A New Stability Criterion for a Partial Element Equivalent Circuit Model of Neutral Type

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***Abstract*—This brief is concerned with stability for a partial element equivalent circuit model of neutral type. First, the relationship between two recently established integral inequalities is presented. Second, a new Lyapunov–Krasovskii functional is introduced based on the fact that the delay interval is nonuniformly divided into multiple subintervals, and different functionals are chosen on different subintervals. Then, some new delay-dependent criteria are derived. Finally, a numerical example is given to show that the results obtained by the new stability criteria can significantly improve some existing results.**

***Index Terms*—Delay decomposition approach, integral inequality, neutral systems, partial-element equivalent-circuit (PEEC) model, stability.**

I. Introduction

PARTIAL ELEMENT equivalent circuits (PEECs) are playing an increasingly important role in many practical applications, particularly in combined electromagnetic and circuit analysis (see [1] and [2] and the references therein). In this brief, we consider a small PEEC model for metal strip in Fig. 1(b) and borrow this model from Bellen *et al.* [3]. This small PEEC model represents a full-wave equivalent circuit for the small metal strip that is discretized into two cells, as shown in Fig. 1(a).

Considering retarded mutual coupling between partial inductances of the form $Lp_{ij}(d/dt)(i_{L_j}(t-\tau))$ and retarded dependent current sources of the form $p_{ij}/p_{ii}i_{cj}(t-\tau)$, we have the following differential equation [4] by applying Kirchhoff's voltage law and Kirchhoff's current law:

$$\begin{cases} -Av_n(t)-Ri_L(t)-Lp_{nr}\frac{di_L(t)}{dt}=v_s(t)+Lp_r\frac{di_L(t-\tau)}{dt} \\ C_s\frac{dv_n(t)}{dt}+P_{n,nr}G_lv_n(t)-P_{n,nr}A^Ti_L(t) \\ \quad =P_ni_s(t-\tau)-P_{n,r}G_lv_n(t-\tau)+P_{n,r}A^Ti_L(t-\tau) \end{cases} \tag{1}$$

where v_n denotes the potentials to infinity, i_L is the (inductive) current flowing in volume cells, v_s is the vector of voltages induced by the external electric field, i_s is the vector of lumped

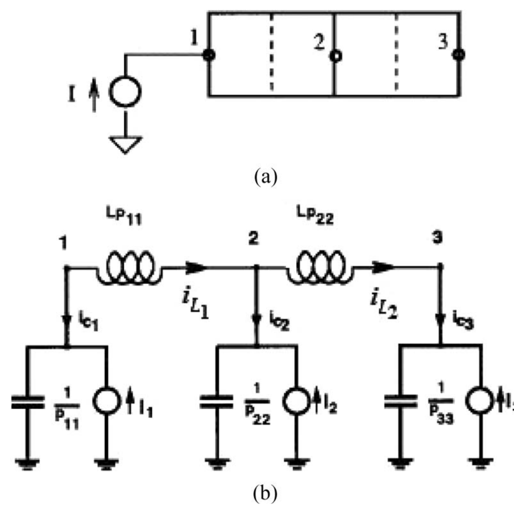

Fig. 1. (a) Metal strip with two L_p cells (three capacitive cells, dashed). (b) Small PEEC model for the metal strip [3].

current sources, A is the connectivity matrix, R is a diagonal matrix containing the resistances of volume cells, C_s is the diagonal matrix containing the pseudo self-capacitances, and G_l represents the matrix of lumped conductances. $Lp=Lp_{nr}+Lp_r$ is the partial inductance matrix, and $P_n=P_{n,nr}+P_{n,r}$ is the dense matrix containing the normalized coefficients of potentials, where the suffixes nr and r mean *not retarded* and *retarded*, respectively. Let

$$y(t)=\begin{bmatrix} v_n(t) \\ i_L(t)\end{bmatrix} \quad u(t,t-\tau)=\begin{bmatrix} v_s(t) \\ i_s(t-\tau)\end{bmatrix}.$$

Rewrite (1) as

$$C_0\dot{y}(t)+G_0y(t)+C_1\dot{y}(t-\tau)+G_1y(t-\tau)=Bu(t,t-\tau) \tag{2}$$

where C_0, C_1, G_0, G_1, and B are some matrices of appropriate dimensions, and the delay τ is a positive constant. The initial condition is defined as $y(t)=g(t)$, which is a continuously differential vector-valued function on the interval $[t_0-\tau,t_0]$, where t_0 is the starting time. The associated delay differential equation of neutral type is

$$\begin{cases} \dot{y}(t)=Ly(t)+My(t-\tau)+N\dot{y}(t-\tau), & t\ge t_0 \\ y(t)=g(t), & t\in[t_0-\tau,t_0] \end{cases} \tag{3}$$

where L, M, and N are the coefficient matrices of appropriate dimensions. In what follows, without loss of generality, we set $t_0=0$.

In the last decade, the delay-dependent stability for a neutral system, such as (3), has widely been studied by employing the Lyapunov–Krasovskii functional method. The main aim

Manuscript received March 2, 2009; revised June 14, 2009 and August 7, 2009. First published October 6, 2009; current version published October 16, 2009. This work was supported in part by the Australian Research Council Discovery Project entitled "Variable Structure Control Systems in Networked Environments" (DP0986376) and in part by Central Queensland University for the Research Advancement Awards Scheme Project entitled "Robust Fault Detection, Filtering and Control for Uncertain Systems with Time-Varying Delay" (Jan. 2006–Dec. 2008) and the Seed Grant Scheme Project entitled "H_∞ Control and Filtering for Networked Control Systems" (Jan. 2009–Dec. 2009). This paper was recommended by Associate Editor I. Belykh.

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Digital Object Identifier 10.1109/TCSII.2009.2030363

is to obtain a maximum admissible upper bound (MAUB) of the delay τ, which guarantees the considered system to be asymptotically stable. Therefore, the MAUB becomes an important performance index to measure the conservatism of the delay-dependent stability criterion. System (3) is investigated in [3] for its contractivity and asymptotic stability by utilizing a suitable reformulation of the system, where some delay-independent stability conditions were obtained. By transforming the system into a descriptor system, some delay-dependent stability conditions were derived in [5]. In [6], some delay-dependent and delay-independent criteria were established by using some bounding techniques for cross terms.

Define $\mathcal{C}$ as the set of continuous-valued functions on the interval $[-\tau, 0]$, and let $y_t \in \mathcal{C}$ be a segment of system trajectory defined as

$$y_t(\theta) = y(t+\theta), \qquad \theta \in [-\tau, 0]. \tag{4}$$

To avoid model transformations and bounding techniques for some cross terms, Han [7] chose the following Lyapunov–Krasovskii functional:

$$\tilde{V}(y_t) = y^T(t)Py(t) + \tau \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{y}^T(s)R\dot{y}(s)dsd\theta + \int_{t-\tau}^{t} y^T(s)Qy(s)ds + \int_{t-\tau}^{t} \dot{y}^T(s)S\dot{y}(s)ds \tag{5}$$

where $P > 0$, $R > 0$, $Q > 0$, and $S > 0$. However, the derived results in [7] have slight improvements over some existing results. *How to significantly improve some existing results motivates this brief.* It seems that, in using the Lyapunov–Krasovskii functional [see (5)], one *cannot* realize the purpose. To significantly improve the existing delay-dependent stability criteria, we should seek a new Lyapunov–Krasovskii functional. Recently, a delay decomposition approach [8], [9], [11] has been proposed to study the stability of linear time-delay systems. The idea of the approach [8], [9] is that the delay interval is uniformly divided into multiple segments, and proper functionals are chosen with different weighted matrices corresponding to different segments in the Lyapunov–Krasovskii functional. Based on the delay decomposition approach, some much less conservative delay-dependent stability criteria [8] for linear time-delay systems are derived. In this brief, we extend the approach to nonuniformly divide the delay interval into multiple segments and to choose different functionals with different weighted matrices for different segments. More specifically, we construct a new Lyapunov–Krasovskii functional as follows. Let q be a finite positive integer, and let τ_j $(j = 0, 1, 2, \ldots, q)$ be nonnegative integers satisfying

$$0 = \tau_0 < \tau_1 < \tau_2 < \cdots < \tau_q = \tau.$$

The delay interval $[-\tau, 0]$ can be expressed as the union of q subintervals, i.e.,

$$[-\tau, 0] = \bigcup_{j=1}^{q} [-\tau_j, -\tau_{j-1}].$$

Then, a new Lyapunov–Krasovskii functional is introduced as

$$\begin{aligned} V(y_t) = &\begin{bmatrix} y(t) \\ y(t-\tau) \end{bmatrix}^T P \begin{bmatrix} y(t) \\ y(t-\tau) \end{bmatrix} \\ &+ \sum_{j=1}^{q} (\tau_j - \tau_{j-1}) \int_{-\tau_j}^{-\tau_{j-1}} \int_{t+\theta}^{t} \dot{y}^T(s)R_j\dot{y}(s)dsd\theta \\ &+ \sum_{j=1}^{q} \int_{t-\tau_j}^{t-\tau_{j-1}} y^T(s)Q_jy(s)ds \\ &+ \int_{t-\tau}^{t} \dot{y}^T(s)S\dot{y}(s)ds \end{aligned} \tag{6}$$

where

$$P = \begin{bmatrix} P_1 & P_2 \\ P_2^T & P_3 \end{bmatrix} \geq 0, \qquad P_1 > 0$$

$R_j > 0$, $Q_j > 0$ $(j = 1, 2, \ldots, q)$, and $S > 0$.

From (6), one can see the following: 1) the first term augments both state $y(t)$ and delayed state $y(t-\tau)$, which is inspired by [10]; 2) compared with the second and third terms in $\tilde{V}(y_t)$, different Lyapunov matrices R_j and Q_j in $V(y_t)$ are chosen on different subintervals $[-\tau_j, -\tau_{j-1}]$; and 3) the last term in $V(y_t)$ is the same as the term in $\tilde{V}(y_t)$ to compensate the neutral term. In this brief, we will apply the new Lyapunov–Krasovskii functional to derive some new delay-dependent stability criteria for the PEEC model.

For simplicity, a symmetric term in a symmetric matrix is denoted by $\star$, e.g.,

$$\begin{bmatrix} X & Y \\ \star & Z \end{bmatrix} = \begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix}$$

and the other notations are routine ones.

II. Two Integral Inequalities and Their Relationship

In this section, we first present two recently established integral inequalities that are useful in deriving delay-dependent stability criteria. Then, we disclose the relationship between the two integral inequalities. For this purpose, we introduce an integral inequality that is slightly different from that in [13].

Lemma 1 [13]: Let $X = X^T > 0$ be a constant real $n \times n$ matrix, and suppose $\dot{x} : [-h, 0] \to \mathbb{R}^n$ with $h > 0$ such that the subsequent integration is well defined. Then, for any constant real $n \times n$ matrices W_1 and W_2, the following integral inequality holds:

$$-h \int_{t-h}^{t} \dot{x}^T(s)X\dot{x}(s)ds \leq \chi^T(t)\mathcal{L}_1\chi(t) \tag{7}$$

where, hereafter, $\chi(t) := [\, x^T(t) \quad x^T(t-h)\,]^T$, and

$$\mathcal{L}_1 := \begin{bmatrix} W_1^T + W_1 & -W_1^T + W_2 \\ \star & -W_2^T - W_2 \end{bmatrix} + \begin{bmatrix} W_1^T \\ W_2^T \end{bmatrix} X^{-1} \begin{bmatrix} W_1^T \\ W_2^T \end{bmatrix}^T. \tag{8}$$

In [7], Han derived the following integral inequality.

Lemma 2 [7]: Let $X = X^T > 0$ be a constant real $n \times n$ matrix, and suppose $\dot{x} : [-h, 0] \to \mathbb{R}^n$ with $h > 0$ such that the subsequent integration is well defined. Then, we have

$$-h \int_{t-h}^{t} \dot{x}^T(s) X \dot{x}(s) ds \leq \chi^T(t) \mathcal{L}_2 \chi(t) \tag{9}$$

where

$$\mathcal{L}_2 := \begin{bmatrix} -X & X \\ \star & -X \end{bmatrix}. \tag{10}$$

We now compare (7) and (9). Notice that if one sets $W_1 = -X$ and $W_2 = X$ in (7), then (7) becomes (9). Notice also that a significant difference between (7) and (9) is that two free matrices are introduced in (7), whereas no free matrix is involved in (9). Therefore, (7) provides a set of upper bounds for the integral term $-h \int_{t-h}^{t} \dot{x}^T(s) X \dot{x}(s) ds$ due to free matrices W_1 and W_2. Denote the set of upper bounds as

$$\mathcal{S} := \left\{ \chi^T(t) \mathcal{L}_1 \chi(t) | \forall\, W_1, W_2 \in \mathbb{R}^{n \times n} \right\}.$$

Then, letting $J := \chi^T(t) \mathcal{L}_1 \chi(t)$, we have

$$-h \int_{t-h}^{t} \dot{x}^T(s) X \dot{x}(s) ds \leq J \in \mathcal{S}. \tag{11}$$

From (9), one can see that it provides an upper bound $J_0 := \chi^T(t) \mathcal{L}_2 \chi(t)$ for the same integral term $-h \int_{t-h}^{t} \dot{x}^T(s) X \dot{x}(s) ds$. The natural question is what kind of relationship exists between J_0 and $\mathcal{S}$. The answer to the question is based on the following two facts.

Fact 1:

$$J_0 \in \mathcal{S}.$$

Proof: If we set $W_1 = -X$ and $W_2 = X$ in $\mathcal{L}_1$, then $\mathcal{L}_1$ reduces to $\mathcal{L}_2$. Thus, $J_0 \in \mathcal{S}$. ■

Fact 2:

$$J_0 = \min_{W_1, W_2 \in \mathbb{R}^{n \times n}} \mathcal{S}.$$

Proof: For the sake of convenience, introduce $W := [W_1 \quad W_2]$ and $E := [I \quad -I]$. Then, $\mathcal{L}_1$ and $\mathcal{L}_2$ can be rewritten as $\mathcal{L}_1 = W^T E + E^T W + W^T X^{-1} W$ and $\mathcal{L}_2 = -E^T X E$. After some simple algebraic manipulation, we have

$$\mathcal{L}_1 - \mathcal{L}_2 = (W^T + E^T X) X^{-1} (W + XE) \geq 0 \tag{12}$$

which means that $\mathcal{L}_2 \leq \mathcal{L}_1$. Consequently, $\chi^T(t) \mathcal{L}_2 \chi(t) \leq \chi^T(t) \mathcal{L}_1 \chi(t)$, from which one can see that the conclusion of Fact 2 is true. ■

From Facts 1 and 2, we conclude that, for the integral term $-h \int_{t-h}^{t} \dot{x}^T(s) X \dot{x}(s) ds$, its upper bound $J_0 := \chi^T(t) \mathcal{L}_2 \chi(t)$ provided by (9) is the least upper bound provided by (7). Thus, when introducing free matrices to derive a stability criterion, one cannot guarantee that the more free matrices, the less conservative stability conditions. In this brief, we will use (9) to investigate delay-dependent stability for the PEEC model [see (3)].

III. Stability Criteria

In this section, we employ the Lyapunov–Krasovskii functional [see (6)] to derive some new delay-dependent stability criteria for the PEEC model. To guarantee the existence and uniqueness of the solution of system [see (3)] [12], we need the following assumption.

Assumption 1: The spectrum radius of matrix N is less than 1, i.e., $\rho(N) < 1$.

Introduce a new vector as

$$\xi(t) = \left[\, y^T(t) y^T(t-\tau_1) \quad \cdots \quad y^T(t-\tau_q) \dot{y}^T(t-\tau) \,\right]^T.$$

Let e_i, $i \in \{1, \ldots, q+2\}$, be row vectors, with the ith block being an identity matrix and the others being zero blocks such that $y(t) = e_1 \xi(t)$, $y(t-\tau_1) = e_2 \xi(t)$, and so on. Then, we rewrite the first equation in (3) as

$$\dot{y}(t) = \Gamma \xi(t) \tag{13}$$

where

$$\Gamma := L e_1 + M e_{q+1} + N e_{q+2}. \tag{14}$$

We now state and establish a new delay-dependent stability criterion.

Preposition 1: Under Assumption 1, for a given scalar $\tau > 0$, the system (3) is asymptotically stable if there exist some real matrices

$$P = \begin{bmatrix} P_1 & P_2 \\ \star & P_3 \end{bmatrix} \geq 0, \qquad P_1 > 0$$

$S > 0$, and $R_j > 0$, $Q_j > 0$ $(j = 1, 2, \ldots, q)$ of appropriate dimensions such that

$$\Phi := \begin{bmatrix} \Xi & \Gamma^T S & (\tau_1 - \tau_0)\Gamma^T R_1 & \cdots & (\tau_q - \tau_{q-1})\Gamma^T R_q \\ \star & -S & 0 & \cdots & 0 \\ \star & \star & -R_1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \star & \star & \star & \cdots & -R_q \end{bmatrix} < 0 \tag{15}$$

where

$$\Xi := \begin{bmatrix} e_1 \\ e_{q+1} \end{bmatrix}^T P \begin{bmatrix} \Gamma \\ e_{q+2} \end{bmatrix} + \begin{bmatrix} \Gamma \\ e_{q+2} \end{bmatrix}^T P \begin{bmatrix} e_1 \\ e_{q+1} \end{bmatrix} + \sum_{j=1}^{q} \left[e_j^T Q_j e_j - e_{j+1}^T Q_j e_{j+1} \right] - e_{q+2}^T S e_{q+2} - \sum_{j=1}^{q} (e_{j+1} - e_j)^T R_j (e_{j+1} - e_j).$$

Proof: Taking the derivative of $V(y_t)$ in (6) with respect to t along the trajectory of the system in (13) yields

$$\dot{V}(y_t) = \xi^T(t) \Omega_1 \xi(t) - \sum_{j=1}^{q} (\tau_j - \tau_{j-1}) \int_{t-\tau_j}^{t-\tau_{j-1}} \dot{y}^T(s) R_j \dot{y}(s) ds \tag{16}$$

where

$$\Omega_1 := \begin{bmatrix} e_1 \\ e_{q+1} \end{bmatrix}^T P \begin{bmatrix} \Gamma \\ e_{q+2} \end{bmatrix} + \begin{bmatrix} \Gamma \\ e_{q+2} \end{bmatrix}^T P \begin{bmatrix} e_1 \\ e_{q+1} \end{bmatrix} + \sum_{j=1}^{q} \left[e_j^T Q_j e_j - e_{j+1}^T Q_j e_{j+1} + (\tau_j - \tau_{j-1})^2 \Gamma^T R_j \Gamma \right] + \Gamma^T S \Gamma - e_{q+2}^T S e_{q+2}.$$

Use the integral inequality in (9) to obtain

$$-(\tau_j - \tau_{j-1}) \int_{t-\tau_j}^{t-\tau_{j-1}} \dot{y}^T(s) R_j \dot{y}(s) ds \leq -\xi^T(t)(e_{j+1} - e_j)^T R_j (e_{j+1} - e_j)\xi(t). \quad (17)$$

Substituting (17) into (16) yields

$$\dot{V}(y_t) \leq \xi^T(t) \Omega \xi^T(t) \quad (18)$$

where

$$\Omega := \begin{bmatrix} e_1 \\ e_{q+1} \end{bmatrix}^T P \begin{bmatrix} \Gamma \\ e_{q+2} \end{bmatrix} + \begin{bmatrix} \Gamma \\ e_{q+2} \end{bmatrix}^T P \begin{bmatrix} e_1 \\ e_{q+1} \end{bmatrix} + \sum_{j=1}^{q} \left[e_j^T Q_j e_j - e_{j+1}^T Q_j e_{j+1} + (\tau_j - \tau_{j-1})^2 \Gamma^T R_j \Gamma \right] + \Gamma^T S \Gamma - e_{q+2}^T S e_{q+2} - \sum_{j=1}^{q} (e_{j+1} - e_j)^T R_j (e_{j+1} - e_j).$$

If (15) is feasible, then applying the Schur complement yields $\Omega < 0$. Then, there exists a scalar $\lambda > 0$ such that $\dot{V}(y_t) \leq \xi^T(t)\Omega\xi^T(t) \leq -\lambda\xi^T(t)\xi(t) \leq -\lambda y^T(t)y(t) < 0$ for $y(t) \neq 0$, from which we can conclude that the system in (3) is asymptotically stable. This completes the proof. ■

As the special case of delay decomposition on the delay interval $[-\tau, 0] = \bigcup_{j=1}^{q} [-\tau_j, -\tau_{j-1}]$, let $\tau_j - \tau_{j-1} = (\tau/q)(j = 1, 2, \ldots, q)$. In this case, we have the following corollary.

Corollary 1: Under Assumption 1, for a given scalar $\tau > 0$, the system in (3) is asymptotically stable if there exist some real matrices

$$P = \begin{bmatrix} P_1 & P_2 \\ \star & P_3 \end{bmatrix} \geq 0, \qquad P_1 > 0$$

$S > 0$, and $R_j > 0$, $Q_j > 0$ $(j = 1, 2, \ldots, q)$ of appropriate dimensions such that

$$\Upsilon := \begin{bmatrix} \Xi & \Gamma^T S & \frac{\tau}{q}\Gamma^T R_1 & \cdots & \frac{\tau}{q}\Gamma^T R_q \\ \star & -S & 0 & \cdots & 0 \\ \star & \star & -R_1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \star & \star & \star & \cdots & -R_q \end{bmatrix} < 0 \quad (19)$$

where Ξ is defined in Proposition 1.

As is well known, uncertainty inevitably exists in a practical system, as does a PEEC model [6]. In the remaining part of this section, we will extend Proposition 1 to formulate a robust stability criterion for a PEEC model with parameter uncertainty. For polytopic uncertainty, it is sufficient for (15) to be satisfied for all the vertices. Considering norm-bounded uncertainty, the PEEC model can be described by

$$\begin{cases} \dot{y}(t) = (L + \Delta L)y(t) + (M + \Delta M)y(t - \tau) \\ \qquad + (N + \Delta N)\dot{y}(t - \tau), \\ y(t) = g(t), \qquad t \in [-\tau, 0] \end{cases} \quad (20)$$

where uncertainty matrices ΔL, ΔM, and ΔN satisfy

$$[\Delta L, \ \Delta M, \ \Delta N] = DF[E_1, \ E_2, \ E_3] \quad (21)$$

where D, E_1, E_2, and E_3 are constant real matrices, and $F := F(t)$ is a time-varying and unknown real matrix with Lebesgue measurable elements satisfying $F^T F \leq I$.

Applying Lemma 1 in [7] and following the same proof process, we can derive a robust stability result for the uncertainty system in (20).

Preposition 2: The system described in (20) and (21) is robustly stable if there exist some real matrices

$$P = \begin{bmatrix} P_1 & P_2 \\ \star & P_3 \end{bmatrix} \geq 0, \qquad P_1 > 0$$

$S > 0$, $R_j > 0$, and $Q_j > 0$ $(j = 1, 2, \ldots, q)$ of appropriate dimensions and scalars $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ such that

$$\begin{bmatrix} -I + \varepsilon_1 E_3^T E_3 & N^T & 0 \\ \star & -I & D \\ \star & \star & -\varepsilon_1 I \end{bmatrix} < 0 \quad (22)$$

$$\begin{bmatrix} \Phi & \Theta_1 & \varepsilon_2 \Theta_2^T \\ \star & -\varepsilon_2 I & 0 \\ \star & \star & -\varepsilon_2 I \end{bmatrix} < 0 \quad (23)$$

where ϕ is defined in (15), and

$$\Theta_1 := \begin{bmatrix} \begin{bmatrix} e_1 \\ e_{q+1} \end{bmatrix}^T P \begin{bmatrix} D \\ 0 \end{bmatrix} \\ SD \\ (\tau_1 - \tau_0) R_1 D \\ \vdots \\ (\tau_q - \tau_{q-1}) R_q D \end{bmatrix} \quad (24)$$

$$\Theta_2 := [\, E_1 e_1 + E_2 e_{q+1} + E_3 e_{q+2} \quad 0 \quad 0 \quad \cdots \quad 0 \,]. \quad (25)$$

Similar to Corollary 1, we have the following result.

Corollary 2: The system described in (20) and (21) is robustly stable if there exist some real matrices

$$P = \begin{bmatrix} P_1 & P_2 \\ \star & P_3 \end{bmatrix} \geq 0, \qquad P_1 > 0$$

$S > 0$, $R_j > 0$, and $Q_j > 0$ $(j = 1, 2, \ldots, q)$ of appropriate dimensions and scalars $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ such that (22) and

$$\begin{bmatrix} \Upsilon & \tilde{\Theta}_1 & \varepsilon_2 \Theta_2^T \\ \star & -\varepsilon_2 I & 0 \\ \star & \star & -\varepsilon_2 I \end{bmatrix} < 0 \quad (26)$$

where Υ and Θ_2 are defined in (19) and (25), respectively, and

$$\tilde{\Theta}_1 := \left[\begin{pmatrix} D \\ 0 \end{pmatrix}^T P \begin{pmatrix} e_1 \\ e_{q+1} \end{pmatrix} \quad D^T S \quad \frac{\tau}{q} D^T R_1 \quad \cdots \quad \frac{\tau}{q} D^T R_q \right]^T.$$

TABLE I
ACHIEVED MAUBs $\tau_{\max}$ FOR DIFFERENT β FOR $\alpha = 0$

β	-2.105	-2.103	-2.1
Han [6]	1.0874	0.3709	0.2433
Yue and Han [5]	1.1413	0.3892	0.2553
Corollary 1 with $q = 2$	1.6086	0.5451	0.3557
Corollary 1 with $q = 3$	1.7053	0.5772	0.3764
Corollary 1 with $q = 4$	1.7399	0.5888	0.3839
Corollary 1 with $q = 5$	1.7560	0.5941	0.3873

TABLE II
ACHIEVED MAUBs $\tau_{\max}$ FOR DIFFERENT β WHEN $\alpha = 2$

β	-2.105	-2.103	-2.1
Han [6]	0.3651	0.2522	0.1906
Yue and Han [5]	0.4064	0.2783	0.2079
Corollary 2 with ($q = 2$)	0.5499	0.3835	0.2877
Corollary 2 with ($q = 3$)	0.5830	0.4061	0.3044
Corollary 2 with ($q = 4$)	0.5949	0.4142	0.3103
Corollary 2 with ($q = 5$)	0.6004	0.4180	0.3131

Remark 1: Propositions 1 and 2 and Corollaries 1 and 2 provide some new stability and robust stability criteria for small PEEC models (3) and (20), respectively. Clearly, these criteria depend on the value of q. The larger q is, the higher the computational complexity becomes. From the numerical results in the next section, one can see that taking $q = 2$ is the tradeoff between less conservative results and computational complexity.

Remark 2: It should be mentioned that the proposed method can be extended to large PEEC models with multiple time delays to formulate some delay-dependent stability criteria. Due to page limitation, this work is omitted.

IV. NUMERICAL EXAMPLE

In this section, we will show significant improvements over some existing results in the literature through a numerical example borrowed from [3] and [6]. Consider the system in (20) with

$$L = 100 \times \begin{bmatrix} \beta & 1 & 2 \\ 3 & -9 & 0 \\ 1 & 2 & -6 \end{bmatrix}$$
$$M = 100 \times \begin{bmatrix} 1 & 0 & -3 \\ -0.5 & -0.5 & -1 \\ -0.5 & -1.5 & 0 \end{bmatrix}$$
$$N = \frac{1}{72} \times \begin{bmatrix} -1 & 5 & 2 \\ 4 & 0 & 3 \\ -2 & 4 & 1 \end{bmatrix}$$
$$D = \alpha I \quad E_1 = I \quad E_2 = I \quad E_3 = 0$$

where $\beta < 0$, and $\alpha \geq 0$.

First, we consider the nominal system, which means that $\alpha = 0$. For $\beta = -7$, the MAUB $\tau_{\max}$ of the delay, which guarantees the asymptotic stability of the system, was given as 0.43 in [14]. It has been concluded in [3] that this system is asymptotically stable independent of the size of delay τ, but when β was increased to -4, the criteria fail to make any conclusion. For various β values, Han [6] and Yue and Han [5] calculated the MAUBs $\tau_{\max}$, which are listed in Table I. For comparison, we use Corollary 1 to calculate the MAUB, and the obtained results are also listed in the table. Clearly, from this table, we found that Corollary 1 in this brief can achieve much less conservative results than those in [5] and [6].

Next, we consider the uncertain system with $\alpha = 2$. Table II lists the MAUBs $\tau_{\max}$ using the criteria in [5] and [6] and Corollary 2 for different β values.

From Tables I and II, one can clearly see that, for this example, the delay-dependent criteria in this brief can provide much less conservative results than those in [3], [5], [6], and [14].

V. CONCLUSION

This brief has investigated the stability of a neural-type PEEC model. The relationship between two integral inequalities was initially discussed. Then, a new Lyapunov–Krasovskii functional has been introduced to derive much less conservative stability conditions. Finally, a numerical example developed from existing research and informed by the discussion in this brief has demonstrated that the new stability criteria can significantly improve some existing results previously outlined in the reference literature.

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