# Optimized Training Design for Wireless Energy Transfer

Yong Zeng, Member, IEEE, and Rui Zhang, Member, IEEE

Abstract-Radio-frequency (RF) enabled wireless energy transfer (WET), as a promising solution to provide cost-effective and reliable power supplies for energy-constrained wireless networks, has drawn growing interests recently. To overcome the significant propagation loss over distance, employing multi-antennas at the energy transmitter (ET) to more efficiently direct wireless energy to desired energy receivers (ERs), termed energy beamforming, is an essential technique for enabling WET. However, the achievable gain of energy beamforming crucially depends on the available channel state information (CSI) at the ET, which needs to be acquired practically. In this paper, we study the design of an efficient channel acquisition method for a point-to-point multipleinput multiple-output (MIMO) WET system by exploiting the channel reciprocity, i.e., the ET estimates the CSI via dedicated reverse-link training from the ER. Considering the limited energy availability at the ER, the training strategy should be carefully designed so that the channel can be estimated with sufficient accuracy, and yet without consuming excessive energy at the ER. To this end, we propose to maximize the net harvested energy at the ER, which is the average harvested energy offset by that used for channel training. An optimization problem is formulated for the training design over MIMO Rician fading channels, including the subset of ER antennas to be trained, as well as the training time and power allocated. Closed-form solutions are obtained for some special scenarios, based on which useful insights are drawn on when training should be employed to improve the net transferred energy in MIMO WET systems.

*Index Terms*—Wireless energy transfer (WET), energy beamforming, channel training, RF energy harvesting.

## I. INTRODUCTION

Wireless energy transfer (WET) has attracted significant interests recently due to its great potential to provide costeffective and reliable power supplies for energy-constrained wireless networks, such as sensor networks [1]. Generally speaking, WET can be implemented based on either the "nearfield" techniques including inductive coupling and magnetic resonance coupling, or the "far-field" electromagnetic (EM) radiation (see e.g. [2], [3] and references therein), which are suitable for short-range (less than tens of centimeters), middlerange (say, a couple of meters), and long-range (up to tens of kilometers) applications, respectively. In this paper, we focus on the EM radiation or radio-frequency (RF) enabled WET, where dedicated energy-bearing signals are transmitted from the energy transmitter (ET) to the energy receiver (ER) for energy harvesting. Besides longer transmission distance, RFenabled WET enjoys other advantages over the two near-field techniques, such as more flexibility in deployment, smaller

receiver size, easier implementation of energy multicasting to a large number of wireless devices distributed in a wide area, and the more convenience to integrate with simultaneous information transmission [4].

The history of microwave enabled wireless energy transmission can be traced back to the early work of Heinrich Hertz in 1880's [5]. Later in 1960's, RF-based WET in free space was extensively studied, with two well-known applications in microwave-powered helicopter [6] and solar power satellite (SPS) [7]. In these WET systems, high transmission power and large antenna apertures were used to overcome the propagation loss for large power reception. During the past decade, the interest in WET has reemerged, which was mainly driven by the increasing need to provide low-cost and perpetual energy supplies to wireless devices with relatively low-power requirements, such as those in wireless sensor, body and personal networks, to replace the traditional battery-powered solution that requires manual battery replacement and/or recharging. Besides, RF-based WET is being actively investigated in wireless communication systems such as cellular networks [8], cognitive radio networks [9], [10], and relay networks [11], [12]. The development in WET technology has also opened up several interesting new applications, e.g., simultaneous wireless information and power transfer (SWIPT) (see e.g. [4], [13]-[15]), where energy and information are jointly transmitted using the same RF waveform, and wireless powered communication networks (WPCN) (see e.g. [16]-[18]), where the communication devices are fully or primarily powered by means of WET.

One critical issue for the design of RF WET systems is to enhance the end-to-end energy transfer efficiency, which is the ratio of the energy harvested by the ER to that spent at the ET. Generally speaking, omni-directional energy transmission could result in very poor efficiency due to the fast power attenuation over distance. On the other hand, uni-directional transmission, which can be achieved by designing the antenna radiation pattern to form sharp beam towards the ER, could significantly enhance the energy transfer efficiency; but it generally relies on the existence of a clear line-of-sight (LOS) link between the ET and ER. For indoor WET applications in rich scattering environment, e.g., wireless sensors for smart buildings [19], the LOS link may not exist, or could be weak as compared to the scattered signal components. In such scenarios, rather than focusing all energy along the LOS link, it is preferable to balance the signal power and phases over the different paths so that the signals could add constructively at the ER for maximum energy delivery. To this end, multiantenna based energy beamforming has been proposed [4],

The authors are with the Department of Electrical and Computer Engineering, National University of Singapore. (e-mail: {elezeng, elezhang}@nus.edu.sg).

where the energy-bearing signals are weighted at the multiple transmit antennas before transmission to achieve energy beamforming gains. In this regard, massive multiple-input multipleoutput (MIMO) technique [20], [21] is particularly suitable for RF WET, where a very large number of antennas are equipped at the ET to achieve enormous beamforming gains; as a result, the end-to-end energy transfer efficiency can be greatly enhanced.

In practice, the benefit of energy beamforming in WET crucially depends on the available channel state information (CSI) at the ET, which needs to be acquired at the cost of additional resources (e.g., time and energy). Similar to wireless communication systems, one straightforward approach to obtain CSI at the ET is by sending pilot signals from the ET to the ER [22]-[24], based on which the ER estimates the channel and sends the estimation back to the ET via a feedback link. However, since the training time required scales up with the number of antennas M at the ET, such a channelacquisition method is not suitable when M becomes large [25]. Furthermore, as pointed out by [26], estimating the channel at the ER requires complex baseband signal processing, which may not be available at the ER due to its practical hardware limitation. A novel channel-learning algorithm for WET by taking into account the practical energy harvesting circuitry at the ER has thus been proposed in [26], which simplifies the processing at the ER and requires only one-bit feedback from the ER per feedback interval. Nevertheless, the number of feedback intervals required for the channel-learning scheme in [26] still increases quickly with M, which may be prohibitive for large M. Furthermore, it is worth noting that for all the aforementioned schemes, feedback signals need to be sent from the ER to the ET using part of the harvested energy. Considering the limited energy availability at the ER, the energy consumed for sending the feedback signals should be taken into account in the channel learning and feedback designs for MIMO WET.

In this paper, we study the important channel acquisition problem for a point-to-point MIMO WET system in Rician fading channels, which are general enough to include a class of MIMO channels ranging from a fully random Rayleigh fading channel to a fully deterministic LOS channel by varying the Rician factor, K. Note that although only pointto-point MIMO system is considered in this paper for ease of exposition, the techniques presented are also applicable to the multiuser setup with simultaneous energy transmission to a group of single- or multi-antenna ERs with similar path losses, i.e., with roughly the same distance from the ET. We assume that the forward-link energy transmission from the ET to the ER and the reverse-link communication from the ER to the ET take place in a time-division duplexing (TDD) manner, so that channel reciprocity holds, i.e., the channel matrices in the forward and reverse links are transpose of each other. Notice that channel reciprocity is a key enabling factor for the implementation of massive MIMO systems to reduce the channel-acquisition overhead for next generation wireless communication systems [27]. Under channel reciprocity, it is a well-known technique in wireless communication that the CSI at the transmitter can be obtained via reverse-link

channel training, where a fraction of the channel coherence time is assigned to the receiver for sending pilot signals to the transmitter to estimate the channel. However, applying this method to WET systems needs a more careful design of the training strategy, since the pilot signals need to be sent using part of the energy harvested by the ER. In particular, the following new trade-offs need to be taken into account for the training design in WET systems: too little training leads to coarsely estimated channel at the ET and hence reduced energy beamforming gain; whereas too much training consumes excessive energy harvested by the ER, and also leaves less time for energy transmission given a finite channel coherence time. To resolve the above trade-offs, we propose to maximize the net harvested energy at the ER, which is the average harvested energy offset by that used for channel training. Specifically, the main contributions of this paper are summarized as follows:

First, we propose a two-phase protocol for MIMO WET by leveraging the channel reciprocity property. In the first phase of  $\tau$  symbol durations, pilot signals are sent from the ER to ET for channel training. Based on the received pilot signals, the ET obtains an estimate of the instantaneous MIMO channel. During the second phase of  $T - \tau$  symbol durations where T denotes the channel coherence time, the ET transmits energy-bearing signals with energy beamforming based on the estimated channel. Compared with the existing feedbackbased channel-acquisition schemes considered in [22], [23], [26], the proposed channel-reciprocity enabled method has the following advantages: (i) it is more efficient for large or massive MIMO system implementation as the training overhead is independent of the number of antennas M at the ET; (ii) it simplifies the processing at the ER since channel estimation and feedback operations are no longer required; and (iii) it explicitly maximizes the net average harvested energy at the ER, which takes into account the time and energy overhead required for channel training.

Second, for the general uncorrelated MIMO Rician fading channels with the proposed two-phase protocol, an optimization problem is formulated to maximize the net average harvested energy at the ER to obtain the optimal training design, which includes the optimal subset of ER antennas to be trained, as well as the training time and power that are allocated.

Third, as finding the optimal solution to the training design problem is challenging in general, we focus on three special scenarios of high practical interests to reveal useful insights, namely, MIMO Rayleigh fading channels as well as MISO and massive MIMO Rician fading channels. For the former two cases, the optimal training designs are obtained via convex optimization techniques. For massive MIMO setup, an approximate solution for the training design problem is obtained based on a newly derived lower bound of the net average harvested energy. For the special case of rank-1 deterministic MIMO Rician channel component, we show that the obtained approximate solution achieves the optimal asymptotic performance scaling as the ideal case assuming perfect CSI at the ET as  $M \to \infty$ .

The rest of this paper is organized as follows. Section II

introduces the system model. Section III discusses the proposed two-phase protocol for MIMO WET and formulates the training optimization problem for MIMO Rician fading channels. In Section IV, the formulated problem is solved for three special scenarios of MIMO Rayleigh fading channels, MISO and massive MIMO Rician fading channels, respectively. In Section V, numerical results are provided to corroborate our study. Finally, we conclude the paper and point out some future working directions in Section VI.

Notations: In this paper, scalars are denoted by italic letters. Boldface lower- and upper-case letters denote vectors and matrices, respectively.  $\mathbb{C}^{M \times N}$  denotes the space of  $M \times N$ complex matrices.  $\mathbb{E}_{\mathbf{X}}[\cdot]$  denotes the expectation with respect to the random matrix **X**.  $I_M$  denotes an  $M \times M$  identity matrix and 0 denotes an all-zero matrix. For an arbitrary-size matrix A, its transpose, Hermitian transpose, and Frobenius norm are respectively denoted as  $\mathbf{A}^T$ ,  $\hat{\mathbf{A}}^H$  and  $\|\mathbf{A}\|_F$ . For a square Hermitian matrix  $\mathbf{S}$ ,  $Tr(\mathbf{S})$  denotes its trace, while  $\lambda_{\max}(\mathbf{S})$  and  $\mathbf{v}_{\max}(\mathbf{S})$  denote its largest eigenvalue and the corresponding eigenvector, respectively. The symbol j represents the imaginary unit of complex number, i.e.,  $j^2 = -1$ .  $[\mathbf{A}]_{nm}$  denotes the (n, m)-th element of matrix  $\mathbf{A}$ , and  $[\mathbf{v}]_{1:N_1}$ denotes a vector consisting of the first  $N_1$  elements of vector v. For a set  $\mathcal{N}$ ,  $|\mathcal{N}|$  denotes its cardinality. Furthermore,  $\mathcal{N} \setminus \mathcal{N}_1$ denotes the complement of set  $\mathcal{N}_1$  in  $\mathcal{N}$ .

## II. SYSTEM MODEL

We consider a point-to-point MIMO WET system as shown in Fig. 1, where an ET with M antennas is employed to deliver wireless energy to the ER, which is equipped with N antennas. We assume a quasi-static flat fading channel model, where the baseband equivalent channel  $\mathbf{H} \in \mathbb{C}^{N \times M}$  from the ET to the ER remains constant within each coherent block of Tsymbol durations, and varies independently from one block to another. We consider the scenarios where a LOS link is generally present between the ET and the ER, for which  $\mathbf{H}$ can be modeled by the MIMO Rician fading channel as [28]

$$\mathbf{H} = \sqrt{\frac{\beta K}{K+1}} \mathbf{\bar{H}} + \sqrt{\frac{\beta}{K+1}} \mathbf{H}_{w}, \tag{1}$$

where  $\tilde{\mathbf{H}}$  is a deterministic  $N \times M$  matrix containing the Rician (including the LOS) components of the channel,  $\mathbf{H}_{w}$  is a random  $N \times M$  matrix with independent and identically distributed (i.i.d.) zero-mean unit-variance circularly symmetric complex Gaussian (CSCG) entries, i.e.,  $[\mathbf{H}_{w}]_{nm} \sim \mathcal{CN}(0, 1)$ , which represent the scattered components of the channel, and  $K \in [0, \infty)$  is the Rician K-factor denoting the power ratio between the Rician and the scattered components. Furthermore, the parameter  $\beta$  models the large-scale fading, which includes the effects of both distance-dependent path loss and shadowing [28]. Note that the deterministic Rician channel component  $\tilde{\mathbf{H}}$  can be modeled as [29]

$$\bar{\mathbf{H}} = \sum_{l=1}^{L} g_l \mathbf{a}_r(\theta_{r,l}) \mathbf{a}_t(\theta_{t,l})^H, \qquad (2)$$

where L is the number of deterministic paths,  $g_l$  is the complex amplitude of the *l*th path, and  $\theta_{r,l}$  and  $\theta_{t,l}$  are the angle of

arrival (AOA) and angle of departure (AOD), respectively. Moreover,  $\mathbf{a}_r(\theta)$  and  $\mathbf{a}_t(\theta)$  are the array responses at the ER and ET, respectively, which are given by [30]

$$\mathbf{a}_{r}(\theta) = \begin{bmatrix} 1, & e^{j\Phi_{1}(\theta)}, & \cdots, & e^{j\Phi_{(N-1)}(\theta)} \end{bmatrix}^{T} \\ \mathbf{a}_{t}(\theta) = \begin{bmatrix} 1, & e^{j\Phi_{1}(\theta)}, & \cdots, & e^{j\Phi_{(M-1)}(\theta)} \end{bmatrix}^{T}, \end{cases}$$
(3)

where  $\Phi_m$ ,  $m = 1, \dots, N-1$  or  $1, \dots, M-1$ , is the phase shift of the *m*th array element with respect to the reference antenna, which depends on the array configuration and is a function of the AOA/AOD. For the simple uniform linear array (ULA) configuration with antenna separation *d* in wavelengths, we have

$$\Phi_m(\theta) = 2\pi m d \sin(\theta), \ \forall m. \tag{4}$$

The total average received power of the channel given in (1) can be expressed as

$$\mathbb{E}\left[\|\mathbf{H}\|_{F}^{2}\right] = \frac{\beta K}{K+1} \operatorname{tr}(\bar{\mathbf{H}}\bar{\mathbf{H}}^{H}) + \frac{\beta}{K+1} \operatorname{tr}\left(\mathbb{E}\left[\mathbf{H}_{w}\mathbf{H}_{w}^{H}\right]\right)$$
$$= \frac{\beta K}{K+1} \operatorname{tr}(\bar{\mathbf{H}}\bar{\mathbf{H}}^{H}) + \frac{\beta}{K+1} MN, \tag{5}$$

where (5) is true since  $\mathbb{E} \left[ \mathbf{H}_{w} \mathbf{H}_{w}^{H} \right] = M \mathbf{I}_{N}$ . Note that in the above expression,  $\operatorname{tr}(\mathbf{\tilde{H}}\mathbf{\tilde{H}}^{H})$  gives the total power gain of the deterministic channel components. Without loss of generality, we assume

$$tr(\bar{\mathbf{H}}\bar{\mathbf{H}}^H) = MN, \tag{6}$$

so that  $\mathbb{E}\left[\|\mathbf{H}\|_{F}^{2}\right] = \beta MN, \forall K \in [0, \infty)$ . Therefore,  $\beta$  can be interpreted as the average channel power per transmit-receive antenna pair.

Note that by varying the Rician K-factor, the model in (1) includes a large class of MIMO wireless channels ranging from a fully random Rayleigh fading MIMO channel (K = 0) to a fully deterministic MIMO channel ( $K \to \infty$ ).

Within one coherent block of T symbols, the input-output relation for the forward link of MIMO WET system can be written as

$$\mathbf{y}[t] = \mathbf{H}\mathbf{x}[t] + \mathbf{n}[t], \ t = 1, \cdots, T,$$
(7)

where  $\mathbf{y}[t] \in \mathbb{C}^{N \times 1}$  is the received signal vector at the ER in the *t*th channel use;  $\mathbf{n}[t]$  denotes the additive Gaussian noise vector at the ER; and  $\mathbf{x}[t] \in \mathbb{C}^{M \times 1}$ ,  $t = 1, \dots, T$ , represents the energy-bearing signal transmitted by the ET with *sample* covariance matrix denoted by **S**, i.e.,

$$\mathbf{S} \triangleq \frac{1}{T} \sum_{t=1}^{T} \mathbf{x}[t] \mathbf{x}[t]^{H}.$$
(8)

Note that if  $\{\mathbf{x}[t]\}_{t=1}^{T}$  are i.i.d. zero-mean random vectors generated from certain distributions, the sample covariance matrix **S** approaches to the statistical covariance matrix as  $T \to \infty$ . In contrast to that in wireless communications where the end-to-end information rate achievable depends on the complete probability distribution (including the transmit covariance matrix) of the transmitted signal vector, it will become clear later (cf. (9)) that only the sample covariance matrix **S** given in (8) affects the amount of energy harvested



Fig. 1: A point-to-point MIMO WET system with reverse-link channel training and forward-link energy transmission.

at the ER during each coherent block in MIMO WET systems. Denote by  $P_f$  the average power constraint imposed at the ET in the forward link transmission. We then have  $tr(\mathbf{S}) \leq P_f$ .

At the ER, the incident RF power captured by each of the N receiving antennas is converted to usable direct current (DC) power by a device called rectifier, which generally consists of a Schottky diode and a lower-pass filter (LPF) [13]. Due to the law of energy conservation, the total harvested RF-band energy, denoted by Q, from all receiving antennas at the ER during each coherent block is proportional to that of the received baseband signal [4], i.e.,

$$Q = \sum_{t=1}^{T} \eta \left\| \mathbf{H} \mathbf{x}[t] \right\|^{2} = \eta T \operatorname{tr} \left( \mathbf{H} \left( \frac{1}{T} \sum_{t=1}^{T} \mathbf{x}[t] \mathbf{x}[t]^{H} \right) \mathbf{H}^{H} \right)$$
$$= \eta T \operatorname{tr} \left( \mathbf{H}^{H} \mathbf{H} \mathbf{S} \right), \tag{0}$$

where  $0 < \eta \leq 1$  denotes the energy harvesting efficiency at the ER. Note that we have ignored the energy harvested from the background noise since it is generally negligible as compared to that harvested from the dedicated energy signals. For convenience, we assume unit symbol duration throughout the paper so that the terms energy and power are used interchangeably. It has been shown in [4] that in the ideal case with perfect CSI H at the ET, the optimal covariance matrix that maximizes the harvested energy Q is

$$\mathbf{S}^{\star} = P_f \mathbf{v}_1 \mathbf{v}_1^H, \tag{10}$$

where  $\mathbf{v}_1 = \mathbf{v}_{\max} (\mathbf{H}^H \mathbf{H})$  denotes the eigenvector corresponding to the dominant eigenvalue of  $\mathbf{H}^H \mathbf{H}$ . The resulting maximum harvested energy is given by

$$Q_{\max} = \eta T P_f \lambda_{\max} \left( \mathbf{H}^H \mathbf{H} \right). \tag{11}$$

Since  $\mathbf{S}^{\star}$  is a rank-one matrix, the transmitted energy-bearing signal  $\mathbf{x}[t]$  satisfying (8) can be obtained with beamforming, i.e.,  $\mathbf{x}[t] = \sqrt{P_f}\mathbf{v}_1 s[t]$ ,  $t = 1, \dots, T$ , where s[t] is an arbitrary random signal with zero mean and unit sample variance satisfying  $1/T \sum_{t=1}^{T} |s[t]|^2 = 1$ .

Note that  $Q_{\text{max}}$  in (11) is generally a random variable depending on the channel realization **H**. The maximum average harvested energy over all random channel realizations can be expressed as

$$\bar{Q}_{\max} = \mathbb{E}_{\mathbf{H}}[Q_{\max}] = \eta T P_f \mathbb{E}_{\mathbf{H}} \left[ \lambda_{\max} \left( \mathbf{H}^H \mathbf{H} \right) \right].$$
(12)

With **H** given by the Rician model in (1), the probability density function (pdf) of the maximum eigenvalue  $\lambda_{\max} (\mathbf{H}^H \mathbf{H})$ has been extensively studied in the literature (see e.g. [31]), based on which the expectation  $\mathbb{E}_{\mathbf{H}} [\lambda_{\max}(\mathbf{H}^H \mathbf{H})]$  can be numerically computed.



Fig. 2: A two-phase protocol for MIMO wireless energy transfer.

In practice, only imperfect CSI can be made available at the ET due to the channel estimation and/or feedback errors. As a consequence, the maximum average harvested energy given in (12) cannot be achieved; instead, it only provides a performance upper bound for practical WET systems. In this paper, by exploiting the channel reciprocity between the forward (from the ET to the ER) and reverse (from the ER to the ET) links, we propose a two-phase protocol for the MIMO WET system for channel training and energy transmission, respectively. As illustrated in Fig. 2, the first phase corresponds to the first  $\tau < T$  symbol durations in each coherent block, where pilot symbols are sent by the ER to the ET for channel training using the energy harvested in previous blocks. Based on the received pilot signals, the ET obtains an estimate of the MIMO channel. In the second phase of the remaining T- $\tau$  symbol durations, based on the estimated channel, the ET transmits the energy-bearing signal with optimized transmit beamforming. These two phases are elaborated in more details in the next section.

## III. TWO-PHASE PROTOCOL

#### A. Reverse-Link Channel Training

We assume that the channel parameters  $\beta$ , K, and H are perfectly known at the ET since they typically vary slowly with time and hence are relatively easy to be estimated. On the other hand, the instantaneous channel realization H, or equivalently  $H_{w}$ , is estimated at the ET via reverse-link channel training. Since the entries in  $\mathbf{H}_{w}$  are i.i.d. random variables, orthogonal pilot signals, which are known to be optimal [32], are assumed here to be sent by the ER. Denote by  $\tau \leq T$  the number of symbol durations used for training in the first phase of each coherent block. To obtain an estimate for the complete channel matrix H, we need at least as many measurements at the ET as unknowns, which implies that  $M\tau \ge MN$  or  $\tau > N$ . Nevertheless, training for all ER antennas can be strictly suboptimal in general, as can be seen by considering the scenario with T = N, in which we have  $\tau = T$  and hence no time is left for energy transmission in the second phase. Therefore, to obtain the optimal training design, we need to consider a more general strategy where possibly only

a subset of the ER antennas is trained, which is denoted by the set  $\mathcal{N}_1 \subset \{1, \dots, N\}$  with  $|\mathcal{N}_1| = N_1$  and  $0 \leq N_1 \leq N$ . Denote the corresponding channel matrix as  $\mathbf{H}_{\mathcal{N}_1} \in \mathbb{C}^{N_1 \times M}$ , i.e.,  $\mathbf{H}_{\mathcal{N}_1}$  is a sub-matrix of  $\mathbf{H}$  obtained by only choosing the rows with indices belonging to  $\mathcal{N}_1$ . Similarly, denote by  $\mathcal{N}_2$  with  $|\mathcal{N}_2| = N_2$  the set of non-trained ER antennas and  $\mathbf{H}_{\mathcal{N}_2} \in \mathbb{C}^{N_2 \times M}$  the corresponding channel matrix. We thus have  $\mathcal{N}_2 = \{1, \dots, N\} \setminus \mathcal{N}_1$  and  $N_2 = N - N_1$ . The channel matrix  $\mathbf{H}$  can then be partitioned as

$$\mathbf{H} = \mathbf{\Pi} \begin{bmatrix} \mathbf{H}_{\mathcal{N}_1} \\ \mathbf{H}_{\mathcal{N}_2} \end{bmatrix}, \tag{13}$$

where  $\Pi$  is an  $N \times N$  permutation matrix that is determined by the choice of  $\mathcal{N}_1$ .

With the Rician channel model given in (1), the sub-matrices  $\mathbf{H}_{\mathcal{N}_1}$  and  $\mathbf{H}_{\mathcal{N}_2}$  can be expressed as

$$\mathbf{H}_{\mathcal{N}_{1}} = \sqrt{\frac{\beta K}{K+1}} \mathbf{\bar{H}}_{\mathcal{N}_{1}} + \sqrt{\frac{\beta}{K+1}} \mathbf{H}_{w,\mathcal{N}_{1}},$$

$$\mathbf{H}_{\mathcal{N}_{2}} = \sqrt{\frac{\beta K}{K+1}} \mathbf{\bar{H}}_{\mathcal{N}_{2}} + \sqrt{\frac{\beta}{K+1}} \mathbf{H}_{w,\mathcal{N}_{2}},$$
(14)

with  $\bar{\mathbf{H}}_{\mathcal{N}_1}$ ,  $\bar{\mathbf{H}}_{\mathcal{N}_2}$ ,  $\mathbf{H}_{w,\mathcal{N}_1}$ , and  $\mathbf{H}_{w,\mathcal{N}_2}$  denoting the corresponding sub-blocks in  $\bar{\mathbf{H}}$  and  $\mathbf{H}_w$ , respectively. During the training phase, pilot symbols are only sent from the  $N_1$  ER antennas in  $\mathcal{N}_1$ , which yields

$$\mathbf{Y}_{\rm tr} = \sqrt{\frac{P_r}{N_1}} \mathbf{\Phi} \mathbf{H}_{\mathcal{N}_1} + \mathbf{Z}_{\rm tr}, \qquad (15)$$

where  $\mathbf{Y}_{tr} \in \mathbb{C}^{\tau \times M}$  contains the received training signals at the ET with  $N_1 \leq \tau \leq T$ ;  $P_r$  is the training power used in the reverse link by the ER;  $\mathbf{\Phi} \in \mathbb{C}^{\tau \times N_1}$  denotes the orthogonal pilot signals sent by the ER with  $\mathbf{\Phi}^H \mathbf{\Phi} = \tau \mathbf{I}_{N_1}$ ; and  $\mathbf{Z}_{tr} \in \mathbb{C}^{\tau \times M}$  represents the noise received at the ET during the training phase with i.i.d. zero-mean CSCG entries each with variance  $\sigma_r^2$ . The energy consumed at the ER due to channel training is given by

$$E_{\rm tr} = \left\| \sqrt{\frac{P_r}{N_1}} \Phi \right\|_F^2 = \frac{P_r}{N_1} {\rm tr}(\Phi^H \Phi) = P_r \tau.$$
(16)

Based on (14) and (15), the minimum mean-square error (MMSE) estimate  $\hat{\mathbf{H}}_{\mathcal{N}_1}$  of  $\mathbf{H}_{\mathcal{N}_1}$  can be expressed as [33]

$$\hat{\mathbf{H}}_{\mathcal{N}_1} = \sqrt{\frac{\beta K}{K+1}} \overline{\mathbf{H}}_{\mathcal{N}_1} + \sqrt{\frac{\beta}{K+1}} \hat{\mathbf{H}}_{\mathrm{w},\mathcal{N}_1},$$

where  $\hat{\mathbf{H}}_{w,\mathcal{N}_1}$  is the MMSE estimate of  $\mathbf{H}_{w,\mathcal{N}_1}$  given by

$$\hat{\mathbf{H}}_{\mathrm{w},\mathcal{N}_{1}} = \frac{\sqrt{P_{r}\beta N_{1}(K+1)}}{P_{r}\tau\beta + \sigma_{r}^{2}N_{1}(K+1)} \boldsymbol{\Phi}^{H} \left( \mathbf{Y}_{\mathrm{tr}} - \sqrt{\frac{P_{r}\beta K}{N_{1}(K+1)}} \boldsymbol{\Phi}\bar{\mathbf{H}}_{\mathcal{N}_{1}} \right).$$
(17)

Let  $\hat{\mathbf{H}}_{w,\mathcal{N}_1}$  denote the estimation error of  $\mathbf{H}_{w,\mathcal{N}_1}$ , i.e.,  $\tilde{\mathbf{H}}_{w,\mathcal{N}_1} = \mathbf{H}_{w,\mathcal{N}_1} - \hat{\mathbf{H}}_{w,\mathcal{N}_1}$ . Based on the well-known orthogonal property of the MMSE estimation for Gaussian random variables [33],  $\hat{\mathbf{H}}_{w,\mathcal{N}_1}$  and  $\tilde{\mathbf{H}}_{w,\mathcal{N}_1}$  are independent. Furthermore, it can be obtained that  $\hat{\mathbf{H}}_{w,\mathcal{N}_1}$  and  $\tilde{\mathbf{H}}_{w,\mathcal{N}_1}$  have i.i.d. zero-mean CSCG entries with variances  $\sigma^2_{\hat{\mathbf{H}}_{w,\mathcal{N}_1}}$  and  $\sigma^2_{\tilde{\mathbf{H}}_{w,\mathcal{N}_1}}$ , respectively, where

$$\sigma_{\hat{\mathbf{H}}_{\mathrm{w},\mathcal{N}_{1}}}^{2} = \frac{\beta P_{r}\tau}{\beta P_{r}\tau + \sigma_{r}^{2}N_{1}(K+1)}$$
(18)

$$\sigma_{\tilde{\mathbf{H}}_{w,N_1}}^2 = \frac{\sigma_r^2 N_1(K+1)}{\beta P_r \tau + \sigma_r^2 N_1(K+1)}.$$
 (19)

With (13), (14) and the identity  $\mathbf{H}_{w,\mathcal{N}_1} = \mathbf{H}_{w,\mathcal{N}_1} + \mathbf{H}_{w,\mathcal{N}_1}$ , the channel matrix  $\mathbf{H}$  can be decomposed as

where  $\hat{\mathbf{H}}$  denotes the CSI known at the ET, including the deterministic Rician complements  $\bar{\mathbf{H}}$  as well as the estimated random components  $\hat{\mathbf{H}}_{w,\mathcal{N}_1}$  corresponding to the  $N_1$  trained ER antennas in  $\mathcal{N}_1$ , and  $\tilde{\mathbf{H}}$  represents the CSI discrepancy from the true channel, including both the channel estimation error  $\hat{\mathbf{H}}_{w,\mathcal{N}_1}$  for the trained ER antennas and the unestimated random components  $\mathbf{H}_{w,\mathcal{N}_2}$  of the  $N_2$  non-trained ER antennas. Note that since  $\hat{\mathbf{H}}_{w,\mathcal{N}_1}$ ,  $\tilde{\mathbf{H}}_{w,\mathcal{N}_1}$ , and  $\mathbf{H}_{w,\mathcal{N}_2}$  are independent random matrices,  $\hat{\mathbf{H}}$  and  $\tilde{\mathbf{H}}$  are independent as well.

## B. Forward-Link Energy Transmission

After the training-based channel estimation, energy-bearing signals are transmitted by the ET based on the estimated channel  $\hat{\mathbf{H}}$  during the remaining  $T - \tau$  symbol durations. Similar to (9), the total harvested energy within one coherent block can be expressed as

$$Q = \eta (T - \tau) \operatorname{tr} \left( \mathbf{H}^{H} \mathbf{HS} \right).$$
(21)

Since the ET only has the knowledge of imperfectly and possibly partially estimated channel matrix  $\hat{\mathbf{H}}$  as given in (20), the optimal transmit covariance matrix  $\mathbf{S}^*$  given in (10) cannot be implemented. In this case, given the knowledge of  $\hat{\mathbf{H}}$ ,  $\mathbf{S}$  is optimized at the ET to maximize the average harvested energy  $\hat{Q} = \mathbb{E}_{\tilde{\mathbf{H}}}[Q|\hat{\mathbf{H}}]$ . With the identity  $\mathbf{H} = \hat{\mathbf{H}} + \tilde{\mathbf{H}}$ , we have

$$\begin{aligned} \widehat{Q} &= \eta (T - \tau) \mathbb{E}_{\widetilde{\mathbf{H}}} \Big[ \operatorname{tr} \big( \mathbf{H}^{H} \mathbf{H} \mathbf{S} \big) \big| \widehat{\mathbf{H}} \Big] \\ &= \eta (T - \tau) \mathbb{E}_{\widetilde{\mathbf{H}}} \Big[ \operatorname{tr} \big( \big( \widehat{\mathbf{H}}^{H} \widehat{\mathbf{H}} + \widetilde{\mathbf{H}}^{H} \widetilde{\mathbf{H}} + \widehat{\mathbf{H}}^{H} \widetilde{\mathbf{H}} + \widetilde{\mathbf{H}}^{H} \widehat{\mathbf{H}} \big) \mathbf{S} \big) \big| \widehat{\mathbf{H}} \Big] \\ &= \eta (T - \tau) \operatorname{tr} \left( \left( \big( \widehat{\mathbf{H}}^{H} \widehat{\mathbf{H}} + \mathbb{E}_{\widetilde{\mathbf{H}}} \Big[ \widetilde{\mathbf{H}}^{H} \widetilde{\mathbf{H}} \Big] \big) \mathbf{S} \right) \end{aligned}$$
(22)

$$= \eta (T - \tau) \operatorname{tr} \left( \left( \widehat{\mathbf{H}}^{H} \widehat{\mathbf{H}} + \frac{\beta}{K+1} \left( N_{1} \sigma_{\widehat{\mathbf{H}}_{w,\mathcal{N}_{1}}}^{2} + N_{2} \right) \mathbf{I}_{M} \right) \mathbf{S} \right),$$
(23)

where (22) follows from the independence between **H** and **H**, and (23) follows from the definition of  $\tilde{\mathbf{H}}$  given in (20). With the average transmit power constraint  $P_f$  imposed at the ET, similar to (10), the harvested energy  $\widehat{Q}$  in (23) is maximized by setting the transmit covariance matrix to be

$$\mathbf{S} = P_f \mathbf{v}_E \mathbf{v}_E^H, \tag{24}$$

where

$$\mathbf{v}_{E} = \mathbf{v}_{\max} \Big( \widehat{\mathbf{H}}^{H} \widehat{\mathbf{H}} + \frac{\beta}{K+1} \big( N_{1} \sigma_{\widetilde{\mathbf{H}}_{w,\mathcal{N}_{1}}}^{2} + N_{2} \big) \mathbf{I}_{M} \Big)$$
(25)  
$$= \mathbf{v}_{\max} \big( \widehat{\mathbf{H}}^{H} \widehat{\mathbf{H}} \big).$$
(26)

The resulting maximum value of  $\widehat{Q}$  is then given by

$$\widehat{Q}^{\star} = \eta (T - \tau) P_f \left( \lambda_{\max} \left( \widehat{\mathbf{H}}^H \widehat{\mathbf{H}} \right) + \frac{\beta}{K+1} \left( N_1 \sigma_{\widehat{\mathbf{H}}_{w,N_1}}^2 + N_2 \right) \right)$$
(27)

It is observed from (26) that, as the channel estimation error  $\widetilde{\mathbf{H}}$  is isotropic, the estimated channel matrix  $\widehat{\mathbf{H}}$  can be treated as the true channel by the ET for maximum energy transfer.

With (27), the average harvested energy  $\bar{Q}$  can be expressed as

$$\bar{Q}(\mathcal{N}_{1},\tau,P_{r}) = \mathbb{E}_{\widehat{\mathbf{H}}}[\widehat{Q}^{\star}] = \eta(T-\tau)P_{f}\left(\mathbb{E}_{\widehat{\mathbf{H}}_{w,\mathcal{N}_{1}}}\left[\lambda_{\max}\left(\widehat{\mathbf{H}}^{H}\widehat{\mathbf{H}}\right)\right] + \frac{\beta}{K+1}\left(N_{1}\sigma_{\widehat{\mathbf{H}}_{w,\mathcal{N}_{1}}}^{2} + N_{2}\right)\right),$$
(28)

where  $\hat{\mathbf{H}}$  is a random matrix dependent on  $\hat{\mathbf{H}}_{w,\mathcal{N}_1}$  given in (20). Note that  $\bar{Q}$  in (28) depends on the subset of the training ER antennas  $\mathcal{N}_1$ , the training duration  $\tau$ , as well as the training power  $P_r$ .

The *net* average harvested energy, i.e., the average harvested energy offset by the energy consumed for sending pilot signals at the ER, is then given by

$$\bar{Q}_{\text{net}}(\mathcal{N}_1, \tau, P_r) = \bar{Q}(\mathcal{N}_1, \tau, P_r) - P_r \tau.$$
(29)

The problem of finding the optimal training design so that  $\bar{Q}_{net}$  is maximized can thus be formulated as

(P1): 
$$\max_{\mathcal{N}_{1},\tau,P_{r}} \bar{Q}_{\text{net}}(\mathcal{N}_{1},\tau,P_{r})$$
  
s.t. 
$$\mathcal{N}_{1} \subset \{1,\cdots,N\},$$
$$N_{1} \leq \tau \leq T,$$
$$P_{r} > 0.$$

Note that there is an implicit constraint  $P_r \tau < \bar{Q}(N_1, \tau, P_r)$ in (P1) to ensure the strict positiveness of the objective value (net average energy). In (P1), we have assumed that there is a sufficiently large initial energy stored at the ER; under this assumption and further considering the fact that our optimized  $\bar{Q}_{net}$  in (P1) is strictly positive and generally large enough (for implementing other functions at the ER such as sensing and data transmission), we can assume that there is always energy available at the ER to send the training signal with power  $P_r$ at all time.

#### IV. OPTIMAL TRAINING DESIGN

Finding the optimal solution to (P1) is challenging in general. One reason is that it involves discrete optimization over the set  $\mathcal{N}_1$ , which in general requires an exhaustive search over  $2^N$  possibilities and thus becomes prohibitive for large N. Besides, even for fixed training set  $\mathcal{N}_1$ , finding the optimal solution for training duration  $\tau$  and training power  $P_r$ 

is non-trivial, which is mainly due to the difficulty in obtaining a closed-form expression for  $\mathbb{E}_{\hat{\mathbf{H}}_{w,\mathcal{N}_1}}\left[\lambda_{\max}(\hat{\mathbf{H}}^H\hat{\mathbf{H}})\right]$  for the general case. In this section, we derive the training strategies by efficiently solving (P1) in three special cases of high practical interests, namely, the MIMO Rayleigh fading channel, as well as the MISO and massive MIMO Rician fading channels.

#### A. MIMO Rayleigh Fading Channel

We first consider the scenario where there is no LOS link between the ET and the ER. In this case, the MIMO Rician fading channel in (1) reduces to MIMO Rayleigh fading channel with K = 0. It then follows from the definition of  $\hat{\mathbf{H}}$  given in (20) that

$$\mathbb{E}_{\hat{\mathbf{H}}_{w,\mathcal{N}_{1}}} \left[ \lambda_{\max}(\widehat{\mathbf{H}}^{H} \widehat{\mathbf{H}}) \right] = \beta \mathbb{E}_{\hat{\mathbf{H}}_{w,\mathcal{N}_{1}}} \left[ \lambda_{\max}(\widehat{\mathbf{H}}^{H}_{w,\mathcal{N}_{1}} \widehat{\mathbf{H}}_{w,\mathcal{N}_{1}}) \right]$$
(30)  
$$= \beta \sigma_{\hat{\mathbf{H}}_{w,\mathcal{N}_{1}}}^{2} \mathbb{E}_{\mathbf{H}_{n}} \left[ \lambda_{\max}(\mathbf{H}_{n}^{H} \mathbf{H}_{n}) \right]$$
(31)

$$\triangleq \beta \sigma_{\hat{\mathbf{H}}_{\mathbf{w},\mathcal{N}_{1}}}^{2} \Lambda(M,N_{1}), \qquad (32)$$

where  $\mathbf{H}_{n} \triangleq \hat{\mathbf{H}}_{w,\mathcal{N}_{1}}/\sigma_{\hat{\mathbf{H}}_{w,\mathcal{N}_{1}}}$  is the normalized channel matrix with i.i.d. zero-mean unit-variance CSCG entries. The pdf of the maximum eigenvalue  $\lambda_{\max}(\mathbf{H}_{n}^{H}\mathbf{H}_{n})$  has been extensively studied in the literature (see e.g. [34]), based on which the expectation  $\mathbb{E}_{\mathbf{H}_{n}} [\lambda_{\max}(\mathbf{H}_{n}^{H}\mathbf{H}_{n})]$  can be computed. Note that  $\mathbb{E}_{\mathbf{H}_{n}} [\lambda_{\max}(\mathbf{H}_{n}^{H}\mathbf{H}_{n})]$  depends only on the dimension of  $\mathbf{H}_{n}$ and hence is denoted as a function  $\Lambda(M, N_{1})$  in (32). In the special cases of  $N_{1} = 1$  or M = 1, it can be easily obtained that  $\Lambda(M, 1) = M$  and  $\Lambda(1, N_{1}) = N_{1}$ . For general M and  $N_{1}$ , no closed-form expression for  $\Lambda(M, N_{1})$  is available, whereas its numerical values can be easily computed, e.g., based on the algorithm proposed in [35].

By substituting (32) into (28) and applying (18) and (19) with K = 0, the average harvested energy for the WET system in MIMO Rayleigh fading channels can be expressed as

$$\bar{Q}^{\mathrm{R}}(N_1, \tau, P_r) = \eta(T - \tau) P_f \beta \Big( \frac{P_r \tau \beta \Lambda(M, N_1) + \sigma_r^2 N_1^2}{P_r \tau \beta + \sigma_r^2 N_1} + N_2 \Big).$$
(33)

Several observations can be made from (33). First, the training antenna subset  $\mathcal{N}_1$  affects the average harvested energy only through its cardinality  $N_1$ . This is expected since for MIMO Rayleigh fading channels without the deterministic LOS component, all the N ER antennas are statistically equivalent. Therefore, determining the optimal training antenna subset  $\mathcal{N}_1$  is tantamount to finding the optimal number of training antennas  $N_1$ . Another observation from (33) is that the average harvested energy  $\bar{Q}^{\mathrm{R}}$  is given by a summation of two terms. The first term, which monotonically increases with the training energy  $P_r \tau$  and the number of ET antennas M, is attributed to the  $N_1$  trained ER antennas whose channel matrix  $\mathbf{H}_{\mathcal{N}_1}$  is estimated at the ET. The second term is attributed to the  $N_2$ non-trained ER antennas, which is independent of the number of ET antennas M since no beamforming gain can be achieved for the energy transmission over the associated channel  $\mathbf{H}_{\mathcal{N}_2}$ . The net average harvested energy, as defined in (29), can then be explicitly written as

$$\bar{Q}_{\text{net}}^{\text{R}}(N_1, \tau, P_r) = \eta (T - \tau) P_f \beta \left( \frac{P_r \tau \beta \Lambda(M, N_1) + \sigma_r^2 N_1^2}{P_r \tau \beta + \sigma_r^2 N_1} + N - N_1 \right) - P_r \tau.$$
(34)

Therefore, the training optimization problem (P1) for the case of MIMO Rayleigh fading channels can be formulated as

$$(P1-R): \max_{N_1,\tau,P_r} Q_{net}^R(N_1,\tau,P_r)$$
  
s.t.  $0 \le N_1 \le N,$   
 $N_1 \le \tau \le T,$   
 $P_r \ge 0.$ 

To optimally solve problem (P1-R), we first show the following lemma.

*Lemma 1:* The optimal solution  $(N_1^{\star}, \tau^{\star}, P_r^{\star})$  to problem (P1-R) satisfies  $\tau^{\star} = N_1^{\star}$ .

*Proof:* Please refer to Appendix A. By applying Lemma 1, (P1-R) can be recast as

$$\max_{N_1, P_r} \quad \bar{Q}_{\text{net}}^{\text{R}}(N_1, P_r)$$
s.t.  $0 \le N_1 \le N, P_r \ge 0,$ 

$$(35)$$

where  $\bar{Q}_{net}^{R}(N_1, P_r)$  is obtained by substituting  $\tau = N_1$  into (34) and is given by

$$\bar{Q}_{\text{net}}^{\text{R}}(N_1, P_r) = \eta (T - N_1) P_f \beta \Big( \frac{P_r \beta \Lambda(M, N_1) + \sigma_r^2 N_1}{P_r \beta + \sigma_r^2} + N - N_1 \Big) - P_r N_1.$$
(36)

To find the optimal solution to problem (35), we first obtain the optimal training power  $P_r$  with  $N_1$  fixed by solving the following optimization problem:

$$\max_{P_r \ge 0} \quad \bar{Q}_{\text{net}}^{\text{R}}(N_1, P_r). \tag{37}$$

Denote the optimal solution and the optimal value of problem (37) as  $P_r^{\rm R}(N_1)$  and  $\bar{Q}_{\rm net}^{\rm R}(N_1)$ , respectively. When  $N_1 = 0$ , it follows trivially that  $P_r^{\rm R}(0) = 0$  and the corresponding energy is

$$\bar{Q}_{\text{net}}^{\text{R}}(0) = \eta T P_f \beta N.$$
(38)

For  $1 \le N_1 \le N$ , it can be verified that  $\bar{Q}_{net}^{R}(N_1, P_r)$  given in (36) is a concave function with respect to  $P_r$ ; hence problem (37) is convex, whose solution can be found as

$$P_{r}^{\mathrm{R}}(N_{1}) = \sqrt{\eta P_{f} \sigma_{r}^{2}} \left[ \sqrt{(T - N_{1}) \left( \frac{\Lambda(M, N_{1})}{N_{1}} - 1 \right)} - \frac{1}{\sqrt{\Gamma}} \right]^{+}$$
(39)

where  $[x]^+ \triangleq \max\{x, 0\}$ , and

$$\Gamma \triangleq \eta P_f \beta^2 / \sigma_r^2 \tag{40}$$

is referred to as the two-way *effective* signal-to-noise ratio (ESNR). Note that the term  $\beta^2$  in  $\Gamma$  captures the effect of twoway signal attenuation due to both the reverse-link training and the forward-link energy transmission. The optimal value of problem (37) for  $1 \leq N_1 \leq N$  can then be expressed as (41) given on top of the next page, where the set  $\mathcal{N}_R$  is defined as

$$\mathcal{N}_{\mathrm{R}} \triangleq \left\{ 1 \le N_1 \le N : (T - N_1) \left( \frac{\Lambda(M, N_1)}{N_1} - 1 \right) > \frac{1}{\Gamma} \right\}.$$
(42)

Finding the optimal solution to problem (35) and that to the original problem (P1-R) now reduces to determining the optimal number of ER antennas  $N_1^{\star}$  to be trained, which can be easily obtained by comparing  $\bar{Q}_{net}^{R}(N_1)$  for the N + 1possible values of  $N_1 \in \{0, 1, \dots, N\}$ . In fact, as evident from (38) and (41), if  $N_1 \ge 1$  and  $N_1 \notin \mathcal{N}_{R}$ , we always have  $\bar{Q}_{net}^{R}(N_1) < \bar{Q}_{net}^{R}(0)$ . Therefore, the searching space for  $N_1^{\star}$ can be reduced as

$$N_{1}^{\star} = \arg \max_{N_{1} \in \{0\} \cup \mathcal{N}_{\mathrm{R}}} \bar{Q}_{\mathrm{net}}^{\mathrm{R}}(N_{1}), \tag{43}$$

which can be readily determined given the closed-form expressions (38) and (41).

It follows from (43) that  $N_1^* > 0$  only when the set  $\mathcal{N}_{\rm R}$ is non-empty. Thus, it can be inferred from (42) that for WET system in MIMO Rayleigh fading channels, channel training helps only if the following conditions are satisfied: (i) the channel coherence time T is sufficiently large; (ii) the ratio  $\Lambda(M, N_1)/N_1$  is sufficiently large, i.e., the number of ET antennas M is large enough; and (iii) the ESNR  $\Gamma$  is sufficiently high; otherwise, the benefit of channel training and hence the energy beamforming based on the estimated channel cannot compensate the time and the energy used for sending the pilot symbols by the ER, and thus no training should be applied; instead, it is optimal to assign all the Tsymbol durations to transmit the energy signals isotropically (since there is no LOS channel in this case), as can be seen from (25) that with  $N_1 = 0$  and hence  $\mathbf{H} = \mathbf{0}$ , the energy beamforming vector  $\mathbf{v}_E$  can be set as any arbitrary unit-norm vector.

Note that for the special case of MISO setup with N = 1, by applying the identity  $\Lambda(M, 1) = M$ , the optimal solution to (43) can be found as

$$N_1^{\star} = \begin{cases} 1, & \text{if } TM - T - M > \frac{1}{\Gamma} + \frac{2}{\sqrt{\Gamma}} \\ 0, & \text{otherwise.} \end{cases}$$
(44)

#### B. MISO Rician Fading Channel

In this subsection, we derive the optimal training strategy by solving (P1) for the case of MISO Rician fading channels, i.e., N = 1 and K > 0. In this case, the antenna training set  $\mathcal{N}_1$ in (P1) has only two possibilities:  $\mathcal{N}_1 = \emptyset$  and  $\mathcal{N}_1 = \{1\}$ , or equivalently,  $N_1 = 0$  and  $N_1 = 1$ , i.e., only a binary decision needs to be made at the ER to send the pilot signals or not. Furthermore, the channel matrix **H** given in (1) reduces to a  $1 \times M$  vector, and hence is denoted by a vector notation  $\mathbf{h}$ .<sup>1</sup>

<sup>&</sup>lt;sup>1</sup>Similarly, all other matrix notations defined in the previous sections are replaced by the corresponding vector notations throughout this subsection when MISO channel is considered, e.g.,  $\hat{\mathbf{h}}$  and  $\tilde{\mathbf{h}}$  correspond to  $\hat{\mathbf{H}}$  and  $\tilde{\mathbf{H}}$  defined in (20) with N = 1, respectively.

$$\bar{Q}_{\text{net}}^{\text{R}}(N_1) = \begin{cases} (T - N_1)\eta P_f \beta N + \eta P_f \beta N_1 \left( \sqrt{(T - N_1) \left( \frac{\Lambda(M, N_1)}{N_1} - 1 \right)} - \frac{1}{\sqrt{\Gamma}} \right)^2, & \text{if } N_1 \in \mathcal{N}_{\text{R}} \\ (T - N_1)\eta P_f \beta N, & \text{otherwise,} \end{cases}$$
(41)

The expectation in (28) for both  $N_1 = 0$  and  $N_1 = 1$  can be explicitly expressed as

$$\mathbb{E}_{\hat{\mathbf{h}}_{w,\mathcal{N}_{1}}}\left[\lambda_{\max}(\hat{\mathbf{h}}^{H}\hat{\mathbf{h}})\right] = \mathbb{E}_{\hat{\mathbf{h}}_{w,\mathcal{N}_{1}}}\left[\|\hat{\mathbf{h}}\|^{2}\right]$$
$$= \frac{\beta K}{K+1}\|\bar{\mathbf{h}}\|^{2} + \frac{\beta}{K+1}\mathbb{E}\left[\|\hat{\mathbf{h}}_{w,1}\|^{2}\right]$$
$$= \frac{\beta MK}{K+1} + \frac{\beta MN_{1}}{K+1}\sigma_{\hat{\mathbf{h}}_{w,1}}^{2}, \quad (45)$$

where in (45) we have used the identity (6). By substituting (45) into (28) and (29), and applying (18) and (19), the net average harvested energy for MISO Rician fading channels can be explicitly expressed as (46) given on top of the next page.

The training optimization problem (P1) for the case of MISO Rician fading channels can then be formulated as

$$(P1-MISO): \max_{N_1,\tau,P_r} \bar{Q}_{net}^{MISO}(N_1,\tau,P_r)$$
  
s.t.  $N_1 \in \{0,1\},$   
 $N_1 \le \tau \le T,$   
 $P_r \ge 0.$ 

The optimal solution to (P1-MISO) can be obtained by separately considering the two cases with  $N_1 = 0$  and  $N_1 = 1$ .

Case 1:  $N_1 = 0$ , i.e. no channel training is performed. In this case, (46) reduces to

$$\bar{Q}_{\text{net}}^{\text{MISO}}(0,\tau,P_r) = \eta(T-\tau)P_f\beta \frac{KM+1}{K+1} - P_r\tau.$$
 (47)

It then follows trivially that we should have  $\tau = 0$  and  $P_r = 0$ , which yields

$$\bar{Q}_{\text{net}}^{\text{MISO}}(N_1 = 0) = \eta T P_f \beta \frac{KM + 1}{K + 1}.$$
 (48)

It can be obtained based on (25) and (20) that the average harvested energy given in (48) with no channel training is achieved by the ET sending the energy-bearing signals only based on the LOS component of the channel, which is intuitively understood since the random channel component is isotropic and is not estimated at the ET.

Case 2:  $N_1 = 1$ . In this case, (P1-MISO) reduces to

$$\max_{\tau, P_r} \quad \eta(T-\tau) P_f \beta \left( \frac{KM}{K+1} + \frac{M\beta P_r \tau / (K+1) + \sigma_r^2}{\beta P_r \tau + \sigma_r^2 (K+1)} \right) - I$$
  
s.t.  $1 \le \tau \le T$ ,  
 $P_r \ge 0$ .

(49)

Following similar derivations as in the previous subsection, it can be obtained that the optimal solution  $(\tau^*, P_r^*)$  to problem (49) is

$$\tau^{\star} = 1, \ P_{r}^{\star} = \sqrt{\eta P_{f} \sigma_{r}^{2}} \left[ \sqrt{(T-1)(M-1)} - \frac{K+1}{\sqrt{\Gamma}} \right]^{+},$$
(50)

where  $\Gamma$  is the ESNR defined in (40). The corresponding optimal value of (49) is

$$\bar{Q}_{\text{net}}^{\text{MISO}}(1) = \begin{cases} (T-1)\eta P_f \beta \frac{KM+1}{K+1} + \frac{\eta P_f \beta}{K+1} \left( \sqrt{(T-1)(M-1)} - \frac{K+1}{\sqrt{\Gamma}} \right)^2, & \text{if } (T-1)(M-1) > \frac{(K+1)^2}{\Gamma} \\ (T-1)\eta P_f \beta \frac{KM+1}{K+1}, & \text{otherwise.} \end{cases}$$
(51)

The optimal solution to (P1-MISO) can then be obtained by comparing  $\bar{Q}_{net}^{MISO}(N_1 = 0)$  in (48) and  $\bar{Q}_{net}^{MISO}(N_1 = 1)$  in (51). After some further derivations, the optimal value for  $N_1$ to problem (P1-MISO) can be obtained as

$$N_1^{\star} = \begin{cases} 1, & \text{if } (T-1)(M-1) > \left(\sqrt{KM+1} + \frac{K+1}{\sqrt{\Gamma}}\right)^2\\ 0, & \text{otherwise.} \end{cases}$$
(52)

It is observed from (52) that, similar to the case of MIMO Rayleigh fading channels, channel training helps for WET in MISO Rician fading channels only if T, M, and  $\Gamma$  are sufficiently large. In addition, the Rician factor K needs to be sufficiently small for channel training to be effective; otherwise, it is optimal for the ET to send with the energy beamforming based on the deterministic Rician components  $\bar{\mathbf{h}}$  of the channel.

Note that by setting K = 0, the solution (52) reduces to that given in (44) for the special case of MISO Rayleigh fading channel.

## C. Massive MIMO Rician Fading Channel

Massive MIMO techniques have been recently advanced to tremendously improve the energy and spectrum efficiency of wireless communication networks by deploying a very large number of antennas (say, hundreds or even more) at the base stations [20], [21]. For WET system with a large number of antennas deployed at the ET, it is expected that the endto-end energy transfer efficiency can be similarly enhanced, provided that the channel is properly learned at the ET to  $P_r$  achieve the enormous beamforming gain. In this subsection, we optimize the training design for massive MIMO WET systems by solving the optimization problem (P1) with a large value of M, i.e.,  $M \gg N$ .

Based on the definition of  $\hat{\mathbf{H}}$  in (20) and the law of large numbers, the following asymptotic result holds for large M:

$$\frac{1}{M}\widehat{\mathbf{H}}\widehat{\mathbf{H}}^{H} \stackrel{a.s.}{\to} \frac{1}{M} \frac{\beta K}{K+1} \overline{\mathbf{H}}\overline{\mathbf{H}}^{H} + \frac{\beta}{K+1} \Pi \begin{bmatrix} \frac{1}{M}\widehat{\mathbf{H}}_{\mathrm{w},\mathcal{N}_{1}} \widehat{\mathbf{H}}_{\mathrm{w},\mathcal{N}_{1}}^{H} & \mathbf{0} \\ \mathbf{0} \end{bmatrix} \Pi^{T}$$

$$\stackrel{a.s.}{\to} \frac{1}{M} \frac{\beta K}{K+1} \overline{\mathbf{H}}\overline{\mathbf{H}}^{H} + \frac{\beta \sigma_{\widehat{\mathbf{H}}_{\mathrm{w},\mathcal{N}_{1}}}^{2}}{K+1} \Pi \begin{bmatrix} \mathbf{I}_{N_{1}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \Pi^{T},$$
(53)

$$\bar{Q}_{\text{net}}^{\text{MISO}}(N_1, \tau, P_r) = \eta(T - \tau) P_f \beta \left( \frac{KM}{K+1} + \frac{N_1 \left( M\beta P_r \tau / (K+1) + \sigma_r^2 \right)}{\beta P_r \tau + \sigma_r^2 N_1 (K+1)} + \frac{1 - N_1}{K+1} \right) - P_r \tau.$$
(46)

where  $\stackrel{a.s.}{\rightarrow}$  denotes the almost sure convergence. As a result, we have

$$\mathbb{E}_{\hat{\mathbf{H}}_{\mathrm{w},\mathcal{N}_{1}}}\left[\lambda_{\mathrm{max}}(\hat{\mathbf{H}}^{H}\hat{\mathbf{H}})\right] = \mathbb{E}_{\hat{\mathbf{H}}_{\mathrm{w},\mathcal{N}_{1}}}\left[\lambda_{\mathrm{max}}(\hat{\mathbf{H}}\hat{\mathbf{H}}^{H})\right]$$
(54)

$$\stackrel{a.s.}{\rightarrow} \lambda_{\max} \left( \frac{\beta K}{K+1} \bar{\mathbf{H}} \bar{\mathbf{H}}^{H} + \frac{\beta M \sigma_{\hat{\mathbf{H}}_{w,N_{1}}}^{2}}{K+1} \mathbf{\Pi} \begin{bmatrix} \mathbf{I}_{N_{1}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \mathbf{\Pi}^{T} \right)$$

$$\geq \frac{\beta K \bar{\lambda}}{K+1} + \frac{\beta M \sigma_{\hat{\mathbf{H}}_{w,N_{1}}}^{2}}{K+1} \left\| \left[ \mathbf{\Pi}^{T} \bar{\mathbf{v}} \right]_{1:N_{1}} \right\|^{2},$$
(55)

where  $\bar{\lambda}$  and  $\bar{\mathbf{v}}$  are the dominant eigenvalue and the corresponding eigenvector of the matrix  $\bar{\mathbf{H}}\bar{\mathbf{H}}^{H}$ , respectively, i.e.,  $\bar{\lambda} \triangleq \lambda_{\max}(\bar{\mathbf{H}}\bar{\mathbf{H}}^{H})$ , and  $\bar{\mathbf{v}} \triangleq \mathbf{v}_{\max}(\bar{\mathbf{H}}\bar{\mathbf{H}}^{H})$ . The inequality in (55) is obtained by directly applying the following result for any two Hermitian matrices **A** and **B**:

$$\lambda_{\max}(\mathbf{A} + \mathbf{B}) = \max_{\|\mathbf{v}\|=1} \mathbf{v}^{H}(\mathbf{A} + \mathbf{B})\mathbf{v}$$
(56)

$$\geq \mathbf{v}_{\mathbf{A}}^{H}\mathbf{A}\mathbf{v}_{\mathbf{A}} + \mathbf{v}_{\mathbf{A}}^{H}\mathbf{B}\mathbf{v}_{\mathbf{A}}$$
(57)

$$=\lambda_{\max}(\mathbf{A}) + \mathbf{v}_{\mathbf{A}}^{H} \mathbf{B} \mathbf{v}_{\mathbf{A}}, \qquad (58)$$

where  $v_A$  is the eigenvector corresponding to the dominant eigenvalue of matrix **A**. Note that equality holds in (57) when **A** and **B** have the same dominant eigenvector.

By applying (18), (19), and (55) into (29), it can be obtained that the net average harvested energy for massive MIMO Rician fading channels with large M is lower-bounded as (59) given on top of the next page.

Based on (55) and (57), it can be shown that the above lower bound is tight when  $N_1 = 0$  or  $N_1 = N$ . Note that in (59), the training antenna subset  $\mathcal{N}_1$  affects  $\tilde{Q}_{net}^{large-M}$ via both the corresponding permutation matrix  $\Pi$  and the cardinality  $N_1$ . In fact, as can be seen from (13),  $\mathcal{N}_1$  can be determined once  $\Pi$  and  $N_1$  are obtained, by re-ordering all the N ER antennas based on the permutation matrix  $\Pi^T$  and then selecting the first  $N_1$  elements as the set  $\mathcal{N}_1$ . Therefore, the training optimization problem (P1) in massive MIMO Rician fading channels can be approximately solved by maximizing the lower bound given in (59) as

$$\begin{array}{ll} (\text{P1-large-M}): & \max_{\boldsymbol{\Pi},N_{1},\tau,P_{r}} & \tilde{Q}_{\text{net}}^{\text{large-}M}(\boldsymbol{\Pi},N_{1},\tau,P_{r})\\ & \text{s.t.}: & \boldsymbol{\Pi} \text{ is a permutation matrix}\\ & 0 \leq N_{1} \leq N,\\ & N_{1} \leq \tau \leq T,\\ & P_{r} > 0. \end{array}$$

To find the optimal solution to (P1-large-M), we first determine the optimal permutation matrix  $\mathbf{\Pi}$ . It can be inferred from (59) that for any given  $N_1 \in \{0, 1, \dots, N\}$ ,  $\tilde{Q}_{net}^{large-M}$  is maximized if the absolute values of the elements in the vector  $\mathbf{\Pi}^T \bar{\mathbf{v}}$  are arranged in a non-decreasing order, which thus gives the optimal permutation matrix  $\mathbf{\Pi}^*$  for (P1-large-M). Furthermore, with similar arguments as that for the proof of

Lemma 1, it can be verified that the optimal training duration  $\tau^*$  should be equal to the optimal number of antennas  $N_1^*$  to be trained. As a result, (P1-large-M) reduces to

$$\max_{N_1, P_r} \quad \tilde{Q}_{\text{net}}^{\text{large-}M}(N_1, P_r)$$
  
s.t.  $0 \le N_1 \le N,$   
 $P_r \ge 0,$  (61)

where  $\tilde{Q}_{\text{net}}^{\text{large-}M}(N_1, P_r)$  is obtained by substituting  $\Pi = \Pi^*$ and  $\tau = N_1$  into (59), which is

$$\tilde{Q}_{\text{net}}^{\text{large-}M}(N_1, P_r) = \frac{\eta (T - N_1) P_f \beta}{K + 1} \bigg( K \bar{\lambda} + \frac{M \| [\bar{\mathbf{v}}^*]_{1:N_1} \|^2 \beta P_r + N_1 \sigma_r^2 (K + 1)}{\beta P_r + \sigma_r^2 (K + 1)} + N - N_1 \bigg) - P_r N_1,$$
(62)

with  $\bar{\mathbf{v}}^* \triangleq \mathbf{\Pi}^{*T} \bar{\mathbf{v}}$  obtained by rearranging the elements in  $\bar{\mathbf{v}}$  so that the elements' absolute values are in a non-increasing order.

To find the optimal solution to problem (61), we first obtain the optimal training power with  $N_1$  fixed by solving the following optimization problem:

$$\max_{P_r \ge 0} \quad \tilde{Q}_{\text{net}}^{\text{large-}M}(N_1, P_r).$$
(63)

Denote the optimal solution and the optimal value of problem (63) as  $P_r^{\text{large-}M}(N_1)$  and  $\tilde{Q}_{\text{net}}^{\text{large-}M}(N_1)$ , respectively. When  $N_1 = 0$ , it follows trivially that  $P_r^{\text{large-}M}(0) = 0$  and the objective value is

$$\tilde{Q}_{\text{net}}^{\text{large-}M}(0) = \eta T P_f \beta \frac{K \lambda + N}{K + 1}.$$
(64)

For  $1 \le N_1 \le N$ , it can be verified that problem (63) is convex, whose optimal solution is given by

$$P_{r}^{\text{large-}M}(N_{1}) = \sqrt{\eta P_{f} \sigma_{r}^{2}} \bigg[ \sqrt{(T - N_{1}) \bigg( \frac{M \|[\bar{\mathbf{v}}^{\star}]_{1:N_{1}} \|^{2}}{N_{1}} - 1} \bigg) - \frac{K + 1}{\sqrt{\Gamma}} \bigg]^{+},$$
(65)

and the corresponding optimal value can be expressed as (66) given on top of the next page, where the set  $\mathcal{N}_{\text{large-}M}$  is defined as

$$\mathcal{N}_{\text{large-}M} \triangleq \left\{ 1 \le N_1 \le N : (T - N_1) \left( \frac{M \| [\bar{\mathbf{v}}^{\star}]_{1:N_1} \|^2}{N_1} - 1 \right) \\ > \frac{(K+1)^2}{\Gamma} \right\}.$$
(67)

Finding the optimal solution to problem (61) and hence that to the original problem (P1-large-M) now becomes finding the optimal number of antennas  $N_1^{\star}$  to be trained, which can be straightforwardly obtained by comparing  $\tilde{Q}_{\text{net}}^{\text{large-M}}(N_1)$ 

$$\bar{Q}_{\text{net}}(\mathcal{N}_1,\tau,P_r) \ge \frac{\eta(T-\tau)P_f\beta}{K+1} \left( K\bar{\lambda} + \frac{M \left\| \left[ \mathbf{\Pi}^T \bar{\mathbf{v}} \right]_{1:N_1} \right\|^2 \beta P_r \tau + N_1^2 \sigma_r^2 (K+1)}{\beta P_r \tau + N_1 \sigma_r^2 (K+1)} + N - N_1 \right) - P_r \tau$$
(59)

$$\triangleq \tilde{Q}_{\text{net}}^{\text{large-}M}(\mathbf{\Pi}, N_1, \tau, P_r).$$
(60)

$$\tilde{Q}_{\text{net}}^{\text{large-}M}(N_1) = \begin{cases} (T - N_1)\eta P_f \beta \frac{K\bar{\lambda} + N}{K+1} + \frac{\eta P_f \beta N_1}{K+1} \left( \sqrt{(T - N_1) \left(\frac{M \|[\bar{\mathbf{v}}^\star]_{1:N_1}\|^2}{N_1} - 1\right)} - \frac{K+1}{\sqrt{\Gamma}} \right)^2, & \text{if } N_1 \in \mathcal{N}_{\text{large-}M} \\ (T - N_1)\eta P_f \beta \frac{K\bar{\lambda} + N}{K+1}, & \text{otherwise} \end{cases}$$
(66)

for  $N_1 \in \{0, 1, \cdots, N\}$ . As evident from (64) and (66), if  $N_1 \geq 1$  and  $N_1 \notin \mathcal{N}_{\text{large-}M}$ , we always have  $\tilde{Q}_{\text{net}}^{\text{large-}M}(N_1) < \tilde{Q}_{\text{net}}^{\text{large-}M}(0)$ . Therefore, the optimal  $N_1^{\star}$  can be obtained as

$$N_1^{\star} = \arg \max_{N_1 \in \{0\} \cup \mathcal{N}_{\text{large-}M}} \tilde{Q}_{\text{net}}^{\text{large-}M}(N_1).$$
(68)

Similar observations to those in Section IV-A and Section IV-B can be made based on (67) and (68), i.e., for WET in massive MIMO Rician fading channels, channel training helps if T and  $\Gamma$  are sufficiently large, and K is sufficiently small.

To gain further insights, we consider the case where the deterministic Rician channel component  $\overline{\mathbf{H}}$  is given by a rank-1 matrix. This corresponds to a propagation environment without any fixed scatterer cluster, so that the deterministic Rician channel component is contributed by the LOS path only. In this case, L = 1 in (2) and hence  $\overline{\mathbf{H}}$  can be represented as

$$\bar{\mathbf{H}} = \mathbf{a}_r(\theta_r) \mathbf{a}_t(\theta_t)^H.$$
(69)

Lemma 2: For WET systems in MIMO Rician fading channels with rank-1 deterministic channel component and sufficiently large channel block length T satisfying  $T > KN^2 + 2N$ , the net average harvested energy  $\bar{Q}_{net}$  with the proposed training-based scheme scales with M asymptotically as

$$\bar{Q}_{\text{net}} \ge \tilde{Q}_{\text{net}}^{\text{large-}M} \to (T-N)\eta P_f \beta \frac{KN+1}{K+1}M, \text{ as } M \to \infty.$$
(70)

#### Proof: Please refer to Appendix B.

As a comparison, we consider in the following the asymptotic scaling for the two benchmark schemes, i.e., the ideal energy beamforming assuming perfect CSI at the ET and the energy beamforming based on the deterministic LOS channel component only.

*Lemma 3:* For WET systems in MIMO Rician fading channels with rank-1 deterministic channel component, the average harvested energy with ideal energy beamforming and that based on the LOS channel component behave as  $M \to \infty$  as

$$\bar{Q}^{\text{ideal}} \to T\eta P_f \beta \frac{KN+1}{K+1} M,$$
 (71)

$$\bar{Q}^{\text{LOS}} \to T\eta P_f \beta \frac{KN}{K+1} M.$$
 (72)



Fig. 3: Net average harvested power versus the number of trained ER antennas  $N_1$  for different block lengths T, with M = 5 and N = 10 in MIMO Rayleigh fading channel.

*Proof:* The proof follows from the similar techniques used in (53), and hence is omitted here for brevity.

The following result immediately follows from Lemma 2 and Lemma 3.

Proposition 1: For WET systems in MIMO Rician fading channels with rank-1 deterministic channel component and sufficiently large channel block length  $T \gg N$ , the net average harvested energy achieved by the optimized training scheme has the same asymptotic scaling with M as that achieved by ideal energy beamforming.

Note that the condition  $T \gg N$  in Proposition 1 generally holds for practical WET systems, since N is usually quite small due to the limited space available at the ER (e.g., wireless sensor nodes), whereas T is usually on the order of hundreds or even thousands in slow fading environment. It is also observed from (71) and (72) that the optimal asymptotic scaling cannot be achieved by the simple LOS-based energy beamforming, except for the extreme case with  $K \to \infty$ .

#### V. NUMERICAL RESULTS

In this section, numerical examples are provided to corroborate our study. We assume that the ET has a maximum transmit power of  $P_f = 1$  Watt. The average signal power attenuation



Fig. 4: Optimal number of trained ER antennas versus block length T for different number of ET antennas M with N = 10 in MIMO Rayleigh fading channels.

is assumed to be 60 dB, i.e.,  $\beta = 10^{-6}$ , which may correspond to an operating distance between the ET and ER around 30 meters with carrier frequency 900 MHz. Furthermore, the noise power received at the ET during training phase is assumed be  $\sigma_r^2 = -90$  dBm and the energy harvesting efficiency at the ER is  $\eta = 0.5$ . For the channel model given in (1), the deterministic component  $\overline{\mathbf{H}}$  is assumed to be rank-1 as given by (69), with AOA and AOD respectively given by  $\theta_r = 0^\circ$  and  $\theta_t = 10^\circ$ . Furthermore, for simplicity, we assume that ULAs with adjacent elements separated by half wavelength are deployed at the ER and ET, so that the phase differences among the antenna elements are given in (4). In the following, we present the numerical results for the three cases of MIMO Rayleigh fading channel, MISO and massive MIMO Rician fading channels, respectively.

#### A. MIMO Rayleigh Fading Channel

First, we consider a WET system in MIMO Rayleigh fading channel with M = 5 and N = 10. In Fig. 3, by varying the number of trained ER antennas  $N_1$ , the net average harvested energy normalized by the channel block length T is plotted for different T values, where the average is taken over 10000random channel realizations. The analytical results derived in Section IV-A, i.e.,  $\bar{Q}_{\rm net}^{\rm R}(N_1)/T$  with  $\bar{Q}_{\rm net}^{\rm R}(N_1)$  given by (38) for  $N_1 = 0$  or by (41) for  $1 \leq N_1 \leq N$ , are also shown in the same figure. It is observed that the simulation and analytical results have perfect match with each other, which thus validates our theoretical studies. Furthermore, for moderate block lengths of T = 25 and T = 50, Fig. 3 clearly shows the trade-offs in selecting the number of ER antennas to be trained. It is observed that the optimal number of training ER antennas is 2 for T = 25 and increases to 5 for T = 50. As the block length T increases to 100 symbol durations, the net average harvested power monotonically increases with  $N_1$ , and hence it is optimal to train all of the N = 10 available ER antennas.

In Fig. 4, the optimal number of ER antennas  $N_1^{\star}$  that



Fig. 5: Optimal training energy versus block length T for different number of ET antennas M with N = 10 in MIMO Rayleigh fading channels.

should be trained is plotted against the channel block length T for different number of ET antennas M. The total number of antennas available at the ER is N = 10. Note that based on Lemma 1,  $N_1^{\star}$  also gives the optimal number of symbol durations  $\tau^*$  that should be allocated for training during each coherent block. It is observed that for M = 1, no training should be applied regardless of the block length T. This is expected since no beamforming gain can be exploited when there is only one antenna at the ET. In contrast, when the ET has multiple antennas, e.g., M = 2 or M = 5, the optimal number of trained ER antennas increases with the block length T. As T becomes sufficiently large, i.e.,  $T \ge 150$  for M = 2and T > 80 for M = 5, all antennas at the ER should be trained. It is also observed from Fig. 4 that more ER antennas should be trained for M = 5 than that for M = 2, which is expected since transmit beamforming is more effective and hence training is more beneficial when more antennas are available at the ET. Similar observations can be made in Fig. 5, where the optimal training energy  $E_r^{\star}$  versus the channel block length T for the same setups as in Fig. 4 is shown. It is noted that for sufficiently large T,  $E_r^{\star}$  increases with T in a squareroot relationship, which implies a diminishing marginal gain offered by channel training as T increases.

In Fig. 6, the net average harvested power of the optimized training-based scheme is plotted against the block length T in a MIMO Rayleigh fading channel with M = N = 5. The performances of two benchmark schemes, namely the ideal energy beamforming assuming perfect CSI at the ET and the isotropic transmission (i.e.,  $\mathbf{S} = (P_f/M)\mathbf{11}^H$  with 1 denoting an all-one vector) with no CSI, are also plotted. It is observed that the training-based WET significantly outperforms isotropic transmission, and its performance improves with the increasing of T. This is expected since for larger block length, it is affordable to have more training as shown in Fig. 4 and Fig. 5, and hence the channel can be more accurately estimated at the ET and the obtained energy beamforming is more effective. It is also observed that with sufficiently large



Fig. 6: Net average harvested power versus block length T for WET in MIMO Rayleigh fading channel with M = N = 5.



Fig. 7: Net average harvested energy versus the Rician factor K for WET in MISO Rician channel with M = 5 and T = 200.

T, the training-based WET approaches the performance upper bound with perfect CSI.

#### B. MISO Rician Fading Channel

Next, we consider the WET systems in MISO Rician fading channels with N = 1 and K > 0. In Fig. 7, the net average harvested energy with the proposed training-based scheme is plotted against the Rician factor K. The number of ET antennas and the channel block length are respectively set as M = 5 and T = 200. The two benchmark schemes are also shown in the same figure, which are the ideal energy beamforming assuming perfect CSI at the ET and that solely based on the LOS component of the channel (i.e.,  $\mathbf{S} = P_f \bar{\mathbf{v}} \bar{\mathbf{v}}^H$ , with  $\bar{\mathbf{v}}$  denoting the dominant eigenvector of  $\bar{\mathbf{H}}^H \bar{\mathbf{H}}$ ). It is observed that the proposed training-based WET significantly outperforms the LOS-based energy beamforming scheme for small K values, for which the power of the scattered signal component is comparable to that of the LOS component. In



Fig. 8: Training and non-training regions for WET in MISO Rician fading channels on the M-versus-T plane.

such scenarios, channel training greatly improves the accuracy of the CSI at the ET, which thus makes energy beamforming more effective than that based on the LOS component only. As K increases, and hence the LOS component becomes dominating, the performance gap between the two schemes diminishes. With sufficiently large K, it is observed that the proposed scheme degenerates to the LOS-based energy beamforming, which approaches to the ideal scenario with perfect CSI at the ET.

In Fig. 8, the optimal training and no-training regions based on (52) are plotted in the *M*-versus-*T* plane for K = 3 dB and K = 10 dB with sufficiently large  $\Gamma$ . Fig. 8 affirms our conclusion drawn in Section IV-B that for fixed Rician factor *K*, training helps if and only if the channel block length *T* and the number of ET antennas *M* are sufficiently large. It is also observed from Fig. 8 that as the Rician factor *K* decreases, the training region enlarges, which is as expected.

#### C. Massive MIMO Rician Fading Channel

Last, we consider the effect of large M on the WET systems in massive MIMO Rician channels as discussed in Section IV-C. In Fig. 9, the net average harvested energy normalized by channel block length T is plotted against the number of ET antennas M, with M ranging from 5 to 300. The channel block length and Rician factor are respectively set as T = 1000 and K = 1. Furthermore, the number of antennas at the ER is set as N = 5. The results for the two benchmark schemes with ideal energy beamforming and LOS-based beamforming are also shown in the figure. It is observed from Fig. 9 that, the proposed training design based on massive MIMO setup achieves a very close performance as the ideal energy beamforming, even for moderate Mvalues. In contrast, the simple LOS-based energy beamforming performs significantly worse than the other two schemes, and the performance gap increases as M gets larger. This implies the necessity for channel training in MIMO WET systems to exploit the scattered channel power, especially when M is large and the potential beamforming gain is significant.



Fig. 9: Net average harvested power versus number of ET antennas M for WET in MIMO Rician fading channels with K = 1, T = 1000 and N = 5.

#### VI. CONCLUSION AND FUTURE WORK

This paper studied the optimal design of channel training for MIMO WET systems in Rician fading channels. Assuming channel reciprocity, the forward link channel of WET can be efficiently estimated at the ET based on the training signals sent by the ER in the reverse communication link. To optimally resolve the tradeoff between channel training and the resulting energy beamforming gain, we proposed a novel design framework to maximize the net harvested energy at the ER to explicitly take into account the time and energy costs for channel training. A training design optimization problem was formulated for the general uncorrelated MIMO Rician fading channels, while the optimal solutions were derived for the special cases of MIMO Rayleigh and MISO Rician fading channels. For massive MIMO Rician fading channels, a highquality approximate solution was obtained, which is shown to achieve the optimal asymptotic scaling with the increasing number of ET antennas.

There are a number of research directions along which the developed results in this paper can be further investigated, as briefly discussed as follows.

- *Correlated Fading Channel:* It is interesting to investigate the optimal training design for correlated MIMO Rician fading channels, which provide more accurate modeling in poor scattering environment and/or when the antennas are not sufficiently separated. In this case, the structure of the training sequence needs to be optimized as well, since orthogonal training sequence may not be optimal in general for correlated MIMO fading channels [36].
- *Multi-User Setup:* More research endeavor is needed to find the optimal training design for the general multiuser setups with near-far ERs. In particular, as evident from (40), the optimal solution needs to resolve the socalled "doubly near-far" problem, where a far ER from the ET suffers from higher propagation loss than a near ER for both reverse-link channel training and forwardlink energy transmission. Note that this phenomenon has

been first revealed in [16] for WPCN.

• *Energy Outage:* In this paper, the net average harvested energy at the ER is maximized by optimizing the training design. In certain applications, a more appropriate performance metric may be the energy outage probability, the minimization of which deserves further study.

## APPENDIX A Proof of Lemma 1

Lemma 1 can be shown by contradiction. Suppose, on the contrary, that the optimal solution  $(N_1^*, \tau^*, P_r^*)$  to (P1-R) satisfies  $\tau^* > N_1^*$ . Then we define a new tuple  $(N_1', \tau', P_r')$ , with  $N_1' = N_1^*$ ,  $\tau' = N_1^*$ , and  $P_r' = P_r^* \tau^* / N_1^*$ . In other words, in the newly defined training design, the number of training antennas  $N_1$  and the total training energy  $P_r \tau$  keep unchanged, while the training duration is decreased to save the time overhead. It can be verified that with such a construction,  $(N_1', \tau', P_r')$  is feasible to (P1-R). Furthermore, due to the reduced time overhead, it follows from (34) that the following inequality holds:

$$\bar{Q}_{\text{net}}^{\text{R}}(N_{1}',\tau',P_{r}') > \bar{Q}_{\text{net}}^{\text{R}}(N_{1}^{\star},\tau^{\star},P_{r}^{\star}).$$
(73)

In other words, there exists another training design  $(N'_1, \tau', P'_r)$  as defined above, which satisfies the constraints of (P1-R) and strictly increases the objective value  $\bar{Q}_{\text{net}}^{\text{R}}$ . This thus contradicts the assumption that  $(N_1^*, \tau^*, P_r^*)$  is optimal. Therefore, the assumption  $\tau^* > N_1^*$  is invalid, or  $\tau^* \le N_1^*$  must hold. Together with the constraint  $\tau \ge N_1$  in (P1-R), we thus have  $\tau^* = N_1^*$  for the optimal solution to (P1-R).

This completes the proof of Lemma 1.

## APPENDIX B Proof of Lemma 2

To show Lemma 2, we need to first obtain the optimal number of ER antennas  $N_1^*$  to be trained by solving (68). With rank-1 deterministic channel component  $\mathbf{\bar{H}}$  given in (69), the solution to (68) can be obtained in closed-form, as we show next. With (69), it can be shown that the dominant eigenvalue  $\bar{\lambda}$  and the corresponding eigenvector  $\mathbf{\bar{v}}$  of  $\mathbf{\bar{H}}^H \mathbf{\bar{H}}$  are respectively given by

$$\bar{\lambda} = \lambda_{\max}(\bar{\mathbf{H}}\bar{\mathbf{H}}^H) = MN, \tag{74}$$

$$\bar{\mathbf{v}} = \mathbf{v}_{\max}(\bar{\mathbf{H}}\bar{\mathbf{H}}^H) = \frac{1}{\sqrt{N}}\mathbf{a}_r(\theta_r).$$
(75)

It follows from (3) that for any AOA  $\theta_r$ , all elements in the vector  $\mathbf{a}_r(\theta_r)$  have absolute values equal to 1. As a result, we have

$$\|[\bar{\mathbf{v}}^{\star}]_{1:N_1}\|^2 = \frac{N_1}{N}, \ 1 \le N_1 \le N.$$
 (76)

Furthermore, in the massive MIMO regime with  $M \gg N$  and with moderate values for K and  $\Gamma$ , we have

$$\sqrt{(T-N_1)\left(\frac{M}{N}-1\right)} \gg \frac{K+1}{\sqrt{\Gamma}}, \ \forall 1 \le N_1 \le N.$$
 (77)

With (74), (76), and (77), by keeping only the dominant terms, the expressions of  $\tilde{Q}_{\text{net}}^{\text{large-}M}(N_1)$  for  $N_1 = 0$  given in (64) and for  $1 \leq N_1 \leq N$  given in (66) can be unified as

$$\tilde{Q}_{\text{net}}^{\text{large-}M}(N_1) \approx \frac{\eta P_f \beta}{K+1} (T-N_1) \left(KN + \frac{N_1}{N}\right) M, \ 0 \le N_1 \le N.$$
(78)

As a consequence, problem (68) reduces to

$$N_1^{\star} = \arg \max_{0 \le N_1 \le N, N_1 \in \mathbb{Z}} \frac{\eta P_f \beta}{K+1} (T-N_1) \left( KN + \frac{N_1}{N} \right) M.$$
(79)

To solve (79), we first obtain the solution  $N_1$  to the relaxed problem without considering the integer constraint  $N_1 \in \mathbb{Z}$ . Then  $N_1^*$  can be obtained as

$$N_{1}^{\star} = \begin{cases} \hat{N}_{1}, & \text{if } \hat{N}_{1} \in \mathbb{Z}, \\ \arg \max \left\{ \tilde{Q}_{\text{net}}^{\text{large-}M} \left( \lfloor \hat{N}_{1} \rfloor \right), \tilde{Q}_{\text{net}}^{\text{large-}M} \left( \lceil \hat{N}_{1} \rceil \right) \right\}, & \text{otherwise.} \end{cases}$$

To capture the most essential insight, we assume that  $\hat{N}_1 \in \mathbb{Z}$  holds. Then the optimal solution  $N_1^*$  to (79) can be easily obtained as

$$N_1^{\star} = \left[\frac{T - KN^2}{2}\right]_0^N,\tag{80}$$

where  $[x]_a^b \triangleq \max\{\min\{x, b\}, a\}$ . In particular, for sufficiently large channel block length T satisfying  $T > KN^2 + 2N$ , we have  $N_1^* = N$ , i.e., all ER antennas should be trained. By substituting  $N_1 = N$  into (78), the result (70) in Lemma 2 follows.

This thus completes the proof of Lemma 2.

#### REFERENCES

- H. J. Visser and R. J. M. Vullers, "RF energy harvesting and transport for wireless sensor network applications: Principles and requirements," *Proceedings of the IEEE*, vol. 101, no. 6, pp. 1410–1423, Jun. 2013.
- [2] S. Bi, C. K. Ho, and R. Zhang, "Wireless powered communication: opportunities and challenges," to appear in *IEEE Commun. Mag.*, available online at http://arxiv.org/abs/1408.2335.
- [3] X. Lu, P. Wang, D. Niyato, D. I. Kim, and Z. Han, "Wireless networks with RF energy harvesting: a contemporary survey," to appear in *IEEE Commun. Surveys Tuts.*, available online at http://arxiv.org/abs/1406.6470.
- [4] R. Zhang and C.-K. Ho, "MIMO broadcasting for simultaneous wireless information and power transfer," *IEEE Trans. Wireless Commun.*, vol. 12, no. 5, pp. 1989–2001, May 2013.
- [5] W. C. Brown, "The history of power transmission by radio waves," *IEEE Trans. on Microwave Theory and Techniques*, vol. MTT-32, no. 9, pp. 1230–1242, Sep. 1984.
- [6] —, "Experiments involving a microwave beam to power and position a helicopter," *IEEE Trans. on Aerospace and Electronic Sys.*, vol. AES-5, no. 692-702, pp. 692–702, Sep. 1969.
- [7] J. O. Mcspadden and J. C. Mankins, "Space solar power programs and microwave wireless power transmission technology," *IEEE Microw. Mag.*, vol. 3, no. 4, pp. 46–57, Dec. 2002.
- [8] K. Huang and V. K. N. Lau, "Enabling wireless power transfer in cellular networks: Architecture, modeling and deployment," *IEEE Trans. Wireless Commun.*, vol. 13, no. 2, pp. 902–912, Feb. 2014.
- [9] S. Lee, R. Zhang, and K. Huang, "Opportunistic wireless energy harvesting in cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 12, no. 9, pp. 4788–4799, Sep. 2013.
- [10] X. Lu, P. Wang, D. Niyato, and E. Hossain, "Dynamic spectrum access in cognitive radio networks with RF energy harvesting," *IEEE Wireless Commun.*, pp. 102–110, Jun. 2014.

- [11] A. A. Nasir, X. Zhou, S. Durrani, and R. A. Kennedy, "Relaying protocols for wireless energy harvesting and information processing," *IEEE Trans. Wireless Commun.*, vol. 12, no. 7, pp. 3622–3636, Jul. 2013.
- [12] Z. Ding, S. M. Perlaza, I. Esnaola, and H. V. Poor, "Power allocation strategies in energy harvesting wireless cooperative networks," *IEEE Trans. Wireless Commun.*, vol. 13, no. 2, pp. 846–860, Feb. 2014.
- [13] X. Zhou, R. Zhang, and C. K. Ho, "Wireless information and power transfer: architecture design and rate-energy tradeoff," *IEEE Trans. Commun.*, vol. 61, no. 11, pp. 4757–4767, Nov. 2013.
- [14] L. Liu, R. Zhang, and K. C. Chua, "Wireless information transfer with opportunistic energy harvesting," *IEEE Trans. Wireless Commun.*, vol. 12, no. 1, pp. 288–300, Jan. 2013.
- [15] D. W. K. Ng, E. S. Lo, and R. Schober, "Wireless information and power transfer: Energy efficiency optimization in OFDMA systems," *IEEE Trans. Wireless Commun.*, vol. 12, no. 12, pp. 6352–6370, Dec. 2013.
- [16] H. Ju and R. Zhang, "Throughput maximization in wireless powered communication networks," *IEEE Trans. Wireless Commun.*, vol. 13, no. 1, pp. 418–428, Jan. 2014.
- [17] L. Liu, R. Zhang, and K. C. Chua, "Multi-antenna wireless powered communication with energy beamforming," to appear in *IEEE Trans. Commun.*, available online at http://arxiv.org/abs/1312.1450.
- [18] G. Yang, C. K. Ho, R. Zhang, and Y.-L. Guan, "Throughput optimization for massive MIMO systems powered by wireless energy transfer," submitted to *IEEE J. Sel. Areas Commun.*, available online at http://arxiv.org/abs/1403.3991.
- [19] D. Snoonian, "Smart buildings," *IEEE Spectrum*, vol. 40, no. 8, pp. 18–23, Aug. 2003.
- [20] F. Rusek, D. Persson, B. K. Lau, E. G. Larsson, T. L. Marzetta, O. Edfors, and F. Tufvesson, "Scaling up MIMO: Opportunities and challenges with very large arrays," *IEEE Signal Process. Mag.*, vol. 30, no. 1, pp. 40–60, Jan. 2013.
- [21] L. Lu, G. Y. Li, A. L. Swindlehurst, A. Ashikhin, and R. Zhang, "An overview of massive MIMO: benefits and challenges," *IEEE J. Sel. Topics Signal Process.*, vol. 8, no. 5, pp. 742–758, Oct. 2014.
- [22] G. Yang, C. K. Ho, and Y.-L. Guan, "Dynamic resource allocation for multiple-antenna wireless power transfer," *IEEE Trans. Signal Process.*, vol. 62, no. 14, pp. 3565 – 3577, Jun. 2014.
- [23] X. Chen, C. Yuen, and Z. Zhang, "Wireless energy and information transfer tradeoff for limited-feedback multiantenna systems with energy beamforming," *IEEE Trans. Veh. Technol.*, vol. 63, no. 1, pp. 407–412, Jan. 2014.
- [24] X. Zhou, "Training-based SWIPT: optimal power splitting at the receiver," to appear in *IEEE Trans. Veh. Technol.*, available online at http://arxiv.org/abs/1405.4623.
- [25] D. J. Love, R. W. Heath Jr., V. K. N. Lau, D. Gesbert, B. D. Rao, and M. Andrews, "An overview of limited feedback in wireless communication systems," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 8, pp. 1341–1365, Oct. 2008.
- [26] J. Xu and R. Zhang, "Energy beamforming with one-bit feedback," *IEEE Trans. Signal Process.*, vol. 62, no. 20, pp. 5370–5381, Oct. 2014.
- [27] T. L. Marzetta, "Noncooperative cellular wireless with unlimited numbers of base station antennas," *IEEE Trans. Wireless Commun.*, vol. 9, no. 11, pp. 3590–3600, Nov. 2010.
- [28] F. R. Farrokhi, G. J. Foschini, A. Lozano, and R. A. Valenzuela, "Link-optimal space-time processing with multiple transmit and receive antennas," *IEEE Commun. Lett.*, vol. 5, no. 3, pp. 85–87, Mar. 2001.
- [29] H. Bolcskei, M. Borgmann, and A. J. Paulraj, "Impact of the propagation environment on the performance of space-frequency coded MIMO-OFDM," *IEEE J. Sel. Areas Commun.*, vol. 21, no. 3, pp. 427–439, Apr. 2003.
- [30] A. Forenza, D. J. Love, and R. W. Heath Jr., "Simplified spatial correlation models for clustered MIMO channels with different array configurations," *IEEE Trans. Veh. Technol.*, vol. 56, no. 4, pp. 1924– 1934, Jul. 2007.
- [31] L. G. Ordonez, D. P. Palomar, and J. R. Fonollosa, "Ordered eigenvalues of a general class of Hermitian random matrices with application to the performance analysis of MIMO systems," *IEEE Trans. Signal Process.*, vol. 57, no. 2, pp. 672–689, Feb. 2009.
- [32] B. Hassibi and B. M. Hochwald, "How much training is needed in multiple-antenna wireless links?" *IEEE Trans. Inf. Theory*, vol. 49, no. 4, pp. 951–963, Apr. 2003.
- [33] S. M. Kay, Fundumentals of Statistical Signal Processing: Estimation Theory. New Jersey: Prentice-Hall, 1993.

- [34] P. A. Dighe, R. K. Mallik, and S. S. Jamuar, "Analysis of transmitreceive diversity in Rayleigh fading," IEEE Trans. Commun., vol. 51, no. 4, pp. 694-703, Apr. 2003.
- 10. 4, pp. 694–705, Apr. 2005.
  [35] A. Maaref and S. Aissa, "Closed-form expressions for the outage and ergodic Shannon capacity of MIMO MRC systems," *IEEE Trans. Commun.*, vol. 53, no. 7, pp. 1092–1095, Jul. 2005.
  [36] J. H. Kotecha and A. M. Sayeed, "Transmit signal design for optimal estimation of correlated MIMO channels," *IEEE Trans. Signal Process.*, vol. 52, no. 2, pp. 546–557, Eds. 2014.
- vol. 52, no. 2, pp. 546-557, Feb. 2014.