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Interference Pricing Mechanism for Downlink Multicell Coordinated Beamforming

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***Abstract*—We consider the downlink coordinated beamforming problem in a cellular network in which the base stations (BSs) are equipped with multiple antennas and each user is equipped with a single antenna. The BSs cooperate in sharing their local interference information, and they aim to maximize the sum-rate of the users in the network. A decentralized interference pricing beamforming (IPBF) algorithm is proposed to identify the coordinated beamformer, where a BS is penalized according to the interference it creates to its peers. We show that the decentralized pricing mechanism converges to an interference equilibrium, which is a KKT point of the sum-rate maximization problem. The proofs rely on the identification of rank-1 solutions of each BSs' interference-penalized rate maximization problem. Numerical results show that the proposed iterative mechanism reduces significantly the exchanged information with respect to other state-of-the-art beamforming algorithms with very little sum-rate loss. The version of the algorithm that limits the coordination to a cluster of base stations (IPBF-L) is shown to have very small sum-rate loss with respect to the full coordinated algorithm with much less backhaul information exchange.**

***Index Terms*—Base station coordination, beamforming, interference equilibirum, multiple input–multiple output (MIMO), non-convex.**

I. Introduction

MULTIPLE input–multiple output (MIMO) communications [1] have been adopted in many recent wireless standards, such as IEEE 802.16 [2] and 3GPP LTE [3], in the aim of boosting the data rates provided to the customers. A promising solution to achieve spectrally-efficient communications is the universal frequency reuse (UFR) scheme, in which all cells operate on the same frequency channel. This scenario is also known as MIMO interference broadcast channel (MIMO-IBC), where the downlink capacity of the conventional cellular systems with UFR is limited by inter-cell interference. As a result, it is necessary to introduce coordination among the base stations (BSs) so that they can jointly manage the interferences in all cells to improve the system performance [4]. Such coordination technique among the BSs in the downlink is also known as *network MIMO* [5] or *Coordinated multipoint* (CoMP) [6].

Cooperative encoding among BSs using the non-linear Dirty Paper Coding (DPC) technique can suppress other cell interference if the interference is known by the transmitters [7]. However, this approach is impractical, so other approaches in the literature have exploited less complex schemes. Linear transmit beamforming has been proven to provide the same sum-rate scaling law as DPC while maintaining low complexity [8] and has attracted great interest as a technique for BS coordination in multicell network [9]–[15]. In this regard, weighted sum-rate maximization (WSRM) for BS coordination with linear beamforming has received increasing attention from the research community. The WSRM problem is a nonconvex and NP-hard problem [16], [17] and, consequently, very challenging from a computational perspective.

Economists use the term *externality* to describe any effect by an economic agent's decision on other economic agents, excepting those mediated by a price mechanism. It is well-known that the competitive equilibrium of an exchange economy with either production or consumption externalities may fail to maximize aggregate surplus. Lindahl [18] proposed an *externality pricing* approach to recover efficiency in a competitive equilibrium by charging consumers (in the case of a consumption externality) the associated marginal benefit reduction. It is in this sense that *interference pricing* has been recently used for the design of linear beamforming for MIMO links [19], multicell networks [20] and single-cell MIMO systems [21], [22]. A distributed coordinated scheduler relying on a master-slave architecture has been proposed in [23] to optimize the network weighted sum-rate with uniform power allocation and fixed beamformers. Interestingly, the problem is maximized over transmission ranks and user scheduling rather than the usual beamforming vectors.

In this paper, we address the WSRM problem for downlink MISO-IBC in a cellular scenario, in which the base stations simultaneously transmit to their respective groups of receivers and impose upon each other an interference externality [24]. We introduce the notion of an *interference equilibrium*, i.e., a 3-tuple of interference prices, beamforming and power allocation strategies (power shadow prices) such that each individual base station cannot strictly improve its own weighted sum rate by unilaterally adopting a different beamforming and/or power

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allocation strategy. We propose a decentralized iterative pricing and beamforming algorithm and show that it converges to an interference equilibrium. Such equilibrium is shown to correspond to a KKT point for the original WSRM problem. The algorithm has low computational and informational requirements (i.e., beamforming updates are of rank 1 and interference price updates are sent to a single base station at each iteration).

The paper is organized as follows. We start by reviewing the existing literature in Section II. In Section III, we give the problem formulation and system description. The interference pricing mechanism framework and convergence of the iterative mechanism are provided in Section IV after relaxing the rank-1 constraint at each BS. In Section V we show that such relaxation is done with no loss of generality as we provide a simple procedure to obtain a rank-1 solution for each BS. The Interference Pricing BeamForming (IPBF) algorithm to solve the complete multicell beamforming problem is developed in Section VI. In Section VII, we provide numerical results to demonstrate the performance of the proposed algorithm and its limited coordination version IPBF-L is presented and evaluated. Finally, we offer concluding remarks in Section VIII.

II. Literature Review and Summary of Contributions

Although the weighted sum-rate maximization (WSRM) for BS coordination with linear beamforming can be globally solved [25], [26], most works have focused on finding suboptimal solutions with less complex algorithms. For instance, mean-square-error (MSE)-based approaches have been proposed for the WSRM problem in MIMO interfered channel. In [27], the MSE criterion is used to design the precoder to solve the WSRM problem. Other works exploit the design of the minimum mean-square error (MMSE) beamformer, in both the transmitter (precoder) and the receiver, to maximize the weighted sum-rate. In [28], the authors propose a set of algorithms based on interference alignment for MIMO point-to-point communications. Also, using interference alignment, in [29] it is studied the maximization of the sum of degrees of freedom for MIMO-IC. The proposed algorithm is semidistributed in terms of communication and needs some information exchange among the nodes, although sharing of users' data streams is not requered. When BSs share users' data streams, it is shown in [30] that partial coordination among BSs is equivalent to the MIMO interference channel, in which each transmitter only knows the message of one user under generalized linear constraints. The main drawback of MSE-based methods for the WSRM problem is that they generally require additional fed back signal coordination from the receivers to the base stations over the iterations of the algorithm.

Previous works have proposed algorithms that reach a stationary point assuming complete users' data sharing. In [17], the authors propose a distributed algorithm to maximize the WSR based on primal decomposition with users served by several BSs. In this case, the convergence of the algorithm is shown with appropriate choice of stopping criteria, although local optimality is not guaranteed since the subproblems are not solved using KKT conditions. The algorithm is defined by the authors as asynchronous in the sense that BSs can solve their local subproblems independently once they have the required information from other BSs. In [31], the authors propose the polite water-filling algorithm, which balances between created interference and link rate, for generalized interference (referred to as B-MAC) networks. The algorithm is based on the duality between the forward and reverse links. This channel reciprocity can be only used for point-to-point communications where the power constraints are symmetric, so it cannot be extended to the case of multicell multiuser networks. In [32], the authors propose a set of centralized and distributed algorithms based on MMSE to maximize the WSR with per antenna power constraints. The algorithms are suitable for both single and multicell scenarios and provide the design of transmitters and receivers, although the decentralized algorithm relies on information exchange among all BSs and users.

However, the limited bandwidth of the backhaul network connecting the base stations may prevent users' data sharing; moreover, users' data sharing can be difficult to implement in real time. More practical models assuming coordinated base stations only exchange channel-state-related information lead to more efficient use of the backhaul bandwidth. For MIMO cognitive radio networks with point-to-point links, the WSRM problem for the secondary users is addressed in [33]. The WSR problem is formulated with constraints in the interference created over primary users to design the link transmitter precoders, and the proposed iterative centralized algorithm is shown to converge to a KKT point. The partially-connected cluster (PCC) model presented in [34] assumes that each user is served by only one BS and that coordinated BSs jointly optimize their beams based on the inter-cell channel gains. They propose the Iterative Coordinated Beamforming (ICBF) centralized algorithm, which also admits a distributed implementation, and the experiments show that the algorithm converges. In the same line, in [35] the authors relate the WSRM problem to the matrix-weighted sum-MSE minimization problem, and present the WMMSE iterative algorithm to calculate the transmit and receive beamformers in the MIMO-IBC. It is shown that the WMMSE algorithm converges to a local optima of the utility optimization problem, and that the algorithm can be extended to other increasing utility functions.

Only a small subset of these papers (e.g., [33] for a cognitive radio scenario in point-to-point communications, and [35] for matrix-weighted sum-MSE minimization) effectively prove convergence of decentralized (or distributed) algorithms to a KKT solution. Based on the interference pricing framework, we develop an algorithm allowing each BS to iteratively optimize its own interference-penalized sum rate. We show the interference equilibrium is equivalent to a KKT point of the original problem, and we prove that the algorithm converges to a KKT point after relaxing the rank-1 constraint at each base station. Such relaxation entails no loss of generality as we describe a simple procedure to obtain the rank-1 closed-form beamforming solution at each base station. The proposed Interference Pricing BeamForming (IPBF) algorithm exhibits the following properties: i) *Decentralization*, i.e., individual BSs optimize their own interference-penalized sum rate; ii) *Low informational requirements*, interference price updates are sent to a single base station at each iteration; iii) *Convergence*, unlike most algorithms in the literature, the IBPF algorithm is guaranteed to converge to a KKT point; iv) *Users' data sharing*, the IPBF algorithm assumes that each user is served by one and only one BS, so no users' data sharing is required.

The proposed Interference Pricing BeamForming (IPBF) algorithm is compared with the Iterative Coordinated Beam-Forming (ICBF) algorithm presented in [34] and with the weighted MMSE (WMMSE) algorithm proposed in [35], which is expected to outperform both IPBF and ICBF algorithms at the expense of higher information exchange. We observe some important differences with respect to these approaches. First, in our approach the BSs update sequentially instead of simultaneously. One important consequence of such difference in updating schedule is the amount of information exchange needed in each iteration; in our algorithm, all BSs only need to send a single copy of their local information to a *single* BS, while in ICBF and WMMSE, they need to send to *all other* BSs. Second, there is no "inner iteration" needed,[1] in which all the BSs can update their beam matrices at the same time to reach some *intermediate convergence* (note that in ICBF and WMMSE the convergence of the inner iteration is *not* guaranteed). Such "inner iteration" is undesirable, because *a*) a finite stopping time necessarily implies the solution identified is only *approximately* optimal, hence the approximation error may significantly deteriorate performance, and *b*) in each of such inner iterations, extra feedback information needs to be exchanged between the BSs and their users. Finally, the IPBF algorithm is proven to converge to a KKT point, while the ICBF algorithm does not possess such convergence guarantee. The simulation results show that the proposed algorithm has similar sum-rate performance that the ICBF and WMMSE algorithms, while requiring significantly less information exchange among the BSs through the backhaul network.

Notation: For a symmetric matrix $\mathbf{X}$, $\mathbf{X} \succeq 0$ signifies that $\mathbf{X}$ is positive semi-definite. We use $\mathrm{Tr}(\mathbf{X})$, $|\mathbf{X}|$, $\mathbf{X}^H$, and $\mathrm{Rank}(\mathbf{X})$ to denote the trace, the determinant, the Hermitian, and the rank of a matrix, respectively. $[\mathbf{X}]_{i,i}$ denotes the (i,i)th element of the matrix $\mathbf{X}$. $\mathbf{I}_n$ is used to represent a $n \times n$ identity matrix. We use $\mathbb{R}^{N\times M}$ and $\mathbb{C}^{N\times M}$ to denote the set of real and complex $N \times M$ matrices; we use $\mathbb{S}^N$ and $\mathbb{S}^N_+$ to identify the set of $N \times N$ Hermitian and Hermitian semi-definite matrices, respectively. Define $t \oslash M \triangleq \{(t+1) \bmod M\} + 1$ as an integer taking values from $\{1, \ldots, M\}$.

III. System Model and Problem Formulation

We consider the interference broadcast channel (IBC) of a multi-cell multiple-input-single-output (MISO) network with M cells, each of them served by a base station (BS) $m \in \mathcal{M} \triangleq \{1, \ldots, M\}$. Each BS m serves a set $\mathcal{N}_m$ with N_m distinctive users and is equipped with K transmit antennas,

[1]In the context of an algorithm that consists of two nested loops, *inner iterations* refer to the iterations executed by the inner loop, and *outer iterations* are the iterations corresponding to the outer loop.

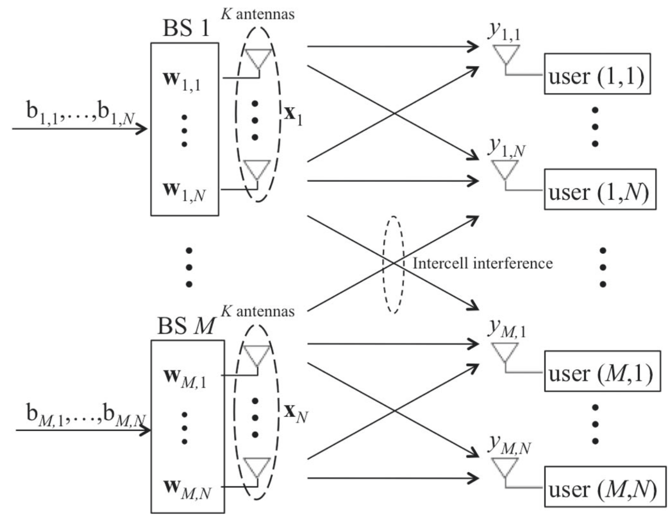

Fig. 1. MISO IFBC system with M base stations, K transmit antennas, and $\mathcal{N}_m = N$ users per BS for all m.

and each user is equipped with a single receive antenna, as shown in Fig. 1. A linear precoder $\mathbf{w}_{m,i} \in \mathbb{C}^K$ with $i \in \mathcal{N}_m$ is used for transmission, and the signal $\mathbf{x}_m \in \mathbb{C}^K$ transmitted by BS m is

$$\mathbf{x}_m = \sum_{i\in\mathcal{N}_m} \mathbf{w}_{m,i} b_{m,i} \tag{1}$$

where $b_{m,i}$ is the complex information symbol to be sent by BS m to user i. We assume that the data symbols are independent and have unit variance: $E[b_{m,i}b^*_{q,j}] = 0$ for all $(m,i) \neq (q,j)$, and $E[b_{m,i}b^*_{m,i}] = 1$, for all (m,i). Each BS $m \in \mathcal{M}$ has a total transmit power constraint such that $\sum_{i\in\mathcal{N}_m} \|\mathbf{w}_{m,i}\|^2 \leq \bar{p}_m$.

We assume the MIMO parallel channels are perfectly uncorrelated assuming the following hypothesis: a) transmit antenna spacing is large enough (for instance, $\lambda/2$); and b) omnidirectional and isotropic antennas, which provides rich scattering environment. Under these ideal conditions, the MIMO channels are i.i.d. frequency-flat Rayleigh fading channels, and the channel vectors $\mathbf{h}_{m,i}$ are modeled as i.i.d. $\mathcal{CN}(0,1)$ random variables. Let $\mathbf{h}_{m,i} \in \mathbb{C}^K$ denote the complex channel between the mth BS and the ith user in mth cell. Let $\mathbf{h}_{q,m_i} \in \mathbb{C}^K$ denote the complex channel between the qth BS and the ith user in mth cell, $q \neq m$. Let $n_{m,i} \in \mathbb{C}$ denote the circularly-symmetric Gaussian noise with variance $c_{m,i}$. We use (m,i) and $-(m,i)$ to denote the ith user in mth cell and all the users except user (m,i), respectively. Then, the matched-filtered, symbol-sampled complex baseband signal received by user (m,i) can be expressed as

$$y_{m,i} = \mathbf{h}^H_{m,i}\mathbf{w}_{m,i}\mathbf{b}_{m,i} + \underbrace{\sum_{j\neq i} \mathbf{h}^H_{m,i}\mathbf{w}_{m,j}\mathbf{b}_{m,j}}_{\text{Intra-cell Interference}} + \underbrace{\sum_{q\neq m, j\in\mathcal{N}_q} \mathbf{h}^H_{q,m_i}\mathbf{w}_{q,j}\mathbf{b}_{q,j}}_{\text{Inter-cell Interference}} + n_{m,i}. \tag{2}$$

The following rate is achievable for user (m, i) by using Gaussian codebooks:

$$\begin{aligned} &R_{m,i}\left(\mathbf{W}_{m,i}, \mathbf{W}_{-(m,i)}\right) \\ &\quad \triangleq \log\left(1 + \frac{\mathbf{h}_{m,i}^H \mathbf{W}_{m,i} \mathbf{h}_{m,i}}{c_{m,i} + \sum\limits_{(q,j)\neq(m,i)} \mathbf{h}_{q,m_i}^H \mathbf{W}_{q,j} \mathbf{h}_{q,m_i}}\right) \\ &\quad = \log\left(1 + \frac{\mathbf{h}_{m,i}^H \mathbf{W}_{m,i} \mathbf{h}_{m,i}}{I_{m,i}\left(\mathbf{W}_{-(m,i)}\right)}\right), \end{aligned} \tag{3}$$

where $\mathbf{W}_{m,i} \triangleq \mathbf{w}_{m,i}\mathbf{w}_{m,i}^H$ is the transmit covariance of user (m, i). Clearly, $\mathbf{W}_{m,i} \succeq 0$ and $\text{Rank}(\mathbf{W}_{m,i}) \leq 1$. Note that $\mathbf{W}_{-(m,i)}$ is short notation for the collection of covariance matrices others than $\mathbf{W}_{m,i}$, i.e., $\mathbf{W}_{-(m,i)} = \{\mathbf{w}_{q,j}\}_{q\neq m, j\neq i}$. Note that throughout this paper we refer to the same set of covariance matrices with two different notations, i.e., we use the compact notation $\mathbf{W}_{-(m,i)}$ to refer to the set of all covariance matrices except $\mathbf{W}_{m,i}$, and we use $\mathbf{w}_{q,j}$ to refer to these covariance matrices individually, for instance in the second and third terms of (2).

The receivers perform single-user detection, i.e., each receiver decodes its intended signal by treating all other interfering signals as noise, so the total interference plus noise experienced at user (m, i) is calculated as

$$\begin{aligned} I_{m,i}\left(\mathbf{W}_{-(m,i)}\right) \triangleq c_{m,i} &+ \sum_{j\neq i} \mathbf{h}_{m,i}^H \mathbf{W}_{m,j} \mathbf{h}_{m,i} \\ &+ \sum_{q\neq m, j\in\mathcal{N}_q} \mathbf{h}_{q,m_i}^H \mathbf{W}_{q,j} \mathbf{h}_{q,m_i}. \end{aligned} \tag{4}$$

We assume that $I_{m,i}(\mathbf{W}_{-(m,i)})$ is perfectly known at user (m, i) and BSs m, but not the neighboring BSs. As suggested by [36], this interference plus noise term can be estimated at each mobile user by many different methods, and fed back to its associated BS.

Then the weighted sum-rate of all users in cell m can be expressed as

$$R(\mathbf{W}) \triangleq \sum_{m\in\mathcal{M}} \sum_{i\in\mathcal{N}_m} \alpha_i R_{m,i}\left(\mathbf{W}_{m,i}, \mathbf{W}_{-(m,i)}\right), \tag{5}$$

where the weight $\alpha_i \geqslant 0$ stands for the priority of user i in the system, and $\mathbf{W}$ represents the total set of covariance matrices, i.e., $\mathbf{W} = \{\mathbf{W}_{m,i}\}_{m\in\mathcal{M}, i\in\mathcal{N}_m}$. User's weights may be assigned according to a number of criteria, e.g., to achieve proportional fairness among the users or to implement max-min fairness. Discussing policies to assign users' weights is out of the scope of this work. Instead, we assume a given set of user priorities $\{\alpha_{i,m}\}$.

In our model, we assume that each BS m knows the channels to all the users in the system. Regarding the information exchange, we opt for an approach where the BSs do not exchange users data to optimize the downlink but only the channel state information (CSI) in designing the downlink beamforming vectors. Since channel variations are much slower than those of data, the amount and the frequency of information exchange is greatly reduced. We note that letting all the BSs coordinate and exchange information induces heavy signaling overhead, especially for large networks. Thus in practice coordination can be limited to neighboring BSs, as we show in Section VII.

Then, we are interested in the following non-convex weighted sum-rate maximization (WSRM) problem:

$$\begin{aligned} \max_{\mathbf{W}} \quad & R(\mathbf{W}) \qquad\qquad \text{(WSRM)} \\ \text{s.t.} \quad & \sum_{i\in\mathcal{N}_m} \text{Tr}[\mathbf{W}_{m,i}] \leq \bar{p}_m, \quad \forall\, m \in \mathcal{M} \\ & \mathbf{W}_{m,i} \succeq 0, \text{Rank}(\mathbf{W}_{m,i}) \leq 1, \quad \forall\, (m, i) \end{aligned}$$

where we want to find the covariance matrices $\mathbf{W}_{m,i}$ that maximize (WSRM). Note that $\mathbf{W}_{m,i}$ are imposed the constraint $\text{Rank}(\mathbf{W}_{m,i}) \leq 1$ for them to be the covariance matrices corresponding to beamformers $\mathbf{W}_{m,i}$. The problem (WSRM) is NP-hard in general [16]. Exact algorithms as the BRB algorithm of [26]) can solve the problem (WSRM) but are computationally intractable. Although optimal algorithms are not applicable in real-time, they can be used to find close approximations. Consequently, the study of heuristic algorithms along with a characterization of their performance with respect to the optimal is a reasonable alternative.

IV. Interference Pricing and Beamforming Mechanism

Interference prices have the ability to force individual base stations to internalize the negative effects (externality) they impose upon other base stations. Consider the following relaxed sum-rate optimization problem (R-WSRM) in which we relax the rank-1 constraint

$$\begin{aligned} \max_{\{\mathbf{W}_{m,i}\}\in\mathcal{F}} \quad & R(\mathbf{W}) \qquad\qquad \text{(R-WSRM)} \\ \text{s.t.} \quad & \sum_{i\in\mathcal{N}_m} \text{Tr}[\mathbf{W}_{m,i}] \leq \bar{p}_m, \quad \forall\, m \in \mathcal{M} \end{aligned}$$

where $\mathcal{F}$ denotes the set of feasible transmit covariance matrices $\mathcal{F} \triangleq \{\mathbf{W}_{m,i} : \mathbf{W}_{m,i} \succeq 0\}$. For the problem (R-WSRM), any optimal point $\mathbf{W}^*$ must satisfy the Karush-Kuhn-Tucker (KKT) conditions, which can be expressed as follows:

$$\nabla_{\mathbf{W}_{m,i}}\left(R(\mathbf{W}^*) + \sum_{m=1}^{M} \lambda_m \left(\bar{p}_m - \sum_{i\in\mathcal{N}_m} \text{Tr}\left[\mathbf{W}_{m,i}^*\right]\right)\right) = 0, \quad \forall\, m, i \tag{6}$$

$$\lambda_m \left(\bar{p}_m - \sum_{i\in\mathcal{N}_m} \text{Tr}\left[\mathbf{W}_{m,i}^*\right]\right) = 0, \quad \forall\, m \tag{7}$$

$$\sum_{i\in\mathcal{N}_m} \text{Tr}\left[\mathbf{W}_{m,i}^*\right] \leq \bar{p}_m, \quad \forall\, m \tag{8}$$

$$\lambda_m \geqslant 0. \tag{9}$$

It can be straightforwardly shown that the KKT conditions (6)–(9) can be expressed as the interference equilibrium defined in the following section.

A. Interference Pricing Equilibrium

We define, for each pair (m,i), a modified objective function $U_{m,i}$ as follows:

$$\begin{aligned} U_{m,i}&\left(\mathbf{W}_{m,i},\mathbf{W}_{-(m,i)},\mathbf{T},\lambda\right)\\ &=R_{m,i}\left(\mathbf{W}_{m,i},\mathbf{W}_{-(m,i)}\right)\\ &\quad-\sum_{(q,j)\neq(m,i)}T^{q,j}_{m,i}(\mathbf{W})G^{q,j}_{m,i}(\mathbf{W}_{m,i})-\lambda_m\mathrm{Tr}[\mathbf{W}_{m,i}], \end{aligned} \tag{10}$$

where $T^{q,j}_{m,i}(\mathbf{W})$ is the *interference price* associated with the interference originated by pair (m,i) on (q,j) so that $T^{q,j}_{m,i}(\mathbf{W})G^{q,j}_{m,i}(\mathbf{W}_{m,i})$ is the "interference payment" due to interference $G^{q,j}_{m,i}(\mathbf{W}_{m,i})=\mathbf{h}^H_{m,q_j}\mathbf{W}_{m,i}\mathbf{h}_{m,q_j}$on pair (q,j), and λ_m is the *shadow price* for transmit power in BS m. While interference prices are used to control the interference the BSs create over each other, shadow prices regulate that users served by a given BS do not consume excessive power.

Let $\lambda\triangleq\{\lambda_m\}_{m=1}^M$ and $\mathbf{T}=\{\mathbf{T}_{m,i}\}\ \forall(m,i)$, where $\mathbf{T}_{m,i}$ is the set of interference prices $T^{q,j}_{m,i}(\mathbf{W})$, i.e., $\mathbf{T}_{m,i}=\{T^{q,j}_{m,i}(\mathbf{W})\}_{q\neq m,q\neq i}$. Let $\mathcal{W}_{m,i}$ denote the set of feasible transmit covariance matrices for (m,i), $\mathcal{W}_{m,i}\triangleq\{\mathbf{W}_{m,i}:\mathrm{Tr}[\mathbf{W}_{m,i}]\leq\bar{p}_m,\mathbf{W}_{m,i}\succeq 0\}$.

Definition 1: The 3-tuple $(\mathbf{W}^*,\mathbf{T}^*,\lambda^*)$ is said to be an *interference pricing equilibrium* if the following conditions are satisfied[2]:

$$\mathbf{W}^*_{m,i}\in\arg\max_{\mathbf{W}_{m,i}\in\mathcal{W}_{m,i}}U_{m,i}\left(\mathbf{W}_{m,i},\mathbf{W}^*_{-(m,i)},\mathbf{T}^*_{m,i},\lambda^*\right) \tag{11}$$

$$\mathbf{T}^*_{m,i}(\mathbf{W}^*)\triangleq-\frac{\partial R_{-(m,i)}(\mathbf{W}^*)}{\partial\mathbf{W}_{m,i}}\geqslant 0 \tag{12}$$

$$\lambda^*_m\left(\bar{p}_m-\sum_{i\in\mathcal{N}_m}\mathrm{Tr}\left[\mathbf{W}^*_{m,i}\right]\right)=0,\ \lambda_m\geqslant 0,\qquad\forall m \tag{13}$$

$$\sum_{i\in\mathcal{N}_m}\mathrm{Tr}\left[\mathbf{W}^*_{m,i}\right]\leq\bar{p}_m,\qquad\forall m. \tag{14}$$

It can be readily seen that (11)–(14) are equivalent to the KKT conditions (6)–(9), and this guarantees that the covariance matrices in the interference equilibrium $\mathbf{W}^*$ are also a KKT point of the sum-rate maximization problem (R-WSRM). In the following we will set out to find such interference equilibrium.

B. Iterative Decentralized Pricing and Beamforming Mechanism

In this section, we describe an iterative decentralized pricing and beamforming mechanism to calculate the tuple $(\mathbf{T}^*,\lambda^*,\mathbf{W}^*)$. The mechanism is decentralized in the sense that as long as the BS m has the information from the other BSs and the channel matrices $\{\mathbf{H}_{m,q_j}\}_{q\neq m}$,[3] it can carry out the computation by itself.

The mechanism consists in iteratively calculating the values of $(\mathbf{T},\lambda,\mathbf{W})$ until convergence. At each iteration t and for a specific user i, a given BS m receives updated interference prices $\mathbf{T}^{m,i,t}_{q,j}(\mathbf{W}^{t-1})$ (15), obtains the optimal $\mathbf{W}^t_{m,i}$ (16), and updates the shadow price λ^t_m (17). Hence, for a chosen pair BS-user (m,i), a basic iteration t comprises the following steps:

1) *Interference prices update*. The interference prices are updated in BSs $q\neq m$ as

$$\begin{aligned} \mathbf{T}^{m,i,t}_{q,j}&(\mathbf{W}^{t-1})\\ &=-\frac{\partial R_{q,j}(\mathbf{W}^{t-1})}{\partial\mathbf{W}_{m,i}}\\ &=\frac{1/\ln(2)}{I_{q,j}\left(\mathbf{W}^{t-1}_{-(q,j)}\right)+\mathbf{h}^H_{q,j}\mathbf{W}^{t-1}_{q,j}\mathbf{h}_{q,j}}\frac{\mathbf{h}^H_{q,j}\mathbf{h}_{q,j}}{I_{q,j}\left(\mathbf{w}^{t-1}_{-(q,j)}\right)} \end{aligned} \tag{15}$$

2) *Information exchange*. BS m collects $\mathbf{W}^{t-1}_{q,j}$ and interference prices $\mathbf{T}^{m,i,t-1}_{q,j}(\mathbf{W}^{t-1})$ from BSs $q\neq m$.
3) *Transmit covariance matrix update*. The following individual optimization problem for user (m,i) is solved:

$$\mathbf{W}^t_{m,i}=\arg\max_{\mathbf{W}_{m,i}}U_{m,i}\left(\left(\mathbf{W}_{m,i},\mathbf{W}^{t-1}_{-(m,i)}\right),\mathbf{T}^t_{m,i},\lambda^t_m\right) \tag{16}$$

with $\mathbf{W}_{m,i}\in\mathcal{W}_{m,i}$.
4) *Transmit power price update*. The transmit power λ_m is updated by BS m as follows:

$$\lambda^t_m=\max\left\{\lambda^{t-1}_m+\rho\left(\sum_{i\in\mathcal{N}_m}\mathrm{Tr}\left[\mathbf{W}^{t-1}_{m,i}\right]-\bar{p}_m\right),0\right\},\rho>0. \tag{17}$$

In our mechanism, in each time instance a single users' maximization problem (16) must be solved, providing a rank-1 solution in accordance with the original (WSRM) problem. In Section V, we find a procedure to calculate a rank-1 solution, showing that the rank relaxation effectuated here does not affect the global solution. The mechanism described above, together with the procedure of Section V, are the basis to design the complete Interference Pricing Beamforming (IPBF) algorithm detailed in Section VI.

C. Convergence of the Pricing and Beamforming Mechanism

We now prove that the previous scheme converges to a KKT point of (R-WSRM), as it is shown by Theorem 1 developed in this section. We first establish the following proposition that characterizes the users' rate (3).

[2]See [37] for a related application of this concept.

[3]Note that channels are assumed to be invariable while solving the WSRM problem. Then, $\{\mathbf{H}_{m,q_j}\}_{q\neq m}$ can be broadcast just once initially.

Proposition 1: $R_{m,i}(\mathbf{W}_{m,i}, \mathbf{W}_{-(m,i)})$ is a convex function of $\mathbf{w}_{q,j} \in \mathbb{S}_+^K$ for all $(q,j) \neq (m,i)$ and for a fixed $\mathbf{W}_{m,i}$, and a concave function of $\mathbf{W}_{m,i} \in \mathbb{S}_+^K$ for fixed $\mathbf{W}_{-(m,i)}$.

Proof: See Appendix A. ■

It is important to note that in classical information theoretical literature we can find some form of concavity w.r.t the precoding matrix, convexity w.r.t the noise interference (see, e.g., [38]). However, it is worth noting that in the context of beamforming, the above property is only true in the space of rank-relaxed covariance matrix $\mathbf{W}_m$, but not in the transmit beamformer space $\mathbf{w}_m$. In the following we state a convergence result for the pricing and beamforming mechanism.

Theorem 1: The sequence of covariance matrices $\mathbf{W}^t$ converges to a KKT solution $\mathbf{W}^*$ of problem (R-WSRM) with probability 1.

Proof: The details of the proof can be found in Appendix B. ■

However, any stationary solution for problem (R-WSRM) may not even be a feasible solution for the original problem (WSRM), due to the rank relaxation performed earlier. In the following section we show that we can in fact find a rank-1 optimal solution for the users' utility maximization problem (16). This result implies that indeed we can find a KKT solution for problem (WSRM).

V. Dealing With the Rank-1 Constraint

In this section, we develop a procedure to find a rank-1 solution to the problem (16) of the pricing and beamforming mechanism. We first show the existence of such rank-1 solution and afterwards we propose a procedure that provides a closed-form solution for the rank-a covariance matrices.

A. Existence of Rank-1 Solution to the User's Utility Maximization Problem

In our mechanism, in each time instance a single users' covariance (say the ith user in mth cell) is computed by solving the following users' utility maximization problem (UUM)

$$\max_{\mathbf{W}_{m,i} \in \mathcal{W}_m} U_{m,i}\left(\mathbf{W}_{m,i}, \mathbf{W}_{-(m,i)}\right). \tag{UUM}$$

We note here that although recent works such as [39], [40] have shown that the WSRM problem must admit rank-1 globally optimal solution, their argument cannot be directly used to show that the (UUM) problem must have rank-1 solution. In fact, the solution of UUM is not directly related to the global optimal solution of WSRM, except for the case where $\mathbf{W}_{-(m,i)}$ is itself part of the globally optimal solution for WSRM. We can identify a special structure of the problem (UUM) that allows it to admit a rank-1 solution. To this end, we tailor the rank reduction procedure (abbreviated as RRP) proposed in [41] to fit our problem. For the sake of readability, we relegate the discussion details to Appendix C.

However, this procedure is not that useful in practice as it requires solving (UUM) to begin with. Therefore, in the following we propose a procedure that directly computes a rank-1 optimal solution to the problem (UUM).

B. Procedure for User's Utility Rank-1 Maximization

The problem (UUM) is a concave determinant maximization (MAXDET) problem [42], and can be solved efficiently using convex program/SDP solvers such as CVX [43]. However, in practice such general purpose solvers may still induce heavy computational burden. Moreover, the resulting optimal solution of the relaxed problem may have rank greater than one. Fortunately, these difficulties can be resolved. We have found an explicit construction that generates a rank-1 solution of the problem (UUM), hence the interference pricing equilibrium covariance matrices $\mathbf{W}^*$ are rank-1. The rank reduction problem of downlink beamforming has been recently studied in [41], [44] and [45]. However, the algorithms proposed in those works cannot be directly used to obtain a rank-1 solution to (UUM): [41] considers problems with linear objective functions; [45] and [44] consider the relaxation of the MAXDET problem *without* the linear penalty terms.[4]

In this section we develop a procedure that provides a rank-1 $\mathbf{W}^*_{m,i}$ for each user (m,i). We first show that $\mathbf{W}^*_{m,i}$ must be diagonal. As a consequence, we can find a closed-form expression for $\mathbf{W}^*_{m,i}$.

The utility function $U_{m,i}$ at time t can be written, for a given $\widehat{\mathbf{W}}^t_{-(m,i)}$, as

$$U^t_{m,i}(\mathbf{W}_{m,i}) = \log\left|\mathbf{I} + \mathbf{W}^t_{m,i}\mathbf{H}_{m,i}\frac{1}{I_{m,i}\left(\widehat{\mathbf{W}}^t_{-(m,i)}\right)}\right| - \mathrm{Tr}\left[\left(\mathbf{A}_{m,i} + \lambda^t_m \mathbf{I}\right)\mathbf{W}^t_{m,i}\right]. \tag{18}$$

We recall that $\lambda^t_m = \max\{\lambda^{t-1}_m + \rho^t(\sum_{i\in\mathcal{N}_m}\mathrm{Tr}[\mathbf{W}^{t-1}_{m,i}] - \bar{p}_m), 0\}$. Notice that $\mathbf{A}_{m,i} \succeq 0$ (see Appendix C), then for any $\lambda^t_m > 0$, we can perform the Cholesky decomposition $\mathbf{A}_{m,i} + \lambda^t_m \mathbf{I} = \mathbf{L}^H\mathbf{L}$, which results in $\mathrm{Tr}[(\mathbf{A}_{m,i} + \lambda^t_m\mathbf{I})\mathbf{W}^t_{m,i}] = \mathrm{Tr}[\mathbf{L}\mathbf{W}^t_{m,i}\mathbf{L}^H]$. Define $\overline{\mathbf{W}}_{m,i} = \mathbf{L}\mathbf{W}^t_{m,i}\mathbf{L}^H$, we have

$$\begin{aligned}
&U^t_{m,i}(\mathbf{W}^t_{m,i})\\
&= \log\left|\mathbf{I} + \mathbf{L}^{-1}\overline{\mathbf{W}}_{m,i}\mathbf{L}^{-H}\mathbf{H}_{m,i}\frac{1}{I_{m,i}\left(\widehat{\mathbf{W}}^t_{-(m,i)}\right)}\right| - \mathrm{Tr}[\overline{\mathbf{W}}_{m,i}]\\
&\overset{(a)}{=} \log\left|\mathbf{I} + \overline{\mathbf{W}}_{m,i}\mathbf{V}\boldsymbol{\Delta}\mathbf{V}^H\right| - \mathrm{Tr}[\overline{\mathbf{W}}_{m,i}]\\
&\overset{(b)}{=} \log\left|\mathbf{I} + \widehat{\mathbf{W}}_{m,i}\boldsymbol{\Delta}\right| - \mathrm{Tr}[\mathbf{V}\widehat{\mathbf{W}}_{m,i}\mathbf{V}^H]\\
&= \log\left|\mathbf{I} + \widehat{\mathbf{W}}_{m,i}\boldsymbol{\Delta}\right| - \mathrm{Tr}[\widehat{\mathbf{W}}_{m,i}] = U_{m,i}(\widehat{\mathbf{W}}_{m,i}),
\end{aligned} \tag{19}$$

[4]In [44], with linear penalty in the form of $-\mathrm{Tr}[\mathbf{A}_m(\mathbf{W}_m - \widehat{\mathbf{W}}_{m,q})]$, equation (43) is no longer equivalent to equation (44).

TABLE I
OPTIMIZATION OF THE USER'S UTILITY

S1) Compute decomposition: $\mathbf{L}^H\mathbf{L} = \mathbf{A}_{m,i} + \lambda_m \mathbf{I}$ $\mathbf{V}\Delta\mathbf{V}^H = \mathbf{L}^{-H}\mathbf{H}_{m,i}\mathbf{L}^{-1}\frac{1}{I_{m,i}(\widehat{\mathbf{W}}_{-(m,i)})}$.
S2) Compute $\widehat{\mathbf{W}}^*_{m,i}$ by (20).
S3) Compute $\mathbf{W}^*_{m,i} = \mathbf{L}^{-1}\mathbf{V}\widehat{\mathbf{W}}^*_{m,i}\mathbf{V}^H\mathbf{L}^{-H}$.

where in (a) we have used the eigendecomposition: $\mathbf{L}^{-H}\mathbf{H}_{m,i}\mathbf{L}^{-1}(1/I_{m,i}(\widehat{\mathbf{W}}^t_{-(m,i)})) = \mathbf{V}\boldsymbol{\Delta}\mathbf{V}^H$; in (b) we have defined $\widehat{\mathbf{W}}_{m,i} = \mathbf{V}^H\overline{\mathbf{W}}_{m,i}\mathbf{V}$.

We now prove that the solution to (19) must be diagonal. Let $\widehat{\mathbf{W}}^*_{m,i}$ denote an optimal solution to the problem $\max_{\widehat{\mathbf{W}}_{m,i}\in\mathcal{W}_m} U_{m,i}(\widehat{\mathbf{W}}_{m,i})$. We claim that there must exist a $\widehat{\mathbf{W}}^*_{m,i}$ that is *diagonal*. Note that $\text{Rank}(\mathbf{H}_{m,i}) = 1$ implies $\text{Rank}(\boldsymbol{\Delta}) \leq 1$. Thus $\widehat{\mathbf{W}}^*_{m,i}\boldsymbol{\Delta}$ has at most a *single column*, and we can remove the off diagonal elements of $\mathbf{I} + \widehat{\mathbf{W}}^*_{m,i}\boldsymbol{\Delta}$ without changing the values of $|\mathbf{I} + \widehat{\mathbf{W}}^*_{m,i}\boldsymbol{\Delta}|$. Consequently, for any given $\widehat{\mathbf{W}}^*_{m,i}$, we can construct a diagonal optimal solution $\widehat{\mathbf{W}}^{*,D}_{m}$ by removing all its off diagonal elements. This operation removes all the off diagonal elements of $\mathbf{I} + \widehat{\mathbf{W}}^*_{m,i}\boldsymbol{\Delta}$, and it does not change either $|\mathbf{I} + \widehat{\mathbf{W}}^*_{m,i}\boldsymbol{\Delta}|$ or $\text{Tr}[\widehat{\mathbf{W}}^*_{m,i}]$. Given that, $\widehat{\mathbf{W}}^{*,D}_{m,i}$ is also optimal. When restricting $\widehat{\mathbf{W}}^*_{m,i}$ to be diagonal, we can straightforwardly derive the closed-form expression of every diagonal element $[\widehat{\mathbf{W}}^*_{m,i}]_{k,k}$ from (19) as

$$\left[\widehat{\mathbf{W}}^*_{m,i}\right]_{k,k} = \begin{cases} \left[\frac{[\boldsymbol{\Delta}]_{k,k}-1}{[\boldsymbol{\Delta}]_{k,k}}\right]^+ & \text{if } [\boldsymbol{\Delta}]_{k,k} \neq 0; \\ 0 & \text{otherwise.} \end{cases} \tag{20}$$

where $[x]^+ = \max\{0, x\}$. Then we can obtain $\mathbf{W}^{t*}_{m,i} = \mathbf{L}^{-1}\mathbf{V}\widehat{\mathbf{W}}^*_{m,i}\mathbf{V}^H\mathbf{L}^{-H}$. Combining the fact that $\text{Rank}(\boldsymbol{\Delta}) \leq 1$ with (20) we conclude $\text{Rank}(\widehat{\mathbf{W}}^*_{m,i}) \leq 1$, and consequently $\text{Rank}(\mathbf{W}^{t*}_{m,i}) \leq 1$, *for any* $\lambda_m > 0$.

We can also show that when $\lambda_m = 0$, $\mathbf{A}_{m,i}$ must have full rank. In this case, we can find the Cholesky decomposition $\mathbf{A}_{m,i} = \mathbf{L}\mathbf{L}^H$, and the above construction can still be used to directly obtain $\mathbf{W}^{t*}_{m,i}(0)$, which satisfies $\text{Rank}(\mathbf{W}^{t*}_{m,i}(0)) \leq 1$.

In conclusion, for any $\lambda_m \geq 0$, we obtain $\text{Rank}(\mathbf{W}^*_{m,i}) \leq 1$. Table I summarizes the above procedure.

VI. Interference Pricing Beamforming (IPBF) Algorithm

In this section we develop an algorithm based on the interference pricing and beamforming mechanism of section IV and the procedure described in Section V-B to solve the problem (WSRM), where a BS m optimizes one and only one user i each time. We propose the Interference Pricing Beamforming (IPBF) algorithm for the users to iteratively compute their beamformers. Moreover, the convergence of the IPBF algorithm is guaranteed by Theorem 1.

Algorithm 1 Interference pricing beamforming (IPBF)

Set $t = 0$.

1) **Initialization and BS Selection**: randomly choose a set of feasible covariance matrices. Choose $m \in \mathcal{M}$ such that $m = t \oslash B$, $t \geqslant 1$, and randomly choose $i \in \mathcal{N}_m$. $\mathbf{W}^0_m$, $\forall\, m \in \mathcal{M}$.
2) **Interference Pricing Update and Information Exchange**: Let each BS $q \neq m$ compute $\mathbf{T}^{q,j,t}_{m,i}(\mathbf{W}^{t-1})$ as in (15), and transfer $\{\mathbf{T}^{q,j,t}_{m,i}(\mathbf{W}^{t-1})\}$ and $\mathbf{W}^{t-1}_{q,j}$ to BS m.
3) **Maximization**: BS m uses the procedure in Table I to obtain $\mathbf{W}^{t*}_{m,i} = \max_{\mathbf{W}^t_{m,i}\in\mathcal{W}_m} U^t_{m,i}(\mathbf{W}^t_{m,i}, \mathbf{W}^{t-1}_{-(m,i)})$.
4) **Update**: Let $\mathbf{W}^t = [\mathbf{W}^{t*}_{m,i}, \mathbf{W}^{t-1}_{-(m,i)}]$.
5) **Update**: $\lambda^t_m = \max\{\lambda^{t-1} + \rho^t(\sum_{i\in\mathcal{N}_m}\text{Tr}[\mathbf{W}^{t-1}_{m,i}] - \bar{p}_m), 0\}$, given by (17).
6) **Information Exchange**: Transfer $\mathbf{W}^{t*}_{m,i}$ to $q \neq m$.
7) **Continue**: If $|R(\mathbf{W}^t) - R(\mathbf{W}^{t-M})| < \epsilon$, stop. Otherwise, set $t = t + 1$, go to Step 1).

The sketch of the algorithm is as follows. We initialize the algorithm with a set of feasible covariance matrices and a random selection of a BS-user pair (m, i) in Step 1). In Step 2), each BS $q \neq m$ updates its interference prices $\{\mathbf{T}^{q,j,t}_{m,i}(\mathbf{W}^{t-1})\}$ for all users j of BS q, and send them to BS m together with the covariance matrices $\mathbf{W}^{t-1}_{q,j}$. With this information, BS m optimizes the utility function $U^t_{m,i}$ and obtains the optimal covariance matrix $\mathbf{W}^{t*}_{m,i}$ (Step 3) for the corresponding value of λ_m, which is updated in Step 5). The covariance matrix $\mathbf{W}^{t*}_{m,i}$ is broadcasted from m in Step 6), and the algorithm concludes if the stopping criteria specified in Step 7) is satisfied with the condition $\epsilon > 0$. Otherwise, the algorithm starts a new iteration and chooses a new BS-user pair.

The algorithm is distributed in the sense that as long as the BS m has the information specified in Step 2) and the matrices $\{\mathbf{H}_{m,q_j}\}_{q\neq m}$, it can carry out the computation by itself. The stopping criteria is specified in Step 7) with the condition $\epsilon > 0$.

Convergence of the algorithm. We have shown by Theorem 1 that the interference mechanism converges to a KKT point of the (R-WSRM) problem. Note that the steps of the IPBF algorithm are equivalent to this mechanism once the rank-1 solution procedure has been incorporated into such mechanism. Then, by Theorem 1, the convergence of the IPBF algorithm is guaranteed.

VII. Numerical Results

In this section we analyze the performance of the IBPF algorithm suitably adjusted to make use of the rank-1 solution developed in Section V-A. We consider a network with a set $\mathcal{M}$ of BS, within a square area of appropriate size. We evenly divide the square area into M cells, with a single BS located at the center of each cell and randomly generated user locations. The BS to BS distance is 2 km. The channel coefficients $\mathbf{h}_{m,i}$ between BS m and any user i are modeled as zero

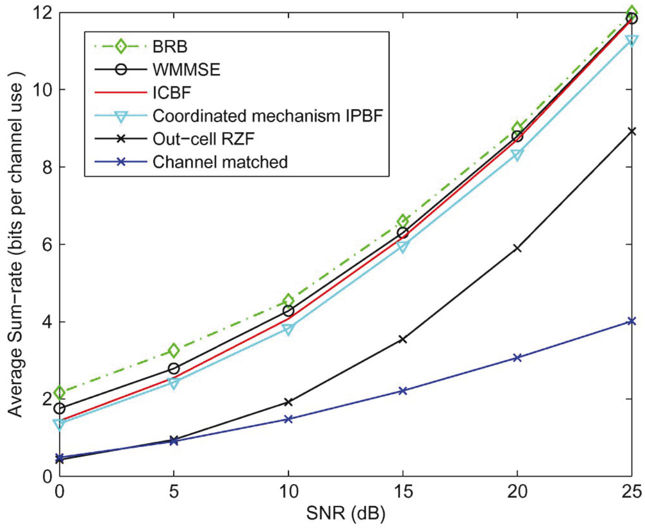

Fig. 2. Comparison of sum-rate of the different algorithms, for $M = 2$, $K = 2$, and $N = 2$. Users $i \in \mathcal{N}_m$ are uniformly placed within $d_{m,i} \in [200, 1000]$ meters within each BS.

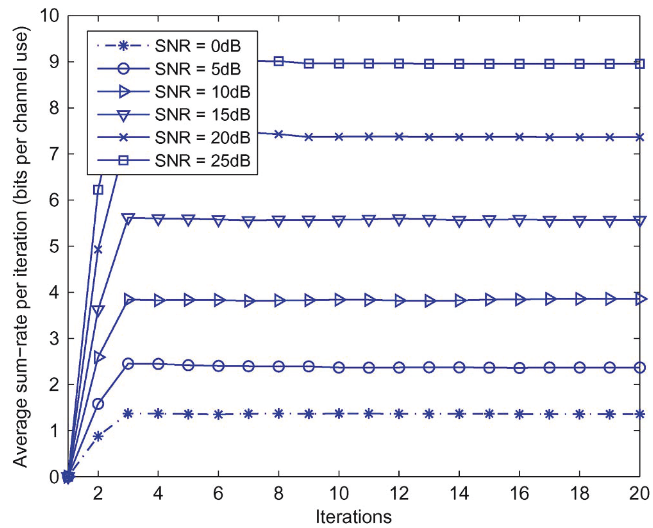

Fig. 3. Convergence of the IPBF algorithm, with different values of SNR from 0 dB to 25 dB.

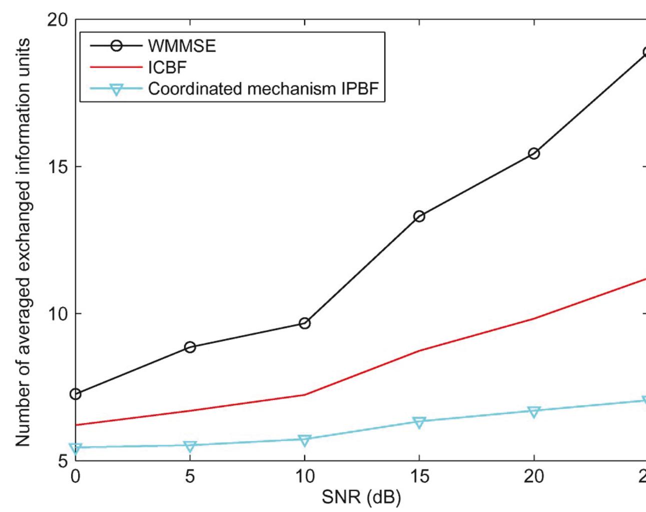

Fig. 4. Comparison of the number of information units needed for convergence for $M = 2$, $K = 2$, and $N = 2$, with stop value of $\epsilon = 10^{-4}$, for the IPBF, ICBF and WMMSE algorithms.

mean circularly symmetric complex Gaussian vector, and these coefficients are calculated as [46]

$$\mathbf{h}_{m,i} = \mathbf{z}_{m,i} \left(\frac{200}{d_{m,i}} \right)^{3.5} L_{m,i},$$

where $z_{m,i}$ is a zero-mean unit-variance circularly symmetric complex Gaussian random vector characterizing Rayleigh fading, $d_{m,i}$ is the distance between BS m and user i, and the large-scale Log-Normal shadowing effect is modeled by $10\log_{10}(L_{m,i})$, which is a real Gaussian random variable modeling with zero mean and standard deviation 8. We take the maximum power per BS $\bar{p}_m = 1$ for all $m \in \mathcal{M}$, assume the same environmental noise power for all $(m,i)c_{m,i} = \sigma^2$ and define the SNR as $10\log_{10}(\bar{p}_m/\sigma^2)$.

We compare our algorithm with relevant algorithms in the recent related literature. The branch-reduce-and-bound (BRB) algorithm [26] provides the optimal solution and is an upper benchmark for the WSR maximization problem; the implementation of the algorithm provided in [47] is utilized. The weighted sum MSE minimization (WMMSE) algorithm [35] has favorably been compared with the interference-based algorithm of [19]. The Iterative Coordinated BeamForming (ICBF) algorithm [34] is a distributed algorithm that has already been compared in terms of sum-rate performance with an upper bound achieved when there is no intercell interference; then, it provides a good benchmark for our algorithm. We also show the results for two non-coordinating schemes, namely regularized zero-forcing beamforming [48] and channel matched scheme (see, for instance, [49]), which are lower bounds.

We first consider a network with two BSs and two users per BS, with two transmit antennas at each BS, i.e., $M = 2$, $K = 2$, $N = 2$, for comparison purposes. For this network topology, the results are obtained by averaging over 200 randomly generated user locations and channel realizations. This network setting is considered as the used BRB algorithm implementation is recommended for less than 6 network users. In Fig. 2, we show the average sum-rate achieved by the different algorithms. We observe that the loss of the IPBF algorithm is small with respect to the WMMSE and ICBF algorithms. At the same time, the difference with respect to the optimal BRB is also small. The IPBF algorithm notoriously outperforms the non-coordinated regularized zero-forcing and channel matched schemes. Fig. 3 reproduces the convergence behavior of the IPBF algorithm for several values of SNR. These curves show that the IPBF algorithm converges in a few steps.

We now compare the amount of inter-cell information needed for the iterative algorithms. We define the *unit of information transfer* as the total information needed from the set of coordinated BSs for updating the beam vectors for *a single BS* $m \in \mathcal{M}$. Clearly, at each iteration of the IPBF algorithm a single unit of information is needed to go through the backhaul network, in the ICBF and WMMSE algorithms M units of information are needed, and $M \times N$ units are required by the BRB algorithm. In Fig. 4 we demonstrate the averaged total units of information needed for the IPBF, ICBF and WMMSE schemes until convergence. Note that the results

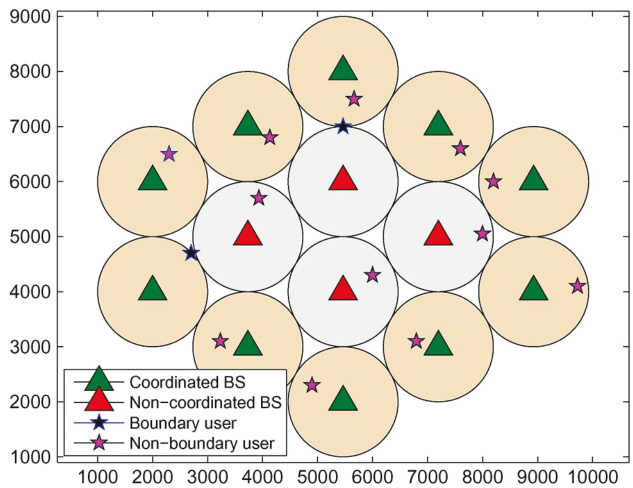

Fig. 5. Example of network with partial coordination, for $M = 14$, $K = 2$, $N = 1$, where only the four central BSs perform coordination. User associated with BS1 is a boundary user.

corresponding to BRB are not included in Fig. 4 for the sake of clarity, since the values are typically above 800 information units. We observe that the total units of information needed for the proposed IPBF algorithm is more than 20% less than the ICBF algorithm for SNR values above 10 dB, and for SNR $= 25$ dB the information excess goes beyond 50%. We also emphasize that typically several *inner iterations* are needed per outer iteration of ICBF and WMMSE algorithms, and we have not counted the extra information needed between the BSs and the users in these inner iterations. We also notice an increase in the number of iterations until sum-rate convergence with SNR. This is due to the fact that the higher the SNR the higher the sum-rate. As $\epsilon = 10^{-4}$ irrespectively of the value of SNR the algorithms converge slower as SNR rises.

Consider now the same network as before but with 14 BSs $(M = 14)$. In practice, as it is mentioned in recent contributions (e.g., [30] and [34]), letting all the BSs coordinate and exchange information induces heavy signaling overhead in the backhaul network, especially when the network size is large. We propose a *limited coordination variant* of IPBF, named as IPBF-L, in which each BS only coordinates with its neighboring BSs. This limited coordination scheme reduces signaling and is reasonable in practice, as the benefit of transmission coordination diminishes when the BSs are far away and create small interference to each other. We compare the results of IPBF with IPBF-L when only 4 of the BSs are coordinated for transmission, being all other BSs' transmission regarded as noise, with 1 user served by each BS and $K = 2$. The system topology of the network is illustrated in Fig. 5, in which we consider the existence of a boundary user for BS1, i.e., the distance between BS1 and this user is 1 km (the cell radius). The other users are non-boundary users, who are randomly distributed within the cell. While the loss in sum-rate is very small (Fig. 6), the speed of convergence is much faster (Fig. 7) and the number of exchanged information units is considerably lower (Fig. 8). We also show in Fig. 9 that boundary users with IPBF-L experience slight degradation with respect to IPBF (full coordination) and limited cooperation practically does not affect non-boundary users.

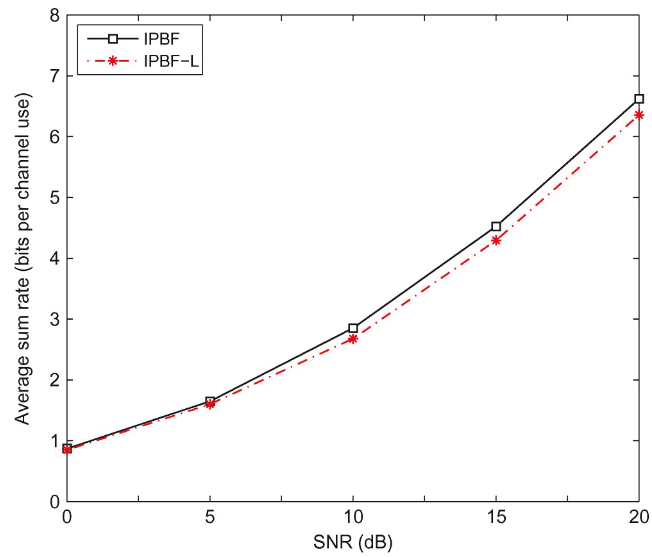

Fig. 6. Comparison of sum-rate of the IPBF and IPBF-L algorithms, for $M = 14$, $K = 2$, $N = 1$. Users $i \in \mathcal{N}_m$ are uniformly placed within $d_{m,i} \in [200, 1000]$ meters within each BS.

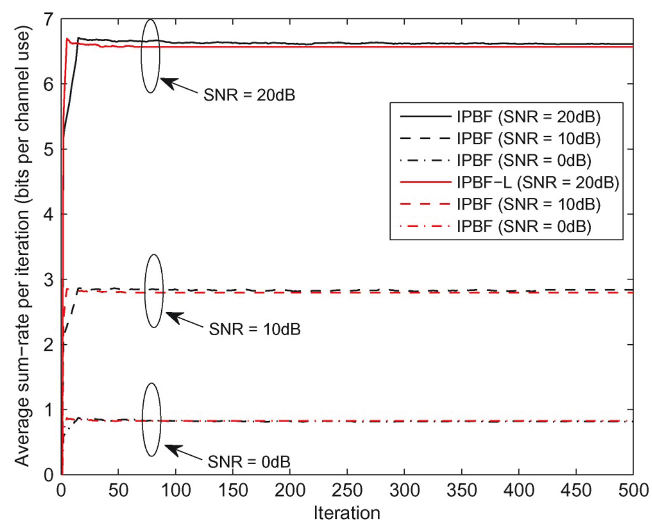

Fig. 7. Comparison of the convergence between IPBF and IPBF-L.

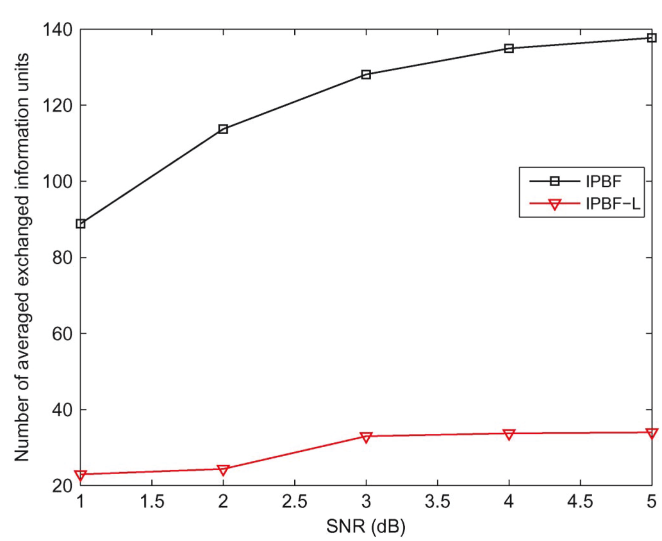

Fig. 8. Comparison of the number of information units needed for convergence, with stop value of $\epsilon = 10^{-4}$, for the IPBF and IPBF-L algorithms.

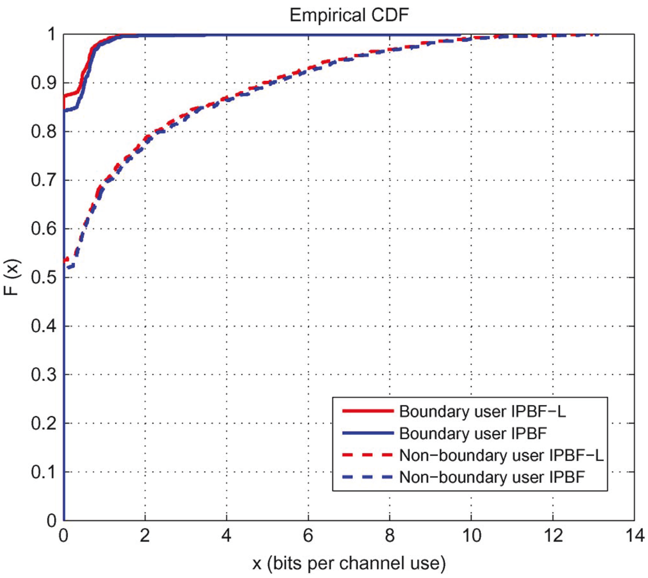

Fig. 9. CDF of user's rate, boundary and non-boundary users, SNR = 20 dB, 1 user per cell.

VIII. Conclusion

In this paper, we propose an interference pricing algorithm to study the non-convex sum-rate maximization problem for coordinated beamforming in a multi-cell multiuser MISO network. We have proved that the interference mechanism converges and we have explored the structure of the problem to identify a procedure that guarantees a rank-1 solution. The IPBF algorithm, which is based on the interference mechanism, is favorably compared with the state-of-the-art ICBF and WMMSE algorithms, achieving almost the same system throughput with notably reduced backhaul information exchange among the BSs, while having little loss with respect to the optimal BRB algorithm.

We also propose the IPBF-L algorithm, a less complex variant of our algorithm that restricts coordination to a cluster of base stations. The IPBF-L algorithm has a very small loss with respect to IPBF while it requires much less information exchange through the backhaul.

Appendix A
Proof of Proposition 1

To show convexity, it is sufficient to prove that whenever $\mathbf{D} \in \mathbb{S}^K$, $\mathbf{D} \neq \mathbf{0}$ and $\mathbf{W}_{q,j} + t\mathbf{D} \succeq 0$, the function $R_{m,i}(t)$ (21) shown at the bottom of the page is convex in t [50, Chapter 3]. Let us simplify the expression a bit by defining the constant $c = \mathbf{h}_{m,i}^H \mathbf{W}_{m,i} \mathbf{h}_{m,i} \geq 0$ (note that $\mathbf{W}_{m,i} \succeq 0$). The first and the second derivatives of $R_{m,i}(t)$ w.r.t. t can be expressed as indicated in (22) and (23) at the bottom of the page. Clearly $I_{m,i}(\mathbf{W}_{-(m,i)}) + t\mathbf{h}_{q,m_i}^H \mathbf{D}\mathbf{h}_{q,m_i} > 0$ for all $\mathbf{W}_{q,j} + t\mathbf{D} \succeq 0$. We also have that $\mathbf{h}_{q,m_i}^H \mathbf{D}\mathbf{h}_{q,m_i}$ is real and $(\mathbf{h}_{q,m_i}^H \mathbf{D}\mathbf{h}_{q,m_i})^2 \geq 0$, due to the assumption that $\mathbf{D} \in \mathbb{S}^K$, and the subsequent implication that $(\mathbf{h}_{q,m_i}^H \mathbf{D}\mathbf{h}_{q,m_i})^H = \mathbf{h}_{q,m_i}^H \mathbf{D}\mathbf{h}_{q,m_i}$. We conclude that whenever $\mathbf{D} \in \mathbb{S}^K$ and $\mathbf{w}_{q,j} + t\mathbf{D} \succeq 0$, $d^2 R_{m,i}(t)/dt^2 \geq 0$, which implies that $R_{m,i}(\mathbf{W}_{m,i}\mathbf{W}_{-(m,i)})$ is convex in $\mathbf{W}_{q,j}$ for all $(q,j) \neq (m,i)$.

The fact that $R_{m,i}(\mathbf{W}_{m,i}, \mathbf{W}_{-(m,i)})$ is concave in $\mathbf{W}_{m,i}$ can be shown similarly as above (see equation at the bottom of the page).

Appendix B
Proof of Theorem 1

Proof: We first prove that if $\mathbf{W}^t \to \mathbf{W}^*$ then $(\mathbf{W}^*, \lambda^*)$ is a KKT point. Given that at time $t > 0$ only one user, say (m,i), is uniformly randomly chosen to optimally react to $\mathbf{W}_{-(m,i)}^t$, it follows that with probability 1 user (m,i) updates $\mathbf{W}_{m,i}$ infinitely often. Let $\{t(n) : n > 0\}$ be the index set of time periods at which user (m,i) is chosen. Hence:

$$U_{m,i}\left(\left(\mathbf{W}_{m,i}^{t(n)+1}, \mathbf{W}_{-(m,i)}^{t(n)}\right), \mathbf{T}_{m,i}^{t(n)}, \lambda_m^{t(n)}\right) \geqslant U_{m,i}\left(\left(\tilde{\mathbf{W}}_{m,i}, \mathbf{W}_{-(m,i)}^{t(n)}\right), \mathbf{T}_{m,i}^{t(n)}, \lambda_m^{t(n)}\right), \ \forall \tilde{\mathbf{W}}_{m,i} \in \mathcal{W}_{m,i}.$$

Hence, in the limit along the sub-sequence $\{t(n) : n > 0\}$ we obtain

$$U_{m,i}\left(\left(\mathbf{W}_{m,i}^*, \mathbf{W}_{-(m,i)}^*\right), \mathbf{T}_{m,i}^*, \lambda_m^*\right) \geqslant U_{m,i}\left(\left(\tilde{\mathbf{W}}_{m,i}, \mathbf{W}_{-(m,i)}^*\right), \mathbf{T}_{m,i}^*, \lambda_m^*\right), \quad \forall \tilde{\mathbf{W}}_{m,i} \in \mathcal{W}_{m,i}. \tag{24}$$

$$R_{m,i}(t) \triangleq \log\left(1 + \frac{\mathbf{h}_{m,i}^H \mathbf{W}_{m,i} \mathbf{h}_{m,i}}{c_{m,i} + \sum_{(p,l)\neq(q,j),(p,l)\neq(m,i)} \mathbf{h}_{p,m_i}^H \mathbf{W}_{p,l} \mathbf{h}_{p,m_i} + \mathbf{h}_{q,m_i}^H (\mathbf{W}_{q,j} + t\mathbf{D})\mathbf{h}_{q,m_i}}\right) \tag{21}$$

$$\frac{dR_{m,i}(t)}{dt} = -\frac{1/\ln(2)}{\left(I_{m,i}\left(\mathbf{W}_{-(m,i)}\right) + t\mathbf{h}_{q,m_i}^H \mathbf{D}\mathbf{h}_{q,m_i} + c\right)} \frac{c\mathbf{h}_{q,m_i}^H \mathbf{D}\mathbf{h}_{q,m_i}}{\left(I_{m,i}\left(\mathbf{W}_{-(m,i)}\right) + t\mathbf{h}_{q,m_i}^H \mathbf{D}\mathbf{h}_{q,m_i}\right)} \tag{22}$$

$$\frac{d^2 R_{m,i}(t)}{dt^2} = \frac{1/\ln(2)}{\left(I_{m,i}\left(\mathbf{W}_{-(m,i)}\right) + t\mathbf{h}_{q,m_i}^H \mathbf{D}\mathbf{h}_{q,m_i} + c\right)^2} \frac{c\left(\mathbf{h}_{q,m_i}^H \mathbf{D}\mathbf{h}_{q,m_i}\right)^2}{I_{m,i}\left(\mathbf{W}_{-(m,i)}\right) + t\mathbf{h}_{q,m_i}^H \mathbf{D}\mathbf{h}_{q,m_i}}$$
$$+ \frac{1/\ln(2)}{I_{m,i}\left(\mathbf{W}_{-(m,i)}\right) + t\mathbf{h}_{q,m_i}^H \mathbf{D}\mathbf{h}_{q,m_i} + c} \frac{c\left(\mathbf{h}_{q,m_i}^H \mathbf{D}\mathbf{h}_{q,m_i}\right)^2}{\left(I_{m,i}\left(\mathbf{W}_{-(m,i)}\right) + t\mathbf{h}_{q,m_i}^H \mathbf{D}\mathbf{h}_{q,m_i}\right)^2} \tag{23}$$

Using (10) we can rewrite (24) as follows:

$$R_{m,i}(\mathbf{W}^*) - \sum_{(q,j)\neq(m,i)} \mathbf{T}_{m,i}^{q,j*} G_{m,i}^{q,j}\left(\mathbf{W}_{m,i}^*\right) - \lambda_m^* \mathrm{Tr}\left[\mathbf{W}_{m,i}^*\right]$$
$$\geqslant R_{m,i}\left(\tilde{\mathbf{W}}_{m,i}, \mathbf{W}_{-(m,i)}^*\right) - \sum_{(q,j)\neq(m,i)} \mathbf{T}_{m,i}^{q,j*} G_{m,i}^{q,j}(\tilde{\mathbf{W}}_{m,i})$$
$$- \lambda_m^* \mathrm{Tr}\left[\tilde{\mathbf{W}}_{m,i}\right], \qquad \forall\, \tilde{\mathbf{W}}_{m,i} \in \mathcal{W}_{m,i}. \tag{25}$$

Consequently, we have

$$R_{m,i}(\mathbf{W}^*) - \lambda_m^* \mathrm{Tr}\left[\mathbf{W}_{m,i}^*\right] \geqslant R_{m,i}\left(\tilde{\mathbf{W}}_{m,i}, \mathbf{W}_{-(m,i)}^*\right)$$
$$+ \sum_{(q,j)\neq(m,i)} \mathbf{T}_{q,j}^{m,i*} \mathbf{h}_{m,q_j}^H \left(\mathbf{W}_{m,i}^* - \tilde{\mathbf{W}}_{m,i}\right) \mathbf{h}_{m,q_j}$$
$$- \lambda_m^* \mathrm{Tr}\left[\tilde{\mathbf{W}}_{m,i}\right] \qquad \forall\, \tilde{\mathbf{W}}_{m,i} \in \mathcal{W}_{m,i}. \tag{26}$$

Since $R_{q,j}(\mathbf{W})$ is convex in $\mathbf{W}_{m,i}, (q,j) \neq (m,i)$ and $\mathbf{h}^H \mathbf{W} \mathbf{h}$ is a linear transformation of $\mathbf{W}$, we have

$$R_{q,j}(\mathbf{W}^*) \geqslant R_{q,j}\left(\tilde{\mathbf{W}}_{m,i}, \mathbf{W}_{-(m,i)}^*\right)$$
$$+ \mathbf{h}_{m,q_j}^H \left(\mathbf{W}_{m,i}^* - \tilde{\mathbf{W}}_{m,i}\right) \mathbf{h}_{m,q_j} \frac{\partial R_{q,j}(\mathbf{W}^*)}{\partial \mathbf{W}_{m,i}}, \tag{27}$$

and if we sum up $\sum_{(q,j)\neq(m,i)} R_{q,j}(\mathbf{W}^*)$ to both sides of (26) and also apply (27) we have

$$\sum_{q,j} R_{q,j}(\mathbf{W}^*) - \lambda_m^* \mathrm{Tr}\left[\mathbf{W}_{m,i}^*\right]$$
$$\geqslant \sum_{q,j} R_{q,j}\left(\tilde{\mathbf{W}}_{m,i}, \mathbf{W}_{-(m,i)}^*\right) - \lambda_m^* \mathrm{Tr}[\tilde{\mathbf{W}}_{m,i}], \tag{28}$$

for all $\tilde{\mathbf{W}}_{m,i} \in \mathcal{W}_{m,i}$. This in turn implies the following first order condition:

$$\frac{\partial R_{m,i}(\mathbf{W}^*)}{\partial \mathbf{W}_{m,i}} = - \sum_{(q,j)\neq(m,i)} \frac{\partial R_{q,j}(\mathbf{W}^*)}{\partial \mathbf{W}_{m,i}} + \lambda_m^*. \tag{29}$$

We now show complementary slackness. If $\sum_{i\in\mathcal{N}_m} \mathrm{Tr}[\mathbf{W}_{m,i}^*] > \bar{p}_m$ then, according to (17), $\lambda_m^t \to \infty$, which would imply $\sum_{i\in\mathcal{N}_m} \mathrm{Tr}[\mathbf{W}_{m,i}^*] = 0$, a contradiction. Finally, if $\lambda_m^t \to \lambda_m^* > 0$ then $\sum_{i\in\mathcal{N}_m} \mathrm{Tr}[\mathbf{W}_{m,i}^*] = \bar{p}_m$.

We conclude verifying that the sequence $\{(\mathbf{W}^t, \lambda^t) : t > 0\}$ converges with probability 1. By compactness, the sequence has a converging sub-sequence $\{(\mathbf{W}^{t(n)}, \lambda^{t(n)}) : n > 0\}$ with limit point $(\hat{\mathbf{W}}, \hat{\lambda})$. For every pair (m,i) at time $t(n)$ either user (m,i) is selected for updating $\mathbf{W}_{m,i} \in \mathcal{W}_{m,i}$ in which case

$$\mathbf{W}_{m,i}^{t(n)+1} = \arg\max_{\mathbf{W}_{m,i}} U_{m,i}\left(\left(\mathbf{W}_{m,i}, \mathbf{W}_{-(m,i)}^{t(n)}\right), \mathbf{T}_{m,i}^{t(n)}, \lambda_m^{t(n)}\right)$$

and $\mathbf{W}_{q,j}^{t(n)+1} = \mathbf{W}_{q,j}^{t(n)}$ for all $(q,j) \neq (m,i)$ or user (m,i) is *not* selected for updating $\mathbf{W}_{m,i}$ in which case, $\mathbf{W}_{m,i}^{t(n)+1} = \mathbf{W}_{m,i}^{t(n)}$. Since either case occurs infinitely often we conclude $\mathbf{W}_{m,i}^{t(n)+1} \to \hat{\mathbf{W}}_{m,i}$ with probability one and

$$\hat{\mathbf{W}}_{m,i} = \arg\max_{\mathbf{W}_{m,i}} U_{m,i}\left(\left(\mathbf{W}_{m,i}, \hat{\mathbf{W}}_{-(m,i)}\right), \hat{\mathbf{T}}_{m,i}, \hat{\lambda}_m\right)$$

Now consider the subsequence $\{\mathbf{W}_{m,i}^{t(n)+2} : n > 0\}$. Here, again, either $\mathbf{W}_{m,i}^{t(n)+2} = \mathbf{W}_{m,i}^{t(n)+1}$ because user (m,i) is not chosen to update at time $t(n)+1$ or

$$\mathbf{W}_{m,i}^{t(n)+2}$$
$$= \arg\max_{\mathbf{W}_{m,i}} U_{m,i}\left(\left(\mathbf{W}_{m,i}, \mathbf{W}_{-(m,i)}^{t(n)+1}\right), \mathbf{T}_{m,i}^{t(n)+1}, \lambda_m^{t(n)+1}\right).$$

Thus, we conclude $\mathbf{W}_{m,i}^{t(n)+2} \to \hat{\mathbf{W}}_{m,i}$. The same argument can now be recursively applied to show that $\mathbf{W}_{m,i}^{t(n)+k} \to \hat{\mathbf{W}}_{m,i}$ for any $k \geq 2$. Thus, the sequence $\{(\mathbf{W}^t, \lambda^t) : t > 0\}$ converges. ■

Appendix C
Existence of Rank-1 Solution to the UUM Problem

Let us define $\mathbf{A}_{m,i} = \sum_{(q,j)\neq(m,i)} \mathbf{T}_{m,i}^{q,j} \mathbf{H}_{m,q_j} \succeq 0$, then the utility for user (m,i) has the form

$$U_{m,i}\left(\mathbf{W}_{m,i}, \mathbf{W}_{-(m,i)}\right) = R_{m,i}\left(\mathbf{W}_{m,i}, \mathbf{W}_{-(m,i)}\right)$$
$$- \mathrm{Tr}\left[(\mathbf{A}_{m,i} + \lambda_m \mathbf{I})\mathbf{W}_{m,i}\right]. \tag{30}$$

Problem (UUM) must provide a rank-1 solution, as the (SRM) problem is solved with rank-1 solution [39], [40]. In the following, we identify a special structure of the problem (UUM) that allows it to admit a rank-1 solution. To this end, we tailor the rank reduction procedure (abbreviated as RRP) proposed in [41] to fit our problem.[5] Assume that using standard optimization package we obtain an optimal solution $\widetilde{\mathbf{W}}_{m,i}^*$ to the convex problem (UUM), with $\mathrm{Rank}(\widetilde{\mathbf{W}}_{m,i}^*) = r > 1$. Let $\widetilde{\mathbf{W}}_{m,i}^{(1)} = \widetilde{\mathbf{W}}_{m,i}^*$, and let $r^{(1)} = r$. At iteration t of the the RRP, we perform an eigen decomposition $\widetilde{\mathbf{W}}_{m,i}^{(t)} = \mathbf{V}^{(t)} \mathbf{V}^{(t)H}$, where $\mathbf{V}^{(t)} \in \mathbb{C}^{K \times r^{(t)}}$. If $r^{(t)} > 1$, find $\mathbf{D}^{(t)} \in \mathbb{S}^{r^{(t)}}$ such that the following three conditions are satisfied

$$\mathrm{Tr}\left(\mathbf{D}^{(t)} \mathbf{V}^{(t)H} \mathbf{H}_{m,i} \mathbf{V}^{(t)}\right) = 0 \tag{31}$$

$$\mathrm{Tr}\left(\mathbf{D}^{(t)} \mathbf{V}^{(t)H} \mathbf{A}_{m,i} \mathbf{V}^{(t)}\right) = 0 \tag{32}$$

$$\mathrm{Tr}\left(\mathbf{D}^{(t)} \mathbf{V}^{(t)H} \mathbf{V}^{(t)}\right) = 0. \tag{33}$$

If such $\mathbf{D}^{(t)}$ cannot be found, exit. Otherwise, let $\lambda(\mathbf{D}^{(t)})$ be the eigenvalue of $\mathbf{D}^{(t)}$ with the largest absolute value, and construct $\widetilde{\mathbf{W}}_{m,i}^{(t+1)} = \mathbf{V}^{(t)}(\mathbf{I}_r - (1/\lambda(\mathbf{D}^{(t)}))\mathbf{D}^{(t)})\mathbf{V}^{(t)H} \succeq 0$. Clearly, $\mathrm{Rank}(\mathbf{I}_r - (1/\lambda(\mathbf{D}^{(t)}))) \leq r^{(t)} - 1$, as a result, $\mathrm{Rank}(\widetilde{\mathbf{W}}_{m,i}^{(t+1)}) \leq \mathrm{Rank}(\widetilde{\mathbf{W}}_{m,i}^{(t)}) - 1$, i.e., the rank has been reduced by at least one. Utilizing (31)–(33), we obtain

$$\mathbf{h}_{m,i}^H \widetilde{\mathbf{W}}_{m,i}^{(t+1)} \mathbf{h}_{m,i}$$
$$= \mathrm{Tr}\left[\mathbf{H}_{m,i} \widetilde{\mathbf{W}}_{m,i}^{(t+1)}\right]$$

[5] Note that the RRP procedure in [41] cannot be directly applied to our problem. This is because in [41], the RRP is used to identify rank-1 solution of semidefinite programs with linear objective and constraints. Our problem is different in that the objective function is of a logdet form.

$$= \mathrm{Tr}\left[\mathbf{H}_{m,i}\mathbf{V}^{(t)}\left(\mathbf{I}_r - \frac{1}{\lambda\left(\mathbf{D}^{(t)}\right)}\mathbf{D}^{(t)}\right)\mathbf{V}^{(t)H}\right]$$
$$= \mathrm{Tr}\left[\mathbf{H}_{m,i}\widetilde{\mathbf{W}}_{m,i}^{(t)}\right] = \mathbf{h}_{m,i}^{H}\widetilde{\mathbf{W}}_{m,i}^{(t)}\mathbf{h}_{m,i}^{H} \quad (34)$$
$$\mathrm{Tr}\left[\mathbf{A}_{m,i}\widetilde{\mathbf{W}}_{m,i}^{(t+1)}\right]$$
$$= \mathrm{Tr}\left[\mathbf{A}_{m,i}\mathbf{V}^{(t)}\left(\mathbf{I}_r - \frac{1}{\lambda\left(\mathbf{D}^{(t)}\right)}\mathbf{D}^{(t)}\right)\mathbf{V}^{(t)H}\right]$$
$$= \mathrm{Tr}\left[\mathbf{A}_{m,i}\widetilde{\mathbf{W}}_{m,i}^{(t)}\right] \quad (35)$$
$$\mathrm{Tr}\left[\widetilde{\mathbf{W}}_{m,i}^{(t+1)}\right]$$
$$= \mathrm{Tr}\left[\mathbf{V}^{(t)}\left(\mathbf{I}_r - \frac{1}{\lambda\left(\mathbf{D}^{(t)}\right)}\mathbf{D}^{(t)}\right)\mathbf{V}^{(t)H}\right]$$
$$= \mathrm{Tr}\left[\widetilde{\mathbf{W}}_{m,i}^{(t)}\right]. \quad (36)$$

Equations (34)–(36) together ensure that the value of the utility function does not change, i.e.,

$$U_m\left(\widetilde{\mathbf{W}}_{m,i}^{(t+1)}, \widehat{\mathbf{W}}_{-(m,i)}\right) = U_{m,i}\left(\widetilde{\mathbf{W}}_{m,i}^{(t)}, \widehat{\mathbf{W}}_{-(m,i)}\right).$$

Combined with the fact that $\widetilde{\mathbf{W}}_{m,i}^{(t+1)} \succeq 0$, we have that $\widetilde{\mathbf{W}}_{m,i}^{(t+1)}$ is also an optimal solution to the problem (UUM).

Evidently, performing the above procedure for at most r times, we will obtain a rank-1 solution $\mathbf{W}_{m,i}^{*}$ that solves the problem (UUM). Now the question is that under what condition we can find $\mathbf{D}^{(t)}$ that satisfies (31)–(33) in each iteration t. Note that $\mathbf{D}^{(t)}$ is a $r^{(t)} \times r^{(t)}$ Hermitian matrix, hence finding $\mathbf{D}^{(t)}$ that satisfies (31)–(33) is equivalent to solving a system of three linear equations with $(r^{(t)})^2$ unknowns.[6] As long as $(r^{(t)})^2 > 3$, the linear system is underdetermined and such $\mathbf{D}^{(t)}$ can be found. Consequently, the RRP procedure, when terminated, gives us a $\mathbf{W}_{m,i}^{*}$ with $\mathrm{Rank}^2(\mathbf{W}_{m,i}^{*}) \leq 3$. As the rank of a matrix is an integer, we must have $\mathrm{Rank}(\mathbf{W}_{m,i}^{*}) \leq 1$. It is important to note, however, that the ability of the RRP procedure to recover a rank-1 solution for problem (UUM) lies in the fact that *we only have three linear terms of* $\mathbf{W}_{m,i}$ *in both the objectives and the constraints*. This results in solving a linear system with *three* equations in each iteration of the RRP procedure. If we have an additional linear constraint of the form $\mathrm{Tr}(\mathbf{B}\mathbf{W}_{m,i}) \leq c$ for some constant c, the RRP procedure may produce a solution $\mathbf{W}_{m,i}^{*}$ with $\mathrm{Rank}^2(\mathbf{W}_{m,i}^{*}) \leq 4$, which does not guarantee $\mathrm{Rank}(\mathbf{W}_{m,i}^{*}) = 1$.

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[6]The number of unknowns for the real part of $\mathbf{D}^{(t)}$ is $(r^{(t)}+1)r^{(t)}/2$, and the number of unknowns for the imaginary part of $\mathbf{D}^{(t)}$ is $(r^{(t)}-1)r^{(t)}/2$.

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