Joint Optimization of File Placement and Delivery in Cache-Assisted Wireless Networks with Limited Lifetime and Cache Space

Bojie Lv, Rui Wang, Ying Cui, Yi Gong and Haisheng Tan

Abstract

In this paper, the scheduling of downlink file transmission in one cell with the assistance of cache nodes with finite cache space is studied. Specifically, requesting users arrive randomly and the base station (BS) reactively multicasts files to the requesting users and selected cache nodes. The latter can offload the traffic in their coverage areas from the BS. We consider the joint optimization of the abovementioned file placement and delivery within a finite lifetime subject to the cache space constraint. Within the lifetime, the allocation of multicast power and symbol number for each file transmission at the BS is formulated as a dynamic programming problem with a random stage number. Note that there are no existing solutions to this problem. We develop an asymptotically optimal solution framework by transforming the original problem to an equivalent finite-horizon Markov decision process (MDP) with a fixed stage number. A novel approximation approach is then proposed to address the curse of dimensionality, where the analytical expressions of approximate value functions are provided. We also derive analytical bounds on the exact value function and approximation error. The approximate value functions depend on some system statistics, e.g., requesting users' distribution. One reinforcement learning algorithm is proposed for the scenario where these statistics are unknown.

I. INTRODUCTION

Caching is a promising technology to save the transmission resource and improve the spectrum efficiency for cellular networks. In this paper, we consider a flexible deployment scenario of a

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Part of this work has been accepted in IEEE GLOBECOM 2018 [1]. We have extended the conference paper by revising the low-complexity scheduling policy design in Section IV, developing a novel reinforcement learning algorithm in Section V, improving the bounds on the approximate value functions in Section IV-C, and generating more illustrative simulation results.

cache-enabled cell where there is no wired connection or dedicated spectrum between the base station (BS) and cache nodes. Thus the cache nodes and requesting users receive popular files simultaneously via downlink multicast. Moreover, the timeliness of popular files is considered as in [2], and the transmission within a finite file lifetime is optimized via the approach of finite-horizon *Markov decision process (MDP)* and *reinforcement learning*.

A. Related Works

There have been a number of works on the optimization of file placement with the cache space constraint. It's obvious that cache nodes should store the most popular files if each user can get access to only one cache node. However, the papers [3], [4] showed that caching files randomly with optimized caching probabilities is better than storing the most popular files at each cache node when one user can be served by multiple cache nodes. In [5], the authors took user mobility into consideration, where each user can move among the service regions of different cache nodes. They proposed a file placement policy to improve data offloading rate. With the help of historical file request information, the authors in [6] proposed a file placement and update method at the cache node via predicting the arrival distribution of future file requests. The optimal file placement strategies were designed in [7] in the cases of imperfect and unknown file popularity distributions. In [8], the authors considered unmanned aerial vehicles as the users and designed a probabilistic file placement method to maximize the average successful file download rate. Moreover, there are also some works on the design of coded caching schemes [9], [10]. With cached files, the authors in [11] designed a multicast beamforming policy to minimize the weight sum of the backhaul cost and transmit power at the BS, and the paper [12] formulated the joint minimization of the average delay and power consumption at the BS as a stochastic optimization problem. In all the above works, the cost of file placement at cache nodes is not taken into consideration, as it is assumed to be completed before the phase of file delivery to the requesting users. In practice, however, there may not be sufficient time for file placement before users' requests, e.g., real-time news.

When the phases of file placement (at cache nodes) and delivery (to requesting users) occur simultaneously, joint scheduling of both phases becomes necessary. For example, a file placement and delivery framework for heterogeneous networks was investigated in [13], where cache node association of requesting users and coded file placement are jointly optimized to maximize the overall throughput in each frame. In [14], an optimal caching and user association policy was

proposed to minimize the latency in a cached-enabled heterogeneous network with wireless backhaul. In the above works, the files are delivered to small BSs via dedicated backhaul links, i.e., there is no resource sharing between file placement and delivery. When there is no dedicated link or period for file placement at cache nodes, file placement and delivery can be simultaneously conducted using multicast [15]. This yields a coupling relationship between the transmission resource consumption and file placement. For example, if more resource is spent on downlink multicast, files will be cached in more cache nodes, which may save the downlink resource in future transmissions. As a result, a joint optimization of file placement and delivery with the consideration of the total transmission resource consumption at the BS becomes inevitable. Moreover, it is of practical value to model the file requests as a temporal and spatial random process. Hence, dynamic programming can be utilized to address the joint optimization of file placement and delivery. This issue was initially studied in our previous work [16]. Specifically, we considered a random number of requests on multiple popular files without cache space limitation in [16], where the scheduling design for multiple files can be equivalently decoupled as single-file scheduling problem. The multi-file case with limited cache space at the cache nodes has not addressed.

Dynamic programming via MDP has been considered in resource allocation of wireless systems [17]–[24] or information systems [25]–[27]. For example, infinite-horizon MDP was used to optimize the cellular uplink transmissions [17], [18], downlink transmissions [19], and relay networks [20], [21], where the average transmission delay is either minimized or constrained. Moreover, low-complexity solutions were considered in the abovementioned works to avoid the curse of dimensionality [28]. Note that popular files to be stored at cache nodes usually have a finite lifetime, and hence infinite-horizon MDP adopted in the aforementioned works may not be suitable for joint optimization of file placement and delivery anymore. Nevertheless, the finite-horizon MDP is usually more complicated [29], and designing low-complexity algorithrms for finite-horizon MDP is still an open issue.

B. Our Contributions

In this paper, we consider the downlink transmission of popular files in a cache-enabled cell within a finite file lifetime. The popular files may not be stored in the cache nodes at the very beginning of the lifetime. The arrival of requesting users is random in both temporal and spatial dimensions. When one file is requested, the BS reactively multicasts it to the requesting user

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(a.k.a. file delivery) as well as some chosen cache nodes (a.k.a. file placement) according to the channel and cache status. With the decoded files, the cache nodes can serve the following requesting users in its coverage region via different spectrum from the downlink (e.g., Wi-Fi) as in [30], [31]. Therefore, the current file transmission may lead to the update of the cache status, which affects the future file transmissions in the remaining lifetime. In this paper, the main contributions on the optimization and analysis of the transmission scheduling are summarized below.

- We consider the joint optimization of the file placement and delivery within a finite lifetime subject to the cache space constraint, and propose a novel optimization framework. In particular, we formulate the scheduling of multicast power and symbol number at the BS as a dynamic programming problem, where the goal is to minimize the average transmission cost (weighted sum of the transmission energy and symbol number) at the BS by offloading the traffic to the cache nodes. Due to the cache space limitation, less popular files stored at one cache node in the early stage of the lifetime and may be replaced with more popular files. This complicated replacement has not been address in our previous work [16], which focused on the transmission scheduling of one file only.
- Since the number of file requests in the lifetime is random, the dynamic programming problem formulation has a random stage number, and there are no existing solutions to this problem to our best knowledge. The main difficulty is that the remaining number of stages in the Bellman's equations is unknown. We address this issue by proposing a novel framework to equivalently transform the original problem to a finite-horizon MDP with a fixed number of stages. In [16], however, the dynamic programming problem with a random stage number is transformed to a finite-horizon MDP in a heuristic way, losing the optimality.
- In order to address the curse of dimensionality, a novel approach of value function approximation is proposed for the abovementioned finite-horizon MDP, where the approximate value functions can be calculated using analytical expressions efficiently and effectively. Instead of numerical algorithms that are computationally expensive. We also provide tight analytical upper and lower bounds on the exact value function (which represent the minimum average transmission cost). The approach of value function approximation in [16] cannot be applied in this paper, due to the different definitions of value functions. Moreover, it is difficult to obtain an analytical upper bound on the minimum average transmission cost via



Fig. 1. Illustration of network model with one BS, multiple wireless cache nodes and a file database, where one example of random spatial and temporal arrivals of requesting users is provided.

the approach in [16].

• The expressions of approximate value functions rely on some system statistics, e.g., the distribution of requesting users and the popularities of files. In the case where the priori knowledge on these system statistics is not available, a novel reinforcement learning algorithm is proposed to evaluate the approximate value functions in an online manner. The issue of unknown file popularity is not addressed in [16].

It is shown by simulations that, compared with some baseline schemes, the proposed lowcomplexity algorithm based on value function approximation can significantly reduce the average transmission cost at the BS.

II. SYSTEM MODEL

A. Cache-Enabled Network Model

As illustrated in Fig. 1, we consider the downlink file transmission in a cell with one BS and N_C cache nodes. The BS has N_T transmission antennas. Each cache node has a receiving antenna. Let $C \subset \mathbb{R}^2$ be the service region of the cell, C_c be the service region of the *c*-th cache node and $C_0 \triangleq C - C^*$ be the region not served by any cache node, where $c = 1, 2, ..., N_C$ and $C^* \triangleq \bigcup_{c=1}^{N_C} C_c$. It is assumed that the service regions of any two cache nodes are not overlapped, i.e., $C_i \cap C_j = \emptyset$ for all $i \neq j, i, j = 1, ..., N_C$. A library with M_F files, denoted as $\mathcal{F} \triangleq \{1, 2, ..., M_F\}$, is accessible to the BS. For convenience of illustration, it is assumed that each file consists of R_F information bits, and the *c*-th cache node can store at most M_c files ($M_c \leq M_F$). Our proposed algorithms can be easily extended to the scenario with different file sizes. In addition, it is assumed that there is at most one user requesting a file in each frame. The locations of the requesting users are independent and identically distributed (i.i.d.) in the cell according to a certain spatial distribution \mathcal{D} . The distribution intensity at location $\mathbf{l} \in \mathcal{C}$ is denoted as $\rho_{\mathcal{D}}(\mathbf{l})$, and the probability that a requesting user falls in an area $\mathcal{S} \subseteq \mathcal{C}$ is $\int_{\mathcal{S}} \rho_{\mathcal{D}}(\mathbf{l}) ds(\mathbf{l})$.

The probability of the *f*-th file being requested by one user in each frame is βp_f , where $\beta \in [0,1]$ is the probability that there is one user request in one frame,¹ and $p_f \in (0,1)$ is the probability that the requested file is the *f*-th one (the popularity of the *f*-th file). Note that $\sum_{f=1}^{M_F} p_f = 1$. Without loss of generality, we assume $1 \ge p_1 \ge p_2 \ge ... \ge p_{M_F} \ge 0$. In this paper, we do not have any restriction on the popularity distribution $\{p_f | \forall f\}$. For example, it can be a Zipf distribution as in [32].

We consider a finite common lifetime \mathfrak{L} for the file library, which usually lasts over several hours. The concept of lifetime captures the practical scenario where the popularity of a file (e.g. video news) may drop down quickly after a certain period. Suppose that there are L frames in the common file lifetime \mathfrak{L} , i.e. $\mathfrak{L} \triangleq \{1, 2, ..., L\}$. L is usually large. Let N_R be the total number of file requests during the lifetime \mathfrak{L} , which is a random variable with probability mass function (PMF) $\Pr(N_R = n) = {L \choose n} \beta^n (1 - \beta)^{L-n}, \forall n = 0, 1, ..., L$.

At the beginning of the lifetime \mathfrak{L} , the cache nodes may be empty or have stored some of the files. We have no requirement on the cache status at the beginning of the lifetime. A requesting user will download the requested file from one cache node (corresponding to traffic offloading from the BS) if it is in the coverage region of that cache node and the requested file has already been stored there; otherwise, it will download the file from the BS. In order to save downlink transmission resource, the BS can simultaneously transmit (i.e. multicast) the requested file to the user and some cache nodes. Hence, there are two types of file transmissions in the network, namely *BS multicast* and *cache node unicast*. The former refers to the downlink file delivery and placement from the BS to both a requesting user and possibly some selected cache nodes. The later is for the file delivery from any cache node to a requesting user using Wi-Fi, bluetooth,

¹ in practice, the transmission of one file usually lasts over a large number of frames. The probability of more than one new request per frame is assumed to be negligible. Otherwise, the traffic of the network cannot be stabilized.



Fig. 2. Illustration of multicast frame allocation for one file starting at the k-th frame.

or other air interfaces, which is in different spectrum from the downlink transmission [30], [31]. Compared with many existing literature on static caching, the receiving cache nodes optimization of BS multicast in the whole lifetime \mathfrak{L} is a dynamic caching problem with a random number of stages (requests): the cached files will be updated after each multicast, and the status of cached files affects the future transmission cost.

B. Physical Layer Model of BS Multicast

In this paper, it is assumed that the file size R_F is large, and the downlink transmission of each file is over a large number of physical-layer frames (we shall refer to the physical-layer frame as frame in the remaining of this paper). In practical systems, the downlink frame is not dedicated for one popular file's multicast. It may also carry transmission symbols of other traffics as illustrated in Fig. 2. In this paper, we do not specify the frame allocation of each file multicast. Instead, we focus on the scheduling of total number of symbols for each file multicast, which is in a larger time granularity than frame allocation. Because the file multicast consumes a large number of frames, the ergodic capacity averaged over small-scale channel fading can be achieved at each receiver. At the BS, each requested file is encoded in a rateless manner to an arbitrary number of modulation symbols. The BS determines the power and the number of modulation symbols to be sent for the multicast of each requested file. Let N_{τ} be the number of downlink multicast symbols allocated for the τ -th file request, and P_{τ} be the multicast power of these symbols. The following peak power constraint shall be satisfied.

Peak power constraint:
$$P_{\tau} \leq P_B, \forall \tau$$
, (1)

where P_B is the maximum transmission power for file multicast at the BS.

The requesting user or a cache node is able to decode the file as long as its corresponding ergodic capacity of N_{τ} modulated symbols is greater than the file size R_F . In this paper, we do not have any restriction on the file transmission delay, i.e, there is no upper-bound constraint on N_{τ} ($\forall \tau$).

The space-time block code (STBC) with full diversity (e.g., Alamouti code) is used in the physical layer for BS multicast for the following two reasons. Firstly, it does not rely on the channel state information (CSI) at the transmitter (CSIT). Secondly, the diversity gain can be achieved at all the receivers. We refer to the user sending the τ -th request as the τ -th user. Let ρ_{τ} and ρ^c be the pathlosses from the BS to the τ -th user and the *c*-th cache node respectively, and let η_{τ} and η^c_{τ} be the corresponding shadowing coefficients during the τ -th file transmission. Assume that ρ_{τ} , η_{τ} and η^c_{τ} are quasi-static within one file transmission, and change independently and identically over different transmissions. Following the capacity of full diversity STBC in [33], the maximum number of information bits that can be delivered from the BS to the τ -th user given transmission parameters (P_{τ} , N_{τ}) is written as

$$R_{\tau} = N_{\tau} \mathbb{E}_{\mathbf{h}_{\tau}} \left[\alpha \log_2 \left(1 + \frac{||\mathbf{h}_{\tau}||^2 P_{\tau}}{N_T \sigma_z^2} \right) \right], \tag{2}$$

where α is the code rate of the adopted full-diversity STBC. For example, when $N_T = 2$ and the Alamouti code is used, $\alpha = 1$; α is usually less than 1 for $N_T > 2$. σ_z^2 is the average power of noise and inter-cell interference, and $\mathbf{h}_{\tau} \in \mathbb{C}^{N_T}$ is the i.i.d. channel vector from the BS to the τ -th user. Each element of $\mathbf{h}_{\tau} \in \mathbb{C}^{N_T}$ follows a complex Gaussian distribution with zero mean and variance $\rho_{\tau}\eta_{\tau}$. Equation (2) is the ergodic channel capacity over a large number of frames, the randomness in small-scale fading is then averaged. R_{τ} depends only on the transmission parameters (P_{τ}, N_{τ}) and the large-scale fading coefficient $\rho_{\tau}\eta_{\tau}$. Hence, the following decoding constraint should be satisfied at the τ -th user if it cannot receive the requested file from any cache node.

Downlink decoding constraint:
$$R_{\tau} \ge R_F, \forall \tau.$$
 (3)

Similarly, the maximum number of information bits that can be delivered from the BS to the *c*-th cache node given transmission parameters (P_{τ}, N_{τ}) is written as

$$R_{\tau}^{c} = N_{\tau} \mathbb{E}_{\mathbf{h}_{\tau}^{c}} \left[\alpha \log_{2} \left(1 + \frac{||\mathbf{h}_{\tau}^{c}||^{2} P_{\tau}}{N_{T} \sigma_{z}^{2}} \right) \right],$$

where $\mathbf{h}_{\tau}^{c} \in \mathbb{C}^{N_{T}}$ is the i.i.d. channel vector from the BS to the *c*-th cache node with each element following the complex Gaussian distribution with zero mean and variance $\rho^{c}\eta_{\tau}^{c}$. R_{τ}^{c} depends only on the transmission parameters (P_{τ}, N_{τ}) and the large-scale fading coefficient $\rho^{c}\eta_{\tau}^{c}$. The *c*-th cache node can decode the file only when

$$R_{\tau}^c \ge R_F$$

The expressions of R_{τ} and R_{τ}^{c} depend on the pathloss and shadowing of the corresponding link. Hence, to decode one file in a BS multicast, the requesting user and cache nodes may need to accumulate different numbers of multicast symbols. By adjusting P_{τ} and N_{τ} , the BS can control the set of receiving cache nodes. Moreover, different frame allocation schemes for these N_{τ} symbols lead to different multicast transmission delay. For example, the transmission delay is large if each frame carries a small number of multicast symbols. We do not directly optimize the delay performance in this work since we focus on a scheduling granularity larger than frame.

C. Cache Dynamics

Let $\{\mathcal{B}_{f,\tau}^c | \forall f\}$ be the cache state information (CaSI) of the *c*-th cache node at the beginning of the τ -th file request, where $\mathcal{B}_{f,\tau}^c \in \{0,1\}$. $\mathcal{B}_{f,\tau}^c = 1$ means that the *f*-th file has been stored at the *c*-th cache node before the τ -th request, and $\mathcal{B}_{f,\tau}^c = 0$ otherwise. Let $\Delta \mathcal{B}_{f,\tau}^c = \mathcal{B}_{f,\tau+1}^c - \mathcal{B}_{f,\tau}^c$, be the corresponding update of the CaSI for the *f*-th file at the *c*-th cache node after the transmission of the τ -th requested file. $\Delta \mathcal{B}_{f,\tau}^c = -1$ means that the *f*-th file cached at the *c*-th cache node is replaced after the transmission of the τ -th requested file, and $\Delta \mathcal{B}_{f,\tau}^c = 1$ means that the *f*-th file is cached at the *c*-th cache node after the transmission of the τ -th requested file. Hence, we have the following constraint due to limited cache size.

Cache size constraint:
$$\sum_{f=1}^{M_F} \left(\mathcal{B}_{f,\tau}^c + \Delta \mathcal{B}_{f,\tau}^c \right) \le M_c, \ \forall c, \tau.$$
(4)

Moreover, letting A_{τ} be the index of the τ -th requested file, we have the following constraints on the update of CaSI.

CaSI update on the decoded file: $\Delta \mathcal{B}_{\mathcal{A}_{\tau},\tau}^{c} \in \{0, \mathbf{I}[R_{\tau}^{c} \geq R_{F}]\}, \forall c, \tau,$ (5)

CaSI update on the cached files:
$$\Delta \mathcal{B}_{f,\tau}^c \in \{0, -\mathcal{B}_{f,\tau}^c\}, \forall c, \tau, f \neq \mathcal{A}_{\tau},$$
 (6)

where I(.) denotes the indicator function. Note that (5) is about the decision on whether to cache the decoded file. For instance, if the *c*-th cache node is able to decode the multicasted file $(I[R_{\tau}^{c} \geq R_{F}] = 1)$, it may store the file $(\Delta \mathcal{B}_{\mathcal{A}_{\tau},\tau}^{c} = 1)$ for possible later transmissions or discard the file $(\Delta \mathcal{B}_{\mathcal{A}_{\tau},\tau}^{c} = 0)$ due to the cache size constraint in (4). Moreover, (6) is about the decision on whether to remove one file from a cache node due to limited cache space.

D. System State and Scheduling Policy

When a requesting user (say the τ -th user) cannot be served by its nearby cache node, the BS should determine the downlink multicast power P_{τ} and the number of transmission symbols N_{τ} for BS multicast. In order to formulate the downlink scheduling problem, we first define the system state S and scheduling policy Ω as follows.

Definition 1 (System State). At the τ -th request arrival, the system state is represented by $S_{\tau} \triangleq (\mathcal{A}_{\tau}, B_{\tau}, \zeta_{\tau})$, consisting of

- Index of the requested file: $A_{\tau} \in \mathcal{F}$.
- CaSI: $B_{\tau} \triangleq \{\mathcal{B}_{f,\tau}^c | \forall f \in \mathcal{F}, c = 1, ..., N_C\}.$
- Statistical channel state information (SCSI): the pathloss and shadowing coefficients of the channel from the BS to the τ -th requesting user and the shadowing coefficients of the channels from the BS to all the cache nodes, denoted as $\zeta_{\tau} \triangleq \{(\eta_{\tau}, \eta_{\tau}^{c}, \rho_{\tau}) | \forall c = 1, ..., N_{C}\}$.

Definition 2 (Scheduling Policy). At the τ -th request arrival, the scheduling policy Ω_{τ} for the τ -th requesting user is a mapping from the system state S_{τ} to the following scheduling actions: BS multicast power P_{τ} and symbol number N_{τ} ; cache update $\{\Delta \mathcal{B}_{f,\tau}^{c} | \forall f \in \mathcal{F}, c = 1, 2, ..., N_{C}\}$. Meanwhile, the constraint (1) on peak power, the constraint (3) on successful decoding at the τ -th requesting user, cache size constraint in (4), and cache update constraints in (5)-(6) should be satisfied. Given a scheduling policy, the system state evolves as a Markov chain. In this paper, we shall minimize the average transmission resource consumptions for BS multicasts, including the multicast power and the number of transmission symbols, by optimizing the scheduling policy for all file requests $\{\Omega_{\tau} | \forall \tau\}$.

Remark 1 (Transmission Time of Each File Multicast). In this paper, we assume that each file multicast can be completed within the coherent time of shadowing effect, which is usually a few seconds. This is suitable for the transmission of short video clips. For the transmission of larger files, they can be divided into a number of segments, each of which can be delivered within the coherent time of shadowing effect. Hence, we can either treat each segment transmission as one new file request, or extend the scheduling policy for one file request in Definition 2 by adopting different scheduling parameters for different segments. The latter extension can follow the similar approach to our previous work [16].

Remark 2 (Overlapped Transmission Period). It is possible that one file request (say the $(\tau + 1)$ th file request) arrives in the transmission period of the previous file request (the τ -th request). It may not be necessary to postpone the $(\tau + 1)$ -th file transmission in this case. For example, the N_{τ} and $N_{\tau+1}$ transmission symbols for both file multicasts may share the same frames (in Fig. 2, one frame may carry symbols for both file multicasts). Hence, the transmission periods for the two file requests can overlap, as long as each file can be delivered within the coherent time of shadowing effect. In order to maintain the latest system state in such concurrent file transmission, the BS can update the CaSI from B_{τ} to $B_{\tau+1}$ immediately after the decision on $\{\Delta \mathcal{B}_{f,\tau}^c | \forall c\}$ is made. The postpone of the $(\tau + 1)$ -th file transmission is necessary when it relies on the previous file transmission. For example, if the $(\tau + 1)$ -th file transmission is from one cache node to the requesting user, and this cache node is receiving the same file from the BS on the τ -th file request. It is clear that the chance of this situation is very small.

III. PROBLEM FORMULATION AND OPTIMAL POLICY STRUCTURE

A. Problem Formulation

In practice, the BS should deliver both popular and dedicated data in downlink as in Fig. 2. The former can be assisted by cache nodes; and the latter, e.g., video call is usually dedicated to one particular user, has to be served by the BS. In this paper, we shall minimize the transmission resource consumption at the BS by offloading traffic for popular files to the cache nodes, so that more downlink transmission resource can be spared for the delivery of dedicated data.

Specifically, let $C_{f,\tau} \triangleq \bigcup_{\{c \mid \forall \mathcal{B}_{f,\tau}^c = 1\}} C_c$ be the coverage area of the cache nodes which have already decoded the *f*-th file before the τ -th file request, and \mathbf{l}_{τ} be the location of the τ -th user. The resource consumption at the τ -th file transmission, measuring the weighted sum of the transmission energy and the number of transmission symbols, is given by

$$g_{\tau}\left(S_{\tau},\Omega_{\tau}(S_{\tau})\right) \triangleq \left(P_{\tau}N_{\tau}+wN_{\tau}\right) \times \mathbf{I}[\mathbf{l}_{\tau} \notin \mathcal{C}_{\mathcal{A}_{\tau},\tau}],$$

where w is the weight on the number of transmission symbols. The average total cost of the BS in the whole lifetime is given by

$$\overline{G}(\{\Omega_1,\Omega_2,\ldots\}) \triangleq \mathbb{E}_{\mathcal{A},\zeta,N_R} \bigg\{ \sum_{\tau=1}^{N_R} g_\tau \Big(S_\tau,\Omega_\tau(S_\tau)\Big) \bigg\},\,$$

where $\zeta \triangleq \{\zeta_{\tau} | \forall \tau = 1, ..., N_R\}$ and $\mathcal{A} \triangleq \{\mathcal{A}_{\tau} | \forall \tau = 1, ..., N_R\}$. The expectation is taken over all possible large-scale channel fading, requested files and the total request number in the lifetime N_R . As a result, the transmission design in this paper can be formulated as the following stochastic optimization problem.

Problem 1 (Optimization with Random Number of Requests).

$$\min_{\{\Omega_1,\Omega_2,\ldots\}} \quad \overline{G}(\{\Omega_1,\Omega_2,\ldots\}) = \mathbb{E}_{\mathcal{A},\zeta,N_R} \left\{ \sum_{\tau=1}^{N_R} g_\tau \left(S_\tau,\Omega_\tau(S_\tau)\right) \right\}$$

s.t. Constraints in (1), (3) - (6).

Remark 3 (Interpretation of Problem 1). Problem 1 is about the BS's scheduling of file multicast within a finite lifetime, where the file requests arrive randomly. The file transmissions are offloaded from the BS, if the requesting users can be served by some cache nodes. Otherwise, the BS should make sure the requesting users are able to decode the requested file from downlink multicast, as shown in the hard decoding constraint (3). Meanwhile, the BS can also choose some cache nodes as the receivers of the downlink multicast. Hence, each file multicast updates the cached files at the cache nodes, which affects the probability of traffic offloading in the future. Different choices of receiving cache nodes in each downlink multicast may lead to different probability of future traffic offloading and different overall transmission cost (objective)

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of Problem 1). Problem 1 is to find the best selection policy of receiving cache nodes for all file multicasts, via the adaptation of multicast power and symbol number.

B. Structure of Optimal Scheduling Policy

Since the number of stages N_R in Problem 1 is random, the standard approach for finite-horizon MDPs with a fixed number of stages [34] cannot be adopted here. An intuitive explanation is as follows: in conventional finite-horizon MDP, the policy at one stage is determined according to the system state and the optimized cost of the remaining stages; in Problem 1, the number of remaining stages is random, and the optimized cost of the remaining stages cannot be derived by the backward induction. Hence, we first revise the definition of *value function* of MDP such that the Bellman's equations can be extended to the case of a random stage number. Specifically, let $W_k(S_k)$ be the revised value function for the k-th file request and system state S_k , which measures the minimum average remaining cost from the k-th request to the last (N_R -th with N_R being a random variable) request given the current system state. That is,

Revised Value Function:
$$W_k(S_k) \triangleq \min_{\{\Omega_k, \Omega_{k+1}, \dots\}} \mathbb{E}_{\mathcal{A}, \zeta, N_R} \left\{ \sum_{\tau=k}^{N_R} g_\tau(S_\tau, \Omega_\tau) \mathbf{I}(k \le N_R) \middle| S_k \right\}$$

s.t. Constraints in $(1), (3) - (6)$.

Compared to the conventional definition of value function, the above definition of $W_k(S_k)$ involves an extra indicator function $I(k \le N_R)$ and an expectation on N_R , which count for the randomness of N_R and the situation that N_R may be smaller than k. Then, the optimal solution of Problem 1 can be deduced via the following revised Bellman's equations.

Lemma 1 (Revised Bellman's Equations for MDP with Random Number of Stages). *The value* function $W_k(S_k), \forall S_k, k = 1, 2, ..., L$, satisfy the following Bellman's equations.

$$W_{k}(S_{k}) = \min_{\Omega_{k}(S_{k})} \left\{ g_{k} \left(S_{k}, \Omega_{k}(S_{k}) \right) \Pr(N_{R} \ge k) + \sum_{S_{k+1}} W_{k+1}(S_{k+1}) \Pr\left[S_{k+1} \middle| S_{k}, \Omega_{k}(S_{k}) \right] \right\},$$

s.t. Constraints in (1), (3) - (6),

where S_{k+1} denotes the system state at the (k+1)-th request, and $W_{L+1}(.) \equiv 0$ for notation convenience.

Proof. Please refer to Appendix A.

The value functions $\{W_k | \forall k\}$ are the functions of both CaSI and SCSI. As the space of the latter is continuous, the calculation of value functions for all system states is intractable. In this paper, by exploiting the independence between the distributions of the large-scale fading and requested files, and adopting the approach proposed in [21], we reduce the system state space. Note that the system state at the k-th request can be represented by $S_k = [\mathcal{A}_k, \mathcal{B}_k, \zeta_k]$ and the distributions of \mathcal{A}_k and ζ_k in each request are independent, we can obtain the following equivalent Bellman's equations with reduced state space by taking expectation over \mathcal{A}_k and ζ_k .

$$\widetilde{W}_{k}(B_{k}) = \min_{\Omega_{k}(B_{k})} \mathbb{E}_{\mathcal{A}_{k},\zeta_{k}} \left\{ g_{k} \left(S_{k}, \Omega_{k}(S_{k}) \right) \Pr(N_{R} \ge k) + \sum_{B_{k+1}} \widetilde{W}_{k+1}(B_{k+1}) \Pr\left[B_{k+1} \left| S_{k}, \Omega_{k}(S_{k}) \right| \right\},$$
(7)

where $\widetilde{W}_k(B_k) \triangleq \mathbb{E}_{\mathcal{A}_k,\zeta_k} \left[W_k(S_k) \middle| B_k \right]$ is the new value function, and $\Omega_k(B_k) \triangleq \{\Omega_k(S_k) | \forall \mathcal{A}_k, \zeta_k\}$ is the aggregation of scheduling policy for all possible requested files, pathlosses and shadowing coefficients of the requesting users and the cache nodes. B_{k+1} denotes the CaSI at the (k+1)-th request. Moreover, we have the following conclusion.

Lemma 2 (Optimal Scheduling Policy). The optimal scheduling policy for Problem 1 is

$$\Omega_k^*(S_k) = \arg\min_{\Omega_k(S_k)} \left\{ g_k \left(S_k, \Omega_k(S_k) \right) \Pr(N_R \ge k) + \sum_{B_{k+1}} \widetilde{W}_{k+1}(B_{k+1}) \Pr\left[B_{k+1} \left| S_k, \Omega_k(S_k) \right| \right\}, \quad (8)$$

Proof. The proof is similar to that of Lemma 1 of [16] and is omitted due to page limitation. \Box

In the standard optimal solution algorithm for finite-horizon MDP, we could start by evaluating the value function for the last stage (the *L*-th stage) via

$$\widetilde{W}_{L}(B_{L}) = \min_{\Omega_{L}(B_{L})} \mathbb{E}_{\mathcal{A}_{L},\zeta_{L}} \bigg\{ g_{L} \bigg(S_{L}, \Omega_{L}(S_{L}) \bigg) \Pr(N_{R} \ge L) \bigg\},\$$

and then evaluate the value function of its previous stage $(\widetilde{W}_{L-1}(B_{L-1}))$ via

$$\widetilde{W}_{L-1}(B_{L-1}) = \min_{\Omega_{L-1}(B_{L-1})} \mathbb{E}_{\mathcal{A}_{L-1},\zeta_{L-1}} \left\{ g_{L-1} \left(S_{L-1}, \Omega_{L-1}(S_{L-1}) \right) \Pr(N_R \ge L-1) + \sum_{B_L} \widetilde{W}_L(B_L) \Pr\left[B_L \middle| S_{L-1}, \Omega_{L-1}(S_{L-1}) \right] \right\}.$$

By such backward induction from the last stage to the first stage, the value functions $\widetilde{W}_k(B_k)$, $\forall k = 1, ..., L$, can all be calculated, and the optimal scheduling policy can be derived via (8). However, this optimal algorithm usually suffers from the curse of dimensionality, which will be further explained in the following section.

IV. LOW-COMPLEXITY SCHEDULING POLICY

The computation complexity of the value functions \widetilde{W}_k (k = 1, ..., L) defined in (7) is huge due to the following two reasons. First, the maximum possible stage number L is usually huge, and the value functions for all stages should be evaluated, unlike the infinite-horizon MDP considered in most of the existing literature [18], [20], [35]. Consider one example with a lifetime of 24 hours and a frame duration of 10 milliseconds. The number of frames in the lifetime is $L = 100 \times 60 \times 60 \times 24 \approx 8.6 \times 10^6$. Moreover, the space of B_k grows exponentially with respect to the number of cache nodes and the number of files. Note that the conventional approaches for approximate MDP are mostly designed for infinite-horizon MDP. In this section, we shall propose a novel framework to approximate the value functions of finite-horizon MDP (with a random stage number), such that the computation complexity of the value functions can be significantly reduced.

Specifically, regarding the above two causes for prohibitive complexity, the approximation of the value functions consists of the two steps.² Firstly, we propose a flexible approximation framework in Section IV-A, where the value functions are approximated by their lower-bounds, and the approximation error can be adjusted as a trade-off of the computation complexity. Secondly, the lower-bounds are further decoupled for each file and each cache node via a novel linear approximation method in Section IV-B. In addition, the overall approximation error is analyzed in Section IV-C, and an online scheduling algorithm based on the approximate value functions is elaborated in Section IV-D.

A. Flexible Approximation Framework for Stage Number Reduction

We first introduce the following bounds on the value functions \widetilde{W}_k , k = 1, ..., L.

Lemma 3. Let $M_R^{\epsilon} \triangleq \max\{M_R^{\epsilon} \in \mathbb{Z}_+ | \Pr(N_R > M_R^{\epsilon}) \le \epsilon\}$. An upper-bound on $\widetilde{W}_k(B_k)$ $(\forall k \le M_R^{\epsilon})$ is given by

$$\widetilde{W}_{k}(B_{k}) \leq \min_{\{\Omega_{\tau} \mid \forall \tau\}} \left\{ \mathbb{E}_{\mathcal{A},\zeta} \left[\sum_{\tau=k}^{M_{R}^{\epsilon}} g_{\tau}(S_{\tau},\Omega_{\tau}) \operatorname{Pr}(\tau \leq N_{R}) \middle| B_{k} \right] \right\} + \sum_{\tau=M_{R}^{\epsilon}+1}^{L} \overline{g}_{max} \operatorname{Pr}(\tau \leq N_{R}) \triangleq \mathcal{U}_{k}(B_{k})$$

² Note that the average number of file requests βL is usually much smaller than the total number of frames L (there will be traffic overflow otherwise), it may be costly and inefficient to evaluate the value functions for all k = 1, 2, ..., L.

where $\overline{g}_{max} \triangleq \mathbb{E}_{\eta_{\tau}} \left[F(\theta, P_B) \middle| \rho_{\tau} = \rho_{min} \right]$ denotes an upper-bound on the average cost of each stage, $F(\theta, P_B)$ is given by

$$F(\theta, P_B) \triangleq \begin{cases} \frac{w \ln(2)R_F}{\alpha W(\frac{2\theta w}{e})}, & \frac{w}{W(\frac{2\theta w}{e})} < P_B \\ \frac{(P_B + w)R_F}{\log_2(P_B) + \theta}, & \frac{w}{W(\frac{2\theta w}{e})} \ge P_B \end{cases}$$

$$(9)$$

 $\theta \triangleq \mathbb{E}_{\mathbf{h}_{\tau}} \left[\log_2 \left(\frac{\|\mathbf{h}_{\tau}\|^2}{N_T \sigma_z^2} \right) \right]$, and $\mathbb{W}(x)$ is the Lambert-W function [36], ρ_{min} is the pathloss from the BS to the farthest location of the cell. Moreover, a lower-bound on $\widetilde{W}_k(B_k)$ ($\forall k \leq M_R^\epsilon$) is given by

$$\widetilde{W}_{k}(B_{k}) \geq \min_{\{\Omega_{\tau} \mid \forall \tau\}} \mathbb{E}_{\mathcal{A},\zeta} \left\{ \sum_{\tau=k}^{M_{R}^{\epsilon}} g_{\tau}(S_{\tau}, \Omega_{\tau}^{*}) \operatorname{Pr}(\tau \leq N_{R}) \middle| B_{k} \right\} \triangleq \mathcal{L}_{k}(B_{k}).$$
(10)

Proof. Please refer to Appendix B.

As a remark, notice that \overline{g}_{max} is actually the minimum average transmission cost to one requesting user located at the farthest position from the BS. It is clear that when increasing M_R^{ϵ} , the lower-bound \mathcal{L}_k and the upper-bound \mathcal{U}_k tend to the exact function \widetilde{W}_k , i.e. the approximation error tends to zero, at the cost of computation complexity and storage increase. The approximation error, in terms of ϵ , will be discussed in Section IV-C. In this paper, we shall use the lower-bound to approximate the value functions when $k \leq M_R^{\epsilon}$, and zero to value functions when $k > M_R^{\epsilon}$, i.e.,

$$\widetilde{W}_{k}(B_{k}) \approx \begin{cases} \mathcal{L}_{k}(B_{k}) & k \leq M_{R}^{\epsilon} \\ 0 & k > M_{R}^{\epsilon} \end{cases}$$
(11)

Direct evaluation of \mathcal{L}_k for $k = 1, 2, ..., M_R^{\epsilon}$ is still intractable due to the huge space of B_k . In the following part, we shall further decouple \mathcal{L}_k via a novel linear approximation method.

B. Linear Approximation of Value Function

Let $\{\Omega_{\tau}^{\dagger} | \tau = 1, ..., M_R^{\epsilon}\} = \arg \min_{\{\Omega_{\tau} | \tau = 1, ..., M_R^{\epsilon}\}} \mathbb{E}_{\mathcal{A}, \zeta} \left\{ \sum_{\tau=1}^{M_R^{\epsilon}} g_{\tau}(S_{\tau}, \Omega_{\tau}) \operatorname{Pr}(\tau \leq N_R) \right\}$ be the optimal scheduling policy with fixed M_R^{ϵ} stages. The value function for $k = 1, ..., M_R^{\epsilon}$ and CaSI

 B_k can be written as

$$\widetilde{W}_{k}(B_{k}) \approx \mathcal{L}_{k}(B_{k}) = \sum_{f=1}^{M_{F}} \sum_{c=0}^{N_{C}} \underbrace{\mathbb{E}_{\mathcal{A},\zeta} \left[\sum_{\tau=k}^{M_{R}^{\epsilon}} g_{\tau}(S_{\tau}, \Omega_{\tau}^{\dagger}) \operatorname{Pr}(\tau \leq N_{R}) \mathbf{I}(\mathcal{A}_{\tau} = f) \mathbf{I}(\mathbf{l}_{\tau} \in \mathcal{C}_{c}) \middle| B_{k} \right]}_{\widetilde{W}_{k,f}(B_{k})}$$
$$= \sum_{f=1}^{M_{F}} \widetilde{W}_{k,f}(B_{k}) = \sum_{f=1}^{M_{F}} \sum_{c=0}^{N_{C}} \widetilde{W}_{k,f,c}(B_{k}), \tag{12}$$

where $\widetilde{W}_{k,f}(B_k)$ is named as the *per-file value function*, approximating the average cost for the f-th file since the k-th stage under the optimal policy $\{\Omega_k^* | \forall k\}$ defined in (8). $\widetilde{W}_{k,f,c}(B_k)$ is named as the *per-file per-region value function*, approximating the average cost for the f-th file in the region C_c since the k-th stage under the optimal policy $\{\Omega_k^* | \forall k\}$. The ways to approximate value function, as introduced in [28], are general but short at exploiting the problem structure. Moreover, they usually require value iteration and may not be applicable for finite-horizon MDP. In the following, we shall propose a problem-specific approximation of value functions with analytical expressions. With such analytical expressions, computationally expensive numerical algorithms such as value iteration can be bypassed.

We refer to $\mathcal{F}_c^H \triangleq \{1, 2, ..., M_c\}, c = 1, 2, \cdots, N_C$, and $\mathcal{F}_c^L \triangleq \{M_c + 1, ..., M_F\}$ as the highpopularity and low-popularity file sets in the region \mathcal{C}_c , respectively. If the f-th file $(f \in \mathcal{F}_c^H)$ has been stored, it will never be replaced by other files. On the other hand, it is possible that f-th file $(f \in \mathcal{F}_c^L)$ stored at the c-th cache node will be replaced by other files in the future transmission due to limited cache space. The approximations of the per-file per-region value functions $\widetilde{W}_{k,f,c}$, $\forall k, f, c$, which are denoted as $J_{k,f,c}$, are elaborated below respectively.

1) Approximation of $\widetilde{W}_{k,f,0}$: $\widetilde{W}_{k,f,0}$ is the transmission cost spent for any requesting users located in the region without cache nodes C_0 . In the approximation, we assume that the BS spends just sufficient transmission cost to ensure the file delivery to any requesting users in C_0 . Hence, the approximation of $\widetilde{W}_{k,f,0}$, denoted as $J_{k,f,0}$, can be written as

$$J_{k,f,0} \triangleq \min_{\{P_{\tau},N_{\tau}|\tau=k,\dots,M_{R}^{\epsilon}\}} \sum_{\tau=k}^{M_{R}^{\epsilon}} p_{f} \Pr[N_{R} \ge \tau] \Pr[\mathbf{l}_{\tau} \in \mathcal{C}_{0}] \times \mathbb{E}_{\zeta_{\tau}} \left[P_{\tau}N_{\tau} + wN_{\tau} \middle| \mathbf{l}_{\tau} \in \mathcal{C}_{0} \right], \quad (13)$$

where the constraints in (1) and (3) should be satisfied. It is clear that³

$$J_{k,f,0} \approx \sum_{\tau=k}^{M_R^{\epsilon}} p_f \Pr[N_R \ge \tau] \underbrace{\Pr[\mathbf{l}_{\tau} \in \mathcal{C}_0] \times \mathbb{E}_{\zeta_{\tau}} \left[F(\theta, P_B) \middle| \mathbf{l}_{\tau} \in \mathcal{C}_0 \right]}_{\text{Denoted as } \mu_0}, \tag{14}$$

where the approximation is for high SINR region, $F(\theta, P_B)$ is given by (9) and $\theta = \mathbb{E}_{\mathbf{h}_{\tau}} \left[\log_2 \left(\frac{||\mathbf{h}_{\tau}||^2}{N_T \sigma_z^2} \right) \right]$. If the statistics of large-scale fading and file popularity are known, the above expectation can be calculated. Otherwise, a learning-based approach is introduced in the next section to evaluate the approximate value function.

2) Value Function Approximation for High-Popularity Files: Let $\mathbf{b}_{f}^{c} \triangleq \{\mathcal{B}_{f,k}^{c} = 0\} \cup \{\mathcal{B}_{f,k}^{i} = 1 | \forall i \neq c\}$ be the CaSI where only the *c*-th cache node has not decoded the *f*-th file. We first define $\widehat{W}_{k,f}(\mathbf{b}_{f}^{c})$ as follows.

$$\widehat{W}_{k,f}(\mathbf{b}_{f}^{c}) \triangleq \min \mathbb{E}_{\mathcal{A},\zeta} \left\{ \sum_{\tau=k}^{M_{R}^{e}} g_{\tau} \mathbf{I}(\mathcal{A}_{\tau}=f) \operatorname{Pr}(\tau \leq N_{R}) \middle| \mathbf{b}_{f}^{c} \right\}$$
(15)
s.t. Constraints in (1), (3) - (6); $\Delta \mathcal{B}_{f,\tau}^{i} = 0, \forall i \neq c, \tau \geq k.$

 $\widehat{W}_{k,f}(\mathbf{b}_{f}^{c})$ approximates the transmission cost for the *f*-th file when the *c*-th the cache node has not decoded the *f*-th file, i.e., $\widetilde{W}_{k,f,c} + \widetilde{W}_{k,f,0}$. Since $J_{k,f,0}$ approximates the cost $\widetilde{W}_{k,f,0}$, the per-file per-region value function $\widetilde{W}_{k,f,c}(B_{k})$ for all c > 0 and $f \in \mathcal{F}_{c}^{H}$ can be approximated by

$$\widetilde{W}_{k,f,c}(B_k) \approx J_{k,f,c}(B_k) \triangleq \begin{cases} 0, & \text{when } \mathcal{B}_{f,k}^c = 1; \\ \widehat{W}_{k,f}(\mathbf{b}_f^c) - J_{k,f,0}, & \text{when } \mathcal{B}_{f,k}^c = 0. \end{cases}$$
(16)

Moreover, $\widehat{W}_{k,f}(\mathbf{b}_f^c)$ can be calculated via the following backward induction:

- Step 1: Let $k = M_R^{\epsilon} + 1$, and initialize $\widehat{W}_{k,f}(\mathbf{b}_f^c) = 0$.
- Step 2: Let k = k 1, and calculate $\widehat{W}_{k,f}(\mathbf{b}_f^c)$ as follows.

$$\widehat{W}_{k,f}(\mathbf{b}_f^c) = (1 - p_f) \times \widehat{W}_{k+1,f}(\mathbf{b}_f^c) + p_f \times v_{k,f,c}$$
(17)

³Please refer to [16] for the derivation of the expression.

$$\begin{aligned}
\upsilon_{k,f,c} &= \Pr\left[\mathbf{l}_{k} \in \mathcal{C}_{f,k}(\mathbf{b}_{f}^{c})\right] \times \widehat{W}_{k+1,f}(\mathbf{b}_{f}^{c}) \\
&+ \Pr\left[\mathbf{l}_{k} \notin \mathcal{C}_{f,k}(\mathbf{b}_{f}^{c})\right] \times \left\{\mathbb{E}_{\zeta_{k}}\left[\overline{G}_{k,f}^{1}\middle|R_{k} \leq R_{k}^{c}\right]\Pr\left[R_{k} \leq R_{k}^{c}\right] \\
&+ \mathbb{E}_{\zeta_{k}}\left[\min\{\overline{G}_{k,f}^{2}, \overline{G}_{k,f}^{3}\}\middle|R_{k} > R_{k}^{c}\right]\Pr\left[R_{k} > R_{k}^{c}\right]\right\},
\end{aligned}$$
(18)

where $C_{f,k}$ is the area (given CaSI \mathbf{b}_{f}^{c}) in which the users are able to receive the *f*-th file from one of the cache nodes, and $\overline{G}_{k,f}^{1}, \overline{G}_{k,f}^{2}, \overline{G}_{k,f}^{3}$ are given by

$$\overline{G}_{k,f}^{1} \triangleq F(\theta, P_{B}) \operatorname{Pr}[N_{R} \ge k] + J_{k+1,f,0}, \overline{G}_{k,f}^{2} \triangleq F(\theta, P_{B}) \operatorname{Pr}[N_{R} \ge k] + \widehat{W}_{k+1,f}(\mathbf{b}_{f}^{c}),$$

$$\overline{G}_{k,f}^{3} \triangleq F(\theta^{c}, P_{B}) \operatorname{Pr}[N_{R} \ge k] + J_{k+1,f,0}, \text{ where } \theta^{c} \triangleq \mathbb{E}_{\mathbf{h}_{k}^{c}} \left[\log_{2} \left(\frac{||\mathbf{h}_{k}^{c}||^{2}}{N_{T} \sigma_{z}^{2}} \right) \right].$$
(19)

• Step 3: If k > 1, go to step 2. Otherwise, terminate.

3) Value Function Approximation for Low-Popularity Files: In order to approximate $\widetilde{W}_{k,f,c}$, $f \in \mathcal{F}_c^L$, we first define the following notations.

• Let $\eta_{k,f,c}$ be the minimum transmission cost for the BS to ensure successful file transmission when the requesting user is in the region C_c since the τ -th frame, given that the *f*-th file has not been cached at the *c*-th cache node, i.e.,

$$\eta_{k,f,c} \triangleq \min_{\{P_k,N_k\}} p_f \Pr[N_R \ge k] \times \Pr[\mathbf{l}_k \in \mathcal{C}_c] \times \mathbb{E}_{\zeta_k} \Big[P_k N_k + w N_k \Big| \mathbf{l}_k \in \mathcal{C}_c \Big],$$

where the constraints in (1) and (3) should be satisfied. It is clear that

$$\eta_{k,f,c} \approx p_f \Pr[N_R \ge \tau] \Pr[\mathbf{l}_k \in \mathcal{C}_c] \mathbb{E}_{\zeta_k} \Big[F(\theta, P_B) \Big| \mathbf{l}_k \in \mathcal{C}_c \Big]$$

for the high SINR region.

• Let $\mathbb{P}_{k,n}^{f,c}(B_k)$ be the probability that there are $M_c - \sum_{m=1}^{f-1} \mathcal{B}_m^c - 1$ requests for the files $\{1, 2, ..., f-1\}$ from the k-th request to the (k + n - 1)-th request, and the (k + n)-th request is also for these files, given the current CaSI B_k . Hence,

$$\mathbb{P}_{k,n}^{f,c}(B_k) \triangleq \begin{cases} 0, & n < M_c - \sum_{m=1}^{f-1} \mathcal{B}_m^c \\ \binom{n-1}{M_c - \sum_{m=1}^{f-1} \mathcal{B}_m^c - 1} \varphi^{M_c - \sum_{m=1}^{f-1} \mathcal{B}_m^c} (1 - \varphi)^{n - M_c + \sum_{m=1}^{m-1} \mathcal{B}_m^c}, & n \ge M_c - \sum_{m=1}^{f-1} \mathcal{B}_m^c \end{cases}$$
where $\varphi \triangleq \sum_{m=1}^{r} \varphi^{m}_{m}(x_m, x_m) \mathcal{B}_m^{r}$

where $\varphi \triangleq \sum_{\{f \mid f \in \mathcal{F}_c^H, \mathcal{B}_f^c = 0\}} p_f$.

True value function $\widetilde{W}_k(B_k)$ defined in (7)

First approximation

 $\widetilde{W}_k(B_k) \approx \mathcal{L}_k(B_k)$, where $\mathcal{L}_k(B_k)$ defined in (10) is the lower-bound of true function

Second approximation

$$\mathcal{L}_{k}(B_{k}) = \sum_{f=1}^{M_{F}} \sum_{c=0}^{N_{C}} \widetilde{W}_{k,f,c}(B_{k}) \approx \sum_{f=1}^{M_{F}} J_{k,f,0}(B_{k}) + \sum_{f=1}^{M_{F}} \sum_{c=1}^{N_{C}} J_{k,f,c}(B_{k})$$

If $c = 0$, $J_{k,f,0}(B_{k})$ is given by (14)
If the file is high-popularity, $J_{k,f,c}(B_{k})$ is given by (16)

If the file is low-popularity, $J_{k,f,c}(B_k)$ is given by (21)

Fig. 3. Illustration of overall value function approximation procedure.

For $f \in \mathcal{F}_c^L$, the per-file per-region value function is approximated as follows.

$$\widetilde{W}_{k,f,c}(B_k) \approx J_{k,f,c}(B_k) \triangleq \begin{cases} \sum_{\tau=k}^{M_R^{\epsilon}} \eta_{\tau,f,c} - \sum_{n=0}^{M_R^{\epsilon}-k} \sum_{\tau=k}^{k+n} \eta_{\tau,f,c} \mathbb{P}_{k,n}^{f,c}(B_k), & \text{when } \mathcal{B}_{f,k}^c = 1\\ \sum_{\tau=k}^{M_R^{\epsilon}} \eta_{\tau,f,c}, & \text{when } \mathcal{B}_{f,k}^c = 0 \end{cases}$$
(21)

In summary, the overall value function approximation procedure is illustrated in Fig. 3. Compare with the conventional optimal solution, the complexity of value function evaluation is dramatically reduced. In the above approximation approach, the computation complexity of value function calculation is $\mathcal{O}(M_R^{\epsilon}N_CM_F)$. In order to store these values, the required memory space is also $\mathcal{O}(M_R^{\epsilon}N_CM_F)$. On the other hand, the optimal solution of MDP suffers from the curse of dimensionality. Specifically, the computation complexity of the conventional value iteration algorithm is $\mathcal{O}(LN_C 2^{M_FN_C})$, and the memory requirement is $\mathcal{O}(L2^{M_FN_C})$.

C. Bounds on Approximate Value Functions

As elaborated in the above two parts, the approximation of value function \widetilde{W}_k is made via two steps. Firstly, the random stage number N_R is transformed to a fixed stage number M_R^{ϵ} , and the value function \widetilde{W}_k is approximated by its lower-bound \mathcal{L}_k . Then, \mathcal{L}_k is further approximated by J_k , i.e

$$J_k(B_k) \triangleq \sum_{f=1}^{M_F} J_{k,f,0}(B_k) + \sum_{c=1}^{N_C} \left\{ \sum_{f=1}^{M_F} J_{k,f,c}(B_k) \right\} \approx \widetilde{W}_k(B_k), \forall B_k$$

From Lemma 3, it is straightforward that the approximation error of the first step is upperbounded as follows.

$$\widetilde{W}_{k}(B_{k}) - \mathcal{L}_{k}(B_{k}) \leq \mathcal{U}_{k}(B_{k}) - \mathcal{L}_{k}(B_{k}) = \sum_{\tau=M_{R}^{\epsilon}+1}^{L} \overline{g}_{max} \operatorname{Pr}(\tau \leq N_{R}) < \epsilon L \overline{g}_{max}.$$
(22)

The error upper-bound $\epsilon L \overline{g}_{max}$ tends to 0 when M_R^{ϵ} tends to L. Since $\beta \ll 1$, ϵL can be arbitrarily small even when $\beta L < M_R^{\epsilon} \ll L$. As \overline{g}_{max} is a constant, we can choose one $M_R^{\epsilon} \ll L$ such that the the upper bound $\epsilon L \overline{g}_{max}$ of $|\widetilde{W}_k(B_k) - \mathcal{L}_k(B_k)|$ is negligible compared with $\mathcal{L}_k(B_k)$. Thus, the first approximation step of the value function can be tight. In order to analyze the overall approximation error (i.e. $|\widetilde{W}_k(B_k) - J_k(B_k)|$), we first introduce the following bounds.

Lemma 4 (Upper-Bound on Value Function). The value function $\widetilde{W}_k(B_k)$ is upper-bounded as

$$\widetilde{W}_{k}(B_{k}) \leq \mathcal{U}_{k}(B_{k}) \leq J_{k}(B_{k}) + \sum_{\tau=M_{R}^{\epsilon}+1}^{L} \overline{g}_{max} \operatorname{Pr}(\tau \leq N_{R}) \triangleq \widetilde{\mathcal{U}}_{k}(B_{k}), \quad \forall k.$$
(23)

Proof. Please refer to Appendix C.

Let $B_{\tau+1}^{\pi} \triangleq \pi(B_{\tau})$ be the CaSI of the $(\tau + 1)$ -th stage after applying the operator π on the CaSI B_{τ} ($\forall \tau$). Specifically, the cache status at the *c*-th cache node ($\forall c$) in $B_{\tau+1}^{\pi}$ is given by

• When $\sum_{f} \mathcal{B}_{f,\tau}^c < M_c$,

$$\mathcal{B}_{f,\tau+1}^{c} = \begin{cases} 1, & f = f_{min}^{c} \\ \mathcal{B}_{f,\tau}^{c}, & f \neq f_{min}^{c} \end{cases}$$
(24)

where $f_{min}^c \triangleq \min\{f | \forall \mathcal{B}_{f,\tau}^c = 0\}$ is the index of the most popular file which has not been cached at the *c*-th cache node.

• When $\sum_{f} \mathcal{B}_{f,\tau}^{c} = M_{c}$ and $f_{min}^{c} < f_{max}^{c}$, where $f_{max}^{c} \triangleq \max\{f | \forall \mathcal{B}_{f,\tau}^{c} = 1\}$ is the index of

$$\square$$

the least popular file cached at the c-th cache node,

$$\mathcal{B}_{f,\tau+1}^{c} = \begin{cases} 1, & f = f_{min}^{c} \\ \mathcal{B}_{f,\tau}^{c}, & f \notin \{f_{min}^{c}, f_{max}^{c}\} \\ 0, & f = f_{max}^{c} \end{cases}$$
(25)

• When
$$\sum_{f} \mathcal{B}_{f,\tau}^{c} = M_{c}$$
 and $f_{min}^{c} > f_{max}^{c}, \mathcal{B}_{f,\tau+1}^{c} = \mathcal{B}_{f,\tau}^{c}, \forall f$.

Moreover, let $\pi^k(B_{\tau})$ be the CaSI of the $(\tau + k)$ -th stage after applying the operator π on the CaSI of the τ -th stage B_{τ} for k times ($\forall \tau$). A lower-bound of value function is described below.

Lemma 5 (First Lower-Bound on Value Function). One lower-bound of $\widetilde{W}_k(B_k)$ is given by

$$\widetilde{W}_{k}(B_{k}) \geq \mathcal{L}_{k}(B_{k}) \geq \sum_{\tau=k}^{M_{R}^{e}} \bar{g}_{min}(\pi^{\tau-k}(B_{k})) \operatorname{Pr}(\tau \leq N_{R}) \triangleq \widetilde{\mathcal{L}}_{k}^{1}(B_{k}),$$
(26)

where

$$\bar{g}_{min}(B_{\tau}) \triangleq \min_{\Omega_{\tau}} \mathbb{E}_{\mathcal{A}_{\tau},\zeta_{\tau}} \{ g_{\tau}(S_{\tau},\Omega_{\tau}) | B_{\tau} \}$$

s.t. Constraint in (1), (3),

denotes the minimum transmission cost for the BS to guarantee the successful reception of the requesting user.

Proof. Please refer to Appendix D.

The above lower-bound may be loose when most of the files are of high popularity, as it underestimates the cost of file placement. By relaxing the cache size constraint in (4), we have another lower-bound as follows.

Lemma 6 (Second Lower-Bound on Value Function). $\widetilde{W}_k(B_k)$ is lower-bounded by

$$\widetilde{W}_{k}(B_{k}) \geq \sum_{f=1}^{M_{F}} \sum_{n=1}^{L-k+1} \binom{L-k+1}{n} (\beta p_{f})^{n} (1-\beta p_{f})^{L-n-k+1} \left(\bar{g}_{min}^{f}(B_{k}) + (n-1)\mu_{0} \right) \triangleq \widetilde{\mathcal{L}}_{k}^{2}(B_{k}),$$

where μ_0 is given by (14), and $\bar{g}_{min}^f(B_k) \triangleq \mathbb{E}_{\zeta_k} \left[F(\theta, P_B) \middle| B_k, \mathbf{l}_k \in \mathcal{C} - \mathcal{C}_{f,k} \right] \Pr(\mathbf{l}_k \in \mathcal{C} - \mathcal{C}_{f,k}).$

Proof. This lower-bound is obtained by assuming that there is sufficient memory space in each cache node, so that all the files can be stored without being replaced. Hence, the conclusion of

this lemma directly follows Lemma 5 in [37].

Hence, a tighter lower-bound of $\widetilde{W}_k(B_k)$ is given by

$$\widetilde{\mathcal{L}}_{k}(B_{k}) \triangleq \max\left(\widetilde{\mathcal{L}}_{k}^{1}(B_{k}), \widetilde{\mathcal{L}}_{k}^{2}(B_{k})\right).$$
(27)

Note that $\{J_k | \forall k\}$ are the proposed approximation of value functions, and the upper-bounds of their approximation errors can be calculated via

$$|J_k(B_k) - \widetilde{W}_k(B_k)| \le \widetilde{\mathcal{U}}_k(B_k) - \widetilde{\mathcal{L}}_k(B_k), \forall k, B_k,$$

where both $\widetilde{\mathcal{U}}_k(B_k)$ and $\widetilde{\mathcal{L}}_k(B_k)$ can be calculated analytically. According to the definition of the value function, the optimal average cost of the whole lifetime with initial CaSI B_1 is $\widetilde{W}_1(B_1)$, which can be bounded as $\widetilde{\mathcal{L}}_1(B_1) \leq \widetilde{W}_1(B_1) \leq \widetilde{\mathcal{U}}_1(B_1)$.

D. Scheduling Policy with Approximate Value Functions

In this part, we optimize the multicast power P_k , symbol number N_k and the decisions on cache update $\{\Delta \mathcal{B}_{f,k}^c | \forall f, c\}$ for the k-th ($\forall k$) multicast, when the k-th file request cannot be served by any cache node (i.e., $\mathbf{l}_k \notin C_{\mathcal{A}_k,k}$). Given P_k , N_k and the system state, the cache nodes which can successfully decode the \mathcal{A}_k -th file are determined, so are $\{\Delta \mathcal{B}_{f,k}^c | \forall f, c\}$. Hence, with the approximate value functions $\{J_{k,f,c} | \forall k, f, c\}$, the optimization problem in (8) can be rewritten as follows.

Problem 2 (Scheduling with Approximate Value Functions).

$$Q_k^* \triangleq \min_{P_k, N_k} \left\{ g_k \left[S_k, \Omega_k(S_k) \right] \Pr(N_R \ge k) + J_{k+1}(B_{k+1}) \right\}$$

s.t. Constraints in (1), (3) - (6), (28)

where B_{k+1} represents the CaSI after the k-th multicast. Specifically, in the cache update from B_k to B_{k+1} , a cache node stores the A_k -th file when (1) it has decoded this file, and (2) there is spare memory or cached files with lower popularity (than A_k).

Because of the factor $J_{k+1}(B_{k+1})$ in the objective, Problem 2 is a mixed continuous and discrete optimization problem: the cache nodes for receiving the A_k -th file should be selected; for given the receiving cache nodes, the transmission power and symbol number should be optimized accordingly. Its optimal solution is summarized below.

Lemma 7 (Optimal Solution of Problem 2). Let $d_1, d_2, ..., d_{N_C}$ be the indexes of the cache nodes, whose large-scale fading coefficients satisfy $\rho^{d_1}\eta_k^{d_1} \leq \rho^{d_2}\eta_k^{d_2} \leq ... \leq \rho^{d_{N_C}}\eta_k^{d_{N_C}}$ and $\rho^{d_m}\eta_k^{d_m} \leq \rho_k\eta_k \leq \rho^{d_{m+1}}\eta_k^{d_{m+1}}$. Define $J_k^c(B_k) \triangleq \sum_{f=1}^{M_F} J_{k,f,c}(B_k)$,

$$Q_{d_{i}}^{*} \triangleq \min\left\{ (P_{k}N_{k} + wN_{k}) \Pr(N_{R} \ge k) + \sum_{c=d_{i}}^{d_{N_{C}}} J_{k+1}^{c}(B_{k+1}) + \sum_{c=d_{1}}^{d_{i-1}} J_{k+1}^{c}(B_{k}) \right\}$$

s.t. Constraints in (1), (3) - (6); $R_{k}^{d_{i}} \ge R_{F}$

The minimized objective of Problem 2 is given by

$$Q_k^* = \min_{i=1,2,\dots,m+1} Q_{d_i}^*.$$
(29)

Moreover, $[P_k^{d^*}, N_k^{d^*}] = \left[\min(\frac{w}{\mathbb{W}(\frac{2\theta^{d^*}w}{e})}, P_B), \max(\frac{R_F \ln(2)}{\alpha[\mathbb{W}(\frac{2\theta^{d^*}w}{e})+1]}, \frac{R_F}{\alpha[\theta^{d_*} + \log_2(P_B)]})\right]$, where $d^* = \arg\min_{d_i} Q_{d_i}^*$ and $\theta^{d^*} = \mathbb{E}_{\mathbf{h}_k^{d^*}} \left[\log_2\left(\frac{||\mathbf{h}_k^{d^*}||^2}{N_T \sigma_z^2}\right)\right]$, is the asymptotically optimal solution of Problem 2 in high SINR region.

Proof. The conclusion of (29) is straightforward by noticing that $Q_{d_i}^*$ is the optimal value when the d_i -th cache node and the cache nodes with better downlink channel than the d_i -th one can decode the multicasted file. Moreover, the expressions of $P_k^{d^*}$ and $N_k^{d^*}$ can be derived similarly to (14).

It is clear from the above lemma that, the asymptotically optimal solution of Problem 2 can be obtained via a one-dimensional search: calculate $Q_{d_i}^*$ for the d_i -th cache node, and find a cache node with minimum $Q_{d_i}^*$. The computation complexity is low.

V. REINFORCEMENT LEARNING ALGORITHM FOR UNKNOWN SYSTEM STATISTICS

In Section IV-B, the approximate per-file per-region value functions $J_{k,f,c}(B_k)$ ($\forall k, f, c$) are evaluated analytically by assuming the knowledge on the distribution of the requesting users \mathcal{D} and the file popularity $\{p_f | \forall f\}$. In practice, however, the former distribution may not be available at the BS, and the initial estimation of file popularity may not be accurate. In this section, a reinforcement learning algorithm is proposed to estimate the approximate value function for the above practical scenario in an online way.

For elaboration convenience, $\forall c = 0, 1, 2..., N_C$, we define

$$\mu_c \triangleq \min_{\{P_k, N_k\}} \Pr[\mathbf{l}_k \in \mathcal{C}_c] \times \mathbb{E}_{\zeta_k} \Big[P_k N_k + w N_k \Big| \mathbf{l}_k \in \mathcal{C}_c \Big],$$

where the constraints in (1) and (3) should be satisfied. Hence, we have

$$\mu_c \approx \Pr[\mathbf{l}_k \in \mathcal{C}_c] \times \mathbb{E}_{\zeta_k} \Big[F(\theta, P_B) \Big| \mathbf{l}_k \in \mathcal{C}_c \Big]$$

in the high SINR region. It can be observed from (14), (16) and (21) that, the approximate per-file per-region value functions depend on $\{\mu_c | \forall c = 0, 1, 2, ..., N_C\}$, $\{\upsilon_{k,f,c} | \forall k, f, c\}$ (defined in (18)) and $\{p_f | \forall f\}$. Instead of the learning of the value functions (e.g., the Q-learning method in [34]) directly, we propose to learn $\{\mu_c | \forall c = 0, 1, 2, ..., N_C\}$, $\{\upsilon_{k,f,c} | \forall k, f, c\}$ and $\{p_f | \forall f\}$, and calculate the per-file per-region value functions in the following algorithm. To facilitate more efficient learning on files' popularity, we extend the data collection scope from single cell to a network. Specifically, it is assumed that the file popularity is homogeneous within N_{cell} cells, and each BS can collect the history data of file requests from all these BSs.

Algorithm 1 (Reinforcement Learning for Per-File Per-Region Value Functions).

- Step 1: Let t = 0. Initialize the values of J_{k,f,c}(B_k), p_f, μ_c, υ_{k,f,c} (∀k, f, c) according to certain assumptions on user arrival and popularity distributions, denoted as J⁰_{k,f,c}(B_k), p⁰_f, μ⁰_c, υ⁰_{k,f,c} respectively.
- Step 2: Let t = t + 1 on each new file request arrival. Update $p_f^t, \mu_c^t, v_{k,f,c}^t(\forall k, f, c)$ as follows.
 - $p_f^t = \frac{t}{t+1}p_f^{t-1} + \frac{1}{t+1}\frac{N_{R,f}^t}{\beta N_{cell}T_F^t}$, where $N_{R,f}^t$ is the total number of requests on the f-th file in the N_{cell} cells during T_F^t frames.
 - $\mu_c^t = \frac{t}{t+1}\mu_c^{t-1} + \frac{1}{t+1}\mathbf{I}[\mathbf{l}_m \in \mathcal{C}_c] \times F(\theta_m, P_B)$, where \mathbf{l}_m is the location of the requesting user, and $\theta_m = \mathbb{E}_{\mathbf{h}_m}\left[\log_2\left(\frac{||\mathbf{h}_m||^2}{N_T\sigma_z^2}\right)\right]$.
 - Moreover,

$$v_{k,f,c}^{t} = \frac{t}{t+1} v_{k,f,c}^{t-1} + \frac{1}{t+1} \bigg\{ \mathbf{I}[\mathbf{l}_{m} \in \mathcal{C}_{f,m}(\mathbf{b}_{f}^{c})] \times \widehat{W}_{k+1,f}^{t}(\mathbf{b}_{f}^{c}) + \mathbf{I}[\mathbf{l}_{m} \notin \mathcal{C}_{f,m}(\mathbf{b}_{f}^{c})] \times \bigg[\overline{G}_{k,f}^{1} \mathbf{I}[R_{m} \le R_{m}^{c}] + \min\{\overline{G}_{k,f}^{2}, \overline{G}_{k,f}^{3}\} \mathbf{I}[R_{m} > R_{m}^{c}] \bigg] \bigg\},$$

where $\overline{G}_{k,f}^1, \overline{G}_{k,f}^2, \overline{G}_{k,f}^3$ are given by (19).

- Step 3: Calculate $J_{k,f,c}^t(B_k)$ ($\forall k, f, c$) according to (14), (16) and (21), respectively.
- Step 4: If $\max_{f,c} |J_{k,f,c}^t(B_k) J_{k,f,c}^{t-1}(B_k)|$ is smaller than one threshold, terminate. Otherwise, go to Step 2.



Fig. 4. The average total cost versus the expectation of request times and Zipf parameter, where the geometrical distribution of requesting users is uniform, and the file popularity $\{p_f | \forall f\}$ is known to the BS.



Fig. 5. The average total cost and hitting rate versus the cache size $(\frac{M_c}{M_F})$, where $\gamma = 1$, $\beta L = 100$, the distribution of requesting users is uniform, and the file popularity $\{p_f | \forall f\}$ is known to the BS.

In the above algorithm, the unbiased estimations of p_f, μ_c and $v_{k,f,c}(\forall k, f, c)$ are utilized to update the learning results. Hence, the learning procedure always converges, and the mean squared errors of the estimated p_f^t, μ_c^t and $v_{k,f,c}^t(\forall k, f, c)$ decrease with the order of $\mathcal{O}(1/t)$.

VI. SIMULATION RESULTS

In the simulation, the radius of the cell is 500 meters, 21 cache nodes are deployed in the cell-edge region, each with a service radius of 90 meters. The number of antennas at the BS



Fig. 6. Illustration of a non-uniform geometrical distribution of the requesting users \mathcal{D} , where whole cell area is divided into three regions, the users' distribution in each region is uniform, and the probabilities one requesting user falls into these three regions are 8%, 74%, and 18% respectively.



Fig. 7. Illustration of the converge of approximated value function via reinforcement learning approach (Algorithm 1), when $k = 1, \gamma = 1.3, \beta L = 100$ and $\frac{M_c}{M_F} = 0.6$ ($\forall c$).

is 8. The downlink pathloss exponent is 3.76, and the standard deviation of the shadowing effect is 10dB. Each file consists of 14M information bits, and the transmission bandwidth is 20MHz. The power constraint at the base station is $P_B = 47dBm$. The lifetime and the frame duration are 24 hours and 10 milliseconds respectively, and the number of frames in the lifetime $L = 100 \times 60 \times 60 \times 24 \approx 8.6 \times 10^6$. It is assumed that $\{p_f | \forall f\}$ follows a Zipf distribution with skewness factor γ [32]. The following three baseline schemes are compared with the proposed scheduling scheme.



Fig. 8. Illustration of the performance of approximated value function via reinforcement learning approach (Algorithm 1), when $\gamma = 1.3$ and $\beta L = 100$.

Baseline 1. The BS only ensures the file delivery to the requesting user in each transmission. The cache nodes with better channel conditions to the BS can also decode the file. The decoded file will be stored at the cache node if there is spare memory or files with lower popularity at the cache node.

Baseline 2. The BS ensures that all the cache nodes can decode each file in its first transmission. Files with lowest popularity will be replaced at cache nodes if the caches are fully occupied.

Baseline 3. In the first transmission of the *f*-th file, the BS ensures that the cache nodes, where *f*-th file is of high-popularity, can decode the *f*-th file. After that, the BS only ensures the delivery of the *f*-th file to the requesting user. The stored file with the lowest popularity at a cache node will be replaced if the cache node is fully occupied and a newly received file has higher popularity.

With the knowledge on uniform distribution of the requesting users and the file popularity $\{p_f | \forall f\}$ at the BS, the average transmission cost versus the average number of file requests in the whole lifetime βL is illustrated in Fig. 4 (a), where each cache node can store 60% popular files. It can be observed that the proposed scheme (Lemma 7) consumes less transmission resource than both baselines. The gap between the lower-bound (27) and the performance of the proposed algorithm is small, demonstrating the tightness of the lower-bound and the good performance of the proposed algorithm. Moreover, the performance gain tends to be a constant when βL is

large. This is because all the four schemes (proposed scheme, Baseline 1, 2 and 3) have the same performance if all the high-popularity files have been stored in cache nodes. In other words, the gain of the proposed scheme lies in the phase of cache placement.

In Fig. 4 (b), the performance of the four schemes are compared for different skewness factors of Zipf distribution. The proposed algorithm consumes less transmission resource than both baselines. It can be observed that the gain of the proposed scheme over Baseline 2 is more significant when the popularity of the files is close to uniform. This is because the transmission coupling between the high-popularity and low-popularity files are better exploited in the proposed scheme. Moreover, the gap between the cost lower-bound and the cost of the proposed algorithm is smaller for larger skewness factor. In other words, the proposed scheme is close to the optimal solution when the Zipf distribution is steeper.

In Fig. 5 (a), the effect of the cache size is evaluated for the skewness factor $\gamma = 1$, where the average system cost versus the M_c/M_F (the ratio of high-popularity files) is plotted. It can be observed that when increasing the cache size, more traffic can be offloaded to cache nodes, which leads to less transmission resource consumption at the BS in all three schemes. Moreover, the reduction of average system cost becomes slow in all the schemes and the cost lower-bound when $M_c/M_F > 50\%$. This is because the increased cache space is for the files with small request probability, and the average transmission resource saving is also small. It can also be observed that the gap between the lower-bound and the proposed scheme is small, and the proposed scheme has better utilization of the caches than both baseline schemes especially for large cache size. Finally, the performance of Baseline 3 is close to that of Baseline 1 with scarce cache space, and close to that of Baseline 2 with rich cache space.

In Fig. 5 (b), the hitting rate, defined as

Hitting rate
$$\triangleq \mathbb{E}^{\Omega}_{\{S_{\tau} | \forall \tau\}, N_R} \left[\frac{1}{N_R} \sum_{\tau=1}^{N_R} \mathbf{I}[\mathbf{l}_{\tau} \in \mathcal{C}_{\mathcal{A}_{\tau}, \tau}] \right],$$

are compared among the four schemes. It can be observed that the BS becomes more conservative on exploiting the cache nodes in traffic offloading from Baseline 2, 3, proposed scheme to Baseline 1. Their hitting rates are therefore in descending order. Although Baseline 2 and 3 have higher hitting rates than that of the proposed scheme, their costs are greater than the proposed scheme, as illustrated in Fig. 5 (a). Hence, maximizing the hitting rate is not a good strategy if the concern is average transmission cost of the BS. In the simulation of the previous figures, the distribution of requesting users is uniform, and the file popularity is also known to the BS. When the non-uniform spatial distribution of the requesting users \mathcal{D} and the file popularity $\{p_f | \forall f\}$ are not available at the BS, the reinforcement learning algorithm (Algorithm 1) can be used in the downlink scheduling. In simulation of both Fig. 7 and 8, the actual distribution of requesting users is non-uniform as in Fig. 6. Fig. 7 illustrates the convergence of reinforcement learning algorithm (Algorithm 1) for some CaSIs. In Fig. 8, the performance of three baselines, the proposed scheme in Lemma 7 with inaccurate values of $\{p_f | \forall f\}$ and the wrong assumption of uniform user distribution, and the proposed scheme with reinforcement learning (Algorithm 1) are compared. It can be observed that the proposed learning algorithm has the best performance. The gap between the cost lower-bound and the proposed learning algorithm is small. Thus, the proposed learning algorithm is close to the optimal solution.

Finally, in order to compare the computation times of value function evaluation between the proposed scheme and the optimal solution, we consider the following simplified scenario⁴. There are 2 cache nodes ($N_C = 2$) in the cell, each can store 1 popular file. The library is with $M_F = 2$ popular files. The computation time of the optimal value iteration is around 10.7 times larger than that of the proposed schema. This ratio will be much larger if we increase M_F or N_C .

VII. CONCLUSION

In this paper, we consider the downlink file transmission with the assistance of cache nodes within a finite lifetime. The BS multicasts files to the requesting users and the selected cache nodes reactively, and the cache nodes with decoded files can help offload the traffic via other air interfaces. We formulate the joint optimization of such file placement and delivery as a dynamic programming problem with a random number of stages, and propose an asymptotically optimal way to transform the original problem into a finite-horizon MDP with a fixed number of stages. In order to avoid the curse of dimensionality, we also introduce a low-complexity sub-optimal solution based on linear approximation of the value functions, which can be calculated analytically. The bound of the approximation error is derived. Finally, a reinforcement learning algorithm is proposed to obtain the approximate value functions in the practical scenario, where system statistics is not available.

⁴The computation of the optimal solution is prohibitive with the previous simulation configuration.

APPENDIX A: PROOF OF LEMMA 1

The expression of $W_k(S_k)$ can be rewritten as

$$W_k(S_k) = \min_{\{\Omega_k, \Omega_{k+1}, \dots\}} \mathbb{E}_{\mathcal{A}, \zeta, N_R} \{ \sum_{\tau=k}^{N_R} g_\tau(S_\tau, \Omega_\tau) \mathbf{I}(k \le N_R) | S_k \}$$
$$= \min_{\{\Omega_k, \Omega_{k+1}, \dots\}} \mathbb{E}_{\mathcal{A}, \zeta, N_R} \{ \sum_{\tau=k}^{N_R} g_\tau(S_\tau, \Omega_\tau) | N_R \ge k, S_k \} \Pr(N_R \ge k).$$

Given system state S_k , we have

$$W_{k}(S_{k}) = \min_{\{\Omega_{k},\Omega_{k+1},\dots\}} \mathbb{E}_{\mathcal{A},\zeta,N_{R}} \{g_{k}(S_{k},\Omega_{k}) + \sum_{\tau=k+1}^{N_{R}} g_{\tau}(S_{\tau},\Omega_{\tau})\mathbf{I}(k+1 \leq N_{R}) | N_{R} \geq k\} \Pr(N_{R} \geq k)$$

$$= \min_{\{\Omega_{k},\Omega_{k+1},\dots\}} \left\{ g_{k}(S_{k},\Omega_{k}) \Pr(N_{R} \geq k) + \sum_{S_{k+1}} \Pr[S_{k+1}|S_{k},\Omega_{k}] \mathbb{E}_{\mathcal{A},\zeta,N_{R}} \{ \sum_{\tau=k+1}^{N_{R}} g_{\tau}(S_{\tau},\Omega_{\tau})\mathbf{I}(k+1 \leq N_{R}) | N_{R} \geq k\} \Pr(N_{R} \geq k) \right\}$$

$$= \min_{\{\Omega_{k},\Omega_{k+1},\dots\}} \left\{ g_{k}(S_{k},\Omega_{k}) \Pr(N_{R} \geq k) + \sum_{S_{k+1}} \Pr[S_{k+1}|S_{k},\Omega_{k}] W_{k+1}(S_{k+1}) \right\}, \quad (30)$$

where the last step is because

$$W_{k+1}(S_{k+1}) = \min_{\{\Omega_{k+1},\dots\}} \left\{ \mathbb{E}_{\mathcal{A},\zeta,N_R} \{ \sum_{\tau=k+1}^{N_R} g_{\tau}(S_{\tau},\Omega_{\tau}) \mathbf{I}(k+1 \le N_R) | N_R \ge k, S_{k+1} \} \Pr(N_R \ge k) + \mathbb{E}_{\mathcal{A},\zeta,N_R,S_{k+1}} \{ \sum_{\tau=k+1}^{N_R} g_{\tau}(S_{\tau},\Omega_{\tau}) \mathbf{I}(k+1 \le N_R) | N_R < k, S_{k+1} \} \Pr(N_R < k) \right\}_{=0}^{N_R}$$

APPENDIX B: PROOF OF LEMMA 3

1) Proof of upper-bound: Because $\overline{g}_{max} = \max_{\rho_k, B_k} \min_{\Omega_k} \mathbb{E}_{\mathcal{A}, \eta_k} \Big\{ g_k(S_k, \Omega_k) \Big\} \ge \mathbb{E}_{\mathcal{A}, \zeta_k} \Big\{ g_k(S_k, \Omega_k^*) \Big\},$ $\forall S_k, k$, we have

$$\widetilde{W}_{k}(B_{k}) = \mathbb{E}_{\mathcal{A},\zeta} \left\{ \sum_{\tau=k}^{L} g_{\tau}(S_{\tau}, \Omega_{\tau}^{*}) \operatorname{Pr}(\tau \leq N_{R}) \middle| B_{k} \right\}$$
$$\leq \min_{\{\Omega_{\tau} \mid \forall \tau\}} \mathbb{E}_{\mathcal{A},\zeta} \left\{ \sum_{\tau=k}^{M_{R}^{\epsilon}} g_{\tau}(S_{\tau}, \Omega_{\tau}) \operatorname{Pr}(\tau \leq N_{R}) \middle| B_{k} \right\} + \sum_{\tau=M_{R}^{\epsilon}+1}^{L} \overline{g}_{max} \operatorname{Pr}(\tau \leq N_{R}),$$

where the upper-bound is due to non-optimal policy. Similar to [16], [37], the analytical expression of \overline{g}_{max} is straightforward.

2) Proof of lower-bound:

$$\widetilde{W}_{k}(B_{k}) \geq \mathbb{E}_{\mathcal{A},\zeta} \left\{ \sum_{\tau=k}^{M_{R}^{\epsilon}} g_{\tau}(S_{\tau}, \Omega_{\tau}^{*}) \operatorname{Pr}(\tau \leq N_{R}) \middle| B_{k} \right\} \geq \min_{\{\Omega_{\tau} \mid \forall \tau\}} \mathbb{E}_{\mathcal{A},\zeta} \left\{ \sum_{\tau=k}^{M_{R}^{\epsilon}} g_{\tau}(S_{\tau}, \Omega_{\tau}) \operatorname{Pr}(\tau \leq N_{R}) \middle| B_{k} \right\}.$$

APPENDIX C: PROOF OF LEMMA 4

Due to page limitation, we only provide the sketch of the proof. In order to prove Lemma 4, we only need to prove $\mathcal{L}_k(B_k) \leq \sum_{f,c} J_{k,f,c}(B_k)$ for all k and B_k . Hence, we first introduce the following heuristic scheduling policy $\{\Omega_{\tau}^* | \tau = 1, 2, ..., M_R^{\epsilon}\}$.

Policy 1 (Heuristic Scheduling Policy). The scheduling actions from the first request to the M_R^{ϵ} -th request are provided below.

- When (a) the requesting user is in the coverage area of one cache node and (b) the requested file is low-popular for that cache node and it has not been cached, the BS only deliver the file to the requesting user. Thus in this case, ΔB^c_{f,k} = 0 and {P^{*}_k, N^{*}_k} = arg min(P_kN_k + wN_k)I[l_k ∉ C<sub>A_k,k], where the constraints in (1) and (3) should be satisfied.
 </sub>
- Otherwise, the transmission is optimized such that the average total cost on remaining transmissions is minimized. Thus, let $\Omega_{\tau}^{\star,H}$ be the scheduling policy on remaining transmissions, we have

$$\{\Omega_{\tau}^{\star,H} | \forall \tau = 1, ..., M_R^{\epsilon}\} = \arg\min \mathbb{E}_{\mathcal{A},\zeta} \sum_{\tau=1}^{M_R^{\epsilon}} \sum_{c=0}^{N_C} g_{\tau}(S_{\tau}, \Omega_{\tau}) \operatorname{Pr}(\tau \le N_R) \mathbf{I}(\mathcal{A}_{\tau} \in \mathcal{F}_c^H \cap \mathbf{l}_{\tau} \in \mathcal{C}_c)$$

where the constraints in (1,3,4,5,6) should be satisfied, and $\mathcal{F}_0^H \triangleq \{1, ..., M_F\}$ for notation convenience.

With the Policy 1, let $\mathcal{U}_k^*(B_k)$ be the average cost from the k-th stage to the M_R^{ϵ} -th stage. Note that the optimal policy is used in \mathcal{L}_k , $\mathcal{L}_k(B_k) \leq \mathcal{U}_k^*(B_k)$. Moreover, it is clear that $\mathcal{U}_k^*(B_k) \leq \sum_{f,c} J_{k,f,c}(B_k)$, the conclusion of Lemma 4 can be obtained.

APPENDIX D: PROOF OF LEMMA 5

Due to page limitation, we only provide the sketch of the proof. First, $\widetilde{W}_k(B_k) \geq \mathcal{L}_k(B_k)$ is given by (10). In order to prove $\widetilde{\mathcal{L}}_k^1(B_k) \leq \mathcal{L}_k(B_k)$, the approach of mathematical induction shall be used.

• Step 1: When $\tau = M_R^{\epsilon}$,

$$\begin{aligned} \widetilde{\mathcal{L}}_{M_{R}^{\epsilon}}^{1}(B_{M_{R}^{\epsilon}}) &= \bar{g}_{min}(B_{M_{R}^{\epsilon}}) \operatorname{Pr}(M_{R}^{\epsilon} \leq N_{R}) = \min_{\Omega_{M_{R}^{\epsilon}}} \mathbb{E}_{\mathcal{A},\zeta} \{ g_{M_{R}^{\epsilon}}(S_{M_{R}^{\epsilon}}, \Omega_{M_{R}^{\epsilon}}) | B_{M_{R}^{\epsilon}} \} \operatorname{Pr}(M_{R}^{\epsilon} \leq N_{R}) \\ &= \mathbb{E}_{\mathcal{A},\zeta,N_{R}} \left\{ g_{M_{R}^{\epsilon}}(S_{M_{R}^{\epsilon}}, \Omega_{M_{R}^{\epsilon}}^{\dagger}) \mathbf{I}(M_{R}^{\epsilon} \leq N_{R}) | B_{M_{R}^{\epsilon}} \right\} = \mathcal{L}_{M_{R}^{\epsilon}}(B_{M_{R}^{\epsilon}}). \end{aligned}$$

• Step 2: Suppose the lower-bound holds for $\tau = k$. When $\tau = k - 1$,

$$\widetilde{\mathcal{L}}_{k-1}^{1}(B_{k-1}) = \bar{g}_{min}(B_{k-1}) \operatorname{Pr}(k-1 \le N_{R}) + \widetilde{\mathcal{L}}_{k}^{1}(\pi(B_{k-1}))$$
$$\mathcal{L}_{k-1}(B_{k-1}) = \mathbb{E}_{\mathcal{A},\zeta,N_{R}} \left\{ g_{k-1}(S_{k-1},\Omega_{k-1}^{\dagger})\mathbf{I}(k-1 \le N_{R}) | B_{k-1} \right\} + \sum_{S_{k}} \mathcal{L}_{k}(B_{k}) \operatorname{Pr}\left(B_{k} | B_{k-1},\Omega_{k-1}^{\dagger}\right)$$
$$\bar{g}_{min}(B_{k-1}) \operatorname{Pr}(k-1 \le N_{R}) \le \mathbb{E}_{\mathcal{A},\zeta,N_{R}} \left\{ g_{k-1}(S_{k-1},\Omega_{k-1}^{\dagger})\mathbf{I}(k-1 \le N_{R}) | B_{k-1} \right\}.$$

Given B_{k-1} , we have $\widetilde{\mathcal{L}}_k^1(\pi(B_{k-1})) \leq \mathcal{L}_k(\pi(B_{k-1})) \leq \mathcal{L}_k(B_k), \forall B_k$. Hence, $\widetilde{\mathcal{L}}_k^1(B_k^\pi) \leq \sum_{B_k} \mathcal{L}_k(B_k) \Pr\left(B_k | B_{k-1}, \Omega_{k-1}^\dagger\right)$. As a result, $\widetilde{\mathcal{L}}_{k-1}^1(B_{k-1}) \leq \mathcal{L}_{k-1}(B_{k-1}), \forall B_{k-1}$.

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