Successive Interference Cancellation in Heterogeneous Networks

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***Abstract*—At present, operators address the explosive growth of mobile data demand by densification of the cellular network so as to reduce the transmitter-receiver distance and to achieve higher spectral efficiency. Due to such network densification and the intense proliferation of wireless devices, modern wireless networks are interference-limited, which motivates the use of interference mitigation and coordination techniques. In this work, we develop a statistical framework to evaluate the performance of multi-tier heterogeneous networks with successive interference cancellation (SIC) capabilities, accounting for the computational complexity of the cancellation scheme and relevant network related parameters such as random location of the access points (APs) and mobile users, and the characteristics of the wireless propagation channel. We explicitly model the consecutive events of canceling interferers and we derive the success probability to cancel the n-th strongest signal and to decode the signal of interest after n cancellations. When users are connected to the AP which provides the maximum average received signal power, the analysis indicates that the performance gains of SIC diminish quickly with n and the benefits are modest for realistic values of the signal-to-interference ration (SIR). We extend the statistical model to include several association policies where distinct gains of SIC are expected: (i) maximum instantaneous SIR association, (ii) minimum load association, and (iii) range expansion. Numerical results show the effectiveness of SIC for the considered association policies. This work deepens the understanding of SIC by defining the achievable gains for different association policies in multi-tier heterogeneous networks.**

***Index Terms*—Successive interference cancellation, multi-tier heterogeneous network, stochastic geometry, association policy.**

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I. Introduction

SMALL cell networks are an important trend in current wireless networks that increase the density of transmitters and result in interference-limited networks where thermal noise is negligible with respect to the interference [1]. The motivation of small cell networks stems from the idea to reduce the distance between transmitter and receiver by deploying additional base stations to increase the spectral efficiency. Yet, as the network interference is the most important obstacle for successful communication, effective interference management schemes are essential to further enhance the performance of dense networks [2]–[4]. These mechanisms impose the orthogonality between transmitted signals in frequency, time, or space, and include adaptive spectrum allocation policies, (MAC) schemes, spatial interference mitigation by means of zero-forcing beamforming, and signal processing algorithms usually referred to as (IC) techniques [5]–[10].

Signal processing techniques such as (JD) or successive interference cancellation (SIC) reduce the interference power by decoding and canceling interfering signals [5]. In this work, we focus on the SIC receiver which decodes signals according to descending signal power and subtracts the decoded signal from the received multi-user signal, so as to improve the SIR. The process is repeated until the SoI is decoded. A common approach in literature is to consider an exclusion region around the receiver. The methodology based on the exclusion region leads to the definition of lower and upper bounds of the outage probability. In [6], [7] for instance, a stochastic geometric model is adopted to capture the spatial distribution of the interfering nodes, accounting for cancellation and decoding errors. The key idea of this work is the division into near field and far field interferers, where every near field interferer is able to cause outage at the reference receiver. Building on this work, [8] proposes bounds of the outage/success probability including the effects of the fading channel, while [10] includes accurately the consecutive steps of the SIC scheme. In [11], a Gaussian approximation is proposed for the sum of interfering signals, while [12] performs an asymptotic analysis of the interference distribution in cognitive radio networks. None of these works concerns a specific cancellation technique, since the order statistics of the received signal power are disregarded, which is an essential aspect in the analysis of SIC. The ordering of the received signal power depends on the transmission power of the network nodes, the spatial distribution of the active transmitters, and the propagation channel conditions.

Specifically, the inclusion of SIC in the network performance analysis requires to characterize the fundamental information-theoretic metric, the SIR, and model the network interference as a trimmed sum of order statistics. In [9], closed-form expressions are presented for the outage probability accounting for the order statistics, assuming that all interferers are at the same distance from the intended receiver, while [13] derives a lower bound of the outage probability based on the order statistics of the strongest uncanceled and partially canceled signals accounting for distance and fading. Recently, some information theoretic contributions can be found that include both topology and fading effects in the description of sum of order statistics of the received signal power [14], [15].

A unified approach to describe the performance of SIC, which jointly accounts for the interference cancellation scheme, network topology, channel fading, and the specific aspects of multi-tier networks, is still evasive. In this work, we develop an analytical framework that describes the success probability for transmissions in multi-tier networks with SIC capabilities, accounting for different association policies. The main contributions of this work are listed as follows.

- We show that the order statistics of the received signal power are dominated by path loss attenuation and derive the probability of successfully canceling the n-th interferer. Our proposed framework reflects how the effectiveness of the SIC scheme depends on the path loss exponent, the density of users and APs, and the maximum number of cancellations.
- Unlike previous work, we provide an analytic framework that is flexible enough to adjust to the network heterogeneity that characterizes future wireless networks. In this regard, we include different association policies for multi-tier networks for which SIC yields distinct performance gains. In particular, we include the maximum instantaneous SIR policy, the minimum load association policy, and range expansion in the analysis.

The proposed framework accounts for all essential network parameters and provides insight in the achievable gains of SIC in multi-tier heterogeneous networks. The maximum instantaneous SIR policy is relevant to define the achievable capacity in multi-tier networks [16], the minimum load policy can be used to enhance the feasibility of load balancing [17], and range expansion with SIC capabilities can allow for efficient traffic offloading [18].

The remainder of the paper is organized as follows. In Section II, the system model is introduced. In Section III, the success probability of transmissions in multi-tier heterogeneous networks with SIC capabilities is defined. In Section IV, several association policies different from the maximum average SINR policy are introduced, where SIC gives rise to a substantial increase in the success probability or the rate distribution. Conclusions are drawn in Section V.

II. System Model

We consider a multi-tier heterogeneous network composed of K tiers. For every tier $k \in \mathcal{K} = \{1,\dots,K\}$, the access points (APs) are distributed according to a homogeneous Poisson point process (PPP) Φ_k in the Euclidean plane with density λ_k such that $\Phi_k \sim \mathrm{PPP}(\lambda_\mathrm{k})$. While it is natural to use the Poisson model as the underlying spatial stochastic process for irregularly deployed APs such as picocells and femtocells, modeling the location of regularly deployed macrocell base stations (MBSs) by means of a PPP has been empirically validated and yields conservative bounds on the network performance [19]. More recently, also theoretical evidence has been given for modeling the deterministic locations of MBSs by means of a PPP, provided there is sufficiently strong log-normal shadowing [20]. All APs apply an open access policy, such that users can be served by each AP of each tier. Each AP of tier k transmits with power P_k over the total bandwidth W. The total available spectrum W is divided in subchannels by aggregating a fixed number of consecutive subcarriers of bandwidth B, such that the total number of available subchannels equals $\lfloor W/B \rfloor$.[1] We denote the subchannel index as j, where $j \in \mathcal{J} = \{1,2,\dots,\lfloor W/B \rfloor\}$. To maximize frequency reuse and throughput, each AP has access to the entire available spectrum. We represent the i-th AP of tier k as $x_{k,i}$. Hence, denoting the available channels of $x_{k,i}$ as $\mathcal{J}^{(x_{k,i})}$, we have $\mathcal{J}^{(x_{k,i})} = \mathcal{J}, \forall i,k$. A user receives a signal from $x_{k,i}$ with signal power $P_k h_{x_{k,i}} g_\alpha(u - x_{k,i})$, where $h_{x_{k,i}}$ represents the power fading coefficient for the link between the user u and $x_{k,i}$, and $g_\alpha(x) = \|x\|^{-\alpha}$ is the power path loss function with path loss exponent α. For notational convenience, u and x will be used to denote network nodes as well as their location. We consider an orthogonal multiple access scheme, where fairness between users is accomplished by proportional allocation of the time and frequency resources. As the multiple access scheme precludes more than one user to be active on channel j within the same cell, the locations of active mobile users and APs are coupled. It can be shown that this dependence has negligible effects on the performance analysis [21]. Therefore, in the sequel we will assume independent PPPs for APs and active users to maintain the tractability of the system model. Considering a K-tier network, each tier $k \in \mathcal{K}$ is characterized by the (UL) transmission power Q_k, DL transmission power P_k, AP density λ_k, and associated user density μ_k. The sets of transmission powers and densities are denoted as $\mathbf{Q} = \{Q_1,\dots,Q_K\}$, $\mathbf{P} = \{P_1,\dots,P_K\}$, $\lambda = \{\lambda_1,\dots,\lambda_K\}$, and $\mu = \{\mu_1,\dots,\mu_K\}$, respectively. As interference dominates noise in modern cellular networks, we consider the network to be interference-limited. For the link between user u and base station $x_{k,i}$, we define the signal-to-interference ratio (SIR) on channel j in DL as

$$\mathrm{SIR}_j(x_{k,i} \to u) = \frac{P_k h_\mathrm{u} g_\alpha(x_{k,i} - u)}{\sum_{k \in \mathcal{K}} \sum_{v \in \Phi_{k,j} \setminus \{x_{k,i}\}} P_k h_v g_\alpha(v - u)}, \tag{1}$$

where $\Phi_{k,j}$ denotes the point process of APs active on channel j in tier k, and h_u represents the channel between the user and the transmitter of interest. In interference-limited networks, a transmission is successful if the SIR of the intended link exceeds a prescribed threshold η_t, which reflects the required quality-of-service (QoS) in terms of transmission rate. Hence, the success probability can be written as $\mathbb{P}_\mathrm{s}(\eta_\mathrm{t}) = \Pr\{\mathrm{SIR}_j(x_{k,i} \to u) \geq \eta_\mathrm{t}\}$.

[1] Without loss of generality, we assume $B = 1$.

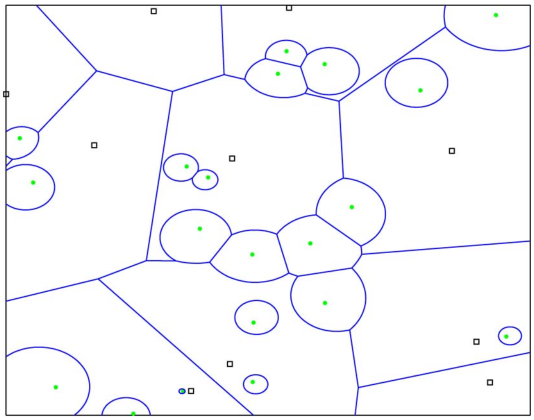

Fig. 1. Multiplicatively weighted Voronoi tessellation for a two-tier network. Macrocell base stations and small cell access points are represented by hollow squares and green circles, respectively. The association region for macrocell and small cell base stations are delimited by the blue lines such that the plot represents the coverage area of each access point.

III. SUCCESSIVE INTERFERENCE CANCELLATION

In this section, we study how SIC affects the success probability in multi-tier heterogeneous networks. The analysis will be presented for UL transmissions but the results similarly apply for DL transmissions. This choice is motivated by the higher computational capabilities of APs in comparison with the mobile nodes, which is essential to successfully implement advanced signal processing techniques, as well as power consumption considerations which are less restrictive for APs. The concept of SIC is to decode the strongest signal and subtract it from the incoming signal which yields an increase of the SIR. In the analysis, we explicitly model the sequence of events in the cancellation process. We define the success probability as a function of the threshold, the number of canceled interferers, and all relevant system parameters such as the interferer density, transmission power, path loss exponent, and channel fading.

The notion of user association is essential to define the set of interfering nodes and the distance distribution to the serving AP. The association of the user u to the AP $x_{k,i}$ is based on an association metric

$$(l,j) = \arg\max_{k,i} A_k \|u - x_{k,i}\|^{-\alpha}, \tag{2}$$

where A_k reflects the association rule. If all $A_k = 1$, the user is associated to the nearest base station. We consider the generic association policy $A_k = P_k$, such that the association is based on the maximum average received signal power, where averaging is done with respect to the fading parameter h_u. Using this association rule, the set of APs forms a multiplicatively weighted Voronoi tessellation on the two dimensional plane, where each cell $C_{k,i}$ consists of those points which have a higher average received signal power from $x_{k,i}$ than from any other AP, as depicted in Fig. 1. Formally, we define the cells as

$$C_{k,i} = \Big\{ y \in \mathbb{R}^2 |\ \|y - x_{k,i}\| \le (A_k/A_l)^{1/\alpha} \|y - x\|, \\ \forall x \in \Phi_l \setminus \{x_{k,i}\},\ l \in \mathcal{K} \Big\}. \tag{3}$$

Within a Voronoi cell, mobile users are independently and uniformly distributed over the cell area. According to the association rule in (2), users will connect to different tiers and active users on channel j connected to tier k can be modeled as $\Psi_{k,j} \sim \mathrm{PPP}(\mu_k)$. We consider a fully loaded network, which ensures that at any given time and channel, only a single user is active per cell, such that $\mu_k = \lambda_k$.

Owing to constraints on computational complexity and delay, the number of interferers that can be canceled is limited to $N \in \mathbb{N}$.[2] The received signal power at the typical AP can be ordered as $\{X_{(1)}, X_{(2)}, \ldots\}$ such that $X_{(i)} \ge X_{(j)}$, with $i \le j$ and $X_{(i)} = Q_i h_i v_i^{-\alpha}$. The AP with SIC capabilities attempts first to decode the SoI without any interference cancellation. If an outage occurs, the AP seeks to decode the strongest signal, subtract it from the incoming signal, and performs a new attempt to decode the SoI [10]. The same actions are repeated until the SoI is decoded while satisfying the constraint on the maximum number of cancellations. Hence, UL transmission is successful as long as one of the following events is successful:

$$\begin{aligned}
0 &: \left(\frac{Q_u h_\mathrm{u} u^{-\alpha}}{I_{\Omega_j^0}} \ge \eta_\mathrm{t} \right) \\
1 &: \left(\frac{Q_u h_\mathrm{u} u^{-\alpha}}{I_{\Omega_j^0}} < \eta_\mathrm{t} \right) \bigcap \left(\frac{X_{(1)}}{I_{\Omega_j^1}} \ge \eta_\mathrm{t} \right) \bigcap \left(\frac{Q_u h_\mathrm{u} u^{-\alpha}}{I_{\Omega_j^1}} \ge \eta_\mathrm{t} \right) \\
&\vdots \\
N &: \left(\bigcap_{n=0}^{N-1} \frac{Q_u h_\mathrm{u} u^{-\alpha}}{I_{\Omega_j^n}} < \eta_\mathrm{t} \right) \bigcap \left(\bigcap_{n=1}^{N} \frac{X_{(n)}}{I_{\Omega_j^n}} \ge \eta_\mathrm{t} \right) \\
&\quad \bigcap \left(\frac{Q_u h_\mathrm{u} u^{-\alpha}}{I_{\Omega_j^N}} \ge \eta_\mathrm{t} \right),
\end{aligned} \tag{4}$$

where the set of interferers on subchannel j after cancellation of the n strongest interferers is represented by $\Omega_j^n = \Omega_j \setminus \{X_{(1)}, \ldots, X_{(n)}\}$, and $\Omega_j = \cup_{k \in \mathcal{K}} \Psi_{k,j} \setminus \{u\}$. The aggregate interference after cancellation is given by

$$I_{\Omega_j^n} = \sum_{i=n+1}^{\infty} X_{(i)}. \tag{5}$$

The first and third factor in the n-th event of (4) represent outage and success for decoding the SoI when $n-1$ and n interferers are canceled, respectively. The second factor in the n-th event of (4) represents the event of successfully canceling the n-th interferer. The cancellation order is based on the received signal power and is independent of the tier to which the interferers belong. Since $X_{(i)}$ can originate from different tiers with different transmission power Q_i, (5) represents the sum of order statistics, where the transmission power, fading parameter, and path loss component are random variables (r.v.'s). Interference cancellation is inevitably imperfect due to inaccuracies in the channel estimation at the receiver and the corresponding imprecise interference reconstruction. Note that (4) and (5) reflect the assumption of perfect interference cancellation in our model, which has also been assumed in prior work [8], [9], [14]. Furthermore, the receiver requires the knowledge of the signal signature and coding scheme, which requires a limited amount

[2]For a precise analysis of the computational complexity of SIC in single-antenna and multiple-antenna OFDM systems, we refer to [22].

of side information feedback. We consider in this work an uncoded system, an assumption that has been adopted in the relevant prior literature [6]–[9], [13], [14].

Remark 1: Due to the displacement theorem ([23], Theorem 1.10), a generic heterogeneous network of transmitters can be represented by an equivalent network where all system parameters such as the transmission power, fading parameter, and path loss exponent are set to constants, while the determinative parameter is an isotropic (possibly non-homogeneous) transmitter density [24], [25]. For a constant path loss exponent, the isotropic density of the equivalent network reduces to a homogeneous value. Applying Campbell's theorem, in a fully loaded network the total set of transmitters active on channel j can be represented by the stochastic equivalent network with corresponding density $\mu = \sum_{k\in\mathcal{K}} \lambda_k Q_k^{2/\alpha}$ and transmission power equal to one.[3] This generalizes previous results on the characterization of network interference by means of a stable distribution, where the interferer density in the expression of the dispersion is scaled with one specific moment of the transmission power and the fading distribution [26].

In the remaining part of Section III, we will make use of the equivalent network as introduced in Remark 1. In Section III-A, we define the probability of successfully decoding the SoI after canceling n interferers and in Section III-B, the success probability of canceling the n-th interferer is derived.

A. Success Probability After Interference Cancellation

We define the success probability of a link in an interference-limited network after successfully canceling n interferers as

$$\begin{aligned}\mathbb{P}_{\mathrm{s,IC}}(\eta_{\mathrm{t}}, n) &= \Pr\left[\sum_{i=n+1}^{\infty} X_{(i)} < Q_{\mathrm{u}} h_{\mathrm{u}} u^{-\alpha}/\eta_{\mathrm{t}}\right] \\ &= \Pr\left[I_{\Omega_j^n} < Q_{\mathrm{u}} h_{\mathrm{u}} u^{-\alpha}/\eta_{\mathrm{t}}\right]. \end{aligned} \tag{6}$$

Note that the calculation of the success probability for decoding the SoI is based on the distribution of the sum of order statistics, which requires the joint distribution of infinitely many r.v.'s. There are several possibilities to handle this problem, which have in common to limit the summation of order statistics to the M strongest interferers, generally denoted as a trimmed sum. Since order statistics are mutually dependent, the cumulative distribution function (CDF) of their sum is hard to characterize. The CDF of the sum of order statistics can be found by the inverse Laplace transform of the moment generating function (MGF). In case of exponentially distributed r.v.'s, it can be shown that the transformation of the dependent order statistics to the spacing between the order statistics results in independent r.v.'s, which alleviates the complexity and allows to calculate the MGF [27]. Exponential r.v.'s appear in case of Rayleigh fading with all interferers at equal distance [9], or considering the difference of ordered square distances in a PPP without fading [28]. However, the computational complexity to calculate the CDF of a trimmed sum of order statistics including both topology and fading is prohibitive. An alternative promising approach to characterize the sum of order statistics is to consider the asymptotic distribution of the sum $T_M(m,k) = \sum_{i=m}^{M-k} X_{(i)}$, where a fixed number $k \geq 0$ of the smallest values and a fixed number $m \geq 0$ of the largest values is trimmed, denominated as lightly trimmed sums [27], [29].

In our model, we include the effects of both fading and topology, yet, we assume that the order statistics are dominated by the distance. This can be understood by considering that the order statistics of the distance outweigh the fading effects, which have an effect on a much shorter time scale. There is a more formal motivation why we assume that the order statistics are dominated by the distance rather than the fading distribution. The following lemma, which defines the distribution of the received signal power, makes this assumption more rigorous.

Lemma 1: Let X represent the distance between a uniformly distributed node in a circular area and the origin, and consider the Gamma r.v. $h \sim \Gamma(m, \theta)$, then $Y = hX^{-\alpha}$ follows a Pareto distribution with CDF

$$F_Y(y) = 1 - \frac{\Gamma(2/\alpha + m)}{\Gamma(m)} \frac{(my)^{-2/\alpha}}{R^2} \tag{7}$$

where R is the maximum considered range for the position of interferers.

Proof: See Appendix A. □

Lemma 1 shows that the distribution of the received signal power is not affected under the class of Nakagami-m fading. Note that $X^{-\alpha}$ follows a Pareto distribution, which belongs to the class of subexponential distributions. According to the theorem of Breiman, which states that the class of subexponential distributions is closed under the product convolution, the multiplication of $X^{-\alpha}$ with the gamma r.v. h does not change the distribution [30]. As compared to the distribution of $X^{-\alpha}$ in the proof of Lemma 1, we observe in (7) that the multiplication of $X^{-\alpha}$ with the fading power h has only effect on the parameter of the Pareto distribution. For the special case where $\alpha = 2$ and $m = 1$, i.e., Rayleigh fading, we note that the distribution of the received signal power remains unaffected by the multiplication with h. Considering the class of Nakagami-m fading, Lemma 1 indicates that the order statistics of the received signal power are dominated by the distance.

Remark 2: In the remainder, we will often make use of integrals of the form $\int 1/(1 + w^{\alpha/2})\mathrm{d}w$. For the integration interval $[b, \infty)$, we define

$$\begin{aligned} C(b,\alpha) &= \int_b^{\infty} \frac{1}{1 + w^{\alpha/2}}\,\mathrm{d}w \\ &= 2\pi/\alpha \csc(2\pi/\alpha) - b\,{}_2F_1\left(1, 2/\alpha; (2+\alpha)/\alpha; -b^{\alpha/2}\right), \end{aligned} \tag{8}$$

where ${}_2F_1(.)$ is the Gaussian hypergeometric function. For special cases, we have $C(0,\alpha) = 2\pi/\alpha \csc(2\pi/\alpha)$ and $C(b,4) = \arctan(1/b)$.

Lemma 2: A mobile user is connected to a typical AP, which has successfully canceled n interferers. In case of Rayleigh fading, the success probability of UL transmission in the presence of network interference is given by (9), shown at the bottom of the next page, where $R_I(n)$ is the cancellation radius that defines the area around the victim receiver without interferers.

[3] In this work, we assume a constant path loss exponent α for all tiers.

Proof: We define the UL coverage probability conditioned on the distance of the intended link after successfully canceling n interferers as

$$\begin{aligned}\mathbb{P}_{\mathrm{s,IC}}(\eta_{\mathrm{t}},n|\,u) &= \mathbb{P}^{!u}\left\{\frac{h_{\mathrm{u}}u^{-\alpha}}{\sum_{v\in\Omega_j^n\setminus\{u\}} h_v v^{-\alpha}} \geq \eta_{\mathrm{t}}\right\}\\ &\overset{(a)}{=} \mathbb{E}_{I_{\Omega_j^n}}\left\{\mathbb{P}\left[h_{\mathrm{u}} > \eta_{\mathrm{t}}u^{\alpha}I_{\Omega_j^n}\right]\right\}\\ &\overset{(b)}{=} \mathbb{E}_{I_{\Omega_j^n}}\left\{\exp(-\eta_{\mathrm{t}}u^{\alpha}I_{\Omega_j^n}\right\}\\ &= \mathcal{L}_{I_{\Omega_j^n}}(\eta_{\mathrm{t}}u^{\alpha}), \end{aligned} \quad (10)$$

where $\mathbb{P}^{!u}$ is the reduced Palm distribution conditioned on the user u, (a) holds because of Slivnyak's theorem, and where (b) assumes a Rayleigh fading channel. The Laplace transform of $I_{\Omega_j^n}$ is denoted as $\mathcal{L}_{I_{\Omega_j^n}}(s)$. Similar to [19], the Laplace transform of the partially canceled interference is obtained by applying the probability generating functional (PGFL), and assuming that the order statistics are dominated by the distance, the conditional coverage probability in (10) can be written as

$$\mathbb{P}_{\mathrm{s,IC}}(\eta_{\mathrm{t}},n|u) = \exp\left(-2\pi\mu\int_{R_{\mathrm{I}}(n)}^{\infty}\frac{v}{1+\frac{v^{\alpha}}{\eta_{\mathrm{t}}u^{\alpha}}}\,\mathrm{d}v\right), \quad (11)$$

where $R_{\mathrm{I}}(n)$ is the distance from the origin to the n-th interferer. By change of variable $w = v^2/(\eta_{\mathrm{t}}^{2/\alpha}u^2)$, we can express (11) as

$$\begin{aligned}\mathbb{P}_{\mathrm{s,IC}}(\eta_{\mathrm{t}},n|u) &= \exp\left(-\pi\mu\eta_{\mathrm{t}}^{2/\alpha}u^2\int_{b(u)}^{\infty}\frac{1}{1+w^{\alpha/2}}\,\mathrm{d}w\right)\\ &= \exp\left(-\pi\mu\eta_{\mathrm{t}}^{2/\alpha}C(b(u),\alpha)u^2\right), \end{aligned} \quad (12)$$

where $b(u) = R_{\mathrm{I}}(n)^2/\eta_{\mathrm{t}}^{2/\alpha}u^2$. Considering the association rule based on the maximum average received signal power, each user in the equivalent network connects to the closest AP and hence, we take the expectation over the distance D between the typical AP and the associated user with PDF given by $f_D(u) = 2\mu\pi u\exp(-\mu\pi u^2)$ [31]. As the SIC procedure cancels at each step the signal with the strongest power and in view of the distance dominated order statistics, we have that $u \in [R_I(n),\infty)$.The integration interval and the function $C(b(u),\alpha)$ depend on $R_{\mathrm{I}}(n)$, which has the following probability density function (PDF) [31]

$$f_{R_{\mathrm{I}}(n)}(r) = \exp(-\mu\pi r^2)\frac{2(\mu\pi r^2)^n}{r\Gamma(n)}. \quad (13)$$

The unconditional success probability can be found by taking the expectation over $R_I(n)$, which yields (9) and concludes the proof. □

To reduce the computational complexity, $R_{\mathrm{I}}(n)$ can be approximated by the cancellation radius $R_{\mathrm{I},n} = \sqrt{n/\mu\pi}$, which encloses on average n mobile users.

B. Interference Cancellation

In the following, the success probability to cancel the n-th strongest signal is derived, assuming that the interference from the $n-1$ strongest signals has been previously canceled. This result can be achieved by (i) building on the PGFL and distance dominated order statistics, or (ii) using the truncated stable distribution (TSD). The approach based on the PGFL is more general, while the approach based on the TSD gives more insight in terms of the convergence of the interference distribution.

Probability Generating Functional Approach:

Lemma 3: Considering a typical AP that successfully canceled the $n-1$ strongest signals, the success probability to cancel the n-th strongest signal is given by

$$\mathbb{P}_{\mathrm{s,can}}(\eta_{\mathrm{t}},n) = \frac{1}{\left(1+\eta_{\mathrm{t}}^{2/\alpha}C\left(1/\eta_{\mathrm{t}}^{2/\alpha},\alpha\right)\right)^n}. \quad (14)$$

Proof: In the SIC scheme, interferers are canceled according to descending signal power. We consider the order statistics $X_{(i)}$ of the received signal power to define the probability of successfully decoding the n-th strongest signal. After successfully decoding and subtracting $n-1$ signals from the received signal, the success probability can be written as

$$\Pr\left[\frac{X_{(n)}}{\sum_{i\in\Omega_j^n}X_{(i)}} \geq \eta_{\mathrm{t}}\right] = \Pr\left[\sum_{i=n+1}^{\infty}X_{(i)} \leq X_{(n)}/\eta_{\mathrm{t}}\right]. \quad (15)$$

Building on Lemma 1, the order statistics of the received signal power are dominated by the distance, such that $X_{(j)} \geq X_{(i)}$ with $i < j$ is equivalent to $v_j \leq v_i$. When the SoI is the strongest signal of the received multi-user signal corresponding to distance v_j, then the remaining interferers will be located in the interval $[v_j,\infty)$. This leads to a remarkable simplification for the calculation of the probability of successfully decoding the n-th interferer conditioned on v_n, which can be expressed as

$$\begin{aligned}\mathbb{P}_{\mathrm{s,can}}(\eta_{\mathrm{t}},n|v_n) &= \Pr\left(\frac{X_{(n)}}{I_{\Omega_j^n}} \geq \eta_{\mathrm{t}}\right)\\ &= \exp\left(-\pi\mu\eta_{\mathrm{t}}^{2/\alpha}v_n^2\int_{R_{\mathrm{I}}(n)^2/(\eta^{2/\alpha}v_n^2)}^{\infty}\frac{1}{1+w^{\alpha/2}}\,\mathrm{d}w\right)\\ &= \exp\left(-\pi\mu\eta_{\mathrm{t}}^{2/\alpha}v_n^2C(1/\eta_{\mathrm{t}}^{2/\alpha},\alpha)\right). \end{aligned} \quad (16)$$

$$\mathbb{P}_{\mathrm{s,IC}}(\eta_{\mathrm{t}},n) = \mathbb{E}_{R_I(n)}\left[\int_{R_I(n)}^{\infty}\exp\left(-\pi\mu\eta_{\mathrm{t}}^{2/\alpha}u^2C\left(R_{\mathrm{I},n}^2/\left(\eta_{\mathrm{t}}^{2/\alpha}u^2\right),\alpha\right)\right)2\pi\mu u\exp(-\mu\pi u^2)\,\mathrm{d}u\right] \quad (9)$$

Note that the residual interferers are located outside the circular area with radius v_n, where $R_{\rm I}(n) = v_n$ and the function $C(1/\eta_{\rm t}^{2/\alpha}, \alpha)$ is independent of v_n. Using (13) and (16), we get

$$\begin{aligned}\mathbb{P}_{\rm s,can}(\eta_{\rm t}, n) &= \int_0^\infty \exp\left(-\pi\mu\eta_{\rm t}^{2/\alpha} C\left(1/\eta_{\rm t}^{2/\alpha}, \alpha\right) v_n^2\right) \\ &\quad \times \exp\left(-\pi\mu v_n^2\right) \frac{2\left(\pi\mu v_n^2\right)^n}{v_n \Gamma(n)} \, dv_n \\ &= \frac{(\pi\mu)^n}{\Gamma(n)} \int_0^\infty \exp\left(-\pi\mu\left(1+\eta_{\rm t}^{2/\alpha} C\left(1/\eta_{\rm t}^{2/\alpha}, \alpha\right)\right) v_n^2\right) \\ &\qquad v_n^{2n-2} \, dv_n^2 \\ &= \frac{(\pi\mu)^n}{\Gamma(n)} \int_0^\infty \exp\left(-\pi\mu\left(1+\eta_{\rm t}^{2/\alpha} C\left(1/\eta_{\rm t}^{2/\alpha}, \alpha\right)\right) w\right) \\ &\qquad w^{n-1} \, dw \\ &= \frac{(\pi\mu)^n}{\Gamma(n)} \left(\pi\mu\left(1+\eta_{\rm t}^{2/\alpha} C\left(1/\eta_{\rm t}^{2/\alpha}, \alpha\right)\right)\right)^{-n} \Gamma(n) \\ &= \frac{1}{\left(1+\eta_{\rm t}^{2/\alpha} C\left(1/\eta_{\rm t}^{2/\alpha}, \alpha\right)\right)^n}. \end{aligned} \tag{17}$$

□

It is worthwhile to note that (17) is independent of μ, which can be understood by the opposing effects of the decreasing distance to the n-th interferer and the increasing aggregate interference as a function of the user density. This is in line with [14] where the authors prove that the probability to successfully cancel at least n interferers is scale invariant with respect to the density as long as the analysis is restricted to the power-law density case.

1) Truncated Stable Distribution Approach: In the unbounded path loss model $g_\alpha(x) = \|x\|^{-\alpha}$, the aggregate interference power generated by the interfering nodes scattered over $\mathbb{R}^2$ can be modeled by a skewed stable distribution $I_{\Omega_j} \sim \mathcal{S}(\alpha_I = 2/\alpha, \beta = 1, \gamma = \pi\lambda C_{2/\alpha}^{-1} \mathbb{E}\{P_i^{2/\alpha}\})$ [26]. Since the singularity at 0 in the unbounded path loss model can have significant effects on the interference distribution [32], a bounded path loss model based on the truncated stable distribution (TSD) has been proposed to avoid the singularity for zero distance by restricting the interferers to a ring structure with a minimum range $d_{\min}$ and maximum range $d_{\max}$ [33]. The ring structure can be applied similarly to represent the guard zone around the victim receiver in the SIC scenario, where the inner radius corresponds to the interference cancellation radius and $d_{\max} = \infty$. In the bounded path loss model, the aggregate interference power is distributed according to a skewed TSD with CF given by [33]

$$\psi_{I_{\Omega_j^n}}(j\omega) = \exp\left(\gamma' \Gamma(-\alpha_I)\left[(g - j\omega)^{\alpha_I} - g^{\alpha_I}\right]\right), \tag{18}$$

where the parameters α_I, g, and γ' determine the shape of the TSD.

Lemma 4: For $\alpha = 4$, the success probability to cancel the n-th strongest signal after $n-1$ successful cancellations is given by

$$\mathbb{P}_{\rm s,can}(\eta_{\rm t}, n|\alpha = 4) = \frac{1}{(\sqrt{9/4 + 3\eta_{\rm t}} - 1/2)^n}. \tag{19}$$

Proof: See Appendix B. □

Note that (19) has the same structure as (17) and similarly shows that the probability of canceling the n-th interferer is independent of the interferer density.

Remark 3: The use of the TSD approach is beneficial since it can provide insight how fast the aggregate network interference converges to a Gaussian distribution by calculating the kurtosis after canceling n interferers. Conditioned on the exclusion radius r, we have

$$\gamma_2 = \frac{\kappa(4)}{\kappa(2)^2} = \frac{6(\alpha-1)^2}{(2\alpha-1)} \frac{1}{\mu\pi r^2}, \tag{20}$$

where $\gamma_2 = 0$ represents the case of a normal distribution. By averaging over r, the kurtosis after canceling n interferers is given by

$$\begin{aligned}\gamma_2(\alpha, n) &= \frac{6(\alpha-1)^2}{(2\alpha-1)} \int_0^\infty \frac{1}{\mu\pi r^2} \exp(-\mu\pi r^2) \frac{2(\mu\pi r^2)^n}{r\Gamma(n)} \, dr \\ &= \frac{6(\alpha-1)^2}{(2\alpha-1)} \frac{1}{\Gamma(n)} \int_0^\infty \exp(-\mu\pi r^2)\left(\mu\pi r^2\right)^{n-2} \, d\mu\pi r^2 \\ &= \frac{6(\alpha-1)^2}{(2\alpha-1)} \frac{\Gamma(n-1)}{\Gamma(n)} \\ &= \frac{6(\alpha-1)^2}{(2\alpha-1)} \frac{1}{n-1}. \end{aligned} \tag{21}$$

This expression shows that the distribution of the aggregate network interference converges logarithmically to a Gaussian distribution with the number of canceled interferers.

C. Success Probability With SIC

Theorem 1: The coverage probability $\mathbb{P}_{\rm s,SIC}$ for a receiver that applies SIC with a maximum of N interferers being canceled is given by (22), shown at the bottom of the page, where $\mathbb{P}_{\rm s,IC}(\eta_{\rm t}, n)$ and $\mathbb{P}_{\rm s,can}(\eta_{\rm t}, n)$ are given in (9) and (14), respectively.

Proof: Assuming independence between the events in (4), the proof follows directly from the sequence of events defined in (4) and the results of $\mathbb{P}_{\rm s,IC}$ and $\mathbb{P}_{\rm s,can}$ in Lemma 2 and 3. □

In the following, we validate the approximations that have been assumed and we provide some numerical results that illustrate the effectiveness of SIC for the association policy based on the maximum average received signal power. We consider a network of APs arranged over a two-dimensional plane with density $\lambda_{\rm m} = 10^{-4}/{\rm m}^2$. We assume a fully loaded

$$\mathbb{P}_{\rm s,SIC}(\eta_{\rm t}, N) = \mathbb{P}_{\rm s,IC}(\eta_{\rm t}, 0) + \sum_{i=1}^{N} \left(\prod_{n=0}^{i-1} (1 - \mathbb{P}_{\rm s,IC}(\eta_{\rm t}, n))\right) \left(\prod_{n=1}^{i} \mathbb{P}_{\rm s,can}(\eta_{\rm t}, n)\right) \mathbb{P}_{\rm s,IC}(\eta_{\rm t}, i) \tag{22}$$

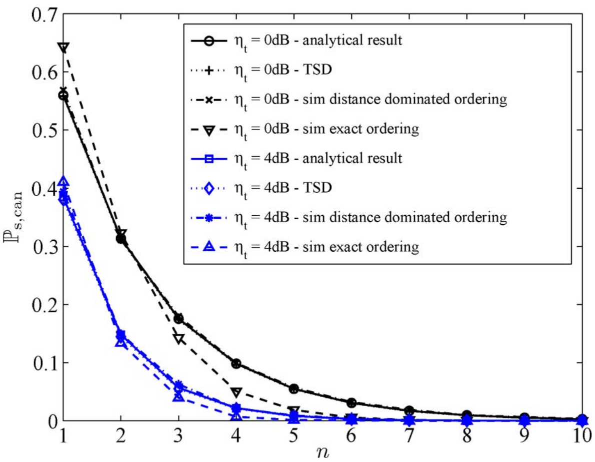

Fig. 2. Comparison of $\mathbb{P}_{s,can}$ using analytical and simulation results.

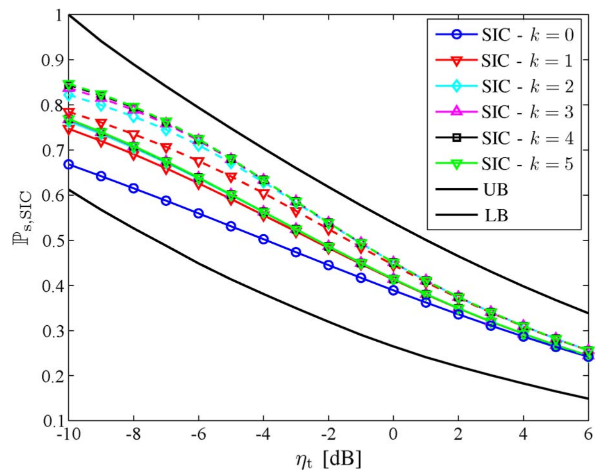

Fig. 3. Coverage probability in the presence of SIC for different values of the maximum number of cancellations. The blue curve represents the success probability when no IC technique is used. Theoretical results are represented by solid lines, while simulations results are represented by dashed lines.

network, where each cell allocates every subchannel to an active user at a given time. Hence, the density of mobile users on subchannel j is given by $\mu = 10^{-4}/\mathrm{m}^2$.

In Fig. 2, $\mathbb{P}_{s,can}$ is depicted for different values of the threshold as a function of the order of the canceled interferer. This figure illustrates that the success probability to cancel the n-th interferer decreases quickly as n and the target SIR increase. Simulation results are added to validate the model. When the received signal power is ordered only with respect to the distance, the simulations coincide with the analytical results based on the PGFL approach and the TSD approach. Moreover, a good agreement between analysis and simulations is achieved even when the ordering is performed based on the instantaneous received signal power. From the numerical results depicted in Fig. 2, we observe that the approximation deteriorates for lower values of the SIR threshold. As the SIR threshold decreases, we resort to values drawn from the central part of the PDF of the SINR, whereas the theorem of Breiman, necessary to assume that the order statistics are dominated by the distance, holds for the tail of the distribution.

Fig. 3 illustrates the success probability including SIC as a function of the SINR target for different values of the maximum number of cancellations. The theoretical results use the approximation for the cancellation radius introduced after Lemma 2. To validate our analysis, we compare the results with bounds that have been proposed in [8] for the scenario of spectrum sharing between cellular and mobile ad hoc networks (MANET), which can be shaped to represent a single-tier cellular network. In this work, the authors present bounds based on the separation of interferers into groups of strong and weak interferers where each strong interferer alone can cause outage. Interference cancellation is performed in descending order of received signal power accounting both for the effects of the spatial distribution of the nodes and the fading affecting each link, and the received power of each interferer intended for cancellation must exceed the SoI signal power multiplied by a factor $\kappa > 1$. From Fig. 3, we observe that the curves derived by our analysis strictly fall within the bounds proposed in [8]. In Theorem 1, we assumed independence between the events in (4). As such, the marginal distribution of the distance to the n-th interferer is used, whereas successfully canceling $n-1$ interferers also conveys useful information about the location of the n-th interferer. The good match between the bounds presented in [8] and analytical results supports the use of the PPP model and the marginal distributions. We observe that the theoretical results present a lower bound for the success probability tighter than the bounds previously known from the literature. Furthermore, we observe a modest improvement in the success probability when SIC is applied for threshold values lower than 2 dB, whilst for higher threshold values this improvement is negligible. The numerical results illustrate that the cancellation of the strongest interferer has a sensible effect on the receiver performance, while the cancellation of other interferers yields a marginal improvement of the success probability.[4]

The results presented in this section provide a guideline for the SIC computational requirements by investigating the performance gain from the cancellation of n interferers. Although applicable for UL and DL transmissions, SIC is particularly attractive in UL since it harnesses the processing power of access points to cancel strong interfering signals from nearby transmitters. However, as only the first cancellation has a significant effect on the performance, the computational requirements related to SIC are limited and hence, SIC qualifies also for DL transmissions. Following the association policy where the nodes connect to the AP offering the highest average received signal power, the overall performance gain of SIC is attractive only for poor signal conditions. In the following section, we present several association policies that are beneficial in terms of success probability, rate coverage, and offloading capability. These association policies can entail poor signal conditions, where the performance gain of SIC is far more appreciable.

[4]Note that the considered association policy based on DL signals applies to both DL and UL transmissions. Thus, different UL transmission powers in multi-tier networks can give rise to situations where the received signal power at the small cell AP of the macro-tier user exceeds the received signal power of the associated user. This scenario is an interesting use case for SIC. This effect is further amplified if power control and small cells at the cell edge are considered.

IV. Association Policies and SIC Gains

In this section, we illustrate the potential of SIC in multi-tier heterogeneous networks. We show that SIC can have distinct advantages for multi-tier networks when the association policy can lead to low signal quality. We first discuss UL association based on maximum instantaneous received signal power. In addition, we present two common DL scenarios where the received SIR is low by construction. Specifically, we demonstrate the benefits of SIC in the context of load balancing and range expansion.

A. Maximum Instantaneous Received Signal Power

Connecting to the AP that yields instantaneously the highest SIR can be of interest for a mobile node to obtain the maximum data rate [16], or to reduce the local delay $\tau = \mathbb{E}\{1/\mathbb{P}_{\mathrm{s}}(\eta t)\}$, which is defined as the mean time until a packet is successfully received [34]. In the following, we will evaluate how SIC can affect the UL success probability when the maximum instantaneous SIR policy is applied.

For this association policy with $A_k = h_{x_{k,i}} P_k$ in (2), we assume that (i) the initial AP load distribution is determined by the association rule based on the maximum DL average received signal power, (ii) all tiers have equal SIR target values, and (iii) the maximum SIR in DL is also the max SIR in UL, which occurs when the network operates in time division duplex (TDD) mode such that we can make use of channel reciprocity. We first determine the node densities μ_k connected to tier k, based on the maximum DL average received signal power. This approach is realistic since it does not require extra signaling from the APs. The user density is given by $\mu_k = p_{\mathrm{a},k}\mu$, where $p_{\mathrm{a},k}$ is the association probability to tier k given by

$$\begin{aligned} p_{\mathrm{a},k} &= \Pr\left\{\bigcap_{i\neq k}\left(\max_{\Phi_k}\mathrm{SIR}_j > \max_{\Phi_i}\mathrm{SIR}_j\right)\right\} \\ &= \prod_{i\neq k}\Pr\left\{P_k x_k^{-\alpha} > P_i x_i^{-\alpha}\right\} \\ &= \prod_{i\neq k}\Pr\left\{x_i > x_k\left(P_i/P_k\right)^{1/\alpha}\right\} \\ &= \int_0^\infty \exp\left(-\pi x_k^2\sum_{i\neq k}\lambda_i\left(P_i/P_k\right)^{2/\alpha}\right)2\pi\lambda_k x_k \\ &\quad\times\exp\left(-\lambda_k\pi x_k^2\right)\mathrm{d}x_k \\ &= \frac{\lambda_k}{\sum_{i\in\mathcal{K}}\lambda_i\left(\frac{P_i}{P_k}\right)^{2/\alpha}}, \end{aligned} \tag{23}$$

where x_i represents the distance to the closest AP of tier i and we consider that the maximum SIR corresponds to the maximum received signal power.

For mobile nodes applying the maximum instantaneous SIR association policy, the outage probability for UL transmissions in a multi-tier heterogeneous network without IC capabilities is given by

$$\begin{aligned} &\mathbb{P}_{\mathrm{out}}(\eta_{\mathrm{t}},\lambda,\mu,\mathbf{Q}) = \Pr\left\{\max_{k\in\mathcal{K},x_{k,i}\in\Phi_k}\mathrm{SIR}_j(u\to x_{k,i}) < \eta_{\mathrm{t}}\right\} \\ &= \bigcap_{k\in\mathcal{K},x_{k,i}\in\Phi_k}\Pr\left\{\mathrm{SIR}_j(u\to x_{k,i}) < \eta_{\mathrm{t}}\right\} \\ &\overset{(a)}{=} \bigcap_{k\in\mathcal{K}}\mathbb{E}_{\Phi_k}\left\{\prod_{x_{k,i}\in\Phi_k}\mathbb{P}^{!u}\left(\frac{Q_k h_{\mathrm{u}} u^{-\alpha}}{I_{\Omega_j}} < \eta_{\mathrm{t}}\right)\right\} \\ &= \prod_{k\in\mathcal{K}}\mathbb{P}_{\mathrm{out}}^{(k)}(\eta_{\mathrm{t}}), \end{aligned} \tag{24}$$

where $\mathbb{P}_{\mathrm{out}}^{(k)}$ is the outage probability of a typical user connected to the AP of tier k that yields the maximum instantaneous SIR. Note that we assume in (a) that all $\mathrm{SIR}_j(u\to x_{k,i})$ are independent.[5] We can further develop $\mathbb{P}_{\mathrm{out}}^{(k)}$ as

$$\begin{aligned} &\mathbb{P}_{\mathrm{out}}^{(k)}(\eta_{\mathrm{t}},\lambda_k,\mu,\mathbf{Q}) \\ &= \mathbb{E}_{\Phi_k}\left[\prod_{x_{k,i}\in\Phi_k}1-\mathbb{E}_{I_\Omega}\left[\exp\left(-\eta_{\mathrm{t}}\frac{u^\alpha}{Q_k}I_{\Omega_j}\right)\right]\right] \\ &= \exp\left(-2\pi\lambda_k\int_0^\infty\mathbb{E}_{I_{\Omega_j}}\left[\exp\left(-\eta_{\mathrm{t}}\frac{u^\alpha}{Q_k}I_{\Omega_j}\right)\right]u\,\mathrm{d}u\right) \\ &= \exp\Bigg(-2\pi\lambda_k \\ &\quad\times\int_0^\infty\exp\left(-\pi\eta_{\mathrm{t}}^{2/\alpha}C(0,\alpha)u^2\sum_{i=1}^K\mu_i\left(\frac{Q_i}{Q_k}\right)^{2/\alpha}\right)u\,\mathrm{d}u\Bigg) \\ &= \exp\left(\frac{-\lambda_k}{\eta_{\mathrm{t}}^{2/\alpha}C(0,\alpha)\sum_{i=1}^K\mu_i\left(Q_i/Q_k\right)^{2/\alpha}}\right). \end{aligned} \tag{25}$$

Combining (24) and (25), the outage probability of a typical user that connects to the AP which yields the maximum instantaneous SIR is given by

$$\mathbb{P}_{\mathrm{out}}(\eta_{\mathrm{t}},\lambda,\mu,\mathbf{Q}) = \exp\left(\frac{-\sum_{j=1}^K\lambda_j Q_j^{2/\alpha}}{\eta_{\mathrm{t}}^{2/\alpha}C(0,\alpha)\sum_{i=1}^K\mu_i Q_i^{2/\alpha}}\right). \tag{26}$$

From (25), we can conclude that the density of the superposition of network nodes with different transmit powers is equal to a weighted sum of the densities $\tilde{\mu}_k = \sum_{i=1}^K\mu_i(Q_i/Q_k)^{2/\alpha}$.

Lemma 5: The success probability for UL transmissions with SIC is given by

$$\begin{aligned} \mathbb{P}_{\mathrm{s}}^{\mathrm{SIC}}(\eta_{\mathrm{t}},\lambda,\mu,\mathbf{Q}) &= 1-\mathbb{P}_{\mathrm{out}}(\eta_{\mathrm{t}},\lambda,\mu,\mathbf{Q}) \\ &\quad\times\prod_{k\in\mathcal{K}}\left\{\exp\left(-2\pi\lambda_k\int_0^\infty\mathbb{P}_{\mathrm{s,SIC}}^{\mathrm{gain}}(\eta_{\mathrm{t}},N|u)u\mathrm{d}u\right)\right\} \end{aligned} \tag{27}$$

[5] In [16], the authors limit the range of the SIR threshold to $\eta_t > 1$ to avoid the dependence, while an exact approach is proposed in [35] where the joint SINR distribution is presented.

where

$$\mathbb{P}^{\mathrm{gain}}_{\mathrm{s,SIC}}(\eta_{\mathrm{t}},N|u) = \sum_{i=1}^{N}\left(\prod_{n=0}^{i-1}\left[1-\exp\left(-\pi\eta_{\mathrm{t}}^{2/\alpha}C\left(R_{\mathrm{I,n}}^2/\eta_{\mathrm{t}}^{2/\alpha}u^2,\alpha\right)\tilde{\mu}_j u^2\right)\right]\right) \times\left(\prod_{n=1}^{i}\mathbb{P}_{\mathrm{s,can}}(\eta_{\mathrm{t}},n)\right)\exp\left(-\pi\eta_{\mathrm{t}}^{2/\alpha}C\left(R_{\mathrm{I},i}^2/\eta_{\mathrm{t}}^{2/\alpha}u^2,\alpha\right)\tilde{\mu}_j u^2\right). \tag{28}$$

Proof: When a typical user is associated to the AP corresponding to the highest instantaneous SIR, we can write

$$\mathbb{P}^{\mathrm{SIC}}_{\mathrm{out}}(\eta_{\mathrm{t}},\lambda,\mu,\mathbf{Q}) = \bigcap_{k\in\mathcal{K}}\mathbb{E}_{\Phi_k}\left\{\prod_{x_{k,i}\in\Phi_k}\mathbb{P}\left(\frac{Q_k h_{\mathrm{u}} u^{-\alpha}}{I_{\Omega_j^n}}<\eta_{\mathrm{t}}\right)\right\} \tag{29}$$

where $\mathbb{P}(Q_k h_{\mathrm{u}} u^{-\alpha}/I_{\Omega_j^n}<\eta_{\mathrm{t}})$ is the outage probability conditioned on the distance u and n interferers being canceled. Using the PGFL for a PPP, (29) can be written as

$$\mathbb{P}^{\mathrm{SIC}}_{\mathrm{out}}(\eta_{\mathrm{t}},\lambda,\mu,\mathbf{Q}) = \prod_{k\in\mathcal{K}}\left\{\exp\left(-2\pi\lambda_k\int_0^\infty \underbrace{\mathbb{E}_{I_{\Omega_j^n}}\left[\exp(-\eta_{\mathrm{t}}u^{-\alpha}I_{\Omega_j^n}/Q_k)\right]}_{\mathbb{P}^{(k)}_{\mathrm{s,SIC}}(\eta_{\mathrm{t}},n|u)} u\mathrm{d}u\right)\right\} \tag{30}$$

where the typical user is assumed to be at the origin and the APs at a distance u from the origin. Still, the interference cancellation radius for each AP is considered with respect to the AP under consideration, which does not induce any problems since we consider stationary point processes. Therefore, the success probability $\mathbb{P}^{(j)}_{\mathrm{s,SIC}}(\eta_{\mathrm{t}},n|u)$ referenced to tier j is invariant under translation. The success probability including SIC conditioned on the distance can further be written as

$$\begin{aligned}\mathbb{P}^{(j)}_{\mathrm{s,SIC}}(\eta_{\mathrm{t}},n|u) &= \mathbb{P}_{\mathrm{s}}(\eta_{\mathrm{t}}|u)+\sum_{i=1}^{N}\left(\prod_{n=0}^{i-1}(1-\mathbb{P}_{\mathrm{s,IC}}(\eta_{\mathrm{t}},R_{\mathrm{I},n}|u))\right) \\ &\quad\times\left(\prod_{n=1}^{i}\mathbb{P}_{\mathrm{s,can}}(\eta_{\mathrm{t}},n)\right)\mathbb{P}_{\mathrm{s,IC}}(\eta_{\mathrm{t}},R_{\mathrm{I},i}|u) \\ &= \exp\left(-\pi\eta_{\mathrm{t}}^{2/\alpha}C(0,\alpha)\tilde{\mu}_j u^2\right) \\ &\quad+\sum_{i=1}^{N}\left(\prod_{n=0}^{i-1}\left[1-\exp\left(-\pi\eta_{\mathrm{t}}^{2/\alpha}C\left(R_{\mathrm{I},n}^2/\eta_{\mathrm{t}}^{2/\alpha}u^2,\alpha\right)\tilde{\mu}_j u^2\right)\right]\right) \\ &\quad\times\left(\prod_{n=1}^{i}\mathbb{P}_{\mathrm{s,can}}(\eta_{\mathrm{t}},n)\right)\exp\left(-\pi\eta_{\mathrm{t}}^{2/\alpha}C\left(R_{\mathrm{I},i}^2/\eta_{\mathrm{t}}^{2/\alpha}u^2,\alpha\right)\tilde{\mu}_j u^2\right).\end{aligned} \tag{31}$$

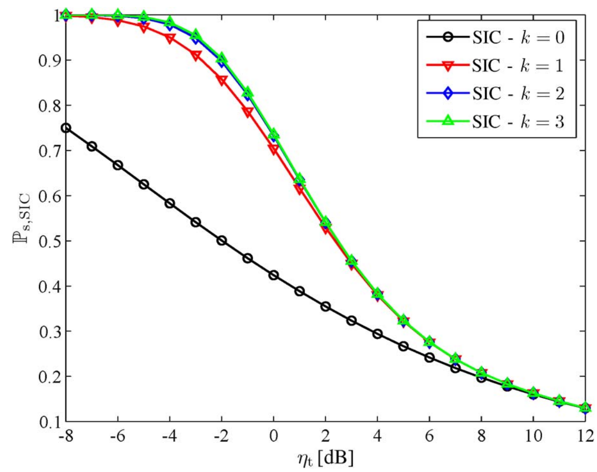

Fig. 4. The gain in success probability that can be achieved by successively canceling interferers.

Note that in (31) the single-tier equivalent network normalized with respect to the transmission power of tier j is used. Substituting (31) in (30), the proof is concluded. □

Considering network infrastructure with SIC capabilities, the typical user will connect to the AP that offers the highest instantaneous SIR after canceling n interferers. The maximum instantaneous SIR association policy that accommodates for SIC, does not necessarily connect the mobile node to the AP which yields the maximum average SIR nor to the closest one.

Fig. 4 shows the success probability for a typical user that connects to the AP that provides the highest instantaneous SIR, with and without SIC. We considered a two-tier network with densities $\lambda_1 = 10^{-5}$ m^{-2} and $\lambda_2 = 10^{-4}$ m^{-2}, respectively, while the user density is given by $\mu = 10^{-4}$ m^{-2}. The ratio between transmission powers is given by $P_1/P_2 = 10$ and $Q_1/Q_2 = 10$. From the figure, we observe an increase of the success probability up to 20% in the range of interest, i.e., for values of the threshold above 0 dB. However, this scheme requires additional signaling and centralized control to connect the user to the AP which provides instantaneously the highest SIR.

B. Minimum Load Association Policy

Considering uniform resource allocation between the users within a cell, it can be preferential for data-sensitive applications to connect to the AP with the lowest load, rather than to the AP that offers the highest SIR. The same observation holds for networks that apply a load balancing policy and where users are actively transferred to lightly loaded APs different from the AP of their own Voronoi cell [17]. In the following, we consider the association policy where a user connects to the AP with the lowest load for a given connectivity range R_{con} with respect to the user. In this scenario, the performance metric of interest is the rate per user, which reflects the quality of service (QoS) and depends on the AP load, defined as the number of users M connected to the AP. This scenario leads to interesting tradeoffs between APs where the loss of SIR can be compensated by the gain of available resource blocks per user. We consider a single tier network and we model explicitly the load of the APs by considering the marked PPP $\tilde{\Phi} = \{(X_i, M_i)|X_i \in \Phi(\lambda), M_i \sim f_M(m)\}$, with M_i the load of X_i and $f_M(m)$ the load distribution. We consider a typical user at the origin of the Euclidean plane and we compare the performance of the maximum average SIR policy with the minimum load association policy for DL transmissions in terms of rate per user. Let $\mathcal{R}$ denote the rate per user and we define the rate coverage as the CCDF of the rate $\mathbb{P}_c(\rho) = \mathbb{P}[\mathcal{R} > \rho]$ [18], which is given by

$$\mathbb{P}_c(\rho) = \Pr\left[\frac{1}{M}\log(1+\text{SIR}) > \rho\right] = \Pr[\text{SIR} > 2^{M\rho} - 1] = \mathbb{E}_M\left[\mathbb{P}_s(2^{M\rho}-1)\right]. \quad (32)$$

To calculate the rate coverage, we need to characterize the distribution of M. The load of a cell depends on the area distribution of the Voronoi cell area $\mathcal{A}$, represented by $f_{\mathcal{A}}(x)$, for which an approximation has been proposed in [36]. Using this approximation, the probability mass function of M is given by

$$f_M(m) = \int_0^\infty \Pr[M = m|\mathcal{A} = x] f_{\mathcal{A}}(x)\,\mathrm{d}x = \frac{3.5^{3.5}}{m!}\frac{\Gamma(m+3.5)}{\Gamma(3.5)}\left(\frac{\mu}{\lambda}\right)^m\left(3.5+\frac{\mu}{\lambda}\right)^{-(m+3.5)}. \quad (33)$$

For the association policy based on the maximum average received signal power, the rate coverage conditioned on the number of associated users is given by

$$\mathbb{P}_c^{(\text{MAX SIR})}(\rho|M) = \int_0^\infty 2\lambda\pi r \exp\left(-\pi\lambda r^2\left(1+\varsigma^{2/\alpha}C(1/\varsigma^{2/\alpha},\alpha)\right)\right)\mathrm{d}r = \frac{1}{1+\varsigma^{2/\alpha}C(1/\varsigma^{2/\alpha},\alpha)}, \quad (34)$$

where $\varsigma = 2^{M\rho} - 1$. Taking the expectation over M, the rate coverage can be written as

$$\mathbb{P}_c^{(\text{MAX SIR})}(\rho) = \sum_{m\geq 0} f_M(m)\mathbb{P}_c^{(\text{MAX SIR})}(\rho|m+1), \quad (35)$$

where the load of the cell under consideration includes the admitted user.

Lemma 6: For a typical user that connects to the AP with the lowest load within the range R_{con}, the rate coverage is given by

$$\mathbb{P}_c^{(\text{MINL})}(\rho) = \sum_{m\geq 0} f_{M_{(1)}}(m)\mathbb{P}_c^{(\text{MINL})}(\rho|m+1), \quad (36)$$

where $f_{M_{(i)}}(m)$ represents the probability mass function (PMF) of the i-th order statistic of the load.

Proof: For the minimum load association policy, the typical user is appointed to the AP with the lowest load which is uniformly distributed over $b(0, R_{\text{con}})$ with distance distribution $f_R(r) = 2r/R_{\text{con}}^2$. We assume that there are $N = \lfloor\lambda\pi R_{\text{con}}^2\rfloor$ APs within the connectivity range. The coverage probability for the minimum load scheme conditioned on the load is given by

$$\mathbb{P}_c^{(\text{MINL})}(\rho|M) = 1/R_{\text{con}}^2\int_0^{R_{\text{con}}^2}\exp\left(-\pi\lambda\varsigma^{2\alpha}C(0,\alpha)r^2\right)2r\,\mathrm{d}r = \frac{1-\exp\left(-\pi\lambda\varsigma^{2/\alpha}C(0,\alpha)R_{\text{con}}^2\right)}{\pi\lambda\varsigma^{2/\alpha}C(0,\alpha)R_{\text{con}}^2}. \quad (37)$$

The i-th order statistic of the load is given by [27]

$$f_{M_{(i)}}(m) = \frac{1}{\mathcal{B}(i, M-i+1)}\int_{F_M(m-1)}^{F_M(m)} w^{i-1}(1-w)^{m-i}\,\mathrm{d}w, \quad (38)$$

where $\mathcal{B}(a,b)$ represents the beta function. Taking the expectation over the first order statistic of the load in (37), we get (36), which concludes the proof. □

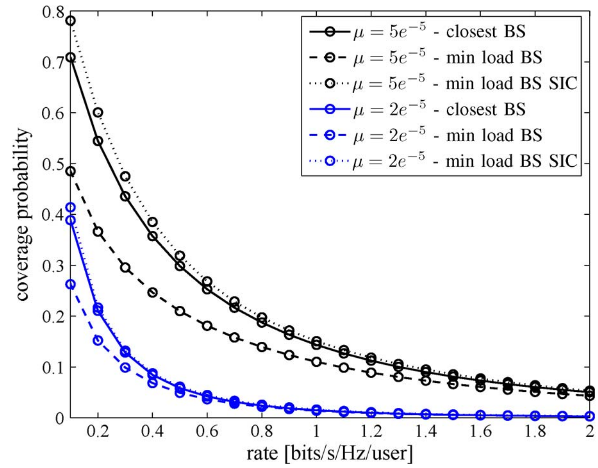

Fig. 5. The coverage probability is depicted for the max-SIR association policy (solid lines), the minimum load association policy (dashed lines), and the minimum load policy with SIC (dotted lines) for $\lambda = 10^{-5}$ and $\alpha = 4$.

Fig. 5 depicts the rate coverage and compares the max-SIR association policy with the minimum load association policy. From the numerical results, we see that the rate coverage decreases significantly for the minimum load policy when no SIC is applied. This means that the loss in SIR cannot be compensated by the lower load of the AP. However, when SIC is applied ($n = 1$), the minimum load association outperforms max-SIR association in terms of rate per user. From this figure, we conclude that when SIC is applied, users can be offloaded to nearby APs while increasing the rate, which paves the way for more advanced load balancing techniques.

C. Range Expansion

While the higher tiers in a multi-tier network are intended to offload data traffic from the macrocell network, this target is impeded considerably due to the relatively small coverage area of the higher tiers, which are usually denoted as small cells. To encourage users to connect to the small cells, range expansion has been proposed which applies an association policy based on a biased received signal power [1]. Specifically, the association rule is adjusted to accommodate for cell range expansion by defining $A_k = b_k P_k$ in (2), where b_k represents an association bias for tier k. Although range expansion mitigates the UL cross-tier interference, users in DL experience bad signal conditions in the range expanded areas since they are not connected to the base station that provides the highest average SIR. It is therefore meaningful to study the benefit of SIC in DL for those users located in the range expanded areas (REAs). To calculate the benefit of canceling the strongest interferer for the users located in the REA, we provide the following lemma.

Lemma 7: After canceling the strongest AP, the success probability of the users located in $C_k^{(\mathrm{RE})}$ is given by

$$\mathbb{P}_{\mathrm{s,IC}}\left(\eta_{\mathrm{t}},1|x_k\in C_k^{(\mathrm{RE})}\right)=\frac{1}{p_{\mathrm{a},k}^{(\mathrm{RE})}}\times\left[\frac{1}{\sum_{i\in\mathcal{K}}\left(\frac{\lambda_i}{\lambda_k}\right)\left(\frac{P_i}{P_k}\right)^{2/\alpha}\left(\eta_{\mathrm{t}}^{2/\alpha}C\left((1/\eta_{\mathrm{t}})^{2/\alpha},\alpha\right)+\left(\frac{b_i}{b_k}\right)^{2/\alpha}\right)}-\frac{1}{\sum_{i\in\mathcal{K}}\left(\frac{\lambda_i}{\lambda_k}\right)\left(\frac{P_i}{P_k}\right)^{2/\alpha}\left(\eta_{\mathrm{t}}^{2/\alpha}C\left((1/\eta_{\mathrm{t}})^{2/\alpha},\alpha\right)+1\right)}\right],\tag{39}$$

where $B=\{b_k\}$ is the set of bias factors corresponding to each tier.

Proof: To calculate the success probability of the users belonging to the REA, we first define the distance distribution of these users with respect to the serving AP. A user that belongs without biasing to tier $i\neq k$, is located in the REA $C_k^{(\mathrm{RE})}$ of tier k if the relationship $P_kx_k^{-\alpha}<P_ix_i^{-\alpha}<b_kP_kx_k^{-\alpha}$ holds. Extending the analysis of [37], [38] to a K-tier network, the distance distribution of users located in the REA to the serving AP is given by

$$f_{X_k^{(\mathrm{RE})}}(x)=\frac{2\pi\lambda_k}{p_{\mathrm{a},k}^{(\mathrm{RE})}}x\left[\exp\left(-\pi\sum_{i\in\mathcal{K}}\lambda_i\left(\frac{P_i}{P_k}\frac{b_i}{b_k}\right)^{2/\alpha}x^2\right)-\exp\left(-\pi\sum_{i\in\mathcal{K}}\lambda_i\left(\frac{P_i}{P_k}\right)^{2/\alpha}x^2\right)\right]\tag{40}$$

where the association probability to the REA of tier k is $P_a^{RE}=1-\sum_{i\neq k}p_{\mathrm{a},i}(b)-p_{\mathrm{a},k}(B|b_k=1)$ and the association probability to the k-th tier is given by

$$p_{\mathrm{a},k}(B)=\frac{\lambda_k}{\sum_{i\in\mathcal{K}}\lambda_i\left(\frac{P_i}{P_k}\frac{b_i}{b_k}\right)^{2/\alpha}}.\tag{41}$$

The success probability of a mobile node belonging to $C_k^{(\mathrm{RE})}$ conditioned on the distance can be written as

$$\mathbb{P}_{\mathrm{s}}\left(\eta_{\mathrm{t}}|x_k\in C_k^{(\mathrm{RE})},x_k\right)=\prod_{i\in\mathcal{K}}\mathcal{L}_{I_{\Phi_i}}\left(\frac{\eta_{\mathrm{t}}x_k^{\alpha}}{P_k}\right),$$

where

$$\mathcal{L}_{I_{\Phi_i}}\left(\frac{\eta_{\mathrm{t}}x_k^{\alpha}}{P_k}\right)=\exp\left(-\pi\lambda_i\eta_{\mathrm{t}}^{\frac{2}{\alpha}}\left(\frac{P_i}{P_k}\right)^{2/\alpha}C\left((b_i/\eta_{\mathrm{t}}b_k)^{2/\alpha},\alpha\right)x_k^2\right).$$

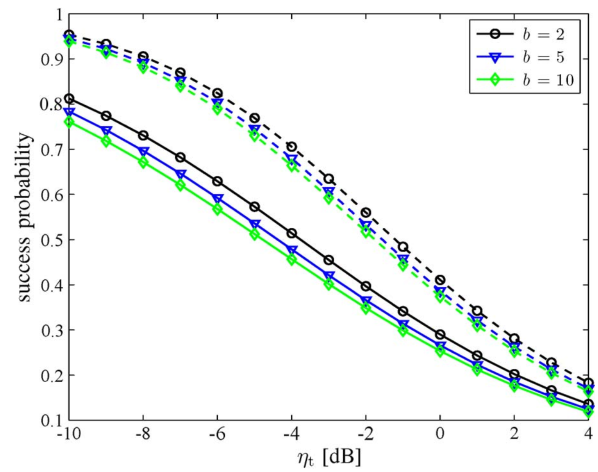

Fig. 6. Success probability for users belonging to the range expansion region. Solid lines represent the success probability without interference cancellation, whereas the dashes lines represent the success probability when the closest AP is canceled. The power ratio $P_1/P_2=10$.

Note that for a user located in the REA $C_k^{(\mathrm{RE})}$ of tier k the following inequality $P_kx_k^{-\alpha}<P_ix_i^{-\alpha}<b_kP_kx_k^{-\alpha}$ holds. As such, we can define the interferer exclusion region $x_i>(P_i/b_kP_k)^{1/\alpha}x_k$, which is useful to determine the integration interval of the integral $C(b,\alpha)$. Applying the change of variables $(P_k/\eta_tP_i)^{2/\alpha}(x_i/x_k)^2\to u$ and taking the expectation over x_k, we can write

$$\begin{aligned}&\mathbb{P}_{\mathrm{s}}\left(\eta_{\mathrm{t}}|x_k\in C_k^{(\mathrm{RE})}\right)\\&=\int_0^{\infty}\exp\left(-\pi\eta_{\mathrm{t}}^{2/\alpha}\sum_{i\in\mathcal{K}}\lambda_i(P_i/P_k)^{2/\alpha}C\left((b_i/\eta_{\mathrm{t}}b_k)^{2/\alpha},\alpha\right)x_k^2\right)\\&\times f_{X_k^{(\mathrm{RE})}}(x_k)\,\mathrm{d}x_k\\&=\frac{1}{p_{\mathrm{a},k}^{(\mathrm{RE})}}\left(\frac{1}{\sum_{i\in\mathcal{K}}\left(\frac{\lambda_i}{\lambda_k}\right)\left(\frac{P_i}{P_k}\right)^{2/\alpha}\left(\eta_{\mathrm{t}}^{2/\alpha}C\left((b_i/\eta_{\mathrm{t}}b_k)^{2/\alpha},\alpha\right)+\left(\frac{b_i}{b_k}\right)^{2/\alpha}\right)}\right.\\&\left.-\frac{1}{\sum_{i\in\mathcal{K}}\left(\frac{\lambda_i}{\lambda_k}\right)\left(\frac{P_i}{P_k}\right)^{2/\alpha}\left(\eta_{\mathrm{t}}^{2/\alpha}C\left((b_i/\eta_{\mathrm{t}}b_k)^{2/\alpha},\alpha\right)+1\right)}\right).\end{aligned}\tag{42}$$

Applying SIC to a user located in the REA, the highest unbiased received signal power of tier i is canceled. As a result the interference cancellation radius relative to the i-th tier increases from $(P_i/b_kP_k)^{1/\alpha}x_k$ to $(P_i/P_k)^{1/\alpha}x_k$, and hence, the success probability after canceling the strongest AP can be written as (39). □

From Lemma 7, the bias factors of the different tiers can be determined to guarantee a given performance for the mobile users belonging to the REA.

Fig. 6 depicts the success probability of a typical user in the REA in a two-tier network with densities $\lambda_1=10^{-5}$ m^{-2} and $\lambda_2=10^{-4}$ m^{-2} for different values of the bias factor b. The figure illustrates how the success probability decreases as the REA gets larger with increasing values of b. Moreover, from the numerical results we observe that the increase of success probability due to SIC is substantial. This scenario is a realistic example where SIC can provide high performance gain.

V. Conclusion

In this work, we developed a probabilistic framework for the performance analysis of multi-tier heterogeneous networks with SIC capabilities. The framework accounts for the consecutive cancellations of the IC scheme, network topology, propagation effects, and the association policy. For users connected to the AP that provides the maximum average received signal power, performance benefits diminish quickly with the number of cancellations n and SIC is only attractive in case of poor signal conditions. There are several deployment scenarios for future multi-tier networks that provide low signal quality. Specifically, our analysis demonstrates that SIC yields distinct performance gains for the maximum instantaneous SIR policy, the minimum load association policy, and range expansion. The proposed framework can be applied to evaluate the achievable performance of multi-tier networks with SIC capabilities. There are several avenues for future research to extend the current framework. It is relevant to incorporate inaccurate channel estimation and the corresponding imperfect interference cancellation. Another interesting line of research is to propose a unified framework consisting of SIC and channel coding strategies.

Appendix A
Proof of Lemma 1

We calculate the distribution of $Y = hX^{-\alpha}$, with $h \sim \Gamma(k,\theta)$. The parameters of the Gamma distribution can be related to the Nakagami-m distribution as $k = m$ and $\theta = \Omega/m$ with Ω the variance of the Nakagami-m r.v. equal to one. For a node uniformly distributed in a circular area, the distribution of the distance X with respect to the origin is given by $F_X(x) = x^2/R^2$, where R is the maximum considered range where nodes make a contribution to the aggregate interference. The distribution of $W = X^{-\alpha}$ is given by

$$\begin{aligned}F_W(w) &= \mathbb{P}[X^{-\alpha} < w]\\ &= \mathbb{P}\left[X \geq (1/w)^{1/\alpha}\right]\\ &= 1 - \frac{w^{-2/\alpha}}{R^2}\end{aligned}$$

which corresponds to the CDF of a Pareto distribution. The distribution of the product of W and the gamma r.v. h with PDF $f_H(h;m,\theta) = h^{m-1}\exp(-h/\theta)/(\theta^m\Gamma(m))$ is now given by

$$\begin{aligned}F_Y(y) &= \mathbb{E}\{\mathbb{P}[W \leq y/h]\}\\ &= \int_0^\infty \left(1 - \frac{(y/h)^{-2/\alpha}}{R^2}\right)\frac{h^{m-1}\exp(-h/\theta)}{\theta^m\Gamma(m)}\mathrm{d}h\\ &= 1 - \frac{\theta^{2/\alpha}y^{-2/\alpha}}{R^2\Gamma(m)}\int_0^\infty (h/\theta)^{2/\alpha+m-1}\exp(h/\theta)\,\mathrm{d}h/\theta\\ &= 1 - \frac{\Gamma(2/\alpha+m)}{\Gamma(m)}\frac{(my)^{-2/\alpha}}{R^2},\end{aligned}$$

which concludes the proof.

Appendix B
Proof of Lemma 4

In (18), α_I is chosen equal to the characteristic exponent of the skewed stable distribution in the unbounded path loss model. The parameters of the TSD can be found using the method of the cumulants. From (18), the cumulants of the truncated stable distribution can be expressed as

$$\begin{aligned}\kappa_I(k) &= \frac{1}{j^k}\frac{d^k}{d\omega^k}\ln\psi_{I_{\Omega_j^n}}(j\omega)\Big|_{\omega=0}\\ &= (-1)^k\gamma'\Gamma(-\alpha_I)g^{\alpha_I-k}\Pi_{i=0}^{k-1}(\alpha_I - i).\end{aligned} \tag{43}$$

Building on Campbell's theorem [33], the cumulants of the aggregate interference can be expressed as

$$\kappa(k) = Q^k\frac{2\pi\mu}{k\alpha-2}d_{\min}^{2-k\alpha}\mu_{h^2}(k). \tag{44}$$

Using (43) and (44), the parameters γ' and g can be written as a function of the first two cumulants as follows

$$\begin{aligned}\gamma' &= \frac{-\kappa(1)}{\Gamma(-\alpha_I)\alpha_I\left(\frac{\kappa(1)(1-\alpha_I)}{\kappa(2)}\right)^{\alpha_I-1}}\\ g &= \frac{\kappa(1)(1-\alpha_I)}{\kappa(2)}.\end{aligned} \tag{45}$$

The success probability of successfully canceling the strongest interferer can be written as

$$\begin{aligned}\mathbb{P}_{\mathrm{s,can}}(\eta_{\mathrm{t}},n|r) &= \mathbb{P}\left(\frac{X_{(n)}}{I_{\Omega_j^n}} \geq \eta_{\mathrm{t}}\right)\\ &= \mathcal{L}_{I_{\Omega_j^n}}(\eta_{\mathrm{t}}r^\alpha)\\ &= \exp\left(\gamma'\Gamma(-\alpha_I)\left[(g+\eta_{\mathrm{t}}r^\alpha)^{\alpha_I} - g^{\alpha_I}\right]\right).\end{aligned} \tag{46}$$

For the special case where $\alpha = 4$, we get $\alpha_I = 2/\alpha = 1/2$, and $\mathbb{P}_{\mathrm{s,can}}(\eta_t,n|r)$ conditioned on the distance of the strongest node is given by

$$\begin{aligned}&\mathbb{P}_{\mathrm{s,can}}(\eta_{\mathrm{t}},n|r,\alpha=4)\\ &= \exp\left(\gamma'\Gamma(-1/2)\sqrt{g}\left[\sqrt{1+\frac{\eta_{\mathrm{t}}}{g}r^4} - 1\right]\right)\\ &= \exp\left(-2\kappa(1)\sqrt{\frac{\kappa(1)}{2\kappa(2)}}\sqrt{\frac{\kappa(1)}{2\kappa(2)}}\left[\sqrt{1+\frac{2\eta_{\mathrm{t}}r^4\kappa(2)}{\kappa(1)}} - 1\right]\right)\\ &= \exp\left(-\frac{\kappa(1)^2}{\kappa(2)}\left[\sqrt{1+\frac{2\eta_{\mathrm{t}}r^4\kappa(2)}{\kappa(1)}} - 1\right]\right)\\ &\overset{(a)}{=} \exp\left(-3/2\mu\pi r^2[\sqrt{1+4\eta_{\mathrm{t}}/3} - 1]\right),\end{aligned} \tag{47}$$

where (a) follows from the fact that $d_{\min}$ corresponds to the distance r of the n-th interferer, and we assume Rayleigh fading such that $\mathbb{E}[h^k] = k/\lambda^k$ with $\lambda = 1$ the intensity of the exponential distribution. The unconditional success probability can now be written as

$$\begin{aligned}
&\mathbb{P}_{\mathrm{s,can}}(\eta_{\mathrm{t}}, n|\alpha = 4)\\
&= \int_0^\infty \exp(-3/2\mu\pi r^2[\sqrt{1+4\eta_{\mathrm{t}}/3} - 1])\\
&\qquad \times \exp(\mu\pi r^2)\frac{2(\mu\pi r^2)^n}{r\Gamma(n)}\,\mathrm{d}r\\
&= \int_0^\infty \exp\left(\mu\pi r^2[\sqrt{9/4+3\eta_{\mathrm{t}}} - 3/2 + 1]\right)\frac{2(\mu\pi)^n r^{2n-1}}{\Gamma(n)}\,\mathrm{d}r\\
&= \int_0^\infty \exp(\mu\pi r^2[\sqrt{9/4+3\eta_{\mathrm{t}}} - 1/2])\frac{(\mu\pi)^{n-1} r^{2n-2}}{\Gamma(n)}\,\mathrm{d}\mu\pi r^2\\
&= \int_0^\infty \exp\left([\sqrt{9/4+3\eta_{\mathrm{t}}} - 1/2]w\right)\frac{w^{n-1}}{\Gamma(n)}\mathrm{d}w\\
&= \frac{1}{(\sqrt{9/4+3\eta_{\mathrm{t}}} - 1/2)^n}.
\end{aligned} \tag{48}$$

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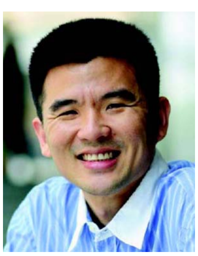

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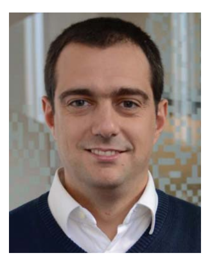

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