Outage Performance Analysis of Imperfect-CSI-Based Selection Cooperation in Random Networks

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***Abstract*—Selection of relays is central to efficient utilization of cooperative diversity gains when multiple relays are available in the network. Such selection is generally based on some form of channel state information (CSI), which is always imperfect in practice. The effects of using imperfect CSI in a relay selection process have been generally considered in existing literature without the account for spatial distribution of relays, while current works on relay selection in random networks mainly assume perfect CSI. In this paper, we analyze the outage performance of a single source–destination pair communicating through a decode-and-forward relay, chosen from a Poisson point process (PPP) of candidate relays using perfect and imperfect CSI. We derive exact outage probability expressions for the selection cooperation strategy. Closed-form expressions are provided for special cases, and asymptotic analysis is conducted to highlight the high-SNR system behavior.**

***Index Terms*—Relay selection, cooperative diversity, stochastic geometry, imperfect channel estimation, decode-and-forward.**

I. Introduction

Relay selection (RS) reduces coordination overhead in cooperative systems with multiple relays while achieving full diversity [1], [2]. The objective of RS is to select one relay with the best channel state to the source and/or destination from a set of candidates. With careful selection, same diversity gain as in the case of coordinated all-relay transmission can be achieved [2]. However, channel quality measurements are necessary at the relays or at a central decision center to realize distributed or centralized RS protocols (e.g., opportunistic relaying or selection cooperation [3]–[7]). In practice such measurements are subject to errors, leading to suboptimal selection decisions and to performance loss in the employed RS strategy [8], [9]. In this paper we aim to assess outage performance of selection cooperation strategy in a scenario where the channel state information (CSI) used in the selection process is imperfect, while taking into account random spatial distribution of candidate relays.

RS mechanisms and analysis of their performance are especially important for emerging dense, multi-hop, multi-tier and decentralized network deployments to meet the expected exponential growth in mobile user traffic [10]. Some motivating application scenarios include uplink or downlink of a macro base stations assisted by femto-access points, picocells or relays with possibly wireless backhaul. Other applications of relay-assisted source–destination communications include sensor networks, emergency or battlefield communications.

The problem of RS based on imperfect CSI has been studied in literature, e.g., [8], [9], [11]–[14]. Considered imperfection models include feedback delays, where the measured CSI becomes outdated at the selection instant [8], [9], [11]–[13], and noisy estimation where the estimated CSI contains unknown channel noise component [9], [14]. The effect of these imperfections and their combinations on outage probability and diversity order has been analyzed extensively [8], [9], [11]–[14]. However the impact of the spatial randomness of relay positions on RS system performance with imperfect CSI has not been considered to date. In particular, references [15]–[23] have investigated RS with account for network topology, but assuming perfect CSI. In this paper we analyze outage performance of RS with combined effects of spatially random relays and imperfect CSI used in the selection process.

The importance of inclusion of spatial node distribution in performance analysis of wireless networks in general has been advocated in [24], and specialized to RS in [15]–[23]. The main reason being that inter-node distances contribute to the selection decision, and are subject to being random as is small-scale fading. To this end, in [15] distance distribution to the selected relay was derived, which allowed obtaining exact outage probability expressions for an RS system with spatially random decode-and-forward (DF) relays. In [16] authors investigate the trade-off between the number of candidate relays participating in selection process and outage performance, and propose a region-based spatial selection process. References [17], [18], [23] apply thinning operation on point processes to

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derive outage probabilities for the cases of DF and AF relays. Energy-fair relay selection in sensor networks has been studied in [19] for the case of destination located in the far-field of the source and cooperating AF relays. A multi-cell environment with other-cell interference has been considered in [20] and asymptotic results have been derived using stochastic geometry instruments. Decentralized and uncoordinated RS algorithms have been studied recently in [21], [22]. However, all above works assume perfect CSI available in the selection process. Scenarios with network topology-inclusive RS based on imperfect CSI have not been covered in existing literature to the best of our knowledge. Few available works on random networks with imperfect CSI used in the communication process do not consider RS [25], [26]. In all above works, Poisson point process is used to model node distributions in networks with different degrees of planning, e.g., macro-cellular systems and small-cell networks [10]. While a PPP is only an approximation of the real node deployments, tractable performance analysis is possible in contrast to uniform or regular node placement models, e.g., [27].

In this paper we consider a two-hop interference-free communication via a DF relay selected from a Poisson field of candidate relays using selection cooperation (SC) [6] strategy. In SC one relay with the strongest channel to the destination is selected from a set of relays that have decoded the source transmission. Chosen relay is referred to as "best relay" [28, p.530]. The selection of the best relay can be performed using training sequence-based channel measurements, that are subject to small-scale Rayleigh fading, propagation path loss and AWGN, so that a relay chosen based on perceived channel quality may not be the best. Channel reciprocity is assumed in this paper, so that channel gain from relay to destination is equivalent to channel gain from destination to relay. In this way, results in this work are applicable to time division duplex (TDD) systems.

We derive exact outage probability expressions for the cases of imperfect and perfect CSI using thinning operation on point processes [29], and study the diversity behavior of the considered scenario. One key advantage of our analytical approach is that explicit derivation of distance distributions as in [15], [17] is not required. Contributions of this paper can be summarized as follows:

- Exact outage probability expressions are derived for the cases of perfect and imperfect CSI used in the selection of the best relay from the pool of spatially distributed candidates. Closed-form expressions are provided for special cases;[1]
- A simplified analytical approach is demonstrated. Our method (a) bypasses complicated calculation of distance distributions for the considered scenario, (b) is applicable both to the cases of perfect and imperfect CSI, and (c) allows extensions to different channel imperfection models.
- Asymptotic analysis is conducted to highlight the impact of system parameters on outage performance at high SNR.

[1] For simplicity of exposition throughout the paper we assume that decoding set is non-empty, i.e. that at least one relay always decodes source transmission.

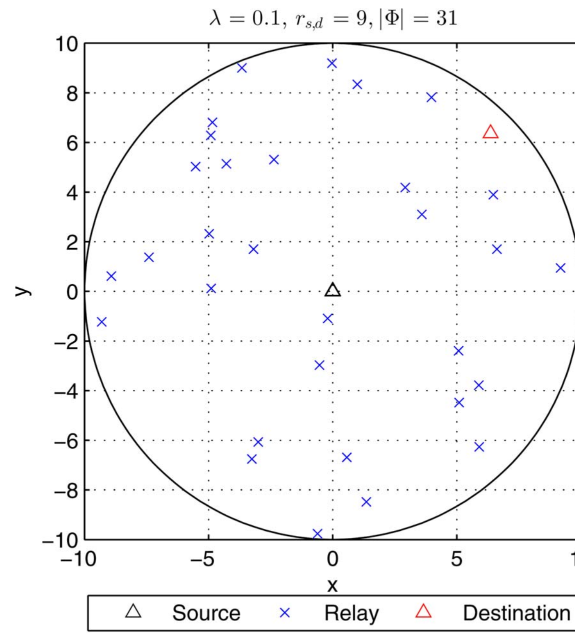

Fig. 1. Network model: source aims to communicate with destination in presence of a realization of the process Φ of candidate relays.

To focus on the system performance in the case of imperfect CSI, and in order to make the discussion specific, we use channel estimation error model from [30]–[32] rather than the actual channel estimation process (e.g., MMSE). Another important point is that our exact solutions require numerical integration, which however can be easily accomplished using available software packages.

This paper is organized as follows. System model is presented in Section II, where we propose and discuss a unified formulation of outage probability for the cases of perfect/imperfect CSI at relays. Sections III and IV respectively present performance analysis of relay selection for the cases of perfect and imperfect CSI about the relay–destination channels. Numerical results, corroborating presented analysis are provided and discussed in Section V, and Section VI presents concluding remarks.

II. System Model

In this section we present network, signal and CSI error models, define point processes used in this paper, and formulate outage probability expression for an arbitrary CSI model. Thinning operation operation on point processes is also described. Presented outage formulation is then analyzed to (a) yield a simple outage probability expression for the case of perfect CSI and (b) highlight the fact that an increase in size of the decoding set can either increase or decrease outage probability. Namely, a larger decoding set may have larger outage probability since there can be more relays with incorrect channel measurements. Exact analysis of perfect and imperfect CSI cases is presented in Sections III and IV, respectively.

A. General Setup

Consider a scenario depicted in Fig. 1, where a single source s communicates with a destination d with assistance from a set of idle users acting as relays. Relays form a spatial Poisson point process (PPP) $\Phi(W)$ with a uniform intensity function λ. We consider communication in a circular region W with radius R. The region W can represent a single cell in a cellular system or coverage area of an access point (femto, pico or other type of a small cell), and can be applicable to scenarios discussed in the Introduction. For compactness the references to cell W will be suppressed in the rest of the paper.

Interference-free scenario is assumed, so that all nodes communicate over orthogonal channels. Half-duplex communication is considered as in [33], so that nodes cannot transmit and receive at the same time, and direct source–destination link is assumed to be unavailable.

In the first time slot the source broadcasts a message x_s, and each candidate relay $j \in \Phi$ receives

$$y_j = \sqrt{\kappa P} g_{s,j} x_s + n_j, \tag{1}$$

where P is the total available power, κ is the share of the power P allocated for source transmission, n_j is the AWGN with variance σ_n^2, coefficient $g_{s,j} = h_{s,j}/\sqrt{1+r_{s,j}^{\alpha}}$ is the channel coefficient incorporating the small scale Rayleigh fading through $h_{s,j} \sim \mathcal{CN}(0, \sigma_h^2)$ and path loss effects through the bounded path loss model $l(r_{s,j}) = 1 + r_{s,j}^{\alpha}$ with $r_{s,j}$ standing for the source–relay distance, and $\alpha \in [2, 6]$ is the path loss exponent. Note that for a given relay j, $g_{s,j} \sim \mathcal{CN}(0, \sigma_{g_{s,j}}^2)$ and has variance $\sigma_{g_{s,j}}^2 = \sigma_h^2/(1 + r_{s,j}^{\alpha})$ [34, p.154]. The relays that receive the source message x_s correctly, form a realization of the decoding set Φ_d:[2]

$$\Phi_d = \left\{ j \in \Phi : |g_{s,j}|^2 \geq \frac{\theta}{\kappa} \right\}, \tag{2}$$

where $\theta = 2^{2\mathcal{R}-1}/(P/\sigma_n^2)$ and $\mathcal{R}$ is the target data rate.

In the second time slot the destination broadcasts a training sequence that allows each relay $j \in \Phi_d$ to obtain an estimate $\hat{g}_{j,d}$ for the channel to the destination d. An MMSE estimate of the channel can be expressed as [30]–[32]

$$\hat{g}_{j,d} = g_{j,d} + \epsilon, \tag{3}$$

where $\epsilon \sim \mathcal{CN}(0, \sigma_\epsilon^2)$ is the estimation error component. In this paper we focus on the effect estimation error on outage probability of relays selection, rather than on performance of channel estimation methods. As in [30], σ_ϵ^2 is assumed to be given *a priori*, through, for example, channel estimation using training sequence. Two simple models for estimation error variance σ_ϵ^2 are employed [30], [32]: (a) SNR-independent model, and (b) a relay transmission power-dependent model $\sigma_\epsilon^2 = \sigma_u^2 + \sigma_n^2/(1-\kappa)P$. In the latter σ_u^2 can be treated as a prediction error due to time variability of the channel, σ_n^2 is the measurement error caused by AWGN, and $(1-\kappa)P$ is the transmit power used in estimation process, assumed to be equal to the relay transmit power.

[2]Note that for sufficiently high source power $P_s = \kappa P$, it can be shown that $\Pr(\Phi_d = \emptyset) \to 0$. Therefore in the following the case $\Phi_d = \emptyset$ will be ignored as this will substantially simplify exposition.

Selection cooperation (SC) strategy [6] is considered in this paper. According to SC, from the set Φ_d of relays that decode the source transmission successfully, one relay J that has the best *perceived* channel gain for the relay–destination channel is selected to forward the message. The selected relay J satisfies

$$J = \arg\max_{j \in \Phi_d} |\hat{g}_{j,d}|^2, \tag{4}$$

where $|\hat{g}_{j,d}|^2$ is the perceived channel gain. If the channel estimation is perfect, i.e. $\hat{g}_{j,d} = g_{j,d},\ \forall j \in \Phi_d$, then the selected relay J indeed has the best channel to the destination among all other relays. However, for the case of non-zero error ϵ, the relay J with the best estimate does not necessarily have the best channel to the destination.

The signal model for the transmission from the selected relay J can be written along similar to (1) as

$$y_{J,d} = \sqrt{(1-\kappa)P} g_{J,d} x_s + n_d, \tag{5}$$

where $1-\kappa$ is the power share allocated to the relay transmission. Relays in the cell W that have reliable links both to the source and to the destination form a set of qualified relays $\Phi_q \subseteq \Phi_d$:

$$\Phi_q = \left\{ j \in \Phi : |g_{s,j}|^2 \geq \frac{\theta}{\kappa}, |g_{j,d}|^2 \geq \frac{\theta}{1-\kappa} \right\}. \tag{6}$$

In general, outage probability for relay-assisted communication is defined as the probability that the end-to-end SNR γ_J falls below some predefined threshold θ, i.e. $\Pr(\gamma_J < \theta)$, were J is the index of relay selected for forwarding the source message to the destination [5], [9]. For the case of selection cooperation, outage probability can be defined as the conditional probability that for a given size of decoding set Φ_d, the channel gain $|g_{J,d}|^2$ for the chosen relay–destination channel falls below a threshold θ, averaged over all possible sizes of the decoding set [9], [15], [18]:

$$\begin{aligned} P_o &= \Pr(\Phi_d = \emptyset) + \sum_{l=1}^{\infty} \Pr\left(|\Phi_d| = l\right) \\ &\quad \times \Pr\left(|g_{J,d}|^2 < \frac{\theta}{1-\kappa} \big| |\Phi_d| = l\right) \\ &= \Pr(\Phi_d = \emptyset) + \Pr(\mathcal{O} | \Phi_d \neq \emptyset) \approx \Pr(\mathcal{O}) \end{aligned} \tag{7}$$

where the approximation in the last step is made for sufficiently high source transmission power, such that $\Pr(\Phi_d = \emptyset) \to 0$, and the event $\mathcal{O}$ can be defined as

$$\begin{aligned} \mathcal{O} &= \Big(\nexists R_j : |g_{s,j}|^2 > \frac{\theta}{\kappa}, |g_{j,d}|^2 > \frac{\theta}{1-\kappa}, \\ &\qquad |\hat{g}_{j,d}|^2 = \max_{i \in \Phi_d} |\hat{g}_{i,d}|^2 \Big) \\ &= \Big(\exists R_j : |g_{s,j}|^2 > \frac{\theta}{\kappa}, |g_{j,d}|^2 < \frac{\theta}{1-\kappa}, \\ &\qquad |\hat{g}_{j,d}|^2 > \max_{i \in \Phi_q} |\hat{g}_{i,d}|^2 \Big) \end{aligned} \tag{8}$$

where the first step corresponds to the event that there is no relay with a reliable connection both to the source and the destination, and with the largest channel estimate $|\hat{g}_{j,d}|^2$. The second step corresponds to the event that there exists a relay with reliable connection to the source, but not to destination, yet with the channel estimate larger than any estimate at the relays in the set of qualified relays Φ_q.

B. Thinning Operation

In its simplest form, thinning is realized by associating each point of a point process with a probability of retention p that is independent of point location and respective locations of other points of the point process [29]. For example, each point of the parent process Φ could be deleted in a random way with probability $1-p$. However, we are interested in a more advanced type of thinning, termed as $p(r)$-*thinning*, where the probability of retention of a point depends on the location r of this point. It is important to note, that p- and $p(r)$-thinnings of a PPP produce point processes that are still Poisson [29], [35], although such processes may not retain stationarity and/or isotropy properties, as we shall see later.

Specifically, to obtain the set of nodes (points) in (2) from the original PPP of candidate relays Φ, one can apply a location-dependent thinning $p_d(r_{s,j})$ that will select candidate relays from Φ with respect to the connectivity of relays to the source. In particular, the relay j located at distance $r_{s,j}$ from the source will be retained with probability

$$p_d(r_{s,j})=\Pr\left(\frac{|h_{s,j}|^2}{1+r_{s,j}^{\alpha}}\geq\frac{\theta}{\kappa}\right)=\exp\left(-\left(1+r_{s,j}^{\alpha}\right)\frac{\theta}{\kappa}\right). \quad (9)$$

The intensity measure Λ_d of such new thinned process Φ_d can be found from the intensity measure Λ of the original point process Φ as [29]:

$$\Lambda_d=\lambda\int_W p(w)\Lambda(\mathrm{d}w), \quad (10)$$

where $p_d(w)$ is equivalent to $p_d(r_{s,j})$ since the unique location w in the region W can be defined in polar coordinates through the distance $r_{s,j}$ from the source to relay j and the angle φ between some reference direction and the line connecting s and j. Connectivity between relays and the source is independent of orientation φ, hence the angle φ is omitted from (9). Element area $\mathrm{d}w$ of the region W can be represented as $\mathrm{d}w=r_{s,j}\mathrm{d}r_{s,j}\mathrm{d}\varphi$. The intensity measure Λ_d can be interpreted as an average number of relays satisfying the condition in p_d within certain area W. This metric is not to be confused with intensity function $\lambda_d(w)$, which denotes the average number of points of the process Φ_d per unit area (length or volume) with location w.

On the other hand, location-dependent thinning $p_q(r_{j,d})$ that will retain relays from Φ with respect to the connectivity to the destination can be applied to obtain the PPP Φ_q of relays connected both to the source and to the destination:

$$p_q(r_{j,d})=\exp\left(-\left(1+r_{j,d}^{\alpha}\right)\frac{\theta}{1-\kappa}\right), \quad (11)$$

TABLE I
POINT PROCESSES AND THEIR PROPERTIES

Notation	Explanation	Relay j qual. if
Φ	All candidate relays	Always
Φ_d	Relays with reliable connections to source (**decoding set**)	$|g_{s,j}|^2\geq\frac{\theta}{\kappa}$
Φ_q	Relays with reliable connections both to source and destination (**qualified set**)	$|g_{s,j}|^2\geq\frac{\theta}{\kappa}$, $|g_{j,d}|^2\geq\frac{\theta}{1-\kappa}$
$\hat{\Phi}_q(x)$	Relays with reliable connections both to source and destination, and with channel estimate $|\hat{g}_{j,d}|^2$ to destination larger than x	$|g_{s,j}|^2\geq\frac{\theta}{\kappa}$, $|g_{j,d}|^2\geq\frac{\theta}{1-\kappa}$, $|\hat{g}_{j,d}|^2>x$
Φ_u	Relays with reliable connection to source, but not to destination	$|g_{s,j}|^2\geq\frac{\theta}{\kappa}$, $|g_{j,d}|^2<\frac{\theta}{1-\kappa}$
$\hat{\Phi}_u(x)$	Relays with reliable connection to source, but not to destination, and with channel estimate $|\hat{g}_{j,d}|^2$ larger than x	$|g_{s,j}|^2\geq\frac{\theta}{\kappa}$, $|g_{j,d}|^2<\frac{\theta}{1-\kappa}$, $|\hat{g}_{j,d}|^2>x$

where

$$r_{j,d}^2=r_{s,j}^2+r_{s,d}^2-2r_{s,j}r_{s,d}\cos(\varphi). \quad (12)$$

Note that the two thinning stages are conditionally independent for any given relay j, and can be applied in arbitrary order on the original PPP of candidate relays Φ.

Based on the two thinning stages described above, in following subsections we formulate and discuss the general expression for the probability of outage in SC, valid for arbitrary CSI imperfection and processing models.

C. Outage Probability Formulation

Let $\Lambda_q(\lambda,\alpha,R)$ be the intensity measure of the process Φ_q, which we will denote as Λ_q for compactness. Further, let $\hat{\Lambda}_q(x)$ be the intensity measure the process $\hat{\Phi}_q(x)$ of such relays in Φ_q that also have the estimation function $|\hat{g}_{j,d}|^2>x$, where $x\in[0,\infty)$. Note that $\hat{\Lambda}_q(x)$ is a decreasing function of x. Similarly, let Λ_u be the intensity measure of the process Φ_u of relays in that have reliable connections to the source but *not* to the destination, so that $\hat{\Lambda}_u(x)$ is the intensity measure of relays in the process $\hat{\Phi}_u(x)$ that also have the function $|\hat{g}_{j,d}|^2>x$, where $x\in[0,\infty)$. Table I summarizes the properties of the point processes processes used above.

Then the probability of outage for the source–destination communication via the set of candidate relays can be expressed as in the following proposition.

Proposition 1 (Outage Probability Formulation): For sufficiently high SNR, so that $\Pr(\Phi_d=\emptyset)\to 0$, the outage probability for SC strategy based on imperfect CSI can be expressed as

$$P_o=1+\int_0^{\infty}\exp\left(-\hat{\Lambda}_u(x)-\hat{\Lambda}_q(x)\right)\hat{\Lambda}_q'(x)\,\mathrm{d}x, \quad (13)$$

where $(\cdot)'$ denotes first derivative.

Proof: Please see Appendix A. ■

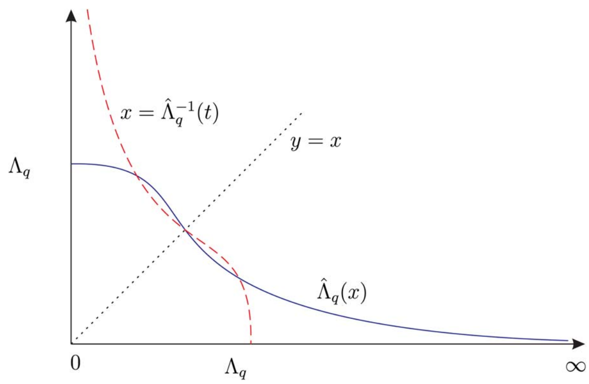

Fig. 2. Schematic plot of the intensity measure $\hat{\Lambda}_q(x)$ (solid blue) and its inverse $x = \Lambda_q^{-1}(t)$ (dashed red). There is a one-to-one correspondence between the threshold x for estimated channel gain $|\hat{g}_{j,d}|^2$ and the average number $\hat{\Lambda}_q(x)$ of qualified relays with the estimates $|\hat{g}_{j,d}|^2$ above this threshold x.

D. Discussion of the Formulation

In this section we present an initial asymptotic analysis of the outage probability formulation in (13).

First, note that in the case of no estimation error, there are no unqualified relays in Φ_u that have estimates for the channel to the destination larger than any estimate at the relays from the qualified set Φ_q, hence $\hat{\Lambda}_u(x) = 0$ for $\forall\, x$. Then (13) can be rewritten as

$$P_{\text{o,imperf}} = 1 + \int_0^\infty \exp\left(-\hat{\Lambda}_q(x)\right) \hat{\Lambda}_q'(x)\,\mathrm{d}x$$
$$\stackrel{(a)}{=} 1 - \int_0^{\Lambda_q} \exp(-t)\,\mathrm{d}t = \exp(-\Lambda_q), \qquad (14)$$

where in step (a) integration by substitution was used:

$$\int_a^b f\left(g(x)\right) g'(x)\,\mathrm{d}x = \int_{g(a)}^{g(b)} f(y)\,\mathrm{d}y.$$

Likewise, the penalty for erroneous relay selection originates from the term $\hat{\Lambda}_u(x)$, which reduces the value of the integrand in (13), and consequently increases the outage probability P_o.

Secondly, note that outage probability for the general case can be expressed using integration by substitution as

$$P_{\text{o,imperf}} = 1 - \int_0^{\Lambda_q} \exp\left(-\hat{\Lambda}_u\left(\hat{\Lambda}_q^{-1}(t)\right) - t\right)\,\mathrm{d}t, \qquad (15)$$

where $\hat{\Lambda}_u(\hat{\Lambda}_q^{-1}(t)) = \hat{\Lambda}_u(x)$, however the former was used to show the dependence of the integrand on variable t. Then the asymptotic behavior of the integrand in (13) can be determined by inspection of the inverse function $\hat{\Lambda}_q^{-1}(t)$, depicted schematically in Fig. 2. In particular, note that for sufficient power level,[3] as intensity $\hat{\Lambda}_q(x) \to 0$ we have the threshold $x \to \infty$, so that $\hat{\Lambda}_u(x) \to 0$. Therefore the integrand in (13) tends to one as $\hat{\Lambda}_q(x) \to 0$.

On the other hand, as the intensity measure $\hat{\Lambda}_q(x) \to \Lambda_q$ we have $x \to 0$, hence $\hat{\Lambda}_u(x) \to \Lambda_u$ (because any unqualified relay has positive estimate of channel gains to the destination). Therefore, the integrand in (13) tends to $e^{-\Lambda_d}$ as $\hat{\Lambda}_q(x) \to \Lambda_q$, where Λ_d is the mean number of relays in the decoding set.

This implies that the size of the decoding set can (a) increase the probability of outage by reducing the integrand $e^{-\Lambda_d}$, and (b) improve the probability of successful communication by potentially increasing the upper integration bound Λ_q in (13)—we will revisit this trade-off in Section V. In the following Sections III and IV we investigate exact system performance for the cases of perfect and imperfect channel estimation at relays.

III. Outage Analysis: Perfect CSI Case

In this section we focus on the case when relays possess exact CSI for the channels to the destination. Using thinning operation on point processes, we demonstrate a simplified analytical framework that provides exact results that are easy to evaluate, and closed-form expressions for special cases. Asymptotic analysis is also presented to characterize the high-SNR system performance in terms of diversity.

A. Exact Analysis

The expression for outage probability $P_{\text{o,perf}}$ is given in (14). To estimate $P_{\text{o,perf}}$ the intensity measure Λ_q must be found. Following proposition provides a general expression for Λ_q that can be evaluated numerically. Closed-form results for special cases are presented and discussed afterwards.

Proposition 2 (Mean Number of Qualified Relays): The intensity measure Λ_q of the PPP Φ_q of relays with reliable links both to the source s and destination d is

$$\Lambda_q = \lambda e^{-\frac{\theta}{\kappa(1-\kappa)}} \int_0^R \int_0^{2\pi} r_{s,j} \exp\left(-\frac{\theta r_{s,j}^\alpha}{\kappa}\right)$$
$$\times \exp\left(-\frac{\theta\left(r_{s,j}^2 + r_{s,d}^2 - 2 r_{s,j} r_{s,d}\cos(\varphi)\right)^{\frac{\alpha}{2}}}{1-\kappa}\right)\mathrm{d}r_{s,j}\,\mathrm{d}\varphi, \qquad (16)$$

where λ is the intensity function of the process Φ of all candidate relays, $\theta = 2^{2\mathcal{R}} - 1/(P/\sigma_n^2)$, P is the total transmission power, κ and $1-\kappa$ are respectively the source and relay power shares, $\varphi \in [0, 2\pi)$ is the angle between the relay j and the destination, $r_{s,j}$ and $r_{s,d}$ are distances from the source to the relay j and the destination respectively, and R is the cell radius.

Proof: See Appendix B. ■

Following corollaries provide closed-form solutions for special cases of system parameters.[4]

[3]By sufficient power level we mean such combination of total power P and power allocation ratio κ that the mean measure of qualified relays $\Lambda_q \neq 0$, i.e., that even a small number of true qualified relays exist in the network.

[4]Note that we now use full notation for the intensity measure $\Lambda(\lambda, \alpha, R)$ to highlight the parameters for special cases.

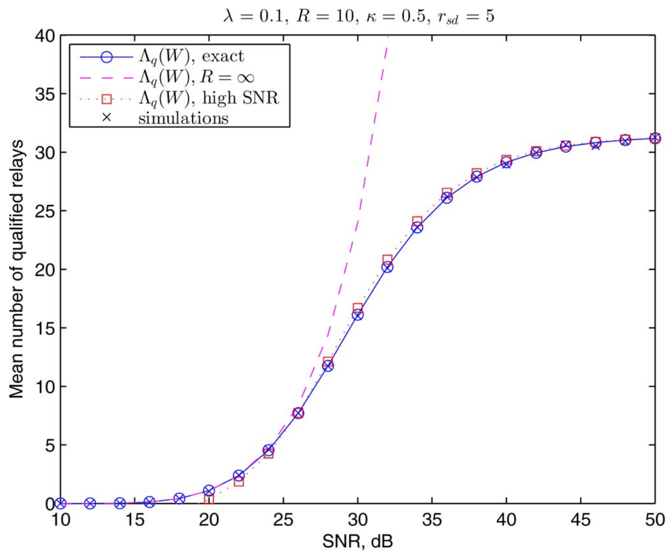

Fig. 3. Intensity measure $\Lambda_q(x)$ for cases of $R = 10$, $R = \infty$ and high SNR.

Corollary 3 (Special Case of $\alpha = 2$): For the special case of the path loss exponent $\alpha = 2$ the intensity measure of the process Φ_q can be expressed as

$$\Lambda_q(\lambda, 2, R) = \frac{\kappa(1-\kappa)\pi\lambda}{\theta} \exp\left(-\frac{\theta}{\kappa(1-\kappa)} - \theta r_{s,d}^2\right) \times \left(1 - Q_1\left(r_{s,d}\sqrt{\frac{2\kappa\theta}{1-\kappa}}, R\sqrt{\frac{2\theta}{\kappa(1-\kappa)}}\right)\right), \quad (17)$$

where $Q_1(\cdot,\cdot)$ is the first-order Marcum-Q function [36], [37].

Proof: See Appendix C. ■

Corollary 4 (Special Case of $\alpha = 2$ and $R \to \infty$): For the special cases of path loss exponent $\alpha = 2$ and $R \to \infty$ the intensity measure of the process Φ_q simplifies to

$$\Lambda_q(\lambda, 2, \infty) = \frac{\kappa(1-\kappa)\pi\lambda}{\theta} \exp\left(-\frac{\theta}{\kappa(1-\kappa)} - \theta r_{s,d}^2\right). \quad (18)$$

Proof: The proof is obtained similarly to Corollary 3 using [38, 6.614.1]. ■

Note that the effect of limited cell size is captured in (17) through the Macrum-Q function, which is not involved in (18) where the cell size and supply of candidate relays are unlimited.

B. Asymptotic Analysis and Diversity Behavior

For high total power P, we have $\theta \to 0$, so that the argument in the exponent in (17) and (18) becomes small. Hence using Taylor expansion we can further approximate (17) as

$$\Lambda_q(\lambda, 2, R) \approx \left(\frac{\kappa(1-\kappa)\pi\lambda}{\theta} - \pi\lambda\left(1 + \kappa(1-\kappa)r_{s,d}^2\right)\right) \times \left(1 - e^{-\frac{\theta R^2}{\kappa(1-\kappa)}}\right), \quad (19)$$

where the last step is obtained using $Q_1(0, x) = e^{-(x^2/2)}$ [37]. Fig. 3 illustrates the behavior of expressions for the intensity measure Λ_q derived in this section ($SNR = P/\sigma_n^2$).

The diversity order of the considered communication scheme can be shown to be infinite or equal to $\lambda\pi R^2$ for the case of infinite and finite[5] sizes of the cell respectively. This can be explained by the fact that with increase of available power, the number of qualified relays increases infinitely in an infinite cell. In other words, as for the cases of infinite cell dimensions and increasing power, the supply of relays should increase as well. However in a finite cell, the number of qualified relays becomes capped with the total number of candidate relays, with average of $\lambda\pi R^2$.

IV. Outage Analysis: Imperfect CSI Case

In this section outage probability of SC is analyzed with account for imperfect channel estimates at randomly distributed relays. As in previous section, we first derive exact expressions that can be numerically evaluated, and discuss asymptotic system performance afterwards.

A. Outage Probability Analysis

Outage probability expressions (13) and (15) offer initial intuition on the impact of important factors on communication outage probability, however it may be difficult to obtain the inverse function $\hat{\Lambda}_u(\hat{\Lambda}_q^{-1}(t))$. Nevertheless, it is possible to obtain an exact outage probability expression by utilizing (13) in the form of

$$P_o = 1 + \int_0^\infty \exp\left(-\hat{\Lambda}_d(x)\right) \frac{\mathrm{d}}{\mathrm{d}x}\hat{\Lambda}_q(x)\,\mathrm{d}x, \quad (20)$$

where $\hat{\Lambda}_d(x)$ is the intensity measure of the process of relays in the decoding set with estimates of the channel to the destination larger than some value x. Similarly, $\hat{\Lambda}_q(x)$ is the intensity measure of the process of qualified relays with channel estimates to the destination larger than x. In the following we derive the quantities $\hat{\Lambda}_d(x)$ and $(\mathrm{d}/\mathrm{d}x)\hat{\Lambda}_q(x)$ individually.

B. Intensity Measure $\hat{\Lambda}_d(x)$

In this subsection we derive the intensity measure $\hat{\Lambda}_d(x)$ of relays that are in the decoding set and have estimates of the channel to the destination $|\hat{g}_{j,d}|^2 > x$ (see Fig. 4). The quantity $\hat{\Lambda}_d(x)$ can be expressed as [29]:

$$\hat{\Lambda}_d(x) = \lambda \int_W p_{dx}(w)\,\mathrm{d}w, \quad (21)$$

where $p_{dx}(w)$ is the probability that a relay j at some location w will (a) be in the decoding set, i.e., $|g_{s,j}|^2 > (\theta/(1-\kappa))$, and (b) have the estimate $|\hat{g}_{j,d}|^2 > x$:

$$p_{dx}(w) = \Pr\left(|g_{s,j}|^2 > \frac{\theta}{1-\kappa}, |\hat{g}_{j,d}|^2 > x\right) = \exp\left(-\frac{1 + r_{s,j}^\alpha}{\sigma_h^2}\frac{\theta}{1-\kappa}\right)\Pr\left(|\hat{g}_{j,d}|^2 > x\right) \quad (22)$$

[5] By infinite and finite cells we mean respectively cells with infinite and finite radii R.

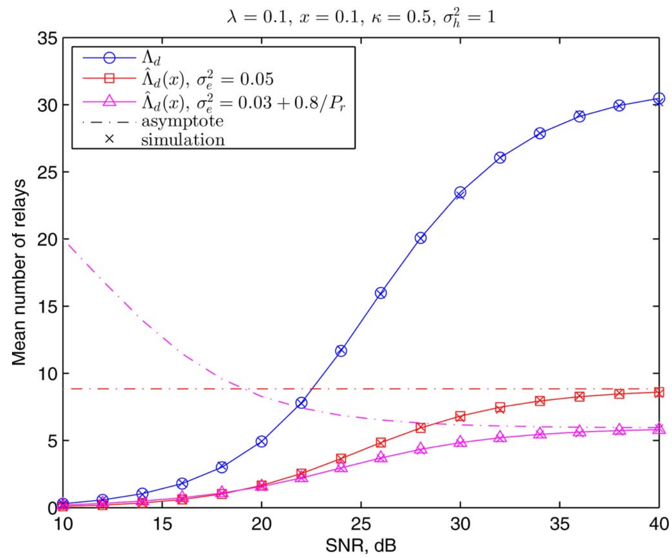

Fig. 4. Exact and asymptotic plots of the intensity measure $\hat{\Lambda}_d(x)$. Intensity measure Λ_d is provided for reference.

where the last step follows from the fact that the two events in the probability are conditionally independent given the location w. The estimate of the channel $\hat{g}_{j,d} = g_{j,d} + \epsilon$ is distributed as $\hat{g}_{j,d} \sim \mathcal{CN}(0, \sigma_{\hat{g}}^2)$, where $\sigma_{\hat{g}}^2 = (\sigma_h^2/(1 + r_{j,d}^\alpha)) + \sigma_\epsilon^2$. Therefore, the second probability can be expressed as

$$\Pr\left(|\hat{g}_{j,d}|^2 > x\right) = \exp\left(-\left(1 - \frac{\sigma_h^2}{\sigma_h^2 + \sigma_\epsilon^2\left(1 + r_{j,d}^\alpha\right)}\right)\frac{x}{\sigma_\epsilon^2}\right). \quad (23)$$

Then the intensity $\hat{\Lambda}_d(x)$ can be rewritten as

$$\hat{\Lambda}_d(x) = \lambda e^{-\frac{x}{\sigma_\epsilon^2} - \frac{\theta}{\sigma_h^2(1-\kappa)}} \int\limits_W e^{-\frac{r_{s,j}^\alpha}{\sigma_h^2}\frac{\theta}{1-\kappa} + \frac{\sigma_h^2}{r_{j,d}^\alpha + 1 + \sigma_h^2/\sigma_\epsilon^2}\frac{x}{\sigma_\epsilon^4}}\, \mathrm{d}w. \quad (24)$$

Above integral can be easily evaluated using numerical methods, while closed-form solutions for general α can be infeasible.

C. Derivative $(\mathrm{d}/\mathrm{d}x)\hat{\Lambda}_q(x)$

In this section we obtain the derivative of the intensity measure $\hat{\Lambda}_q(x)$ of relays that are qualified to retransmit the source message to the destination, and have estimates of the channels to the destination $|\hat{g}_{j,d}|^2 > x$:

$$\frac{\mathrm{d}}{\mathrm{d}x}\hat{\Lambda}_q(x) = \lambda \int\limits_W \frac{\mathrm{d}}{\mathrm{d}x} p_{qx}(w)\mathrm{d}w, \quad (25)$$

where $p_{qx}(w)$ is the probability that a relay j at some location w will (a) be qualified for end-to-end transmission of the message, i.e., $|g_{s,j}|^2 > (\theta/(1-\kappa))$, $|g_{j,d}|^2 > (\theta/\kappa)$, and (b) have the estimate $|\hat{g}_{j,d}|^2 > x$:

$$\begin{aligned} p_{qx}(w) &= \Pr\left(|g_{s,j}|^2 > \frac{\theta}{\kappa}, |g_{j,d}|^2 > \frac{\theta}{1-\kappa}, |\hat{g}_{j,d}|^2 > x\right) \\ &= \exp\left(-\frac{1 + r_{s,j}^\alpha}{\sigma_h^2}\frac{\theta}{1-\kappa}\right) \\ &\quad \times \Pr\left(|g_{j,d}|^2 > \frac{\theta}{\kappa}, |\hat{g}_{j,d}|^2 > x\right). \end{aligned} \quad (26)$$

Hence the quantity of our interest can be expressed as

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}x}\hat{\Lambda}_q(x) &= \lambda e^{-\frac{\theta}{\sigma_h^2(\kappa)}} \int\limits_W e^{-\frac{\theta}{\sigma_h^2(\kappa)} r_{s,j}^\alpha} \\ &\quad \times \underbrace{\frac{\mathrm{d}}{\mathrm{d}x}\Pr\left(|g_{j,d}|^2 > \frac{\theta}{1-\kappa}, |\hat{g}_{j,d}|^2 > x\right)}_{f'} \mathrm{d}w, \end{aligned} \quad (27)$$

where the derivative f' of the probability can be found as

$$\begin{aligned} f' &= \frac{\mathrm{d}}{\mathrm{d}x} \int\limits_{\theta/(1-\kappa)}^{\infty} \int\limits_x^{\infty} f_{|\hat{g}_{j,d}|^2||g_{j,d}|^2}(y|t) f_{|g_{j,d}|^2}(t)\,\mathrm{d}y\,\mathrm{d}t \\ &= -\int\limits_{\theta/(1-\kappa)}^{\infty} f_{|\hat{g}_{j,d}|^2||g_{j,d}|^2}(x|t) f_{|g_{j,d}|^2}(t)\,\mathrm{d}t \end{aligned} \quad (28)$$

The conditional PDF $f_{|\hat{g}_{j,d}|^2||g_{j,d}|^2}(x|t)$ can be found following the methodology in [9], [11], [39] as

$$f_{|\hat{g}_{j,d}|^2||g_{j,d}|^2}(x|t) = \frac{1}{\sigma_\epsilon^2}\exp\left(-\frac{x+t}{\sigma_\epsilon^2}\right) I_0\left(\frac{2}{\sigma_\epsilon^2}\sqrt{xt}\right), \quad (29)$$

where $I_0(\cdot)$ is zero-order Bessel function of imaginary argument [38, 8.447]. The second PDF can be expressed as

$$f_{|g_{j,d}|^2}(t) = \frac{1 + r_{j,d}^\alpha}{\sigma_h^2}\exp\left(-\frac{1 + r_{j,d}^\alpha}{\sigma_h^2}t\right). \quad (30)$$

Substituting the PDFs (29) and (30) into (28), the exact expression for the derivative of the intensity measure $\hat{\Lambda}_q(x)$ can be obtained as

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}x}\hat{\Lambda}_q(x) &= -\lambda \exp\left(-\frac{\theta}{\sigma_h^2\kappa} - \frac{x}{\sigma_\epsilon^2}\right) \int\limits_W \frac{1 + r_{j,d}^\alpha}{\sigma_h^2}\beta \\ &\times \exp\left(-\frac{\theta}{\sigma_h^2\kappa} r_{s,j}^\alpha + \beta\frac{x}{\sigma_e^2}\right) Q_1\left(\sqrt{\frac{2\beta x}{\sigma_e^2}}, \sqrt{\frac{1}{\beta\sigma_e^2}\frac{2\theta}{1-\kappa}}\right)\mathrm{d}w, \end{aligned} \quad (31)$$

where $Q_1(\cdot,\cdot)$ is the Marcum Q-function of first kind, and $\beta = \sigma_h^2/(\sigma_h^2 + \sigma_e^2(1 + r_{j,d}^\alpha))$.

Finally, the desired communication outage probability can be obtained numerically by substituting (24) and (31) into (20).

D. Asymptotic Analysis

To understand the asymptotic outage performance in the case of relay selection with imperfect CSI, let us consider the high-SNR behavior of the components $\hat{\Lambda}_d(x)$ and $\hat{\Lambda}_q(x)$ of the outage probability expression (20). When available transmission power P is large, the mean number Λ_q of relays in the qualified set Φ_q approaches the mean number Λ_d of relays in the decoding set Φ_d. Consequently, $\hat{\Lambda}_q(x) \to \hat{\Lambda}_d(x)$. Then, at high SNR, (20) can be rewritten as

$$P_o \approx 1 + \int\limits_0^{\infty} \exp\left(-\hat{\Lambda}_q(x)\right)\mathrm{d}\hat{\Lambda}_q(x) = \exp(-\Lambda_q). \quad (32)$$

Therefore, outage probability of imperfect CSI-based RS at high SNR approaches outage probability of perfect CSI-based selection. In the next section we provide numerical study of outage performance based on the relations derived in Sections III and IV.

V. Numerical Results and Discussion

In this section we consider outage performance of a system depicted in Fig. 1. Monte Carlo simulations with 10^6 iterations were conducted to model the outcome of message transmission via a DF relay, selected from a realization of the decoding set Φ_d. Objectives of these simulations are to (a) verify analytical results presented in Sections III and IV, and (b) investigate the outage behavior of DF relaying with random spatial distribution of relays as a function of available power P, power allocation κ and channel estimation error variance σ_ϵ^2. Analytical results in this section were obtained from expressions for outage probability using numerical integration in Matlab. For the case of SC with perfect CSI, we compare our results with [23, Eq(14a, 16)].

Two estimation error variance models are used as in [30], [32]. For the SNR-independent model, the estimation error variance $\sigma_\epsilon^2 = \{0.05, 0.1\}$, and for the power-dependent estimation error $\sigma_\epsilon^2 = 0.03 + 0.8/(1-\kappa)P$. Small-scale fading variance σ_h^2 is set to unity as in [32].

Three series of outage event simulations were conducted: (a) for different levels of total available power P and equal power allocation $\kappa = 0.5$ between the source and relay transmissions, (b) for a fixed $SNR = 20$ dB (where $SNR = P/\sigma_n^2$) and variable power allocation coefficient κ, and (c) for a fixed $SNR = 20$ dB and variable channel estimation error σ_ϵ^2. The source–destination distance is set as $r_{s,d} = 5$, candidate relay PPP intensity is $\lambda = 1$ relay per unit area in a cell with radius $R = 10$. In all three cases presented analytical results are fully corroborated by simulations. In the following we discuss results of each simulation scenario in detail.

Outage probability for the case of blind selection, where the retransmitting relay is selected randomly from the decoding set, are included to put the performance of SC in perspective. Analytical results for blind selection can be obtained following the same line of development as in [40].

Fig. 5 illustrates the outage probability behavior as a function of SNR. As expected, in the case of exact CSI available at relays, outage probability drops with growing speed as available power increases. This can be explained by the fact that with growing available power it becomes increasingly unlikely that none of the candidate relays has reliable connections both to the source and the destination. Note that our results are in good agreement with results in [23, Eqs. (14a, 16)]. On contrary, blind selection with no information about the relay–destination link leads to a very slow decay in outage probability as diversity gains become unavailable.

When the CSI at relays contains a random error, the relay selection outcome may be sub-optimal. The outage probability curves for different levels of the channel estimation error variance σ_ϵ^2 are shown in Fig. 5. One can observe that even for error variance of $\sigma_\epsilon^2 = 0.05$ outage probability deviates

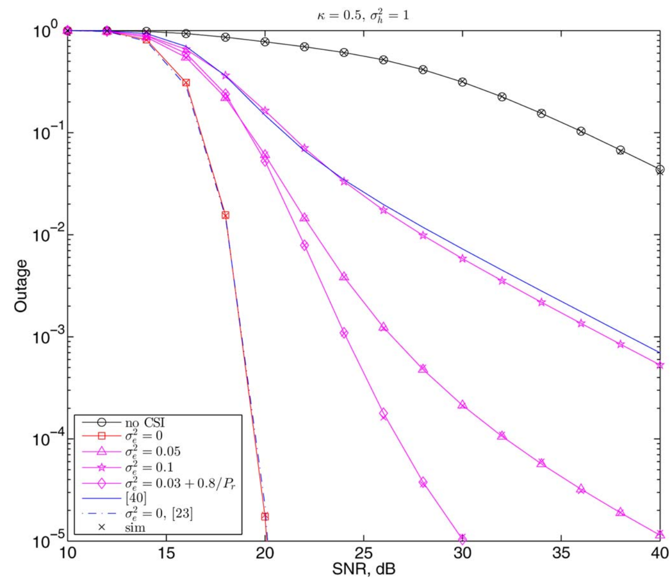

Fig. 5. Outage probability P_o as a function of SNR ($SNR = P/\sigma_n^2$) for equal power distribution between transmission stages.

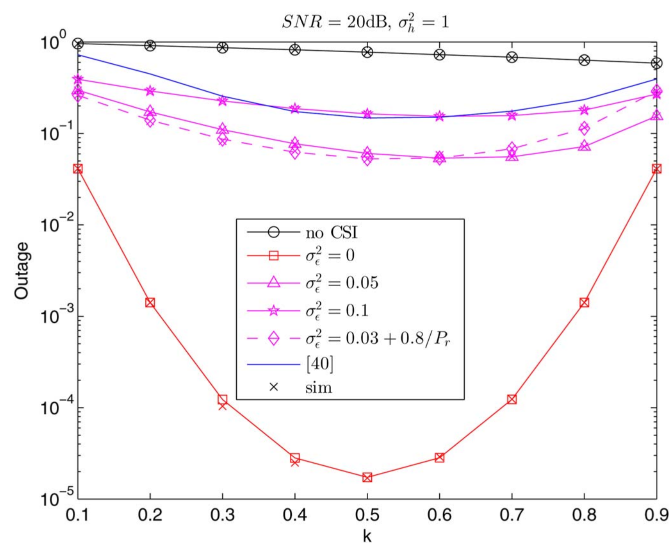

Fig. 6. Outage probability P_o as a function of power distribution coefficient κ for the SC strategy.

significantly from the curve for perfect CSI-based selection. For larger error variances, outage probability deteriorates significantly, so that for $\sigma_\epsilon^2 = 0.1$ erroneous CSI-based selection is equivalent to position-based selection (results from [40]). For the case of power-dependent error model, outage probability is high at lower SNR values, however as the measurement error component diminishes, outage behavior follows the case with power-independent error of $\sigma_\epsilon^2 = \sigma_u^2$.

Fig. 6 depicts the dependence of outage probability on power allocation coefficient κ. One can observe that by investing more power in source transmission, better performance in the case of blind relay selection can be achieved; however in the considered case of perfect CSI, equal power allocation between stages is optimal. In presence of channel estimation errors the role of power allocation is mixed, as pointed out in Section II-D. In particular, for the cases of $\sigma_\epsilon^2 = 0.05$ and $\sigma_\epsilon^2 = 0.1$, the minimum of outage probability is shifted to $\kappa > 0.5$, i.e., more

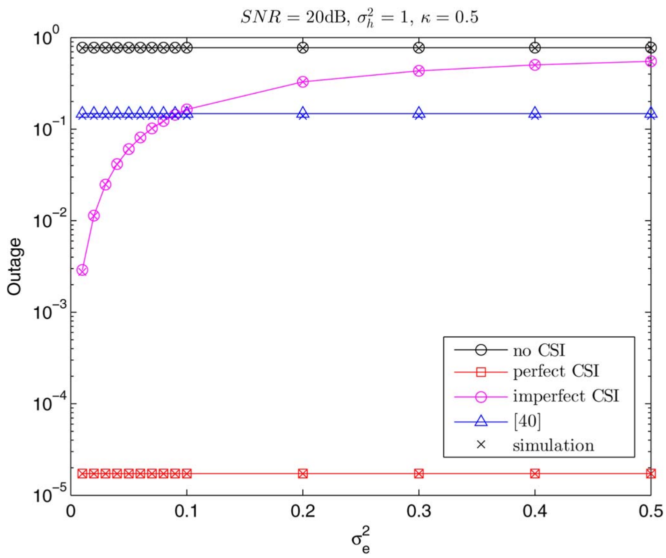

Fig. 7. Outage probability P_o as a function of channel estimation error variance σ_ϵ^2 for the SC strategy.

powerful source transmission is favorable. On the other hand, for $\sigma_\epsilon^2 = 0.03 + 0.8/(1-\kappa)P$, higher source power leads to an increase in the outage probability, which is due to larger contribution of the measurement error. In addition, as the decoding set becomes larger, more relays contend for retransmission based on the channel measurements to the destination. Then for large estimation error and large decoding set, more relays wrongly perceive themselves qualified for the retransmission. Conversely, with small error and larger source power, more relays can become truly qualified, while only few have their CSI corrupted enough to cause outage.

Fig. 7 illustrates the impact of the variance of channel estimation error on outage performance. In general, system performance deteriorates rapidly as the CSI estimation error variance σ_ϵ^2 increases, so that statistical CSI-based relay selection [40] outperforms noisy CSI-based selection from $\sigma_\epsilon^2 \approx 0.1$ onwards for the considered case. However, selection cooperation still provides lower outage rate compared to blind selection across a wide range of estimation error variances.

VI. Conclusion

In this paper we analyzed outage performance of selection cooperation strategy where the retransmitting DF relay was chosen using CSI, which is not necessarily perfect. Different from existing works on imperfect CSI-based relay selection, we explicitly considered network topology, and unlike current literature on RS in random networks, we considered the case of imperfect CSI. Using thinning operation on point processes allowed bypassing some analytical complexity involved in previous works, such as derivation of distance distributions. In this way, we obtained exact outage probability expressions for the considered RS scenario, and provided closed-form expressions for special cases of system parameters.

Asymptotic analysis has shown that in the case of perfect CSI, the diversity order rapidly increases, potentially up to the number of available relay candidates in the cell. Similar diversity behavior is present in the case of imperfect CSI-based selection, however, for SNR values, where most relay candidates become qualified for the retransmission. In low SNR region, the channel estimation error variance σ_ϵ^2 has been shown to significantly affect outage performance, so that for high σ_ϵ^2 outage rate becomes comparable to those of blind or position-based relay selection schemes.

The focus of this paper was on systems with channel reciprocity, applicable to TDD systems. In frequency division duplex (FDD) mode correlation between uplink and downlink channels gains need to be established, which may result in a different channel estimation error model. In the case of Gaussian error, presented results are technically applicable to FDD case, however an explicit account for FDD channel estimation specifics is left as future work.

Appendix

A. Proof of Proposition 1

Proof: Outage probability can be expressed as

$$\begin{aligned} P_o \approx \Pr(\mathcal{O}) &= \int_0^\infty \Pr(\mathcal{O}|M=x) f_M(x)\,\mathrm{d}x \\ &= 1 - \int_0^\infty \Pr\left(\Phi_u(x) = \emptyset | M = x\right) f_M(x)\,\mathrm{d}x, \end{aligned} \quad (33)$$

where $\mathcal{O}$ is defined in (8), and $M = \max_{j\in\Phi_q}(|\hat{g}_{i,d}|^2)$ is the maximal value of the function of the channel estimate $|\hat{g}_{i,d}|^2$ among all qualified relays in the process Φ_q. The estimate $|\hat{g}_{j,d}|^2$ at some unqualified relay $j \in \Phi_u$ may still be larger than M because it is a *perceived* channel state at the relay. In other words, the outage event in Proposition 1 corresponds to the case when there exists at least one unqualified relay, whose perceived estimate of the channel to the destination is greater than any of the perceived estimates at qualified relays, provided the decoding set is non-empty.

The probability density function (PDF) $f_M(x)$ can be written as

$$\begin{aligned} f_M(x) &= \frac{\mathrm{d}}{\mathrm{d}x}\Pr\left(\max_{j\in\Phi_q}\left(|\hat{g}_{i,d}|^2\right) < x\right) \\ &= \frac{\mathrm{d}}{\mathrm{d}x}\Pr\left(\Phi_q(x) = \emptyset\right). \end{aligned} \quad (34)$$

The PDF of the outcome of a general Poisson point processes Φ_i [29] is

$$\Pr\left(|\Phi_i| = k\right) = e^{-\Lambda_i}\frac{(\Lambda_i)^k}{k!}. \quad (35)$$

Hence substituting (34) into (33) and invoking (35) on both processes $\Phi_q(x)$ and $\Phi_u(x)$, we obtain the result in (13) as

$$\begin{aligned} P_o &= 1 - \int_0^\infty \Pr\left(\Phi_u(x) = \emptyset | x\right)\frac{\mathrm{d}}{\mathrm{d}x}\Pr\left(\Phi_q(x)\right)\mathrm{d}x \\ &= 1 + \int_0^\infty \exp\left(-\hat{\Lambda}_u(x) - \hat{\Lambda}_q(x)\right)\frac{\mathrm{d}}{\mathrm{d}x}\hat{\Lambda}_q(x)\,\mathrm{d}x. \end{aligned} \quad (36)$$

■

B. Proof of Proposition 2

Following the logic of SC strategy, we will first apply the first thinning stage with retention probability defined in (9) to obtain the process of relays connected to the source (the decoding set Φ_d), and then apply the second stage characterized in (11). Then the mean number of relays that are retained after these two thinning stages can be found as

$$\Lambda_q = \int\limits_W p_q(w)\Lambda_d(\mathrm{d}w)$$
$$\stackrel{a}{=} \lambda \int\limits_0^R \int\limits_0^{2\pi} r_{s,j} p_q(r_{s,j},\varphi) p_d(r_{s,j})\, \mathrm{d}r_{s,j}\, \mathrm{d}\varphi, \qquad (37)$$

where $\Lambda_d(\mathrm{d}w)$ is the mean number of relays in the decoding set in a small region $\mathrm{d}w$ of the cell W, and in step (a) $\Lambda_d(\mathrm{d}w) = \lambda_d(w)\cdot \mathrm{d}w = \lambda p_d(w)\mathrm{d}w$ was used [29].

Note that the probability of retention $p_q(r_{s,j},\varphi)$ in (37) is equivalent to $p_q(r_{j,d})$ in (11) with the distance $r_{j,d}$ represented as in (12). Then by substituting (9) and (11) into (37) we obtain the result in (16). Outage probability $P_{\mathrm{o,perf}}$ is then obtained from (14).

C. Proof of Corollary 3

Proof: For the case of $\alpha = 2$ the relation (16) reads as

$$\Lambda_q(\lambda, 2, R) = 2\pi\lambda e^{-\frac{\theta\left(\kappa^{-1}-r_{s,d}^2\right)}{1-\kappa}}$$
$$\times \int\limits_0^R r_{s,j} e^{-\frac{\theta r_{s,j}^2}{\kappa(1-\kappa)}} I_0\left(\frac{2\theta r_{s,j} r_{s,d}}{1-\kappa}\right) \mathrm{d}r_{s,j}, \qquad (38)$$

where [38, 3.364.2] was used. Introducing a change of variables $r_{s,j} = t\sqrt{\kappa(1-\kappa)/2\theta}$ in the last integral and after some algebra we obtain (17). ■

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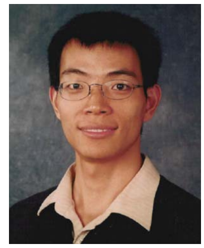

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