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Robust Direct Adaptive Regulation of Unknown Disturbances in the Vicinity of Low Damped Complex Zeros - Application to Active Vibration Control

Abraham Castellanos Silva*, Ioan Doré Landau*,
Petros Ioannou†.

***Abstract*—Adaptive feedback control approaches have been widely used to address the issue of rejecting multiple narrow band disturbances with unknown and time varying characteristics (frequency, phase and amplitude), in Active Vibration Control (AVC) and Active Noise Control (ANC). These approaches are based directly or indirectly on the use of the Internal Model Principle and the Youla-Kučera parametrization combined with an adaptive law. All the algorithms associated with these approaches make the assumption that the plant zeros are different from the poles of the disturbance model in order to achieve disturbance compensation. However in practice the problem is more intricate since it is not clear what happens if the plant has very low damped complex zeros (often encountered in mechanical structures) and the frequency of the disturbance is close to the anti-resonance frequency (the resonance frequency of the plant zeros).**

In this paper we evaluate comparatively in simulation and in real time on a benchmark test bed, two different approaches to deal with the low damped complex zeros of the plant. An evaluation of the combination of the two approaches is also presented.

***Index Terms*—Adaptive Regulation, Active Vibration Control, Inertial Actuators, Multiple Narrow Band Disturbances, Youla-Kučera Parametrization, Internal Model Principle**

I. Introduction

The basic problem in active vibration control (AVC) and active noise control (ANC) is the *strong* attenuation of multiple narrow band disturbances[1] with unknown and varying frequencies. The problem has been approached both in continuous-time ([23], [24] among other references) and in discrete-time ([26], [21] among other references). This paper consider the discrete-time case.

The disturbance model is assumed to be either a function equal to the sum of sinusoidal with unknown frequencies, amplitudes and phases or equivalently, a transfer function with unknown complex poles on the unit circle with white noise or a Dirac pulse as an input. When information about such model is available, it is possible to design an appropriate controller. If the model of a disturbance is considered, one has to address two issues: *1)* its structure (complexity, order of the parametric model) and *2)* the values of the parameters of the model. In general, one can assess from data the structure for such *model of disturbance* (using spectral analysis or order estimation techniques). However, the parameters of the model are unknown and may be time varying. This will require the use of an adaptive feedback approach in order to adapt to changes in parameters. An adaptive feedback approach, called *adaptive regulation* (known plant model and unknown/time varying disturbance model) is now widely accepted as the most effective approach for solving this class of problems.

It is also assumed that the plant model is stable and this property could be the result of a robust control design that is already incorporated in the system under consideration. The problem of disturbance rejection and adaptive regulation as defined above has been previously addressed in a number of papers ([3], [1], [7], [10], [11], [21], [16], [15], [6], [8], [2], [5]) among others. More recent references on the subject are [25], [17].

The *Internal Model Principle* implemented through a Youla-Kučera parametrization – also known as *Q*-parametrization – arises as a very attractive and efficient solution, since it allows the incorporation of the model of the disturbance in the controller without modifying the *desired* closed loop poles, defined by the designer ([26], [21], [4]) when a finite impulse response (FIR) filter structure is considered for the *Q*-filter. This parametrization allows one to have a two-stage controller: *1)* a central controller for the stabilization and broadband disturbance rejection and *2)* the adaptive part which deals with the rejection of the narrow band disturbance effects. The number of parameters to adapt is defined by the complexity of the assumed disturbance model. An international benchmark on adaptive rejection of narrow band disturbances has been organized and the results are published in a special issue of the European Journal of Control [17].

As indicated earlier the disturbance is considered to be periodic, i.e. the poles of the disturbance models are on the unit circle. All the stability proofs for the adaptation algorithms make the assumption that the plant zeros are different from the poles of the disturbance model in order to achieve disturbance compensation. However in practice the situation is more intricate since it is not clear what happens if the plant has very low damped complex zeros (often encountered in mechanical structures) and the frequency of the disturbance is close to the anti-resonance frequency (the resonance frequency of the plant zeros). Obviously even in the linear case with known parameters the design of the controller in this situation is difficult for robustness reasons. Finding a good control solution for this situation in an adaptive context is very challenging. In the international benchmark dedicated to adaptive regulation in the presence of unknown time varying disturbances [17] such a situation has been explicitly considered. Several solutions

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[1]Called *tonal* disturbances in active noise control.

have been proposed and the most successful has been based on the appropriate choice of the desired closed loop poles to be achieved by the Youla-Kučera central controller and by using an adjustable Q-FIR filter with the minimum number of parameters [4]. Recently in [13] it was suggested that over parametrization of the Q (FIR) filter can enhance the robustness of the linear and adaptive scheme in the vicinity of plant complex zeros[2].

The aim of the article is to present simulation and real-time comparisons of these two approaches on a relevant example (The EJC Benchmark for adaptive regulation benchmark test bed [17]) and to evaluate also the combination of the two approaches. The paper is organized as follows: Section II-A presents the active vibration control system used for the real-time experiments. Section II-B introduces the plant/disturbance representation and the controller structure used in this work. The influence of low damped complex zeros for disturbance rejection is discussed in Section III. In this section the two approaches for improving the behavior of the system in the presence of periodic disturbances located near low damped complex zeros are also briefly presented. The direct adaptive feedback regulation scheme, based on a Youla-Kučera parametrization is recalled briefly in Section IV. The comparison between the two approaches presented in [4] and in [13] and their combination is done in Section V through simulation and real-time results. Some concluding remarks are presented in Section VI.

II. Plant description and controller structure

A. System structure

The photo of the active vibration control experimental setup used in this study is presented in Fig. 1. The shaker acts as a disturbance source by introducing vibration forces and the inertial actuator can be used to counteract them by introducing vibrational forces in the opposite direction (inertial actuators use a similar principle as loudspeakers). This test bed was used as international benchmark in adaptive regulation. A detailed description together with the results were published in [17]. For sake of completeness some features are recalled here. The test bed consists of a passive damper, an inertial actuator, a mechanical structure, a transducer for the measurement of the residual force, a controller, a power amplifier and a shaker. The mechanical structure is such that the vibrations produced by the shaker, fixed to the ground, are transmitted to the upper side, on top of the passive damper. The inertial actuator is fixed to the chassis where the vibrations should be attenuated. The equivalent control scheme is shown in Fig. 2. The system input, $u(t)$ is the position of the mobile part (magnet) of the inertial actuator, the output $y(t)$ is the residual force measured by a force sensor. The plant transfer function ($G = q^{-d}\frac{B}{A}$) between the input of the inertial actuator, $u(t)$, and the residual force is called *secondary path*.

The control objective is to reject the effect of unknown narrow band disturbances on the output of the system (residual force), i.e. to attenuate the vibrations transmitted from the machine to the chassis. This requires that the compensator system (the secondary path) has enough gain in the frequency range where the narrow band disturbances are located [19]. The physical parameters of the system are not available. The system has to be considered as a *black box* and the corresponding models for control design should be identified. The sampling frequency is $F_s = 800$ Hz.

[2]This idea has not been explored by the participants to the benchmark. Note that the over parametrization of the Q filter for robustness with respect to uncertainties in the plant model has been proposed in [26], [13], however, here the objective of over parametrization is different.

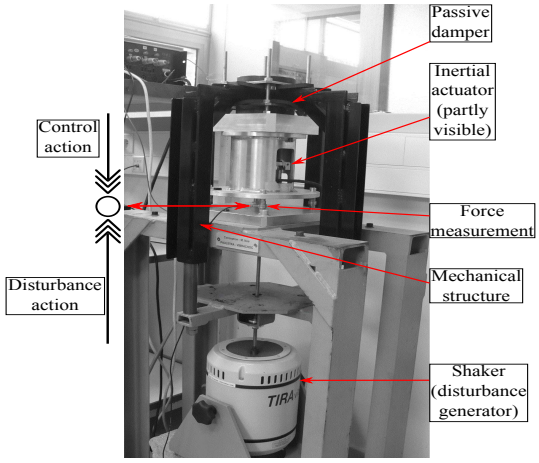


Fig. 1. Active vibration control using an inertial actuator (photo).

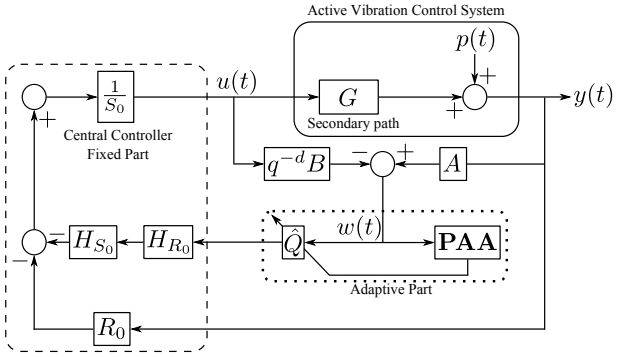


Fig. 2. Direct adaptive scheme using a YK-parametrization of the controller. Dashed line: fixed part, dotted line: adaptive part, solid line: active vibration control set up.

Fig. 3 gives the frequency characteristics of the identified parametric model for the secondary path (the excitation signal was a PRBS). The system itself in the absence of the disturbances features a number of low damped vibration modes as well as low damped complex zeros (anti-resonance). This makes the design of the controller difficult for rejecting disturbances close to the location of low damped complex zeros. The most significant are those located at 46.45, 100.50 and 111.55 Hz (see the zoom of the frequency characteristics of the secondary path in Fig. 3). Note that the design of a linear controller for rejecting a disturbance at 50 or 95 Hz (as required by the benchmark) is difficult since each one of these frequencies is close to a pair of low damped complex zeros (damping around 0.005). The parametric model of the secondary path has a high order, $n_A = 22$ and $n_B = 25$. The system has a double differentiator behavior.

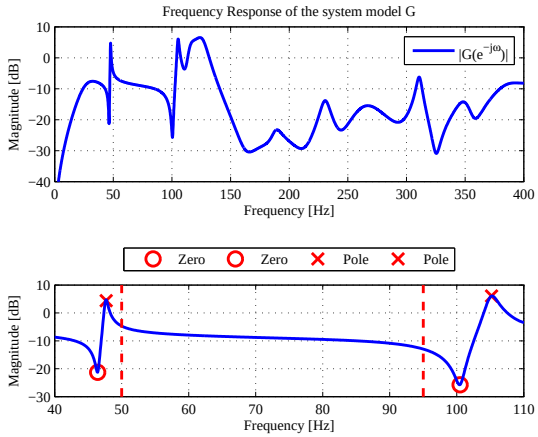


Fig. 3. Magnitude of the frequency response for secondary path model (top). Zoom at the low damped complex poles and zeros (bottom).

B. Plant/disturbance representation and controller structure

The structure of the linear time invariant discrete time model of the plant - the secondary path - used for controller design is:

$$G(z^{-1}) = \frac{z^{-d}B(z^{-1})}{A(z^{-1})} = \frac{z^{-d-1}B^*(z^{-1})}{A(z^{-1})} \ , \qquad (1)$$

where d is equal to the plant integer time delay (number of sampling periods) and

$$A(z^{-1}) = 1 + a_1 z^{-1} + \cdots + a_{n_A} z^{-n_A} \ ;$$
$$B(z^{-1}) = b_1 z^{-1} + \cdots + b_{n_B} z^{-n_B} = z^{-1} B^*(z^{-1}) \ ;$$
$$B^*(z^{-1}) = b_1 + \cdots + b_{n_B} z^{-n_B+1} \ ,$$

are polynomials in the complex variable z^{-1} and n_A, n_B and $n_B - 1$ represent their orders[3]. The model of the plant may be obtained by system identification. Details on system identification of the models considered in this paper can be found in [22], [20], [19].

Since the control objective is focused on regulation, the controller to be designed corresponds to a *RS* polynomial digital controller, ([18], [22] - see also Fig. 2). The controller has the form $K = \frac{R}{S}$, where $R(z^{-1})$ and $S(z^{-1})$, are polynomials in z^{-1}. Under the YK-parametrization (when a Q-FIR filter is considered) they have the following expressions:

$$R(z^{-1}) = R_0 + AH_{S_0}H_{R_0}Q, \qquad (2)$$
$$S(z^{-1}) = S_0 - z^{-d}BH_{S_0}H_{R_0}Q. \qquad (3)$$

The YK-parametrization used here is depicted in Fig. 2, where both fixed and adaptive parts are shown. In this paper a YK-parametrization using an equation-error disturbance observer is implemented along with a FIR filter representation of the optimal Q filter

$$Q(z^{-1}) = q_0 + q_1 z^{-1} + \cdots + q_{n_Q} z^{-n_Q}. \qquad (4)$$

It can be shown [21] that for any arbitrary FIR $Q(z^{-1})$ filter, the closed loop poles remain unchanged. They are defined by the stable polynomial

$$P(z^1) = A(z^{-1})S_0(z^{-1}) + z^{-d}B(z^{-1})R_0(z^{-1}), \qquad (5)$$

whose order is $n_P \leq n_A + n_B + n_{H_{R_0}} + n_{H_{S_0}} + d - 1$ and where R_0 and S_0 are minimal degree solutions to the previous Bezout equation, whose structure is defined by the following equations:

$$R_0(z^{-1}) = r_0^0 + r_1^0 z^{-1} + \ldots + r_{n_{R_0}}^0 z^{-n_{R_0}} = R_0' \cdot H_{R_0}; \qquad (6)$$
$$S_0(z^{-1}) = 1 + s_1^0 z^{-1} + \ldots + s_{n_{S_0}}^0 z^{-n_{S_0}} = S_0' \cdot H_{S_0}, \qquad (7)$$

where H_{R_0} and H_{S_0} are fixed parts used to open the loop at some frequency or incorporate the model of the disturbance, respectively. Eqs. (6) and (7) and Fig. 2 describe the structure of the central controller. Pole Placement or other design technique can be used to place the poles.

The output of the plant $y(t)$ and the input $u(t)$ may be written as:

$$y(t) = \frac{q^{-d}B(q^{-1})}{A(q^{-1})} \cdot u(t) + p(t) \ ; \qquad (8)$$
$$u(t) = -\frac{R(q^{-1})}{S(q^{-1})} \cdot y(t) \ , \qquad (9)$$

where $p(t)$ is the resulting additive disturbance on the output of the system.

We define the following sensitivity functions[4]:

- Output sensitivity function (the transfer function between the disturbance $p(t)$ and the output of the system $y(t)$):

$$S_{yp}(z^{-1}) = \frac{AS}{P} = \frac{A\left[S_0 - z^{-d}BH_{S_0}H_{R_0}Q\right]}{P} \ ; \qquad (10)$$

- Input sensitivity function (the transfer function between the disturbance $p(t)$ and the input of the system $u(t)$):

$$S_{up}(z^{-1}) = -\frac{AR}{P} = -\frac{A\left[R_0 + AH_{S_0}H_{R_0}Q\right]}{P} \ , \qquad (11)$$

Using equations (8) and (9), one can write the output of the system as:

$$y(t) = \frac{A(q^{-1})S(q^{-1})}{P(q^{-1})} \cdot p(t) = S_{yp}(q^{-1}) \cdot p(t) \ . \qquad (12)$$

For more details on *RS*-type controllers and sensitivity functions see [22].

Suppose that $p(t)$ is a deterministic disturbance, so it can be written as

$$p(t) = \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t) \ , \qquad (13)$$

where $\delta(t)$ is a Dirac pulse and $N_p(z^{-1})$, $D_p(z^{-1})$ are coprime polynomials in z^{-1}, of degrees n_{N_p} and n_{D_p}, respectively. In the case of persistent (stationary) disturbances the roots of $D_p(z^{-1})$ are on the unit circle and they are complex and non repeated. The energy of the disturbance is essentially represented by D_p. The contribution of the terms of N_p is weak compared to the effect of D_p, so N_p can be neglected.

[3]The complex variable z^{-1} will be used for characterizing the system's behavior in the frequency domain and the delay (shift) operator q^{-1} ($x(t) = q^{-1}x(t+1)$) will be used for describing the system's behavior in the time domain.

[4]The arguments (z^{-1}) and (q^{-1}) will be omitted in some of the following equations to make them more compact.

III. Constraints for adaptive regulation due the presence of low damped complex zeros

It follows from eq. (12) that a complete rejection at a certain frequency ω_k can be achieved when $S_{yp}(e^{-j\omega_k}) = 0$, which is possible if $S(e^{-j\omega_k}) = 0$, i.e. the roots of $D_p(z^{-1})$ are contained in $S(z^{-1})$. This is known as the internal model principle [9]. In this case the modulus of the input sensitivity function (eq. (11)) becomes equal to the inverse of the plant gain which implies that if $B(z^{-1})$ has a pair of low damped complex zeros close to ω_k, the plant input will reach very high values, putting an important stress on the actuator. Another implication is that the modulus of the input sensitivity function will become very large and therefore the tolerance with respect to additive uncertainties will become very low. Therefore, the cancellation (or in general an important attenuation) of disturbance effect on the output should be done only in frequency regions where the system gain is large enough.

It is also well known that the use of the internal model for total rejection (asymptotically) of the disturbance raises the maximum value of the modulus of the output sensitivity function [4], also known as "waterbed" effect. This may lead to unacceptable values for the modulus margin[5] – defined as $\Delta M = \left|S_{yp}(e^{-j\omega})\right|^{-1}_{\max}$ – and the delay margin if the controller design is not appropriately done [22].

To illustrate the above, first consider the case when a double narrow band disturbance placed at 50 and 70 Hz must be canceled, (in this case $D_p(z^{-1})$ is known). In order that the polynomial $Q(z^{-1})$ introduces the internal model of $D_p(z^{-1})$ in $S(z^{-1})$, the following Diophantine equation must be satisfied

$$D_p S' + z^{-d} B H_{S_0} H_{R_0} Q = S_0 \tag{14}$$

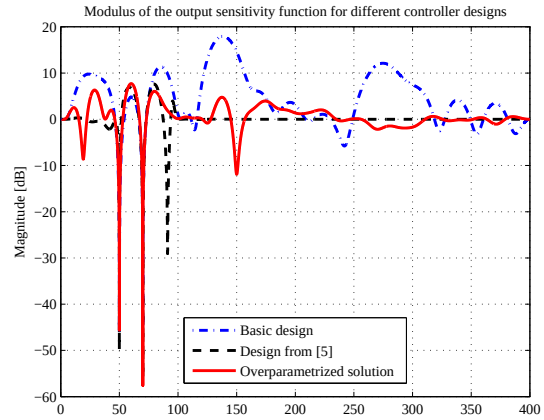
where D_p, H_{S_0}, H_{R_0}, d, B and S_0 are known, and S' and Q are unknown. Eq. (14) has a unique and minimal degree solution for S' and Q with $n_{S_0} \leq n_{D_p} + n_B + n_{H_{S_0}} + n_{H_{R_0}} + d - 1$, $n_{S'} = n_B + d + n_{H_{S_0}} + n_{H_{R_0}} - 1$ and $n_Q = n_{D_p} - 1$.

Remark: It is assumed that D_p and B do not have common factors but nothing is said of the feasibility of the solution if some roots of D_p are very close to some of the roots of B. In such case, the solution of (14) will be computationally challenging due to ill conditioning. Furthermore, if $n_Q > n_{D_P} - 1$ then the solution is not unique leading to overparametrization.

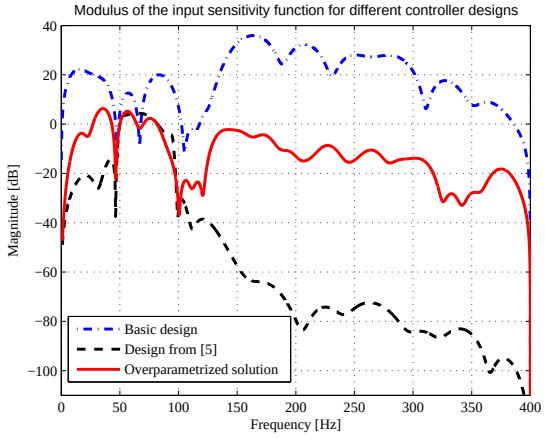
The central controller (R_0, S_0) has been computed by Poles Placement and the desired poles have been selected using "Internal model control" [22]. The system has n_A poles. The closed loop will have $n_P \leq n_A + n_B + n_{H_{R_0}} + n_{H_{S_0}} + d - 1$ poles. They are assigned as follows:

- The plant is stable and all plant poles (22) are assigned as desired closed loop poles.
- Among the remaining poles, 10 real poles are assigned at 0.3 for robustness reasons (robustifying poles called also robustifying filter) [22], the other being at 0.
- The loop is open at $0F_s$ and $0.5F_s$ by choosing $H_{R_0}(z^{-1}) = 1 - z^{-2}$ (controller gain equal to zero in steady state and at the Nyquist frequency).

[5]The modulus margin is the minimum distance between the open loop transfer function hodograph and the Nyquist point [22].



(a) S_{yp}



(b) S_{up}

Fig. 4. Modulus of the sensitivity functions for different cases. Dash-dot line: basic design, dashed line: design from [4], solid line: overparametrized solution.

- No fixed parts are considered for $S_0(z^{-1})$, i.e. $H_{S_0}(z^{-1}) = 1$.

Using this central controller the resulting modulus margin will have unacceptable low values (the maximum of the modulus of the sensitivity functions is too high) as shown in Fig. 4(a). For the two sinusoidal disturbances at 50 Hz and 70 Hz one gets $\Delta M = 0.127$ (17.9 dB). These values are far from the recommended value of $\Delta M = 0.5 - 0.4$. One observes also an important increase in the modulus of the input sensitivity function outside the attenuation band (50 to 95 Hz) in Fig. 4(b). This is why in [4], the central controller design was considered as a major problem.

A. Improving the central controller design by poles selection [4]

In [4] it was shown that using the plant model information (frequency characteristics), it is possible to *keep* the modulus of $S_{yp}(z^{-1})$ under an imposed maximum value by choosing appropriately some of the desired closed loop poles in $P(z^{-1})$. In this approach the minimal degree for the polynomial $\hat{Q}(z^{-1})$ is preserved and an "equation error" Youla-Kučera observer

is used. Basically it consist to place in addition of the plant poles, two pairs of complex poles close to the frequency region limits (50 and 95 HZ), and two other pairs of complex poles at 65 Hz and 80 Hz (these last values are not critical). The idea behind is that this will create "band stop filters " on the sensitivity functions around these frequencies which will allow to reduce significantly the waterbed effect (for more details on the effect of band stop filters see [22]). The damping of these poles is chosen in relation with the attenuation imposed at the corresponding frequencies. In addition, the shaping of $S_{up}(z^{-1})$ outside the attenuation zone was considered in order to lower down its magnitude. This was done again by the technique of band stop filters. The sensitivity functions obtained with this design are also shown in Fig. 4 (dashed line).

B. Improving robustness by increasing the number of parameters in the Q filter [13]

In (14) the order of Q can be $n_Q > n_{D_p} - 1$. In this case the solution of (14) for Q, S' is not unique and an infinite number of polynomials with order $n_Q > n_{D_p} - 1$ satisfy (14). The approach of [13] is to consider the Q that minimizes S_{yp} subject to the constraint of (14). The rationale behind this approach is that in the presence of noise or other uncertainties, denoted by $v(t)$, we have

$$y(t) = S_{yp}(q^{-1})p(t) + S_{yp}(q^{-1})v(t), \tag{15}$$

since S_{yp} contains the internal model D_p of $p(t)$, $S_{yp}(q^{-1})p(t)$ will converge to zero asymptotically with time and $y(t)$ will be driven by the term $S_{yp}(q^{-1})v(t)$. The additional objective is to keep the modulus of S_{yp} as low as possible. This gives rise to the following optimization problem

$$\min_{Q} J(Q) = \left\| \frac{H_{S_0} H_{R_0} q^{-d} BQ}{S_0} \right\|_\infty \tag{16}$$

subject to (14) and $n_Q > n_{D_p} - 1$. However another significant advantage of $n_Q > n_{D_p} - 1$ is that the ill conditioning that arises when the zeros of $D_p(z^{-1})$ are close to the zeros of $B(z^{-1})$ is reduced as the order of Q increases. The details of the solution of the optimization problem in (16) are presented in [14]. We should also note that the solution of the optimization problem shows the existence of an optimum Q, but in order to solve (16) we need to know the internal model $D_p(z^{-1})$.

The adaptive law that estimates the coefficients of Q search for the optimum Q given that it exists and does not require the a priori knowledge of the disturbances. The unknown optimal value is considered to be the desired one in the analysis of the adaptive scheme.

Note that the controller considered in [13] uses an "output error" type Youla-Kučera disturbance observer, however the methodology of [13] as indicated above extends to other types of disturbance observers without any significant difference[6]. The sensitivity functions obtained with this design are also shown in Fig. 4 (solid line).

[6]For a definition of the various types of Youla-Kučera disturbance observers see [17].

C. Combination of the two approaches

It is obvious that the two approaches can be combined, i.e; one can consider to use the approach presented in III-A by using the improved controller design and augmenting the order of the Q filter.

IV. Adaptive Feedback Regulation

In this section the direct adaptive algorithm – based on the YK-paremetrized controller – is recalled and used as basis for both comparison and combination of the two approaches to be evaluated. It can be used for the case of the minimal order of the polynomial Q (when $n_Q = n_{D_p} - 1$) or for the augmented case (when $n_Q > n_{D_p} - 1$).

As pointed out in [21], [19], the *Internal Model Principle* along with the YK-parametrization can be used to develop an algorithm to incorporate into the controller the assumed model of the disturbance (13) without changing the desired closed loop poles. From Fig. 2 we have that

$$w(t) = A(q^{-1})y(t) - q^{-d}B^*(q^{-1})u(t-1) = A(q^{-1})p(t) \tag{17}$$

and replacing (3) in (12), the output of the closed loop system can be express as

$$y(t) = \frac{\left[S_0 - q^{-d} B H_{S_0} H_{R_0} Q\right]}{P} \cdot w(t). \tag{18}$$

From the previous equation it is possible to derive the direct adaptive algorithm from [21], which uses the following *a posteriori* adaptation error

$$\varepsilon(t+1) = \left[\theta^T - \hat{\theta}^T(t+1)\right] \cdot \phi(t) + v(t+1), \tag{19}$$

with the following definitions

$$\theta = \left[q_0, q_1, \cdots, q_{n_Q}\right]^T, \tag{20}$$

$$\hat{\theta}(t+1) = \left[\hat{q}_0(t+1), \hat{q}_1(t+1), \cdots, \hat{q}_{n_Q}(t+1)\right]^T, \tag{21}$$

$$\phi(t) = \left[w_2(t), w_2(t-1), \cdots, w_2(t-n_Q)\right]^T, \tag{22}$$

$$w_2(t) = \frac{q^{-d} H_{R_0} H_{S_0} B^*}{P} \cdot w(t), \tag{23}$$

$$v(t+1) = \frac{S' D_p}{P} \cdot w(t+1) = \frac{S' A N_p}{P} \cdot \delta(t+1), \tag{24}$$

where $v(t+1)$ tends asymptotically towards zero (an asymptotically stable system excited by a Dirac pulse).

The associated standard parameter adaptation algorithm (PAA) used is [19],[18]:

$$\hat{\theta}(t+1) = \hat{\theta}(t) + F(t)\phi(t)\varepsilon(t+1) \tag{25}$$

$$\varepsilon(t+1) = \frac{\varepsilon^0(t+1)}{1 + \phi^T(t)F(t)\phi(t)} \tag{26}$$

$$\varepsilon^0(t+1) = w_1(t+1) - \hat{\theta}^T(t)\phi(t) \tag{27}$$

$$w_1(t+1) = \frac{S_0}{P} \cdot w(t+1) \tag{28}$$

$$F(t+1)^{-1} = \lambda_1(t)F(t)^{-1} + \lambda_2(t)\phi(t)\phi^T(t) \tag{29}$$

$$0 < \lambda_1(t) \leq 1,\ 0 \leq \lambda_2(t) < 2,\ F(0) > 0 \tag{30}$$

where $\lambda_1(t)$, $\lambda_2(t)$ allow to obtain various profiles for the evolution of the adaptation gain $F(t)$ (for details see [22][18]).

For a stability proof under the hypothesis *model = plant*, see [21][7].

V. Comparison of the two approaches and their combination - simulation and real time results

As seen in Section III, the standard central controller design using "internal model control" does not achieve a robust modulus margin, either for a single or double narrow band disturbances in the vicinity of low damped complex zeros, when the minimal degree for Q ($n_Q = n_{D_p} - 1$) is used. An improved central controller design has been proposed for this case. See III-A(this will be called "Case 1"). Alternatively using the standard central controller design the augmentation of the order of the Q filter has been considered (this will be called "Case 2"). Finally augmenting the order of the Q polynomial for the improved central controller design has also been considered (this will be called "Case 3").

Due to space constraints, only some of the results will be presented. To determine a relevant value of the degree of polynomial Q for the augmented solutions, simulations were carried out.The performance specifications ans constraints as well as the measurements procedures defined [17] are considered. Nevertheless, not all the criteria will be presented. Special attention is given to: global attenuation (GA) of the disturbance, disturbance attenuation (DA) and the maximum amplification (MA) along with the transient evaluation (TE). For the case of over parametrization a normalized parameter adaptation algorithm should be used [18] and [12]. However the normalization slow down significantly the adaptation transients and therefore for the purpose of comparison with the minimal parametrization approach the unnormalized adaptation algorithm has been used for all the cases.

Two different narrow band disturbances are tested on the simulator and experimental setup (Section II-A). The first disturbance corresponds to a double narrow band disturbance located at 50 and 70 Hz and the second is when the disturbance frequency corresponds to 75 and 95 Hz. This two set of frequencies were chosen due to the proximity that they have to the plant low damped complex zeros.
For all simulations ans experiments, the parameter adaptation algorithm given in Section IV has been used. λ_1 and λ_2 have been updated in order to obtain a time decreasing adaptation gain combined with a constant trace adaptation gain (residual gain assuring the adaptive behavior). The value of the constant trace was 0.004xN where N is the number of parameters to be adapted. The corresponding updating formula can be found in [18].

A. Performance criteria for simulation and real-time experiments

Simulations **(SIM)** and real-time **(RT)** experiments were conducted using both approaches and their combinations. The results were classified in frequency and in time domain. In frequency domain the objective is to strongly attenuate the disturbance with a limited amplification of the other frequencies. To evaluate the performance three indicators have been defined together with three target values according to [17]: Disturbance Attenuation (**DA**) (min = 40 dB), Global Attenuation (**GA**) (min = 30 dB), and Maximum Amplification (**MA**) (max = 7 dB)[8].

In time domain the Transient Evaluation (**TE** in %) is considered. The transient evaluation criterion requires that the transient duration when a disturbance is applied, has to be smaller than 2 s. A percentage was established for the fulfillment of this criterion. TE = 0% indicates a transient duration of 4 s and TE = 100% a transient duration smaller or equal to 2 s. The percentage is assigned using the ratio (α) of the truncated two norm (N^2T) of the residual force evaluated in two periods of time. This means that 2 s after application of a disturbance the N^2T of the output has to be equal or smaller than 1.21 of the steady state value of the N^2T of the residual force. The N^2T is evaluated over an interval of 3 s both for transient and steady state, taking into account that the disturbance is applied at $t = 5$ and the steady state is evaluated between 27 and 30 s. The performance variables are calculated using the following equations:

$$N^2T(i:m) = \sum_i^m y(i)^2 \tag{31}$$

$$\alpha = \frac{N^2T(7:10)}{N^2T(27:30)} \tag{32}$$

$$\delta Trans = \alpha - 1.21 \quad \text{if} \quad \alpha > 1.21 \tag{33}$$

$$\delta Trans = 0 \quad \text{if} \quad \alpha \leq 1.21 \tag{34}$$

$$\text{TE} = \left(\frac{1.21 - \delta Trans}{1.21}\right) 100\% \tag{35}$$

Another very important indicator is the complexity of the solutions proposed. This complexity is evaluated in terms of Computation Time (**CT** in μs). The computation time is calculated from the Task Execution Time evaluated in the MATLAB©'s xPC-Target environment. Since the computation time is measured only in the real-time application, no simulation results are provided. The computational time only consider the closed loop calculations[9].

B. Results in simulation and real-time experiments

Table I summarizes the results in frequency domain when a double disturbance at 50 and 70 Hz is introduced into the system. The results correspond to simulations and real-time experiments. For the over parametrized solutions the persistent excitation condition is not preserved.

As is seen from the results, the solution used in *Case 1* shows its effectiveness by achieving very good results for GA and DA and being slightly over the limit in MA (for the RT). Since the objective is to compare the three cases,

[7]Convergence of the adaptation error for any initial conditions, boundedness of all the variables, convergence of the estimated parameters toward the optimal values for $n_Q = n_{D_p} - 1$

[8]GA and MA give indication about the quality of the control which is supposed to introduce a very limited amplification at frequencies different from the frequency of the disturbance.

[9]The CT for an open loop test is 12.9 μs is subtracted from the total computation time

TABLE I
FREQUENCY DOMAIN RESULTS IN SIMULATION AND REAL TIME FOR A DOUBLE SINUSOIDAL DISTURBANCE AT 50 AND 70 HZ

Case	n_Q	GA dB		DA dB		MA dB	
		SIM	RT	SIM	RT	SIM	RT
1	3	36.1	35.4	45.3-42.7	42.8-43.6	9.8	7.5
2	21	36.2	36.3	42.6-44.1	42.1-47.8	10.8	11.7
3	4	36.6	34.7	46.3-45.0	41.5-44.2	9.0	7.6

RT: Real time, SIM: Simulation, GA: Global attenuation, DA: Disturbance attenuation, MA: Maximum amplification.

TABLE II
FREQUENCY DOMAIN RESULTS IN SIMULATION AND REAL TIME FOR A DOUBLE SINUSOIDAL DISTURBANCE AT 75 AND 95 HZ

Case	n_Q	GA dB		DA dB		MA dB	
		SIM	RT	SIM	RT	SIM	RT
1	3	35.4	33.7	48.0-37.0	39.3-37.4	7.3	9.4
2	21	34.7	-	45.6-35.0	-	9.0	-
	26	35.2	32.8	47.5-39.6	39.5-37.6	7.6	9.3
3	4	35.5	-	48.0-37.1	-	7.3	-
	7	35.7	33.6	51.9-36.5	41.9-38.0	6.8	7.9

RT: Real time, SIM: Simulation, GA: Global attenuation, DA: Disturbance attenuation, MA: Maximum amplification.

the number of parameters for *Case 2* was augmented until the results were close to the ones of the *Case 1*. For this disturbance, 22 parameters (nb. of parameters $= n_Q + 1$) were necessary to achieve almost the same level of effectiveness. Nevertheless, the MA shows that keeping the modulus of S_{yp} under the settled limit is difficult. It was found that even though the number of parameters could be increased, this have a negative impact either over the maximum amplification or the disturbance attenuation. So 22 parameters seems to be a good compromise. Finally, *Case 3* shows that augmenting from 4 to 5 parameters, better GA (in SIM) and MA (in SIM) are achieved. However, the Disturbance attenuation (DA) in real time (RT) is slightly less good. In Fig. 5 the achieved modulus margin (ΔM) is shown for all the cases. Since in all the cases the DA and GA were achieved, the figure aims to show the amplifications due to the waterbed effect. *Case 1* and *3* handle in a better way this effect, while the *Case 2* achieves almost the same profile but not for the amplifications at other frequencies different from 50 and 70 Hz.

Table II shows the results when the disturbance is located at 75 and 95 Hz. As in the previous test, the *Case 1* shows simulation results close to the benchmark specifications. The RT results shows some differences, the most important in DA and MA. Taking these results as a reference, it results that one need 27 parameters for the *Case 2* in order to achieve or improve the performance obtained in *Case 1* (simulation results with 22 parameters shows less good results). As for the previous test, higher orders for $\hat{Q}(z^{-1})$ allows to achieve better DA for 95 Hz but not for 75 Hz. Also, MA achieves its minimum using 27 parameters but if more parameters are used, this result is degraded. Although simulations results where better, the real-time results were quite close to the ones obtained for the *Case 1*. Regarding the real-time results, the *Case 3* achieves the best ones, showing clearly that the combination of both approaches allows to improve the result from a minimal degree solution (*Case 1*), without increasing significantly the number of parameters (*Case 2*). Fig. 6 displays the estimated output sensitivity function for the three cases, using the estimated parameters of $\hat{Q}(z^{-1})$. The figure is zoomed in order to show the amplifications due to the waterbed effect. It is clear that the *Case 3* shows the lowest amplifications (higher ΔM).

The transient evaluation results are shown in Table III. All the cases except *Case 2* with $n_Q = 26$ for a disturbance located at [75,95] Hz pass the transient evaluation. For *Case 2* with $n_Q = 26$ for a disturbance located at [75,95] Hz on achieves 94.2% and 99.7% in simulation and real-time respectively. These results are close to the imposed specifications.

TABLE III
TIME DOMAIN RESULTS IN SIMULATION AND REAL TIME FOR A DOUBLE SINUSOIDAL DISTURBANCE

Disturbance Hz	Case	n_Q	TE %	
			SIM	RT
[50,70]	1	3	100	100
	2	21	100	100
	3	4	100	100
[75,95]	1	3	100	100
	2	26	94.2	99.7
	3	7	100	100

TE: Transient evaluation criterion.

The complexity of the various solutions are evaluated in Table IV by comparing the Computation Time (CT). As expected, the *Case 1* shows the lowest CT since it uses the minimum number of parameters to be adapted. By augmenting the number of parameters, as in *Case 3*, the CT is increased. The most significant increases occur for the *Case 2* for [50,70] Hz and and for [75,95] Hz. The computation time increases by a factor over 12 with respect to the *Case 1*.

VI. CONCLUDING REMARKS

Two approaches and their combination for handling the problem of attenuation of unknown narrow band disturbances in the vicinity of low damped plant complex zeros have

TABLE IV
COMPUTATION TIME RESULTS FOR A DOUBLE SINUSOIDAL DISTURBANCE

Disturbance (Hz)	Case	n_Q	CT (μs)
[50,70]	1	3	3.6
	2	21	44
	3	4	5
[75,95]	1	3	3.6
	2	26	61.3
	3	7	7.5

CT: Computation Time.

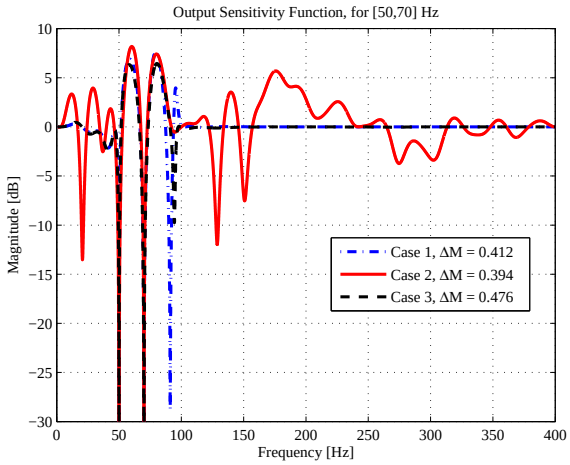


Fig. 5. Output sensitivity function comparison, for rejecting a double disturbance located at [50,70] Hz.

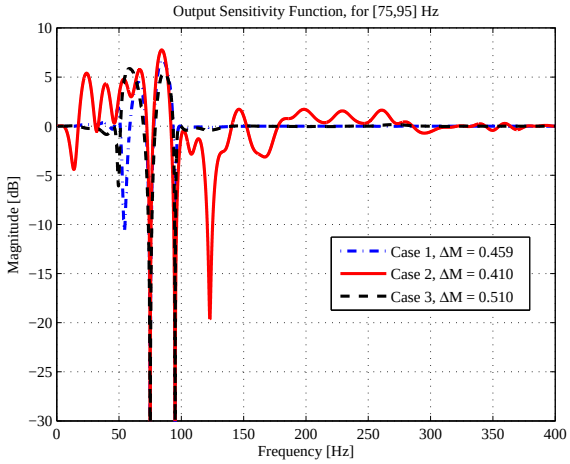


Fig. 6. Output sensitivity function comparison, for rejecting a double disturbance located at [75,95] Hz.

thoroughly evaluated. One approach emphasizes the need for a careful selection of some of the desired closed poles in order to use a minimal number of parameters to adapt. The other approach overcome the problem of a careful design of the central controller by a significant over parametrization of the compensator filter to be adapted. The price to pay is a significant increase in the computer load. Combination of the two approaches can be considered. A small increase of the size of the minimal order of the filter to be adapted combined with an improved central controller design can further slightly improve the performance. It is however important to recall that strictly speaking the over parametrization requires to use a normalized parameter adaptation algorithm and this will augment significantly the duration of the adaptation transients.

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