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Energy Management Considering Unknown Dynamics Based on Extremum Seeking Control and Particle Swarm Optimization

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***Abstract*— This brief studies an energy management (EM) problem with unknown dynamics of consumer appliances. A two-level optimization model is established between the utility company and the consumers. In this model, the utility company maximizes its profit by setting the electricity price, and the consumers respond to the price by regulating power usage to minimize their costs. The aforementioned process is performed in multiple stages. In each stage, the consumer response is formulated as a constrained optimization problem, which can be transformed into an unconstrained optimization problem using the penalty function method, and then an extremum seeking control (ESC) algorithm is developed to search for the quasi-optimal power consumption of the consumers. The ESC algorithm has noncontinuous first and second derivatives with respect to the variables. We propose an approximation method to make the ESC algorithm continuous and prove that the algorithm is semiglobally practically asymptotically (SPA) stable. After the consumer response in the same stage, the utility company updates the electricity price by the particle swarm optimization (PSO) algorithm. Then, we give an EM algorithm that integrates the ESC with PSO. In simulations, the algorithm is applied to achieve EM of heating, ventilation, and air conditioning (HVAC) systems, and the results show that the algorithm can converge to a neighborhood of the optimal solution and reduce the peak load and daily load.**

***Index Terms*— Energy management (EM), extremum seeking control (ESC), heating, ventilation, and air conditioning (HVAC), particle swarm optimization (PSO), semiglobally practically asymptotically (SPA) stability.**

I. Introduction

ENERGY MANAGEMENT (EM) plays an important role in smart grids [1] in order to achieve frequency regulation [2], economic dispatch [3], and voltage balance [4]. Generally, the objective of EM is to maximize the social welfare of electricity markets. Specifically, the utility company sets the electricity price to maximize its profit, and then, the consumers determine the power consumption to minimize their costs according to the published electricity price. The decision making of the utility company and consumers can be formulated by game models, such as noncooperative game [5], [6], hierarchical game [7], [8], and cooperative game [9]. In these game models, the profit maximization or cost minimization can be achieved by different types of optimization algorithms. For example, gradient-based algorithms are used in [5], [6], and [10]–[12], numerical algorithms are considered in [13] and [14], and bioinspired algorithms are applied in [15] and [16]. It is noted that unknown dynamics exist in the operation of some typical appliances, such as the heating, ventilation, and air conditioning (HVAC) system [17] and the lighting system [18]. In this case, the gradient-based algorithms cannot be used, and the numerical algorithms and bioinspired algorithms are not suitable for the dynamic system. To address this problem, this brief proposes an optimization algorithm based on the extremum seeking control (ESC), which is an online optimization method for seeking the extremum of a dynamic system with unknown dynamics [19], [20].

In this brief, a two-level optimization model is established between the utility company and the consumers, and the decision making of the utility company and the consumers is formulated as constrained optimization problems. We use the ESC algorithm to search for the quasi-optimal power consumption of the consumers and prove that the ESC algorithm is semiglobally practically asymptotically (SPA) stable. Then, we optimize the electricity price of the utility company to maximize its profit by using the particle swarm optimization (PSO) algorithm, which was applied in the home EM system to solve a combinational optimization problem in our previous work [16]. The novelty of this brief is to apply the ESC algorithm to the EM system with unknown dynamics and give the convergence conditions of the ESC algorithm. The contributions are as follows.

1) An EM algorithm that integrates the ESC with PSO is proposed.
2) An approximation method is developed to achieve the continuity for the ESC algorithm with unknown dynamics.
3) The SPA stability is proved for the approximated ESC algorithm.

The rest of the brief is organized as follows. Section II gives some notations and definitions. Section III presents the problem formulation. Section IV describes the EM algorithm

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in detail. Section V shows the simulation results. Finally, the conclusions are summarized in Section VI.

II. Preliminaries

In this section, we introduce some notations and definitions that will be used in this brief. Given a vector x, we define $\|x\|$ as the Euclidean norm.

Definition 1 [21]: A continuous function $\boldsymbol{\beta} : [0, a) \times [0, \infty) \to [0, \infty)$ is of class $\mathscr{KL}$ if, for each fixed s, $\boldsymbol{\beta}(r, s)$ is strictly increasing with r and $\boldsymbol{\beta}(0, s) = 0$, for each fixed r, $\boldsymbol{\beta}(r, s)$ is decreasing with s and converging to zero as $s \to \infty$.

Definition 2 [21]: A vector function $f(x, \boldsymbol{\varepsilon}) \in R^n$ is said to be $O(\boldsymbol{\varepsilon})$ if for any compact set $\mathscr{D}$ there exist positive constants k and $\boldsymbol{\varepsilon}^*$ such that $\|f(x, \boldsymbol{\varepsilon})\| \leq k\boldsymbol{\varepsilon}$, for $\boldsymbol{\varepsilon} \in (0, \boldsymbol{\varepsilon}^*] and x \in \mathscr{D}$.

Definition 3 [22]: Given a parameterized family of systems

$$\dot{x} = f(t, x, \boldsymbol{\varepsilon}) \tag{1}$$

where $x \in R^n$, $t \in R_+$, and $\boldsymbol{\varepsilon} \in R_+^l$ are the state vector, the time variable, and the parameter vector, respectively. The system (1) is said to be SPA stable in $(\boldsymbol{\varepsilon}_1, \ldots, \boldsymbol{\varepsilon}_l)$, if there exists $\boldsymbol{\beta} \in \mathscr{KL}$ and constructed parameters $\boldsymbol{\varepsilon} = (\boldsymbol{\varepsilon}_1, \ldots, \boldsymbol{\varepsilon}_l)$ that meet the following properties. For each pair of strictly positive real numbers (Δ, υ), there exists $\boldsymbol{\varepsilon}_1^* > 0$ for any $\boldsymbol{\varepsilon}_1 \in (0, \boldsymbol{\varepsilon}_1^*)$, $\boldsymbol{\varepsilon}_2^* = \boldsymbol{\varepsilon}_2^*(\boldsymbol{\varepsilon}_1) > 0$ for any $\boldsymbol{\varepsilon}_2 \in (0, \boldsymbol{\varepsilon}_2^*)$, $\boldsymbol{\varepsilon}_3^* = \boldsymbol{\varepsilon}_3^*(\boldsymbol{\varepsilon}_1, \boldsymbol{\varepsilon}_2) > 0$ for any $\boldsymbol{\varepsilon}_3 \in (0, \boldsymbol{\varepsilon}_3^*), \ldots, \boldsymbol{\varepsilon}_l^* = \boldsymbol{\varepsilon}_l^*(\boldsymbol{\varepsilon}_1, \boldsymbol{\varepsilon}_2, \ldots, \boldsymbol{\varepsilon}_{l-1}) > 0$ for any $\boldsymbol{\varepsilon}_l \in (0, \boldsymbol{\varepsilon}_l^*)$, respectively, and the solution of (1) satisfy

$$\|x\| \leq \boldsymbol{\beta}(\|x(0)\|, (\boldsymbol{\varepsilon}_1 \cdot \boldsymbol{\varepsilon}_2 \cdot \cdots \cdot \boldsymbol{\varepsilon}_l)(t - t_0)) + \upsilon \tag{2}$$

for all $t \geq t_0 \geq 0$ and the initial state $x(t_0) = x(0)$ with $\|x(0)\| \leq \Delta$.

III. Problem Formulation

We consider an EM system consisting of one utility company and N consumers. A two-level optimization model is established between the utility company and the consumers. The utility company determines the pricing strategy to maximize its profit. According to the electricity price announced by the utility company, the consumers determine the power consumption to minimize their costs.

A. Cost Model of Consumers

The cost function $U_i(y_i)$ of the consumer i ($i \in \mathbb{N} = \{1, \ldots, N\}$) is defined as the summation of the discomfort costs and the payments to utility company. We denote the discomfort cost as $g_i(y_i)$, where y_i is the operating state of consumer i's appliance. The discomfort cost is caused by changing the desired operating state y_i^N to the actual operating state y_i. The operating state is a general concept that denotes the working status of the appliances, such as the temperature setting of the HVAC system and the brightness of the lighting system. Thus, the cost function can be defined as: $U_i(y_i) = g_i(y_i) + px_i$, where p is the electricity price announced by the utility company and x_i is the power consumption of consumer i. The dynamics of the appliances are characterized as $\mathrm{d}y_i/\mathrm{d}t = \gamma_i s_i(x_i, y_i)$, where γ_i is a coefficient of operating state change caused by the characteristics of the electric equipment and external environment. A consumption-to-operating state function $y_i = f_i(x_i)$ is obtained from $s_i(x_i, y_i) = 0$, which denotes the relationship between the operating states and the power consumption at the equilibrium. In practice, the consumption-to-operating state function $f_i(x_i)$ could be a combination of multiple step functions, which can be approximated to a continuous convex function [11].

The objective of the consumer i is to minimize its cost subject to the power limitation and the appliance dynamics. It can be formulated as the following optimization problem:

$$\begin{aligned} \text{(P1)} \quad \min \quad & U_i(y_i) \\ \text{s.t.} \quad & x_i^{\min} \leq x_i \leq x_i^{\max}, i \in \mathbb{N}, \\ & \mathrm{d}y_i/\mathrm{d}t = \gamma_i s_i(x_i, y_i), i \in \mathbb{N} \end{aligned}$$

where $x_i^{\min}$ and $x_i^{\max}$ are the minimum and maximum power consumption of the consumer i, respectively. Before proceeding further, we give the following assumptions.

Assumption 1: $U_i(y_i)$ is continuous, increasing, and convex, and $f_i(x_i)$ is convex.

Assumption 2: The dynamics of the appliances are fast enough such that they can be separated from the EM algorithm, and the equilibrium $y_i = f_i(x_i)$ is globally asymptotically stable along the dynamics $\mathrm{d}y_i/\mathrm{d}t = \gamma_i s_i(x_i, y_i)$.

The first assumption is made to guarantee a globally optimal solution, and the second assumption is necessary to analyze the stability of the algorithm with dynamic mappings. These two assumptions are commonly used in the research of EM and ESC algorithm [23]–[26]. Next, we address the optimization problem with the equilibrium constraint $y_i = f_i(x_i)$ and transform (P1) to

$$\begin{aligned} \text{(P2)} \quad \min \quad & U_i(f_i(x_i)) \\ \text{s.t.} \quad & x_i^{\min} \leq x_i \leq x_i^{\max}, i \in \mathbb{N}. \end{aligned}$$

Using the penalty function method [27, Ch. 9], we transform the above problem to the following unconstrained problem:

$$\begin{aligned} \text{(P3)} \quad \min \quad P(x_i, m) = & \; U_i(f_i(x_i)) \\ & + m_1\big[\max(x_i^{\min} - x_i, 0)\big]^2 \\ & + m_2\big[\max(x_i - x_i^{\max}, 0)\big]^2 \end{aligned}$$

where m_1 and m_2 are the penalty factors, and the last two parts are the penalty terms. When x_i is out of the upper and lower bounds, the objective function is penalized, and the punishment is more severe as the penalty factors getting greater.

B. Profit of the Utility Company

The utility company purchases power from the electricity markets and sells them to the consumers. We assume that the power supply is equal to the summation of the power consumption of all consumers. Hence, the profit of the utility company is defined as the difference between the revenue obtained from the consumers and the costs of purchasing power from the electricity markets. The utility company's profit is maximized by the optimization problem as follows:

$$\text{(P4)} \quad \max \quad W(p) = p\sum_{i\in\mathbb{N}} x_i - \left(\rho_1\Big(\sum_{i\in\mathbb{N}} x_i\Big)^2 + \rho_2 \sum_{i\in\mathbb{N}} x_i + \rho_3\right)$$

where ρ_1, ρ_2, and ρ_3 are the cost coefficients, which are determined by the generation costs. The first part is the revenue obtained from the residential consumers and the second part is the cost of purchasing power.

IV. Energy Management Algorithm

In this section, we propose an EM algorithm, including the ESC algorithm and the PSO algorithm. Specifically, the utility company announces the optimal retail price to maximize its profit based on PSO algorithm, and the residential consumers' power consumption is optimized by the ESC algorithm. The utility company and the consumers communicate with each other based on a two-way communication network [28], [29].

A. Optimizing Power Consumption of Consumers

For the consumers, we use a subgradient algorithm to solve the optimization problem (P3) and model the dynamics of the power consumption as

$$\frac{\mathrm{d}x_i}{\mathrm{d}t} = k_i\left(-\frac{\mathrm{d}U_i(f_i(x_i))}{\mathrm{d}f_i(x_i)}\cdot\frac{\mathrm{d}f_i(x_i)}{\mathrm{d}x_i} + 2m_1[x_i^{\min} - x_i]^+ - 2m_2[x_i - x_i^{\max}]^+\right) \tag{3}$$

where k_i is the adaptive gain, $[\phi]^+ = \phi$ if $\phi > 0$, and $[\phi]^+ = 0$ if $\phi \le 0$.

In (3), we need to obtain the gradient of $f_i(x_i)$. In practice, it is hard to obtain the accurate formulation of $f_i(x_i)$. For example, the actual temperature setting of the HVAC is varying with the weather conditions, the occupant behavior, and so on. Therefore, the relationship between the temperature setting and the power consumption is unknown. Next, we utilize the ESC algorithm to estimate the gradient of $f_i(x_i)$. The core idea of the ESC algorithm is to estimate the gradient by adding dither signals to the input of the system (i.e., power consumption) and multiplying dither signals to the output of the system (i.e., operating states). The ESC algorithm based on the estimated gradient is developed as

$$\frac{\mathrm{d}\hat{x}_i}{\mathrm{d}t} = \hat{k}_i\left(-\frac{\mathrm{d}g_i(f_i(\hat{x}_i))}{\mathrm{d}f_i(\hat{x}_i)}\cdot\xi_i - p + 2m_1\big[\hat{x}_i^{\min} - \hat{x}_i\big]^+ - 2m_2[\hat{x}_i - \hat{x}_i^{\max}]^+\right) \tag{4}$$

$$\frac{\mathrm{d}\xi_i}{\mathrm{d}t} = -\hat{\omega}_i^{\xi}\left(\xi_i - \frac{2}{a}f_i(\hat{x}_i + a\sin(\omega t))\sin(\omega t)\right) \tag{5}$$

where $\hat{k}_i$ is the control gains of the ESC algorithm, $\hat{\omega}_i^{\xi}$ is the adaptive gain, $\sin(\omega t)$ is the dither signal, a is the signal amplitude, and ξ_i is a filtered signal that represents the gradient estimation of $f_i(x_i)$. To separate the dynamics of ξ_i from the other variables, we assume that $\hat{\omega}_i^{\xi}$ is much larger than $\hat{k}_i$, $\hat{\omega}_i^{\xi} = \omega_L\omega_i^{\xi}$, $\hat{k}_i = \delta\hat{\omega}_i^x$, and $\hat{\omega}_i^x = \omega_L\omega_i^x$, where ω_L and ω_i^x are both positive and real numbers.

The first and second derivatives of the right-hand side function of (4) with respect to x_i is noncontinuous because of the projection in (4). We construct an approximated continuous ESC algorithm as

$$\frac{\mathrm{d}\hat{x}_i}{\mathrm{d}t} = \hat{k}_i\Big(-\frac{\mathrm{d}g_i(f_i(\hat{x}_i))}{\mathrm{d}f_i(\hat{x}_i)}\cdot\xi_i - p + 2m_1\big([\hat{x}_i^{\min} - \hat{x}_i]^+ + q_1(\hat{x}_i)\big) - 2m_2\big([\hat{x}_i - \hat{x}_i^{\max}]^+ + q_2(\hat{x}_i)\big)\Big), \tag{6}$$

$$\frac{\mathrm{d}\xi_i}{\mathrm{d}t} = -\hat{\omega}_i^{\xi}(\xi_i - \frac{2}{a}f_i(\hat{x}_i + a\sin(\omega t))\sin(\omega t)) \tag{7}$$

where $q_1(\hat{x}_i)$ and $q_2(\hat{x}_i)$ are continuous functions that are defined in $[\hat{x}_i^{\min} - \epsilon_1, \hat{x}_i^{\min} + \epsilon_2]$ and $[\hat{x}_i^{\max} - \epsilon_1, \hat{x}_i^{\max} + \epsilon_2]$, respectively. The continuous functions $q_1(\hat{x}_i)$ and $q_2(\hat{x}_i)$ satisfy the following properties:

$$\begin{cases} q_1(\hat{x}_i^{\min} - \epsilon_1^2) = 0 \\ q_1(\hat{x}_i^{\min} + \epsilon_2^2) = 0 \\ Dq_1(\hat{x}_i^{\min} - \epsilon_1^2) = -1 \\ Dq_1(\hat{x}_i^{\min} + \epsilon_2^2) = 0 \\ D^2q_1(\hat{x}_i^{\min} - \epsilon_1^2) = 0 \\ D^2q_1(\hat{x}_i^{\min} + \epsilon_2^2) = 0 \end{cases} \tag{8}$$

and

$$\begin{cases} q_2(\hat{x}_i^{\max} - \epsilon_1^2) = 0 \\ q_2(\hat{x}_i^{\max} + \epsilon_2^2) = 0 \\ Dq_2(\hat{x}_i^{\max} - \epsilon_1^2) = 0 \\ Dq_2(\hat{x}_i^{\max} + \epsilon_2^2) = 1 \\ D^2q_2(\hat{x}_i^{\max} - \epsilon_1^2) = 0 \\ D^2q_2(\hat{x}_i^{\max} + \epsilon_2^2) = 0 \end{cases} \tag{9}$$

where D and D^2 denote the first and second derivatives of the functions with respect to the variables, respectively. Defining $\epsilon = \max\{\epsilon_1, \epsilon_2\}$, we prove the SPA stability of the approximated ESC algorithm [see (6) and (7)]. It is intuitive that there always exist polynomial functions that satisfy the above properties given any $\epsilon > 0$ and $\rho > 0$. For example, we use the following polynomial functions:

$$q_1(\hat{x}_i) = b_1(\hat{x}_i - \hat{x}_i^{\min})^8 + o_1(\hat{x}_i - \hat{x}_i^{\min})^7 + e_1(\hat{x}_i - \hat{x}_i^{\min})^6 + n_1(\hat{x}_i - \hat{x}_i^{\min})^5 + w_1(\hat{x}_i - \hat{x}_i^{\min})^4 + h_1(\hat{x}_i - \hat{x}_i^{\min})^3 \tag{10}$$

and

$$q_2(\hat{x}_i) = b_2(\hat{x}_i^{\max} - \hat{x}_i)^8 + o_2(\hat{x}_i^{\max} - \hat{x}_i)^7 + e_2(\hat{x}_i^{\max} - \hat{x}_i)^6 + n_2(\hat{x}_i^{\max} - \hat{x}_i)^5 + w_2(\hat{x}_i^{\max} - \hat{x}_i)^4 + h_2(\hat{x}_i^{\max} - \hat{x}_i)^3 \tag{11}$$

where

$$\begin{cases} b_1 = \dfrac{3(2\epsilon_1+\epsilon_2)}{\epsilon_1^4(\epsilon_1+\epsilon_2)\left(\epsilon_1^3+3\epsilon_1^2\epsilon_2+3\epsilon_1\epsilon_2^2+\epsilon_2^3\right)} \\ o_1 = \dfrac{13\epsilon_1^2-11\epsilon_1\epsilon_2-9\epsilon_2^2}{\epsilon_1^4(\epsilon_1+\epsilon_2)\left(\epsilon_1^3+3\epsilon_1^2\epsilon_2+3\epsilon_1\epsilon_2^2+\epsilon_2^3\right)} \\ e_1 = \dfrac{7\epsilon_1^3-35\epsilon_1^2\epsilon_2-3\epsilon_1\epsilon_2^2+9\epsilon_2^3}{\epsilon_1^4(\epsilon_1+\epsilon_2)\left(\epsilon_1^3+3\epsilon_1^2\epsilon_2+3\epsilon_1\epsilon_2^2+\epsilon_2^3\right)} \\ n_1 = \dfrac{3\left(-7\epsilon_1^3\epsilon_2+9\epsilon_1^2\epsilon_2^2+5\epsilon_1\epsilon_2^3\right)-\epsilon_2^4}{\epsilon_1^4(\epsilon_1+\epsilon_2)\left(\epsilon_1^3+3\epsilon_1^2\epsilon_2+3\epsilon_1\epsilon_2^2+\epsilon_2^3\right)} \\ w_1 = \dfrac{21\epsilon_1^2\epsilon_2^2-\epsilon_1\epsilon_2^3-7\epsilon_2^4}{\epsilon_1^3(\epsilon_1+\epsilon_2)\left(\epsilon_1^3+3\epsilon_1^2\epsilon_2+3\epsilon_1\epsilon_2^2+\epsilon_2^3\right)} \\ h_1 = \dfrac{-4\epsilon_2^4-7\epsilon_1\epsilon_2^3}{\epsilon_1^2(\epsilon_1+\epsilon_2)\left(\epsilon_1^3+3\epsilon_1^2\epsilon_2+3\epsilon_1\epsilon_2^2+\epsilon_2^3\right)} \end{cases} \tag{12}$$

and

$$\begin{cases} b_2 = \dfrac{-3(\epsilon_1+2\epsilon_2)}{\epsilon_2^3(\epsilon_1+\epsilon_2)^2\left(\epsilon_1^2\epsilon_2+2\epsilon_1\epsilon_2^2+\epsilon_2^3\right)} \\ o_2 = \dfrac{9\epsilon_1^2+11\epsilon_1\epsilon_2-13\epsilon_2^2}{\epsilon_2^3(\epsilon_1+\epsilon_2)^2\left(\epsilon_1^2\epsilon_2+2\epsilon_1\epsilon_2^2+\epsilon_2^3\right)} \\ e_2 = \dfrac{-9\epsilon_1^3+3\epsilon_1^2\epsilon_2+35\epsilon_1\epsilon_2^2-7\epsilon_2^3}{\epsilon_2^3(\epsilon_1+\epsilon_2)^2\left(\epsilon_1^2\epsilon_2+2\epsilon_1\epsilon_2^2+\epsilon_2^3\right)} \\ n_2 = \dfrac{3\epsilon_1\left(\epsilon_1^3-5\epsilon_1^2\epsilon_2-9\epsilon_1\epsilon_2^2+7\epsilon_2^3\right)}{(\epsilon_1+\epsilon_2)\left(\epsilon_1^3\epsilon_2^4+3\epsilon_1^2\epsilon_2^5+3\epsilon_1\epsilon_2^6+\epsilon_2^7\right)} \\ w_2 = \dfrac{\epsilon_1^2\left(7\epsilon_1^2+\epsilon_1\epsilon_2-21\epsilon_2^2\right)}{(\epsilon_1+\epsilon_2)\left(\epsilon_1^3\epsilon_2^3+3\epsilon_1^2\epsilon_2^4+3\epsilon_1\epsilon_2^5+\epsilon_2^6\right)} \\ h_2 = \dfrac{\epsilon_1^3(4\epsilon_1+7\epsilon_2)}{(\epsilon_1+\epsilon_2)\left(\epsilon_1^3\epsilon_2+3\epsilon_1^2\epsilon_2^2+3\epsilon_1\epsilon_2^3+\epsilon_2^4\right)}. \end{cases} \tag{13}$$

Theorem 1: The closed-loop system under the ESC algorithm [see (6) and (7)] is SPA stable at the optimal solution of (P3), uniformly in (a, ϵ, δ), if $dg_i(y_i)/dy_i \le \mu_i$ and $d^2g_i(y_i)/dy_i^2 \ge \eta_i$ for $i \in \mathbb{N}$, where μ_i and η_i are the positive scalars.

Proof: The proof is provided in Appendix A.

Next, we turn to the optimization problem (P1) with unknown appliance dynamics and solve it by the following algorithm:

$$\frac{\mathrm{d}\hat{x}_i}{\mathrm{d}t} = \hat{k}_i\Bigg(-\frac{\mathrm{d}g_i(f_i(\hat{x}_i))}{\mathrm{d}f_i(\hat{x}_i)}\cdot\xi_i - p + 2m_1\left[\hat{x}_i^{\min}-\hat{x}_i\right]^+ + q_1(\hat{x}_i) - 2m_2\left[\hat{x}_i-\hat{x}_i^{\max}\right]^+ + q_2(\hat{x}_i)\Bigg) \tag{14}$$

$$\frac{\mathrm{d}\xi_i}{\mathrm{d}t} = -\hat{\omega}_i^{\xi}\left(\xi_i - \frac{2}{a}\hat{y}_i\sin(\omega t)\right) \tag{15}$$

$$\frac{\mathrm{d}\hat{y}_i}{\mathrm{d}t} = \gamma_i s_i(\hat{x}_i + a\sin(\omega t), \hat{y}_i). \tag{16}$$

Following the second assumption, we define $\gamma_i = \zeta_i^{-1}\hat{\omega}_i^{\xi}$, which is much larger than $\hat{\omega}_i^{\xi}$ and k_i. The results of Theorem 1 can be extended to the power consumption problem with unknown appliance dynamics by analyzing the system in two separated time scales. Before presenting the theorem, we define $\zeta = \max\{\zeta_1, \zeta_2, \ldots, \zeta_N\}$.

Theorem 2: The closed-loop system under the ESC algorithm [see (14)–(16)] is SPA stable at the optimal solution of (P1), uniformly in $(a, \epsilon, \zeta, \delta)$, if $dg_i(y_i)/dy_i \le \mu_i$ and $d^2g_i(y_i)/dy_i^2 \ge \eta_i$ for $i \in \mathbb{N}$, where μ_i and η_i are the positive scalars.

Proof: The proof is provided in Appendix B.

Remark 1: The upper bounds of parameters a and ϵ are determined in the proof in Appendix A, and the existence and calculation of the upper bounds of parameters ζ and δ can be referred to [21, Th. 11.3].

B. Optimizing Price of the Utility Company

We utilize the PSO algorithm to solve the profit optimization problem (P4). Assume that the size of particles is Z. $(P_1, \ldots, P_i, \ldots, P_Z)$ and $(V_1, \ldots, V_i, \ldots, V_Z)$ are the position and velocity of particles, respectively. In D-dimensional space, $P_i = (p_i^1, p_i^2, \cdots, p_i^d, \cdots, p_i^D)$, where $p_i^d \in [p^{\min}, p^{\max}]$ and $V_i = (v_i^1, v_i^2, \cdots, v_i^d, \cdots, v_i^D)$, where $v_i^d \in [v^{\min}, v^{\max}]$ and $d \in [1, D]$. The velocity and position updating strategy of the ith particle are expressed as follows:

$$v_i^d \leftarrow v_i^d + c_1\cdot rand1_i^d\cdot\left(pbest_i^d - p_i^d\right) + c_2\cdot rand2_i^d\cdot\left(gbest^d - p_i^d\right) \tag{17}$$

$$p_i^d = p_i^d + v_i^d \tag{18}$$

where c_1 and c_2 indicate the learning factors that adjust the step size. c_1 represents the step size that the ith particle tracks its own historical optimal value $pbest_i$, and c_2 represents the step size that the ith particle tracks the globally optimal value $gbest$. Within each iteration, $pbest_i$ and $gbest$ are updated according to the fitness value of each particle. $rand1_i^d$ and $rand2_i^d$ are the two uniform random numbers over [0, 1]. The convergence value of p_i^d represents the electricity price p announced by the utility company. It will be substituted into the ESC algorithm [see (6) and (7)] to obtain the optimal power consumption, and the initial value of p_i^d is selected in the range $[p^{\min}, p^{\max}]$.

C. Implementation of Energy Management Algorithm

The EM algorithm includes the ESC algorithm and the PSO algorithm. First, the price is initialized by the PSO algorithm, and the power consumption of the consumers is obtained by substituting the price into the ESC algorithm [see (6) and (7)]. Consequently, the fitness value can be calculated by the PSO algorithm. Then, the price is updated by the PSO algorithm based on (17) and (18). The PSO algorithm and the ESC algorithm are repeated until the price and the power consumption converge or the maximum number of iterations is reached. Especially, the time scales of the ESC algorithm and the PSO algorithm are different. The PSO algorithm starts to run after the ESC algorithm converges. For the PSO algorithm, the fitness value denotes the profit of the utility company. The pseudocode of the EM algorithm is given in Algorithm 1.

V. Simulation Results

We apply the algorithm to achieve EM of residential consumers with distributed HVAC systems. The operating state

Algorithm 1 EM Algorithm

Input: Z: size of the whole population; iter-max: maximum iterations; Initialize each particle's position p_i^d and velocity v_i^d.

Output: each particle's position p_i^d.

1: **for** iter=1: iter-max **do**

2: Substitute p_i^d into the ESC algorithm (6)–(7), and obtain $\sum_{i\in\mathbb{N}} x_i$.

3: Calculate their fitness values and update $pbest_i$, $gbest$;

4: Update each particle using (17) and (18) and revise v_i^d, p_i^d using v_i^d=min($v^{\max}$, max($v^{\max}$, v_i^d)), p_i^d=min($p^{\max}$, max($p^{\max}$, p_i^d));

5: **end for**

6: Substitute p_i^d into the ESC algorithm, and update x_i and $\sum_{i\in\mathbb{N}} x_i$.

is the temperature setting T_i in the HVAC system, and the discomfort cost function $g_i(T_i)$ is defined as

$$g_i(T_i) = \theta_i(T_i - T_i^N)^2 \tag{19}$$

where θ_i is a constant coefficient used to describe the discomfort cost. Here, the Fanger thermal comfort model can be applied to describe the human body's thermal comfort and obtain the corresponding θ_i by quadratic fitting of the PPD-PWV index [30].

We assume that the dynamics of the HVACs are denoted as $\mathrm{d}T_i/\mathrm{d}t = \gamma_i s_i(l_i, T_i)$, where l_i is the power consumption. The relationship between the temperature settings and the power consumption is defined by a convex function $T_i = f_i(l_i)$. In practice, the dynamics of the EM algorithm should be carefully set up, such that the dynamics of the HVAC can be separated from it. Thus, the second assumption is satisfied, and the optimization problem is as follows:

$$\begin{aligned} \text{(P5)} \quad \min \quad & \theta_i\big(f_i(l_i) - T_i^N\big)^2 + pl_i \\ \text{s.t.} \quad & l_i^{\min} \le l_i \le l_i^{\max}, i \in \mathbb{N}. \end{aligned}$$

A quadratic function is used to approximate the relationship between the temperature setting and the power consumption [11]

$$l_i = \lambda_1(30 - T_i)^2 + \lambda_2(30 - T_i) + \lambda_3 \tag{20}$$

where $\lambda_1 = \varphi\Phi\varsigma A_i H_i I_1$, $\lambda_2 = \varphi\varpi S_i + \varphi\Phi\varsigma A_i I_0$, and $\lambda_3 = \varphi Q^{\mathrm{sil}}$, and the parameter settings are given in Table I.

In the simulations, we consider an EM system consisting of one utility company and ten residential consumers with HVAC systems. Without loss of generality, the HVAC is assumed to be a cooler. The desired temperature settings are assumed to be 24 °C, and the parameters of the ESC algorithm are $k_i = 0.001$, $a = 0.1$, $\hat{\omega}_i^{\zeta} = 2$, and $\omega = 20$. The initial values of l_i and p are randomly selected in three ranges and have no impact on the convergence of the algorithm. The simulations are performed based on MATLAB (R2010b).

Using the sixth, eighth, and tenth residential consumers as an example, the SPA stability is demonstrated in Figs. 1 and 2. It is shown that the ESC algorithm converges to a neighborhood of the optimal power consumption and temperature setting. The convergence bound of the temperature setting is 0.3 $°C$. The quasi-optimal power consumption and temperature settings of ten residential consumers are shown in Table II.

The solar and internal load is assumed to be 300 W from 19 : 00 to 7 : 00 in the next day, within [3000 W, 3600 W] from 7 : 00 to 8 : 00 and from 18 : 00 to 19 : 00, and within [3000 W, 4500 W] from 8 : 00 to 18 : 00. The daily outdoor temperature in the summer is shown in Fig. 3, and the indoor temperature of the sixth consumer in a day is shown in Fig. 4. The prices, profits, and the total power consumption in a day are shown in Figs. 5–7, respectively. We can observe that the EM algorithm can effectively reduce the peak load

TABLE I
PARAMETER SETTINGS

Outdoor temperature (°C)	$T_0 = 30$
Desired temperature (°C)	$T_i^N = 24$
Transmission area (m^2)	$S_i \in [30, 60]$
Heat transfer constant (W/m^2 · °C)	$\varpi = 15$
Specific heat of air (kJ/kg · °C)	$\Phi = 1.006$
Air density (kg/m^3)	$\varsigma = 1.1839$
Wind speed coefficient	$I_0 = 0.343$
Outdoor temperature coefficient	$I_1 = 1.12$
Effective infiltration area (m^2)	$A_i \in [15, 45]$
Building height (m)	$H_i \in [8, 15]$
Solar and internal load (W)	$Q^{sil} \in [300, 4500]$

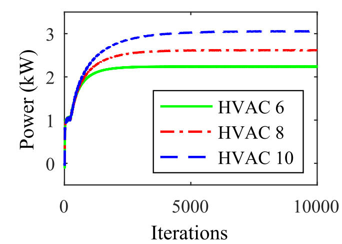

Fig. 1. Convergence of the power consumption.

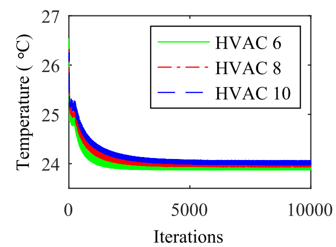

Fig. 2. Convergence of the temperature setting.

TABLE II
POWER CONSUMPTION AND THE TEMPERATURE SETTING

Residential consumer	1	2	3	4	5
Power consumption (kW)	1.12	1.28	1.48	1.88	2.09
Temperature setting (°C)	23.88	23.89	23.90	23.93	23.95
Residential consumer	6	7	8	9	10
Power consumption (kW)	2.24	2.45	2.62	2.81	3.05
Temperature setting (°C)	23.95	23.97	23.98	24.00	24.02

This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.

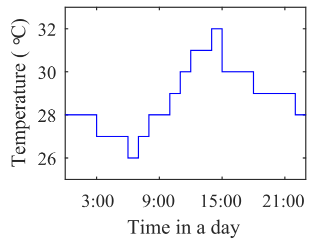

Fig. 3. Outdoor temperature in a day.

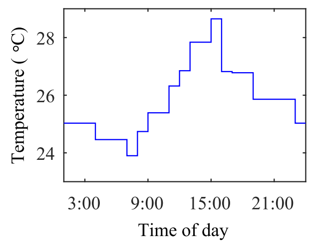

Fig. 4. Indoor temperature of the sixth consumer in a day.

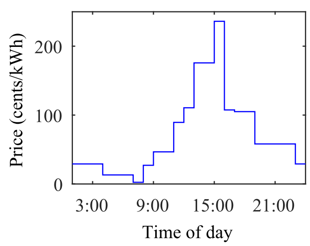

Fig. 5. Price of the utility company in a day.

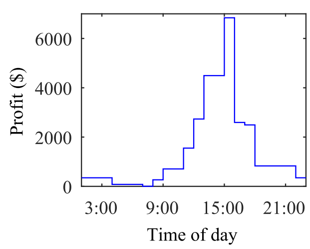

Fig. 6. Profit of the utility company in a day.

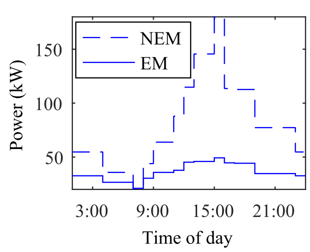

Fig. 7. Total power consumption in a day.

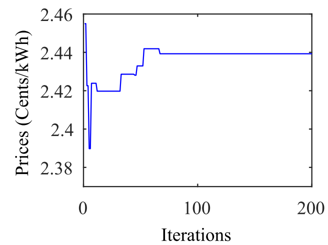

Fig. 8. Convergence curve of the electricity price.

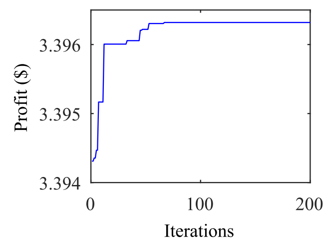

Fig. 9. Convergence curve of the profit of the utility company.

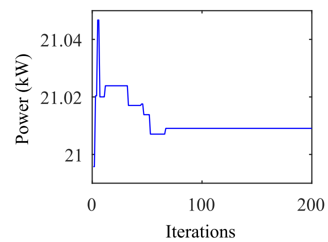

Fig. 10. Convergence curve of the total power consumption.

TABLE III
PERFORMANCE OF EM

	The proposed algorithm	No energy management
Daily EC (kWh)	862	1895
EC reduction	54.51%	/
PAR	1.37	2.28

compared with no EM (NEM) scheme. The NEM scheme means that the consumers use the appliances without sacrificing any comfort. The electricity price announced by the utility company, the profit of the utility company, and the total power consumption of all residential consumers can be obtained from the PSO algorithm, and the convergence results are shown in Figs. 8–10. The overall performance of the EM algorithm is given in Table III, from which we observe that the daily energy consumption (EC) is reduced by 54.51%. The PAR is reduced from 2.28 to 1.37 using the proposed EM algorithm.

VI. CONCLUSION

In this brief, we formulate a two-level optimization model between the utility company and the residential consumers. An ESC algorithm is utilized to study the EM problem with unknown dynamics of the residential consumers. Due to upper and lower limits on the power consumption, the positive projection will be introduced to the ESC algorithm. We propose an approximation method to make the algorithm continuous and prove the stability of the algorithm. Using the PSO algorithm, we optimize the electricity price of the utility company to maximize its profit. It is shown that the algorithm can converge to a small neighborhood of the optimal solution with unknown dynamics of the appliances, and the optimal profit of the utility company can be obtained.

Appendix A
Proof of Theorem 1

Proof: $q_1(\hat{x}_i)$ and $q_2(\hat{x}_i)$ can be denoted as the infinitesimal of higher order that are denoted as $O(\epsilon^2)$ [31]. Then, the extremum seeking algorithm is approximated to

$$\begin{aligned}\frac{\mathrm{d}\hat{x}_i}{\mathrm{d}t} = \hat{k}_i\bigg(&-\frac{\mathrm{d}g_i(f_i(\hat{x}_i))}{\mathrm{d}f_i(\hat{x}_i)}\cdot\xi_i - p\\ &+2m_1\big([\hat{x}_i^{\min}-\hat{x}_i]^+ + O(\epsilon^2)\big)\\ &-2m_2\big([\hat{x}_i-\hat{x}_i^{\max}]^+ + O(\epsilon^2)\big)\bigg)\end{aligned} \tag{21}$$

$$\frac{\mathrm{d}\xi_i}{\mathrm{d}t} = -\hat{\omega}_i^{\xi}\Big(\xi_i - \frac{2}{a}f_i(\hat{x}_i + a\sin(\omega t))\sin(\omega t)\Big). \tag{22}$$

If $\tau = \omega_L t$, then we obtain the ESC algorithm in the new time scale τ

$$\begin{aligned}\frac{\mathrm{d}\hat{x}_i}{\mathrm{d}\tau} = \delta\omega_i^{x}\bigg(&-\frac{\mathrm{d}g_i(f_i(\hat{x}_i))}{\mathrm{d}f_i(\hat{x}_i)}\cdot\xi_i - p\\ &+2m_1\big([\hat{x}_i^{\min}-\hat{x}_i]^+ + O(\epsilon^2)\big)\\ &-2m_2\big([\hat{x}_i-\hat{x}_i^{\max}]^+ + O(\epsilon^2)\big)\bigg)\end{aligned} \tag{23}$$

$$\frac{\mathrm{d}\xi_i}{\mathrm{d}\tau} = -\omega_i^{\xi}\bigg(\xi_i - \frac{2}{a}f_i(\hat{x}_i + a\sin(\omega t))\sin(\omega t)\bigg). \tag{24}$$

According to the averaging method of the dynamic system [21], the dynamic system with periodic disturbance can be approximated by its average system

$$\begin{aligned}\frac{\mathrm{d}\hat{x}_i^A}{\mathrm{d}\tau} = \delta\omega_i^{x}\bigg(&-\frac{\mathrm{d}g_i(f_i(\hat{x}_i^A))}{\mathrm{d}f_i(\hat{x}_i^A)}\cdot\xi_i^A - p\\ &+2m_1\big([\hat{x}_i^{\min}-\hat{x}_i^A]^+ + O(\epsilon^2)\big)\\ &-2m_2\big([\hat{x}_i^A-\hat{x}_i^{\max}]^+ + O(\epsilon^2)\big)\bigg)\end{aligned} \tag{25}$$

$$\frac{\mathrm{d}\xi_i^A}{\mathrm{d}\tau} = -\omega_i^{\xi}\big(\xi_i^A - \frac{2}{a}f_i^A\big) \tag{26}$$

where $\hat{x}_i^A$ and ξ_i^A denote the averaging variable with respect to $\hat{x}_i$ and ξ_i, respectively, and f_i^A is defined as

$$f_i^A = \frac{1}{2\pi}\int_0^{2\pi} f_i(\hat{x}_i + a\sin(\omega t))\sin(\omega t)\mathrm{d}t. \tag{27}$$

Approximating $f_i(\hat{x}_i + a\sin(\omega t))$ with Taylor series, we have

$$\begin{aligned}\frac{2}{a}f_i^A &= \frac{1}{a\pi}\int_0^{2\pi}\bigg(f_i(\hat{x}_i) + a\sin(\omega t)\frac{\mathrm{d}f_i(\hat{x}_i)}{\mathrm{d}\hat{x}_i}\\ &\qquad + \sum_{n=2}^{\infty}\frac{(a\sin(\omega t))^n}{n!}\cdot\frac{\mathrm{d}^n f_i(\hat{x}_i)}{\mathrm{d}(\hat{x}_i)^n}\bigg)\sin(\omega t)\mathrm{d}t\\ &= \frac{\mathrm{d}f_i(\hat{x}_i^A)}{\mathrm{d}\hat{x}_i^A} + O_i^f(a^2)\end{aligned} \tag{28}$$

where $O_i^f(a^2)$ denotes the infinitesimal of a^2.

If $\alpha = \delta\tau$ and substituting (28) into the average systems (25) and (26), we have the dynamic system in time scale α

$$\begin{aligned}\frac{\mathrm{d}\hat{x}_i^A}{\mathrm{d}\alpha} = \omega_i^{x}\bigg(&-\frac{\mathrm{d}g_i(f_i(\hat{x}_i^A))}{\mathrm{d}f_i(\hat{x}_i^A)}\cdot\xi_i^A - p\\ &+2m_1\big([\hat{x}_i^{\min}-\hat{x}_i^A]^+ + O(\epsilon^2)\big)\\ &-2m_2\big([\hat{x}_i^A-\hat{x}_i^{\max}]^+ + O(\epsilon^2)\big)\bigg)\end{aligned} \tag{29}$$

$$\delta\frac{\mathrm{d}\xi_i^A}{\mathrm{d}\alpha} = -\omega_i^{\xi}\bigg(\xi_i^A - \frac{\mathrm{d}f_i(\hat{x}_i^A)}{\mathrm{d}\hat{x}_i^A} - O_i^f(a^2)\bigg). \tag{30}$$

The systems (29) and (30) are the standard singular perturbation forms with fast dynamics ξ_i^A when δ is small [21]. "Freezing" the dynamics (30) at the equilibrium $\xi_i^{A*} = \mathrm{d}f_i(\hat{x}_i^A)/\mathrm{d}\hat{x}_i^A + O_i^f(a^2)$, we obtain the reduced system

$$\begin{aligned}\frac{\mathrm{d}\hat{x}_i^r}{\mathrm{d}\alpha} = \omega_i^{x}\bigg(&-\frac{\mathrm{d}g_i(f_i(\hat{x}_i^r))}{\mathrm{d}f_i(\hat{x}_i^r)}\cdot\Big(\frac{\mathrm{d}f_i(\hat{x}_i^r)}{\mathrm{d}\hat{x}_i^r} + O_i^f(a^2)\Big) - p\\ &+2m_1\big([\hat{x}_i^{\min}-\hat{x}_i^r]^+ + O(\epsilon^2)\big)\\ &-2m_2\big([\hat{x}_i^r-\hat{x}_i^{\max}]^+ + O(\epsilon^2)\big)\bigg)\end{aligned} \tag{31}$$

where $\hat{x}_i^r$ denotes the variable in the reduced system with respect to $\hat{l}_i^A$. Next, we will prove the practically asymptotic stability of the reduced system. We assume that $\hat{x}^{r*}$ is the optimal solution of the optimization problem in (P2) and define $\hat{x}^r = (\hat{x}_1^r, \ldots, \hat{x}_i^r, \ldots, \hat{x}_N^r)^T$, $\hat{x}^{r*} = (\hat{x}_1^{r*}, \ldots, \hat{x}_i^{r*}, \ldots, \hat{x}_N^{r*})^T$, and $\tilde{x}^r = \hat{x}^r - \hat{x}^{r*}$. We choose a candidate Lyapunov function

$$V = \frac{1}{2}\tilde{x}^{rT}\Phi^{-1}\tilde{x}^r \tag{32}$$

where $\Phi = \mathrm{diag}\{k_i\}$ is a diagonal matrice. Define $R_N = (1, \ldots, 1)^T$ with $|R_N| = N$. Then, the derivative of the Lyapunov function along the reduced system (31) is denoted as

$$\begin{aligned}\dot{V} = \tilde{x}^{rT}\bigg(&-g'(\hat{x}^r) - O^f(g_i'(f_i(\hat{x}_i^r)), a^2) - R_N^T p\\ &+2m_1\big([\hat{x}^{\min}-\hat{x}^r]^+ + O(\epsilon^2)\big)\\ &-2m_2\big([\hat{x}^r-\hat{x}^{\max}]^+ + O(\epsilon^2)\big)\bigg)\end{aligned} \tag{33}$$

where $g'(\hat{x}^r) = (\mathrm{d}g_1(\hat{x}_1^r)/\mathrm{d}\hat{x}_1^r, \ldots, \mathrm{d}g_i(\hat{x}_i^r)/\mathrm{d}\hat{x}_i^r, \ldots, \mathrm{d}g_N(\hat{x}_N^r)/\mathrm{d}\hat{x}_N^r)^T$, $\mathrm{d}g_i(\hat{x}_i^r)/\mathrm{d}\hat{x}_i^r = \mathrm{d}g_i(f_i(\hat{x}_i^r))/\mathrm{d}f_i(\hat{x}_i^r)\cdot\mathrm{d}f_i(\hat{x}_i^r)/\mathrm{d}\hat{x}_i^r$, and $O^f(g_i'(f_i(\hat{x}_i^r)), a^2) = (g_1'(f_1(\hat{x}_1^r))\cdot O_1^f(a^2), \ldots, g_i'(f_i(\hat{x}_i^r))\cdot O_i^f(a^2), \ldots, g_N'(f_N(\hat{x}_N^r))\cdot O_N^f(a^2))^T$.

At the equilibrium, we have $g'(\hat{x}^{r*}) = -R_N^T p + 2m_1[\hat{x}^{\min} - \hat{x}^{r*}]^+ - 2m_2[\hat{x}^{r*} - \hat{x}^{\max}]^+$. Then, we can obtain

$$\begin{aligned}\dot{V} &= \tilde{x}^{rT}\Big(- g'(\hat{x}^r) + g'(\hat{x}^{r*}) - O^f\big(g_i'(f_i(\hat{x}_i^r)), a^2\big) \\ &\quad -2m_1[\hat{x}^{\min} - \hat{x}^{r*}]^+ + 2m_2[\hat{x}^{r*} - \hat{x}^{\max}]^+ \\ &\quad +2m_1\big([\hat{x}^{\min} - \hat{x}^r]^+ + O(\epsilon^2)\big) \\ &\quad -2m_2\big([\hat{x}^r - \hat{x}^{\max}]^+ + O(\epsilon^2)\big)\Big) \\ &= \tilde{x}^{rT}\Big(- g'(\hat{x}^r) + g'(\hat{x}^{r*}) - O^f(g_i'(f_i(\hat{x}_i^r)), a^2) \\ &\quad - 2m_1\big[\hat{x}^{\min} - \hat{x}^{r*}\big]^+ + 2m_2\big[\hat{x}^{r*} - \hat{x}^{\max}\big]^+ \\ &\quad + 2m_1\big[\hat{x}^{\min} - \hat{x}^r\big]^+ - 2m_2\big[\hat{x}^r - \hat{x}^{\max}\big]^+ \\ &\quad + 2(m_1 - m_2)O(\epsilon^2)\Big). \end{aligned} \tag{34}$$

Using the mean value theorem, we have

$$R_N g'(\hat{x}^r) - R_N g'(\hat{x}^{r*}) = g''(\hat{x}^m)^T \tilde{x}^r \tag{35}$$

where $\hat{x}^m = (\hat{x}_1^m, \ldots, \hat{x}_i^m, \ldots, \hat{x}_N^m)^T$ such that $\hat{x}_i^m \in [\hat{x}_i^r, \hat{x}_i^{r*}]$ or $\hat{x}_i^m \in [\hat{x}_i^{r*}, \hat{x}_i^r]$ and $g''(\hat{x}^r) = (\mathrm{d}g_1'(\hat{x}_1^r)/\mathrm{d}\hat{x}_1^r, \ldots, \mathrm{d}g_i'(\hat{x}_i^r)/\mathrm{d}\hat{x}_i^r, \ldots, \mathrm{d}g_N'(\hat{x}_N^r)/\mathrm{d}\hat{x}_N^r)^T$.

Substituting (35) into (34) and combining with the lower bounds of $g_i''(\hat{x}_i^r) = \mathrm{d}g_i'(\hat{x}_i^r)/\mathrm{d}\hat{x}_i^r$, we can obtain that when $\hat{x}^r < \hat{x}^{\min}$, the derivative of the Lyapunov function can be bounded by

$$\begin{aligned}\dot{V} &\le \tilde{x}^{rT}\big(- g'(\hat{x}^r) + g'(\hat{x}^{r*}) - O^f(g_i'(f_i(\hat{x}_i^r)), a^2) \\ &\quad - 2m_1\tilde{x}^r\big) + 2\tilde{x}^{rT}(m_1 - m_2)O(\epsilon^2) \\ &\le -(\eta + 2m_1)\|\tilde{x}^r\|^2 - \tilde{x}^{rT} O^f\big(g_i'\big(f_i(\hat{x}_i^r)\big), a^2\big) \\ &\quad +2\tilde{x}^{rT}(m_1 - m_2)O(\epsilon^2) \end{aligned} \tag{36}$$

where $\eta = \{\eta_1, \eta_2, \ldots, \eta_N\}$ is the lower bound of $g''(\hat{x}^r)$.

When $\hat{x}^r > \hat{x}^{\max}$, the derivative of the Lyapunov function can be bounded by

$$\begin{aligned}\dot{V} &\le \tilde{l}^{rT}\big(- g'(\hat{x}^r) + g'(\hat{x}^{r*}) - O^f(g_i'(f_i(\hat{x}_i^r)), a^2) \\ &\quad - 2m_2\tilde{x}^r\big) + 2\tilde{x}^{rT}(m_1 - m_2)O(\epsilon^2) \\ &\le -(\eta + 2m_2)\|\tilde{x}^r\|^2 - \tilde{x}^{rT} O^f\big(g_i'(f_i(\hat{x}_i^r)), a^2\big) \\ &\quad +2\tilde{x}^{rT}(m_1 - m_2)O(\epsilon^2). \end{aligned} \tag{37}$$

When $\hat{x}^{\min} \le \hat{x}^r \le \hat{x}^{\max}$, the derivative of the Lyapunov function can be bounded by

$$\begin{aligned}\dot{V} &\le \tilde{x}^{rT}\big(- g'(\hat{x}^r) + g'(\hat{x}^{r*}) - O^f(g_i'(f_i(\hat{x}_i^r)), a^2)\big) \\ &\quad +2\tilde{x}^{rT}(m_1 - m_2)O(\epsilon^2) \\ &\le -\eta\|\tilde{x}^r\|^2 - \tilde{x}^{rT} O^f(g_i'(f_i(\hat{x}_i^r)), a^2) \\ &\quad +2\tilde{x}^{rT}(m_1 - m_2)O(\epsilon^2). \end{aligned} \tag{38}$$

Under the condition of $\hat{x}^r < \hat{x}^{\min}$, there exists a positive scalar η^* such that

$$\eta^* V = 2(\eta_1 + 2m_1)\omega_{\min}^x V \tag{39}$$

where $\omega_{\min}^x = \min\{\omega_1^x, \ldots, \omega_N^x\}$. When $\eta \in [0, \eta^*]$, we have

$$\begin{aligned}\dot{V} &\le -\eta V - \tilde{x}^{rT} O^f(g_i'(f_i(\hat{x}_i^r)), a^2) \\ &\quad +2\tilde{x}^{rT}(m_1 - m_2)O(\epsilon^2). \end{aligned} \tag{40}$$

For sufficiently small a, there exists μ such that $\mu V^{(1/2)} \ge -\tilde{x}^{rT} O^f(g_i'(f_i(\hat{x}_i^r)), a^2) + 2\tilde{x}^{rT}(m_1 - m_2)O(\epsilon^2)$, i.e., $\dot{V} \le -\eta V + \mu V^{(1/2)}$. Following the comparison principle in [21], we have:

$$\|W\| \le e^{-\frac{\eta}{2}\alpha}\|W(0)\| + \frac{2}{\eta}\mu \tag{41}$$

where $W = \sqrt{V}$. If $\tilde{z}^r = (\tilde{x}_1^r, \ldots, \tilde{x}_N^r)^T$, then we obtain

$$\|\tilde{z}^r\| \le \sqrt{2\omega_{\max}}\|W\| \le \sqrt{2\omega_{\max}}\left(e^{-\frac{\eta}{2}\alpha}\|W(0)\| + \frac{2}{\eta}\mu\right) \tag{42}$$

where $\omega_{\max} = \max\{\omega_1^x, \ldots, \omega_N^x\}$. Similar to $\hat{x}^r < \hat{x}^{\min}$, when $\hat{x}^r > \hat{x}^{\max}$ or $\hat{x}^{\min} \le \hat{x}^r \le \hat{x}^{\max}$, we can also have the above conclusions. Thus, the reduced system (31) is SPA stable, uniformly in (a, ϵ). The upper bound of a and ϵ can be determined by $\mu V^{(1/2)} \ge -\tilde{x}^{rT} O^f(g_i'(f_i(\hat{x}_i^r)), a^2) + 2\tilde{x}^{rT}(m_1 - m_2)O(\epsilon^2)$, which can be transformed to

$$k\mu \ge \mu O(a^2) + 2(m_1 - m_2)O(\epsilon^2) \tag{43}$$

where $k = (k_1, k_2, \ldots, k_N)$, $\mu = \{\mu_1, \mu_2, \ldots, \mu_N\}$, and $\mu_i = \max_{f_i(\hat{x}_i^r)} g_i'(f_i(\hat{x}_i^r))$. Define the boundary system as $e_i^x = \xi_i^A - 2/af_i^A$ for $i = 1, \ldots, N$. According to (30), the boundary system is globally asymptotically stable. Combining with the SPA stability of the reduced system and [22, Lemma 2], the average systems (25) and (26) are SPA stable, uniformly in (a, ϵ, δ) in τ-time scale. Following [22, Lemma 1], the original systems (6) and (7) are SPA stable, uniformly in (a, ϵ, δ).

Appendix B
Proof of Theorem 2

Proof: The systems (14)–(16) are the standard singular perturbation forms with fast dynamics $\mathrm{d}y_i/\mathrm{d}t = \gamma_i s_i(\hat{x}_i + a\sin(\omega t), \hat{y}_i)$ when ε is small. "Freezing" the dynamics (16) at the equilibrium $y_i = f_i(x_i + a\sin(\omega t))$, we obtain the reduced systems (6) and (7), which are proved to be SPA stable, uniformly in (a, ϵ, δ), in Theorem 1. Following Assumption 2, the boundary system of (14)–(16) is globally asymptotically stable, and we conclude that the systems (14)–(16) are SPA stable, uniformly in $(a, \epsilon, \delta, \zeta)$.

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