Design, analysis and experimental validation of a distributed platoon protocol in the presence of time-varying heterogeneous delays

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Abstract

This paper presents a novel control design framework for vehicle platooning along with its experimental validation. The problem of controlling the vehicles within a platoon, to converge to their desired velocity and inter-vehicle distances, is formulated as a networked, high-order consensus control problem. By resorting to Lyapunov-Razumikhin functions, convergency of the platoon to the consensus speed and spacing is proven under both fixed and switching communication network topologies, thus showing the capability of the proposed approach of coping with joining/leaving maneuvers and communication failures. The string stability of the platoon is studied under the proposed control law and conditions are provided for tuning the controller parameters to achieve both consensus and string stability. Finally, the proposed approach is validated in both numerical simulations and in-vehicle experiments in a three vehicle platoon demonstrating its capability of simultaneously accommodate consensus and string stability requirements.

I. Introduction

Autonomous driving is the next major step in road transportation technologies. This is clearly shown by the many demonstrators highlighted everyday in the media [1]–[3]. Automakers' efforts to rapidly deploy autonomous driving vehicles are propelled by society's demand for efficient and safer transportation as high-tech vehicles promise safer highways and roads, shorter traveling time, lower energy consumption and emission reduction. To achieve this final and ambitious goal, vehicles will integrate automation, information and communication technologies to gather, analyze and use information about the surrounding environment. In particular, a variety of on-board sensors will contribute to build and update online a map of the surrounding environment, while the vehicle itself will be able to wirelessly broadcast its driving behavior and intention [4], [5].

A significant contribution to safer and more efficient driving, definitely beyond human drivers' capabilities, is expected from the vehicles' capability of coordinating their driving tasks with other surrounding vehicles. In particular, by exploiting wireless communication, vehicles can exchange their driving intentions thus enabling *cooperative driving* to, e.g., coordinate the crossing of a traffic intersection to maximize the throughput while guaranteeing individual quality of services requirements (to, e.g., emergency vehicles or public transport), establish vehicle formations in highways with small inter-vehicle distances to reduce air-drag and save fuel (*vehicle platooning*).

The idea of *vehicle platooning* dates back to the eighties [6], when the California's Partners for Advanced Transit and Highways (PATH) program was established to study and develop vehicle-highway cooperation and communication systems [7]. The basic idea is to enable the communication and cooperation among neighboring vehicles traveling in a string, in order to safely reduce their mutual distance (more vehicles without increasing the road capacity) and suppress traffic shockwaves (thus saving fuel and reducing pollutant emissions). The core of such cooperative driving systems is a set of algorithms, deployed on the vehicles and controlling their longitudinal motion based on the behavior of the surrounding vehicles.

Low cost and reliable communication systems have recently renewed interest in cooperative vehicle-highway systems [8]. In the *2011 Grand Cooperative Driving Challenge* (GCDC) [8], a number of *heterogeneous* vehicles have cooperated in platoons in urban and highway driving scenarios. Evaluation criteria were based on the vehicles' ability of minimizing their distances from the preceding vehicle, while attenuating accelerations shockwaves. Designing a control algorithm for such realistic cooperative driving application presents several challenges including, among others, the heterogeneity of the vehicles behaviors (i.e., each vehicle runs its own control algorithm and its specific communication hardware). In this case, establishing global properties of the whole platoon can be difficult.

Platooning problems have been widely studied in control and important fundamental properties of the platoon, like string stability, have been thoroughly analyzed for both homogeneous [9] and heterogeneous [10] platoons. String stability is the capability of vehicles within a platoon of attenuating the propagation of motion perturbations toward the tail of the platoon.

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Motion perturbations of interest are spacing errors or accelerations. It is well known that string stability cannot be achieved when constant spacing policies are enabled, without establishing a communication link with the platoon leader [9]. Nevertheless, a speed dependent spacing policy, based on a *headway time*, leads to a string stable platoon for large enough headway time [11]. While well established design and analysis tools are available to study the string stability of vehicle platoons in a variety of scenarios, tools are lacking when impairments and limitations of the communication networks are considered. For example, under communication range limitations, it can be difficult to establish a reliable communication link with the leading vehicle. Moreover, the time varying delays affecting the information received via wireless communication may destroy the string stability properties achieved by standard design tools [12]. With this respect, reconfigurable communication network topologies may intuitively allow the recovery of string stability properties under failures of communication links.

In this paper, the problem of establishing platooning is formulated, and analytically solved, as a high-order consensus control problem. The presence of different time-varying delays introduced by the wireless vehicular communication network is considered and a control law is designed as result of two actions: a local action depending on the vehicle state variables and a *cooperative* action depending on information received from the neighboring vehicles (e.g., within the transmission range). The problem of controlling a vehicle platoon has been formulated as a networked consensus control problem also in [13], where a leaderless strategy is proposed for three autonomous vehicles ideally moving in a circle and sharing information across an all-to-all communication network topology affected by a constant and common delay. Platooning as a weighted and constrained consensus control problem is also discussed in [14] where the aim is understanding the influence of time-varying network topologies on the platooning dynamics by using a discrete-time Markov chain based approach (in absence of communication delay). In both cases consensus has been only numerically validated. The main contribution of this paper is to extend the approach initially presented in our paper providing that it guarantees convergence to the desired spacing policy while ensuring robustness with respect to time-varying topology due to, e.g., vehicles joining or leaving a platoon, or to the loss of communications links. The stability proof is based on the use of a quadratic Lyapunov-Razumikhin function and holds for both fixed and switched network communication topologies. Note that switching topologies in network control design are used to model and compensate the effect of packet losses, communications failures or automated vehicle maneuvers, like joining/leaving the platoon. Also, string stability is analyzed for the proposed control law and criteria for selecting tuning parameters leading to both convergency and string stability are proposed.

Most notably the effectiveness of the control approach is demonstrated here for the first time by performing its experimental validation by in-vehicle testing with a three-vehicle platoon. Experiments demonstrate the effectiveness of the approach in *creating*, *maintaining*, and *joining* platoon maneuvers and show how velocity and acceleration fluctuations are attenuated downstream the string of vehicles, thus fulfilling string stability requirements.

The paper is structured as follows. Section II introduces the notation and recall the mathematical background used in the rest of the paper. In Section III, the platooning control problem is stated and formulated as a consensus problem and a simple control law is proposed. Section IV derives the closed-loop model of the vehicular network that is studied in Section V, to derive the convergency conditions under fixed communication network topology. The convergency is then analyzed for a switching network topology in Section VI. String stability properties of the proposed control law are analyzed in Section VII. The whole design frameworks is validated and demonstrated in simulation and experiments in Sections VIII and IX, respectively, while concluding remarks in Section X close the manuscript.

II. Notation and mathematical preliminaries

Information exchange among agents can be modeled by a graph where every agent is regarded as a *node* and the communication links by *edges*. (Some basic notions on graph theory can be found in [15] and references therein).

A platoon of N vehicles can be described by a weighted directed graph (*digraph*) $\mathcal{G} = (\mathcal{V}_N, \mathcal{E}_N, \mathcal{A}_N)$ of order N characterized by a set of nodes $\mathcal{V}_N = \{1, \dots, N\}$ and a set of edges $\mathcal{E}_N \subseteq \mathcal{V}_N \times \mathcal{V}_N$. The topology of the graph is associated with a weighted adjacency matrix with nonnegative elements $\mathcal{A}_N = [a_{N,ij}]_{N\times N}$. In general, we assume $a_{N,ii} = 0$ (i.e., self-edges (i,i) are not allowed unless indicated otherwise). The presence of edge (i,j) in the edge set denotes that vehicle i can obtain information from vehicle j, but not necessarily *vice versa*.

The set of neighbors of node i is denoted as $\mathcal{N}_i = \{i \in \mathcal{V}_N : e_{ij} = (i,j) \in \mathcal{E}_N, j \neq i\}$, $\mathcal{E}_N \subseteq \mathcal{V}_N \times \mathcal{V}_N$. A sequence $1, 2, ..., l$ of distinct nodes is a *directed path* if $(i-1, i) \in \mathcal{E}_N$, $i = 2, \dots, l$. We say that j is *reachable* from i if there exists a path from node i to node j. A *cluster* is any subset $\mathcal{V}_N^s \subset \mathcal{V}_N$ of the nodes of the digraph. Defining the degree matrix as $D = \mathrm{diag}\{\overline{d}_1, \overline{d}_2, ..., \overline{d}_N\}$, with $\overline{d}_i = \sum\limits_{j\in\mathcal{N}_i} a_{N,ij}$, the Laplacian of the weighted directed graph $\mathcal{G}$ can be defined as $L = D - \mathcal{A}_N$.

Definition 1. *A graph $\mathcal{G}$ is balanced if and only if all of its nodes are balanced, $\sum_j a_{ij} = \sum_j a_{ji}$ $\forall i,j \in \mathcal{N}_i$ [16].*

In this paper, we consider N vehicles (agents) together with a leader vehicle considered as an additional agent labelled with the index zero i.e., node 0. We use an augmented weighted directed graph $\overline{\mathcal{G}}$ to model the network topology. We assume node 0 is *globally reachable* in $\overline{\mathcal{G}}$ if there is a path in $\overline{\mathcal{G}}$ from every node i in $\mathcal{G}$ to node 0 [17].

Moreover, we report here a result on the stability of delayed systems which will be useful in the rest of the paper.

Given a system of the form:

$$\begin{array}{c} \dot{x} = f(x_t), t > 0, \\ x(\theta) = \varphi(\theta),\ \theta \in [-r, 0], \end{array} \tag{1}$$

where $x_t(\theta) = x(t+\theta), \forall \theta \in [-r, 0]$ and $f(0) = 0$. Let $C([-r, 0], \mathbb{R}^n)$ be a Banach space of continuous functions defined on an interval $[-r, 0]$, taking values in $\mathbb{R}^n$ with a norm $||\varphi||_c = \max_{\theta \in [-r,0]} ||\varphi(\theta)||$, $||\cdot||$ being the Euclidean norm.

Theorem 1. *(Lyapunov-Razumikhin) [18]. Given system (1), suppose that the function $f : C([-r, 0], \mathbb{R}^n) \to \mathbb{R}^n$ maps bounded sets of $C([-r, 0], \mathbb{R}^n)$ into bounded sets of $\mathbb{R}^n$. Let ψ_1, ψ_2, and ψ_3 be continuous, nonnegative, nondecreasing functions with $\psi_1(s) > 0, \psi_2(s) > 0, \psi_3(s) > 0$ for $s > 0$ and $\psi_1(0) = \psi_2(0) = 0$. If there exists a continuous function $V(t, x)$ such that:*

$$\psi_1(||x||) \leq V(t, x) \leq \psi_2(||x||), t \in \mathbb{R}, x \in \mathbb{R}^n. \tag{2}$$

In addition there exists a continuous nondecreasing function $\psi_4(s)$ with $\psi_4(s) > s, s > 0$ such that :

$$\dot{V}(t, x) \leq -\psi_3(||x||) \text{ if } \quad V(t+\theta, x(t+\theta)) < \psi_4(V(t, x(t))), \theta \in [-r, 0], \tag{3}$$

then the solution $x = 0$ is uniformly asymptotically stable.

$V(t, x)$ is a Lyapunov-Razumikhin function if it satisfies conditions (2)-(3) in Theorem 2.
Finally, the following definition is given:

Definition 2. *A complex square matrix is said to be negative stable [positive stable] if its spectrum lies in the open left [right] half of the complex plane [19].*

III. Platooning as a consensus problem

In this paper we focus on the longitudinal control of a vehicle platoon. The platoon consists of a string of *N* vehicles, as sketched in Fig. 1, where the leading vehicle, w.l.o.g. assumed to be the first vehicle in the string, sets the reference speed for the whole platoon.
The objective is to regulate velocity and relative distance of each vehicle from its predecessor to the leader's speed and a desired distance, respectively [20], [21]. We assume that each vehicle within the platoon is equipped with onboard sensors measuring position, velocity, acceleration and relative position and velocity w.r.t. the preceding vehicle. Such set of measurements requires Inertial Measurements Units (IMU), Global Positioning Systems (GPS) and radars, which are commonly available on road vehicles. Each vehicle is also equipped with wireless V2V communication hardware to share information with its neighbors and receive reference signals.

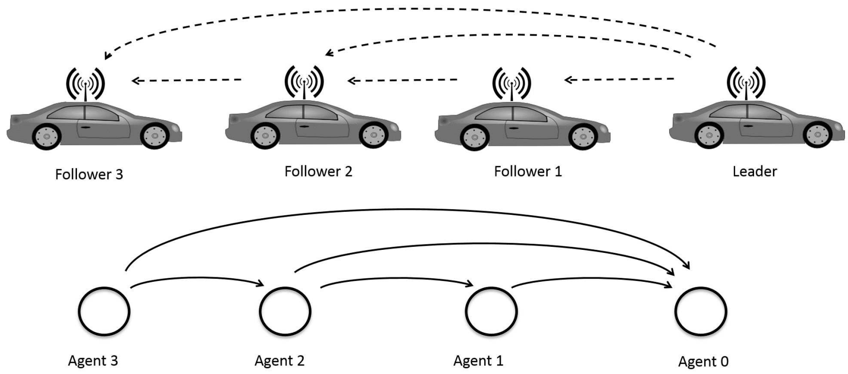


Fig. 1. A three vehicle platoon and the corresponding network graph. Note that the arrows in the upper sketch denote the information flow among vehicles and with the leader, while those in the associated network graphs indicate edges directed according to the definition given in Section II and used in the literature (e.g. [22], [23]).

A. Problem statement

Consider the platoon in Fig. 1. As in [24], the behavior of the generic i-th vehicle is mathematically described as the following inertial agent ($i = 1, \ldots, N$):

$$\begin{array}{l} \dot{r}_i(t) = v_i(t), \\ \dot{v}_i(t) = \frac{1}{M_i} u_i(t), \end{array} \tag{4}$$

where r_i [m] and v_i [m/s] are the i-th vehicle position and velocity, measured with respect to a road reference frame, M_i [kg] is the i-th vehicle mass assumed to be constant and the propelling force u_i denotes the control input to be appropriately chosen to achieve the desired position, maintain a desired speed and perform braking maneuvers. We assume that a lower level control exists on each vehicle delivering the demanded force u_i.
We assume a constant velocity for the leader so that the leader dynamics can be expressed as:

$$\begin{aligned} &\dot{r}_0(t) = v_0; \\ &\dot{v}_0 = 0. \end{aligned} \tag{5}$$

Given (4) - (5), the problem of maintaining a desired inter-vehicle spacing policy and a common velocity can be rewritten as a second order consensus problem, where the consensus positions and velocities are given by

$$\begin{aligned} &r_i(t) \to \frac{1}{d_i}\left\{\sum_{j=0}^{N} a_{ij}\cdot(r_j(t)+d_{ij})\right\} \\ &v_i(t) \to v_0. \end{aligned} \tag{6}$$

and d_{ij} is the desired distance between vehicles i and j; a_{ij} models the network topology emerging from the presence/absence of a communication link between vehicles i and j; $d_i = \sum_{j=0}^{N} a_{ij}$ is the degree of vehicle/agent i, i.e., the number of vehicles establishing a communication link with vehicle i. Note that a_{ij} are the nonnegative elements of the weighted adjacency matrix associated to the network directed graph, say $\overline{\mathcal{G}}$ (see section II for definitions and further mathematical details.) Furthermore, we assume $a_{0j} = 0$ ($\forall j = 0, \ldots, N$), since the leader does not receive data from any other vehicle.
According to [25] the desired spacing d_{ij} can be expressed as $d_{ij} = h_{ij}v_i + d_{ij}^{st}$, where h_{ij} is the *constant time headway* (i.e., the time necessary for vehicle $i-$th to travel the distance to its predecessor), and d_{ij}^{st} is the distance between vehicles i and j at standstill (see Appendix A for further details). By setting $h_{ij} = -h_{ji}$ as in [26], the consensus variables (6) can be easily rewritten in a more compact form as:

$$\begin{aligned} &r_i(t) \to r_0(t) + d_{i0} \\ &v_i(t) \to v_0. \end{aligned} \tag{7}$$

where d_{i0} is the desired distance of vehicle i from the leader.
The platooning consensus problem (7) is solved here by using the following decentralized coupling protocol embedding the spacing policy information as well as all the time-varying communication delays:

$$u_i = -b\left[v_i\left(t\right)-v_0\right] - \frac{1}{d_i}\sum_{j=0}^{N} k_{ij}a_{ij}\left[r_i\left(t\right)-r_j\left(t-\tau_{ij}\left(t\right)\right)-\tau_{ij}\left(t\right)v_0-h_{ij}v_i-d_{ij}^{st}\right], \tag{8}$$

where k_{ij} and b are control gains to be appropriately tuned to achieve the consensus positions and velocities (6); $\tau_{ij}(t)$ and $\tau_{i0}(t)$ are the unavoidable time-varying communication delays when information is transmitted to vehicle i from its neighbor j and from the leader, respectively. (Note that in general $\tau_{ij}(t) \neq \tau_{ji}(t)$.) Moreover, the delay $\tau_{ij}(t)$ can be assumed to be a bounded piecewise continuous function such that $0 \leq \tau_{ij}(t) \leq \tau$ [27]. Although the delay $\tau_{ij}(t)$ is unknown, it is assumed to be measurable. In particular, the communication delay over a link can be evaluated at a vehicle when information is received, since it is assumed that each vehicle transmits a timestamp $\bar{t}$ (i.e., the time instant when the information is sent) [28] [29]. Finally, note that, when a vehicle is equipped with onboard sensors (like, e.g., radars), the delay affecting the state measurements of the preceding vehicle is negligible, hence for that link $\tau_{ij}(t) = 0$. We remark that our approach allows the integration of both sensor-based and communication-based vehicle technologies [4]. Specifically, information on the preceding vehicles can be obtained both via sensors and/or via V2V wireless communication (different types of link with or without associated delays can be used to account for the different devices used to gather information). Note that, differently from [24], the control protocol proposed here allows a spacing policy dependent on the vehicle velocity and also standstill requirements [30]. Furthermore, different time-varying propagation and/or packet-losses delays are considered, thus relaxing the requirement made in [24] that at every timestamp each vehicles computes onboard a unique aggregate time delay deriving by fusing the delays coming from different sources. Finally we wish to emphasize that the previous version of the algorithm which was presented in [24] was not experimentally validated, but only numerical simulations were used to assess its performance.

IV. Closed-loop vehicular network

To prove consensus (7) for system (4)-(5) under the action of the coupling protocol (6)-(8), we define the following position and velocity errors with respect to the reference signals $r_0(t), v_0$ ($i = 1, \ldots, N$):

$$\begin{aligned} &\bar{r}_i = (r_i(t) - r_0(t) - h_{i0}v_i - d_{i0}^{st}), \\ &\bar{v}_i = (v_i(t) - v_0). \end{aligned} \tag{9}$$

Position and velocity errors can be more compactly recast as:
$\bar{r} = [\bar{r}_1, \ldots, \bar{r}_i \ldots, \bar{r}_N]^\top$, $\bar{v} = [\bar{v}_1, \ldots, \bar{v}_i \ldots, \bar{v}_N]^\top$.

To derive the expression of the closed-loop vehicular network, next we first rewrite the coupling protocol u_i in terms of the state errors $\bar{r}_i$ and $\bar{v}_i$ as defined in (9). By expressing both the headway constants h_{ij} and the standstill distances d_{ij}^{st} between vehicles i and j in terms of the leading vehicle ones, namely $h_{ij} = h_{i0} - h_{j0}$ and $d_{ij}^{st} = d_{i0}^{st} - d_{j0}^{st}$ (see Appendix A), the control action u_i in (8) can be rewritten as

$$u_i(t) = -\tfrac{1}{d_i}\sum_{j=1}^{N} k_{ij}a_{ij}\left[r_i(t) - r_0(t) - h_{i0}v_i - d_{i0}^{st}\right] - \tfrac{1}{d_i}\sum_{j=1}^{N} k_{ij}a_{ij}\left[-r_j(t-\tau_{ij}(t)) + r_0(t-\tau_{ij}(t)) + h_{j0}v_i + d_{j0}^{st}\right] + \\ -\tfrac{1}{d_i}k_{i0}a_{i0}\left[r_i(t) - r_0(t-\tau_{i0}(t)) - \tau_{i0}(t)v_0 - h_{i0}v_i - d_{i0}^{st}\right] - \tfrac{1}{d_i}\sum_{j=1}^{N} k_{ij}a_{ij}\left[r_0(t) - r_0(t-\tau_{ij}(t)) - \tau_{ij}(t)v_0\right] - b(v_i(t) - v_0). \tag{10}$$

Since $r_0(t) = r_0(t-\tau_{ij}(t)) + \tau_{ij}(t)v_0$ $(j = 0, \ldots, N)$, from (9) and (10), some algebraic manipulations lead to:

$$u_i(t) = -b\bar{v}_i - \frac{1}{d_i}\sum_{j=1}^{N} k_{ij}a_{ij}\left[\bar{r}_i(t) - \bar{r}_j(t-\tau_{ij}(t))\right] - \frac{1}{d_i}k_{i0}a_{i0}\bar{r}_i. \tag{11}$$

Hence, the closed-loop dynamics of the error variables under the control action (11) can be written for a generic i-th vehicle in the platoon $(i = 1, \ldots, N)$ as

$$\begin{cases} \dot{\bar{r}}_i &= \bar{v}_i, \\ M_i\dot{\bar{v}}_i &= -\tfrac{1}{d_i}(k_{i0}a_{i0} + \sum_{j=1}^{N} k_{ij}a_{ij})\bar{r}_i - b\bar{v}_i(t) + \tfrac{1}{d_i}\sum_{j=1}^{N} k_{ij}a_{ij}\left[\bar{r}_j(t-\tau_{ij}(t))\right]. \end{cases} \tag{12}$$

To recast the closed-loop network dynamics in presence of the time-varying delays associated to different links in a compact form, we now define the augmented error state vector as $\bar{x}(t) = \left[\begin{array}{cc} \bar{r}^\top(t) & \bar{v}^\top(t) \end{array}\right]^\top$ and define $\tau_p(t)$, $p = 1, 2, ..., m$, with $m \leq N(N-1)$ as an element of the sequence of time-delays $\{\tau_{ij}(t) : i, j = 1, 2, ..., N, i \neq j)\}$ $(0 \leq \tau_p(t) \leq \tau)$.

Remark 1. Note that m is the total number of different time delays and it is equal to its maximum, $N(N-1)$, if the network is represented by a directed complete graph and all time delays are different.

From (12), the dynamics of the closed loop vehicular network can be now written as:

$$\dot{\bar{x}}(t) = A_0\bar{x}(t) + \sum_{p=1}^{m} A_p\bar{x}(t-\tau_p(t)), \tag{13}$$

where

$$A_0 = \left[\begin{array}{cc} 0_{N\times N} & I_{N\times N} \\ -M\widetilde{K} & -M\widetilde{B} \end{array}\right] \quad \text{and} \quad A_p = \left[\begin{array}{cc} 0_{N\times N} & 0_{N\times N} \\ M\widetilde{K}_p & 0_{N\times N} \end{array}\right] \tag{14}$$

with

$$M = \operatorname{diag}\left\{\frac{1}{M_1}, \ldots, \frac{1}{M_N}\right\} \in \mathbb{R}^{N\times N}; \qquad \widetilde{B} = \operatorname{diag}\{b, \ldots, b\} \in \mathbb{R}^{N\times N}; \tag{15}$$

$$\widetilde{K} = \operatorname{diag}\left\{\tilde{k}_{11}, \ldots, \tilde{k}_{NN}\right\} \in \mathbb{R}^{N\times N}, \text{ with } \tilde{k}_{ii} = \frac{1}{d_i}\sum_{j=0}^{N} k_{ij}a_{ij}; \tag{16}$$

and $\tilde{K}_p = [\bar{k}_{pij}] \in \mathbb{R}^{N\times N}$ $(p = 1, \ldots, m)$ being the matrix defined according to the formalism adopted in [31] as:

$$\bar{k}_{pij} = \begin{cases} \frac{a_{ij}k_{ij}}{d_i}, & j \neq i, \tau_p(\cdot) = \tau_{ij}(\cdot), \\ 0, & j \neq i, \tau_p(\cdot) \neq \tau_{ij}(\cdot). \\ 0, & j = i. \end{cases} \tag{17}$$

Remark 2. Matrix $\widetilde{K}$ in (16) can be written as follows:

$$\widetilde{K} = K + \overline{K} \tag{18}$$

where

$$K = \operatorname{diag}\{k_1, \ldots, k_N\}, \quad \text{being} \quad k_i = \frac{k_{i0}a_{i0}}{d_i} \qquad (i = 1, \ldots, N), \tag{19}$$

and

$$\overline{K} = \operatorname{diag}\{\bar{l}_{11}, \ldots, \bar{l}_{NN}\} \tag{20}$$

$\overline{l}_{ii}$ being the diagonal elements of the normalized weighted Laplacian matrix $\overline{L}$ associated to the graph $\mathcal{G}$ defined as [22] :

$$\overline{l}_{ii} = \frac{1}{d_i} l_{ii} = \frac{1}{d_i} \sum_{j=1}^{N} k_{ij} a_{ij} \qquad (i = 1, \ldots, N), \tag{21}$$

with l_{ii} diagonal elements of the weighted Laplacian matrix L of $\mathcal{G}$ (see section II).

Remark 3. From the definition of $\widetilde{K}_p$ and $\overline{K}$ given in (17) and (20), respectively, the normalized weighed Laplacian $\overline{L}$ can also be expressed as:

$$\overline{L} = \left[\overline{l}_{ij}\right]_{N \times N} = \overline{K} - \sum_{p=1}^{m} \widetilde{K}_p \quad (i, j = 1, \ldots, N;\ j \neq 0). \tag{22}$$

V. Convergence analysis

Now, before solving the consensus problem in the presence of time-varying communication delays, we introduce a model transformation. From the Leibniz-Newton formula. It is known that [32]:

$$\bar{x}\left(t - \tau_p(t)\right) = \bar{x}\left(t\right) - \int_{-\tau_p(t)}^{0} \dot{\bar{x}}\left(t + s\right) ds. \tag{23}$$

Hence, substituting expression (13) in (23) we have:

$$\bar{x}\left(t - \tau_p(t)\right) = \bar{x}\left(t\right) - \sum_{q=0}^{m} A_q \int_{-\tau_p(t)}^{0} \bar{x}\left(t + s - \tau_q\left(t + s\right)\right) ds, \tag{24}$$

where matrices $A_0, A_1, \ldots, A_m$ are defined in (14) and $\tau_0\left(t + s\right) \equiv 0$.
Expressing the delayed state as in (24), the time-delayed model (13) can be transformed into:

$$\dot{\bar{x}}\left(t\right) = A_0 \bar{x}\left(t\right) + \sum_{p=1}^{m} A_p \bar{x}\left(t\right) - \sum_{p=1}^{m} \sum_{q=0}^{m} A_p A_q \int_{-\tau_p(t)}^{0} \bar{x}\left(t + s - \tau_q\left(t + s\right)\right) ds. \tag{25}$$

From (14), it follows that $A_p A_q = 0$ when $p = 1, \ldots m$ and $q = 1, \ldots, m$ ($q \neq 0$). Hence system (13) can be rewritten as:

$$\dot{\bar{x}}\left(t\right) = F \bar{x}\left(t\right) - \sum_{p=1}^{m} C_p \int_{-\tau_p(t)}^{0} \bar{x}\left(t + s\right) ds \tag{26}$$

where

$$C_p = A_p A_0 = \begin{bmatrix} 0_{N \times N} & 0_{N \times N} \\ 0_{N \times N} & M \widetilde{K}_p \end{bmatrix}, \tag{27}$$

and

$$F = A_0 + \sum_{p=1}^{m} A_p = \begin{bmatrix} 0_{N \times N} & I_{N \times N} \\ -M \widehat{K} & -M \widetilde{B} \end{bmatrix}, \tag{28}$$

with

$$\widehat{K} = -\sum_{p=1}^{m} \widetilde{K}_p + \widetilde{K}. \tag{29}$$

Before giving the proof of convergence we introduces some preliminary Lemmas.

Lemma 1. *Supposing $k_i \geq 0$ in (19) $(i = 1, \ldots, N)$, the matrix $\widehat{K}$ in (29) is positive stable if and only if node 0 is globally reachable in $\overline{\mathcal{G}}$.*

Remark 4. Notice that according to Lemma 1 the following matrix

$$\widehat{K}_M = M \widehat{K} \tag{30}$$

is also positive stable if $M > 0$ (15).

Lemma 2. *Let F be the matrix defined in (28). F is Hurwitz stable if and only if $\widehat{K}_M$ defined as in (30) is positive stable and*

$$b > \max_i \left\{ \frac{|Im(\mu_i)|}{\sqrt{Re(\mu_i)}} M_i \right\} \tag{31}$$

where μ_i is the i-th eigenvalue of $\widehat{K}_M$ $(i = 1, \ldots, N)$.

Lemmas 1 and 2 can be easily proved extending the proof in [24] to the more general case considered here where the closed loop matrices depend on $m \leq N(N-1)$ heterogeneous time-varying delays.

Once the problem has been recast as in Sec.IV, the same framework used in [24] can be used to prove convergence. We point out once more that, different from [24], the approach proposed in this paper does not require any aggregation of delays from different sources for each node.

Theorem 2. *Consider system (13) with the control parameters in (8) chosen as $k_{ij} > 0$ and b such that*

$$b > b^\star = \max_i \left\{ \frac{|Im(\mu_i)|}{\sqrt{Re(\mu_i)}} M_i \right\} \tag{32}$$

where $\widehat{K}_M$ is defined in (30). Then, there exists a constant $\tau^\star > 0$ such that, when $0 \leq \tau_p(t) \leq \tau < \tau^\star$ $(p = 1, \ldots, m)$,

$$\lim_{t\to\infty} \overline{x}(t) = 0, \tag{33}$$

if and only if node 0 is globally reachable in $\overline{\mathcal{G}}$.

Proof. (*Sufficiency*). Since node 0 is globally reachable in $\overline{\mathcal{G}}$, from Lemma 1 it follows that the matrix $\widehat{K}_M$ is positive stable. Setting b as in (32), the hypothesis of Lemma 2 is satisfied, hence the matrix F defined in (28) is Hurwitz stable and from Lyapunov theorem there exists a positive definite matrix $P \in \mathbb{R}^{2N\times 2N}$ such that

$$PF + F^\top P = -Q; \quad Q = Q^\top > 0. \tag{34}$$

Consider the following Lyapunov-Razumikhin candidate function (e.g. satisfying condition (2) of Lyapunov-Razumikin Theorem 1 in Section II)

$$V(\overline{x}) = \overline{x}^\top P \overline{x}. \tag{35}$$

From equation (26), differentiating V along (13) gives

$$\dot{V}(\overline{x}) = \overline{x}^\top (PF + F^\top P)\overline{x} - \sum_{p=1}^{m} 2\overline{x}^\top PC_p \int_{-\tau_p(t)}^{0} \overline{x}(t+s)ds. \tag{36}$$

Now for any positive definite matrix Ξ it is possible to show that $2a^\top c \leq a^\top \Xi a + c^\top \Xi^{-1} c$ according to [17]. Therefore, setting $a^\top = -\overline{x}^\top PC_p$, $c = \overline{x}(t+s)$, $\Xi = P^{-1}$, and integrating both sides of the inequality, we can write

$$\dot{V}(\overline{x}) \leq \overline{x}^\top (PF + F^\top P)\overline{x} + \sum_{p=1}^{m} [\tau_p(t)\overline{x}^\top PC_p P^{-1} C_P^\top P\overline{x} + \int_{-\tau_p(t)}^{0} \overline{x}^\top(t+s)P\overline{x}(t+s)ds]. \tag{37}$$

Let

$$\tau < \tau^\star = \frac{\lambda_{\min}(Q)}{\lambda_{\max}(H)}; \tag{38}$$

where $\lambda_{\min}(Q)$ is the minimun eigenvalue of Q; $\lambda_{\max}(H)$ the maximum eigenvalue of the matrix H defined as $H = \sum_{p=1}^{m} PC_p P^{-1} C_P^\top P + qP$, and, according to the hypotheses of the Lyapunov-Razumikin Theorem (section II), choose and consider the following continuing non decreasing function $\psi_4(s) = qs$ (for some constant $q > 1$) and the continuous, nonnegative, nondecreasing function $\psi_3(s) = (\lambda_{\min}(Q) - \tau\lambda_{\max}(H))s^2$.
Simple algebraic manipulations yield that when

$$V(\overline{x}(t+\theta)) < \psi_4(V(\overline{x})) = qV(\overline{x}(t)), \ -\tau \leq \theta \leq 0, \tag{39}$$

from (37) it follows

$$\dot{V}(\overline{x}) \leq -(\lambda_{\min}(Q) - \tau\lambda_{\max}(H))||\overline{x}||^2 = -\psi_3(||\overline{x}||). \tag{40}$$

Thus, the sufficient condition is proven.

(*Necessity*). System (13) is asymptotically stable for any time delay $\tau_p(t) \leq \tau < \tau^\star(p = 1, \ldots, m)$. Letting $\tau_p(t) \equiv 0$ $(p = 1, \ldots, m)$ in (13), from (26) it follows that system $\dot{\overline{x}} = F\overline{x}$, with F defined in (28), is asymptotically stable. As all the eigenvalues of F have negative real parts, Lemma 2 implies that $(M\widehat{K})$ is positive stable. Now, applying Lemma 1, the theorem is proven. ∎

VI. PLATOONING IN THE PRESENCE OF A SWITCHED TOPOLOGY

In this section we analytically investigate the ability of the proposed control law (8) to cope with loss and recovery of communication links. Communication failures and recovery are modeled here by letting the communication network switching between different topologies. Since V2V communication links repeatedly form and break, thus inducing changes in the vehicular network structure [33]. Switching network topologies well describe the highly dynamical nature of platoon communication links [14].

To model switching structures we introduce a switching signal $\sigma(t):[0,\infty)\to\phi_\Gamma=\{1,2,\dots,G\}$ that determines the coupling topology we denote $\Gamma=\{\overline{\mathcal{G}}_1,\overline{\mathcal{G}}_2,\dots,\overline{\mathcal{G}}_G\}$ as a finite collection of graphs with a common node set $\overline{\mathcal{V}}$ describing all the possible topologies that can be obtained by varying the communication links [34]. (G denotes the total number of all possible digraphs.) $\sigma(t)$ determines the index of the active graph at time instant t that we assume piecewise constant and continuous from the right. Moreover, we assume that two consecutive switching instants are separated by some finite dwell-time. This guarantees that the switching frequency remains bounded so that Zeno behavior cannot occur [35].

Taking into account the switched interconnecting graph, the closed-loop vehicular network (13) can be expressed as the following switched delayed system [23]:

$$\dot{\bar{x}}(t)=A_{0,\sigma}\bar{x}(t)+\sum_{p=1}^{m}A_{p,\sigma}\bar{x}(t-\tau_p(t)), \tag{41}$$

where

$$A_{0,\sigma}=\begin{bmatrix}0_{N\times N} & I_{N\times N}\\ -M\widetilde{K}_\sigma & -M\widetilde{B}\end{bmatrix}\quad\text{and}\quad A_{p,\sigma}=\begin{bmatrix}0_{N\times N} & 0_{N\times N}\\ M\widetilde{K}_{p,\sigma} & 0_{N\times N}\end{bmatrix}, \tag{42}$$

whose solutions are defined in the sense of Caratheodory [36].

System (41) can be written in compact form in [34]:

$$\dot{\bar{x}}(t)=F_\sigma\bar{x}(t)-\sum_{p=1}^{m}C_{p,\sigma}\int_{-\tau_p(t)}^{0}\bar{x}(t+s)\,ds, \tag{43}$$

where

$$C_{p,\sigma}=A_{p,\sigma}A_{0,\sigma}=\begin{bmatrix}0_{N\times N} & 0_{N\times N}\\ 0_{N\times N} & M\widetilde{K}_{p,\sigma}\end{bmatrix}, \tag{44}$$

and

$$F_\sigma=A_{0,\sigma}+\sum_{p=1}^{m}A_{p,\sigma}=\begin{bmatrix}0_{N\times N} & I_{N\times N}\\ -M\widehat{K}_\sigma & -M\widetilde{B}\end{bmatrix}, \tag{45}$$

with

$$\widehat{K}_\sigma=-\sum_{p=1}^{m}\widetilde{K}_{p,\sigma}+\widetilde{K}_\sigma. \tag{46}$$

Let $\mathcal{I}=\{i|k_{i0}a_{i0}>0, i\in\mathcal{V}_N\}$ denote the index set of the vertex whose neighbors include vertex 0, the following Lemma holds (see [37] for the proof).

Lemma 3. *Suppose vertex* 0 *is globally reachable in* $\overline{\mathcal{G}}_\sigma$ *and the weights for the edges of* $\overline{\mathcal{G}}_\sigma$ *satisfy the following conditions*

$$\begin{array}{l}\sum\limits_{j\in\mathcal{N}_i}a_{ij}\geq\sum\limits_{j\in\mathcal{N}_i}a_{ji},\ i\notin\mathcal{I},\ i\in\mathcal{V}_N,\\ 2k_{i0}a_{i0}+\sum\limits_{j\in\mathcal{N}_i}a_{ij}>\sum\limits_{j\in\mathcal{N}_i}a_{ji},\ i\in\mathcal{I},\end{array} \tag{47}$$

then $\widehat{K}_\sigma+\widehat{K}_\sigma^T$ *is positive definite.*

By exploiting Lemma 3 and Lyapunov-Razumikhin functional techniques [34], [38], [39], we now prove consensus of the platoon in presence of switching interconnection topology and heterogeneous time-varying delays. Note that inequalities (47) in Lemma 3 are conditions on the in degree and out degree of nodes in the digraph $\overline{\mathcal{G}}_\sigma$ that are weaker than the classical assumption of balanced graph $\mathcal{G}_\sigma$ (see Sec. II for mathematical definition), often made when a common Lyapunov-Razumikin function is used [37]. Conditions (47) are fulfilled in vehicular networks based on, for example, topologies that arise in absence of broadcast communication with the leader or when temporary leader communication is lost and/or a vehicle is unable to share information with its closest neighbors (see section VIII-B for an illustrative example).

Theorem 3. *Assume that $\overline{\mathcal{G}}_\sigma \in \Gamma$ fulfills the hypothesis of Lemma 3. Consider the closed-loop system (43) and choose the control parameters in (8) as $k_{ij} > 0$ and b such that*

$$b > b_1^\star = \left\{ \frac{\widehat{\mu}}{2\widehat{\lambda}} + 1 \right\} M_i, \tag{48}$$

with $\widehat{\mu} = \max_\sigma\{\lambda_{\max}(\widehat{K}_\sigma \widehat{K}_\sigma^T)\}$ and $\widehat{\lambda} = \min_\sigma\{\lambda_{\min}(\widehat{K}_\sigma + \widehat{K}_\sigma^T)\}$.
Then, there exists a constant $\tau_1^\star > 0$ such that, when $0 \leq \tau_p(t) \leq \tau < \tau_1^\star$ $(p = 1, \ldots, m)$, the origin is globally asymptotically stable.

Proof. Consider the following Lyapunov-Razumikhin candidate function

$$V(\overline{x}) = \overline{x}^\top P \overline{x}, \tag{49}$$

with positive definite matrix ($i = 1, \ldots, N$)

$$P = \begin{bmatrix} bMI_{N\times N} & I_{N\times N} \\ I_{N\times N} & I_{N\times N} \end{bmatrix}, \quad \frac{b}{M_i} > 1. \tag{50}$$

Similar to the analysis in the proof of Theorem 2 for the case of fixed interconnection topology, we obtain

$$\dot{V}(\overline{x}) \leq \overline{x}^\top (PF_\sigma + F_\sigma^\top P)\overline{x} + \sum_{p=1}^{m} [\tau_p(t)\overline{x}^\top PC_{p,\sigma}P^{-1}C_{p,\sigma}^\top P\overline{x} + \int_{-\tau_p(t)}^{0} \overline{x}^\top (t+s)P\overline{x}(t+s)ds]. \tag{51}$$

Choose now the following continuos, non decreasing function $\psi_4(s) = qs$ (for some constant $q > 1$), after some simple algebraic manipulations, when

$$V(\overline{x}(t+\theta)) < \psi_4(V(\overline{x})) = qV(\overline{x}(t)), \; -\tau \leq \theta \leq 0, \tag{52}$$

inequality (51) becomes

$$\dot{V}(\overline{x}) \leq -\overline{x}^\top Q_\sigma \overline{x} + \tau \sum_{p=1}^{m} \overline{x}^\top (PC_{p,\sigma}P^{-1}C_{p,\sigma}^\top P + qP)\overline{x} \tag{53}$$

where, from (45), we have

$$Q_\sigma = -(PF_\sigma + F_\sigma^\top P) = \begin{bmatrix} M(\widehat{K}_\sigma + \widehat{K}_\sigma^T) & M\widehat{K}_\sigma^T \\ M\widehat{K}_\sigma & 2(Mb-1)I_{N\times N} \end{bmatrix}. \tag{54}$$

From Lemma 3 $\widehat{K}_\sigma + \widehat{K}_\sigma^T$ is positive definite. Hence, according to Schur complements theorem [17], matrix Q_σ is positive definite if b fulfills conditions (48).
Let $H_\sigma = \sum_{p=1}^{m} (PC_{p,\sigma}P^{-1}C_{p,\sigma}^\top P + qP)$, now from (53) if

$$\tau < \tau_1^\star = \frac{\min_\sigma(\lambda_{\min}(Q_\sigma))}{\max_\sigma(\lambda_{\max}(H_\sigma))}, \tag{55}$$

then $\dot{V}(\overline{x}) \leq -\eta \overline{x}^\top \overline{x}$ for some constant $\eta > 0$. Hence asymptotic stability follows from Lyapunov-Razumikhin Theorem. □

VII. Disturbance propagation through the string

As mentioned in Section I, a vehicle platoon control system should be designed to meet *string stability* requirements. This is the capability of vehicles within a platoon of attenuating the propagation of traffic shockwaves [10], [21], [30]. In particular, the vehicles of a string stable platoon attenuate the propagation of acceleration and/or spacing errors generated by speed change maneuvers of the leading vehicle. In general, string stability can be defined w.r.t. spacing errors [10] or w.r.t. vehicles accelerations [40].
Well known fundamental limitations in the design of a string stable platoon with fixed spacing policy are explained in [9], where it is shown that communication with the leading vehicle must be established in order to achieve string stability, defined in terms of the 2-norm of the spacing errors/accelerations signals. On the other hand, communication with the leading vehicle is not necessary if a varying spacing policy is adopted, where the desired spacing increases linearly with the vehicle speed [41]. The objective of this section is to analyze the string stability properties of a vehicle platoon under our control action (8) with the time-varying delays set to a unique constant value τ (that may also correspond to their maximum [30], i.e. $\tau_i(t) = \tau \leq \tau^\star$). Following the approach used in the GCDC [40], string stability is defined here w.r.t. the vehicles accelerations, as

$$\left\| \frac{A_i(s)}{A_{i-1}(s)} \right\|_\infty \leq 1, \tag{56}$$

where $A_i(s)$ is the Laplace transform of the vehicle acceleration.
Writing the vehicles dynamics (4) and the control action (8) in the Laplace domain and approximating the constant time delay τ by using a first-order Padé approximation, after some algebraic manipulations, the acceleration can be computed in terms of the sensitivity functions $W_i^{\alpha}(s)(i = 2, \dots, N$ and $\alpha = 0, 1)$ as:

$$A_i(s) = W_i^0(s)A_{i-1}(s) + W_i^1(s). \tag{57}$$

See Appendix B for details on the mathematical derivation.
The string stability requirement reduces to $|W_i^0(j\omega)| < 1$ for all frequencies of interest [10].
In summary, the tuning parameters in the control law (8) must be chosen in order to guarantee consensus, according to Theorems 2 and 3, while satisfying the string stability requirement $|W_i^0(j\omega)| < 1$. Illustrative control designs are illustrated in Section VIII.

VIII. Numerical Validation

A. Consensus in nominal conditions

As a first validation of our platooning strategies, some numerical investigations performed in Matlab/Simulink are described. Next we consider a platoon of four vehicles and a leader, where the leader communicates with all the other vehicles, while every vehicle shares information with its neighbors [20], [42].
The simulation scenario has been set according to [43] where the leader vehicle imposes a common and constant fleet velocity equal to 20 [m/s] (i.e., 72 [km/h]) along a single lane road. The spacing policy requires a constant time headway $h_{01} = h_{12} = h_{23} = h_{34} = 0.8$ [s] for all vehicles in the platoon with $h_{ij} = -h_{ji}$. Furthermore, without loss of generality we consider the case of homogeneous traffic i.e., $M_i = M$ $(i = 0, 1, 2, 3, 4, 5)$. The time-varying delays $\tau_{ij}(t)$ are taken as random variables with uniform probability in the range $[\tau_{\min}, \tau_{\max}]$ with $\tau_{\min} = 0$ [s] and $\tau_{\max} \leq \tau^\star \cong 1.1 \cdot 10^{-1}$ [s] with the theoretical upper bound $\tau^\star$ computed as in Theorem 2 (choosing $q = 1.02 > 1$). Note that $\tau^\star$ is within the typical bound of the IEEE 802.11p standard for vehicular communication networks [44]. Control parameters values have been tuned according to Theorem 2 and conditions of Lemma 2, to achieve acceptable transient performance (as also discussed in section VII).
As expected from the theoretical analysis, simulation results depicted in Fig. 2 confirm formation of the platoon and its maintenance despite the presence of different time-varying delays affecting the information exchange among vehicles.

B. Robustness with respect to perturbations and communication failures

Numerical simulations were also performed to investigate the robustness of the proposed approach in the presence of perturbations and/or communication losses.
The first goal was to assess if and how velocity and acceleration were amplified downstream the traffic flow in the presence of perturbations of the leader motion and time-varying communication delays. Results in Figure 3 show the platoon robustness for a sudden variation in the leader motion. Note that the presence of noise in Figure 3 depend on the rapid piecewise nature of the time varying delay as considered in the simulation run.
Furthermore, we investigated the effect of the simultaneous presence of a sinusoidal perturbation acting on the leader motion (due, for example, to the human leader driver, namely $\delta(t) = A\sin\omega t$ being $A = 0.8$ and $\omega = \pi/15$ (see section VII) and sudden inter-vehicles communication losses, and their subsequent recovery. Note that communication losses and their subsequent recovery are modeled in terms of switchings of the network topology as described in section VI.
As a representative case of study, we consider the platoon depicted in Figure 4. (Note that a periodic disturbance $\delta(t)$ on the leader vehicle acceleration is added to the leader dynamics at $t = 50$ [s].) At time instant $t = 75$ [s], the last follower (number 4 in the platoon) looses connection with the leader and, therefore, the interconnection topology of the information exchange switches from topology 1) to topolgy 2) in Figure 4. Then, at time instant $t = 100$ [s], a new communication failure happens and follower 3 also looses connection with the leader (network commuting from topology 2) to topology 3) in Figure 4). At time instant $t = 115$ [s] follower 2 stops exchanging information with its predecessor and, hence, topology 4) in Figure 4 arises. Finally, the initial topology 1) in Figure 4 is recovered at time instant 170 [s] by switching backward through topologies 2) and 3) in Figure 4 at time instants $t = 130, 150$ [s], respectively. Controller parameters have been tuned inside the consensus regions defined by both Theorems 2 and 3. The selected control parameter values have been used for the experimental validation whose results are described in the following section. Moreover, in the simulation runs again $\tau_{ij}(t) \leq {\tau_1}^\star$, where the theoretical upper bound ${\tau_1}^\star$ is computed as in Theorem 3. Note that, also in this case, the computed bound ${\tau_1}^\star$ is below the average end-to-end communication delay typical of the IEEE 802.11p standard for vehicular communication networks.
As predicted by the theoretical analysis, results depicted in Fig. 5 confirm robustness of our platooning strategy despite temporary connection losses. Note that the network switches among topologies (depicted in Figure 4) that fulfill the assumptions of Theorem 3.

IX. Experimental Implementation and Validation

This section presents the results of the in-vehicle experimental validation of the proposed platooning control law. The experiments have been executed with the experimental setup described in Section IX-A.

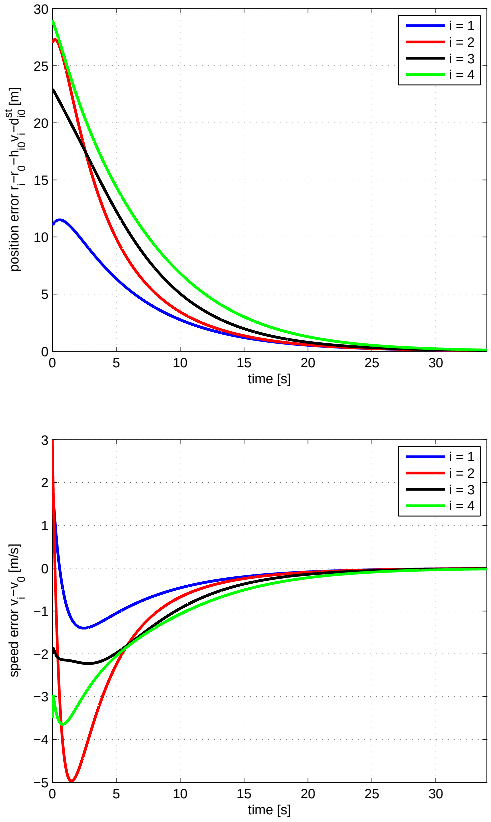


Fig. 2. Platooning in the presence of *time-varying* heterogeneous delays $\tau_{ij}(t)$. Upper plot: Time history of the position error. Lower plot: Time history of the velocity error.

A. Description of the Experimental Set-up

The experiments were performed at Chalmers University in a three vehicle platoon, with a leader (hereafter referred to as **L**) and two followers (hereafter referred to as **F1** and **F2**, respectively). **L** is a Volvo S80, equipped with

- a ublox EVK-5H GPS evaluation kit, based on a ublox LEA-5H GPS receiver module, updating the position with an accuracy of $2.5\ m$ at a maximum frequency of $4\ Hz$,
- a communication box based on a PC Engines Alix3d2 board, a Mikrotik 802.11a/b/g wireless MiniPCI card with an Atheros AR5414 chipset and an output power up to $350\ mW$ and a $6\ dBi$ radio antenna,
- an ethernet gateway, forwarding signals from the vehicle CAN bus to a Local Area Network,
- a notebook forwarding signals, sent by the GPS through a USB interface, to the LAN,
- an ethernet switch.

The signals from the CAN bus that are broadcast, together with the vehicle global position and the corresponding timestamp, are the vehicle longitudinal speed and acceleration and the yaw rate. The vehicle is driven manually. i.e., a driver is instructed to follow a given speed profile.

The vehicles **F1** and **F2** are two identical Volvo S60 and, as shown in Figure 6, are equipped with

- a Trimble GPS receiver SPS85, updating the position with a maximum frequency of $20\ Hz$ and an accuracy of $< 1\ m$ and interfaced to the rest of the system through an ethernet port,
- the same communication box as the vehicle **L**,
- a dSpace MicroAutoBox II, a rapid prototyping system based on a IBM PowerPC processor running at 900 MHz,
- an ethernet switch.

Communication is established among the three vehicles through the protocol IEEE 802.11p, implemented in a OpenWrt environment. This is a Linux distribution for embedded systems. Vehicles **F1** and **F2**, fuse the information from their onboard sensors and GPSs with the information shared via V2V communication through standard Kalman filters outlined in [40], to obtain their relative positions and velocities within the platoon.

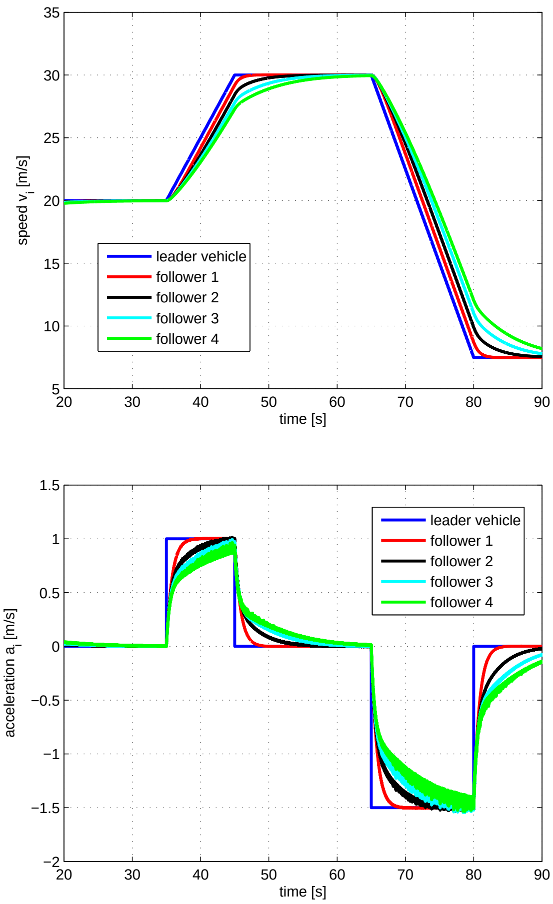


Fig. 3. Performance of the platooning algorithm in the presence of sudden variation in the leader motion). Time history of the velocity (Top panel) and acceleration (Bottom panel) attenuating along the string.

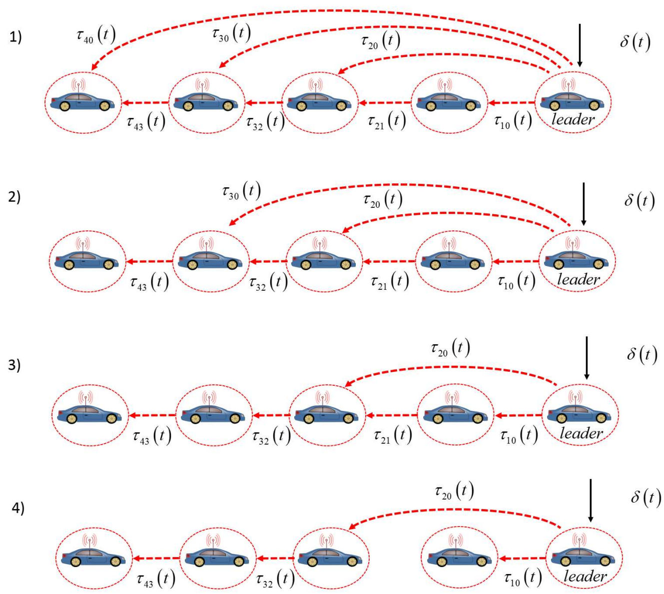


Fig. 4. Schematics of a platoon in the simultaneous presence of a sinusoidal perturbation on the leader motion and sudden inter-vehicles communication losses.

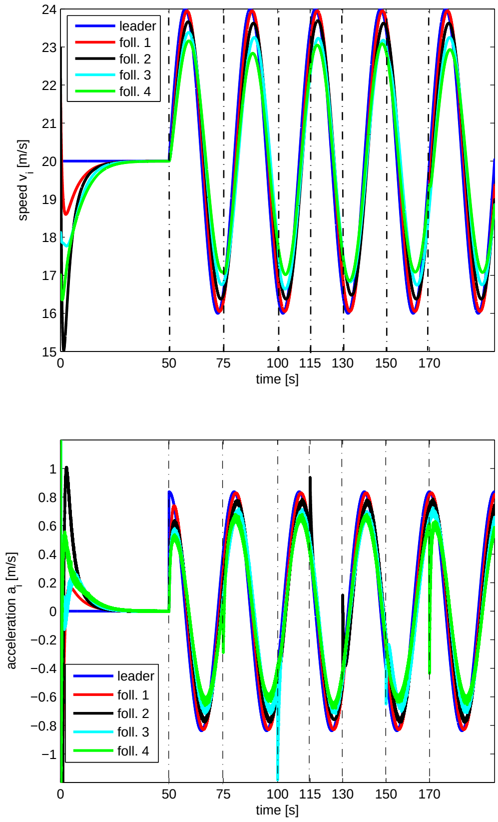


Fig. 5. Performance of the platooning algorithm in the simultaneous presence of a periodic disturbance acting on the leader motion and communication losses (as depicted in 4). Time history of the velocity (Top panel) and acceleration (Bottom panel) attenuating along the string.

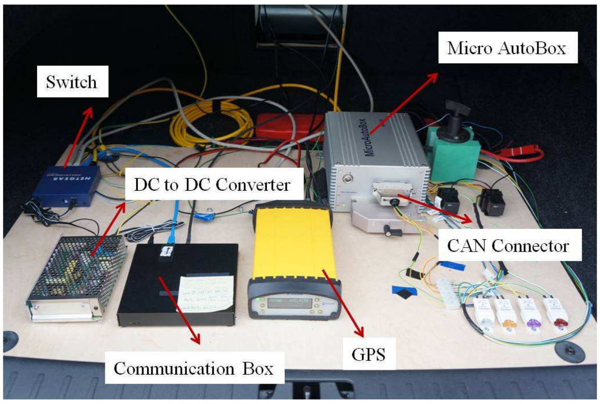


Fig. 6. The truck of one the following vehicles.

B. *Experimental Results*

In this section we show experimental results obtained in experimental tests with the three prototypal vehicles described above. (Experimental scenario parameters are summarized in Table I. See also Section A for further details on platoon spacing policy). Control gains and spacing policy parameters have been set as in the numerical simulations to ensure convergence and string stable behavior with respect to both velocity and acceleration.

Experimental results refer to a single lane scenario where leader and followers share information via wireless communication as depicted in Fig. 7 and described in section IX-A. The first goal is to investigate the effectiveness of the consensus based strategy in *creating* and *maintaining the platoon*. Vehicles start from different velocities and positions and as shown in Fig. 8 are able to converge to the desired velocity and inter-vehicular distance.

Further experiments have been performed with the objective of testing the ability of the approach to perform a *joining* maneuvre. In this case, it is assumed that F2 has to automatically join the platoon composed by L and F1, traveling together with a common

TABLE I
EXPERIMENTAL SCENARIO PARAMETERS

Vehicles Parameters	**Value**
Numbers of Cars	3
Leader Mass [kg]	1661
Followers Mass [kg]	1545
Leader Length [m]	4.820
Followers Length [m]	4.628
Spacing Policy Parameters	**Value**
Headway time $h_{10} = h_{21}$ [s]	0.8
Headway time h_{20} [s]	1.6
Distance at Standstill $d_{10}^{st} = d_{21}^{st}$ [m]	15
Distance at Standstill d_{20}^{st} [m]	30
Target	**Value**
Leader Reference Speed - creating and maintaining the platoon [m/s]	9.2
Leader Reference Speed - joining the platoon $[m/s]$	9.3
Tracking - max leader acceleration $[m/s^2]$	0.75
String stability - max/min leader acceleration $[m/s^2]$	$\pm$ 1.5

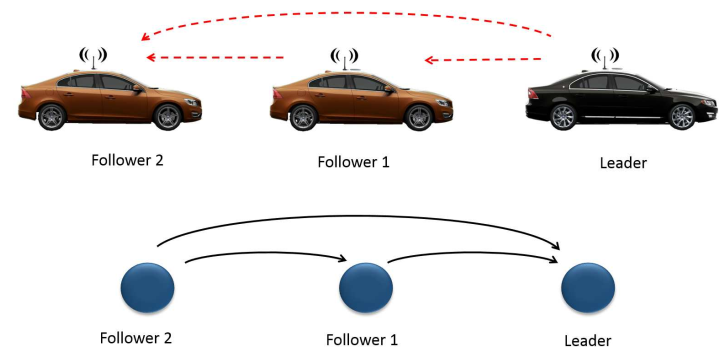


Fig. 7. Schematics of experimental scenario. Three vehicles moving along a single lane. Leader and followers share information via the wireless communication.

velocity of $9.3[m/s]$. Results in Fig. 9 show how the vehicle equipped with the algorithm described in this paper automatically performs the engaging maneuver and reaches the desired position and velocity.

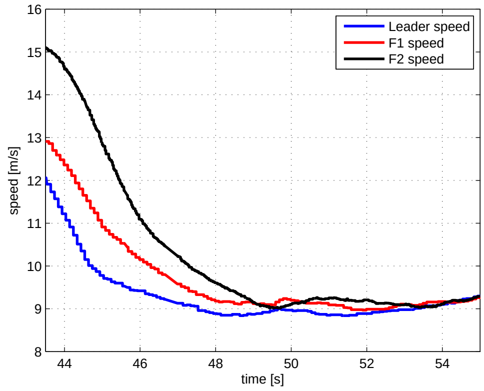
Although consensus is theoretically guaranteed only in the case of a constant leader velocity, further experiments have been devoted to test the ability of the strategy in *tracking the leader* during speed transients, when moving from rest to its final velocity v_0. To this aim, the three vehicles are at rest at the beginning of the experiments, with non zero spacing errors. Results in Fig. 10 show that the proposed control law is able to achieve tracking bringing all vehicles to the desired position. Note that the small peaks in the velocity profiles are due to gear changing which is automatically performed by the on board control units of the cars.

Note that in all cases, according to the theoretical derivation, the control effort converges to zero once the control goal is achieved (see, as an example, results in Figure 11).

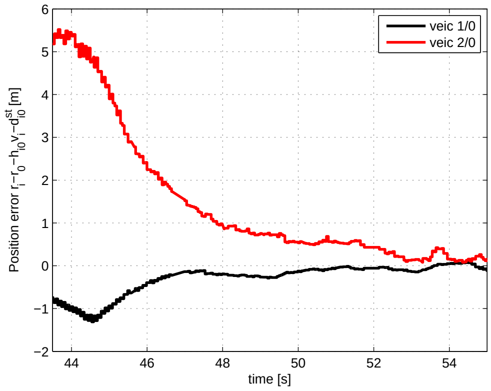
Finally, additional experiments have been dedicated to investigate the string stability of the platoon and validate the strategy in accelerating maneuvers of the leader. Results in Fig. 12 confirm that velocity and acceleration fluctuations are attenuated downstream the string of vehicles as expected. Norms of signals are in Tab. II.

X. CONCLUSIONS

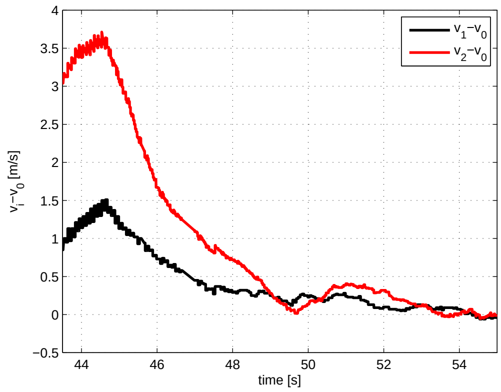
We have presented a platooning strategy based on the idea of trating the problem as that of achieving high order consensus in a network of dynamical agents in the presence of heterogeneous time varying delays. After discussing the control protocol the proof of convergence originally presented in [24] is extended removing the need of aggregating information on delays from different sources at each vehicle. Also the case of possibly switching network topologies is explicitly taken into account. Most notably an experimental validation of the strategy is presented that was carried out on a platoon of three vehicles at Chalmers University of Technology, Göteborg, Sweden. These experimental results reported here for the first time show the effectiveness of the algorithm creating and maintaining a platoon of vehicles travelling in a single lane. Moreover, experimental tests confirm that the strategy guarantees string stable behaviour despite the presence of heterogeneous delays.



(a)



(b)

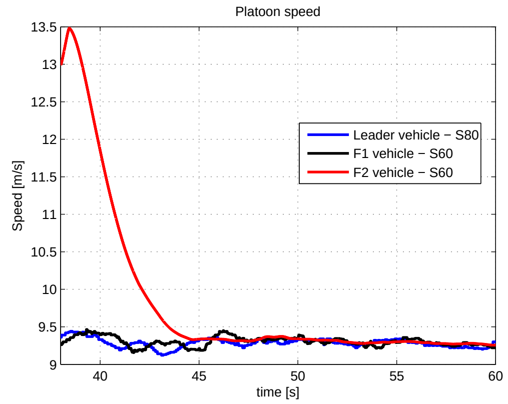


(c)

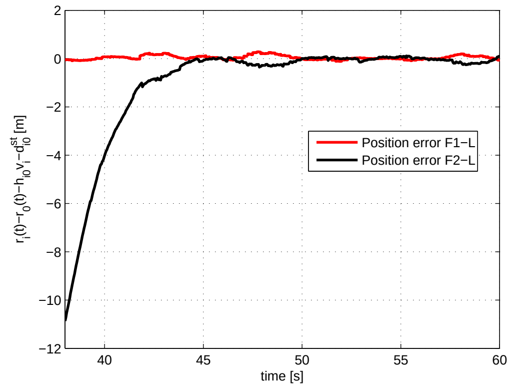
Fig. 8. Convergence to platooning in the scenario depicted in Fig. 7. (a): vehicle speed. (b): position error. (c): speed error

TABLE II
NORMS OF SIGNALS

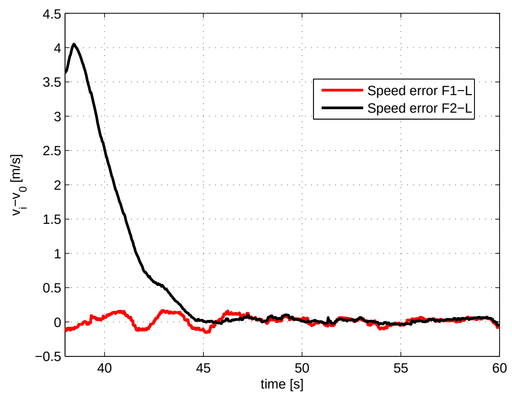
2-Norm	**Value**	**∞-Norm**	**Value**
$\|\|v_0(t)\|\|_2$	1024	$\|\|v_0(t)\|\|_\infty$	10.48
$\|\|v_1(t)\|\|_2$	1012	$\|\|v_1(t)\|\|_\infty$	10.47
$\|\|v_2(t)\|\|_2$	1004	$\|\|v_2(t)\|\|_\infty$	10.41
$\|\|a_0(t)\|\|_2$	81.4736	$\|\|a_0(t)\|\|_\infty$	1.69
$\|\|a_1(t)\|\|_2$	73.1254	$\|\|a_1(t)\|\|_\infty$	1.62
$\|\|a_2(t)\|\|_2$	55.7812	$\|\|a_2(t)\|\|_\infty$	1.1372



(a)



(b)



(c)

Fig. 9. (a) : vehicle speed; (b): position error; (c): speed error when follower nr. 2 has to join the platoon formed by the leader and follower nr. 1.

XI. Acknowledgement

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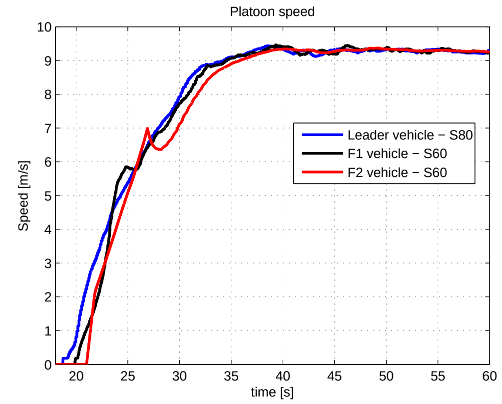
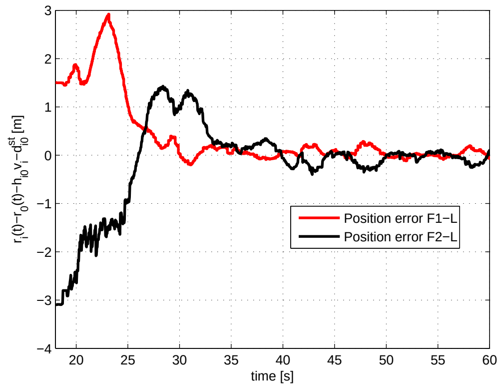
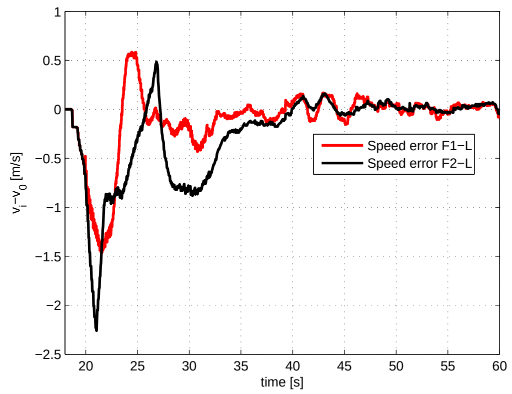


(a)



(b)



(c)

Fig. 10. Leader tracking maneuver: (a): vehicle speed; (b): position error; (c): speed error.

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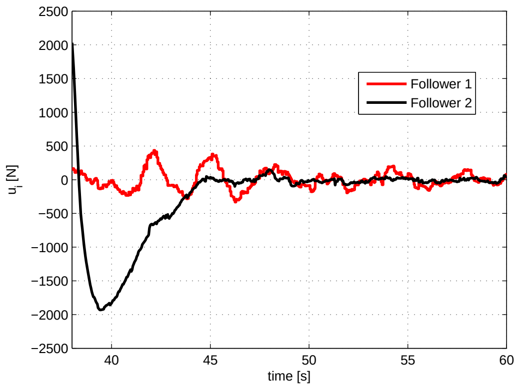

Fig. 11. Time history of the control signal during the platoon joining manouver.

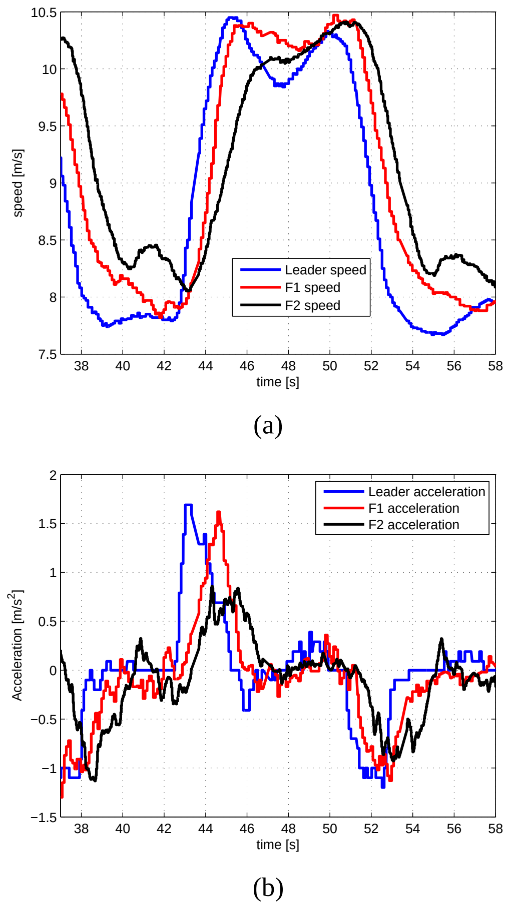

Fig. 12. String Stability of the platoon when the leader speed is perturbed. (a): vehicles speed. (b): vehicles acceleration.

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APPENDIX

A. Spacing policy

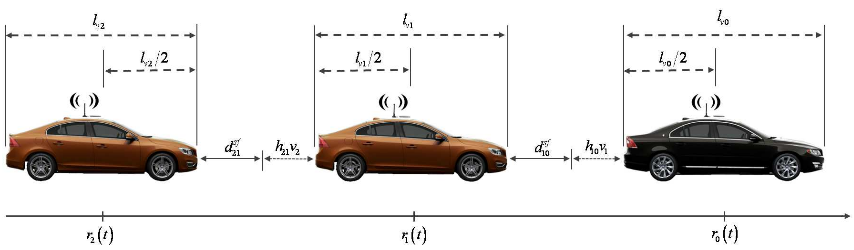

Fig. 13. Schematics of the spacing policy. Leader L and two followers $F1 - F2$.

Consider the parameters relative to any pair of vehicles in the platoon according to the schematic in Fig 13. The distance between two adjacent vehicles, the i-th vehicle and its preceding, at standstill can be easily expressed in terms of both the vehicle lengths and the safety distance required as [11]:

$$d^{st}_{ii-1} = l_{vi}/2 + l_{vi-1}/2 + d^{sf}_{ii-1} \tag{58}$$

where d^{sf}_{ii-1} is the safety distance (i.e. the minimum distance to be guaranteed between two adjacent vehicles) from vehicle $i-1$ to vehicle i and l_{vi}, l_{vi-1} are the vehicles lengths (we assume the reference is located at half the length of each vehicle. Note that different choices can be also made as for example in [11] where the vehicle reference is located on the front of each vehicle).

Generalizing expression (58), the distance at standstill between the i-th vehicle and any of its preceding vehicles (not necessarily the adjacent) along the string ($i > j$), is:

$$d^{st}_{ij} = \sum_{p=j+1}^{i} d^{st}_{pp-1}. \tag{59}$$

Analogously, the distance between a vehicle i and any of its followers j (not necessarily the adjacent) at standstill ($j > i$) can be expressed as:

$$d^{st}_{ji} = \sum_{p=i+1}^{j} d^{st}_{pp-1}. \tag{60}$$

Note that exploiting the above expressions the distance between vehicles i and j can be recast in terms of standstill distances with respect to the leading vehicle as $d^{st}_{ij} = d^{st}_{i0} - d^{st}_{j0}$. Moreover, $d^{st}_{ij} = -d^{st}_{ji}$. Furthermore, following [25], in this paper we assume that the desired following distance is linearly proportional to the vehicle velocity, $d_{ij} = h_{ij} v_i + d^{st}_{ij}$, and that the constant headway time of vehicle i with respect to vehicle j can be computed from the headway time with respect to the leading vehicle as $h_{ij} = h_{i0} - h_{j0}$.

B. *Further details on String Stability*

Writing the vehicles dynamics (4) and the control action (8) in the Laplace domain and approximating the constant time delay τ by using a first-order Padé approximation, after some algebraic manipulations the transfer functions $W^0_i(s)$ and $W^1_i(s)$ in (57) can be derived as:

$$W^0_i(s) = T^0_i(s) + T^1_i(s) G^{-1}(s) F(s); \quad W^1_i(s) = T^2_i(s) - \frac{k_{(i-1)0} d_i}{M_i} G^{-1}(s) \tag{61}$$

where $T^0_i(s) = V_i(s) C_i(s)$, $T^1_i(s) = V_i(s) D_i(s)$, $T^2_i(s) = V_i(s)(H_i(s) + Y_i(s))$, with

$$\begin{aligned}
&V_i(s) = [1 - B_i(s)]^{-1};\\
&B_i(s) = \tfrac{1}{M_i}\left[\tfrac{k_{i0}}{d_i}\left(\tfrac{h_{i0}}{s} - \tfrac{1}{s^2}\right) + \tfrac{k_{i(i-1)}}{d_i}\left(\tfrac{h_{i(i-1)}}{s} - \tfrac{1}{s^2}\right) - \tfrac{b}{s}\right];\\
&C_i(s) = \tfrac{1}{M_i}\left(\tfrac{k_{i(i-1)}}{d_i}\right)\tfrac{e^{-\tau s}}{s^2};\\
&D_i(s) = \tfrac{1}{M_i}\left[\tfrac{k_{i0}}{d_i}\left(\tfrac{e^{-\tau s}}{s^2} + \tfrac{\tau}{s}\right) + \tfrac{k_{i(i-1)}}{d_i}\tfrac{\tau}{s} + \tfrac{b}{s}\right];\\
&H_i(s) = \tfrac{1}{M_i}\left(\tfrac{k_{i0}}{d_i}\tfrac{d^{st}_{i0}}{s} + \tfrac{k_{i(i-1)}}{d_i}\tfrac{d^{st}_{i(i-1)}}{s}\right);\\
&Y_i(s) = \tfrac{1}{M_i}\left[\tfrac{k_{i0}}{d_i}\left(h_{i0} d_{i0} - \tfrac{d^{st}_{i0}}{s}\right) + \tfrac{k_{i(i-1)}}{d_i}\left(\tfrac{d^{st}_{(i-1)0}}{s} e^{-\tau s} - \tfrac{d^{st}_{i0}}{s} + h_{i(i-1)} d^{st}_{i0}\right)\right].
\end{aligned} \tag{62}$$

Note that:

$$\left\| \frac{A_i(s)}{A_{i-1}(s)} \right\|_\infty = \sup_{a_{i-1}(t) \in \mathcal{L}_2} \frac{\|a_i(t)\|_2}{\|a_{i-1}(t)\|_2} \tag{63}$$

with $\mathcal{L}_2$ the set of signals with 2-norm.