiple-output (MIMO) systems, spatial modulation (SM), space shift keying (SSK), Hamming code, Huffman code.

I. INTRODUCTION

SPATIAL modulation (SM) is an emerging transmission technique that specifically exploits the deployment of multiple antennas in multiple-input multiple-output (MIMO) wireless communications. Unlike conventional phase and amplitude modulation such as quadrature amplitude modulation (QAM), SM encodes the source information partially or fully in the indices of the activated and idle transmit antennas. In the original SM scheme proposed in [1], both the index of a single activated transmit antenna and the data symbol sent via the activated antenna carry information. The single activated antenna consideration is generalized to multiple activated antennas in [2]–[5] with sophisticated bit mapping rules. The limitation that the total number of transmit antennas has to be a power of two in the original SM scheme is relaxed in [6], [7] by proposing novel ways of fractional bit mapping. Trellis coded spatial modulation (TCSM) [8], [9] improves the performance of SM in correlated channel conditions. Space-time block coded SM (STBC-SM) [10] combines space-time block coding (STBC) and SM. Analytical performance results

for various SM schemes in different channel conditions are given in [11]–[15].

A class of SM that eliminates the use of phase and amplitude modulation and encodes the source information fully in the indices of transmit antennas has become a promising implementation of SM for future MIMO wireless communications. Space shift keying (SSK) [16] activates a single transmit antenna as in the original SM, which is generalized to a fixed number of multiple activated antennas in the generalized SSK (GSSK) [17]. Hamming code-aided SSK (HSSK) [18] links the constellation design of HSSK with the codeword construction technique of Hamming codes (in general, binary linear block codes) [19] and adopts a varying number of multiple activated antennas. Space-time shift keying (STSK) [20], [21] and space-time SSK (ST-SSK) [22] extend SSK to both space and time dimensions by combining SSK with STBC. Opportunistic power allocation for SSK is suggested in [23] to improve the performance of SSK. SSK-type modulation presents an attractive class of modulation techniques, especially for future large-scale MIMO systems, for several reasons. First, the hardware cost in the radio frequency (RF) section [24] is reduced since only a subset of the transmit antennas are switched on for data transmission. The same hardware cost reduction benefits as in transmit antenna selection [25] are achieved. Second, the detection complexity is lowered and the receiver design is simpler, since the information is contained entirely in the indexing of the antennas. Third, the transceiver requirement such as synchronization is reduced due to the absence of phase and amplitude modulation [16]. One disadvantage of SSK-type modulation is that the data rates increase only logarithmically with the number of transmit antennas, resulting in lower transmission rates as compared with conventional modulation. This problem can be alleviated by employing a large antenna array and activating multiple antennas, as well as by the efficient use of the set of antenna indices. An overview of the various aspects of SM (including SSK-type modulation) is available in [26].

In this paper, we consider energy-efficient (green) communication using SSK-type modulation. Essentially, energy efficiency is achieved by nonequiprobable signaling where less power-consuming modulation symbols are used more frequently to transmit a given amount of information. We first derive the optimal transmission strategy that guarantees minimum average symbol power consumption provided that the spectral-efficiency, performance, and hardware constraints are met. The theoretical optimum is then achieved by the proposed energy-efficient HSSK (EE-HSSK) modulation scheme. The

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main contributions of this paper are summarized as follows.

- The dependency of the error performance of SSK-type modulation on the minimum Hamming distance between arbitrary two modulation symbols has been previously reported (see, e.g., [18]). In this paper, we show that the employment of a fixed number of activated antennas poses a fundamental limit on the performance of GSSK (including SSK) modulation. Specifically, we show that, among other properties of GSSK, the minimum Hamming distance between arbitrary two GSSK symbols is always equal to 2.
- We investigate the important issue of energy-efficient transmission using SSK-type modulation. The proposed design framework considers a varied number of activated antennas, thus overcoming the limitation of GSSK modulation as mentioned above. The optimal transmission strategy is derived theoretically, which is achieved by the proposed practical EE-HSSK modulation scheme. The design of EE-HSSK is facilitated by a novel use of the bit-mapping technique (rather than the compression capability) of Huffman coding. The energy efficiency of EE-HSSK in comparison with existing schemes is verified by simulation. Various implementation issues of EE-HSSK are discussed.

This paper is organized as follows. Sec. II presents the system description. The design problem and theoretical results for energy-efficient transmission are presented in Sec. III. The practical energy-efficient modulation scheme is described in Sec. IV. Performance results are demonstrated in Sec. V. Conclusion is given in Sec. VI.

Notations: In this paper, \mathbf{I}_N is the $N \times N$ identity matrix, $\|\cdot\|$ the l_2 -norm of a vector, $(\cdot)^T$ and $(\cdot)^H$ the matrix/vector transpose and conjugate matrix/vector transpose, respectively, $Q(x)$ the Q-function defined as $\frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-(\alpha^2/2)} d\alpha$, $|\cdot|$ the cardinality of a set, \log the natural logarithm, \log_2 the base-2 logarithm, and $\lfloor x \rfloor$ the floor operation giving the largest integer not greater than x .

II. TRANSMISSION SYSTEMS WITH SSK-TYPE MODULATION

A. System Model

We consider an uncoded MIMO system with N_T transmit antennas and N_R receive antennas (denoted by an $N_T \times N_R$ system). The system employs an SSK-type modulation scheme that uses solely the antenna indices to carry information. The complex baseband signal model is given by

$$\mathbf{y} = \mathbf{H}\sqrt{E_s}\tilde{\mathbf{x}} + \mathbf{v} \quad (1)$$

where $\mathbf{y} \in \mathbb{C}^{N_R \times 1}$ is the received signal, $\tilde{\mathbf{x}}$ is the $N_T \times 1$ transmitted symbol comprised of 1's (corresponding to activated antennas) and 0's (corresponding to idle antennas), $\mathbf{H} \in \mathbb{C}^{N_R \times N_T}$ is the flat-fading channel, $\mathbf{v} \in \mathbb{C}^{N_R \times 1}$ is the additive white Gaussian noise (AWGN), and E_s is the transmit power at each antenna. Channel matrix \mathbf{H} has independent and identically distributed (i.i.d.) complex Gaussian entries with zero mean and covariance matrix $\sigma_H^2 \mathbf{I}_{N_R}$, where $\sigma_H^2 = 1$. The channel information is assumed perfectly known to the receiver. Noise \mathbf{v} has i.i.d. complex elements with zero mean

and covariance matrix $(N_0/2)\mathbf{I}_{N_R}$. Transmitted symbol $\tilde{\mathbf{x}}$ is selected from the modulation alphabet (constellation set) \mathcal{A} with *a priori* probability $P(\tilde{\mathbf{x}})$, where $\sum_{\tilde{\mathbf{x}} \in \mathcal{A}} P(\tilde{\mathbf{x}}) = 1$.

B. System Performance

Given the signal model in (1), optimal detection is the maximum *a posteriori* probability (MAP) estimate, i.e.,

$$\tilde{\mathbf{x}}_{\text{MAP}} = \arg \max_{\tilde{\mathbf{x}} \in \mathcal{A}} p(\mathbf{y}|\tilde{\mathbf{x}})P(\tilde{\mathbf{x}}) \quad (2)$$

or equivalently

$$\tilde{\mathbf{x}}_{\text{MAP}} = \arg \max_{\tilde{\mathbf{x}} \in \mathcal{A}} \log \left(p(\mathbf{y}|\tilde{\mathbf{x}})P(\tilde{\mathbf{x}}) \right) \quad (3)$$

where

$$p(\mathbf{y}|\tilde{\mathbf{x}}) = \frac{1}{\pi^{N_R} \left(\frac{N_0}{2}\right)^{N_R}} \exp \left(-\frac{2}{N_0} \left\| \mathbf{y} - \mathbf{H}\sqrt{E_s}\tilde{\mathbf{x}} \right\|^2 \right). \quad (4)$$

Therefore,

$$\tilde{\mathbf{x}}_{\text{MAP}} = \arg \min_{\tilde{\mathbf{x}} \in \mathcal{A}} \left(\left\| \mathbf{y} - \mathbf{H}\sqrt{E_s}\tilde{\mathbf{x}} \right\|^2 - \log P(\tilde{\mathbf{x}}) \right). \quad (5)$$

When alphabet \mathcal{A} contains equiprobable elements, i.e., $P(\tilde{\mathbf{x}}) = 1/|\mathcal{A}|$ for all $\tilde{\mathbf{x}}$, MAP detection reduces to maximum likelihood (ML) detection which does not incorporate the *a priori* probabilities in its detection metric [27].

The system performance based on MAP detection can be quantified by first deriving the pairwise error probability (PEP) and then averaging the PEP over all pairwise symbol combinations. Let \mathbf{x}_i and \mathbf{x}_j be two distinct symbols in \mathcal{A} . Following the same derivation approach in [17], [18], the PEP of deciding on \mathbf{x}_j given that $\mathbf{x}_i = \tilde{\mathbf{x}}$ is transmitted is given by

$$P(\mathbf{x}_i \rightarrow \mathbf{x}_j | \mathbf{x}_i = \tilde{\mathbf{x}}) = \int_0^\infty Q\left(\sqrt{x} + \frac{L(\mathbf{x}_i, \mathbf{x}_j)}{N_0\sqrt{x}}\right) f_Z(x) dx \quad (6)$$

where $L(\mathbf{x}_i, \mathbf{x}_j) \triangleq \log(P(\mathbf{x}_i)/P(\mathbf{x}_j))$ is the log-ratio between the *a priori* probability of \mathbf{x}_i and \mathbf{x}_j , f_Z is the probability density function (pdf) of $Z = \sum_{k=1}^{2N_R} z_k^2$, where z_k 's are i.i.d. $\mathcal{N}(0, \sigma_z^2)$ with $\sigma_z^2 = E_s d(\mathbf{x}_i, \mathbf{x}_j)/(2N_T N_0)$, and $d(\mathbf{x}_i, \mathbf{x}_j)$ is the Hamming distance between \mathbf{x}_i and \mathbf{x}_j . The system error probability P_s can be derived by averaging the result in (6) over all pairwise combinations, i.e.,

$$P_s = \sum_{\mathbf{x}_i \neq \mathbf{x}_j \in \mathcal{A}} P(\mathbf{x}_i) P(\mathbf{x}_j | \mathbf{x}_j \neq \mathbf{x}_i) P(\mathbf{x}_i \rightarrow \mathbf{x}_j | \mathbf{x}_i = \tilde{\mathbf{x}}). \quad (7)$$

The dominant terms in (7) involve the error events of symbol \mathbf{x}_i with the largest *a priori* probability $P(\mathbf{x}_i)$. The PEP in these dominant terms can be obtained by substituting the known expression of f_Z into (6) and using the Chernoff bound $Q(x) \leq (1/2) \exp(-x^2/2)$, $x \geq 0$ in (6) (since

$L(\mathbf{x}_i, \mathbf{x}_j) \geq 0$), i.e.,

$$\begin{aligned}
 & P(\mathbf{x}_i \rightarrow \mathbf{x}_j | \mathbf{x}_i = \tilde{\mathbf{x}}) \\
 & \leq \int_0^\infty \frac{1}{2} \exp\left(-\frac{x}{2}\right) \exp\left(-\frac{L(\mathbf{x}_i, \mathbf{x}_j)}{N_0}\right) \\
 & \quad \cdot \exp\left(-\frac{L(\mathbf{x}_i, \mathbf{x}_j)^2}{2N_0^2 x}\right) f_Z(x) dx \\
 & \leq \frac{1}{2} \exp\left(-\frac{L(\mathbf{x}_i, \mathbf{x}_j)}{N_0}\right) \int_0^\infty \exp\left(-\frac{x}{2}\right) f_Z(x) dx \\
 & = \frac{1}{2} \exp\left(-\frac{L(\mathbf{x}_i, \mathbf{x}_j)}{N_0}\right) \cdot (\sigma_z^2 + 1)^{-N_R} \\
 & = \frac{1}{2} \exp\left(-\frac{L(\mathbf{x}_i, \mathbf{x}_j)}{N_0}\right) \cdot \left(\frac{E_s}{N_0} \cdot d(\mathbf{x}_i, \mathbf{x}_j) \cdot \frac{1}{2N_T} + 1\right)^{-N_R}
 \end{aligned} \tag{8}$$

where for the second inequality we have used the fact that $\exp(-L(\mathbf{x}_i, \mathbf{x}_j)^2/(2N_0^2 x)) \leq 1$ in the domain of integration. Note that the bound in (8) has been shown for equiprobable signaling [18] to be loose in low signal-to-noise ratio (SNR) and tighter in high SNR regimes. Substituting (8) in (7) it is clear that at a given operating E_s/N_0 the system performance is dominated by pairs of symbols with large *a priori* probabilities and with small pairwise Hamming distances. When the *a priori* probabilities are all equal, the minimum Hamming distance between arbitrary two distinct symbols, i.e., $d_{\min} = \min_{\mathbf{x}_i \neq \mathbf{x}_j \in \mathcal{A}} d(\mathbf{x}_i, \mathbf{x}_j)$, determines the system performance. When the *a priori* probabilities are not all equal where symbols with large (small) *a priori* probabilities have smaller (larger) pairwise Hamming distances, again, the minimum Hamming distance between arbitrary two distinct symbols determines the system performance. In our energy-efficient consideration for SSK-type modulation enabled transmission (Sec. III), symbols with large *a priori* probabilities typically have smaller pairwise Hamming distances, since creating large Hamming distances requires the employment of large numbers of 1's in the resultant more power-consuming modulation symbols. Note that the analytical insights established here apply to the uncoded system. For the performance analysis for coded systems, the reader is referred to [8], [16]. Specifically, the convolutional coded bit error probability is shown to be bounded by a function of PEP in [8, Sec. IV-B] and [16, Sec. IV-D], with the PEP analyzed through the Hamming distance.

III. ENERGY-EFFICIENT HSSK TRANSMISSION: THEORETICAL RESULTS

A. Motivation of the Proposed Design

A well-known SSK-type modulation is GSSK modulation [17]. To support m bits/s/Hz transmission in a system with N_T transmit antennas, GSSK modulation employs a fixed number of n_t 1's and $N_T - n_t$ 0's in each $N_T \times 1$ modulation symbol, where n_t is chosen such that $\binom{N_T}{n_t-1} < |\mathcal{A}^{(\text{GSSK})}| = 2^m \leq \binom{N_T}{n_t}$ and $n_t \leq N_T/2$. The employment of n_t being the minimum number of 1's required to encode the given amount of information ensures energy efficiency and conservation of the hardware expenses (i.e., the required number of RF chains at the transmitter). An example of the GSSK alphabet is shown in Table I for a system with $N_T = 5$ and $n_t = 2$ to support $m = 3$ bits/s/Hz transmission. If $|\mathcal{A}^{(\text{GSSK})}| < \binom{N_T}{n_t}$ as

TABLE I
EXAMPLE OF GSSK ($n_t = 2$) MODULATION ALPHABET AND BIT MAPPING FOR 3 BITS/S/Hz TRANSMISSION IN A SYSTEM WITH $N_T = 5$

Source bits	GSSK symbols $\in \mathcal{A}^{(\text{GSSK})}$
000	$[0, 0, 0, 1, 1]^T$
001	$[0, 0, 1, 0, 1]^T$
010	$[0, 1, 0, 0, 1]^T$
011	$[1, 0, 0, 0, 1]^T$
100	$[0, 0, 1, 1, 0]^T$
101	$[0, 1, 0, 1, 0]^T$
110	$[1, 0, 0, 1, 0]^T$
111	$[0, 1, 1, 0, 0]^T$

in this example, $\mathcal{A}^{(\text{GSSK})}$ is not unique and may be chosen lexicographically as shown in Table I or based on some optimization criterion at an extra computational cost [17].

While useful for improving the transmission rates of SSK modulation (a special case of GSSK with $n_t = 1$), GSSK modulation presents several limitations.

- 1) *Transmission rate*: The employment of a fixed number of n_t 1's in each modulation symbol results in an inefficient utilization of the set of antenna indices and poses limitation on the attainable transmission rates of GSSK.
- 2) *Symbol selection*: When the size of the GSSK modulation alphabet $|\mathcal{A}^{(\text{GSSK})}| = 2^m$ is smaller than the number of available symbols $\binom{N_T}{n_t}$, a sophisticated selection of symbols into $\mathcal{A}^{(\text{GSSK})}$ (as compared with plain lexicographic selection) typically requires a computationally intense search over the $\binom{N_T}{n_t}$ possibilities of the alphabet.
- 3) *System performance*: The GSSK modulation alphabet, constructed based on lexicographic selection or any other criterion, always leads to $d_{\min} = \min_{\mathbf{x}_i \neq \mathbf{x}_j \in \mathcal{A}^{(\text{GSSK})}} d(\mathbf{x}_i, \mathbf{x}_j) = 2$. This limits the performance of GSSK modulation as discussed previously in Sec. II-B. Furthermore, an increasingly small subset of the total $\binom{N_T}{n_t}$ available symbols can have a pairwise Hamming distance larger than 2 as N_T increases. These performance-related properties of GSSK modulation are analytically explored in the Appendix.

The perceived limitations of GSSK modulation can be overcome by fully utilizing the set of antenna indices, as implemented by HSSK modulation [18]. HSSK modulation employs a varied number of 1's in each modulation symbol based on the Hamming code (in general, binary linear block code) construction technique. For example, to support 4 bits/s/Hz transmission in a system with $N_T = 5$, the HSSK modulation alphabet incorporates the 16 codewords of the $(5, 4)$ code¹ with the last bit of each codeword complemented, as shown in Table II. Note that 4 bits/s/Hz transmission is achievable with HSSK but not with GSSK in a system with $N_T = 5$. Furthermore, as opposed to GSSK with $d_{\min} = 2$ always, HSSK can be configured with a flexible d_{\min} . For example, the HSSK modulation alphabet may incorporate the four codewords of the $(5, 2)$ code $[0, 0, 0, 0, 1]^T$, $[0, 0, 1, 1, 0]^T$,

¹We use the notation (n, k) for binary linear block codes with block length n , message length k , and an implicit minimum distance d . For example, $(n, n-1)$ is binary parity check code with $d = 2$, and $(n, 1)$ is repetition code with $d = n$.

TABLE II
EXAMPLE OF HSSK MODULATION ALPHABET AND BIT MAPPING FOR 4
BITS/s/Hz TRANSMISSION IN A SYSTEM WITH $N_T = 5$

Source bits	HSSK symbols $\in \mathcal{A}^{(\text{HSSK})}$
0000	$[0, 0, 0, 0, 1]^T$
0001	$[0, 0, 0, 1, 0]^T$
0010	$[0, 0, 1, 0, 0]^T$
0011	$[0, 1, 0, 0, 0]^T$
0100	$[1, 0, 0, 0, 0]^T$
0101	$[0, 0, 1, 1, 1]^T$
0110	$[0, 1, 0, 1, 1]^T$
0111	$[1, 0, 0, 1, 1]^T$
1000	$[0, 1, 1, 0, 1]^T$
1001	$[1, 0, 1, 0, 1]^T$
1010	$[1, 1, 0, 0, 1]^T$
1011	$[0, 1, 1, 1, 0]^T$
1100	$[1, 0, 1, 1, 0]^T$
1101	$[1, 1, 0, 1, 0]^T$
1110	$[1, 1, 1, 0, 0]^T$
1111	$[1, 1, 1, 1, 1]^T$

$[1, 1, 0, 0, 0]^T$, and $[1, 1, 1, 1, 1]^T$ to achieve $d_{\min} = 3$ in support of 2 bits/s/Hz transmission in a system with $N_T = 5$. Clearly, there is a tradeoff in terms of achievable rate, d_{\min} , power and hardware costs in the selection of design options. For a detailed coverage on the comparison of HSSK and GSSK, the reader is referred to [18].

Since HSSK modulation employs a varied number of 1's in the modulation symbols to fully utilize the set of antenna indices, some modulation symbols naturally require higher transmission power and more activated transmit antennas than others. This motivates an energy-efficient and hardware-cost-aware refinement of HSSK-enabled transmission that maintains the advantages of HSSK modulation. The design problem and theoretical results are presented next, and a practical energy-efficient HSSK transmission scheme is presented in Sec. IV.

B. Problem Formulation

Let \mathcal{U}^s be the N_T -dimensional binary symbol universe, i.e., $\mathcal{U}^s = \{[x_1, x_2, \dots, x_{N_T}]^T | x_j \in \{0, 1\}, j = 1, \dots, N_T\}$, and \mathcal{U}_i ($i = 0, 1, \dots, N_T$) be a subset of \mathcal{U}^s containing symbols with exactly i 1's, i.e., $\mathcal{U}_i = \{[x_1, x_2, \dots, x_{N_T}]^T | x_j \in \{0, 1\}, j = 1, \dots, N_T, \sum_{j=1}^{N_T} x_j = i\}$. The HSSK modulation alphabet with a specified d_{\min} property incorporates selected codewords in the code $\mathcal{C} = \bigcup_i \mathcal{C}_i$ with minimum distance $d = d_{\min}$, where $\mathcal{C}_i \subseteq \mathcal{U}_i$ ($i \in \{1, \dots, N_T\}$) is the nonempty set of codewords containing exactly i 1's. The all-zero symbol in \mathcal{U}_0 is not used in the modulation alphabet and is reserved for special use (see Sec. IV-B). For example, in the previously described example, the HSSK modulation alphabet incorporates all codewords in the (5, 2) code $\mathcal{C} = \mathcal{C}_1 \cup \mathcal{C}_2 \cup \mathcal{C}_5$ with $d = d_{\min} = 3$ to achieve 2 bits/s/Hz transmission in a system with $N_T = 5$, where $|\mathcal{C}_1| = |\mathcal{C}_5| = 1$ and $|\mathcal{C}_2| = 2$. It is often useful to further restrict the required number of RF chains at the transmitter to M ($1 \leq M \leq N_T$) for the modulation scheme. Therefore, the objective of energy-efficient transmission based on HSSK modulation is to design an alphabet and the symbol *a priori* probabilities so that minimum average symbol power per

transmission $\mathbb{E}[\tilde{\mathbf{x}}^H \tilde{\mathbf{x}}]$ is achieved, while the target transmission rate (spectral-efficiency constraint), the minimum Hamming distance property (performance constraint), and the maximum required number of RF chains (hardware constraint) are met. Given a code $\mathcal{C} = \bigcup_i \mathcal{C}_i$ with the specified minimum distance property, and given the fact that each element in \mathcal{C}_i requires i RF chains at the transmitter and consumes power equal to i , the design problem is mathematically formulated as

$$\begin{aligned} \min_{P_i} \quad & \sum_{\substack{i: \mathcal{C}_i \subseteq \mathcal{C} \\ i \leq M}} i |\mathcal{C}_i| P_i \\ \text{s.t.} \quad & \sum_{\substack{i: \mathcal{C}_i \subseteq \mathcal{C} \\ i \leq M}} |\mathcal{C}_i| P_i = 1 \\ & \sum_{\substack{i: \mathcal{C}_i \subseteq \mathcal{C} \\ i \leq M}} |\mathcal{C}_i| r(P_i) \geq m \end{aligned} \quad (9)$$

where P_i is the *a priori* probability of each symbol in \mathcal{C}_i , i.e., $P_i \triangleq P(\mathbf{x}), \mathbf{x} \in \mathcal{C}_i$, and

$$r(P_i) = \begin{cases} -P_i \log_2 P_i, & \text{if } P_i > 0 \\ 0, & \text{if } P_i = 0 \\ -\infty, & \text{otherwise} \end{cases} \quad (10)$$

The first constraint of Problem (9) states that the *a priori* probabilities of all symbols in the alphabet sum to one, and implies that $P_i = 0$ for i for which $\mathcal{C}_i \not\subseteq \mathcal{C}$ or $i > M$. The second constraint states that the target information rate of m bits is met, described by Shannon's entropy formula. Clearly, the optimal power yielded in Problem (9) with a larger M is no greater than with a smaller M .

Consider an example. Suppose that it is desired to design a modulation alphabet with $d_{\min} = 2$. The largest set of length- N_T codewords with $d = d_{\min} = 2$ is the $(N_T, N_T - 1)$ binary parity check code. More specifically, the code \mathcal{C} is given by $\mathcal{C}_1 \cup \mathcal{C}_3 \cup \mathcal{C}_5 \cup \dots \cup \mathcal{C}_{N_T}$ if N_T is odd and $\mathcal{C}_1 \cup \mathcal{C}_3 \cup \mathcal{C}_5 \cup \dots \cup \mathcal{C}_{N_T-1}$ if N_T is even, where $\mathcal{C}_i = \mathcal{U}_i$. Since $|\mathcal{C}_i| = |\mathcal{U}_i| = \binom{N_T}{i}$, the design problem is specifically formulated as

$$\begin{aligned} \min_{\{P_1, P_3, P_5, \dots\}} \quad & \sum_{i=1,3,5,\dots}^M i \binom{N_T}{i} P_i \\ \text{s.t.} \quad & \sum_{i=1,3,5,\dots}^M \binom{N_T}{i} P_i = 1 \\ & \sum_{i=1,3,5,\dots}^M \binom{N_T}{i} r(P_i) \geq m. \end{aligned} \quad (11)$$

The first constraint of Problem (11) implies that P_2, P_4, \dots are all equal to zero, meaning that symbols in $\mathcal{U}_2, \mathcal{U}_4, \dots$ are not included in the modulation alphabet.

C. Optimal Solution

Note that Problem (9) has a linear objective subject to affine equality and convex inequality constraints. Therefore, they are convex optimization problems with a globally optimal solution, and can be solved using the Lagrange multiplier

method [28]. The Lagrangian function for Problem (9) is given by

$$\Lambda(P_i, \lambda_1, \lambda_2) = \sum_{\substack{i: \mathcal{C}_i \subseteq \mathcal{C} \\ i \leq M}} i|\mathcal{C}_i|P_i + \lambda_1 \left(\sum_{\substack{i: \mathcal{C}_i \subseteq \mathcal{C} \\ i \leq M}} |\mathcal{C}_i|P_i - 1 \right) + \lambda_2 \left(\sum_{\substack{i: \mathcal{C}_i \subseteq \mathcal{C} \\ i \leq M}} |\mathcal{C}_i|P_i \log_2 P_i + m \right) \quad (12)$$

where λ_1 and λ_2 are the Lagrange multipliers. The optimal solution P_i^* and the associated Lagrange multipliers λ_1^* and λ_2^* satisfy the following Karush-Kuhn-Tucker (KKT) necessary conditions

$$\begin{cases} \partial \Lambda(P_i^*, \lambda_1^*, \lambda_2^*) / \partial P_i = 0, & i: \mathcal{C}_i \subseteq \mathcal{C}, i \leq M \\ \lambda_2^* \geq 0 \\ \lambda_2^* = 0, & \text{if the inequality constraint of (9) is inactive.} \end{cases} \quad (13)$$

By directly computing the first condition in (13) we have

$$i|\mathcal{C}_i| + \lambda_1^*|\mathcal{C}_i| + \lambda_2^*|\mathcal{C}_i| \left(\log_2 P_i^* + \frac{1}{\log 2} \right) = 0. \quad (14)$$

Arranging the terms yields $P_i^* = \alpha \beta^i$, where $\alpha = (1/e)2^{-\lambda_1^*/\lambda_2^*}$ and $\beta = 2^{-1/\lambda_2^*}$. Normalizing P_i^* 's according to the equality constraint of Problem (9), we obtain

$$P_i^* = \frac{\beta^i}{\sum_{i: \mathcal{C}_i \subseteq \mathcal{C}, i \leq M} |\mathcal{C}_i| \beta^i}, \quad 0 < \beta \leq 1. \quad (15)$$

The value of β determines the optimal *a priori* probabilities for the alphabet. If $\beta = 1$, we have $P_i^* = 1/(\sum_{i: \mathcal{C}_i \subseteq \mathcal{C}, i \leq M} |\mathcal{C}_i|)$ for all i for which $\mathcal{C}_i \subseteq \mathcal{C}$ and $i \leq M$, i.e., all codewords in \mathcal{C} are included in the alphabet equiprobably to achieve the highest information rate of $\log_2 (\sum_{i: \mathcal{C}_i \subseteq \mathcal{C}, i \leq M} |\mathcal{C}_i|)$ bits at the cost of the largest average symbol power consumption. If $\beta = 0^+$, we have $P_i^* = 1/|\mathcal{C}_i|$ for the smallest i for which $\mathcal{C}_i \subseteq \mathcal{C}$ and $P_i^* = 0$ otherwise, i.e., only the least power-consuming codewords in \mathcal{C} are included in the alphabet equiprobably to support an information rate as high as $\log_2 |\mathcal{C}_i|$ bits, where i is the smallest index for which $\mathcal{C}_i \subseteq \mathcal{C}$. If the target information rate m is less than this rate, the inequality constraint of Problem (9) becomes inactive at the same optimal solution point.

The solution to Problem (11) is given similarly:

$$P_i^* = \frac{\beta^i}{\sum_{i=1,3,5,\dots}^M \binom{N_T}{i} \beta^i}, \quad 0 < \beta \leq 1, i = 1, 3, 5, \dots \quad (16)$$

Note that the solution in (16) coincides with the original GSSK and HSSK ($d_{\min} = 2$) modulation in scenarios in which the original GSSK or HSSK is energy-optimal. For example, to support 4 bits/s/Hz transmission in a system with $N_T = 5$, the solution to Problem (11) with $M = N_T = 5$ and $m = 4$ is given by $P_1^* = P_3^* = P_5^* = 1/16$ in (16) with $\beta = 1$. This coincides with the original HSSK alphabet shown in Table II incorporating symbols in $\mathcal{U}_1, \mathcal{U}_3$, and \mathcal{U}_5 equiprobably. For another example, to support 2 bits/s/Hz transmission in a system with $N_T = 4$, the solution to Problem (11) with $M = N_T = 4$ and $m = 2$ is given by $P_1^* = 1/4$ and $P_3^* = 0$ in (16) with $\beta = 0^+$, which coincides with the SSK alphabet incorporating symbols in \mathcal{U}_1 equiprobably.

Problems (9) and (11) can alternatively be formulated so that the objective is to maximize the transmission rate under the average symbol power constraint (with other constraints unchanged). It is not difficult to see that the same solution set as (15)–(16) will be yielded. In other words, the proposed design scheme will either achieve higher transmission rate at the same symbol power consumption, or achieve less symbol power consumption at the same target transmission rate, than any other SSK-type modulation scheme. The theoretical study in this section provides the optimal symbol *a priori* probabilities. However, no information is given for accomplishing the bit mapping as in Tables I–II. In practice, the technique of variable-length coding is useful for creating an efficient bit-string representation of symbols with unequal *a priori* probabilities, as presented next.

IV. ENERGY-EFFICIENT HSSK TRANSMISSION: PRACTICAL SCHEMES

In this section, we present the proposed practical EE-HSSK transmission scheme and discuss its implementation issues.

A. EE-HSSK Transmission Model

Huffman coding [29] is an entropy coding algorithm for encoding a source of symbols with unequal symbol frequencies. The algorithm achieves a mapping from symbols to bit strings that has the smallest average bit-string length among all binary codes. Therefore, it has been widely used for data compression. The algorithm first generates a symbol frequency-sorted binary tree (i.e., the Huffman tree), and then assigns each symbol with a unique bit string based on the structure of the tree. Here, instead of using the compression capability of Huffman coding, we apply its bit-mapping technique in the proposed modulation scheme.

The system model for EE-HSSK enabled transmission is shown in Fig. 1. At the transmitter, Problem (9) is first solved to obtain the optimal symbol probabilities. Then, the Huffman tree is built for these symbol probabilities, and the Huffman code table (used as the modulation mapping table) is constructed. The Huffman decoder then maps the input bit sequence to the modulation symbols according to the mapping table, and the EE-HSSK modulator and the RF front-end transmit the signal using the corresponding modulation symbols. At the receiver, the EE-HSSK demodulator detects the signal using the MAP criterion, and the Huffman encoder maps each detected symbol to bits according to the mapping table to generate the output bit sequence.

B. Implementation Details

An example of EE-HSSK modulation alphabet is shown in Table III for (approximately) 3 bits/s/Hz transmission in a system with $N_T = 5$. This alphabet is given by first solving Problem (11) with $M = 3$ to obtain the optimal symbol probabilities and then using Huffman coding to approximate the symbol probabilities. As can be clearly seen, unlike standard ASCII coding-based bit mapping in Tables I–II, a Huffman code-aided bit mapping is employed here where the length of the bit strings is roughly reversely proportional to the

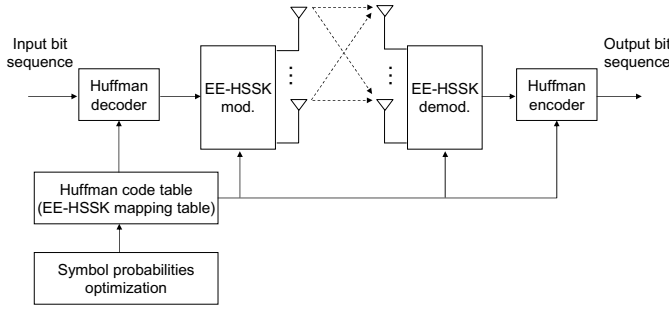


Fig. 1. System model for EE-HSSK modulation enabled transmission.

TABLE III
EXAMPLE OF EE-HSSK ($M = 3$) MODULATION ALPHABET AND BIT MAPPING FOR APPROXIMATELY 3 BITS/S/Hz TRANSMISSION IN A SYSTEM WITH $N_T = 5$

Source bits	EE-HSSK symbols $\in \mathcal{A}^{(\text{EE-HSSK})}$
00	$[0, 0, 0, 0, 1]^T$
01	$[0, 0, 0, 1, 0]^T$
100	$[0, 0, 1, 0, 0]^T$
101	$[0, 1, 0, 0, 0]^T$
110	$[1, 0, 0, 0, 0]^T$
111000	$[0, 0, 1, 1, 1]^T$
111001	$[0, 1, 0, 1, 1]^T$
111010	$[1, 0, 0, 1, 1]^T$
111011	$[0, 1, 1, 0, 1]^T$
111100	$[1, 0, 1, 0, 1]^T$
111101	$[1, 1, 0, 0, 1]^T$
1111100	$[0, 1, 1, 1, 0]^T$
1111101	$[1, 0, 1, 1, 0]^T$
1111110	$[1, 1, 0, 1, 0]^T$
1111111	$[1, 1, 1, 0, 0]^T$

symbol power. Since longer bit strings appear less frequently in a random input sequence, symbols more power-consuming are used less frequently to achieve energy efficiency. Note that the optimal *a priori* probability given by Problem (11) for a symbol is only approximately matched by probability 2^{-k} using Huffman coding, where k is the length of the bit string associated with the symbol. Therefore, EE-HSSK modulation may achieve a slightly different transmission rate than is configured in Problem (9) or (11), and may incur an energy penalty compared to the optimal theoretical results. The energy penalty, however, is generally very small (see Sec. V).

Due to the unequal length of bit strings, if an error occurs in the detector, the Huffman encoder at the receiver will map the incorrect symbol to a bit string that is different from the transmitted one and possibly of a different length. The mismatch in the bit length will result in an error propagation effect affecting all subsequent symbols regardless of whether they are correctly detected. Among other solutions, the following methods may help resolve this issue.

- 1) *Frame-based operation*: In the frame (packet)-based operation of EE-HSSK scheme, the input bit sequence is divided into frames. Each frame is transmitted using multiple EE-HSSK modulation symbols. Different frames are separated by the all-zero (“idle”) symbol which signals the beginning/end of the transmission of a frame. If the mismatch effect occurs, it affects the current frame but

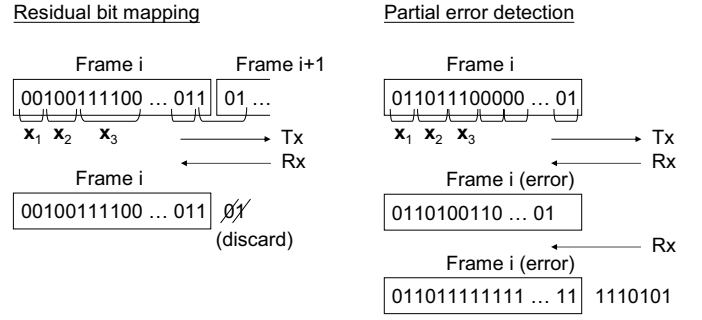


Fig. 2. Residual bit mapping and partial error detection in the frame-based EE-HSSK transmission.

not the next frame.

- 2) *Channel coding and interleaving*: If the information sequence is protected by the channel encoder and the interleaver before entering the Huffman decoder, the bit errors resulted from the mismatch effect in a frame will be deinterleaved to non-contiguous locations and could be corrected by the channel decoder.

We consider the frame-based implementation of EE-HSSK enabled transmission in the uncoded scenarios. Operation details and an embedded capability of this implementation are described below.

- 1) *Residual bit mapping*: In the frame-based implementation, if the residual bits at the end of a frame are not sufficient to map to an EE-HSSK symbol, the initial bits of the next frame are borrowed to complete the mapping. At the receiver, extra bits beyond a frame are discarded. This is illustrated in Fig. 2, where the exemplary EE-HSSK mapping in Table III is adopted.
- 2) *Partial error detection (ED)*: The frame-based implementation enables partial ED at no extra cost. Specifically, when the receiver recovers a bit sequence shorter than the frame size, or longer than the frame size plus the maximum length of EE-HSSK symbols minus one, the receiver detects an incorrect frame being received. This is illustrated in Fig. 2. This embedded partial ED mechanism can be used in conjunction with automatic repeat request (ARQ) to improve the frame transmission performance, as will be demonstrated in Sec. V.

V. SIMULATION RESULTS

In this section, we show the simulation results for the performance of the proposed energy-efficient transmission scheme in comparison with the theoretical results and the conventional schemes. The frame size is set to 100 bits in simulation, and enough frames are simulated to count 300 frame errors. The lexicographic constellation design is adopted for GSSK modulation for computational efficiency without much loss of performance [18]. The detection algorithm is MAP for all schemes.

In Fig. 3, we plot the average symbol power per transmission $\mathbb{E}[\tilde{\mathbf{x}}^H \tilde{\mathbf{x}}]$ versus the transmission rate for different SSK-type modulation schemes. First, we see that a more flexible utilization of the antenna indices allows HSSK to achieve different d_{\min} values while GSSK always has $d_{\min} = 2$. The

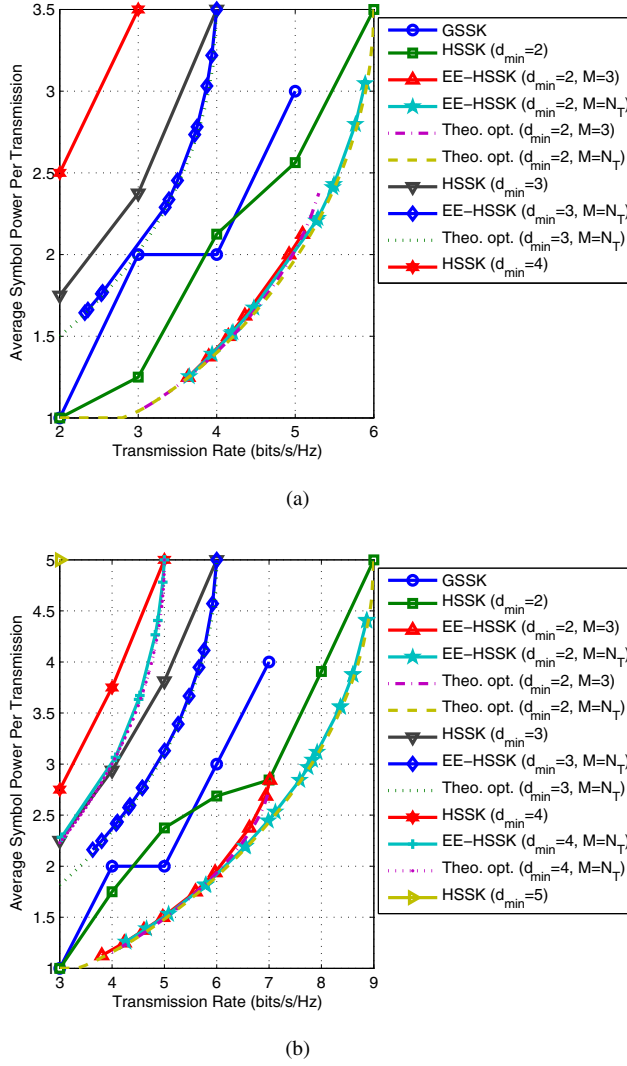


Fig. 3. Average symbol power per transmission versus transmission rate for SSK-type modulation schemes in a system with (a) $N_T = 7$ and (b) $N_T = 10$.

adoption of a larger d_{\min} consumes higher average symbol power but contributes to a better error performance for HSSK (the tradeoff to be examined in Figs. 4–5). Second, HSSK can support transmission rates unachievable with GSSK, possibly using even lower average symbol power (e.g., 8 bits/s/Hz transmission for HSSK versus 7 bits/s/Hz transmission for GSSK in Fig. 3(b)) due to a more efficient utilization of the antenna indices. Third, the proposed EE-HSSK scheme maintains the advantages of HSSK modulation while demonstrates better energy efficiency as it nearly overlaps with the theoretical optimum for various d_{\min} configurations. The effect of imposing an RF-chain constraint $M < N_T$ is examined for the $d_{\min} = 2$ configuration. As can be seen, imposing a hardware constraint for EE-HSSK generally demonstrates negligible energy penalty. A bigger energy penalty is observed when the transmission rate approaches the capacity (maximum supportable rate) of the modulation scheme under the specified RF-chain constraint. For example, for a system with $N_T = 10$ the capacity of EE-HSSK ($M = 3$) is

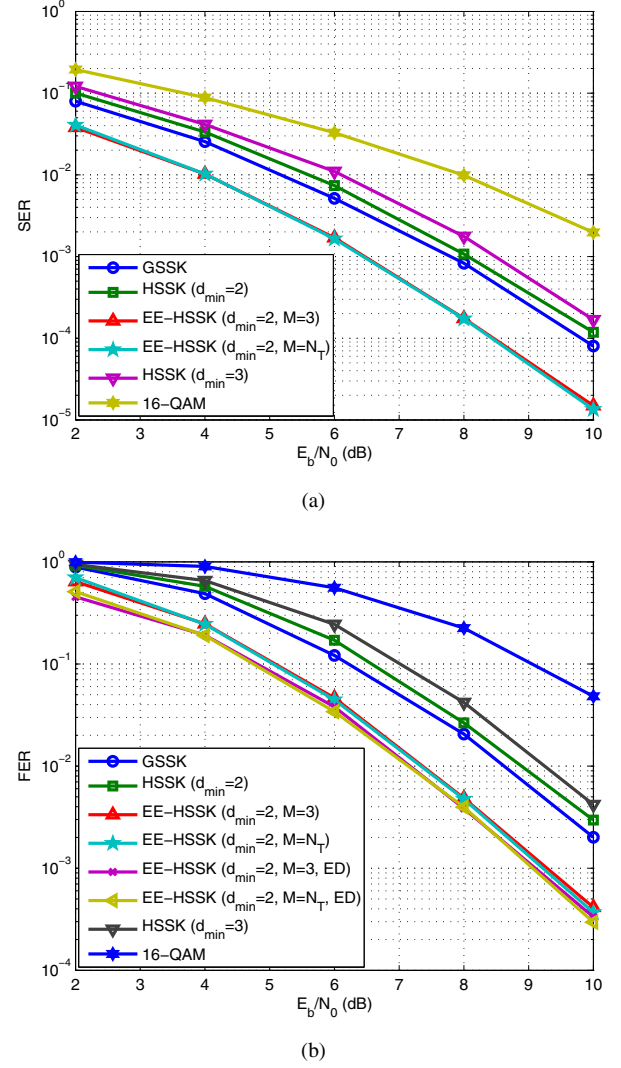


Fig. 4. (a) SER and (b) FER performance for SSK-type modulation schemes (7×7 system) and 16-QAM (1×7 system), for 4 bits/s/Hz transmission.

$130 \left[- (1/130) \log_2(1/130) \right] = 7.0224$ bits/s/Hz, incorporating all the $\binom{10}{1} + \binom{10}{3} = 130$ symbols in \mathcal{U}_1 and \mathcal{U}_3 equiprobably in the alphabet. At the capacity-achieving rate of 7 bits/s/Hz, a minor energy gap is observed between adopting $M = 3$ and $M = N_T = 10$ for EE-HSSK, as shown in Fig. 3(b). Also, at this rate EE-HSSK ($M = 3$) and HSSK nearly overlap, because HSSK's alphabet with equiprobable symbols is nearly energy-optimal in this particular case. The simulation results motivate the use of EE-HSSK ($M < N_T$) scheme in practice to reduce the number of required RF chains from N_T to M at little loss of energy performance.

In Figs. 4–5, we plot the symbol-error-rate (SER) and frame-error-rate (FER) versus E_b/N_0 performance at fixed transmission rates, where E_b represents the average energy of a single bit and is given by $E_s \cdot \mathbb{E}[\tilde{\mathbf{x}}^H \tilde{\mathbf{x}}]/m$ for m bits/s/Hz transmission. The target transmission rates are 4 bits/s/Hz for a system with $N_T = 7$ in Fig. 4, and 6 bits/s/Hz for a system with $N_T = 10$ in Fig. 5. We consider symmetric systems ($N_T = N_R$) for SSK-type modulation schemes, although this is not a constraint on the proposed energy-

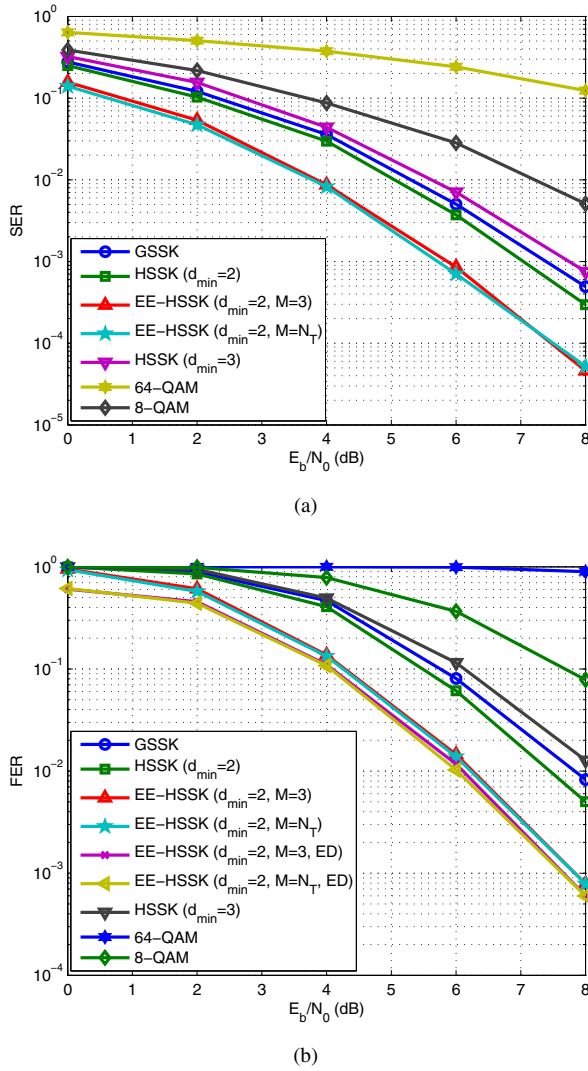


Fig. 5. (a) SER and (b) FER performance for SSK-type modulation schemes (10×10 system), 64-QAM (1×10 system), and 8-QAM (2×10 system), for 6 bits/s/Hz transmission.

efficient transmission strategy. We see that EE-HSSK with $M = 3$ and $M = N_T$ have almost identical performance, both outperforming the original GSSK/HSSK and the conventional QAM (with gray-coded bit mapping). HSSK with $d_{\min} = 3$ configuration exhibits a slight disadvantage of performance as compared with $d_{\min} = 2$ configuration, suggesting that in these particular scenarios the improved error performance due to a larger d_{\min} does not outweigh the extra power spent for creating a larger d_{\min} . The performance gap between EE-HSSK and GSSK/HSSK is about 1.5–2 dB at $\text{SER} = 10^{-3}$ and $\text{FER} = 10^{-2}$ in both Figs. 4 and 5. The embedded ED mechanism working in conjunction with ARQ-based transmission can further improve the FER performance of EE-HSSK. With the implementation of ED, a frame in error, when detected, is retransmitted so that with high probability it will be received correctly with joint detection of the two transmissions (we assume that the first retransmission is always successful in our simulation). The performance improvement is however slight, since not all frame error patterns can be detected by

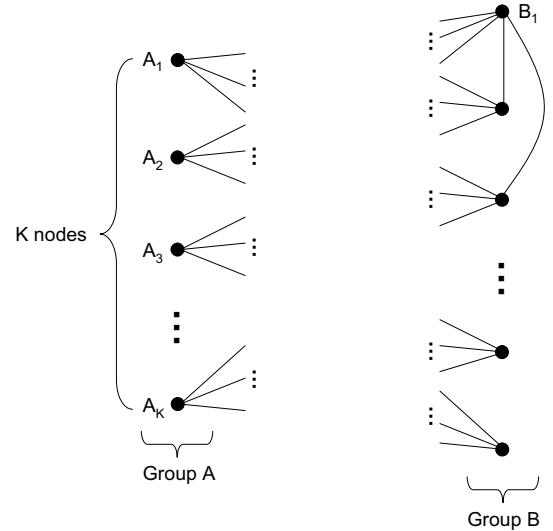


Fig. 6. Graph constructed for Proof of Theorem 1.

the embedded ED mechanism (e.g., if no bit-length mismatch occurs, the frame error cannot be detected).

VI. CONCLUSION

The energy efficiency of SSK-type modulation enabled transmission over MIMO wireless channels has been studied. A theoretical framework for the design of energy-efficient communication forms the foundation for the development of a practical energy-efficient modulation scheme. The proposed modulation scheme incorporates a novel use of the Hamming code and Huffman code techniques in the alphabet and bit-mapping designs. The proposed modulation scheme carries an energy-saving potential unachievable in previous schemes, as demonstrated by extensive simulations, and shows great promise for next-generation green MIMO communications.

APPENDIX

PROPERTIES OF GSSK MODULATION

Recall that, to support m bits/s/Hz transmission in a system with N_T transmit antennas, GSSK modulation adopts a modulation alphabet $\mathcal{A}^{(\text{GSSK})}$ with $2^m N_T \times 1$ symbols each comprised of n_t 1's and $N_T - n_t$ 0's, where n_t is chosen such that

$$\binom{N_T}{n_t - 1} < |\mathcal{A}^{(\text{GSSK})}| = 2^m \leq \binom{N_T}{n_t} \quad \text{and} \quad n_t \leq \frac{N_T}{2}. \quad (17)$$

Lemma 1: The Hamming distance between arbitrary two distinct GSSK symbols $\mathbf{x}_i, \mathbf{x}_j \in \mathcal{A}^{(\text{GSSK})}$ is a positive even number, i.e., $d(\mathbf{x}_i, \mathbf{x}_j) = 2, 4, 6, \dots$

Proof: Assume \mathbf{x}_i and \mathbf{x}_j differ in k positions, $1 \leq k \leq N_T$. Thus, $d(\mathbf{x}_i, \mathbf{x}_j) = k$. Since the numbers of 1's in \mathbf{x}_i and \mathbf{x}_j are equal, and \mathbf{x}_i and \mathbf{x}_j are identical in the other $N_T - k$ positions, \mathbf{x}_i and \mathbf{x}_j contain the same number of 1's, say l , where $1 \leq l \leq n_t$, in these k positions. Since \mathbf{x}_i and \mathbf{x}_j differ in these k positions, we must have $k = 2l$. ■

Theorem 1: For any $\mathcal{A}^{(\text{GSSK})}$, the minimum Hamming distance between arbitrary two distinct GSSK symbols is 2, i.e., $d_{\min} = \min_{\mathbf{x}_i \neq \mathbf{x}_j \in \mathcal{A}^{(\text{GSSK})}} d(\mathbf{x}_i, \mathbf{x}_j) = 2$.

Proof: If $n_t = 1$, it is straightforward that GSSK (i.e., SSK) modulation has $d_{\min} = 2$. Therefore, we consider $n_t \geq 2$. Assume the opposite; that is, by Lemma 1, GSSK modulation has $d_{\min} = 4$. Let \mathcal{U}_{n_t} be the set of symbol candidates from which the GSSK modulation symbols are selected, where $|\mathcal{U}_{n_t}| = \binom{N_T}{n_t}$. We construct an undirected graph with $\binom{N_T}{n_t}$ nodes, each node representing a symbol candidate in \mathcal{U}_{n_t} . Two nodes are connected by an edge if they represent symbols that have a Hamming distance of 2, and unconnected otherwise. Nodes in the graph are divided into two groups, where group A contains $K \triangleq 2^m$ nodes that represent the GSSK modulation symbols and group B contains the rest of nodes, as shown in Fig. 6. Since we assume that GSSK modulation has $d_{\min} = 4$, there is no edge between nodes in group A . There is, however, possibly an edge between a node in group A and a node in group B , as well as between two nodes in group B , as depicted in Fig. 6. Suppose that, without loss of generality, node A_1 in group A represents the following symbol²

$$\underbrace{111 \dots 1}_{n_t-1} \underbrace{000 \dots 0}_{N_T-n_t} 1. \quad (18)$$

Symbols in \mathcal{U}_{n_t} that have a Hamming distance of 2 from the symbol in (18) must be represented by nodes in group B . These symbols can be produced by switching positions of an 1 and a 0 in (18), and therefore there are $n_t(N_T - n_t)$ such symbols. By our graph construction rule, there is an edge between node A_1 and each of these $n_t(N_T - n_t)$ nodes in group B . Thus, node A_1 has $n_t(N_T - n_t)$ edges going out to group B . Likewise, for each node A_2, A_3, \dots, A_K in group A , there are also $n_t(N_T - n_t)$ edges going out to group B . As a result, the total number of edges going out from group A to group B is $Kn_t(N_T - n_t)$.

Suppose that, without loss of generality, node B_1 in group B represents the following symbol

$$\underbrace{111 \dots 1}_{n_t} \underbrace{000 \dots 0}_{N_T-n_t}. \quad (19)$$

If a node in group A and node B_1 are connected by an edge, this node in group A represents a symbol that has a Hamming distance of 2 from the symbol in (19) and can be produced by switching positions of an 1 and a 0 in (19). That is, it is a concatenation of a row in \mathbf{G}_1 and a row in \mathbf{G}_2 :

$$\begin{bmatrix} 111 \dots 110 \\ 111 \dots 101 \\ 111 \dots 011 \\ \vdots \\ 011 \dots 111 \end{bmatrix} \begin{bmatrix} 000 \dots 001 \\ 000 \dots 010 \\ 000 \dots 100 \\ \vdots \\ 100 \dots 000 \end{bmatrix}. \quad (20)$$

$\mathbf{G}_1: n_t \times n_t \quad \mathbf{G}_2: (N_T - n_t) \times (N_T - n_t)$

Suppose that there are two nodes in group A that are each connected with node B_1 by an edge. Then, these two nodes must each represent a symbol comprised of a row in \mathbf{G}_1 and a row in \mathbf{G}_2 , as discussed above. Furthermore, since these two nodes in group A represent symbols with a Hamming distance of 4 or more in between, they must each represent a symbol

comprised of a different row in \mathbf{G}_1 and a different row in \mathbf{G}_2 because otherwise they would have a Hamming distance of 2 rather than 4. For this reason, node B_1 can be connected by an edge with at most $\min(n_t, N_T - n_t) = n_t$ nodes in group A , or equivalently, node B_1 has at most n_t edges going out to group A . Likewise, each of the other nodes in group B has at most n_t edges going out to group A .

Since the total number of edges going out from group A to group B is $Kn_t(N_T - n_t)$, and each node in group B can have at most n_t edges going out to group A , there must be at least $Kn_t(N_T - n_t)/n_t = K(N_T - n_t)$ nodes in group B . Since the combined total number of nodes in group A and group B is $\binom{N_T}{n_t}$, we have

$$\underbrace{K}_{\text{\# nodes in group } A} + \underbrace{K(N_T - n_t)}_{\text{min. \# nodes in group } B} \leq \binom{N_T}{n_t} \quad (21)$$

or

$$\begin{aligned} K &\leq \frac{1}{N_T - n_t + 1} \binom{N_T}{n_t} \\ &= \frac{1}{n_t} \binom{N_T}{n_t - 1}. \end{aligned} \quad (22)$$

Since $n_t \geq 2$, (22) and (17) are in contradiction, which completes the proof. ■

Corollary 1: For any $\mathcal{A}^{(\text{GSSK})}$, the maximum number of GSSK symbols with a pairwise Hamming distance of 4 or more is $\left\lfloor 1/(N_T - n_t + 1) \binom{N_T}{n_t} \right\rfloor$.

Proof: This result follows from (22) in Proof of Theorem 1. ■

Corollary 2: For any $\mathcal{A}^{(\text{GSSK})}$, the maximum number of GSSK symbols with a pairwise Hamming distance of 6 or more is $\left\lfloor 1/(N_T n_t - n_t^2 + 1) \binom{N_T}{n_t} \right\rfloor$.

Proof: Following Proof of Theorem 1 and considering that the K nodes in group A have a pairwise Hamming distance of at least 6, it is clear that node B_1 can be connected by an edge with at most one node in group A . This is because otherwise the two or more nodes in group A connected with B_1 must each represent a symbol comprised of a row in \mathbf{G}_1 and a row in \mathbf{G}_2 in (20) and therefore cannot have a Hamming distance of 6 or more in between. Since each node in group B has at most one edge going out to group A , there must be at least $Kn_t(N_T - n_t)$ nodes in group B . Thus,

$$K + Kn_t(N_T - n_t) \leq \binom{N_T}{n_t} \quad (23)$$

or equivalently

$$K \leq \frac{1}{N_T n_t - n_t^2 + 1} \binom{N_T}{n_t}. \quad (24)$$

Corollary 1 and Corollary 2 have an implication for GSSK-based transmission in large MIMO systems. Specifically, for any $\mathcal{A}^{(\text{GSSK})}$ the number of GSSK symbols with a pairwise Hamming distance of 4 (or 6) or more compared with the total number of symbol candidates $|\mathcal{U}_{n_t}| = \binom{N_T}{n_t}$ is no more than the ratio of $2/N_T$ (or $4/N_T^2$) when N_T is large and $n_t \approx N_T/2$. This suggests that an increasingly small

²For notational convenience, we use 101 to denote symbol $[1, 0, 1]^T$, etc.

subset of the available symbol candidates can have a pairwise Hamming distance larger than 2 as N_T increases, a result of the inefficiency of the fixed n_t construction in GSSK modulation.

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