Performance Analysis of Series Elastic Actuator based on Maximum Torque Transmissibility

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Abstract—The use of the Series Elastic Actuator (SEA) system as an actuator system equipped with a compliant element has contributed not only to advances in human interacting robots but also to a wide range of improvements in the robotics area. Nevertheless, there are still limitations in its performance; the elastic spring that is adopted to provide compliance is considered to limit the actuator performance thus lowering the frequency bandwidth of force/torque generation, and the bandwidth decreases even more when it is supposed to provide large torque.

This weakness is in turn owing to the limitations of motor and motor drives such as torque and velocity limits. In this paper, mathematical tools to analyze the impact of these limitations on the performance of SEA as a transmission system are provided. A novel criterion called Maximum Torque Transmissibility (MTT)is defined to assess the ability of SEA to fully utilize maximum continuous motor torque. Moreover, an original frequency bandwidth concept, maximum torque frequency bandwidth, which can indicate the maximum frequency up to which the SEA can generate the maximum torque, is proposed based on the proposed MTT. The proposed MTT can be utilized as a unique criterion of the performance, and thus various design parameters including the load condition, mechanical design parameters, and controller parameters of a SEA can be evaluated with its use. Experimental results under various conditions verify that MTT can precisely indicate the limitation of the performance of SEA, and that it can be utilized to accurately analyze the limitation of the controller of SEA.

I. INTRODUCTION

The necessity for compliant actuation has emerged as a key technology requirement in the field of robotics in order to achieve safe interactions with humans while achieving given tasks. There are many approaches investigated to achieve safe robot interaction with humans, such as link-mass reduction, impedance control, and increased compliance [1]–[3].

In the wake of these research approaches, SEA, which embeds a compliant element, has been developed [4]. Specifically, this element is a spring set in series between the motor and the load, which facilitates force sensing without extra sensors, as it enables control of the force to the load through spring deformation.

SEA has been developed as an ideal force source, which has low output impedance with low reflected inertia and low friction [5] for safety and high force fidelity. The advantages of SEA over conventional actuators also include energy storage capability, low cost force measurement, low cost transmission, and better force control stability [6], [7]. Having these beneficial characteristics, SEA has been applied to various robotic devices, such as rehabilitation robots [8], humanoid robots [9], quadrupedal robots [10], robotic prosthesis [11], [12], and industrial robots [13]. As the use of SEA increases, many robotics researchers have demonstrated the performance of SEA as the next generation actuator system, and the results of their studies in this regard have been reported in many recent papers [14]–[19].

In spite of the many benefits of SEA, mechanical complexity has been its most problematic issue since its dynamics consist of a combination of motors as well as gears and compliant components. This mechanical complexity in the dynamics of SEA leads to various limitations including the difficulty of high performance controller design [20]–[23]. A number of research studies have investigated the ability and limit of the mechanical characteristics of SEA to address this problem [24]–[26].

One of the main limitations of SEA, which is widely accepted among SEA researchers, and has been demonstrated and analyzed by many researchers [11], [23], [27], is its reduced bandwidth [28] particularly when the magnitude of the force that the SEA is supposed to provide is large [4], [29].

In addition to this bandwidth limitation, the deterioration of large/maximum torque generation has been an issue in SEA applications. [30] has analyzed this problem in detail based on the electro-mechanical characteristics of a motor; the torque of a motor decreases as its speed increases, which suggests that the motor velocity limitation can hinder the large/maximum torque generation of SEA. This has been a significant issue in SEA, and several succeeding studies have accepted this idea and utilized it as a design criterion in SEA systems [11], [27], [31].

The actual limitation of motor and motor drive, however, does not always follow the constant electro-mechanical characteristics utilized in [30], [31], particularly when it is under current control. Moreover, the analyses in [30], [31] did not take into account the controller design nor the load conditions. In other words, the studies did not fully consider the dynamics of SEA, and the results of the studies provided more qualitative discussion than quantitative ones. Therefore, it is difficult to use the results as criterion for the mechanical or control design of SEA. This paper, therefore, proposes a more practical analysis of the limitation of large/maximum torque generation in SEA taking into consideration all the feedback controllers and load conditions.

An accurate and practical analytic tool is proposed to address the large force generation problem of SEA in this paper; a novel criterion, called Maximum Torque Transmissibility (MTT), is proposed to assess the ability of SEA to fully utilize the maximum continuous motor torque. By using

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the proposed MTT, it can be quantitatively shown that the performance of SEA is either maintained or deteriorated when the desired torque becomes large and how the mechanical design parameters and controller parameters affect large force generation performance.

The proposed MTT is a function of frequency and it can analyze the performance of SEA in the frequency domain, and thus, the accurate frequency bandwidth can be determined. This bandwidth is named maximum torque bandwidth and can specify the frequency bandwidth up to which the transmission of the maximum motor torque is guaranteed.

In the proposed MTT, not only the torque limit but also the velocity limit of the motor drive is taken into account as well, thus providing the comprehensive analysis of the limitation of the SEA performance caused by the motor drive. In addition, the proposed MTT can be applied to force generation during dynamic motions, which has not been studied in previous research.

Focusing on the foregoing, this paper will make the following contributions.

- 1) A novel criterion, MTT is proposed to assess the ability of SEA to transmit the maximum motor torque.
- 2) The maximum torque bandwidth of SEA is defined using the proposed MTT.
- 3) Using the proposed maximum torque bandwidth as criterion, the influence of the load condition, the mechanical parameters, and the controllers of SEA are evaluated, and design guidelines are given based on it.

This paper is organized as follows. Section II discusses the necessity and definition of the proposed MTT. The large torque generation problem is first introduced, then the dynamic characteristic of SEA (including environments) is explained. MTT is derived based on the dynamic characteristic of SEA. In Sec. III, the proposed MTT is verified through experiments. Through the experiments, it is verified that the MTT can identify the relationship between gear ratio and large torque generation capability. Section IV shows that the proposed MTT can be utilized as guide for SEA mechanical and controller design. Concluding discussions are given in Sec. V.

II. MAXIMUM TORQUE TRANSMISSIBILITY OF SEA

This section describes the deterioration of force control performance when SEA provides a large force and explains the background that needs to be analyzed to address this phenomenon. To solve this problem, Maximum Torque Transmissibility (MTT), defined as a quantitative criterion that evaluates SEA force control performance considering motor torque limit and velocity limit, is utilized.

A. Large Torque Generation Performance Deterioration

As SEA can be considered a transmission system consisting of a spring and a reduction gear, it is supposed to be capable of transmitting the maximum continuous motor torque multiplied by the gear ratio. This torque transmission or generation of SEA is realized through the control of spring deformation, which guarantees the transmission performance up to a certain



Fig. 1: Force tracking performance with two different desired torque magnitudes. (a) has the desired torque magnitude set to $0.6 \times$ the gear ratio \times the maximum continuous motor torque, while (b) has the desired torque magnitude set to the gear ratio \times the maximum continuous motor torque. The upper graphs show torque reference tracking results and errors, whereas the lower graphs show the current references and actual currents of the motor driver.

frequency bandwidth. However, this bandwidth is degenerated when SEA is controlled to provide the maximum torque.

Figure 1 shows the experimental result of force generation/tracking of SEA, where the force controller is designed to provide up to 8 Hz frequency bandwidth. Five Hz sinusoidal torque patterns with two different magnitudes were applied as the desired torque reference: one was 60% of the rated motor torque (multiplied by the gear ratio), and the other was 100% of the rated motor torque.

The result shows that the SEA failed to generate the desired torque at 5 Hz when the reference magnitude is set to the maximum level. It can be noticed that the controller is designed to guarantee tracking at 5 Hz, which can be verified in Fig. 1. (a). The measured current that is actually provided to the motor (the bottom figures of Fig. 1) explains the cause of this performance deterioration; the current is limited by the motor drive when the desired torque magnitude is set large, and fails to track the current reference set by the force controller of the SEA. It then becomes necessary to know when and how this maximum torque generation failure start



Fig. 2: Continuous operating range of a DC motor considering various conditions

to happen.

B. Limitation of Motor and Drive System

In most cases, the limitation of the motor torque and velocity is determined by the motor drive system, which is far below the torque/speed curve of the motor [32]. Figure 2 shows a typical operation range of a DC motor along with the torque/speed curve of the motor. Notice that the torque limitation (the nominal torque, $\mathcal{T}_{nominal}$) and the velocity limitation (the maximum permissible speed V_p) are set independently from the torque/speed curve.

In [33], the velocity limit was considered to follow the torque/speed curve (the thick line in Fig. 2)implying that it varies with regard to the torque output. The actual torque and velocity limits, however, are not set according to this line but according to constant values independent of each other [32]. Therefore, the discussion in [33] cannot be regarded as effective in practical cases.

The torque and velocity that are generated by a DC or BLDC motor are limited by its electro-mechanical dynamics. In addition to this limitation, the motor driver also sets limitations on the torque and velocity.

The nominal torque of a DC motor is determined by the thermal condition, i.e., the nominal current of the motor is selected so that the winding temperature is kept under the maximum temperature.

In Fig. 2, the thermal condition is depicted as a red solid line on the torque/speed curve, and the nominal current value is determined by the intersection point between the torque/speed curve and the thermal curve. The maximum continuous torque of motor $\mathcal{T}_{nominal}$ is determined by the nominal current multiplied by the torque constant.

The motor speed v_m is limited due to various reasons: the mechanical wear and the electro-erosion of brushes and commutators of a brushed DC motor, and the service life of the bearings [33]. Figure 2 shows the limitation of the motor velocity which is also known as the maximum permissible speed V_p . Note that V_p is constant and not related to the motor torque output.

These limitations of the motor lead to the limitation in the performance of SEA as a transmission system. It is required that SEA fully utilize the continuous operation range in Fig. 2 of the motor as an efficient transmission. The proposed maximum torque transmissibility, which is derived by taking into consideration these motor limitations, can be a criterion to indicate this effectiveness.

C. Generalized Dynamic Model of SEA

In order to accurately explore and exploit the best performance of SEA, it is required to understand the dynamic characteristics of SEA in terms of motion and force/torque generation.

The mechanical structure of SEA becomes more complicated than a conventional rigid actuator system as SEA consists of several dynamic components such as an electric motor, a spring for elasticity, reduction gears, and the load.

As the spring connects the motor and the load in SEA, it can be modeled as a two-mass system. The left figures in Fig. 3 illustrates the dynamic model of SEA as a two-mass system, where the motor $P_m(s)$ and the load $P_l(s)$ are connected through the spring K_s . $P_m(s)$ and $P_l(s)$ are modeled as

$$P_m(s) = \frac{1}{J_m s^2 + B_m s}, P_l(s) = \frac{1}{J_l s^2 + B_l s},$$
(1)

where J_m and B_m represent the moment of inertia and the damping coefficient of the motor, respectively, and J_l and B_l represent those of the load. In this paper, load side dynamics $P_l(s)$ is modeled as (1), whereas SEA can be contacted with a variety of external environments, which can be modeled as closely as possible to a real contact environment [18].

The output torque of SEA as a transmission is τ_{out} which is transmitted to the load, and it is determined by $K_s\theta_d$, the product of the spring deformation and the stiffness, respectively. Therefore, the dynamic characteristic of SEA can be defined as the transfer function from the motor torque τ_c to the output torque τ_{out} , given as

$$P_{dynamic}(s) = \frac{N_m^{-1}K_s P_m(s)}{1 + P_l(s)K_s + N_m^{-2}P_m(s)K_s}.$$
 (2)



Fig. 3: Block diagram describing SEA dynamics. The left side figures are cases where SEA moves freely connected to a load, and the right side figures are cases where SEA is in contact with a stiff environment. $P_m(s)$ and $P_l(s)$ represent the dynamics of the motor and the load in the SEA, respectively. K_s is the stiffness of the spring, N_m is the gear ratio, and τ_c is the motor torque input to the SEA. θ_m and θ_l are the angles of the motor and the load, respectively, and their difference $\theta_d = N_m^{-1}\theta_m - \theta_l$ represents the spring deflection. v_m is the motor velocity output.

This transfer function can be interpreted as the transmissibility of SEA representing the relationship between the input torque and the output torque of SEA [34].

This derived dynamic characteristic changes when the SEA contacts stiff environments as shown in the right side figures in Fig. 3. In this case, SEA is supposed to provide force/torque directly to the environment rather than to generate motions. By limiting J_l and B_l in the load dynamics (2) to infinity, the transfer function of SEA can be seamlessly shifted to the high impedance environment case, given as follows.

$$P_{static}(s) = \frac{N_m^{-1} K_s P_m(s)}{1 + N_m^{-2} P_m(s) K_s}.$$
(3)

Notice that the spring deflection θ_d becomes equal to the motor angle θ_m multiplied by N_m^{-1} . The block diagram of SEA in this case can be derived in the same way, and is given in the right side figures of Fig. 3.

The transfer function to the motor velocity v_m is also required when analyzing the maximum torque generation. Based on the model in the left side figures of Fig. 3, the transfer function from the motor torque τ_c to the motor velocity can be derived as

$$P_V(s) = \frac{P_m(s)[1 + K_s P_l(s)]s}{1 + P_l(s)K_s + N_m^{-2}P_m(s)K_s}.$$
(4)

In the remainder of the paper, the case when SEA is freely moving with a dynamic load is referred to as the *dynamic load case*, whereas it is called the *static load case* when SEA contacts a stiff environment. In this paper, both cases are considered in the analysis of force generation performance, which is different from other studies where only the static load case was discussed [11], [27], [31], [35].

D. Control Input and Motor Velocity under Force Feedback Control

For SEA to generate the desired torques τ_d , the output torque τ_{out} (which is spring deformation \times spring stiffness) should be controlled. Figure 4 illustrates the SEA system P(s)under the force control C(s) to achieve this. Note that P(s)can be modeled as $P_{dynamic}(s)$ of (2) in the dynamic load case and it can be modeled as $P_{static}(s)$ of (3) in the static load case.

It is important to investigate the control input τ_c and the motor velocity v_m during this force control because it should



Fig. 4: Configuration of SEA force feedback control to provide desired torque τ_d from SEA. P(s) is the transfer function of SEA given in (2) or (3) according to the load condition. $P_v(s)$ is the transfer function from control input to the motor output velocity in (4). C(s) is a feedback controller.

be checked whether τ_c and v_m exceed their limiting values or not.

First, the transfer function from the desired torque output τ_d to the control input τ_c is derived as (5) based on Fig. 4.

$$T_c(s) = \frac{N_m^{-1}C(s)}{1 + N_m^{-1}C(s)P(s)} \mathcal{T}_d(s)$$
(5)

where $T_c(s)$ and $\mathcal{T}_d(s)$ are the control input (i.e., motor torque) and the desired torque output in the Laplace domain, respectively.

Then, the transfer function to the motor velocity v_m under the force control is derived as (6).

$$V_m(s) = P_V(s)T_c(s)$$
(6)
= $P_V(s)\frac{N_m^{-1}C(s)}{1+N_m^{-1}C(s)P(s)}\mathcal{T}_d(s)$

where $V_m(s)$ is the motor velocity in the Laplace domain.

Note that the configuration in Fig. 3 is general without specifying the type of SEA or the type of controller C(s) as long as they are stable. That is, the proposed MTT, which will be defined based on (5) and (6) can be applied to any types of SEA and controller.

In this paper, a P or PD controller is utilized as C(s), the gains of which are designed to guarantee the stability of the closed loop system. Even though a more complicated controller can be utilized, a P or PD controller is utilized here as it is the most widely employed controller.

E. Maximum Torque Transmissibility based on Maximum Continuous Motor Torque

Based on the configuration of the force controller in Sec. II-D, the following three conditions are considered in order to define MTT:

- 1) The nominal torque $\mathcal{T}_{nominal}$ of a motor is adopted as the maximum continuous motor torque $\mathcal{T}_{m.c}$ [33], [35].
- 2) The motor velocity is considered to be restricted when it exceeds the maximum permissible velocity V_p of a motor [33], [35].
- 3) The SEA is controlled to provide the desired torque τ_d by the feedback control of the spring deformation θ_d as shown in Fig. 4 [35].

The maximum torque output \mathcal{T}_d^{max} of SEA is supposed to be the product of the maximum continuous motor torque $\mathcal{T}_{m.c}$ and the gear ratio N_m , as follows.

$$\mathcal{T}_d^{max} = N_m \mathcal{T}_{m.c} \tag{7}$$

Notice that \mathcal{T}_d^{max} is the maximum torque value that can be expected from SEA when it works ideally as a transmission system.

The control input τ_c , which is also the motor torque required to achieve the maximum SEA output torque \mathcal{T}_d^{max} can be derived as follows, based on the relationship between τ_d and τ_c in (5).

$$T_{c}(s) = \frac{\left[1 + K_{s}\left(P_{l}(s) + N_{m}^{-2}P_{m}(s)\right)\right]C(s)N_{m}^{-1}}{1 + K_{s}\left[P_{l}(s) + N_{m}^{-2}P_{m}(s)(1 + C(s))\right]}\mathcal{T}_{d}^{max}$$
(8)

In this equation, the dynamic load model (2) is adopted as the plant model P(s) to describe the dynamics of SEA.

If τ_c in (8) exceeds the maximum continuous motor torque $\mathcal{T}_{m.c}$, SEA cannot generate \mathcal{T}_d^{max} , which leads to the definition of MTT; MTT is defined as the ratio of the magnitude of τ_c in (8) to the maximum continuous motor torque $\mathcal{T}_{m.c}$. By replacing \mathcal{T}_d^{max} in (8) with $N_m \mathcal{T}_{m.c}$ as in (7), the proposed MTT is finally derived as follows.

$$MTT_{\tau} = \frac{1}{\mathcal{T}_{m.c}} |T_{c}(s)| \\ = \left| \frac{\left[1 + K_{s} \left(P_{l}(s) + N_{m}^{-2} P_{m}(s) \right) \right] C(s)}{1 + K_{s} \left[P_{l}(s) + N_{m}^{-2} P_{m}(s) (1 + C(s)) \right]} \right|$$
(9)

Notice that MTT_{τ} in (9) has the following features

- 1) MTT_{τ} is a non-dimensional functional value of s.
- 2) MTT_{τ} is interpreted as the required control input normalized by $\mathcal{T}_{m.c.}$.
- 3) MTT_{τ} can be utilized to analyze the frequency characteristic of the transmissibility of maximum torque.

The magnitude of 1 dB or 0 dB of MTT_{τ} represents the critical level; if the magnitude of MTT_{τ} is larger than 1 dB or 0 dB at a certain frequency, it means that the motor cannot produce the required torque for SEA to generate the maximum torque \mathcal{T}_d^{max} in that frequency, which may cause force generation error or even instability.

F. Maximum Torque Transmissibility based on Maximum Permissible Velocity

As explained in Sec. I, the maximum velocity of the motor is also limited by the drive system, which should be taken into account in MTT [35]. To this end, the motor velocity output to achieve the maximum desired torque output \mathcal{T}_d^{max} is calculated using (6) and (7) as follows.

$$V_m(s) = \frac{N_m^{-1}C(s)P_m(s)[1+K_sP_l(s)]s}{1+K_s\left[P_l(s)+N_m^{-2}P_m(s)(1+C(s))\right]}\mathcal{T}_d^{max},$$
(10)

Another Maximum Torque Transmissibility can be defined by assessing this motor velocity with regard to the maximum permissible velocity V_p , which is given as MTT_V in (11).

$$MTT_{V} = \frac{1}{V_{p}} |V_{m}(s)|$$

$$= \left| \frac{P_{m}(s)C(s)[1 + K_{s}P_{l}(s)]s}{1 + K_{s} \left[P_{l}(s) + N_{m}^{-2}P_{m}(s)(1 + C(s))\right]} \right| \frac{\mathcal{T}_{m.c}}{V_{p}}$$
(11)

Notice that (7) is also utilized in this derivation.

As MTT_{τ} in (9), 0 dB of MTT_V is the critical level over which the motor torque will be restricted by the motor drive, and subsequently, the desired maximum torque cannot be provided by the SEA. It is noticeable that MTT_V includes $\mathcal{T}_{m.c}$ and V_p , which are determined mostly by the intrinsic property of a motor, taking into consideration the thermal, electrical and mechanical properties of the motor.



Fig. 5: (a) MTT_{τ} and (b) MTT_V with various load inertia conditions including the static case. Other parameters are from Table I, and the feedback controller C(s) is designed as a proportional control with the gain $K_p = 1$.

The defined MTT_{τ} in (9) and MTT_V in (11) contain the load model $P_l(s)$. As explained in Sec. II-C, $P_l(s)$ can be modeled in various forms considering the environments in which the SEA contacts. In particular, there are many studies on SEA modeling the contact environment with high impedance described in Sec. II-C as *static load case*. The MTTs of the SEA in the *static load case* are derived by limiting J_l and B_l to infinity, which are given as:

$$MTT_{\tau}^{sta} = \left| \frac{\left[1 + N_m^{-2} K_s P_m(s) \right] C(s)}{1 + N_m^{-2} K_s P_m(s) (1 + C(s))} \right|, \tag{12}$$

$$MTT_{V}^{sta} = \left| \frac{P_{m}(s)C(s)s}{1 + N_{m}^{-2}K_{s}P_{m}(s)(1 + C(s))} \right| \frac{\mathcal{T}_{m.c}}{V_{p}}.$$
 (13)

G. Maximum Torque Bandwidth

Utilizing the proposed MTT, a novel frequency bandwidth, the maximum torque frequency bandwidth can be defined, in which the maximum torque generation of SEA is guaranteed. The maximum torque frequency bandwidth can play a key role in the evaluation of mechanical design and controller design as an analysis tool that can indicate the influence of design parameters on MTT.

Figure 5 (a) shows the MTT_{τ} with various load conditions. The maximum torque bandwidth $\omega_{MT_{\tau}}$ in this figure is defined as the lowest frequency where MTT_{τ} meets 0 dB. This

$$\left| \frac{\left[1 + K_s \left(P_l(s) + N_m^{-2} P_m(s) \right) \right] C(s)}{1 + K_s \left[P_l(s) + N_m^{-2} P_m(s) (1 + C(s)) \right]} \right|_{s=j\omega_{MT_\tau}} = 1.$$
(14)

Figure 5 (a) shows how the maximum torque bandwidth changes as J_l changes; the critical frequency lowers as J_l increases, and it can be regarded that the static load condition is the poorest case in terms of the maximum torque bandwidth.

 MTT_V also defines the frequency bandwidth over which SEA cannot generate the desired maximum torque due to the motor velocity limitation. This bandwidth can be considered the maximum velocity bandwidth ω_{MT_V} , where the motor is required to rotate at its maximum speed to provide \mathcal{T}_{max}^d . ω_{MT_V} can be derived using the proposed MTT_V in (11) as follows.

$$\left| \frac{C(s)P_m(s)[1+K_sP_l(s)]s}{1+K_s\left[P_l(s)+N_m^{-2}P_m(s)(1+C(s))\right]} \frac{\mathcal{T}_{m.c}}{V_p} \right|_{s=j\omega_{MT_V}} = 1$$
(15)

Figure 5 (b) shows MTT_v with various load conditions including the static case. In this figure, ω_{MT_V} is defined as the lowest frequency where MTT_v meets 0 dB, which changes with varying J_l . Similar to ω_{MT_τ} , ω_{MT_V} decreases when J_l increases.

The decrease in the maximum torque frequency bandwidth with regard to the increasing load inertia is due to the decrease in the plant frequency bandwidth given in (2), namely, the load condition affects the maximum torque frequency bandwidth mostly in a similar way it affects the plant frequency bandwidth. This is not the case with other parameters, which is elaborated in Sec III-B.

Between $\omega_{MT_{\tau}}$ and $\omega_{MT_{V}}$, the smaller frequency is a more critical condition for maximum torque generation of SEA, and thus the final maximum torque frequency bandwidth is determined as (16).

$$\omega_{MT} = \min(\omega_{MT_{\tau}}, \omega_{MT_{V}}) \tag{16}$$

III. VERIFICATION OF MTT THROUGH EXPERIMENTS USING VARYING GEAR TRANSMISSION

In this section, the proposed MTT is verified through various experiments, and it is demonstrated that the proposed MTT can be utilized to analyze the force generation performance of SEA.

A. Experimental Setup with Varying Gear Transmission

In order to investigate whether the derived MTT_{τ} and MTT_{v} can identify the performance limitation precisely under various conditions of SEA and how MTT reflects the effect of the mechanical parameter of SEA, a varying gear ratio transmission set is developed and utilized in the following case study.

Figure 6 is SEA with varying reduction gears, which is called Varying Gear Transmission (VGT). It consists of a

motor with a varying gear stage composed of timing pulleys with timing belts (for low backlash and friction) and a spring. The power source of VGT is a Maxon BLDC motor with the fixed 6:1 reduction ratio. An encoder to measure the motor angle θ_m is attached to the motor.

The second gear stage is the varying gear stage that can shift its ratio from 6:1 to 1:6 by changing the timing belt, which means the whole gear ratio from the motor to the spring changes from 1:1 to 36:1 (1:1, 2.4:1, 4.5:1, 8:1, 15:1, and 36:1) depending on the position of the second timing belt. This varying gear ratio corresponds to N_m in Fig. 3. The shaft of the second gear is connected to a spring with stiffness K_s and the external load, which corresponds to J_l and B_l , can be attached to the other end of the spring.

In order to obtain the dynamic model (3) of the experimental setup, the frequency response from the motor torque to the spring torque is measured using an FFT analyzer (ONO-SOKKI CF-9400). Table I shows the parameters of the setup estimated in this way.

B. Verification of MTT in the Static Load Case

Experiments have been conducted to verify the following points with the load side fixed to simulate the static load case.

- Reliability of the proposed Maximum Torque Transmissibility with various mechanical parameters
- 2) The maximum torque bandwidth ω_{MT} as an analysis tool to evaluate SEA design parameters

To verify the proposed MTTs in (9) and (11) which are functions of frequency, SEA with the VGT is controlled to follow desired torques while the load side of the VGT is fixed. Chirp signals were employed as the desired torque reference, and the motor torque τ_c and the motor velocity were measured



Fig. 6: Varying Gear Transmission for MTT experiments

TABLE I: Identified system parameters.

Parameters	Notations	Identified value
Motor inertia	J_m	$0.000075 \text{ kg} \cdot \text{m}^2$
Load inertia	J_l	0.005 kg· m ²
Motor damping	B_m	0.0006 N· s/m
Load damping	B_l	0.08 N· s/m
Spring stiffness	K_s	1.1 N· m /rad
Maximum continuous torque	$\mathcal{T}_{m.c}$	0.0315 N·m
Maximum permissible velocity	V_p	10.472 rad/s
Spring stiffness Maximum continuous torque Maximum permissible velocity	$ \begin{array}{c} K_s \\ \mathcal{T}_{m.c} \\ V_p \end{array} $	1.1 N· m /rad 0.0315 N·m 10.472 rad/s

and normalized by $\mathcal{T}_{m.c}$ and V_p , respectively. The magnitude of the chirp signals were determined based on (7), which change according to the selected gear ratio. The frequency range of the chirp signals was set from 0 rad/s to 50 rad/s. A proportional force feedback control with the gain K_p was employed as C(s) in the experiments. The normalized torque and velocity measurements are compared with the proposed MTT_{τ} and MTT_V in (12) and (13),respectively, to verify that they precisely portray the transmissibility or the SEA.

Experiments were conducted with two different sets of reduction gear ratio, which are 1:1 and 36:1. Figures 7 (a) and (b) show the experimental results. The thin red dotted lines are the measured control input τ_c normalized by $\mathcal{T}_{m.c}$ and the bold solid green lines are MTT_{τ} calculated from (9) in the upper graphs of (a) and (b). The thin black dotted lines are the velocity output normalized by V_p , and the bold solid blue lines are MTT_V from (11) in the middle graphs. The dashed lines in all the graphs represent the maximum boundary for torque and velocity. Lastly, the black bold lines in the lower graphs are torque control errors normalized by the desired torque output \mathcal{T}_d^{max} .

From these results, it is verified that the proposed MTT_{τ} and MTT_V can successfully portray the required motor torque and velocity, respectively, and thus can be utilized to express maximum torque transmissibility. Even though the motor driver occasionally allows the motor to generate torque over the maximum continuous torque (based on motor temperature and duration time), the measured actual motor torques were mostly bounded by the maximum continuous motor torque.

Restriction by the maximum permissible velocity is done in a different way from the torque limitation: when the motor velocity exceeds maximum permissible velocity, the velocity is not directly restricted to be the maximum value, but the current or the motor torque decreases instead. This is why the actual normalized motor velocity can occasionally go over 1, the maximum permissible velocity level in Fig. 7 (b). However, due to the limitation, the motor torque decreases when the motor velocity becomes greater than the maximum permissible velocity, and thus cannot reach the required level, as can be verified by an examination of the normalized motor input in Fig. 7 (b).

Any of both limitations causes large torque control error from the desired torque output: when the gear ratio N_m is small ($N_m = 1$), $\omega_{MT_{\tau}}$ determines the maximum torque bandwidth, beyond which the motor torque is limited and the torque error increases as shown in Fig. 7 (a). As the gear ratio becomes large ($N_m = 36$), ω_{MT_V} becomes smaller than $\omega_{MT_{\tau}}$ and determines the maximum torque bandwidth as shown in Fig. 7 (b).

Large torque errors caused by the limitations in this experiment imply that the frequency bandwidth estimated without consideration of the motor limit can be erroneous. On the other hand, the proposed maximum torque bandwidth can precisely indicate the frequency up to which the torque control performance is guaranteed and the maximum torque transmission is achieved.

The MTT for the dynamic load case is verified through the experiments in the following section.



 $\tau_c/\tau_{m.c}$

 V_m/V_p

2 0

2

-26

10

Normalized error



30

40

50

60

20



(b) Normalized motor input and velocity with MTT_{τ} and MTT_V $(N_m=36)$

Fig. 7: Experimental results of MTT verification with various gear ratios



Fig. 8: MTT_{τ} and $\omega_{MT_{\tau}}$ with regard to various K_p values.

IV. DISCUSSION- NOVEL BANDWIDTH CRITERION USING MTT

In this section, it is shown that MTT, in particular the maximum torque bandwidth ω_{MT} , can be utilized as a criterion to evaluate the mechanical and control design parameters of SEA.

In the dynamic case where the torque of SEA is transmitted to the load and generates dynamic motions, MTT becomes more complicated and it becomes necessary to take load dynamics into account. Even under the dynamics case, the proposed large torque bandwidths can be calculated as (14) and (15), which are functions of the mechanical design parameters N_m , K_s , control parameters K_p , K_d , and load environment parameters J_l , B_l .

As large torque bandwidth ω_{MT} is a metric to indicate the performance of SEA as a transmission, it can offer novel insights on how the parameters affect the large force transmissibility and be utilized as a guideline for the design of SEA; the following three points are new findings that can be drawn only through the proposed maximum force bandwidth.

- 1) There is a certain force feedback gain K_p that decreases ω_{MT} abruptly (discussed in Sec. IV-A)
- 2) There is a certain gear ratio N_m from which the velocity limit becomes a more significant limitation for maximum force generation (discussed in Sec. IV-B)
- 3) There is a certain spring constant K_s that decreases ω_{MT} abruptly (discussed in Sec. IV-C)

The parameters K_p , N_m , and K_s are usually set as high as possible for better force generation performance. In this session, however, it is shown that they are to be limited in terms for MTT and the bandwidth ω_{MT} . This point is demonstrated through analytical discussions and experiments.

The parameters utilized in the following subsections are from Table I with the default gear ratio set to $N_m = 8$ and the default PD controller ($C(s) = K_p + K_d s$) gains are set to $K_p = 0.8$ and $K_d = 0.05$. Notice that K_p and K_d used in this section are selected using Matlab graphical tuning method to have a closed loop control bandwidth of 5 Hz.



Fig. 9: Maximum torque tracking performance comparison between two different gains

A. Limitation of Feedback Gain in Terms of ω_{MT}

Figure 8 (a) shows the three dimensional plot of MTT_{τ} with respect to the change in gain K_p and frequency ω . As explained in Sec. II-G, SEA cannot generate the maximum torque in the area where MTT_{τ} exceeds 1 in Fig. 8. The bottom view of the three dimensional MTT_{τ} , given in Fig. 8 (b), clearly displays how the maximum torque bandwidth $\omega_{MT_{\tau}}$ changes with regard to K_p .

The area where the grid is disclosed in Fig. 8 (b) is where SEA cannot generate the maximum torque and $\omega_{MT_{\tau}}$ is determined based on this area. The thick solid line in this figure indicates the relationship between $\omega_{MT_{\tau}}$ and K_p , which shows that $\omega_{MT_{\tau}}$ drops to 0 Hz from a certain gain K_p .

This relationship implies that the maximum torque generation performance deteriorates abruptly when the gain K_p is set too high. Figure 9 shows experimental results to verify this point; SEA is controlled to generate the maximum torque at 1 Hz frequency with two different gains. K_p is set to 2 in the left case, and K_p is set to 4 in the right case.

Contrary to common sense, the results show that tracking performance deteriorates with higher feedback gain.



Fig. 10: MTT_{τ} and MTT_{V} with regard to N_{m}



Fig. 11: Large torque bandwidth $\omega_{MT_{\tau}}$ (solid red) and $\omega_{MT_{V}}$ (dashed blue) with regard to N_m

The proposed equation (14) can be utilized to precisely examine the relationship between the gain, and $\omega_{MT_{\tau}}$ and K_p . For example, the gain value from which ω_{MT} suddenly drops to 0 Hz can be specified from the DC component of (14) as follows.

$$K_p = 1 + N_m^{-2} \frac{B_l}{B_m}$$
(17)

This value can be utilized as marginal gain when the large torque generation performance is considered significant.

B. Influence of Gear Ratio on Dynamic Maximum Torque Generation

The gear ratio in the SEA is usually set large to generate a large torque. However, the proposed MTT analysis reveals that large gear ratio reduces the maximum torque bandwidth ω_{MT} , which means a large gear ratio can increase the magnitude of the torque output while it decreases the response time of the large torque.

Figure 10 shows the three dimensional plots of MTT_{τ} and MTT_{V} with regard to gear ratio N_{m} and frequency ω , where it can be found that both bandwidths $\omega_{MT_{\tau}}$ and $\omega_{MT_{V}}$ decrease as N_{m} increases.

Although it is well understood that the velocity limitation of a motor becomes a more significant limitation whenever a very large reduction gear is employed, there has not been a clear standard indicating the gear ratio from which the velocity limitation plays a significant role. The proposed large torque bandwidth can be utilized to assess whether the torque limit is critical or the velocity limit is critical.



Fig. 12: MTT_{τ} and $\omega_{MT_{\tau}}$ with regard to K_s

Figure 11 shows the relationship among $\omega_{MT_{\tau}}$, $\omega_{MT_{V}}$, and N_m . The lower graph between the two in Fig. 11 is the dominant maximum torque bandwidth as shown in (16), which can identify from what gear ratio the velocity limitation becomes the more dominant factor. This assessment method can be utilized when selecting a motor and gear ratio in terms of maximum torque generation.

C. Assessment of Spring Stiffness in Terms of ω_{MT}

It is generally accepted knowledge that the natural frequency of SEA increases as the spring constant becomes large, which can thus enhance the bandwidth of force control. The maximum torque bandwidth ω_{MT} reveals a different aspect of large spring constants.

The three dimensional plot of MTT_{τ} with regard to the spring stiffness K_s is given in Fig. 12 (a), and the bottom view of the three dimensional plot is Fig. 12 (b), which displays the relationship between the large torque bandwidths $\omega_{MT_{\tau}}$ and K_s .

As earlier explained, SEA cannot provide the maximum torque in the gridded area in Fig. 12 (b), and the frequency where the gridded area starts is the bandwidth $\omega_{MT_{\tau}}$. The thick solid line in this figure indicates the relationship between $\omega_{MT_{\tau}}$ and K_s , which shows that 1) $\omega_{MT_{\tau}}$ increases as K_s increases, and 2) too large a spring stiffness suddenly drops $\omega_{MT_{\tau}}$. The second point shows that a high spring coefficient does not always increase the bandwidth, which is contrary to common knowledge.

The spring coefficient where $\omega_{MT_{\tau}}$ suddenly drops varies depending on the load condition. Figure 13 illustrates the relationships between $\omega_{MT_{\tau}}$ and K_s under two load conditions: the left plot is with low load inertia ($J_l = 0.003 \text{ kg} \cdot \text{m}^2$), and the right plot is with high load inertia ($J_l = 0.007 \text{ kg} \cdot \text{m}^2$). From the comparison between these two plots, it can be concluded that large load inertia can increase large torque bandwidth whenever the same spring stiffness is employed.

To validate this aspect experimentally, maximum torque tracking control is applied to the VGT under two different load conditions: $J_l = 0.003 \text{ kg} \cdot \text{m}^2$ and $J_l = 0.007 \text{ kg} \cdot \text{m}^2$. The spring of 20 Nm/rad is utilized for this experiment, and other parameters are set the same as in Table I. The reference



(a) Low load inertia $J_l = 0.003 \text{ kg} \cdot \text{m}^2$ (b) High load inertia $J_l = 0.007 \text{ kg} \cdot \text{m}^2$

Fig. 13: $\omega_{MT_{\tau}}$ with regard to K_s under two different load inertia conditions



(b) High load inertia $J_l = 0.007 \text{ kg} \cdot \text{m}^2$

Fig. 14: Large torque tracking performance with two load conditions.

torque is set to a sinusoidal signal with the frequency of 5 Hz and the maximum magnitude of 0.252 Nm (= $T_{m.c} \cdot N_m$)

The stiffness in this experiment $K_s = 20$ Nm/rad corresponds to the red thickly drawn circles in Fig. 13. Even though the stiffness is set the same in both cases, it is in the gridded area in Fig. 13 (a), and the experimental result in Fig. 14 (a) validates that SEA cannot generate the maximum torque with low load inertia. On the other hand, the same stiffness is

outside the gridded area in Fig. 13 (b), and correspondingly, SEA can generate the maximum torque in Fig. 14 (b).

It is verified through the experiments that the proposed MTT and ω_{MT} can precisely represent the maximum torque transmissibility as a dynamic characteristic of SEA, and thus can be utilized as a guideline and standard for various mechanical/controller parameters when designing SEA.

V. CONCLUSION

In this paper, Maximum Torque Transmissibility (MTT) is proposed as a mathematical tool to assess the ability of SEA to fully utilize the maximum continuous motor torque. Moreover, the influence of load condition, mechanical parameters, and controller gains on the ω_{MT} is analyzed using the proposed MTT.

The discussion of the experiments given in this paper can be summarized as follows.

- 1) Maximum Torque Transmissibility $(MTT_{\tau} \text{ and } MTT_{v})$ is proposed as a mathematical tool to analyze the influence of the maximum continuous motor torque and the maximum permissible velocity of a motor on SEA performance.
- Effectiveness of the proposed MTT is verified through experiments under various conditions.
- 3) Maximum torque frequency bandwidth ω_{MT} can be derived based on the proposed MTT, in which the transmission of the maximum motor torque by the SEA is guaranteed.
- 4) It is shown that the proposed ω_{MT} can be a novel criterion to evaluate SEA design factors such as control gain, gear ratio, and spring stiffness.
- 5) Experimental results verify the above.

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