

# On the Optimal Memory-Load Tradeoff of Coded Caching for Location-Based Content

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**Abstract**—Caching at the wireless edge nodes is a promising way to boost the spatial and spectral efficiency, for the sake of alleviating networks from content-related traffic. Coded caching originally introduced by Maddah-Ali and Niesen significantly speeds up communication efficiency by transmitting multicast messages simultaneously useful to multiple users. Most prior works on coded caching are based on the assumption that each user may request all content in the library. However, in many applications the users are interested only in a limited set of content that depends on their location. For example, assisted self-driving vehicles may access super High-Definition maps of the area through which they are travelling. Motivated by these considerations, this paper formulates the coded caching problem for location-based content with edge cache nodes. The considered problem includes a content server with access to  $N$  location-based files (e.g., High-Definition maps),  $K$  edge cache nodes located at different regions, and  $K$  users (i.e., vehicles) each of which is in the serving region of one cache node and can retrieve the cached content of this cache node with negligible cost. Depending on the location, each user only requests a file from a location-dependent subset of the library. The objective is to minimize the worst-case load (i.e., the worst-case number of broadcasted bits from the content server among all possible demands). For this novel coded caching problem, we propose a highly non-trivial converse bound under uncoded cache placement (i.e., each cache node directly copies some library bits in its cache), which shows that a simple achievable scheme is optimal under uncoded cache placement. In addition, this achievable scheme is also proved to be generally order optimal within a factor of 3. Finally, we extend the coded caching problem for location-based content to the multiaccess coded caching topology originally proposed by Hachem et al., where each user is connected to  $L$  nearest cache nodes. When  $L \geq 2$ , we characterize the exact optimality on the worst-case load.

**Index Terms**—Coded caching, location-based content, edge cache nodes, uncoded cache placement.

## I. INTRODUCTION

Caching reduces peak traffic by taking advantage of devices' memories distributed across the network to duplicate content

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during off-peak hours, such that the network traffic is shifted from peak to off-peak hours. A caching system is operated in two phases: i) *placement phase*: each user stores some bits in its cache without knowledge of later demands; ii) *delivery phase*: after each user has made its request and according to cached content, the server transmits packets in order to satisfy the user demands. The goal is to minimize the transmission load such that user demands can be satisfied.

Information theoretic coded caching was originally proposed by Maddah-Ali and Niesen (MAN) in [1] for a shared-link caching system where a server with a library of  $N$  equal-length files is connected to  $K$  users through a noiseless shared link and each user can store  $M$  files in its local cache. Each user demands an arbitrary file in the library during the delivery phase. The MAN scheme uses a combinatorial design in the placement phase such that each multicast message transmitted during the delivery phase simultaneously satisfies the demands of multiple users. Under the constraint of uncoded cache placement (i.e., each user directly caches a subset of the library bits) and for the worst-case load among all possible demands, the MAN scheme was proved to be optimal when  $N \geq K$  [2], [3]. Provided the observation that some MAN linear combinations are redundant if there exist files demanded by several users, the authors in [3] improved the MAN delivery scheme and achieved the optimal worst-case load under the constraint of uncoded cache placement for any  $K$ . It was also proved in [4] that the multiplicative gap between the optimal caching scheme with uncoded cache placement and any caching scheme with coded cache placement is at most 2.

Caching at the wireless edge nodes reduces both the backhaul traffic and the transmission time for high-volume data delivery [5], [6]. Although extensively considered in the literature, the end-user-caches, e.g. mobile devices, have some limitations as they have typically small storage size (compared to the library size) and are useful only for one user device. By contrast, the caches at edge devices, e.g. small base stations, have larger storage capability and can be accessed by multiple users. In addition, location-based content could be placed into the edge caching devices at different locations, such that when the mobile clients enter one area, they can retrieve the location-based content for this area from the corresponding edge caching devices.

*Coded caching for location-based content*: Recently the emerging vehicular applications (such as autonomous vehicles, intelligent transportation, high-quality Internet navigation and entertainment, etc.) bring a revolutionary change to the traditional vehicular transportation, and meanwhile lead to a

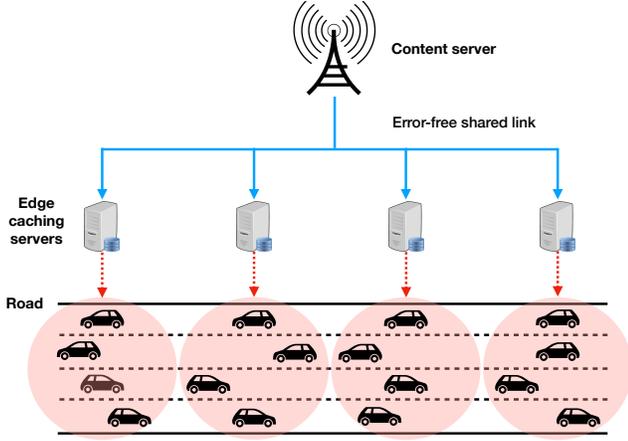


Fig. 1: Vehicular network coded caching problem for location-based content.

dramatically increasing number of demands on data services. To scope with the large volumes of data, edge caching was widely used in the vehicular networks, to list a subset of literature [7]–[14]. Motivated by this type of applications, in this paper we focus on the coded caching problem for location-based content. In order to keep the problem tractable and nevertheless provide some fundamental insight, we consider a very simplistic model of vehicular network with edge cache nodes as illustrated in Fig. 1, including one content server with access to  $N$  location-based files such as High-Definition (HD) maps and/or location-based advertising, entertainment, and services. Following the original MAN coded caching model, we assume that each content file has equal size of  $B$  bits. The content server is connected to  $K$  edge cache nodes through an error-free shared link. We assume that each cache node is in a fixed assigned location and has a local cache with size  $MB$  bits; for example, the cache nodes could be roadside units (RSUs), mobile edge caching (MEC) servers, unmanned aerial vehicles (UAVs) hovering on assigned geographic areas. Each cache node is accessible to the vehicles in one area without load cost;<sup>1</sup> that is, we count only the transmission load from the content server. The whole road modelled as a ring is divided into  $K$  non-overlapping regions of equal size, each of which is connected to one cache node.<sup>2</sup> Each vehicle in one region demands one location-based file corresponding to its region. The set of possible demanded files in the  $k^{\text{th}}$  region is denoted by  $\mathcal{D}_k$ , for each  $k \in [K]$ , where the set  $\mathcal{D}_k \subseteq [N]$  is formed by three subsets: two subsets of equal size  $a$  that represent files also present in the neighbouring regions to the left  $\mathcal{D}_{k-1}$  and to the right  $\mathcal{D}_{k+1}$ , respectively; a subset of size

<sup>1</sup> The cost-free access between edge caches and users can be justified by dedicated local links of very high capacity (e.g. wideband mmWave proximity links). Models of this kind are widely assumed in the literature of coded edge caching systems such as FemtoCaching [15], multiaccess coded caching [16], and coded caching with shared-caches [17].

<sup>2</sup> The ring networks are very popular and widely studied model in the literature, such as circular Wyner model for interference networks with limited interference from neighbours [18], [19], multiaccess coded caching [16], etc. A ring network is interesting in the theoretic sense, because the boundary effect at both ends is ignored.

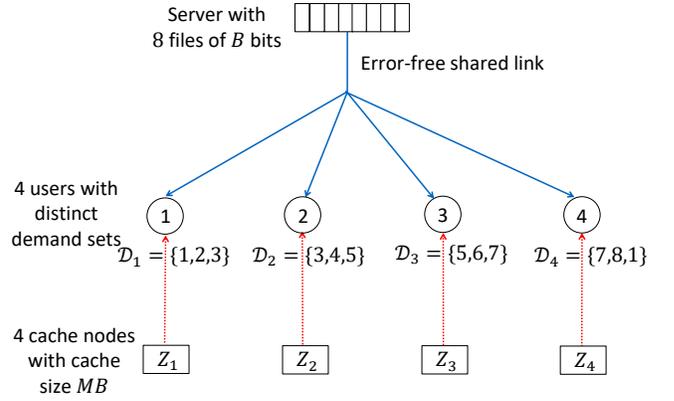


Fig. 2: The information theoretic model of the considered coded caching problem for location-based content with  $K = 4$ ,  $N = 8$ , and  $a = b = 1$ .

$b$  that represent files uniquely present in  $\mathcal{D}_k$ .

We treat the considered problem as an information theoretic coded caching problem with  $K$  users (i.e. vehicles) such that each user is located at one distinct region,<sup>3</sup> as illustrated in Fig. 2. In the cache placement phase, each cache node stores some content of the  $N$  files without knowledge of the users' later demands. In the delivery phase, the user at the  $k^{\text{th}}$  region requests a file in  $\mathcal{D}_k$ . According to the users' demands, the central server broadcasts  $RB$  bits to all users, such that each user can recover its demanded file from the broadcasted packets and the stored content of its connected cache node. The objective is to minimize  $R$  for the worst-case demand(s) over the restricted set of possible (location-based) demands.

*Relation to the existing works:* Our considered problem differs from the existing works [7]–[14] for the vehicular networks with edge cache nodes on two aspects: (i) we consider a coded caching problem for which we need to design the cache placement of the cache nodes and the multicast messages transmitted by the content server to minimize the broadcasted load, while the techniques in [7]–[13] are based on uncoded caching; (ii) there exists some overlap on the demand sets of each two neighbouring regions in our problem, which is not considered in [7]–[14]. Coded caching with location-based content delivery was also considered in [20]. Different from our edge caching model, the users in the setting considered in [20] have their own cache memories and are mobile with uniform access probability to each location, where each location corresponds to one specific file and the placement phase is done without knowing the locations of the users.

Our considered problem can be seen as a special case of the coded caching problem with different demand sets, where the set of possible demanded files by each cache-aided user is different. The exact optimality results on the memory-load tradeoff were fully characterized in [21] for the two-user two-

<sup>3</sup> In a more practical scenario with multiple vehicles per region, one direct solution is to divide the whole transmission into multiple rounds, where in each round we serve one user in each region. However, the converse bounds derived in this paper would not directly apply. It is one of our on-going works to characterize the optimal memory-load tradeoff for this case.

file case, and were partially characterized in [22], [23] for the two-user N-file case. For the general case where the users are divided into classes and the users in the same class have the same demand set, some achievable schemes were proposed in [24], [25]. Namely, these schemes let each user use some fraction of its cache to store the common files among different groups and remaining fraction to store the unique files in its group, such that the MAN caching scheme could be used to transmit the common files among different groups and the unique files in each group, respectively. Very recently, by focusing on a special case where there exist the same number of common files in the demand sets of each  $\alpha$ -subset of users (e.g.  $\alpha = K$  reduces to the setup of [1]), the authors in [26] showed that the direct use of the MAN caching scheme regardless the users' demand sets can yield unbounded gains over the best selfish caching scheme which lets each user only cache some bits from the files in its demanded files.

However, except some asymptotic optimality guarantees for some specific cases, the optimality for the coded caching problem with different demand sets is generally open. In our paper, we focus on a special structure of different demand sets, where the number of common demanded files by each two neighbouring users is denoted by  $a$  and the number of unique demanded files by each user is denoted by  $b$ .

*Our Contributions:* In addition to the formulation of on the novel coded caching problem for location-based content, our main contributions are as follows:

- By proposing a highly non-trivial converse bound under uncoded cache placement, we prove that the memory sharing among two simple memory-load tradeoff points  $(0, K)$  and  $(2a + b, 0)$ , and one achieved tradeoff point by the MAN scheme  $(\frac{N}{K}, \frac{K-1}{2})$ , is optimal under uncoded cache placement. The converse strategy in [2], [3] for the MAN caching problem under uncoded cache placement leads to a loose converse in our problem, because it sums many redundant inequalities in order to derive the final lower bound.
- Compared to a novel cut-set converse bound, this achievable scheme is proved to be order optimal within a factor of 3.
- We also formulate our location-based content problem with the multiaccess coded caching topology, as illustrated in Fig. 4. In this multiaccess coded caching topology originally considered in [16], each user is connected to  $L \geq 2$  neighbouring cache nodes. By extending the proposed achievable scheme and the cut-set converse bound to this novel model, we characterize the exact optimality. It is interesting to see that when  $L \geq 2$ , it does not reduce the load if each user is allowed to access more than 2 caches. This result provides a very important insight on the design of edge caching schemes for location-based content, showing that essentially localized access through high-capacity proximity links to the neighbouring caches is indeed sufficient to achieve the optimal load of the (costly) cellular broadcast channel.

*Paper Organization:* The rest of this paper is organized as follows. Section II formulates the coded caching problem for location-based content and reviews some related results.

Section III introduces the main results in this paper. Section IV extends the proposed bounds to the multiaccess coded caching problem for location-based content. Section V concludes the paper, while some proofs can be found in the Appendix.

*Notation Convention:* Calligraphic symbols denote sets, bold symbols denote vectors, and sans-serif symbols denote system parameters. We use  $|\cdot|$  to represent the cardinality of a set or the length of a vector. Sets of consecutive integers are denoted as  $[a : b] := \{a, a + 1, \dots, b\}$  and  $[n] := [1 : n]$ . The symbol  $\oplus$  represents bit-wise XOR.  $a! = a \times (a - 1) \times \dots \times 1$  represents the factorial of  $a$ .  $\langle b \rangle_a$  represents the modulo operation on  $b$  with integer divisor  $a$  and in this paper we let  $\langle b \rangle_a \in \{1, \dots, a\}$  (i.e., we let  $\langle b \rangle_a = a$  if  $a$  divides  $b$ ). We use the convention that  $\binom{x}{y} = 0$  if  $x < 0$  or  $y < 0$  or  $x < y$ .

## II. SYSTEM MODEL AND RELATED RESULTS

### A. System Model

The information theoretic formulation of the  $(K, a, b)$  coded caching problem for location-based content is given as follows, illustrated in Fig. 2. A central server has access to a library of  $N$  location-based files, denoted by  $W_1, \dots, W_N$ , each of which contains  $B$  i.i.d. bits.  $B$  is assumed to be large enough such that any subpacketization on the files is possible. We consider a one-dimensional cyclic route.  $K$  cache nodes are distributed on the route where the distance between two neighbouring cache nodes is identical. Each cache node can cache up to  $MB$  bits. Each user on the route can retrieve the cached content from its nearest cache node. Thus the whole route is divided into  $K$  regions, each of which corresponds to one cache node. Based on the location of the  $k^{\text{th}}$  region, each user connected to the cache node  $k$  is only interested in the files whose indices are in the set  $\mathcal{D}_k$ . Intuitively,  $\mathcal{D}_k$  is the union of three disjoint parts:

- $\mathcal{D}_{k,1} := \mathcal{D}_k \cap \mathcal{D}_{\langle k-1 \rangle_K}$ , representing the  $a$  common files which can also be demanded by the users in the  $k^{\text{th}}$  region and in the left-hand side neighbouring region, i.e., the  $\langle k-1 \rangle_K$  region.
- $\mathcal{D}_{k,2} := \mathcal{D}_k \setminus (\cup_{j \in [K] \setminus \{k\}} \mathcal{D}_j)$ , representing the  $b$  files which can only be demanded by the users in the  $k^{\text{th}}$  region.
- $\mathcal{D}_{k,3} := \mathcal{D}_k \cap \mathcal{D}_{\langle k+1 \rangle_K}$ , representing the  $a$  common files which can also be demanded by the users in the  $k^{\text{th}}$  region and in the right-hand side neighbouring region, i.e., the  $\langle k+1 \rangle_K$  region.

Due to the topology of the one-dimensional cyclic route, there does not exist any file which can be demanded by two users in two non-neighbouring regions, i.e.,  $\mathcal{D}_{k_1} \cap \mathcal{D}_{k_2} = \emptyset$  where  $\langle k_1 - k_2 \rangle_K \in [2 : K - 2]$ . Hence, we have  $N := K(a + b)$  and

$$\begin{aligned} \mathcal{D}_k &:= \underbrace{[(k-1)(a+b) + 1 : ka + (k-1)b]}_{:= \mathcal{D}_{k,1}} \\ &\cup \underbrace{[ka + (k-1)b + 1 : k(a+b)]}_{:= \mathcal{D}_{k,2}} \end{aligned}$$

$$\cup \underbrace{[\langle k(a+b) + 1 \rangle_{\mathcal{K}(a+b)} : \langle (k+1)a + kb \rangle_{\mathcal{K}(a+b)}]}_{:= \mathcal{D}_{k,3}}. \quad (1)$$

The server communicates with  $K$  users through an error-free shared link. Each user connected to cache node  $k$  can retrieve the content stored in cache node  $k$ . In this paper, we focus on the communication bottleneck on the shared link from the server to the users; thus we assume that each user can retrieve the cached content from its connected cache node without any cost.

The system operates in two phases.

*Cache Placement Phase:* During the cache placement phase, each cache node stores information about the  $N$  files in its local cache without knowledge of the users' demands. We denote the cached content of cache node  $k \in [K]$  by  $Z_k = \phi_k(W_1, \dots, W_N)$ , where

$$\phi_k : [0 : 1]^{NB} \rightarrow [0 : 1]^{MB}, \quad k \in [K]. \quad (2)$$

Let  $\mathbf{Z} = (Z_1, \dots, Z_K)$  be the cached content of all cache nodes.

*Delivery Phase:* As explained in Footnote 3, we assume that there is exactly one user in each region who makes the request in the delivery phase, where the user in the  $k^{\text{th}}$  region is called user  $k$ , for each  $k \in [K]$ . The demand vector is defined as  $\mathbf{d} := (d_1, \dots, d_K)$ , where  $d_k \in \mathcal{D}_k$  represents to the index of the file demanded by user  $k \in [K]$ . The demand vector  $\mathbf{d}$  is known to the server and all users. Given  $(\mathbf{Z}, \mathbf{d})$ , the server broadcasts the message  $X = \psi(\mathbf{d}, W_1, \dots, W_K)$ , where

$$\psi : \mathcal{D}_1 \times \dots \times \mathcal{D}_K \times [0 : 1]^{NB} \rightarrow [0 : 1]^{RB}, \quad (3)$$

for some non-negative number  $R$  referred to as load.

*Decoding:* Each user  $k \in [K]$  decodes its desired file  $F_{d_k} = \xi_k(\mathbf{d}, Z_k, X)$ , where

$$\xi_k : \mathcal{D}_1 \times \dots \times \mathcal{D}_K \times [0 : 1]^{MB} \times [0 : 1]^{RB} \rightarrow [0 : 1]^B, \quad k \in [K]. \quad (4)$$

*Objective:* For any cache size  $M \in [0, N]$ , we aim to determine the *minimum worst-case load* among all possible demands, defined as the smallest  $R$  such that there exists an ensemble of placement functions  $\phi_k, k \in [K]$ , encoding function  $\psi$ , and decoding functions  $\xi_k, k \in [K]$ , satisfying all the above constraints. The optimal load is denoted by  $R^*$ .

Note that if  $K = 1$ , we have  $a = 0$  and  $N = b$ . The considered problem becomes the 1-user MAN coded caching problem, where the uncoded caching scheme is optimal. In the rest of this paper, we consider  $K \geq 2$ .

*Uncoded Cache Placement:* The cache placement policy is *uncoded* if the bits of the files are directly copied into the cache nodes. Under the constraint of uncoded cache placement, we can partition each file  $W_i$  where  $i \in [N]$  into subfiles as

$$W_i = \{W_{i,\mathcal{T}} : \mathcal{T} \subseteq [K]\}, \quad (5)$$

where  $W_{i,\mathcal{T}}$  represents the bits of  $W_i$  exclusively cached by the cache nodes in  $\mathcal{T}$ . The optimal load under the constraint of uncoded cache placement is denoted by  $R_u^*$ .

## B. Optimality of the MAN Scheme under Uncoded Cache Placement for the Shared-link Model

In the following, we briefly introduce the MAN coded caching scheme [1] for the shared-link MAN coded caching model, including a server with  $N$  files and  $K$  cache-aided users with cache size  $M$ .

We focus on the memory size  $M = \frac{Nt}{K}$ , where  $t \in [0 : K]$ . By dividing each file  $W_i$  where  $i \in [N]$  into  $\binom{K}{t}$  non-overlapping and equal-length subfiles,  $W_i = \{W_{i,\mathcal{T}} : \mathcal{T} \subseteq [K], |\mathcal{T}| = t\}$ , we let each user  $k \in [K]$  cache  $W_{i,\mathcal{T}}$  where  $k \in \mathcal{T}$ . Hence, each user totally caches  $N \frac{\binom{K-1}{t-1}}{\binom{K}{t}} B = \frac{Nt}{K} B = MB$  bits, satisfying the memory size constraint.

In the delivery phase, we assume that the demand vector is  $\mathbf{d} = (d_1, \dots, d_K) \in [N]^K$ . For each set  $\mathcal{S} \subseteq [K]$  where  $|\mathcal{S}| = t + 1$ , the server broadcasts a multicast message  $X_{\mathcal{S}} = \bigoplus_{k \in \mathcal{S}} W_{d_k, \mathcal{S} \setminus \{k\}}$ . Each user  $k \in \mathcal{S}$  caches all subfiles but  $W_{d_k, \mathcal{S} \setminus \{k\}}$  in  $X_{\mathcal{S}}$ . Since the server transmits  $\binom{K}{t+1}$  multicast messages, each of which contains  $B/\binom{K}{t}$  bits, the achieved memory-load tradeoff is

$$(M, R_{\text{MAN}}) = \left( \frac{Nt}{K}, \frac{K-t}{t+1} \right), \quad \forall t \in [0 : K]. \quad (6)$$

The lower convex envelope of the memory-load tradeoff points in (6) was proved to be optimal under the constraint of uncoded cache placement and  $N \geq K$  [2], [3]. More precisely, for any coded caching scheme with uncoded cache placement  $\mathbf{Z}$ , we can divide each file into  $2^K$  subfiles as in (5). We consider one demand vector  $\mathbf{d} = (d_1, \dots, d_K)$  where  $d_i \neq d_j$  if  $i \neq j$ , and one permutation of  $[K]$  denoted by  $\mathbf{u} = (u_1, \dots, u_K)$ . We then construct a genie-aided super-user with cached content

$$Z' = (Z_{u_1}, Z_{u_2} \setminus (W_{d_{u_1}} \cup Z_{u_1}), \dots, Z_{u_K} \setminus (W_{d_{u_1}} \cup Z_{u_1} \cup W_{d_{u_2}} \cup Z_{u_2} \cup \dots \cup W_{d_{u_{K-1}}} \cup Z_{u_{K-1}})),$$

who is able to recover  $(W_{d_1}, \dots, W_{d_K})$  from  $(X, Z')$ . This is because, this super-user first decodes  $W_{d_{u_1}}$  from  $(X, Z_{u_1})$ , then decodes  $W_{d_{u_2}}$  from  $(X, Z_{u_1}, Z_{u_2} \setminus (W_{d_{u_1}} \cup Z_{u_1}))$ , and does the similar procedure iteratively until decoding  $W_{d_{u_K}}$ . Hence, we have

$$H(W_{d_1}, \dots, W_{d_K} | Z') = H(W_{d_1}, \dots, W_{d_K} | Z', X) + I(W_{d_1}, \dots, W_{d_K}; X | Z') \quad (7a)$$

$$= I(W_{d_1}, \dots, W_{d_K}; X | Z') \leq H(X | Z') \leq H(X), \quad (7b)$$

$$\implies R \geq \sum_{i \in [K]} \sum_{\mathcal{T} \subseteq [K] \setminus \{u_1, \dots, u_i\}} \frac{|W_{d_{u_i}, \mathcal{T}}|}{B}. \quad (7c)$$

By considering all demand vectors in which users have distinct demands and all permutations of users, we sum all the inequalities in the form of (7c). Because of symmetry, for each  $t \in [0 : K]$ , on the right side of the sum of all these inequalities, the coefficients of the term  $|W_{i,\mathcal{T}}|$  are the same, where  $i \in [N]$  and  $|\mathcal{T}| = t$ . Note that in (7c), there are  $\binom{K}{t+1}$  terms with  $|\mathcal{T}| = t$  whose coefficient is 1. Hence, the sum of all inequalities in the form of (7c) is

$$R \geq \sum_{t \in [0:K]} \frac{\binom{K}{t+1}}{N \binom{K}{t}} x_t = \sum_{t \in [0:K]} \frac{K-t}{N(t+1)} x_t, \quad (8a)$$

$$\text{where } x_t := \sum_{i \in [N]} \sum_{\mathcal{T} \subseteq [K]: |\mathcal{T}|=t} \frac{|W_{i,\mathcal{T}}|}{B}. \quad (8b)$$

From the memory size constraint, it should satisfy

$$\sum_{t \in [0:K]} tx_t \leq KM. \quad (9)$$

From the file size constraint, it should satisfy

$$\sum_{t \in [0:K]} x_t = N. \quad (10)$$

Finally, by the Fourier-Motzkin elimination on  $x_q$  where  $q \in [0 : K]$ , we obtain  $R_u^*$  is lower bounded by the lower convex envelope of the memory-load points  $\left(\frac{Nt}{K}, \frac{K-t}{t+1}\right)$ , where  $t \in [0 : K]$ , coinciding with that of the MAN scheme.

### III. MAIN RESULTS

For the considered coded caching problem for location-based content, the K-user MAN scheme could be directly used, which achieves the memory-load tradeoff points in (6). However, in the following theorem, we show that this topology-agnostic scheme is strictly sub-optimal.

**Theorem 1.** *For the  $(K, a, b)$  coded caching problem for location-based content, when  $b(K-1) < 2a$ , the optimal load under the constraint of uncoded cache placement is*

$$R_u^* = \begin{cases} K - \frac{K+1}{2(a+b)}M, & \text{if } 0 \leq M \leq a+b; \\ \frac{(K-1)(2a+b)}{2a} - \frac{K-1}{2a}M, & \text{if } a+b < M \leq 2a+b. \end{cases} \quad (11)$$

When  $b(K-1) \geq 2a$ , the optimal load under the constraint of uncoded cache placement is

$$R_u^* = K - \frac{K}{2a+b}M. \quad (12)$$

□

*Proof: Achievability.* When  $b(K-1) < 2a$ , the optimal load under uncoded cache placement in (11) is achieved by the memory sharing among the memory-load points  $(0, K)$ ,  $(a+b, \frac{K-1}{2})$ , and  $(2a+b, 0)$ . For  $(0, K)$ , we let the server directly transmit  $W_{d_k}$  for each  $k \in [K]$ . For  $(a+b, \frac{K-1}{2}) = (\frac{N}{K}, \frac{K-1}{2})$ , we directly use the MAN scheme with  $t=1$  in (6). For  $(2a+b, 0)$ , we let each cache node  $k$  cache all the  $2a+b$  files in  $\mathcal{D}_k$ . Since the demand of user  $k$  is in  $\mathcal{D}_k$  and user  $k$  can retrieve the cached content of cache node  $k$ , the load in the delivery phase is 0.

When  $b(K-1) \geq 2a$ , the optimal load under uncoded cache placement in (12) is achieved by the memory sharing between the memory-load points  $(0, K)$  and  $(2a+b, 0)$ , which can be achieved as described above. Note that when  $b(K-1) > 2a$ , the MAN scheme is strictly sub-optimal for any  $0 < M \leq 2a+b$ .

*Converse.* The main technical challenge for Theorem 1 is the proof of the converse under uncoded cache placement. Since the set of possible demanded files by each user is a proper subset of  $[N]$ , the converse for the original MAN coded caching problem is not a converse for our considered problem.

Similar to the converse bound for (6), we can consider all possible demand vectors where users have distinct demands and all permutations of the K users, and obtain a lower bound on the load in the form of (7c) for each combination of the aforementioned demand vector and permutation. Together with the memory size and file size constraints, we can obtain a converse bound on  $R_u^*$ , which is a Linear Programming (LP) with the numbers of constraints and of variables exponential to K. To compute the closed-form of the optimal solution for the LP, one idea is to sum all the inequalities in the form of (7c) as we did for the original MAN coded caching problem. However, summing all the inequalities loosens the converse bound in our problem, because some inequalities are redundant.<sup>4</sup> Intuitively, this redundancy is because in this network topology the demand vectors are not symmetric, neither the permutations of users; thus the resulting inequalities are not symmetric. **Instead, our main contribution is to smartly select the non-redundant inequalities. This is done by carefully selecting the demand vectors and specific permutation(s) of users for each selected demand vector.** Then we sum these non-redundant inequalities all together, such that we can obtain a closed-form of the solution for the LP, which is exactly identical to the optimal load in Theorem 1. The detailed proof on the converse bound for Theorem 1 could be found in Sections III-A to III-C. ■

**Remark 1** (Effect of  $a$  and  $b$ ). *It is interesting to see from Theorem 1 that, when  $b \geq \frac{2a}{K-1}$ , under the constraint of uncoded cache placement, coded caching does not have any advantage compared to the uncoded caching scheme (i.e., the memory sharing between  $(0, K)$  and  $(2a+b, 0)$ ). By contrast, when  $a$  increases, coded caching becomes more significant compared to uncoded caching, as illustrated in Fig. 3. The coded caching gain of the proposed scheme for Theorem 1 is no more than 2 compared to the uncoded caching scheme, since we only use the MAN coded caching scheme in (6) with  $t=1$ .* □

By comparing the achieved load in Theorem 1 with a cut-set converse bound, we obtain the following order optimality results, whose proof could be found in Appendix A.

**Theorem 2.** *For the  $(K, a, b)$  coded caching problem for location-based content,*

- if  $K$  is even, we have  $R_u^* \leq 2R^*$ ;
- if  $K$  is odd, we have  $R_u^* \leq 3R^*$ .

□

Theorem 2 shows that the proposed achievable scheme for Theorem 1 is generally order optimal within a factor of 3.

**Remark 2** (Extension to the multiple-input single-output (MISO) broadcast channel). *The proposed achievable scheme in this paper could be directly extended to the case where the server has multiple antennas by using the cache-aided MISO schemes in [27]–[30]. By leveraging the multiplexing*

<sup>4</sup>For example, if we have two lower bounds on  $R$ , say  $R \geq 3$  and  $R \geq 1$ . Obviously,  $R \geq 1$  is redundant. If we sum these two bounds, we have  $R \geq 2$ , which is looser than  $R \geq 3$ .

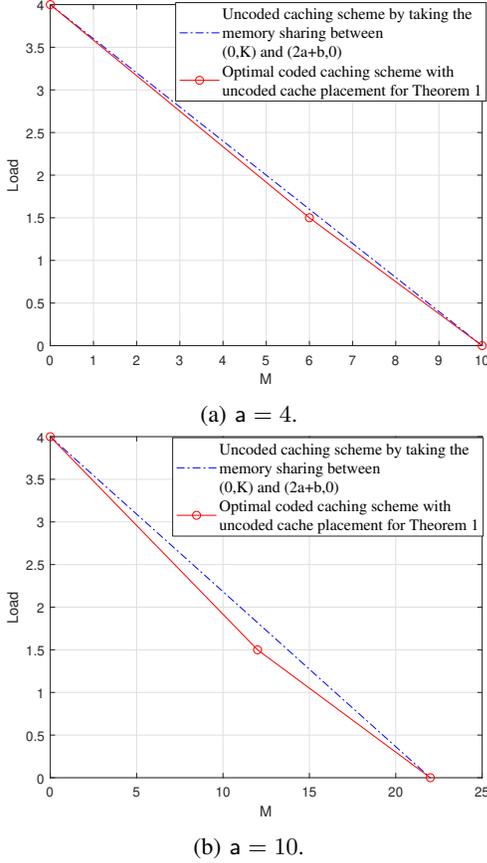


Fig. 3: Coded caching problem for location-based content with  $K = 4$ ,  $b = 2$ , and various values  $a$ .

gain from  $L$  antennas at the server, the achieved corner points become  $(0, \frac{K}{L})$ ,  $(a + b, \frac{K-1}{L+1})$ , and  $(2a + b, 0)$ .  $\square$

**A. Converse Proof of Theorem 1:**  $b(K-1) < 2a$  and  $a + b \leq M \leq 2a + b$

We first focus on the case where  $b(K-1) < 2a$  and  $a + b \leq M \leq 2a + b$ , and use the following example to illustrate the main idea of our proposed converse bound under uncoded cache placement.

**Example 1** ( $(K, a, b) = (3, 2, 1)$  and  $3 \leq M \leq 5$ ). Consider the coded caching problem for location-based content with  $K = 3$ ,  $a = 2$ , and  $b = 1$ . In this example,  $N = K(a + b) = 9$  and

$$\mathcal{D}_1 = \{1, 2, 3, 4, 5\}, \mathcal{D}_2 = \{4, 5, 6, 7, 8\}, \mathcal{D}_3 = \{7, 8, 9, 1, 2\}.$$

It can be seen that the  $N = 9$  files could be divided into two classes, where in the first class denoted by  $\mathcal{C}_1 = \{1, 2, 4, 5, 7, 8\}$ , each file may be demanded by two users; in the second class denoted by  $\mathcal{C}_2 = \{3, 6, 9\}$ , each file can only be demanded by one user. The achieved load by the proposed scheme for Theorem 1 is  $\frac{5}{2} - \frac{M}{2}$  when  $3 \leq M \leq 5$ . In the following, we will prove that it is optimal under uncoded cache placement.

For any caching scheme with uncoded cache placement  $\mathbf{Z}$ , we can divide each file  $W_i$ ,  $i \in [N]$ , into subfiles  $W_i =$

$\{W_{i,\mathcal{T}} : \mathcal{T} \subseteq [3]\}$ , where  $W_{i,\mathcal{T}}$  represents the bits of  $W_i$  exclusively cached by the cache nodes in  $\mathcal{T}$ . Different from the converse proof for the MAN coded caching problem described in Section II-B which considers all possible demand vectors with distinct demands and all permutations of  $K$  users, we will carefully select the demand vectors and user permutations which lead to non-redundant inequalities on the load.

We first fix one permutation  $(u_1, u_2, u_3) = (1, 3, 2)$ . For this permutation of users, we consider the demand vectors  $(d_1, d_2, d_3)$  where  $d_1 \in \{1, 2\}$ ,  $d_2 = 6$  and  $d_3 \in \{7, 8\}$ . More precisely, pick one demand vector  $(d_1, d_2, d_3) = (1, 6, 7)$ , we construct a genie-aided super user with cache

$$\begin{aligned} Z' &= (Z_{u_1}, Z_{u_2} \setminus (Z_{u_1} \cup W_{d_{u_1}}), \\ &Z_{u_3} \setminus (Z_{u_1} \cup W_{d_{u_1}} \cup Z_{u_2} \cup W_{d_{u_2}})) \\ &= (Z_1, Z_3 \setminus (Z_1 \cup W_1), Z_2 \setminus (Z_1 \cup W_1 \cup Z_3 \cup W_7)). \end{aligned}$$

From  $(Z', X)$ , the virtual user can decode  $W_1$ ,  $W_7$ , and  $W_6$ , iteratively. Hence, we have

$$\begin{aligned} &H(W_1, W_7, W_6 | Z') \\ &= H(W_1, W_7, W_6 | Z', X) + I(W_1, W_7, W_6; X | Z') \\ &\leq H(X), \\ \implies R &\geq (|W_{1,\emptyset}| + |W_{1,\{2\}}| + |W_{1,\{3\}}| + |W_{1,\{2,3\}}| \\ &+ |W_{7,\emptyset}| + |W_{7,\{2\}}| + |W_{6,\emptyset}|) / B, \\ \implies R &\geq (|W_{1,\emptyset}| + |W_{1,\{2\}}| + |W_{1,\{3\}}| + |W_{7,\emptyset}| \\ &+ |W_{7,\{2\}}| + |W_{6,\emptyset}|) / B. \end{aligned} \quad (13)$$

Similarly, for this permutation of users, when  $(d_1, d_2, d_3) = (1, 6, 8)$ , we have

$$\begin{aligned} R &\geq (|W_{1,\emptyset}| + |W_{1,\{2\}}| + |W_{1,\{3\}}| + |W_{8,\emptyset}| \\ &+ |W_{8,\{2\}}| + |W_{6,\emptyset}|) / B. \end{aligned} \quad (14)$$

When  $(d_1, d_2, d_3) = (2, 6, 7)$ , we have

$$\begin{aligned} R &\geq (|W_{2,\emptyset}| + |W_{2,\{2\}}| + |W_{2,\{3\}}| + |W_{7,\emptyset}| \\ &+ |W_{7,\{2\}}| + |W_{6,\emptyset}|) / B. \end{aligned} \quad (15)$$

When  $(d_1, d_2, d_3) = (2, 6, 8)$ , we have

$$\begin{aligned} R &\geq (|W_{2,\emptyset}| + |W_{2,\{2\}}| + |W_{2,\{3\}}| + |W_{8,\emptyset}| \\ &+ |W_{8,\{2\}}| + |W_{6,\emptyset}|) / B. \end{aligned} \quad (16)$$

We then fix one permutation  $(u_1, u_2, u_3) = (1, 2, 3)$ . For this permutation of users, we consider the demand vectors  $(d_1, d_2, d_3)$  where  $d_1 \in \{4, 5\}$ ,  $d_2 \in \{7, 8\}$ , and  $d_3 = 9$ . More precisely, when  $(d_1, d_2, d_3) = (4, 7, 9)$ , we have

$$\begin{aligned} R &\geq (|W_{4,\emptyset}| + |W_{4,\{2\}}| + |W_{4,\{3\}}| + |W_{7,\emptyset}| \\ &+ |W_{7,\{3\}}| + |W_{9,\emptyset}|) / B. \end{aligned} \quad (17)$$

When  $(d_1, d_2, d_3) = (4, 8, 9)$ , we have

$$\begin{aligned} R &\geq (|W_{4,\emptyset}| + |W_{4,\{2\}}| + |W_{4,\{3\}}| + |W_{8,\emptyset}| \\ &+ |W_{8,\{3\}}| + |W_{9,\emptyset}|) / B. \end{aligned} \quad (18)$$

When  $(d_1, d_2, d_3) = (5, 7, 9)$ , we have

$$\begin{aligned} R &\geq (|W_{5,\emptyset}| + |W_{5,\{2\}}| + |W_{5,\{3\}}| + |W_{7,\emptyset}| \\ &+ |W_{7,\{3\}}| + |W_{9,\emptyset}|) / B. \end{aligned} \quad (19)$$

When  $(d_1, d_2, d_3) = (5, 8, 9)$ , we have

$$R \geq (|W_{5,\emptyset}| + |W_{5,\{2\}}| + |W_{5,\{3\}}| + |W_{8,\emptyset}| + |W_{8,\{3\}}| + |W_{9,\emptyset}|)/B. \quad (20)$$

By summing (13)-(20), we obtain

$$R \geq \frac{1}{4B} (|W_{1,\emptyset}| + |W_{2,\emptyset}| + |W_{4,\emptyset}| + |W_{5,\emptyset}| + 2|W_{7,\emptyset}| + 2|W_{8,\emptyset}|) + \frac{1}{2B} (|W_{6,\emptyset}| + |W_{9,\emptyset}|) + \frac{1}{4B} \sum_{i \in \{1,2,4,5,7,8\}} \sum_{j \in \{2,3\}} |W_{i,\{j\}}|. \quad (21)$$

Next we fix one permutation  $(u_1, u_2, u_3) = (2, 1, 3)$ . For this permutation of users, we consider the demand vectors  $(d_1, d_2, d_3)$  where  $d_1 \in \{1, 2\}$ ,  $d_2 \in \{4, 5\}$ , and  $d_3 = 9$ . Hence, we can list 4 inequalities. We also fix one permutation  $(u_1, u_2, u_3) = (2, 3, 1)$ . For this permutation of users, we consider the demand vectors  $(d_1, d_2, d_3)$  where  $d_1 = 3$ ,  $d_2 \in \{7, 8\}$ , and  $d_3 \in \{1, 2\}$ . Hence, we can also list 4 inequalities. Then we sum these 8 inequalities to obtain

$$R \geq \frac{1}{4B} (|W_{4,\emptyset}| + |W_{5,\emptyset}| + |W_{7,\emptyset}| + |W_{8,\emptyset}| + 2|W_{1,\emptyset}| + 2|W_{2,\emptyset}|) + \frac{1}{2B} (|W_{9,\emptyset}| + |W_{3,\emptyset}|) + \frac{1}{4B} \sum_{i \in \{1,2,4,5,7,8\}} \sum_{j \in \{1,3\}} |W_{i,\{j\}}|. \quad (22)$$

Finally, we fix one permutation  $(u_1, u_2, u_3) = (3, 2, 1)$ . For this permutation of users, we consider the demand vectors  $(d_1, d_2, d_3)$  where  $d_1 = 3$ ,  $d_2 \in \{4, 5\}$ , and  $d_3 \in \{7, 8\}$ . Hence, we can list 4 inequalities. We also fix one permutation  $(u_1, u_2, u_3) = (3, 1, 2)$ . For this permutation of users, we consider the demand vectors  $(d_1, d_2, d_3)$  where  $d_1 \in \{4, 5\}$ ,  $d_2 = 6$ , and  $d_3 \in \{1, 2\}$ . Hence, we can also list 4 inequalities. Then we sum these 8 inequalities to obtain

$$R \geq \frac{1}{4B} (|W_{7,\emptyset}| + |W_{8,\emptyset}| + |W_{1,\emptyset}| + |W_{2,\emptyset}| + 2|W_{4,\emptyset}| + 2|W_{5,\emptyset}|) + \frac{1}{2B} (|W_{3,\emptyset}| + |W_{6,\emptyset}|) + \frac{1}{4B} \sum_{i \in \{1,2,4,5,7,8\}} \sum_{j \in \{1,2\}} |W_{i,\{j\}}|. \quad (23)$$

By summing (21)-(23), we have

$$\begin{aligned} R &\geq \frac{1}{3} \underbrace{(|W_{1,\emptyset}| + |W_{2,\emptyset}| + |W_{4,\emptyset}| + |W_{5,\emptyset}| + |W_{7,\emptyset}| + |W_{8,\emptyset}|)/B}_{:=\alpha_0} \\ &+ \frac{1}{3} \underbrace{(|W_{3,\emptyset}| + |W_{6,\emptyset}| + |W_{9,\emptyset}|)/B}_{:=\beta_0} \\ &+ \frac{1}{6} \underbrace{\sum_{i \in \{1,2,4,5,7,8\}} \sum_{j \in \{3\}} |W_{i,\{j\}}|/B}_{:=\alpha_1} \\ &= \frac{1}{3}\alpha_0 + \frac{1}{3}\beta_0 + \frac{1}{6}\alpha_1. \end{aligned} \quad (24a)$$

$$= \frac{1}{3}\alpha_0 + \frac{1}{3}\beta_0 + \frac{1}{6}\alpha_1. \quad (24b)$$

By the file size constraint, for the first class of files  $C_1$ , we have

$$6 = (|W_1| + |W_2| + |W_4| + |W_5| + |W_7| + |W_8|)/B \quad (25a)$$

$$= \alpha_0 + \alpha_1 + \sum_{t_1 \in \{2,3\}} \sum_{i_1 \in \{1,2,4,5,7,8\}} \sum_{\mathcal{T}_1 \subseteq [3]: |\mathcal{T}_1|=t_1} \frac{|W_{i_1, \mathcal{T}_1}|}{B}; \quad (25b)$$

and for the second class of files  $C_2$ , we have

$$3 = (|W_3| + |W_6| + |W_9|)/B \quad (26a)$$

$$= \beta_0 + \sum_{t_2 \in [3]} \sum_{i_2 \in \{3,6,9\}} \sum_{\mathcal{T}_2 \subseteq [3]: |\mathcal{T}_2|=t_2} \frac{|W_{i_2, \mathcal{T}_2}|}{B}. \quad (26b)$$

By the memory size constraint, we have

$$3M \geq \alpha_1 + \sum_{t_1 \in \{2,3\}} \sum_{i_1 \in \{1,2,4,5,7,8\}} \sum_{\mathcal{T}_1 \subseteq [3]: |\mathcal{T}_1|=t_1} \frac{t_1 |W_{i_1, \mathcal{T}_1}|}{B} + \sum_{t_2 \in [3]} \sum_{i_2 \in \{3,6,9\}} \sum_{\mathcal{T}_2 \subseteq [3]: |\mathcal{T}_2|=t_2} \frac{t_2 |W_{i_2, \mathcal{T}_2}|}{B}. \quad (27)$$

The next step is to derive the converse bound on  $R$  from the constraints in (24b), (25b), (26b), and (27). More precisely, from (25b) we have

$$\frac{1}{3}(\alpha_0 + \alpha_1) + \frac{1}{3} \sum_{t_1 \in \{2,3\}} \sum_{i_1 \in \{1,2,4,5,7,8\}} \sum_{\mathcal{T}_1 \subseteq [3]: |\mathcal{T}_1|=t_1} \frac{|W_{i_1, \mathcal{T}_1}|}{B} = 2. \quad (28)$$

From (26b) we have

$$\frac{1}{6}\beta_0 + \frac{1}{6} \sum_{t_2 \in [3]} \sum_{i_2 \in \{3,6,9\}} \sum_{\mathcal{T}_2 \subseteq [3]: |\mathcal{T}_2|=t_2} \frac{|W_{i_2, \mathcal{T}_2}|}{B} = \frac{1}{2}. \quad (29)$$

From (27) we have

$$-\frac{1}{6}\alpha_1 - \frac{1}{6} \sum_{t_1 \in \{2,3\}} \sum_{i_1 \in \{1,2,4,5,7,8\}} \sum_{\mathcal{T}_1 \subseteq [3]: |\mathcal{T}_1|=t_1} \frac{t_1 |W_{i_1, \mathcal{T}_1}|}{B} - \frac{1}{6} \sum_{t_2 \in [3]} \sum_{i_2 \in \{3,6,9\}} \sum_{\mathcal{T}_2 \subseteq [3]: |\mathcal{T}_2|=t_2} \frac{t_2 |W_{i_2, \mathcal{T}_2}|}{B} \geq -\frac{M}{2}. \quad (30)$$

We sum (28)-(30) to obtain

$$\begin{aligned} \frac{1}{3}\alpha_0 + \frac{1}{6}\beta_0 + \frac{1}{6}\alpha_1 &\geq \frac{5}{2} - \frac{M}{2} \\ &+ \frac{1}{6} \sum_{t_1 \in \{2,3\}} \sum_{i_1 \in \{1,2,4,5,7,8\}} \sum_{\mathcal{T}_1 \subseteq [3]: |\mathcal{T}_1|=t_1} (t_1 - 2) \frac{|W_{i_1, \mathcal{T}_1}|}{B} \\ &+ \frac{1}{6} \sum_{t_2 \in [3]} \sum_{i_2 \in \{3,6,9\}} \sum_{\mathcal{T}_2 \subseteq [3]: |\mathcal{T}_2|=t_2} (t_2 - 1) \frac{|W_{i_2, \mathcal{T}_2}|}{B} \end{aligned} \quad (31a)$$

$$\geq \frac{5}{2} - \frac{M}{2}. \quad (31b)$$

By taking (31b) into (24b), we have

$$R \geq \frac{1}{3}\alpha_0 + \frac{1}{3}\beta_0 + \frac{1}{6}\alpha_1 \geq \frac{1}{3}\alpha_0 + \frac{1}{6}\beta_0 + \frac{1}{6}\alpha_1 \geq \frac{5}{2} - \frac{M}{2}. \quad (32)$$

Hence, from (32) we have  $R_u^* \geq \frac{5}{2} - \frac{M}{2}$ , which coincides with the achieved load for Theorem 1 when  $3 \leq M \leq 5$ .

Note that if we consider all the possible demand vectors with distinct demands and all permutations of users, and sum all the obtained inequalities from them, the resulting converse bound is not tight; for example, if  $M = 3$ , the resulting non-tight converse bound provides  $R_u^* \geq \frac{54}{95}$ , while the tight converse bound is  $R_u^* \geq \frac{5}{2} - \frac{M}{2} = 1$ .  $\square$

We now generalize the converse bound proof in Example 1 for the case  $b(K-1) < 2a$  and  $a+b \leq M \leq 2a+b$ . Recall that for each user  $k \in [K]$ , the set of possible demanded files by user  $k$  is  $\mathcal{D}_k$  defined in (1), where  $\mathcal{D}_k := \mathcal{D}_{k,1} \cup \mathcal{D}_{k,2} \cup \mathcal{D}_{k,3}$ . By definition,  $\mathcal{D}_{k,1} = \mathcal{D}_{<k-1>_{K,3}}$  and  $\mathcal{D}_{k,3} = \mathcal{D}_{<k+1>_{K,1}}$ . We also have  $|\mathcal{D}_{k,1}| = |\mathcal{D}_{k,3}| = a$  and  $|\mathcal{D}_{k,2}| = b$ . In addition, as in Example 1, we divide all the  $N := K(a+b)$  files into two classes, where

$$\mathcal{C}_1 := \cup_{k \in [K]} \mathcal{D}_{k,1}, \quad (33a)$$

$$\text{and } \mathcal{C}_2 := \cup_{k \in [K]} \mathcal{D}_{k,2}. \quad (33b)$$

For any caching scheme with uncoded cache placement  $\mathbf{Z}$ , we can divide each file  $W_i$ ,  $i \in [N]$ , into subfiles  $W_i = \{W_{i,\mathcal{T}} : \mathcal{T} \subseteq [K]\}$ .

Fix one integer  $k \in [K]$ . For this integer, we consider two permutations of users,  $(k, <k-1>_{K, \dots, <k-K+1>_{K}}$ ) and  $(k, <k+1>_{K, \dots, <k+K-1>_{K}}$ ).

For the first permutation  $(u_1, u_2, \dots, u_K) = (k, <k-1>_{K, \dots, <k-K+1>_{K}}$ ), we consider the demand vectors  $(d_1, \dots, d_K)$  where  $d_{u_j} \in \mathcal{D}_{u_j,1}$  for  $j \in [K-1]$ , and  $d_{u_K} \in \mathcal{D}_{u_K,2}$ , totally  $a^{K-1}b$  demand vectors. For each  $(d_1, \dots, d_K)$ , we construct a genie-aided super user with cache

$$Z' = (Z_{u_1}, Z_{u_2} \setminus (W_{d_{u_1}} \cup Z_{u_1}), \dots, Z_{u_K} \setminus (W_{d_{u_1}} \cup Z_{u_1} \cup \dots \cup W_{d_{u_{K-1}}} \cup Z_{u_{K-1}})). \quad (34)$$

From  $(X, Z')$  we can decode  $W_{d_{u_1}}, \dots, W_{d_{u_K}}$ , iteratively. Hence,

$$H(W_{d_{u_1}}, \dots, W_{d_{u_K}} | Z') = H(W_{d_{u_1}}, \dots, W_{d_{u_K}} | Z', X) + I(W_{d_{u_1}}, \dots, W_{d_{u_K}}; X | Z') \quad (35a)$$

$$= I(W_{d_{u_1}}, \dots, W_{d_{u_K}}; X | Z') \leq H(X), \quad (35b)$$

which leads to

$$R \geq \left( |W_{d_{u_1}, \emptyset}| + \sum_{j_1 \in [K] \setminus \{u_1\}} |W_{d_{u_1}, \{j_1\}}| \right) / B + \left( |W_{d_{u_2}, \emptyset}| + \sum_{j_2 \in [K] \setminus \{u_1, u_2\}} |W_{d_{u_2}, \{j_2\}}| \right) / B + \dots + |W_{d_{u_K}, \emptyset}| / B \quad (36a)$$

$$= \left( |W_{d_k, \emptyset}| + \sum_{j_1 \in [K] \setminus \{k\}} |W_{d_k, \{j_1\}}| \right) / B + \left( |W_{d_{<k-1>_{K, \dots, <k-1>_{K}}, \emptyset}| + \sum_{j_2 \in [K] \setminus \{k, <k-1>_{K, \dots, <k-1>_{K}\}} |W_{d_{<k-1>_{K, \dots, <k-1>_{K}}, \{j_2\}}| \right) / B + \dots + |W_{d_{<k-K+1>_{K, \dots, <k-K+1>_{K}}, \emptyset}| / B. \quad (36b)$$

Considering all the demand vectors  $(d_1, \dots, d_K)$  where  $d_{u_j} \in \mathcal{D}_{u_j,1}$  for  $j \in [K-1]$ , and  $d_{u_K} \in \mathcal{D}_{u_K,2}$ , we list  $a^{K-1}b$

inequalities in the form of (36b) and sum them all together to obtain

$$R \geq \frac{1}{aB} \sum_{i_1 \in \mathcal{D}_{k,1}} \left( |W_{i_1, \emptyset}| + \sum_{j_1 \in [K] \setminus \{k\}} |W_{i_1, \{j_1\}}| \right) + \frac{1}{aB} \sum_{i_2 \in \mathcal{D}_{<k-1>_{K,1}}} \left( |W_{i_2, \emptyset}| + \sum_{j_2 \in [K] \setminus \{k, <k-1>_{K,1}\}} |W_{i_2, \{j_2\}}| \right) + \dots + \frac{1}{aB} \sum_{i_{K-1} \in \mathcal{D}_{<k-K+2>_{K,1}}} \left( |W_{i_{K-1}, \emptyset}| + \sum_{\substack{j_{K-1} \in [K] \setminus \{k, \\ <k-1>_{K, \dots, <k-K+2>_{K}\}}} |W_{i_{K-1}, \{j_{K-1}\}}| \right) + \frac{1}{bB} \sum_{i_K \in \mathcal{D}_{<k-K+1>_{K,2}}} |W_{i_K, \emptyset}|. \quad (37)$$

For the second permutation  $(u_1, u_2, \dots, u_K) = (k, <k+1>_{K, \dots, <k+K-1>_{K}}$ ), we consider the demand vectors  $(d_1, \dots, d_K)$  where  $d_{u_j} \in \mathcal{D}_{u_j,3}$  for  $j \in [K-1]$ , and  $d_{u_K} \in \mathcal{D}_{u_K,2}$ , totally  $a^{K-1}b$  demand vectors. For each of such demand vectors, we construct a genie-aided super user with cache as in (34) and obtain an inequality as in (36a). By summing all the obtained  $a^{K-1}b$  inequalities, we have

$$R \geq \frac{1}{aB} \sum_{i_1 \in \mathcal{D}_{k,3}} \left( |W_{i_1, \emptyset}| + \sum_{j_1 \in [K] \setminus \{k\}} |W_{i_1, \{j_1\}}| \right) + \frac{1}{aB} \sum_{i_2 \in \mathcal{D}_{<k+1>_{K,3}}} \left( |W_{i_2, \emptyset}| + \sum_{j_2 \in [K] \setminus \{k, <k+1>_{K,3}\}} |W_{i_2, \{j_2\}}| \right) + \dots + \frac{1}{aB} \sum_{i_{K-1} \in \mathcal{D}_{<k+K-2>_{K,3}}} \left( |W_{i_{K-1}, \emptyset}| + \sum_{\substack{j_{K-1} \in [K] \setminus \{k, \\ <k+1>_{K, \dots, <k+K-2>_{K}\}}} |W_{i_{K-1}, \{j_{K-1}\}}| \right) + \frac{1}{bB} \sum_{i_K \in \mathcal{D}_{<k+K-1>_{K,2}}} |W_{i_K, \emptyset}| \quad (38a)$$

$$= \frac{1}{aB} \sum_{i_1 \in \mathcal{D}_{<k+1>_{K,1}}} \left( |W_{i_1, \emptyset}| + \sum_{j_1 \in [K] \setminus \{k\}} |W_{i_1, \{j_1\}}| \right) + \frac{1}{aB} \sum_{i_2 \in \mathcal{D}_{<k+2>_{K,3}}} \left( |W_{i_2, \emptyset}| + \sum_{j_2 \in [K] \setminus \{k, <k+1>_{K,3}\}} |W_{i_2, \{j_2\}}| \right) + \dots + \frac{1}{aB} \sum_{i_{K-1} \in \mathcal{D}_{<k+K-1>_{K,1}}} \left( |W_{i_{K-1}, \emptyset}| + \sum_{\substack{j_{K-1} \in [K] \setminus \{k, \\ <k+1>_{K, \dots, <k+K-2>_{K}\}}} |W_{i_{K-1}, \{j_{K-1}\}}| \right) + \frac{1}{bB} \sum_{i_K \in \mathcal{D}_{<k+K-1>_{K,2}}} |W_{i_K, \emptyset}|, \quad (38b)$$

where (38b) comes from that  $\mathcal{D}_{i,3} = \mathcal{D}_{<i+1>_{K,1}}$  for any  $i \in [K]$ .

We sum (37) and (38b) to obtain

$$R \geq \frac{1}{2a} \sum_{k_1 \in \{k, <k+1>_{K}\}} \sum_{i_1 \in \mathcal{D}_{k_1,1}} \frac{|W_{i_1, \emptyset}|}{B} + \frac{1}{a} \sum_{k_2 \in ([K] \setminus \{k, <k+1>_{K}\})} \sum_{i_2 \in \mathcal{D}_{k_2,1}} \frac{|W_{i_2, \emptyset}|}{B}$$

$$\begin{aligned}
& + \frac{1}{2b} \sum_{k_3 \in \{<k-1>_K, <k+1>_K\}} \sum_{i_3 \in \mathcal{D}_{k_3,2}} \frac{|W_{i_3, \emptyset}|}{B} \\
& + \frac{1}{2a} \sum_{i_4 \in \mathcal{C}_1} \sum_{j \in [K] \setminus \{k\}} \frac{|W_{i_4, \{j\}}|}{B}, \tag{39}
\end{aligned}$$

where  $\mathcal{C}_1 = \cup_{k \in [K]} \mathcal{D}_{k,1}$  defined in (33a).

By considering all  $k \in [K]$ , we list  $K$  inequalities in the form of (39), and then sum them all together to obtain

$$\begin{aligned}
R & \geq \underbrace{\frac{K-1}{aK} \sum_{i_1 \in \mathcal{C}_1} \frac{|W_{i_1, \emptyset}|}{B}}_{:=\alpha_0} + \underbrace{\frac{1}{bK} \sum_{i_2 \in \mathcal{C}_2} \frac{|W_{i_2, \emptyset}|}{B}}_{:=\beta_0} \\
& + \underbrace{\frac{K-1}{2aK} \sum_{i_3 \in \mathcal{C}_1} \sum_{j \in [K]} \frac{|W_{i_3, \{j\}}|}{B}}_{:=\alpha_1}. \tag{40}
\end{aligned}$$

By the file size constraint, for the first class of files  $\mathcal{C}_1$ , we have

$$\alpha_0 + \alpha_1 + \sum_{t_1 \in [2:K]} \sum_{i_1 \in \mathcal{C}_1} \sum_{\mathcal{T}_1 \subseteq [K]: |\mathcal{T}_1|=t_1} \frac{|W_{i_1, \mathcal{T}_1}|}{B} = aK; \tag{41}$$

and for the second class of files  $\mathcal{C}_2$ , we have

$$\beta_0 + \sum_{t_2 \in [K]} \sum_{i_2 \in \mathcal{C}_2} \sum_{\mathcal{T}_2 \subseteq [K]: |\mathcal{T}_2|=t_2} \frac{|W_{i_2, \mathcal{T}_2}|}{B} = bK. \tag{42}$$

By the memory size constraint, we have

$$\begin{aligned}
\alpha_1 + \sum_{t_1 \in [2:K]} \sum_{i_1 \in \mathcal{C}_1} \sum_{\mathcal{T}_1 \subseteq [K]: |\mathcal{T}_1|=t_1} \frac{t_1 |W_{i_1, \mathcal{T}_1}|}{B} \\
+ \sum_{t_2 \in [K]} \sum_{i_2 \in \mathcal{C}_2} \sum_{\mathcal{T}_2 \subseteq [K]: |\mathcal{T}_2|=t_2} \frac{t_2 |W_{i_2, \mathcal{T}_2}|}{B} \leq KM. \tag{43}
\end{aligned}$$

We take  $\frac{K-1}{aK} \times (41) + \frac{K-1}{2aK} \times (42) - \frac{K-1}{2aK} (43)$  to obtain

$$\begin{aligned}
& \frac{K-1}{aK} \alpha_0 + \frac{K-1}{2aK} \beta_0 + \frac{K-1}{2aK} \alpha_1 \\
& \geq \frac{(K-1)(2a+b)}{2a} - \frac{K-1}{2a} M \\
& + \frac{K-1}{2aK} \sum_{t_1 \in [2:K]} \sum_{i_1 \in \mathcal{C}_1} \sum_{\mathcal{T}_1 \subseteq [K]: |\mathcal{T}_1|=t_1} (t_1 - 2) \frac{|W_{i_1, \mathcal{T}_1}|}{B} \\
& + \frac{K-1}{2aK} \sum_{t_2 \in [K]} \sum_{i_2 \in \mathcal{C}_2} \sum_{\mathcal{T}_2 \subseteq [K]: |\mathcal{T}_2|=t_2} (t_2 - 1) \frac{t_2 |W_{i_2, \mathcal{T}_2}|}{B} \tag{44a}
\end{aligned}$$

$$\geq \frac{(K-1)(2a+b)}{2a} - \frac{K-1}{2a} M. \tag{44b}$$

By taking (44b) into (40), we have

$$R \geq \frac{K-1}{aK} \alpha_0 + \frac{1}{bK} \beta_0 + \frac{K-1}{2aK} \alpha_1 \tag{45a}$$

$$\geq \frac{K-1}{aK} \alpha_0 + \frac{K-1}{2aK} \beta_0 + \frac{K-1}{2aK} \alpha_1 \tag{45b}$$

$$\geq \frac{(K-1)(2a+b)}{2a} - \frac{K-1}{2a} M, \tag{45c}$$

which leads to  $R_u^* \geq \frac{(K-1)(2a+b)}{2a} - \frac{K-1}{2a} M$ , coinciding with the achieved load for Theorem 1 when  $b(K-1) < 2a$  and  $a+b \leq M \leq 2a+b$ .

**B. Converse Proof of Theorem 1:**  $b(K-1) < 2a$  and  $0 \leq M \leq a+b$

We then focus on the case where  $b(K-1) < 2a$  and  $0 \leq M \leq a+b$ . We go back to Example 1 and consider the memory size regime  $0 \leq M \leq 3$ .

**Example 2** ( $(K, a, b) = (3, 2, 1)$  and  $0 \leq M \leq 3$ ). Recall that in this example we have  $N = K(a+b) = 9$  and  $\mathcal{D}_1 = \{1, 2, 3, 4, 5\}$ ,  $\mathcal{D}_2 = \{4, 5, 6, 7, 8\}$ ,  $\mathcal{D}_3 = \{7, 8, 9, 1, 2\}$ . The achieved load by the proposed scheme for Theorem 1 is  $3 - \frac{2}{3}M$  when  $0 \leq M \leq 3$ . In the following, we will prove that it is optimal under uncoded cache placement.

For any caching scheme with uncoded cache placement  $\mathbf{Z}$ , with the definition of  $\alpha_0$ ,  $\beta_0$ , and  $\alpha_1$  given in (24a), it has been proved in (24b) that

$$R \geq \frac{1}{3} \alpha_0 + \frac{1}{3} \beta_0 + \frac{1}{6} \alpha_1. \tag{46}$$

We will derive another lower bound for  $R$  in terms of  $\alpha_0$ ,  $\beta_0$ , and  $\alpha_1$ , by using another strategy to select demand vectors and permutation of users.

We first fix one permutation  $(u_1, u_2, u_3) = (1, 3, 2)$ . For this permutation of users, we consider the demand vectors  $(d_1, d_2, d_3)$  where  $d_1 \in \{1, 2\}$ ,  $d_2 \in \{4, 5\}$ , and  $d_3 \in \{7, 8\}$ . For each of such 8 demand vectors, we can generate an inequality on  $R$ ; for example if  $(d_1, d_2, d_3) = (1, 4, 7)$ , by generating a genie-aided super user with cache  $Z' = (Z_1, Z_3 \setminus (W_1 \cup Z_1), Z_2 \setminus (W_1 \cup Z_1 \cup W_7 \cup Z_3))$ , we can recover  $(W_1, W_4, W_7)$  from  $(Z', X)$ , and thus

$$\begin{aligned}
R & \geq |W_{1, \emptyset}|/B + |W_{1, \{2\}}|/B + |W_{1, \{3\}}|/B + |W_{7, \emptyset}|/B \\
& + |W_{7, \{2\}}|/B + |W_{4, \emptyset}|/B. \tag{47}
\end{aligned}$$

By considering all such 8 demand vectors, we can list 8 inequalities in the form of (47), and sum them all together to obtain

$$\begin{aligned}
R & \geq \frac{1}{2B} (|W_{1, \emptyset}| + |W_{2, \emptyset}| + |W_{4, \emptyset}| + |W_{5, \emptyset}| + |W_{7, \emptyset}| \\
& + |W_{8, \emptyset}|) + \frac{1}{2B} (|W_{1, \{2\}}| + |W_{1, \{3\}}| + |W_{2, \{2\}}| + |W_{2, \{3\}}| \\
& + |W_{7, \{2\}}| + |W_{8, \{2\}}|). \tag{48}
\end{aligned}$$

We then fix one permutation  $(u_1, u_2, u_3) = (1, 2, 3)$ . For this permutation of users, we consider the demand vectors  $(d_1, d_2, d_3)$  where  $d_1 \in \{4, 5\}$ ,  $d_2 \in \{7, 8\}$ , and  $d_3 \in \{1, 2\}$ . By considering all such 8 demand vectors, we can list 8 inequalities in the form of (47), and sum them all together to obtain

$$\begin{aligned}
R & \geq \frac{1}{2B} (|W_{1, \emptyset}| + |W_{2, \emptyset}| + |W_{4, \emptyset}| + |W_{5, \emptyset}| + |W_{7, \emptyset}| \\
& + |W_{8, \emptyset}|) + \frac{1}{2B} (|W_{4, \{2\}}| + |W_{4, \{3\}}| + |W_{5, \{2\}}| + |W_{5, \{3\}}| \\
& + |W_{7, \{3\}}| + |W_{8, \{3\}}|). \tag{49}
\end{aligned}$$

By summing (48) and (49), we have

$$R \geq \sum_{i \in \{1, 2, 4, 5, 7, 8\}} \left( \frac{1}{2B} |W_{i, \emptyset}| + \frac{1}{4B} \sum_{j \in \{2, 3\}} |W_{i, \{j\}}| \right). \tag{50}$$

Next we fix one permutation  $(u_1, u_2, u_3) = (2, 1, 3)$ , and consider the demand vectors  $(d_1, d_2, d_3)$  where  $d_1 \in \{1, 2\}$ ,  $d_2 \in \{4, 5\}$ , and  $d_3 \in \{7, 8\}$ . Hence, we can list 8 inequalities. We also fix one permutation  $(u_1, u_2, u_3) = (2, 3, 1)$ , and consider the demand vectors  $(d_1, d_2, d_3)$  where  $d_1 \in \{4, 5\}$ ,  $d_2 \in \{7, 8\}$ , and  $d_3 \in \{1, 2\}$ . Hence, we can also list 8 inequalities. Then we sum these 16 inequalities to obtain

$$R \geq \sum_{i \in \{1, 2, 4, 5, 7, 8\}} \left( \frac{1}{2B} |W_{i, \emptyset}| + \frac{1}{4B} \sum_{j \in \{1, 3\}} |W_{i, \{j\}}| \right). \quad (51)$$

Finally we fix one permutation  $(u_1, u_2, u_3) = (3, 2, 1)$ , and consider the demand vectors  $(d_1, d_2, d_3)$  where  $d_1 \in \{1, 2\}$ ,  $d_2 \in \{4, 5\}$ , and  $d_3 \in \{7, 8\}$ . Hence, we can list 8 inequalities. We also fix one permutation  $(u_1, u_2, u_3) = (3, 1, 2)$ , and consider the demand vectors  $(d_1, d_2, d_3)$  where  $d_1 \in \{4, 5\}$ ,  $d_2 \in \{7, 8\}$ , and  $d_3 \in \{1, 2\}$ . Hence, we can also list 8 inequalities. Then we sum these 16 inequalities to obtain

$$R \geq \sum_{i \in \{1, 2, 4, 5, 7, 8\}} \left( \frac{1}{2B} |W_{i, \emptyset}| + \frac{1}{4B} \sum_{j \in \{1, 2\}} |W_{i, \{j\}}| \right). \quad (52)$$

By summing (50)-(52), we obtain

$$R \geq \sum_{i \in \{1, 2, 4, 5, 7, 8\}} \left( \frac{1}{2B} |W_{i, \emptyset}| + \frac{1}{6B} \sum_{j \in \{3\}} |W_{i, \{j\}}| \right) \quad (53a)$$

$$= \frac{1}{2} \alpha_0 + \frac{1}{6} \alpha_1. \quad (53b)$$

We take  $\frac{2}{3} \times (46) + \frac{1}{3} \times (53b)$  to obtain

$$R \geq \frac{2}{9} \alpha_0 + \frac{2}{9} \beta_0 + \frac{1}{9} \alpha_1 + \frac{1}{6} \alpha_0 + \frac{1}{18} \alpha_1 \quad (54a)$$

$$= \frac{7}{18} \alpha_0 + \frac{2}{9} \beta_0 + \frac{1}{6} \alpha_1. \quad (54b)$$

Recall that the file size constraints are given in (25b) and (26b), while the memory size constraint is given in (27). From (25b), we have

$$\frac{7}{18} \alpha_0 + \frac{7}{18} \alpha_1 + \frac{7}{18} \sum_{t_1 \in \{2, 3\}} \sum_{i_1 \in \{1, 2, 4, 5, 7, 8\}} \sum_{\mathcal{T}_1 \subseteq \{3\}: |\mathcal{T}_1| = t_1} \frac{|W_{i_1, \mathcal{T}_1}|}{B} = \frac{7}{3}. \quad (55)$$

From (26b), we have

$$\frac{2}{9} \beta_0 + \frac{2}{9} \sum_{t_2 \in \{3\}} \sum_{i_2 \in \{3, 6, 9\}} \sum_{\mathcal{T}_2 \subseteq \{3\}: |\mathcal{T}_2| = t_2} \frac{|W_{i_2, \mathcal{T}_2}|}{B} = \frac{2}{3}. \quad (56)$$

From (27), we have

$$\frac{2}{9} \alpha_1 + \frac{2}{9} \sum_{t_1 \in \{2, 3\}} \sum_{i_1 \in \{1, 2, 4, 5, 7, 8\}} \sum_{\mathcal{T}_1 \subseteq \{3\}: |\mathcal{T}_1| = t_1} \frac{t_1 |W_{i_1, \mathcal{T}_1}|}{B} + \frac{2}{9} \sum_{t_2 \in \{3\}} \sum_{i_2 \in \{3, 6, 9\}} \sum_{\mathcal{T}_2 \subseteq \{3\}: |\mathcal{T}_2| = t_2} \frac{t_2 |W_{i_2, \mathcal{T}_2}|}{B} \leq \frac{2}{3} M. \quad (57)$$

By taking (55) + (56) - (57), we have

$$\frac{7}{18} \alpha_0 + \frac{2}{9} \beta_0 + \frac{1}{6} \alpha_1 \geq 3 - \frac{2}{3} M. \quad (58)$$

From (54b) and (58), we have

$$R \geq 3 - \frac{2}{3} M. \quad (59)$$

Hence, from (59) we have  $R_u^* \geq 3 - \frac{2}{3} M$ , which coincides with the achieved load for Theorem 1 for  $0 \leq M \leq 3$ .  $\square$

We are now ready to generalize the converse proof in Example 2 for the case where  $b(K-1) < 2a$  and  $0 \leq M \leq a+b$ . For any caching scheme with uncoded cache placement  $\mathbf{Z}$ , with the definition of  $\alpha_0$ ,  $\beta_0$ , and  $\alpha_1$  in (40), it has been proved in (40) that

$$R \geq \frac{K-1}{aK} \alpha_0 + \frac{1}{bK} \beta_0 + \frac{K-1}{2aK} \alpha_1. \quad (60)$$

Now we fix one integer  $k \in [K]$ . For this integer, we consider two permutations of users,  $(k, \langle k-1 \rangle_K, \dots, \langle k-K+1 \rangle_K)$  and  $(k, \langle k+1 \rangle_K, \dots, \langle k+K-1 \rangle_K)$ .

For the first permutation  $(u_1, u_2, \dots, u_K) = (k, \langle k-1 \rangle_K, \dots, \langle k-K+1 \rangle_K)$ , we consider the demand vectors  $(d_1, \dots, d_K)$  where  $d_{u_j} \in \mathcal{D}_{u_j, 1}$  for  $j \in [K]$ , totally  $a^K$  demand vectors. For each  $(d_1, \dots, d_K)$ , we construct a genie-aided super user with cache as in (34) and derive an inequality as in (36b). By considering all such  $a^K$  demand vectors, we list  $a^K$  inequalities, and sum them all together to obtain

$$\begin{aligned} R \geq & \frac{1}{aB} \sum_{i_1 \in \mathcal{D}_{k, 1}} \left( |W_{i_1, \emptyset}| + \sum_{j_1 \in [K] \setminus \{k\}} |W_{i_1, \{j_1\}}| \right) \\ & + \frac{1}{aB} \sum_{i_2 \in \mathcal{D}_{\langle k-1 \rangle_K, 1}} \left( |W_{i_2, \emptyset}| + \sum_{j_2 \in [K] \setminus \{k, \langle k-1 \rangle_K\}} |W_{i_2, \{j_2\}}| \right) \\ & + \dots + \frac{1}{aB} \sum_{\substack{i_{K-1} \in \\ \mathcal{D}_{\langle k-K+2 \rangle_K, 1}}} \left( |W_{i_{K-1}, \emptyset}| + \sum_{\substack{j_{K-1} \in [K] \setminus \{k, \\ \langle k-1 \rangle_K, \dots, \langle k-K+2 \rangle_K\}}} |W_{i_{K-1}, \{j_{K-1}\}}| \right) \\ & + \frac{1}{aB} \sum_{i_K \in \mathcal{D}_{\langle k-K+1 \rangle_K, 1}} |W_{i_K, \emptyset}|. \end{aligned} \quad (61)$$

For the second permutation  $(u_1, u_2, \dots, u_K) = (k, \langle k+1 \rangle_K, \dots, \langle k+K-1 \rangle_K)$ , we consider the demand vectors  $(d_1, \dots, d_K)$  where  $d_{u_j} \in \mathcal{D}_{u_j, 3} = \mathcal{D}_{\langle u_j+1 \rangle_K, 1}$  for  $j \in [K]$ , totally  $a^K$  demand vectors. For each of such demand vectors, we construct a genie-aided super user with cache as in (34) and derive an inequality as in (36b). By summing all the obtained  $a^K$  inequalities, we have

$$\begin{aligned} R \geq & \frac{1}{aB} \sum_{i_1 \in \mathcal{D}_{\langle k+1 \rangle_K, 1}} \left( |W_{i_1, \emptyset}| + \sum_{j_1 \in [K] \setminus \{k\}} |W_{i_1, \{j_1\}}| \right) \\ & + \frac{1}{aB} \sum_{i_2 \in \mathcal{D}_{\langle k+2 \rangle_K, 1}} \left( |W_{i_2, \emptyset}| + \sum_{j_2 \in [K] \setminus \{k, \langle k+1 \rangle_K\}} |W_{i_2, \{j_2\}}| \right) + \dots + \\ & \frac{1}{aB} \sum_{i_{K-1} \in \mathcal{D}_{\langle k+K-1 \rangle_K, 1}} \left( |W_{i_{K-1}, \emptyset}| + \sum_{\substack{j_{K-1} \in [K] \setminus \{k, \\ \langle k+1 \rangle_K, \dots, \langle k+K-2 \rangle_K\}}} |W_{i_{K-1}, \{j_{K-1}\}}| \right) \\ & + \frac{1}{aB} \sum_{i_K \in \mathcal{D}_{\langle k+K \rangle_K, 1}} |W_{i_K, \emptyset}|. \end{aligned} \quad (62)$$

By summing (61) and (62), we obtain

$$R \geq \frac{1}{a} \sum_{i_1 \in \mathcal{C}_1} \frac{|W_{i_1, \emptyset}|}{B} + \frac{1}{2a} \sum_{i_2 \in \mathcal{C}_1} \sum_{j \in [K] \setminus \{k\}} \frac{|W_{i_2, \{j\}}|}{B}. \quad (63)$$

By considering all  $k \in [K]$ , we list  $K$  inequalities in the form of (63), and sum them to obtain

$$R \geq \frac{1}{a} \sum_{i_1 \in \mathcal{C}_1} \frac{|W_{i_1, \emptyset}|}{B} + \frac{K-1}{2aK} \sum_{i_2 \in \mathcal{C}_1} \sum_{j \in [K]} \frac{|W_{i_2, \{j\}}|}{B} \quad (64a)$$

$$= \frac{1}{a} \alpha_0 + \frac{K-1}{2aK} \alpha_1. \quad (64b)$$

Note that, since  $b(K-1) < 2a$ , we have  $1 - \frac{(K+1)b}{2(a+b)} = \frac{2a-b(K-1)}{2(a+b)} > 0$ . Hence, we take  $\frac{(K+1)b}{2(a+b)} \times (60) + \left(1 - \frac{(K+1)b}{2(a+b)}\right) \times (64b)$  to obtain

$$R \geq \frac{2aK + b(K-1)}{2(a+b)aK} \alpha_0 + \frac{K+1}{2(a+b)K} \beta_0 + \frac{K-1}{2aK} \alpha_1. \quad (65)$$

Recall that the file size constraints are given in (41) and (42), while the memory size constraint is given in (43). By taking  $\frac{2aK+b(K-1)}{2(a+b)aK} \times (41) + \frac{K+1}{2(a+b)K} \times (42) - \frac{K+1}{2(a+b)K} \times (43)$ , we obtain

$$\begin{aligned} & \frac{2aK + b(K-1)}{2(a+b)aK} \alpha_0 + \frac{K+1}{2(a+b)K} \beta_0 + \frac{K-1}{2aK} \alpha_1 \\ & \geq K - \frac{K+1}{2(a+b)} M + \frac{1}{2(a+b)aK} \sum_{t_1 \in [2:K]} \end{aligned}$$

$$\begin{aligned} & \sum_{i_1 \in \mathcal{C}_1} \sum_{\mathcal{T}_1 \subseteq [K]: |\mathcal{T}_1|=t_1} ((t_1-2)aK + t_1a - b(K-1)) \frac{|W_{i_1, \mathcal{T}_1}|}{B} \\ & + \frac{K+1}{2(a+b)K} \sum_{t_2 \in [K]} \sum_{i_2 \in \mathcal{C}_2} \sum_{\mathcal{T}_2 \subseteq [K]: |\mathcal{T}_2|=t_2} (t_2-1) \frac{|W_{i_2, \mathcal{T}_2}|}{B} \end{aligned} \quad (66a)$$

$$\geq K - \frac{K+1}{2(a+b)} M, \quad (66b)$$

where (66b) comes from that  $b(K-1) < 2a$ . From (65) and (66b), we have

$$R \geq K - \frac{K+1}{2(a+b)} M, \quad (67)$$

which leads to  $R_u^* \geq K - \frac{K+1}{2(a+b)} M$ , coinciding with the achieved load for Theorem 1 when  $b(K-1) < 2a$  and  $0 \leq M \leq a+b$ .

### C. Converse Proof of Theorem 1: $b(K-1) \geq 2a$

In the end, we focus on the case where  $b(K-1) \geq 2a$ . For any caching scheme with uncoded cache placement  $\mathbf{Z}$ , we will first prove that

$$R \geq \frac{1}{b} \sum_{i \in \mathcal{C}_2} \frac{|W_{i, \emptyset}|}{B} = \frac{1}{b} \beta_0, \quad (68)$$

where  $\mathcal{C}_2$  and  $\beta_0$  are defined in (33b) and (40), respectively.

More precisely, each time we consider a demand vector  $(d_1, \dots, d_K)$ , where  $d_k \in \mathcal{D}_{k,2}$  for each  $k \in [K]$ . Since

$W_{d_1, \emptyset}, \dots, W_{d_K, \emptyset}$  are not cached by any cache node and from  $(Z_1, \dots, Z_K, X)$  we can recover  $W_{d_1, \emptyset}, \dots, W_{d_K, \emptyset}$ , we have

$$R \geq \frac{H(X)}{B} \geq \frac{H(X|Z_1, \dots, Z_K)}{B} \quad (69a)$$

$$= \frac{H(X, W_{d_1, \emptyset}, \dots, W_{d_K, \emptyset} | Z_1, \dots, Z_K)}{B} \quad (69b)$$

$$\geq \frac{H(W_{d_1, \emptyset}, \dots, W_{d_K, \emptyset} | Z_1, \dots, Z_K)}{B} \quad (69c)$$

$$= \frac{H(W_{d_1, \emptyset}, \dots, W_{d_K, \emptyset})}{B} \quad (69d)$$

$$= \frac{|W_{d_1, \emptyset}|}{B} + \dots + \frac{|W_{d_K, \emptyset}|}{B}. \quad (69e)$$

By considering all demand vectors  $(d_1, \dots, d_K)$ , where  $d_k \in \mathcal{D}_{k,2}$  for each  $k \in [K]$ , we list  $b^K$  inequalities in the form of (69e), and sum them all together to obtain (68).

In the following, we will prove the converse bound for case where  $b(K-1) \geq 2a$ , by the help of the derived lower bounds of  $R$  in (40) and (68), the file constraints in (41) and (42), and the memory size constraint in (43).

More precisely, since  $b(K-1) \geq 2a$ , we have  $1 - \frac{2aK}{(K-1)(2a+b)} = \frac{b(K-1)-2a}{(K-1)(2a+b)} \geq 0$ . Hence, by taking  $\frac{2aK}{(K-1)(2a+b)} \times (40) + \left(1 - \frac{2aK}{(K-1)(2a+b)}\right) \times (68)$ , we obtain

$$R \geq \frac{2}{2a+b} \alpha_0 + \frac{1}{2a+b} \beta_0 + \frac{1}{2a+b} \alpha_1. \quad (70)$$

Next, by taking  $\frac{2}{2a+b} \times (41) + \frac{1}{2a+b} \times (42) - \frac{1}{2a+b} \times (43)$ , we obtain

$$\begin{aligned} & \frac{2}{2a+b} \alpha_0 + \frac{1}{2a+b} \beta_0 + \frac{1}{2a+b} \alpha_1 \\ & \geq K - \frac{K}{2a+b} M + \frac{1}{2a+b} \sum_{t_1 \in [2:K]} \sum_{i_1 \in \mathcal{C}_1} \sum_{\mathcal{T}_1 \subseteq [K]: |\mathcal{T}_1|=t_1} (t_1-2) \end{aligned}$$

$$\begin{aligned} & \frac{|W_{i_1, \mathcal{T}_1}|}{B} + \frac{1}{2a+b} \sum_{t_2 \in [K]} \sum_{i_2 \in \mathcal{C}_2} \sum_{\mathcal{T}_2 \subseteq [K]: |\mathcal{T}_2|=t_2} (t_2-1) \frac{|W_{i_2, \mathcal{T}_2}|}{B} \end{aligned} \quad (71a)$$

$$\geq K - \frac{K}{2a+b} M. \quad (71b)$$

From (70) and (71b), we have

$$R \geq K - \frac{K}{2a+b} M, \quad (72)$$

which leads to  $R_u^* \geq K - \frac{K}{2a+b} M$ , coinciding with the achieved load for Theorem 1 when  $b(K-1) \geq 2a$ .

## IV. EXTENSION TO MULTIACCESS CODED CACHING SYSTEMS

An ideal coded edge caching model with a line multiaccess topology, referred to as multiaccess coded caching, was originally introduced in [16]. Different from our considered edge caching topology in Section II where each user is connected to the nearest cache node, in [16] each user is connected to  $L \in [K]$  cache nodes in a cyclic wrap-around fashion. Since [16], the multiaccess coded caching problem was widely considered, where different achievable and converse bounds were proposed in [31]–[36], while the optimality remains open

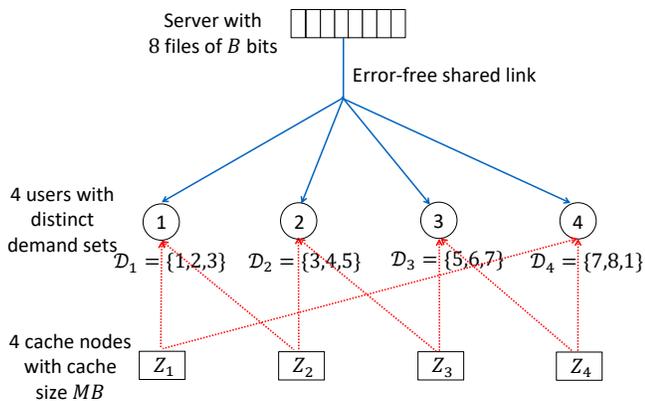


Fig. 4: The information theoretic model of the multiaccess coded caching problem for location-based content with  $K = 4$ ,  $N = 8$ ,  $a = b = 1$ ,  $L = 2$ .

(even for the uncoded cache placement). In the following, we consider the  $(K, a, b, L)$  multiaccess coded caching problem for location-based content, as illustrated in Fig. 4. The only difference from the considered system model in Section II is that, each user  $k \in [K]$  is connected to the  $L$  cache nodes in  $\{k, \langle k+1 \rangle_K, \dots, \langle k+L-1 \rangle_K\}$ , and can retrieve the cached content of its connected cache nodes without any cost. When  $L = 1$ , it reduces to the system model in Section II. Hence, in this section, we consider  $L \in [2 : K]$ . Note that, different from the multiaccess coded caching problem in [16] where each user may request any file in the library, in the considered problem the set of possible demanded files by user  $k$  is  $\mathcal{D}_k$ , where  $\mathcal{D}_k$  is defined in (1).

Our objective in the  $(K, a, b, L)$  multiaccess coded caching problem for location-based content is to design the cache placement and delivery phases, such that the worst-case load among all possible demands is minimized, where the optimal worst-case load is denoted by  $R^*$ .

We characterize the exact optimality for the considered problem in the following theorem.

**Theorem 3.** *For the  $(K, a, b, L)$  multiaccess coded caching problem for location-based content where  $L \in [2 : K]$ , we have*

$$R^* = \begin{cases} K - \frac{K}{a+b}M, & \text{if } 0 \leq M \leq a+b; \\ 0, & \text{if } M \geq a+b. \end{cases} \quad (73)$$

□

*Proof:*

*Achievability.* When  $M = 0$ , obviously we have  $R = K$ . In the following we will show that the memory-load tradeoff  $(M, R) = (a+b, 0)$  is achievable. By the memory sharing between  $(0, K)$  and  $(a+b, 0)$ , we can achieve  $R = K - \frac{K}{a+b}M$  when  $0 \leq M \leq a+b$ , which coincides with (73).

Let us focus on  $M = a+b$ . In the cache placement phase, each cache node  $k \in [K]$  caches  $W_n$  where  $n \in \mathcal{D}_{k,1} \cup \mathcal{D}_{k,2}$ . Since  $|\mathcal{D}_{k,1} \cup \mathcal{D}_{k,2}| = a+b$ , the memory size constraint is satisfied.

In the delivery phase, user  $k \in [K]$  requests  $W_{d_k}$  where  $d_k \in \mathcal{D}_k$ . By (1), we have  $\mathcal{D}_k = \mathcal{D}_{k,1} \cup \mathcal{D}_{k,2} \cup \mathcal{D}_{k,3}$  and  $\mathcal{D}_{k,3} = \mathcal{D}_{\langle k+1 \rangle_K, 1}$ . Hence,  $\mathcal{D}_k = \mathcal{D}_{k,1} \cup \mathcal{D}_{k,2} \cup \mathcal{D}_{\langle k+1 \rangle_K, 1}$ . If  $\mathcal{D}_k \in \mathcal{D}_{k,1} \cup \mathcal{D}_{k,2}$ , user  $k$  can retrieve  $W_{d_k}$  from cache node  $k$ ; otherwise, user  $k$  can retrieve  $W_{d_k}$  from cache node  $\langle k+1 \rangle_K$ . Hence, when  $M = a+b$ , we achieve  $R = 0$ .

*Converse.* Let us focus on the regime  $0 \leq M \leq a+b$ . For any achievable scheme with the memory-load tradeoff  $(M, R)$ , we consider a cut of all  $K$  cache nodes and  $K$  users. Recall that  $\mathcal{D}_k(i)$  denotes the  $i^{\text{th}}$  smallest element in  $\mathcal{D}_k$ . For each  $i \in [a+b]$ , we assume that  $X_i$  is transmitted by the server for the demand vector  $(\mathcal{D}_1(i), \mathcal{D}_2(i), \dots, \mathcal{D}_K(i))$ . Note that

$$\bigcup_{i \in [a+b]} \bigcup_{k \in [K]} \{\mathcal{D}_k(i)\} = \left( \bigcup_{i_1 \in [a]} \bigcup_{k_1 \in [K]} \{\mathcal{D}_{k_1}(i_1)\} \right) \cup \left( \bigcup_{i_2 \in [a+1:a+b]} \bigcup_{k_2 \in [K]} \{\mathcal{D}_{k_2}(i_2)\} \right) \quad (74a)$$

$$= \left( \bigcup_{k_1 \in [K]} \mathcal{D}_{k_1,1} \right) \cup \left( \bigcup_{k_1 \in [K]} \mathcal{D}_{k_1,2} \right) \quad (74b)$$

$$= \mathcal{C}_1 \cup \mathcal{C}_2 = [N], \quad (74c)$$

where  $\mathcal{C}_1$  and  $\mathcal{C}_2$  are defined in (33a) and (33b), respectively. Hence, from  $(Z_1, \dots, Z_K, X_1, \dots, X_{a+b})$ , we can decode  $W_n$  for each  $n \in [N]$ ; thus (recall that  $N := (a+b)K$ )

$$KMB + (a+b)RB \geq H(Z_1, \dots, Z_K, X_1, \dots, X_{a+b}) \quad (75a)$$

$$\geq H(W_1, \dots, W_N), \quad (75b)$$

$$\implies R \geq K - \frac{K}{a+b}M, \quad (75c)$$

$$\implies R^* \geq K - \frac{K}{a+b}M, \quad (75d)$$

which coincides with (73). ■

**Remark 3.** *From Theorem 3, it can be seen that when  $L \in [2 : K]$ , the optimal load for the considered multiaccess coded caching problem for location-based content does not depend on  $L$ . In other words, allowing the users to access more than just their two nearest local caches does not reduce the load of the common bottleneck link.* □

## V. CONCLUSIONS

This paper introduced a novel coded caching problem for location-based content in networks equipped with edge caching nodes. This is motivated, for example, by a vehicular network where self-driving vehicles need to access super High-Definition maps of the region through which they are driving. In the proposed model, each user is connected to the nearest cache node and requests a file in a subset of library depending on its location. Novel information theoretic converse bounds (with or without the constraint of uncoded cache placement) and achievable scheme were proposed, from which we can show the exact optimality on the worst-case load under uncoded cache placement and the general order optimality within a factor of 3. We also extended the coded caching problem for location-based content to the multiaccess coded caching topology, and characterized the exact optimality if each user is connected to at least two nearest cache nodes. On-going works include characterizing the exact optimality without the constraint of uncoded cache placement, considering the model where each region contains more than one users, and studying two-dimensional vehicular networks such as the Manhattan topology.

APPENDIX A  
PROOF OF THEOREM 2

We divide the proof into two cases,  $K$  is even and  $K$  is odd, respectively. For each case, we first propose a general cut-set bound on the optimal load for the considered problem, and then upper bound the multiplicative gap between  $R_u^*$  and this cut-set converse.

A.  $K$  is Even

For any achievable scheme with the memory-load tradeoff  $(M, R)$ , we consider a cut of  $\frac{K}{2}$  cache nodes with the indices in  $\mathcal{V} = \{1, 3, \dots, K-1\}$ , and their connected users. It can be seen that for any  $k_1 \neq k_2$  and  $k_1, k_2 \in \mathcal{V}$ , we have  $\mathcal{D}_{k_1} \cap \mathcal{D}_{k_2} = \emptyset$ . Denote the  $i^{\text{th}}$  smallest element in  $\mathcal{D}_k$  by  $\mathcal{D}_k(i)$ , where  $i \in [2a+b]$  and  $k \in [K/2]$ . For each  $i \in [2a+b]$ , we assume that  $X_i$  is transmitted by the server for the demand vector  $(\mathcal{D}_1(i), \mathcal{D}_3(i), \dots, \mathcal{D}_{K-1}(i))$ . From  $(Z_1, Z_3, \dots, Z_{K-1}, X_1, X_2, \dots, X_{2a+b})$ , we can decode  $W_n$  where  $n \in \cup_{k \in \mathcal{V}} \mathcal{D}_k$ ; thus by the cut-set bound,

$$\begin{aligned} & \frac{K}{2}MB + (2a+b)RB \\ & \geq H(Z_1, Z_3, \dots, Z_{K-1}, X_1, X_2, \dots, X_{2a+b}) \\ & \geq H((W_n : n \in \cup_{k \in \mathcal{V}} \mathcal{D}_k)), \\ & \implies R \geq \frac{K}{2} - \frac{K}{2(2a+b)}M, \\ & \implies R^* \geq \frac{K}{2} - \frac{K}{2(2a+b)}M. \end{aligned} \quad (76)$$

Let us then compare the converse bound in (76) with the achievable bound in Theorem 1.

First, we consider  $b(K-1) < 2a$  and  $0 \leq M \leq a+b$ , for which the achieved load is  $R_u^* = K - \frac{K+1}{2(a+b)}M$ . In this regime, the converse bound in (76) is the memory sharing between  $(0, K/2)$  and  $(a+b, \frac{K}{2} - \frac{K(a+b)}{2(2a+b)})$ , while the achievable bound is the memory sharing between  $(0, K)$  and  $(a+b, \frac{K-1}{2})$ . When  $M = 0$ , the multiplicative gap between the achievable bound and the converse bound is 2. When  $M = a+b$ , the multiplicative gap between the achievable bound and the converse bound is within a factor of 2; this is because

$$\begin{aligned} 2 \left( \frac{K}{2} - \frac{K(a+b)}{2(2a+b)} \right) - \frac{K-1}{2} &= \frac{aK}{2a+b} - \frac{K-1}{2} \quad (77a) \\ &= \frac{2a - b(K-1)}{2(2a+b)} > 0. \quad (77b) \end{aligned}$$

Hence, when  $b(K-1) < 2a$  and  $0 \leq M \leq a+b$ , we can prove  $2R^* \geq R_u^*$ .

Second, we consider  $b(K-1) < 2a$  and  $a+b \leq M \leq 2a+b$ , for which the achieved load is  $R_u^* = \frac{(K-1)(2a+b)}{2a} - \frac{K-1}{2a}M$ . In this regime, the converse bound in (76) is the memory sharing between  $(a+b, \frac{K}{2} - \frac{K(a+b)}{2(2a+b)})$  and  $(2a+b, 0)$ , while the achievable bound is the memory sharing between  $(a+b, \frac{K-1}{2})$  and  $(2a+b, 0)$ . It has been proved that when  $M = a+b$ , the multiplicative gap between the achievable bound and the converse bound is within a factor of 2. In addition, when  $M = 2a+b$ , the achievable bound coincides with the converse

bound. Hence, when  $b(K-1) < 2a$  and  $a+b \leq M \leq 2a+b$ , we can prove  $2R^* \geq R_u^*$ .

Third, we consider  $b(K-1) \geq 2a$ , for which the achieved load is  $R_u^* = K - \frac{K}{2a+b}M$ . When  $0 \leq M \leq 2a+b$ , the converse bound in (76) is the memory sharing between  $(0, K/2)$  and  $(2a+b, 0)$ , while the achievable bound is the memory sharing between  $(0, K)$  and  $(2a+b, 0)$ . Hence, in this case, we have  $2R^* \geq R_u^*$ .

In conclusion, when  $K$  is even, the proposed scheme for Theorem 1 is generally order optimal within a constant of 2.

B.  $K$  is Odd

In the following, we consider  $K$  is odd and  $K \geq 3$ .

For any achievable scheme with the memory-load tradeoff  $(M, R)$ , we consider a cut of  $\frac{K-1}{2}$  cache nodes with the indices in  $\mathcal{V} = \{1, 3, \dots, K-2\}$ , and their connected users. It can be seen that for any  $k_1 \neq k_2$  and  $k_1, k_2 \in \mathcal{V}$ , we have  $\mathcal{D}_{k_1} \cap \mathcal{D}_{k_2} = \emptyset$ . For each  $i \in [2a+b]$ , we assume that  $X_i$  is transmitted by the server for the demand vector  $(\mathcal{D}_1(i), \mathcal{D}_3(i), \dots, \mathcal{D}_{K-2}(i))$ . From  $(Z_1, Z_3, \dots, Z_{K-2}, X_1, X_2, \dots, X_{2a+b})$ , we can decode  $W_n$  where  $n \in \cup_{k \in \mathcal{V}} \mathcal{D}_k$ ; thus by the cut-set bound,

$$\begin{aligned} & \frac{K-1}{2}MB + (2a+b)RB \\ & \geq H(Z_1, Z_3, \dots, Z_{K-2}, X_1, X_2, \dots, X_{2a+b}) \\ & \geq H((W_n : n \in \cup_{k \in \mathcal{V}} \mathcal{D}_k)), \\ & \implies R \geq \frac{K-1}{2} - \frac{K-1}{2(2a+b)}M, \\ & \implies R^* \geq \frac{K-1}{2} - \frac{K-1}{2(2a+b)}M. \end{aligned} \quad (78)$$

We also compare the converse bound in (78) with the achievable bound in Theorem 1.

First, we consider  $b(K-1) < 2a$  and  $0 \leq M \leq a+b$ , for which the achieved load is  $R_u^* = K - \frac{K+1}{2(a+b)}M$ . In this regime, the converse bound in (78) is the memory sharing between  $(0, \frac{K-1}{2})$  and  $(a+b, \frac{K-1}{2} - \frac{(K-1)(a+b)}{2(2a+b)})$ , while the achievable bound is the memory sharing between  $(0, K)$  and  $(a+b, \frac{K-1}{2})$ . When  $M = 0$ , the multiplicative gap between the achievable bound and the converse bound is within a factor  $\frac{2K}{K-1} \leq 3$ . When  $M = a+b$ , the multiplicative gap between the achievable bound and the converse bound is within a factor of 3; this is because

$$\begin{aligned} & 3 \left( \frac{K-1}{2} - \frac{(K-1)(a+b)}{2(2a+b)} \right) - \frac{K-1}{2} \\ &= (K-1) \left( 1 - \frac{3(a+b)}{2(2a+b)} \right) = (K-1) \frac{a-b}{2(2a+b)} \\ &> 0, \end{aligned} \quad (79)$$

where (79) comes from  $b(K-1) < 2a$  and  $K \geq 3$ , which leads to  $a > b$ . Hence, when  $b(K-1) < 2a$  and  $0 \leq M \leq a+b$ , we can prove  $3R^* \geq R_u^*$ .

Second, we consider  $b(K-1) < 2a$  and  $a+b \leq M \leq 2a+b$ , for which the achieved load is  $R_u^* = \frac{(K-1)(2a+b)}{2a} - \frac{K-1}{2a}M$ . In this regime, the converse bound in (78) is the memory sharing between  $(a+b, \frac{K-1}{2} - \frac{(K-1)(a+b)}{2(2a+b)})$  and  $(2a+b, 0)$ , while the

achievable bound is the memory sharing between  $(a + b, \frac{K-1}{2})$  and  $(2a + b, 0)$ . It has been proved that when  $M = a + b$ , the multiplicative gap between the achievable bound and the converse bound is within a factor of 3. In addition, when  $M = 2a + b$ , the achievable bound coincides with the converse bound. Hence, when  $b(K-1) < 2a$  and  $a + b \leq M \leq 2a + b$ , we can prove  $3R^* \geq R_u^*$ .

Third, we consider  $b(K-1) \geq 2a$ , for which the achieved load is  $R_u^* = K - \frac{K}{2a+b}M$ . When  $0 \leq M \leq 2a + b$ , the converse bound in (78) is the memory sharing between  $(0, \frac{K-1}{2})$  and  $(2a + b, 0)$ , while the achievable bound is the memory sharing between  $(0, K)$  and  $(2a + b, 0)$ . It has been proved that when  $M = 0$ , the multiplicative gap between the achievable bound and the converse bound is within a factor of 3. In addition, when  $M = 2a + b$ , the achievable bound coincides with the converse bound. Hence, in this case, we have  $3R^* \geq R_u^*$ .

In conclusion, when  $K$  is even, the proposed scheme for Theorem 1 is generally order optimal within a constant of 3.

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