

NIH Public Access Author Manuscript

IEEE Trans Circuits Syst I Regul Pap. Author manuscript; available in PMC 2014 March 28

Published in final edited form as:

IEEE Trans Circuits Syst I Regul Pap. 2012 September ; 59(9): 2065–2074. doi:10.1109/TCSI. 2011.2180446.

The Circuit Theory Behind Coupled-Mode Magnetic Resonance-Based Wireless Power Transmission

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Abstract

Inductive coupling is a viable scheme to wirelessly energize devices with a wide range of power requirements from nanowatts in radio frequency identification tags to milliwatts in implantable microelectronic devices, watts in mobile electronics, and kilowatts in electric cars. Several analytical methods for estimating the power transfer efficiency (PTE) across inductive power transmission links have been devised based on circuit and electromagnetic theories by electrical engineers and physicists, respectively. However, a direct side-by-side comparison between these two approaches is lacking. Here, we have analyzed the PTE of a pair of capacitively loaded inductors via reflected load theory (RLT) and compared it with a method known as coupled-mode theory (CMT). We have also derived PTE equations for multiple capacitively loaded inductors based on both RLT and CMT. We have proven that both methods basically result in the same set of equations in steady state and either method can be applied for short- or midrange coupling conditions. We have verified the accuracy of both methods through measurements, and also analyzed the transient response of a pair of capacitively loaded inductors. Our analysis shows that the CMT is only applicable to coils with high quality factor (Q) and large coupling distance. It simplifies the analysis by reducing the order of the differential equations by half compared to the circuit theory.

Index Terms

Coupled-mode theory (CMT); near field; quality factor; power transfer efficiency (PTE); reflected load theory; resonance circuits; wireless power transmission

I. Introduction

Inductive power transmission links that utilize a pair of mutually coupled coils have been in use over the last decades to power up radio frequency identification (RFID) transponders and cochlear implants with power consumptions in the range of sub-micro to milliwatts [1], [2]. The use of this technique to wirelessly transfer energy across a short distance is, however, expected to see an explosive growth over the next decade in a much broader range of applications from advanced implantable microelectronic devices (IMD), such as retinal implants and brain–computer interfaces (BCI) to cut the power cord in charging mobile electronic devices, operating small home appliances, and energizing electric cars, which have higher power consumptions in the order of hundreds of milliwatts to kilowatts [3]–[12]. As a result, improving the wireless power transmission efficiency (PTE) in inductive

This paper was recommended by Associate Editor E. Alarcon

links particularly at larger coupling distances (coil separations) without increasing the volume and weight of the coupled coils has gained considerable attention [13]–[19].

Achieving high PTE is necessary in high-power IMDs to not only reduce the size of the external energy source (battery) that should be carried around by the patient but also to limit the tissue exposure to the AC magnetic field, which can result in excessive heat dissipation if it surpasses safe limits, and to minimize interference with near by electronics [20]–[22]. In near field RFID applications, the bottleneck in increasing the reading range without changing the coil size in many cases is the PTE when the received power is no longer sufficient to operate the transponder [2]. Increasing the PTE in higher power applications is also important for generating less heat, cost saving, reducing interference, and improving safety.

Design and optimization of inductive power transmission links has been extensively studied in the literature over the last three decades [2], [5], [23]–[34]. The majority of these approaches model and analyze the inductive link from a circuit perspective, which differs, at least on the surface, from the coupled-mode theory (CMT) that was recently presented in a new form by physicists at MIT [35]. They utilized the CMT approach to propose multi-coil inductive links, which can increase the PTE considerably at large coupling distances [13], [14]. Their 4-coil inductive link has so far been studied from a circuit perspective for power transmission to multiple small receivers, transcutaneous powering, and recharging mobile devices [17]–[19]. We have presented the analysis, modeling, design, and optimization of multi-coil inductive links using the circuit based reflected load theory (RLT) in [36]. However, an in-depth comparison between the coupled-mode and circuit-based theories, which would clarify the relationship between these two methods, often used by physicist and electrical engineers, respectively, is still lacking.

In this paper, first we review the inductive link steady-state analysis using CMT and then prove that both CMT and the more conventional RLT result in the same formulation for the inductive links' key performance measures, particularly the PTE. For the first time, we have derived the PTE equations for multi-coil inductive links via CMT. Our analysis shows that in the steady state mode, contrary to popular belief, both CMT and RLT are applicable to small and large coupling distances, d, as long as the coils interact in the near field regime. Because they are basically the same [37]! We have also presented the measurement results, which show the accuracy of both RLT and CMT models in estimating the PTE. On the other hand, our comparative transient analysis of a pair of capacitively loaded inductors reveals that CMT is accurate only if the mutual coupling, k, is small, e.g., due to large d, and coil quality factors, Q, are large, which limits the applicability of the CMT-based transient analysis to midrange coupling distances of large coils. We have also shown that utilizing CMT reduces the order of the differential equations by half compared to the circuit theory as it only considers a first order equation for each resonant object. Hence, the CMT seems to be helpful in analyzing the transient behavior of complicated multi-coil inductive links despite being less accurate than its circuit-based counterpart. For all other conditions, the RLT and similar circuit based methods will do just fine.

In the following section, the CMT analysis for a pair of capacitively loaded inductors is presented. Section III describes the RLT analysis for 2-coil inductive links. The correspondence between CMT and RLT has been presented in Section IV. The PTE analysis of the multi-coil inductive links utilizing both CMT and RLT are presented in Section V. The calculation and measurement results are compared in Section VI followed by concluding remarks in Section VII.

II. Coupled-Mode Theory

CMT is a framework to analyze energy exchange between two resonating objects [35]. Based on the CMT, the time-do-main field amplitudes of two objects, $a_2(t)$ and $a_3(t)$, which are defined so that the energy contained in them are and $|a_2(t)|^2$ and $|a_3(t)|^2$, respectively, at distance d_{23} can be found from [37]

$$\frac{da_2(t)}{dt} = -(j\omega_2 + \Gamma_2)a_2(t) + jK_{23}a_3(t) + F_S(t) \\ \frac{da_3(t)}{dt} = -(j\omega_3 + \Gamma_3 + \Gamma_L)a_3(t) + jK_{23}a_2(t)$$
(1)

Where ω_2 and ω_3 are the eigen frequencies, Γ_2 and Γ_3 are the resonance widths or rate of intrinsic decay due to the objects' absorption (Ohmic) and radiative losses, Γ_L is the resonance width due to load resistance connected to the second object (proportional to $1/R_L$), $F_S(t)$ is the excitation applied to the first object, and K_{23} is the coupling rate between the two objects. The CMT method has been recently applied to a pair of capacitively loaded conducting-wire loops, spaced by d_{23} , as shown in Fig. 1, forming a conventional inductive power transmission link, in which L_2 is the power transmitter and L_3 is the power receiver inductors, both tuned at the same frequency, $\omega_2 = \omega_3 = \omega$ [14], [37].

A. Steady-State Analysis via Coupled-Mode Theory

In steady state analysis, which applies to the conventional inductive power transmission links, $F_S(t)$ in (1) is a sinusoidal signal described as $A_s e^{-j\omega t}$. In this condition, the alternating field amplitude in the primary inductor is constant, $a_2(t) = A_2 e^{-j\omega t}$, resulting in a constant field amplitude in the secondary inductor, $a_3(t) = A_3 e^{-j\omega t}$. It can be shown from (1) that $A_3/A_2 = jK_{23}/(\Gamma_3 + \Gamma_L)$ [37]. Therefore, the average power at different nodes of the power transmission system can be calculated. The power absorbed by L_2 and the power delivered to L_3 are $P_2 = 2\Gamma_2|A_2|^2$ and $P_3 = 2\Gamma_3|A_3|^2$, respectively. The power delivered to R_L is $P_L = 2\Gamma_L|A_3|^2$ and, therefore, based on the energy conservation theory, if we neglect the radiated power in the near field regime, the total power delivered to the system from source is $P_S = P_2 + P_3 + P_L$. Hence, the PTE of the 2-coil system can be found from

$$\eta_{23} = \frac{P_L}{P_S} = \frac{1}{1 + \frac{\Gamma_3}{\Gamma_L} \left[1 + \frac{1}{fom^2} \left(1 + \frac{\Gamma_L}{\Gamma_3} \right)^2 \right]}$$
(2)

where $fom = K_{23}/\sqrt{\Gamma_2\Gamma_3}$ is the distance-dependent figure-of-merit for energy transmission systems [37]. To maximize the PTE based on (2), *fom* should be maximized and an optimal value should be chosen for Γ_L/Γ_3 . Parameters that affect *fom* are obviously the coupling rate between the two objects, K_{23} , which should be increased and the resonance width of each object (intrinsic loss of each inductor), Γ , which should be decreased. These are quite similar to the coupling coefficient, k_{23} , and inverse of the quality factor, 1/Q, used in conventional methods for optimizing inductive links [33]. The other key parameter, $\Gamma_{L,\text{PTE}}/\Gamma_3$, which shows the effect of R_L in optimizing the PTE, can be found by calculating the derivative of η_{23} in (2) with respect to Γ_L/Γ_3 , resulting in [13]

$$\Gamma_{L,\text{PTE}} = \frac{\Gamma_3}{\left(1 + K_{23}^2 / \Gamma_2 \Gamma_3\right)^{1/2}}.$$
 (3)

B. Transient Analysis via Coupled-Mode Theory

The CMT can also be utilized in analyzing the transient behavior of the resonant-coupled inductors in Fig. 1 by setting $F_S(t) = 0$ in (1) and considering an initial energy stored in L_2 . This analysis provides designers with better understanding of the dynamics of energy exchange in such inductively coupled systems. As the first step, we eliminate $a_3(t)$ in (1) to find an expression for the time varying field in the primary coil

$$\frac{d^2a_2(t)}{dt^2} + (2j\omega + \Gamma_2 + \Gamma_3 + \Gamma_L)\frac{da_2(t)}{dt} + [K_{23}^2 + (j\omega + \Gamma_2)(j\omega + \Gamma_3 + \Gamma_L)]a_2(t) = 0.$$
(4)

For the sake of simplicity, if similar to [14] we assume that the two inductors are identical, i.e., $\Gamma_2 = \Gamma_3 = \Gamma$, and $\Gamma_L = 0$ (no load condition, $R_L = \infty$), then $a_2(t)$ can be found from (4):

$$a_{2}(t) = e^{-j\omega t} \left[e^{-\Gamma t} \left(b_{1} e^{jK_{23}t} + b_{2} e^{-jK_{23}t} \right) \right]$$
(5)

Where b_1 and b_2 are constants, which values depend on the initial conditions. Similarly, $a_3(t)$ can be calculated using (1) and (5) as

$$a_{3}(t) = e^{-j\omega t} \left[e^{-\Gamma t} (d_{1}e^{jK_{23}t} + d_{2}e^{-jK_{23}t}) \right] \quad (6)$$

Where d_1 and d_2 are also constants with values dependent on the initial conditions. The total energy in L_2 and L_3 can be found from $E_2(t) = |a_2(t)|^2$ and $E_3(t) = |a_3(t)|^2$, respectively [24].

If one starts with 100% of the total energy normalized and initially stored in L_2 , i.e., $|a_2(t = 0)|^2 = 1$ and $|a_2(t = 0)|^2 = 0$, the energy stored in each object over time can be found from

$$E_{2}(t) = e^{-2\Gamma t} \cos^{2} K_{23} t$$

$$E_{3}(t) = e^{-2\Gamma t} \sin^{2} K_{23} t.$$
(7)

It should be noted that E_2 and E_2 are not the instantaneous but the peak values (or envelopes) of the energy content stored in L_2 and L_3 , which are in resonance with C_2 and C_3 , respectively, at the rate of 2ω .

III. Reflected Load Theory

Fig. 2(a) shows a pair of inductively coupled coils, which will be referred to as the primary (L_2) and secondary (L_3) . It can be shown that the highest PTE across such link scan be achieved when both LC-tanks are tuned at the same resonance frequency, i.e.,

 $\omega = 1/\sqrt{L_2C_2} = 1/\sqrt{L_3C_3}$ [31].

A. Steady State Analysis via Reflected Load Theory

The inductive link PTE is mainly dependent on the mutual coupling between the coils, k_{23} , and their quality factors, $Q_2 = \omega L_2/R_2$ and $Q_3 = \omega L_3/R_3$ [33]. At resonance frequency, the secondary loop which is connected to R_L can be reflected on to the primary loop and represented by R_{ref} [2]. To find R_{ref} , the secondary loop is modeled with a parallel load as shown in Fig. 2(a). R_3 , the parasitic resistance of L_3 , can be transformed to a parallel resistance equal to $R_{P3} = Q_3^2 R_3$ in parallel with R_L [30]. Hence, if we define, $R_P = R_{P3} \parallel R_L$ then we can reflect the secondary impedance onto the primary side [Fig. 2(b)]:

Where $Q_{3L} = R_P/\omega L_3$ is the loaded quality factor of the secondary coil [9], [30]. At resonance, C_{ref} resonates out with, $k_{23}^2 L_2$, leaving only the reflected resistance, R_{ref} , on the primary side, as shown in Fig. 2(c). In the simplified inductive link model of Fig. 2(c), L_2 and L_3 also resonate out, and the input power provided by V_s simply divides between R_2 and R_{ref} . The power absorbed by R_2 is dissipated as heat in the primary coil and the power delivered to R_{ref} , i.e., the transferred power to the secondary loop, divides between R_3 and R_L , which are the only power consuming components on the secondary side. This will lead to

$$\eta_{23} = \frac{R_{\text{ref}}}{R_2 + R_{\text{ref}}} \frac{R_{P_3}}{R_{P_3} + R_L} = \frac{k_{23}^2 Q_2 Q_{3L}}{1 + k_{23}^2 Q_2 Q_{3L}} \cdot \frac{Q_{3L}}{Q_L} \quad (9)$$

Where $Q_L = R_L/\omega L_3$ is often referred to as the load quality factor, and $Q_{3L} = Q_3Q_L/(Q_3+Q_L)$ [31]. It can be seen from (9) that large k_{23} , Q_2 and Q_3 are needed to maximize the PTE. However, for a given set of Q_2 , Q_3 and k_{23} values, there is an optimal load, $R_{L,PTE} = \omega L_3Q_{L,PTE}$, which can maximize the PTE. $Q_{L,PTE}$ can be found by calculating the derivative of (9) with respect to Q_L from

$$Q_{L,\text{PTE}} = \frac{Q_3}{\left(1 + k_{23}^2 Q_2 Q_3\right)^{1/2}}.$$
 (10)

B. Transient Analysis via Circuit Theory

In this analysis, the primary and secondary loops are considered identical, i.e., $L_2 = L_3 = L$, $C_2 = C_3 = C$, $R_2 = R_3 = R$ and $R_L = \infty$, similar to the CMT criteria in [14]. We have also set $V_s = 0$ V (short circuit) to focus on the transient response while considering an initial condition (current) in L_2 . Primary and secondary loop currents can be found from

$$\frac{1}{C} \int_{0}^{t} I_{2}(t) dt + RI_{2}(t) + L \frac{dI_{2}(t)}{dt} + M \frac{dI_{3}(t)}{dt} = 0$$

$$\frac{1}{C} \int_{0}^{t} I_{3}(t) dt + RI_{3}(t) + L \frac{dI_{3}(t)}{dt} + M \frac{dI_{2}(t)}{dt} = 0$$
(11)

Where $M = k_{23} \times L$ is the mutual inductance between L_2 and L_3 . Taking the derivative of (11) after substituting R/L and 1/LC with ω/Q and ω^2 , respectively, result in

$$\frac{d^{2}I_{2}(t)}{dt^{2}} + \frac{\omega}{Q}\frac{dI_{2}(t)}{dt} + \omega^{2}I_{2}(t) + k_{23}\frac{d^{2}I_{3}(t)}{dt^{2}} = 0$$

$$k_{23}\frac{d^{2}I_{2}(t)}{dt^{2}} + \frac{d^{2}I_{3}(t)}{dt^{2}} + \frac{\omega}{Q}\frac{dI_{3}(t)}{dt} + \omega^{2}I_{3}(t) = 0.$$
(12)

Utilizing Laplace transform, (12) can be written as

$$\left(s^{2} + \frac{\omega}{Q}s + \omega^{2}\right) I_{2}(s) + k_{23}s^{2}I_{3}(s) = 0$$

$$k_{23}s^{2}I_{2}(s) + \left(s^{2} + \frac{\omega}{Q}s + \omega^{2}\right) I_{2}(s) = 0.$$
⁽¹³⁾

Characteristic equation in the S-domain can be found by setting the coefficient matrix determinant in (13) to zero:

 $\left(s^2 + \frac{\omega}{Q}s + \omega^2\right)^2 - k_{23}^2 s^4 = 0.$ (14)

The roots of (14) are

$$s_{1,2} = \frac{-\omega'}{2Q'} \left(1 \pm \sqrt{1 - 4Q'^2} \right) = \frac{-\omega'}{2Q'} \pm j\omega'_d$$

$$s_{3,4} = \frac{-\omega''}{2Q''} \left(1 \pm \sqrt{1 - 4Q''^2} \right) = \frac{-\omega''}{2Q''} \pm j\omega''_d$$
(15)

where

Therefore, $I_2(t)$ and $I_3(t)$ can be calculated from

$$I_{2}(t) = e^{\frac{-\omega'}{2Q'}t} (a_{11}\cos\omega'_{d}t + b_{11}\sin\omega'_{d}t) + e^{\frac{-\omega''}{2Q''}} (a_{12}\cos\omega''_{d}t + b_{12}\sin\omega''_{d}t)$$
(17)
$$I_{3}(t) = e^{\frac{-\omega'}{2Q'}t} (a_{21}\cos\omega'_{d}t + b_{21}\sin\omega'_{d}t) + e^{\frac{-\omega''}{2Q''}} (a_{22}\cos\omega''_{d}t + b_{22}\sin\omega''_{d}t).$$
(18)

In order to find the unknown coefficients in (17) and (18), one should have at least four initial conditions. Two of them are $I_2(0) = I_0$, and $I_3(0) = 0$ A, which indicate that 100% of the total energy is initially stored in L_2 . The other two initial conditions can be found from (11) when t = 0:

$$\frac{dI_2}{dt}(0) = \frac{-\omega}{Q} \frac{I_0}{1 - k_{23}^2}, \frac{dI_3}{dt}(0) = \frac{-\omega}{Q} \frac{k_{23}I_0}{1 - k_{23}^2}.$$
 (19)

IV. Coupled Mode Versus Reflected Load Theories

In this section, we compare the formulation derived from CMT and RLT for the 2-coil inductive link PTE and transient response derived in the previous sections.

A. Two-Coil Inductive Link Power Transfer Efficiency

Resonance widths, $\Gamma_{2,3}$, and coupling rate, K_{23} , in a pair of capacitively loaded inductors in Fig. 1 are equivalent in terms of circuit model parameters in Fig. 2 to $\omega/2Q_{2-3}$ and $\omega k_{23}/2$, respectively [37]. Similarly, the load resonance width, Γ_L , is equal to $\omega/2Q_L$. By substituting these in *fom* and Γ_L/Γ_3

$$fom = \frac{K_{23}}{\sqrt{\Gamma_2 \Gamma_3}} = \frac{\omega k_{23}}{2} \left(\frac{\omega}{2Q_2} \frac{\omega}{2Q_3}\right)^{-1/2}$$
$$= k_{23} \sqrt{Q_2 Q_3} \qquad (20)$$
$$\frac{\Gamma_L}{\Gamma_3} = \frac{\omega}{2Q_L} / \frac{\omega}{2Q_3} = \frac{Q_3}{Q_L}.$$

In the next step, we substitute the CMT parameters from (20) in (2) and recalculate the PTE

$$\eta_{23} = \frac{1}{1 + \frac{Q_L}{Q_3} \left[1 + \frac{1}{k_{23}^2 Q_2 Q_3} \left(1 + \frac{Q_3}{Q_L} \right)^2 \right]} \\ = \frac{Q_3}{Q_3 + Q_L + \frac{Q_2}{k_{23}^2 Q_2 Q_3 Q_L}} \qquad (21)$$
$$= \frac{k_{23}^2 Q_2 Q_3}{Q_L + Q_3 + k_{23}^2 Q_2 Q_3 Q_L} \cdot \frac{Q_L Q_3}{Q_L + Q_3}.$$

After simplification and considering that $Q_{3L} = Q_L Q_3 / (Q_L + Q_3)$, the PTE formula in (21) can be further simplified to

$$\eta_{23} = \frac{k_{23}^2 Q_2 Q_3 Q_L / (Q_L + Q_3)}{1 + k_{23}^2 Q_2 Q_3 Q_L / (Q_L + Q_3)} \cdot \frac{Q_{3L}}{Q_L} = \frac{k_{23}^2 Q_2 Q_{3L}}{1 + k_{23}^2 Q_2 Q_{3L}} \cdot \frac{Q_{3L}}{Q_L}$$
(22)

which is the same as (9) that was derived via RLT.

Similarly, it is straightforward to show that the optimum $\Gamma_{L, PTE}$ in (3) can be linked to $Q_{L, PTE}$ in (10) by substituting the equivalent circuit parameters in (20), leading to

$$\frac{\Gamma_{L,\text{PTE}}}{\Gamma_3} = \frac{Q_3}{Q_{L,\text{PTE}}} = \left(1 + \frac{K_{23}^2}{\Gamma_2\Gamma_3}\right)^{1/2} = \left(1 + k_{23}^2 Q_2 Q_3\right)^{1/2}.$$
(23)

Thus, we have shown mathematically that the CMT and RLT equations for the PTE and optimal loading of 2-coil power transmission links in steady state are basically the same.

B. Two-Coil Inductive Link Transient Response

In order to arrive at the CMT transient response in (7), two assumptions were made. First, the coil quality factors were considered large ($Q \gg 1$), resulting in $\omega'_d \approx \omega'$ and $\omega''_d \approx \omega''$ in (16). Second, the coupling distance, d_{23} , was considered large, resulting in small k_{23} , which simplifies the rest of (16) to

$$\omega'/Q' \approx \omega''/Q'' \approx \omega/Q, \omega' \approx \omega(1+k_{23}/2), \omega'' \approx \omega(1-k_{23}/2) \quad (24)$$

and (19) to

$$\frac{dI_2}{dt}(0) = \frac{-\omega}{Q} I_0, \frac{dI_3}{dt}(0) = 0. \quad (25)$$

Thus, $I_2(t)$ and $I_3(t)$ in (17) and (18) can be approximated as

$$I_2(t) = e^{\frac{-\omega}{2Q}t} \left[a_{11}\cos\omega(1+k_{23}/2)t + b_{11}\sin\omega(1+k_{23}/2)t + a_{12}\cos\omega(1-k_{23}/2)t + b_{12}\sin\omega(1-k_{23}/2)t \right]$$
(26)

$$I_{3}(t) = e^{\frac{2\omega}{2Q}t} [a_{21}\cos\omega(1+k_{23}/2)t + b_{21}\sin\omega(1+k_{23}/2)t + a_{22}\cos\omega(1-k_{23}/2)t + b_{22}\sin\omega(1+k_{23}/2)t].$$
(27)

Unknown coefficients in (26) and (27) can be found by applying the initial conditions, which result in $a_{11} = a_{12} = I_0/2$, $b_{11} = b_{12} = -I_0/4Q$, $a_{21} = -a_{22} = -I_0/2$, and $b_{21} = b_{22} = 0$. When substituting these in (26) and (27)

$$I_2(t) = \frac{I_0}{2} e^{\frac{-\omega}{2Q}t} \left[\cos\omega(1+k_{23}/2)t + \cos\omega(1-k_{23}/2)t - \frac{1}{2Q}\sin\omega(1+k_{23}/2)t - \frac{1}{2Q}\sin\omega(1-k_{23}/2)t\right]$$
(28)

$$I_3(t) = \frac{I_0}{2} e^{\frac{-\omega}{2Q}t} \left[-\cos\omega(1 + k_{23}/2)t + \cos\omega(1 - k_{23}/2)t \right].$$
(29)

The large Q assumption combined with the expansion of the sinusoidal functions in (28) and (29)

$$\cos(\alpha \pm \beta) = \cos\alpha \cos\beta \mp \sin\alpha \sin\beta$$

$$\sin(\alpha \pm \beta) = \sin\alpha \cos\beta \pm \cos\alpha \sin\beta, \quad (30)$$

can further simplify the primary and secondary currents as

$$I_2(t) = I_0 e^{\frac{-\omega}{2Q}t} \cos \frac{\omega k_{23}t}{2} \cos \omega t$$

$$I_3(t) = I_0 e^{\frac{-\omega}{2Q}t} \sin \frac{\omega k_{23}t}{2} \sin \omega t.$$
(31)

Considering that the energy stored in an inductor, L, that carries a current, I, is $0.5LI^2$, the normalized envelope of the energy inside L_2 and L_3 can be expressed as [38]

$$E_{2}(t) = e^{\frac{-\omega}{Q}t} \cos^{2}\left(\frac{\omega k_{23}t}{2}\right),$$

$$E_{3}(t) = e^{\frac{-\omega}{Q}t} \sin^{2}\left(\frac{\omega k_{23}t}{2}\right).$$
(32)

By substituting $2\Gamma = \omega/Q$, and $K_{23} = \omega k_{23}/2$ in (32), we can arrive at (7), which was derived using CMT. Therefore, in calculating the transient response of the inductive links, CMT is only valid for midrange coupling distances (small k_{23}) between large coils (high-Q), as also indicated in [37].

In order to validate our theoretical calculations and demonstrate the level of accuracy (or lack there of) in the transient CMT analysis for various coupling coefficients and Q factors, the 2-coil inductive link in Fig. 2 was simulated in the LT-SPICE circuit simulator (Linear Technology, Milpitas, CA), while setting V_S and initial L_2 current, $I_2(0)$, at 0 V and 1 A, respectively, for the two conditions summarized in Table I. Fig. 3(a) shows the percentage of the energy stored in primary, L_2 , and secondary, L_3 , coils over time for a pair of large identical coils that are placed relatively far from each other (midrange, low k_{23} , high-Q). It can be seen that both CMT (7) and circuit-based (12) equations, solved in MATLAB (MathWorks, Natick, MA), match the LT-SPICE simulation results very well. In Fig. 3(b), however, which represents a small pair of coils that are very close to one another (short

range, high k_{23} , low-Q) the CMT-based formulas have become quite inaccurate in predicting the energy exchange, while the circuit analysis still matches the LT-SPICE simulation output. In this condition, the inductor currents tend to have two harmonics, ω' and ω'' in (17) and (18), one of which has not been predicted by the CMT. Nonetheless, the simplicity of the CMT in analyzing the transient behavior of inductive links can be useful particularly in midrange high-Q conditions. This stems from the fact that CMT models each resonant object (e.g., RLC tank) with a first-order differential equation, while in the circuit analysis the order of the equations increases by each independent energy storage element regardless of being an inductor or a capacitor.

V. Multi-Coil Inductive Power Transfer

CMT-based analysis took circuit designers by surprise when the group physicists at MIT demonstrated a method of achieving high PTE by utilizing multiple coils (3- and 4-coils) for wireless power transmission [13], [14]. Nonetheless, the closed-form CMT formulation presented in the literature was limited to 2-coils. In this section, we derive the closed-form PTE equations for multi-coil inductive links based on the CMT and compare them with parallel equations derived from RLT, particularly for 3- and 4-coil links [36]. We prove that both CMT and RLT result in the same set of equations.

Equations for a pair of capacitively loaded inductors in Fig. 1 can be extended to m inductors, in which the first and mth inductors are connected to the energy source and load, respectively, while all inductors are tuned at the same resonance frequency, f [14]. The time-domain field amplitudes of each inductor, $a_i(t)$, can be expressed as

$$\frac{da_i(t)}{dt} = -(j\omega + \Gamma_i)a_i(t) + jK_{i-1,i}a_{i-1}(t) + jK_{i,i+1}a_{i+1}(t), i=2,3,\dots,m-1 \quad (33)$$

where $K_{i,i+1}$ and Γ_i are the coupling rate between the *i*th and (i + 1)th inductor and resonance width of the *i*th inductor, respectively. For the sake of simplicity, the coupling between non-neighboring inductors has been considered negligible. For the first and *m*th inductors, the field amplitudes are

$$\frac{\frac{da_1(t)}{dt} = -(j\omega + \Gamma_1)a_1(t) + jK_{12}a_2(t) + F_s(t)}{\frac{da_m(t)}{dt} = -(j\omega + \Gamma_m + \Gamma_r)a_m(t) + jK_{m-1}ma_{m-1}(t).}$$
(34)

In the steady state mode, the field amplitudes in each inductor is considered constant, i.e., $a_i(t) = A_i e^{-j\omega t}$. Therefore, the differential equations in (33) and (34) result in a set of m - 1 equations

$$\Gamma_{i}A_{i} - jK_{i-1,i}A_{i-1} - jK_{i,i+1}A_{i+1} = 0, i = 2, 3, \dots, m-1$$

$$(\Gamma_{m} + \Gamma_{L})A_{m} - jK_{m-1,m}A_{m-1} = 0.$$
(35)

One can solve (35) to find A_i constants based on the load field amplitude, A_m . From these values, the average power at different nodes of the inductive power transmission link can be calculated. The absorbed power by the *i*th inductor and the delivered power to R_L can be expressed as $P_i = 2\Gamma_i |A_i|^2$ and $P_L = 2\Gamma_L |A_m|^2$, respectively, from which the total delivered power to the system from source can be found from $P_s = \sum_{i=1}^m P_i + P_L$, using the law of conservation of energy. Finally, the PTE of the *m*-coil system can be found from

$$\eta_{m-\text{coil}} = \frac{P_L}{P_S} = \frac{\Gamma_L}{\Gamma_m + \Gamma_L + \sum_{i=1}^{m-1} \Gamma_i \left| \frac{A_i}{A_m} \right|^2}.$$
 (36)

In an *m*-coil link, the reflected load from the (i + 1)th coil onto the *i*th coil can be found from

$$R_{\text{ref}i,i+1} = k_{i,i+1}^2 \omega L_i Q_{(i+1)L}, i=1,2,\dots,m-1 \quad (37)$$

where $k_{i,i+1}$ is the coupling coefficient between the *i*th and (i + 1)th coils. $Q_{(i+1)L}$ is the loaded quality factor of the (i + 1)th coil, which can be found from

$$Q_{iL} = \frac{\omega L_i}{R_i + R_{\text{ref},i+1}} = \frac{Q_i}{1 + k_{i,i+1}^2 Q_i Q_{(i+1)L}}, i = 1, 2, \dots, m-1 \quad (38)$$

where $Q_i = \omega L_i/R_i$ and R_i are the unloaded quality factor and parasitic series resistance of the *i*th coil (L_i), respectively. It should be noted that for the last coil, which is connected to the load in series, $Q_{mL} = \omega L_m/(R_m + R_L)$. Assuming that the coupling between non-neighboring coils is negligible, the partial PTE from the *i*th coil to (i + 1)th coil can be written as

$$\eta_{i,i+1} = \frac{R_{\text{ref}i,i+1}}{R_i + R_{\text{ref}i,i+1}} = \frac{k_{i,i+1}^2 Q_i Q_{(i+1)L}}{1 + k_{i,i+1}^2 Q_i Q_{(i+1)L}}.$$
 (39)

Using (37)–(39), the overall PTE in such a multi-coil inductive link can be found from

$$\eta_{m-\text{coil}} = \prod_{i=1}^{m-1} \eta_{i,i+1} \cdot \frac{Q_{mL}}{Q_L}.$$
 (40)

A. Three-Coil Power Transfer Inductive Links

The 3-coil inductive power transfer link, shown in Fig. 4(a), was initially proposed in [14] and analyzed based on the CMT. If we ignore K_{24} due to large separation between L_2 and L_4 , the field amplitudes at each inductor can be calculated by solving a set of two equations in (35), which leads to

$$\frac{A_2}{A_4} = \frac{K_{34}^2 + \Gamma_3 \left(\Gamma_4 + \Gamma_L\right)}{K_{23}K_{34}}, \frac{A_3}{A_4} = \frac{\left(\Gamma_4 + \Gamma_L\right)}{jK_{34}}.$$
 (41)

PTE of the 3-coil link can then be found by substituting (41) in (36), which leads to (42) after some minor simplifications, as shown at the bottom of the page.

$$\eta_{3-\text{coil}} = \frac{K_{23}^2 K_{34}^2 \Gamma_L}{\Gamma_2 \left[K_{34}^2 + \Gamma_3 \left(\Gamma_4 + \Gamma_L \right) \right]^2 + K_{23}^2 \left[\Gamma_3 \left(\Gamma_4 + \Gamma_L \right)^2 \right] + K_{23}^2 K_{34}^2 \left(\Gamma_4 + \Gamma_L \right)}$$
(42)

The lumped circuit model for 3-coil inductive link has been shown in Fig. 4(b). In this type of inductive links, k_{34} can be adjusted by changing the distance or geometry of L_4 to match the actual R_L with the optimal $R_{L,PTE}$ in (10) for the L_2-L_3 inductive link. It should be noted

that R_2 , which is the series resistance of L_2 , can also include the source output resistance [36]. The PTE of this circuit can be calculated by reflecting the resistive components of each loop from the load back towards the primary coil loop, one stage at a time, using (37) and calculating the percentage of the power that is delivered from one stage to the next, using (39), until it reaches R_L . According to (40), this procedure leads to

$$\eta_{3-\text{coil}} = \frac{\left(k_{23}^2 Q_2 Q_3\right) \left(k_{34}^2 Q_3 Q_{4L}\right)}{\left[\left(1+k_{23}^2 Q_2 Q_3+k_{34}^2 Q_3 Q_{4L}\right) \left(1+k_{34}^2 Q_3 Q_{4L}\right)\right]} \cdot \frac{Q_{4L}}{Q_L} = \eta_{23}\eta_{34} \quad (43)$$

where k_{24} has been ignored due to large separation between L_2 and L_4 , and

$$\eta_{23} = \frac{k_{23}^2 Q_2 Q_{3L}}{1 + k_{23}^2 Q_2 Q_{3L}} = \frac{k_{23}^2 Q_2 Q_3}{1 + k_{23}^2 Q_2 Q_3 + k_{34}^2 Q_3 Q_{4L}}$$

$$\eta_{34} = \frac{k_{34}^2 Q_3 Q_{4L}}{1 + k_{34}^2 Q_3 Q_{4L}} \cdot \frac{Q_{4L}}{Q_L}.$$
(44)

The resonance widths, Γ_{2-4} , and coupling rates, $K_{23,34}$, in CMT based on circuit parameters are defined as $\omega/2Q_{2-4}$ and $\omega k_{23,34}/2$, respectively [37]. By substituting these parameters in

(42) and multiplying both numerator and denominator with $Q_2 Q_3^2 Q_{4L}^2$, the 3-coil PTE can be found from

$$\eta_{3-\text{coil}} = \frac{k_{23}^2 k_{34}^2 Q_2 Q_3^2 Q_{4L}^2 / Q_L}{\left[\left(k_{34}^2 Q_3 Q_{4L} + 1 \right)^2 + k_{23}^2 Q_2 Q_3 + k_{23}^2 k_{34}^2 Q_2 Q_3^2 Q_{4L} \right]} = \frac{k_{23}^2 k_{34}^2 Q_2 Q_3^2 Q_{4L}}{\left[\left(1 + k_{34}^2 Q_3 Q_{4L} \right)^2 + k_{23}^2 Q_2 Q_3 \left(1 + k_{34}^2 Q_3 Q_{4L} \right) \right]} \cdot \frac{Q_{4L}}{Q_L} \quad (45)$$

where $1/Q_{4L} = 1/Q_4 + 1/Q_L$. It can be seen that (45), which is derived from the CMT is the same as (43), which is based on the RLT. Therefore, these two formulations are not different in the steady state analysis.

B. Four-Coil Power Transfer Inductive Links

Fig. 5(a) shows an inductive power transfer link consisting of four capacitively loaded inductors, in which L_2 and L_3 are the main coils responsible for power transmission, similar to the 2-coil link, while L_1 and L_4 are added for impedance matching [18], [36], [37]. The field amplitudes at each inductor can be found by solving a set of three equations in (35). A_2 and A_3 can be found based on A_4 from (41) and A_1 can be found from

$$A_1/A_4 = -\frac{\Gamma_2\Gamma_3\left(\Gamma_4 + \Gamma_L\right) + K_{34}^2\Gamma_2 + K_{23}^2\left(\Gamma_4 + \Gamma_L\right)}{jK_{12}K_{23}K_{34}}.$$
 (46)

To simplify the analysis, K_{13} , K_{14} and K_{24} have been neglected in comparison to coupling rates between neighboring coils. One can find the 4-coil PTE utilizing CMT by substituting (41) and (46) in (36), which after simplification leads to

$$\eta_{4-\text{coil}} = \frac{K_{12}^2 K_{23}^2 K_{34}^2 \Gamma_L}{D} \quad (47)$$

where

$$D = \Gamma_{1} \left[\Gamma_{2} \Gamma_{3} \left(\Gamma_{4} + \Gamma_{L} \right) + K_{34}^{2} \Gamma_{2} + K_{23}^{2} \left(\Gamma_{4} + \Gamma_{L} \right) \right]^{2} + K_{12}^{2} \Gamma_{2} \left[K_{34}^{2} + \Gamma_{3} \left(\Gamma_{4} + \Gamma_{L} \right) \right]^{2} + K_{12}^{2} K_{23}^{2} \Gamma_{3} \left(\Gamma_{4} + \Gamma_{L} \right)^{2} + K_{12}^{2} K_{23}^{2} K_{34}^{2} \left(\Gamma_{4} + \Gamma_{L} \right).$$

$$(48)$$

In the 4-coil lumped circuit schematic, shown in Fig. 5(b), the PTE can be calculated from (40) if we ignore k_{13} , k_{14} , and k_{24} in comparison to k_{12} , k_{23} and k_{34} [18], shown in (49) at the bottom of the page.

$$\eta_{4-\text{coil}} = \frac{\left(k_{12}^2 Q_1 Q_2\right) \left(k_{23}^2 Q_2 Q_3\right) \left(k_{34}^2 Q_3 Q_{4L}\right)}{\left[\left(1+k_{12}^2 Q_1 Q_2\right) \cdot \left(1+k_{34}^2 Q_3 Q_{4L}\right)+k_{23}^2 Q_2 Q_3\right] \cdot \left[1+k_{23}^2 Q_2 Q_3+k_{34}^2 Q_3 Q_{4L}\right]} \cdot \frac{Q_{4L}}{Q_L} \quad (49)$$

We can prove that CMT and RLT equations in (47) and (49) are basically the same by substituting the resonance widths, Γ_{1-4} , and coupling rates, K_{12} , K_{23} , and K_{34} in (47) and (48) with their equivalent circuit parameters, $\omega/2Q_{1-4}$, $\omega k_{12}/2$, $\omega k_{23}/2$, and $\omega k_{34}/2$, respectively [37]. Once both numerator and denominator of (47) are multiplied by

 $Q_1 Q_2^2 Q_3^2 Q_{4L}^2$, the CMT-based 4-coil PTE leads to (50), shown at the bottom of the next page, where A and B are

$$\eta_{4-\text{coil}} = \frac{k_{12}^2 k_{23}^2 k_{34}^2 Q_1 Q_2^2 Q_3^2 Q_{4L}^2 / Q_L}{A^2 + k_{12}^2 Q_1 Q_2 B^2 + k_{12}^2 Q_1 Q_2 k_{23}^2 Q_2 Q_3 + k_{12}^2 Q_1 Q_2 k_{23}^2 Q_2 Q_3 k_{34}^2 Q_3 Q_{4L}} \quad (50)$$

$$A = B + k_{23}^2 Q_2 Q_3, B = 1 + k_{34}^2 Q_3 Q_{4L}. \quad (51)$$

The third and fourth terms of the denominator in (50) can be written in terms of B as

$$\eta_{4-\text{coil}} = \frac{k_{12}^2 k_{23}^2 k_{34}^2 Q_1 Q_2^2 Q_3^2 Q_{4L}^2 / Q_L}{A^2 + k_{12}^2 Q_1 Q_2 B^2 + k_{12}^2 Q_1 Q_2 k_{33}^2 Q_2 Q_3 B} = \frac{k_{12}^2 k_{23}^2 k_{34}^2 Q_1 Q_2^2 Q_3^2 Q_{4L}^2 / Q_L}{A^2 + k_{12}^2 Q_1 Q_2 B (B + k_{23}^2 Q_2 Q_3)}.$$
(52)

By further manipulation of the numerator and denominator

$$\eta_{4-\text{coil}} = \frac{k_{12}^2 k_{23}^2 k_{34}^2 Q_1 Q_2^2 Q_3^2 Q_{4L}^2 / Q_L}{A^2 + k_{12}^2 Q_1 Q_2 B(A)} \\ = \frac{k_{12}^2 Q_1 Q_2 k_{23}^2 Q_2 Q_3 k_{34}^2 Q_3 Q_{4L}}{A(A + k_{12}^2 Q_1 Q_2 B)} \cdot \frac{Q_{4L}}{Q_L}$$
(53)
$$= \frac{(k_{12}^2 Q_1 Q_2) (k_{23}^2 Q_2 Q_3) (k_{34}^2 Q_3 Q_{4L})}{A[B(1 + k_{12}^2 Q_1 Q_2) + k_{23}^2 Q_2 Q_3]} \cdot \frac{Q_{4L}}{Q_L}$$

Once *A* and *B* are substituted from (51) in (53), it will be identical to (49), which was derived from the RLT.

VI. Modeling Versus Measurement Results

In order to verify the accuracy of the PTE equations derived from the RLT and CMT in measurements, we designed and fabricated three sets of inductive links with 2, 3, and 4 coils. L_2 and L_3 in all three links were identical printed spiral coils (PSCs), fabricated on 1.5-mm-thick FR4 printed circuit boards (PCB) with 1-oz copper (35.6 μ m thick). L_4 and L_1

in the 3- and 4-coil links were made of single filament solid copper wires, placed in the middle of L_3 and L_2 , respectively [see Fig. 6(b)]. Given and L_3 geometries, the identical diameter of L_4 and L_1 were optimized based on the procedure that we presented in [36] for the nominal coupling distance of $d_{23} = 30$ cm, $R_L = 100 \Omega$, and $f_0 = 13.56$ MHz. Table II shows specifications these three inductive links.

Fig. 6(a) shows the measurement setup that we used for the PTE, which is quite suitable for multi-coil inductive links [36]. In this method, resonance capacitors and R_L are added to the driver and load coils, and the entire circuit is considered a 2-port system including the multi-coil inductive link in between. A network analyzer is used to measure the S-parameters, from which the Z-parameters are derived [39]. The PTE can then be found from 2-port equations

$$\text{PTE} = \frac{|Z_{41}|^2}{R_L |Z_{11}| \cos(\angle Z_{11})} \quad (54)$$

where $Z_{11} = V_1/I_1$ and $Z_{41} = V_4/I_1$ are derived when $I_4 = 0$. The $I_4 = 0$ requirement in calculating the Z-parameters ensures that the network analyzer loading (often 50 Ω) on the inductive link does not affect the measurement results. Fig. 6(b) shows the 4-coil inductive link, which coils were held in parallel and perfectly aligned using sheets of Plexiglas. L_4 and L_1 and were placed in the middle of L_3 and L_2 and in a coplanar fashion, respectively, to achieve $k_{12} = k_{34} = 0.09$.

Fig. 7 compares the measured versus calculated values of the PTE versus coupling distance in the 2-, 3-, and 4-coil inductive links. Calculated PTE values are from the RLT and CMT models described in previous sections, while the circuit parameters, such as k and Q, are extracted from the models presented in [33] and [36] for PSCs and wire-wound coils, respectively. It can be seen that both RLT and CMT models, which are basically the same in steady state, estimate the PTE with a high level of accuracy.

It can be seen from Table II that the calculated PTEs of the 3- and 4-coil links at $d_{23} = 30$ cm are 31.4% and 31.3%, which are very close to the measured results, 29.7% and 27.9%, respectively. On the other hand, the measured 2-coil PTE at the same distance (1.37%) shows a greater difference on a percentage basis from the calculated value of 1.1%. This is most likely due to the measurement errors and equipment non-idealities at very low PTE levels. Because, as shown in Fig. 7, the calculated and measured PTEs for the same 2-coil link match very well at higher PTE values. The measured and calculated PTEs in 3- and 4-coil links show slightly more differences at smaller $d_{23} < 5$ cm. This is because for the sake of simplicity, we have neglected non-neighboring coils' coupling in our models in Section V, which are more significant when d_{23} is small. We would also like to point out that the 2-coil PTE at $d_{23} = 30$ cm is ~21 times smaller than that of 3- and 4-coil links because L_3 cannot provide Q_L values close to the $Q_{L,PTE}$ in (10) for $R_L = 100 \Omega$ in the 2-coil link. On the other hand, the 3- and 4-coil links can achieve the optimal $Q_{L,PTE}$ owing to their extra degree of freedom in impedance transformation, provided by k_{34} , which determines the optimal sizing of L_4 and L_1 [36].

VII. Conclusion

Electrical engineers analyze inductively coupled coils, transformers, and RLC tank circuits using lumped models, well known Kirchhoff's current/voltage laws, and a method often referred to as the RLT. Physicists, on the other hand, prefer to consider the energy stored in the system and exchanged between two or more resonating objects (in this case capacitively

loaded conductive-wire loops) and use a method known as the CMT. We have presented detailed formulation for calculating the PTE based on both theories for a conventional 2-coil inductive power transmission link, and extended the solutions in the steady state to 3-, 4-, and *m*-coil systems. In each case, we have proven that despite using different parameters and terminologies, both CMT and RLT lead to the exact same set of equations, and therefore, both are applicable to short- and midrange coil arrangements as long as near-field non-radiative conditions are applicable to the resonating circuits. Moreover, we have derived equations describing the transient behavior of the 2-coil inductive power transmission links using both the CMT and circuit theory. Theoretical analysis and simulation results show that in this case, CMT is only accurate when coils have small coupling and large quality factors. However, it simplifies the analysis by reducing the order of the differential equations by half, compared to the circuit-based approach. The measurement results show that the RLT and CMT equations are both sufficiently accurate for the PTE calculation.

Acknowledgments

The authors would like to thank members of the GT-Bionics lab for their comments and constructive discussions.

This work was supported in part by the National Institutes of Health, NIBIB, under Grant 1R21EB009437-01A1, and in part by the National Science Foundation under Award ECCS-824199.

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A pair of capacitively loaded inductors used for efficiency calculation using the coupledmode theory (CMT).



Fig. 2.

(a) Simplified model of a 2-coil power transmission link with a resistive load. (b) Equivalent circuit seen across the driver. (c) Reflected load onto the primary loop at resonance frequency.



Fig. 3.

Energy stored in the primary and secondary coils (E_2 and E_3) versus time starting from an initial condition, $I_2(0) = 1$ A, based on CMT (7), circuit theory (12), and SPICE simulations for 2-coil inductive links in: (a) midrange, high-Q, and (b) short range, low-Q conditions, as specified in Table I.



Fig. 4.

(a) Three capacitively loaded, mutually coupled, inductors for wireless power transmission in which L_4 and K_{34} serve as impedance matching elements to improve the PTE. (b) Lumped circuit model of the 3-coil inductive link.



Fig. 5.

(a) Four capacitively loaded, mutually coupled, inductors for wireless power transmission, in which $L_1 - K_{12}$ and $L_4 - K_{34}$ serve as impedance matching elements to improve the PTE. (b) Lumped circuit model of the 4-coil inductive link.



(b)

Fig. 6.

(a) PTE measurement setup using a network analyzer with all the coils tuned at the carrier frequency and R_L connected to the load coil. (b) Four-coil inductive link used to measure the PTE (specifications in Table II).





Comparison between measured and calculated (using RLT or CMT equations) values of the PTE versus the coupling distance, d_{23} , for 2-, 3-, and 4-coil inductive links specified in the Table II.

L ₂ and L ₃ Specifications [*]	Symbol	Midrange High-Q	Short Range Moderate-Q
Inductance (µH)	L	96.2	0.962
Series resistance (Ω)	R	0.76	1.08
Resonant capacitance (pF)	С	0.812	81.2
Quality factor	Q	1.4×10^{4}	100
Mutual coupling	k ₂₃	0.153×10 ⁻³	0.4
Resonance width (rad)	Г	4039	5.65×10 ⁵
Coupling rate (rad)	K ₂₃	8652	22.6×10^{6}
Load resistance (Ω)	R_L	∞ (No	load condition)
Resonance frequency (MHz)	f		18

 Table I

 Inductive Link Specifications for Transient Analyses

* Primary and secondary coils are identical.

Table II	Measurements
	er Efficiency
	ower Transf
	ications for F
	Link Specif
	Inductive

P	arameters		Symbols	2-Coil	3-Coil	4-Coil
	Induc	ctance (µH)	L_I/L_4		0.57	0.57
	Outer d	liameter (cm)	D_{ol} / D_{o4}	-	5.2	5.2
	Fi	ll factor	Φ_l / Φ_4	I	0.027	0.027
	Nur	n. of turns	n_I/n_4	ı	2	2
-1/ -14	Line	width (mm)	m_I/m_4	-	0.64	0.64
	Line s	pacing (µm)	s_I/s_4	-	100	100
	Qua	lity factor	$\mathcal{Q}_1/\mathcal{Q}_4$	I	183	183
	Resonar	ice width (rad)	Γ_l / Γ_4	-	2.3×10^{5}	2.3×10^{5}
	Induc	stance (µH)	L_2/L_3		0.9	
	Outer d	liameter (cm)	D_{o2}/D_{o3}		16.8	
	E	ll factor	Φ_2/Φ_3		0.13	
1/ 1	Nur	n. of turns	n_2/n_3		2	
L 2/ L3	Line	width (mm)	w2/w3		8.66	
	Line s	pacing (mm)	s_2 / s_3		2.6	
	Qua	lity factor	$\mathcal{Q}_2/\mathcal{Q}_3$		255	
	Resonar	ice width (rad)	Γ_2/Γ_3		$1.67{\times}10^{5}$	
L_2 and L_3 co	upling dista	nce (cm)	d_{23}		30	
$L_{2}-L_{3}$ n	nutual coupl	ing	k_{23}		6.6×10 ⁻³	
L_{2} - L_{3} co	oupling rate ((rad)	K_{23}		2.8×10^{5}	
$L_{I}-L_{2}/L_{3}$ -	L_4 mutual co	guiling	k_{12}/k_{34}		0.09	0.09
$L_{I}-L_{2}/L_{3}-L$	4 coupling r	ate (rad)	K_{12}/K_{34}	ı	3.8×10 ⁶	3.8×10 ⁶
Power transfer effic	iency (%)	Calculated	2	1.1	31.4	31.3
		Measured	"	1.37	29.7	27.9
Nom	inal load (Ω	()	R_L		100	
Power carri	ier frequency	/ (MHz)	f_0		13.56	

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