A New Settling-time Estimation Protocol to Finite-time Synchronization of Impulsive Memristor-Based Neural Networks

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***Abstract*—In this article, the issues of finite-time synchronization and finite-time adaptive synchronization for the impulsive memristive neural networks (IMNNs) with discontinuous activation functions (DAFs) and hybrid impulsive effects are probed into and elaborated on, where the stabilizing impulses (SIs), inactive impulses (IIs), and destabilizing impulses (DIs) are taken into account, respectively. Not resembling several earlier works, a more extensive range of impulses in the context of impulsive effects has been analyzed without using the known average impulsive interval strategy (AIIS). In light of the theories of differential inclusions and set-valued map, as well as impulsive control, new sufficient criteria with respect to the estimated settling time for synchronization of the related IMNNs are established using two types of switching control approaches, which sufficiently utilize information from not only the SIs, DIs, and DAFs but also the impulse sequences. Two simulation experiments are presented to the efficiency of the proposed results.**

***Index Terms*—Adaptive synchronization, destabilizing impulses (DIs), memristive neural networks (MNNs), settling time.**

I. Introduction

MEMRISTIVE neural networks (MNNs) are one of the state-dependent switching networks, which have been successfully applied to different fields of science and engineering, such as pattern recognition, secure communication, deep learning, signal and image processing, as well as associate memory [1]–[5]. In comparison with conventional recurrent neural networks (RNNs), the MNNs resort to the memristors for the purpose of replacing the resistors in conventional RNNs. On the ground of the variable resistance and memory characteristics of memristors, a neural-network model for emulating the human brain [6] using a memristor element is more feasible in design. Accordingly, it comes to no surprise that the research of the dynamical behavior in MNNs has harvested fruitful results (refer to [7]–[10] and the relevant references therein).

Over the past few years, synchronization of MNNs, an important collective dynamical behavior, has been hotly debated theoretically and observed experimentally on account of its potential engineering applications in various disciplines [11]–[15], such as biological systems [16], [17]; information processing [18]; etc. Nonetheless, what deserves attention shall be attributed to the fact that the convergent speed with respect to the stability and synchronization for the related MNNs in [8], [10]–[12], [15], and [18] were asymptotical or exponential. To put it in another way, most of these published results on asymptotic or exponential synchronization can be derived merely when the time approaches infinity. Considering that the lifespan of apparatus and biology is limited, we always desire to acquire faster or even finite-time convergent speed in practice. Hence, the concept of finite-time synchronization (FTS) has been introduced and widely reported in recent years [19]–[21]. In the meantime, unlike the infinite-time synchronization, the FTS denotes faster convergence speed and exhibits serval other better features, including but not limited to disturbance rejection, better robustness [22], [23], etc. Moreover, as shown in [7], as a result of the impacts exerted by time delay and state-dependent nonlinear switching behaviors, the research of FTS on MNNs has become more intricate than that of classic NNs. Naturally, it is of greater significance to hold that the FTS of MNNs use subject to time delay. In this regard, for example, in [24], sufficient criteria for FTS of delayed MNNs had been established by taking the M-matrix and discontinuous controller into account. By combining the differential inclusions theory and the Lyapunov method, Yang *et al.* [25] derived the FTS criteria for the delayed MNNs, adopting two novel state-feedback control methods. Also in [26], the FTS of delayed MNNs was solved by employing the famous finite-time stability theorem and an adaptive control approach.

On another research front, the impulsive phenomenon is inevitable in the process of signal transmission [27]–[30]

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in that the signals may suddenly change at some discrete-time instants. Moreover, in contrast to continuous control, the impulsive control effectively saves the resources and exhibits other advantages represented by easy installation, high reliability, and high efficiency [31]–[33]. Particularly, with respect to the impulsive strength [34], the impulsive effects can be divided into three categories, which are stabilizing impulses (SIs), inactive impulses (IIs), and destabilizing impulses (DIs). Abundant fruitful achievements on the synchronization of impulsive MNNs (IMNNs) were grounded on SI effects because not only can it improve the speed of convergence and reduce the cost of time but it also can optimize the synchronization process [35]. Since the DIs are usually considered as impulsive disturbances and these impulses will do harm to the synchronization, how to achieve the IMNNs synchronization under different types of impulsive effects has become a difficult and important point of the theoretical study. More recently, Hu *et al.* [36] solved the problem of impulsive stochastic delay differential systems with SIs and DIs through an improved Razumikhin approach. With the aid of the convex combination technique and partitioning impulse interval method, sufficient conditions based on LMIs were acquired to guarantee the fixed-time synchronization in [37], which unifies the SIs and DIs. Afterward, Zhang *et al.* [38] researched the fixed-time synchronization for IMNNs by using the impulsive controller designed in [37].

Although many researchers have devoted themselves to the study of IMNNs, many aspects are still worthy of attention and need to be further improved. First, in these published works [29]–[31], [35], [37], [38], only the positive effects or negative effects of impulses were studied for the final synchronization. Accordingly, the sufficient conditions for synchronization have been found either for the impulsive strength $d_k \in (0, 1)$ or $d_k \in (-2, 0)$. Second, it should be pointed out that these developed FTS results [14], [24]–[26] were independent of the impulse sequences; thus, they are quite conservative in the context of impulsive effects. Furthermore, for reducing the conservatism of works in [14] and [24]–[26], many derived criteria such as [37] and [38] with respect to the settling time are impulse-dependent by using the known average impulsive interval strategy (AIIS). However, there exist some constraints with respect to the impulse sequence $\{t_k\}^{\mathcal{N}}$, that is, $\alpha \le t_k - t_{k-1} \le \beta$. On the other hand, the activation functions (AFs) for IMNNs also need to be further researched. As mentioned by [39] and [40], the discontinuous AFs (DAFs) are of importance and do frequently emerge in reality, especially for the dynamical systems with high-slope nonlinear elements, but this is seldom mentioned in IMNNs. What is more, the analysis of finite-time adaptive synchronization (FTAS) with DIs in IMNNs is usually ignored despite having meaningful significance in the case of the synchronization of networks. All these motivate us to conduct research in this article.

As a result of the above analysis, we studied the problems of FTS and FTAS for a class of IMNNs with DAFs and hybrid impulsive effects, and the main contributions are listed follows.

1) First of all, by employing the differential inclusions theories and combining set-valued map as well as impulsive control, the problems of FTS and FTAS for such general MNNs with DAFs and hybrid impulsive effects are considered.
2) In contrast to several earlier works in [11], [14], [24]–[26], and [38] and references cited therein, a broader range of impulses, DAFs, and the estimated settling time protocol have been analyzed in this article.
3) Without introducing the known AIIS and the comparison principle, new sufficient conditions with respect to the estimated settling time for the synchronization of the related IMNNs are obtained by employing two switching control strategies, which sufficiently utilize the information from not only the SIs, DIs, and DAFs, but also the impulse sequences.

Notations: Let Λ stand for the set $\{1, 2, \ldots, n\}$, $\mathcal{Z}^+$ denotes the positive integers, $\mathbb{R}$ represents the real values, $\mathbb{R}^n$ stands for the n-dimensional Euclidean space, and $\mathbb{R}^+$ is the positive real values. $co[\cdot, \cdot]$ denotes the closure of convex hull. $\overline{b}_{ij} = \max\{\acute{b}_{ij}, \grave{b}_{ij}\}$, $\underline{b}_{ij} = \min\{\acute{b}_{ij}, \grave{b}_{ij}\}$, $\overline{c}_{ij} = \max\{\acute{c}_{ij}, \grave{c}_{ij}\}$, and $\underline{c}_{ij} = \min\{\acute{c}_{ij}, \grave{c}_{ij}\}$.

II. PRELIMINARIES

Consider the following MNN with impulsive effects:

$$\begin{cases} \mathcal{D}z_i(t) = -d_i z_i(t) + \sum_{j=1}^{n} b_{ij}(z_i(t)) f_j\big(z_j(t)\big) \\ \qquad\qquad + \sum_{j=1}^{n} c_{ij}(z_i(t)) g_j\big(z_j\big(t - \tau_j(t)\big)\big), \ t \neq t_k \\ \Delta z_i(t) = q_k z_i\big(t_k^-\big), \ t = t_k, \ k \in \mathcal{Z}^+ \\ z_{i0} \quad = \phi_i \end{cases} \tag{1}$$

where $i \in \Lambda$, $z_i(t)$ stands for the voltage of the capacitor, $d_i > 0$ is the self-feedback coefficient, $f_j(\cdot)$ and $g_j(\cdot)$ are the AFs, $\tau_j(\cdot) : \mathbb{R}^+ \to [0, \tau]$ is the time-varying transmission delay, $z_{i0} = z_i(t_0 + s)$, $s \in [-\tau, 0]$, $\mathcal{D}$ denotes the distributional derivative, and the impulse sequences $\{t_k : k \in \mathcal{Z}^+\}$, short for $\{t_k\}$, are assumed to be strictly increasing on $\mathbb{R}^+$, for example, $\{t_k\}^{\mathcal{N}}$ implies that the impulse sequences satisfy $0 = t_0 < t_1 < \cdots < t_{\mathcal{N}} < \infty$, where $\mathcal{N}$ stands for the number of impulse points. q_k denotes the strength of impulses, and $\Delta z_i(t) = z_i(t_k^+) - z_i(t_k^-)$. Without loss of generality, we assume that the solution $z_i(t)$ is right-continuous at impulse instants, that is, $z_i(t_k^+) = z_i(t_k)$. $\phi_i \in PC([-\tau, 0], \mathbb{R})$ is the initial state of MNN (1) and the connection weight coefficients satisfy the following conditions:

$$b_{ij}(z_i(t)) = \begin{cases} \acute{b}_{ij}, & |z_i(t)| \le \mathcal{T}_i \\ \grave{b}_{ij}, & |z_i(t)| > \mathcal{T}_i \end{cases} \tag{2}$$

$$c_{ij}(z_i(t)) = \begin{cases} \acute{c}_{ij}, & |z_i(t)| \le \mathcal{T}_i \\ \grave{c}_{ij}, & |z_i(t)| > \mathcal{T}_i \end{cases} \tag{3}$$

where the switching jumps $\mathcal{T}_i > 0$ and $\acute{b}_{ij}$, $\grave{b}_{ij}$, $\acute{c}_{ij}$, and $\grave{c}_{ij}$, are real constants.

Assumption 1: For $\forall x, y \in \mathbb{R}, x \neq y$, the neuron AFs $f_j(\cdot)$ and $g_j(\cdot)$ are bounded and satisfy the following conditions:

$$l_j^- \le \frac{f_j(x) - f_j(y)}{x - y} \le l_j^+$$

and

$$\lambda_j^- \le \frac{g_j(x) - g_j(y)}{x - y} \le \lambda_j^+, \quad j \in \mathcal{Z}^+$$

where $f_j(0)=0$, $g_j(0)=0$, λ_j^-, λ_j^+, l_j^-, and l_j^+ are constants.

By employing the differential inclusion theories and the Filippov set-valued maps [41], [42], the MNN (1) is equivalent to the following form:

$$\begin{aligned}\mathcal{D}z_i(t) \in& -d_iz_i(t)+\sum_{j=1}^{n}\overline{co}\big[b_{ij}(z_i(t))\big]f_j\big(z_j(t)\big)\\&+\sum_{j=1}^{n}\overline{co}\big[c_{ij}(z_i(t))\big]g_j\big(z_j(t-\tau(t))\big),\ t\neq t_k\\ z_{i0}=&\ \phi_i,\quad i\in\Lambda\end{aligned}\tag{4}$$

where

$$\overline{co}\big[b_{ij}(z_i(t))\big]=\begin{cases}\acute{b}_{ij}, & |z_i(t)|<\mathcal{T}_i\\ \big[\underline{b}_{ij},\overline{b}_{ij}\big], & |z_i(t)|=\mathcal{T}_i\\ \grave{b}_{ij}, & |z_i(t)|>\mathcal{T}_i\end{cases}$$

$$\overline{co}\big[c_{ij}(z_i(t))\big]=\begin{cases}\acute{c}_{ij}, & |z_i(t)|<\mathcal{T}_i\\ \big[\underline{c}_{ij},\overline{c}_{ij}\big], & |z_i(t)|=\mathcal{T}_i\\ \grave{c}_{ij}, & |z_i(t)|>\mathcal{T}_i.\end{cases}$$

Then, there exist measurable functions $\tilde{b}_{ij}(z_i(t))\in\overline{co}[b_{ij}(z_i(t))]$ and $\tilde{c}_{ij}(z_i(t))\in\overline{co}[c_{ij}(z_i(t))]$ such that

$$\begin{cases}\mathcal{D}z_i(t)=-d_iz_i(t)+\sum_{j=1}^{n}\tilde{b}_{ij}(z_i(t))f_j\big(z_j(t)\big)\\ \qquad\qquad+\sum_{j=1}^{n}\tilde{c}_{ij}(z_i(t))g_j\big(z_j\big(t-\tau_j(t)\big)\big),\ t\neq t_k\\ z_i(t_k)\ =(1+q_k)z_i\big(t_k^-\big),\ k\in\mathcal{Z}^+\\ z_{i0}\ \ =\phi_i, i\in\Lambda.\end{cases}\tag{5}$$

Through the above discussions, the corresponding response system is described by

$$\begin{cases}\mathcal{D}z_i^\dagger(t)=-d_iz_i^\dagger(t)+\sum_{j=1}^{n}\tilde{b}_{ij}\Big(z_i^\dagger(t)\Big)f_j\Big(z_j^\dagger(t)\Big)\\ \qquad\qquad+\sum_{j=1}^{n}\tilde{c}_{ij}\Big(z_i^\dagger(t)\Big)g_j\Big(z_j^\dagger\big(t-\tau_j(t)\big)\Big)+u_i(t)\\ \qquad\qquad t\neq t_k\\ z_i^\dagger(t_k)\ =(1+q_k)z_i^\dagger\big(t_k^-\big),\ k\in\mathcal{Z}^+\\ z_{i0}^\dagger\ \ =\psi_i,\quad i\in\Lambda\end{cases}\tag{6}$$

where $\psi_i\in PC([-\tau,0],\mathbb{R})$ is the initial state of IMNN (6), and $u_i(t)$ is the state-feedback controller.

If the neuron AFs $f_j(\cdot)$ and $g_j(\cdot)$ are discontinuous, we have the following assumptions.

Assumption 2: The DAFs $f_j(\cdot)$ or $g_j(\cdot)$ are bounded and continuous except for a finite set of jump points $\{\mathcal{T}_k\}$, and have the left limits $f_j^-(\mathcal{T}_k)$ (or $g_j^-(\mathcal{T}_k)$) and right limits $f_j^+(\mathcal{T}_k)$ (or $g_j^+(\mathcal{T}_k)$) satisfying the following conditions:

$$\begin{aligned}\overline{co}\big[f_j\big(z_j(t)\big)\big]=\Big[&\min\Big\{f_j^-\big(z_j(t)\big),f_j^+\big(z_j(t)\big)\Big\}\\ &\max\Big\{f_j^-\big(z_j(t)\big),f_j^+\big(z_j(t)\big)\Big\}\Big]\end{aligned}$$

and $\overline{co}[g_j(z_j(t-\tau_j(t)))]=[\min\{g_j^-(z_j(t-\tau_j(t))),g_j^+(z_j(t-\tau_j(t)))\},\max\{g_j^-(z_j(t-\tau_j(t))),g_j^+(z_j(t-\tau_j(t)))\}]$.

Assumption 3: For $\tilde{f}_j(z_j(t))\in\overline{co}[f_j(z_j(t))]$ and $\tilde{g}_j(z_j(t-\tau_j(t)))\in\overline{co}[g_j(z_j(t-\tau_j(t)))]$, we have the following conditions:

$$\Big|\tilde{f}_j\Big(z_j^\dagger(t)\Big)-\tilde{f}_j\big(z_j(t)\big)\Big|\leq\chi_j\Big|z_j^\dagger(t)-z_j(t)\Big|+\hbar_j$$

and

$$\begin{aligned}&\tilde{g}_j\Big(z_j^\dagger\big(t-\tau_j(t)\big)\Big)-\tilde{g}_j\big(z_j\big(t-\tau_j(t)\big)\big)\\ &\quad\leq\chi_j^\dagger\Big|z_j^\dagger(t)-z_j(t)\Big|+\hbar_j^\dagger,\ j\in\mathcal{Z}^+\end{aligned}$$

where χ_j, $\hbar_j$, $\chi_j^\dagger$, and $\hbar_j^\dagger$ are positive constants.

Definition 1: The IMNN (6) is said to be globally synchronization with (5) within a finite time, if there exists T, which is dependent on the initial states z_{i0} and $z_{i0}^\dagger$ such that $\lim_{t\to T}|z_i^\dagger(t)-z_i(t)|=0$ and $|z_i^\dagger(t)-z_i(t)|\equiv 0$ for $t\geq T$ and $i\in\Lambda$. T is called the settling time.

Now, let $e_i(t)=z_i^\dagger(t)-z_i(t)$, $i\in\Lambda$ as the synchronization error, thus the synchronization error system (SES) can be expressed as follows:

$$\begin{cases}\mathcal{D}e_i(t)=-d_ie_i(t)+\sum_{j=1}^{n}\tilde{b}_{ij}(e_i(t))f_j\big(e_j(t)\big)\\ \qquad\qquad+\sum_{j=1}^{n}\tilde{c}_{ij}(e_i(t))g_j\big(e_j\big(t-\tau_j(t)\big)\big)+u_i(t),\ t\neq t_k\\ e_i(t_k)\ =(1+q_k)e_i\big(t_k^-\big),\ k\in\mathcal{Z}^+\\ e_{i0}\ \ =\varphi_i,\quad i\in\Lambda\end{cases}\tag{7}$$

where $\tilde{b}_{ij}(e_i(t))f_j(e_j(t))=\tilde{b}_{ij}(z_i^\dagger(t))f_j(z_j^\dagger(t))-\tilde{b}_{ij}(z_i(t))f_j(z_j(t))$, $\tilde{c}_{ij}(e_i(t))g_j(e_j(t-\tau_j(t)))=\tilde{c}_{ij}(z_i^\dagger(t))g_j(z_j^\dagger(t-\tau_j(t)))-\tilde{c}_{ij}(z_i(t))g_j(z_j(t-\tau_j(t)))$, and $\varphi_i=\psi_i-\phi_i$.

To guarantee the FTS, the following controllers are constructed.

Case 1:

$$\begin{aligned}u_i(t)=&-\alpha_ie_i(t)-\mathrm{sgn}(e_i(t))\big(\delta_i+\gamma|e_i(t)|^\eta\big)\\&-\sum_{j=1}^{n}\xi_{ij}\mathrm{sgn}(e_i(t))\big|e_j\big(t-\tau_j(t)\big)\big|,\quad i\in\Lambda\end{aligned}$$

where α_i, δ_i, and ξ_{ij} are positive constants determined later, γ is the tunable constant parameter, and $\eta\in(0,1)$.

Case 2:

$$\begin{aligned}u_i(t)=&-\alpha_i(t)e_i(t)-\mathrm{sgn}(e_i(t))\big(\delta_i(t)+\gamma|e_i(t)|^\eta\big)\\&-\sum_{j=1}^{n}\xi_{ij}(t)\mathrm{sgn}(e_i(t))\big|e_j\big(t-\tau_j(t)\big)\big|,\quad i\in\Lambda\end{aligned}\tag{8}$$

where the adaptive controller rules satisfy the following conditions $\mathcal{D}\alpha_i(t)=\pi_i|e_i(t)|^p$, $\mathcal{D}\delta_i(t)=\mu_i|e_i(t)|^{p-1}$, and $\mathcal{D}\xi_{ij}(t)=\vartheta_{ij}|e_i(t)|^{p-1}|e_j(t-\tau_j(t))|$, $j\in\Lambda$, and π_i, μ_i, and ϑ_{ij} are some positive constants. Also, γ is the tunable constant parameter, and the real number $\eta\in(0,1)$.

Lemma 1 [43]: If $a_1, a_2, \ldots, a_n\geq 0$, $0<\theta\leq 1$, then the following inequality holds:

$$\sum_{i=1}^{n}a_i^\theta\geq\left(\sum_{i=1}^{n}a_i\right)^\theta.$$

III. Main Results

In this section, by using the Filippov-framework [41] and finite-time stability theory [19], we consider the FTS between the response system (6) and the drive system (5) under different types of impulsive effects.

A. FTS of IMNNs Using SIs

To begin with, we present the following result.

Theorem 1: Given the scalars $p \geq 2$, $\gamma > 0$, $\sigma \in (0,1)$, and $\rho \in (\sigma, 1)$, suppose that Assumption 1 holds, if there exist some constants α_i, δ_i, and ξ_{ij}, such that the following inequalities hold:

$$\begin{cases} \alpha_i \geq -d_i + \varpi_i/p \\ \xi_{ij} \geq \hat{c}_{ij}\lambda_j \\ \delta_i \geq M_j^{\ddagger}, \quad i \in \Lambda,\ j \in \Lambda \end{cases} \tag{9}$$

then, the response system (6) with the FTS controller (8) (case 1) synchronizes to the drive system (5) in finite time. The settling time is: case 1), $T = [(V^{1-q/p}(t_0))/(\beta(1-(q/p)))]$; case 2), $\widetilde{T} = \rho^{\mathcal{N}}T$, which is not only dependent on the initial condition but also the impulse sequences $\{t_k\}^{\mathcal{N}}$, where the impulse sequences $\{t_k\}^{\mathcal{N}}$ satisfying

$$t_{\mathcal{N}} \leq \rho^{\mathcal{N}-1}\frac{(\rho-\sigma)V^{1-q/p}(t_0)}{\beta(1-\sigma)\left(1-\frac{q}{p}\right)} \tag{10}$$

and $M_j^{\ddagger} = M_j|\acute{b}_{ij} - \grave{b}_{ij}| + M_j^{\dagger}|\acute{c}_{ij} - \grave{c}_{ij}|$ and $\varpi_i = \sum_{j=1}^{n}\sum_{l=1}^{p-1}\hat{b}_{ij}^{pw_{l,ij}}L_j^{p\pi_{l,ij}} + \sum_{j=1}^{n}\hat{b}_{ij}^{pw_{p,ij}}L_j^{p\pi_{p,ij}}$, $q = p+\eta-1$, $\beta = p\gamma$.

Proof: Choose the following Lyapunov functional:

$$V(t) = \sum_{i=1}^{n}|e_i(t)|^p. \tag{11}$$

Computing the time derivative of $V(t)$ along the trajectory of the IMNN (7) for $t \neq t_k$, $k \in \mathcal{Z}^+$, yields the following:

$$\begin{aligned}\mathcal{D}V(t) &= p\sum_{i=1}^{n}|e_i(t)|^{p-1}\mathrm{sgn}(e_i(t))\mathcal{D}e_i(t)\\ &= p\sum_{i=1}^{n}|e_i(t)|^{p-2}\mathrm{sgn}^2(e_i(t))e_i(t)\mathcal{D}e_i(t)\\ &= p\sum_{i=1}^{n}|e_i(t)|^{p-2}e_i(t)\\ &\quad\times\Bigg[-d_ie_i(t) + \sum_{j=1}^{n}\tilde{b}_{ij}(e_i(t))f_j\big(e_j(t)\big)\\ &\qquad + \sum_{j=1}^{n}\tilde{c}_{ij}(e_i(t))g_j\big(e_j\big(t-\tau_j(t)\big)\big) + u_i(t)\Bigg].\end{aligned} \tag{12}$$

By using Assumption 1, there exist some constants $M_j > 0$ and $M_j^{\dagger} > 0$ that make $|f_j(\cdot)| < M_j$ and $|g_j(\cdot)| < M_j^{\dagger}$ for $j \in \Lambda$, then

$$\begin{aligned}\mathcal{D}V(t) &\leq -p\sum_{i=1}^{n}d_i|e_i(t)|^p + p\sum_{i=1}^{n}\sum_{j=1}^{n}\hat{b}_{ij}|e_i(t)|^{p-1}L_j\big|e_j(t)\big|\\ &\quad + p\sum_{i=1}^{n}\sum_{j=1}^{n}\hat{c}_{ij}|e_i(t)|^{p-1}\lambda_j\big|e_j\big(t-\tau_j(t)\big)\big|\\ &\quad + p\sum_{i=1}^{n}\sum_{j=1}^{n}|e_i(t)|^{p-1}\Big(M_j\big|\acute{b}_{ij}-\grave{b}_{ij}\big| + M_j^{\dagger}\big|\acute{c}_{ij}-\grave{c}_{ij}\big|\Big)\\ &\quad + p\sum_{i=1}^{n}|e_i(t)|^{p-2}e_i(t)\\ &\quad\times\Bigg(-\alpha_ie_i(t) - sgn(e_i(t))\delta_i\\ &\qquad - \mathrm{sgn}(e_i(t))\gamma|e_i(t)|^{\eta}\\ &\qquad - \sum_{j=1}^{n}\xi_{ij}\mathrm{sgn}(e_i(t))\big|e_j\big(t-\tau_j(t)\big)\big|\Bigg)\end{aligned} \tag{13}$$

where $\lambda_j = \max\{|\lambda_j^+|, |\lambda_j^-|\}$, $L_j = \max\{|l_j^+|, |l_j^-|\}$, $\hat{b}_{ij} = \max\{|\acute{b}_{ij}|, |\grave{b}_{ij}|\}$, and $\hat{c}_{ij} = \max\{|\acute{c}_{ij}|, |\grave{c}_{ij}|\}$.

According to the mean value inequality, we have

$$\begin{aligned}&\sum_{j=1}^{n}p\hat{b}_{ij}L_j|e_i(t)|^{p-1}\big|e_j(t)\big|\\ &= \sum_{j=1}^{n}p\left[\prod_{l=1}^{p-1}\hat{b}_{ij}^{w_{l,ij}}L_j^{\pi_{l,ij}}|e_i(t)|\right]\Big(\hat{b}_{ij}^{w_{p,ij}}L_j^{\pi_{p,ij}}\big|e_j(t)\big|\Big)\\ &\leq \sum_{j=1}^{n}\sum_{l=1}^{p-1}\hat{b}_{ij}^{pw_{l,ij}}L_j^{p\pi_{l,ij}}|e_i(t)|^p\\ &\quad + \sum_{j=1}^{n}\hat{b}_{ij}^{pw_{p,ij}}L_j^{p\pi_{p,ij}}\big|e_j(t)\big|^p\end{aligned} \tag{14}$$

where $\sum_{l=1}^{p}w_{l,ij} = \sum_{l=1}^{p}\pi_{l,ij} = 1$.

Then, based on Lemma 1, we have

$$\begin{aligned}\mathcal{D}V(t) &\leq -p\sum_{i=1}^{n}(d_i+\alpha_i-\varpi_i/p)|e_i(t)|^p\\ &\quad - p\sum_{i=1}^{n}\sum_{j=1}^{n}\big(\xi_{ij}-\hat{c}_{ij}\lambda_j\big)|e_i(t)|^{p-1}\big|e_j\big(t-\tau_j(t)\big)\big|\\ &\quad - p\sum_{i=1}^{n}\sum_{j=1}^{n}\Big(\delta_i - M_j^{\ddagger}\Big)|e_i(t)|^{p-1} - p\gamma\sum_{i=1}^{n}|e_i(t)|^q\\ &\leq -p\gamma\left(\sum_{i=1}^{n}|e_i(t)|^p\right)^{q/p}\\ &= -\beta V^{q/p}(t).\end{aligned} \tag{15}$$

When $t = t_k$, it follows from (7) that:

$$\begin{aligned}V(t_k) &= \sum_{i=1}^{n}\big|(1+q_k)e_i\big(t_k^-\big)\big|^p\\ &\leq |1+q_k|^p\sum_{i=1}^{n}\big|e_i\big(t_k^-\big)\big|^p\\ &= |1+q_k|^pV\big(t_k^-\big).\end{aligned} \tag{16}$$

Case 1: Consider the SIs as follows, that is, $|1+q_k| \in (0,1)$. We first consider the settling time is independent on the impulse sequences $\{t_k\}^{\mathcal{N}}$. Define $T = [(V^{1-q/p}(t_0))/(\beta(1-(q/p)))]$, from (15) and (16), it yields

$$V^{1-\frac{q}{p}}(t) \leq V^{1-\frac{q}{p}}(t_0) - \beta\left(1-\frac{q}{p}\right)t \tag{17}$$

where $t \in [0, t_1 \wedge T)$.

It is evident that when $t_1 \geq T$, we have $V(t) \leq V(t_0)$, for $\forall t \in [0, T]$, and $V(t) \equiv 0$ for all $t \geq T$. In contrast, when $t_1 < T$, we assume that there exist N impulsive points on the interval $[0, T]$, which satisfy the following condition $0 = t_0 < t_1 < \cdots < t_N < T$ for $N \in \mathcal{Z}^+$.

Then, by simple induction, it is calculated that

$$V^{1-\frac{q}{p}}(t) \leq V^{1-\frac{q}{p}}(t_0) - \beta\left(1 - \frac{q}{p}\right)t, \text{ for } t \in \left[t_j, t_{j+1}\right) \tag{18}$$

where $j = 0, 1, \ldots, N$, and for any $\epsilon > 0$, t_{N+1} is defined by $T+\epsilon$. Therefore, it is easy to see that there exists a time instant $t = T$ on the interval $[t_N, t_{N+1})$ such that the right-hand side of the inequality (18) is 0, which implies that $V(t) \leq V(t_0)$ for $t \in [0, T]$ and $V(t) \equiv 0$ for all $t \geq T$.

Based on the aforementioned discussions, one has $\lim_{t\to T} |e_i(t)| = 0$ and $|e_i(t)| \equiv 0$ for $t \geq T$, that is, the IMNN is FTS, and the settling time is independent on the impulse sequences $\{t_k\}^{\mathcal{N}}$.

Case 2: Let $\sigma = |1 + q_k|^p$, since $\rho < 1$ and $t_{\mathcal{N}} \leq \rho^{\mathcal{N}-1}[((\rho - \sigma)V^{1-q/p}(t_0))/(\beta(1 - \sigma)(1 - (q/p)))]$, in view of $t_{\mathcal{N}} < T$, we have

$$t_\ell \leq \frac{\rho^{\mathcal{N}}(1 - \sigma/\rho)}{(1 - \sigma)}T \leq \frac{\rho^{\ell}(1 - \sigma/\rho)}{(1 - \sigma)}T \tag{19}$$

where $\ell = 0, 1, \ldots, \mathcal{N}$.

It indicates that

$$\sigma\rho^{\ell-1} + \frac{t_\ell(1 - \sigma)}{T} \leq \rho^{\ell} \quad \forall\, \ell = 0, 1, \ldots, \mathcal{N}. \tag{20}$$

Also, from (15), we obtain

$$V^{1-\frac{q}{p}}(t) \leq V^{1-\frac{q}{p}}(t_0) - \beta\left(1 - \frac{q}{p}\right)t \quad \forall t \in [0, t_1).$$

Then, combining (16) and (20) yields

$$\begin{aligned} V^{1-\frac{q}{p}}(t) &\leq V^{1-\frac{q}{p}}(t_1) - \beta\left(1 - \frac{q}{p}\right)(t - t_1) \\ &\leq \sigma V^{1-\frac{q}{p}}\left(t_1^-\right) - \beta\left(1 - \frac{q}{p}\right)(t - t_1) \\ &\leq (\sigma + (1 - \sigma)t_1/T)V^{1-\frac{q}{p}}(t_0) - \beta\left(1 - \frac{q}{p}\right)t \\ &= \rho V^{1-\frac{q}{p}}(t_0) - \beta\left(1 - \frac{q}{p}\right)t \quad \forall t \in [t_1, t_2). \end{aligned}$$

By using (20), it is calculated that for any $\ell = 0, 1, \ldots, \mathcal{N}$

$$V^{1-\frac{q}{p}}(t) \leq \rho^{\ell} V^{1-\frac{q}{p}}(t_0) - \beta\left(1 - \frac{q}{p}\right)t \quad \forall t \in \left[t_\ell, t_{\ell+1}\right)$$

where $t_{\mathcal{N}+1}$ is defined by $\rho^{\mathcal{N}}T + \epsilon$, based on the similar arguments as discussed in (18), we can conclude that $V(t) \leq V(t_0)$ for $t \in [0, \rho^{\mathcal{N}}T)$, and $V(t) \equiv 0$ for all $t \geq \rho^{\mathcal{N}}T$. In other words, there exists a settling time $\widetilde{T} = \rho^{\mathcal{N}}T$ such that $\lim_{t\to\widetilde{T}} |e_i(t)| = 0$ and $|e_i(t)| \equiv 0$ for $t \geq \widetilde{T}$, that is, the IMNN is FTS, and the settling time is not only dependent on the initial condition but also the impulse sequences $\{t_k\}^{\mathcal{N}}$. ∎

B. FTS of IMNNs Using DIs

When $|1 + q_k| \in [1, \infty)$, the impulses may destroy the synchronization potentially, we present the following criterion to determine the synchronization between the response system (6) and the drive system (5) in finite time.

Theorem 2: Given the scalars $p \geq 2$, $\gamma > 0$, and $\sigma \in [1, \infty)$, suppose that Assumption 1 holds, if there exist some constants α_i, δ_i, and ξ_{ij} such that the following inequalities hold:

$$\begin{cases} \alpha_i \geq -d_i + \varpi_i/p \\ \xi_{ij} \geq \hat{c}_{ij}\lambda_j \\ \delta_i \geq M_j^{\ddagger}, \quad i \in \Lambda,\ j \in \Lambda \end{cases} \tag{21}$$

then, the response system (6) with the FTS controller (8) (case 1) synchronizes to the drive system (5) in finite time. Moreover, the settling time is $T^{\dagger} = \sigma^{\mathbb{N}-1}T$ with $T = [(V^{1-q/p}(t_0))/(\beta(1 - (q/p)))]$, which is not only dependent on the initial condition but also the impulse sequences $\{t_k\}^{\mathbb{N}}$, where the DIs $\{t_k\}^{\mathbb{N}}$ satisfying

$$\min\left\{\ell \in \mathcal{Z}^+ : \frac{t_\ell}{\sigma^{\ell-1}} \geq \frac{V^{1-\frac{q}{p}}(t_0)}{\beta\left(1 - \frac{q}{p}\right)}\right\} := \mathbb{N} < \infty. \tag{22}$$

Proof: Consider the following DIs effects, that is, $|1 + q_k| \in (1, \infty)$. Let $\sigma = |1 + q_k|^p$, and define $T = [(V^{1-q/p}(t_0))/(\beta(1 - (qp)))]$, from (15) and (16), one has

$$V^{1-\frac{q}{p}}(t) \leq V^{1-\frac{q}{p}}(t_0) - \beta\left(1 - \frac{q}{p}\right)t \tag{23}$$

where $t \in [0, t_1 \wedge T)$.

When $t_1 \geq T$, we see that $\mathbb{N} = 1$, and $V(t) \leq V(t_0)$, for $\forall t \in [0, T]$, and $V(t) \equiv 0$ for all $t \geq T$. While $t_1 < T$, it implies that $\mathbb{N} \geq 2$. According to the definition of impulse sequences $\{t_k\}^{\mathbb{N}}$ in (22), we have $t_\ell < \sigma^{\ell-1}T$, for $\ell = 1, 2, \ldots, \mathbb{N} - 1$, and $t_{\mathbb{N}} \geq \sigma^{\mathbb{N}-1}T$.

Then, in view of $\sigma \geq 1$, it yields

$$\begin{aligned} V^{1-\frac{q}{p}}(t) &\leq V^{1-\frac{q}{p}}(t_1) - \beta\left(1 - \frac{q}{p}\right)(t - t_1) \\ &\leq \sigma\left(V^{1-\frac{q}{p}}(t_0) - \beta\left(1 - \frac{q}{p}\right)t_1\right) \\ &\quad - \beta\left(1 - \frac{q}{p}\right)(t - t_1) \\ &\leq \sigma V^{1-\frac{q}{p}}(t_0) - \beta\left(1 - \frac{q}{p}\right)t, \ t \in [t_1, t_2 \wedge \sigma T). \end{aligned} \tag{24}$$

Moreover, it is mentioned that $t_\ell < \sigma^{\ell-1}T$, for $\ell = 1, 2, \ldots, \mathbb{N} - 1$, thus

$$\begin{aligned} V^{1-\frac{q}{p}}(t) &\leq V^{1-\frac{q}{p}}(t_2) - \beta\left(1 - \frac{q}{p}\right)(t - t_2) \\ &\leq \sigma V^{1-\frac{q}{p}}\left(t_2^-\right) - \beta\left(1 - \frac{q}{p}\right)(t - t_2) \\ &\leq \sigma\left(\sigma V^{1-\frac{q}{p}}(t_0) - \beta\left(1 - \frac{q}{p}\right)t_2\right) \\ &\quad - \beta\left(1 - \frac{q}{p}\right)(t - t_2) \end{aligned}$$

$$\leq \sigma^2 V^{1-\frac{q}{p}}(t_0) - \beta\left(1-\frac{q}{p}\right)t,\ \ t \in \left[t_2, t_3 \wedge \sigma^2 T\right). \tag{25}$$

Then, by simple induction, we obtain

$$V^{1-\frac{q}{p}}(t) \leq \sigma^{\mathbb{N}-1} V^{1-\frac{q}{p}}(t_0) - \beta\left(1-\frac{q}{p}\right)t \quad \text{for } t \in \left[t_{\mathbb{N}-1}, t_{\mathbb{N}-1} \wedge \sigma^{\mathbb{N}-1}T\right). \tag{26}$$

Since $t_{\mathbb{N}-1} < \sigma^{\mathbb{N}-2}T$ and $t_{\mathbb{N}} \geq \sigma^{\mathbb{N}-1}T$, so based on the aforementioned discussions, we can conclude that $V(t) \leq \sigma^{[(\mathbb{N}-1)/(1-q/p)]}V(t_0)$ for $t \in [0, \sigma^{\mathbb{N}-1}T)$, and $V(t) \equiv 0$ for all $t \geq \sigma^{\mathbb{N}-1}T$. Thus, there exists a settling time $T^\dagger = \sigma^{\mathbb{N}-1}T$ such that $\lim_{t\to T^\dagger} |e_i(t)| = 0$ and $|e_i(t)| \equiv 0$ for $t \geq T^\dagger$, that is, the IMNN is FTS, and the settling time is not only dependent on the initial condition but also the DIs $\{t_k\}^{\mathbb{N}}$. The proof is considered complete. ■

Remark 1: In view of the impulsive strength, the impulse effects can be divided into three categories, that is, SIs, IIs, and DIs. Recently, in reference to SIs effects, countless impressive achievements on the IMNNs were explored on the ground that it not only can improve the speed of convergence and reduce the cost of time but also can optimize the stable process [31], [35]. However, in view of the fact that DIs are usually considered as impulsive disturbances which will do harm to synchronization, thus how to achieve the IMNNs synchronization under different types of impulsive effects has become a difficult and important point of the theoretical study. From this point of view, the problems of FTS and IMNNs with both SIs and DIs are studied in this article.

Remark 2: In contrast to most of the existing results [37], [38], the used method on impulsive effects is neither the comparison principle nor the AIIS. Based on the Filippov theory, Lyapunov functional strategies, and finite-time techniques, sufficient conditions with respect to the estimate settling time for synchronization of the related IMNNs are obtained in combination with two switching control approaches, which sufficiently utilize the information from not only the SIs, DIs, and DAFs but also the impulse sequences.

In Theorems 1 and 2, by introducing Assumption 1, a general class of neural AFs is processed. Different from the schemes therein, the adaptive controllers (case 2) in (8) and DAFs are taken into consideration in the following.

For system (1), under Assumption 2, we employ the differential inclusions theories and Filippov set-valued maps [41], [42], and obtain

$$\mathcal{D}z_i(t) \in -d_i z_i(t) + \sum_{j=1}^{n} \overline{co}\left[b_{ij}(z_i(t))\right]\overline{co}\left[f_j\left(z_j(t)\right)\right] + \sum_{j=1}^{n} \overline{co}\left[c_{ij}(z_i(t))\right]\overline{co}\left[g_j\left(z_j(t-\tau(t))\right)\right],\ \ t \neq t_k$$
$$z_{i0} = \phi_i, \quad i \in \Lambda \tag{27}$$

or equivalently

$$\begin{cases} \mathcal{D}z_i(t) &= -d_i z_i(t) + \sum_{j=1}^{n} \tilde{b}_{ij}(z_i(t))\tilde{f}_j\left(z_j(t)\right) \\ & \quad + \sum_{j=1}^{n} \tilde{c}_{ij}(z_i(t))\tilde{g}_j\left(z_j\left(t-\tau_j(t)\right)\right), \quad t \neq t_k \\ z_i(t_k) &= (1+q_k)z_i\left(t_k^-\right),\ \ k \in \mathcal{Z}^+ \\ z_{i0} &= \phi_i, \quad i \in \Lambda \end{cases} \tag{28}$$

where $\tilde{f}_j(z_j(t)) \in \overline{co}[f_j(z_j(t))]$ and $\tilde{g}_j(z_j(t-\tau_j(t))) \in \overline{co}[g_j(z_j(t-\tau_j(t)))]$. Then, we yield the following result.

C. FTAS of IMNNs Using SIs

Theorem 3: Given the scalars $p \geq 2$, $\gamma > 0$, $\sigma \in (0, 1)$, and $\rho \in (\sigma, 1)$, suppose that Assumptions 2 and 3 hold, then, under the adaptive synchronization controller (case 2) in (8), the response system (6) synchronizes to the drive system (5) in finite time. In addition, the settling time is: case 1), $T = [(V^{\dagger 1-q/p}(t_0))/(\beta(1-(q/p)))]$; case 2), $\widetilde{T} = \rho^{\mathcal{N}}T$, it is not only dependent on the initial condition but also the impulse sequences $\{t_k\}^{\mathcal{N}}$, and the impulse sequences $\{t_k\}^{\mathcal{N}}$ satisfying

$$t_{\mathcal{N}} \leq \rho^{\mathcal{N}-1}\frac{(\rho-\sigma)V^{\dagger 1-q/p}(t_0)}{\beta(1-\sigma)\left(1-\frac{q}{p}\right)}. \tag{29}$$

Proof: Consider the following Lyapunov functional:

$$\begin{aligned} V^\dagger(t) = V(t) &+ \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{p}{2n\pi_i}\left(\alpha_i(t)-a_{ij}\right)^2 \\ &+ \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{p}{2\vartheta_{ij}}\left(\xi_{ij}(t)-\zeta_{ij}\right)^2 \\ &+ \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{p}{2n\mu_i}\left(\delta_i(t)-\chi_{ij}\right)^2 \end{aligned} \tag{30}$$

where $V(t)$ is defined in (11) and a_{ij}, ζ_{ij}, and χ_{ij} are positive constants determined later.

Computing the time derivative of $V^\dagger(t)$ for SES (7) with the adaptive synchronization controller (case 2) for $t \neq t_k$, $k \in \mathcal{Z}^+$, one can obtain the following:

$$\begin{aligned} \mathcal{D}V^\dagger(t) = p\sum_{i=1}^{n}&|e_i(t)|^{p-2}e_i(t) \\ &\times \Bigg[-d_i e_i(t) + \sum_{j=1}^{n}\tilde{b}_{ij}(e_i(t))\tilde{f}_j\left(e_j(t)\right) \\ &\qquad + \sum_{j=1}^{n}\tilde{c}_{ij}(e_i(t))\tilde{g}_j\left(e_j\left(t-\tau_j(t)\right)\right) + u_i(t)\Bigg] \\ &+ \sum_{i=1}^{n}\sum_{j=1}^{n}(p/n)\left(\alpha_i(t)-a_{ij}\right)|e_i(t)|^p \\ &+ \sum_{i=1}^{n}\sum_{j=1}^{n}p\left(\xi_{ij}(t)-\zeta_{ij}\right)|e_i(t)|^{p-1}\left|e_j\left(t-\tau_j(t)\right)\right| \\ &+ \sum_{i=1}^{n}\sum_{j=1}^{n}(p/n)\left(\delta_i(t)-\chi_{ij}\right)|e_i(t)|^{p-1}. \end{aligned} \tag{31}$$

In addition, with introducing Assumption 3, we obtain

$$\begin{aligned}&\left|\tilde{b}_{ij}(e_i(t))\tilde{f}_j\big(e_j(t)\big)\right|\\&\quad\le \hat{b}_{ij}\big(\chi_j\big|e_j(t)\big|+\hbar_j\big)+\left|\acute{b}_{ij}-\grave{b}_{ij}\right|M_j\end{aligned}\tag{32}$$

and

$$\begin{aligned}&\left|\tilde{c}_{ij}(e_i(t))\tilde{g}_j\big(e_j\big(t-\tau_j(t)\big)\big)\right|\\&\quad\le \hat{c}_{ij}\Big(\chi_j^{\dagger}\big|e_j\big(t-\tau_j(t)\big)\big|+\hbar_j^{\dagger}\Big)+\left|\acute{b}_{ij}-\grave{b}_{ij}\right|M_j^{\dagger}.\end{aligned}\tag{33}$$

Then, based on the aforementioned discussions and using the same arguments as in Theorem 1, we can obtain the result Theorem 3 immediately. ■

D. FTAS of IMNNs Using DIs

Next, when the DIs are taken into account, by using the same method as in the proof of Theorems 2 and 3, we have the following theorem.

Theorem 4: Given the scalars $p \ge 2$, $\gamma > 0$, and $\sigma \in [1, \infty)$, suppose that Assumptions 2 and 3 hold, then, under the adaptive synchronization controller (case 2) in (8), the response system (6) synchronizes to the drive system (5) in finite time. Moreover, the settling time is $T^{\dagger} = \sigma^{\mathbb{N}-1}T$ with $T = [(V^{\dagger 1-q/p}(t_0))/(\beta(1-(q/p)))]$, which is not only dependent on the initial condition but also the impulse sequences $\{t_k\}^{\mathbb{N}}$, and the DIs $\{t_k\}^{\mathbb{N}}$ satisfying the following condition:

$$\min\left\{\ell \in \mathcal{Z}^{+} : \frac{t_\ell}{\sigma^{\ell-1}} \ge \frac{V^{\dagger 1-\frac{q}{p}}(t_0)}{\beta\Big(1-\frac{q}{p}\Big)}\right\} := \mathbb{N} < \infty. \tag{34}$$

Proof: Consider the same Lyapunov functional as in (30), and using the same arguments as in Theorems 2 and 3, we can obtain the result Theorem 4 immediately. ■

Remark 3: In the previous works [31], [35], [37], [38], only the positive effects or negative effects of impulses for the final synchronization was studied. Accordingly, the sufficient criteria for synchronization have been found either for the impulsive strength $d_k \in (0, 1)$ or $d_k \in (-2, 0)$. However, they are quite conservative in the context of impulsive effects. Recently, Kumar *et al.* [34] extended the range of the impulsive strength d_k by applying the AIIS, a broader range of impulsive effects are classified into two cases one for $d_k \in (-2, 0]$ except $d_k \neq -1$ and another for $d_k \in (-\infty, -2]$ or $d_k \in (0, \infty)$. It, therefore, yielded less conservative criteria contrasted with those studied from [31], [35], [37], and [38]. Moreover, the extended analysis of impulsive effects investigated in [34] is just a special case of our works in the case of $p = 2$.

Remark 4: Compared with the existing results on the assumption that the AFs are continuous or even Lipschitz continuous [24]–[26], [35], [38], the DAFs are of importance and do frequently emerge in practice [39], especially for the dynamical systems with high-slope nonlinear elements [40]. Consequently, most researchers focus their efforts on the study of the dynamical behavior of NNs with DAFs for the reason that the time-cost and difficulty of exploratory step-based DAFs are acceptable, and DAFs outperforms continuous AFs.

Remark 5: On one hand, the settling time of MNNs in [14] and [24]–[26] usually dependent on the initial value is fixed for a given initial state. Whereas, when the system suffers from impulse effects before reaching the settling time, it is possible that the settling time will be changed and becomes impulse-dependent. This is the first reason why the existing results are conservative. On the other hand, for reducing the conservatism of works in [14] and [24]–[26], many derived criteria [37], [38] with respect to the settling time are impulse-dependent by using the known AIIS. However, there exist some constraints with respect to the impulse sequence $\{t_k\}^{\mathcal{N}}$, that is, $\alpha \le t_k - t_{k-1} \le \beta$, and α and β are positive constants. While in this article, the impulsive instants $t_1, t_2, \ldots, t_{\mathcal{N}-1}$ remove such restriction, except the last instant $t_{\mathcal{N}}$ satisfies the condition (10). Thus, it reduces the conservatism to some extent.

IV. Illustrative Examples

Two simulation examples with SIs, DIs, and DAFs are presented to demonstrate the feasibility of the obtained results.

We first consider the design of controller (8) under case 1.

Example 1: Consider the following 2-D IMNN:

$$\begin{cases}\mathcal{D}z_i(t) = -d_iz_i(t) + \sum_{j=1}^{2} b_{ij}(z_i(t))f_j\big(z_j(t)\big)\\ \qquad + \sum_{j=1}^{2} c_{ij}(z_i(t))g_j\big(z_j\big(t-\tau_j(t)\big)\big),\ t \neq t_k\\ \Delta z_i(t) = q_kz_i\big(t_k^{-}\big),\ t = t_k,\ k \in \mathcal{Z}^{+},\ i = 1, 2\end{cases}\tag{35}$$

where $d_1 = d_2 = 1$ and

$$b_{11}(z_1(t)) = \begin{cases}-0.8, & |z_1(t)| \le 1\\ -1, & |z_1(t)| > 1\end{cases}$$

$$b_{12}(z_1(t)) = \begin{cases}1.2, & |z_1(t)| \le 1\\ 0.9, & |z_1(t)| > 1\end{cases}$$

$$b_{21}(z_2(t)) = \begin{cases}1.5, & |z_2(t)| \le 1\\ 1.2, & |z_2(t)| > 1\end{cases}$$

$$b_{22}(z_2(t)) = \begin{cases}-1.5, & |z_2(t)| \le 1\\ -0.9, & |z_2(t)| > 1\end{cases}$$

$$c_{11}(z_1(t)) = \begin{cases}-3.2, & |z_1(t)| \le 1\\ -2.1, & |z_1(t)| > 1\end{cases}$$

$$c_{12}(z_1(t)) = \begin{cases}0.8, & |z_1(t)| \le 1\\ 1.5, & |z_1(t)| > 1\end{cases}$$

$$c_{21}(z_2(t)) = \begin{cases}0.3, & |z_2(t)| \le 1\\ 1.2, & |z_2(t)| > 1\end{cases}$$

$$c_{22}(z_2(t)) = \begin{cases}-2.1, & |z_2(t)| \le 1\\ -1.3, & |z_2(t)| > 1.\end{cases}$$

The AFs are considered as $f_i(x) = \tanh(x)$, $g_i(x) = 1/2(|x+1| - |x-1|)$, and the TVDs $\tau_i(t) = e^t/(1+e^t)$, $i = 1, 2$. The phase plot of (35) is shown in Fig. 1 under the the initial values $z_1(s) = 1.5$ and $z_2(s) = -0.5$ $\forall s \in [-1, 0)$.

In addition, the corresponding response system with the initial conditions $z_1^{\dagger}(s) = -1$ and $z_2^{\dagger}(s) = 0.5$ $\forall s \in [-1, 0)$ is as follows:

$$\begin{cases}\mathcal{D}z_i^{\dagger}(t) = -d_iz_i^{\dagger}(t) + \sum_{j=1}^{2} b_{ij}\Big(z_i^{\dagger}(t)\Big)f_j\Big(z_j^{\dagger}(t)\Big)\\ \qquad + \sum_{j=1}^{2} c_{ij}\Big(z_i^{\dagger}(t)\Big)g_j\Big(z_j^{\dagger}\big(t-\tau_j(t)\big)\Big) + u_i(t)\\ \qquad t \neq t_k\\ z_i^{\dagger}(t_k) \ = (1+q_k)z_i^{\dagger}\big(t_k^{-}\big),\ k \in \mathcal{Z}^{+},\ i = 1, 2.\end{cases}\tag{36}$$

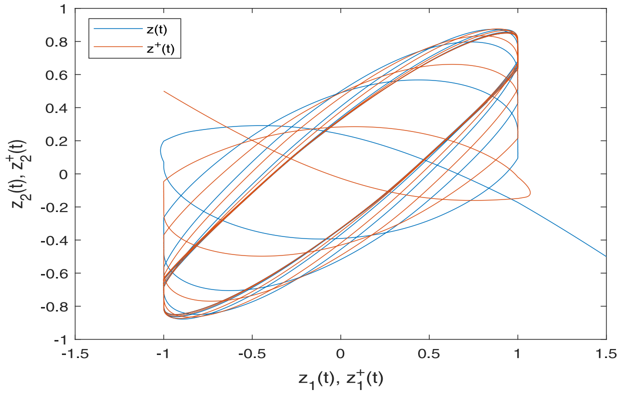

Fig. 1. Phase trajectories of the drive system and response system.

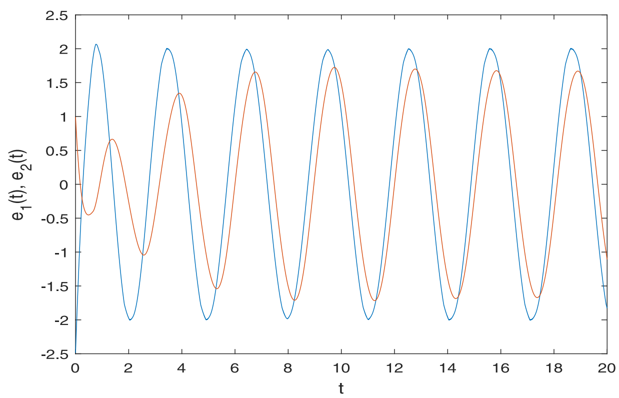

Fig. 2. State trajectories of SES without the controller.

As shown in Fig. 2, the SES of (35) and (36) without controller does not converge to 0 as time goes on.

Case 1 [Synchronization Between (35) and (36) of IMNNs Using SIs]: Let $p = 2$, $\gamma = 1.5$, $\eta = 0.5$, and $\rho = 0.8$, by simple computation, if we take $\alpha_i = 0.5$, $\delta_i = 1.75$, $i = 1, 2$, $\xi_{11} = 3.3$, $\xi_{12} = 1.6$, $\xi_{21} = 1.3$, and $\xi_{22} = 2.2$, the conditions of Theorem 1 are satisfied. Meanwhile, we have found the impulse sequences $\{t_k\}^{\mathcal{N}}$ satisfy $t_{\mathcal{N}} < 0.8^{\mathcal{N}-1} * 2.1879$. When $\mathcal{N} = 3$, that is, $\{t_k\}^3 = \{0.3, 0.5, 0.7\}$, the state trajectories of drive system (35) and response system (36) are given in Figs. 3 and 4. Moreover, Fig. 5 shows the trajectories of SESs with respect to the systems (35) and (36) for the impulse sequences $\{t_k\}^0 = \{\emptyset\}$, $\{t_k\}^3 = \{0.3, 0.5, 0.7\}$, and $\{t_k\}^5 = \{0.25, 0.35, 0.45, 0.5, 0.6\}$, respectively.

Case 2 [Synchronization Between (35) and (36) of IMNNs Using DIs]: In this case, the uncertain parameters are considered the same as the case 1. Then, it follows from Theorem 2 that the response system (36) synchronizes to the drive system (35) in finite time, where the DIs $\{t_k\}^{\mathbb{N}}$ satisfy $\min\{\ell \in \mathcal{Z}^+ : [(t_\ell)/(\sigma^{\ell-1})] \geq [(V^{1-(q/p)}(t_0))/(\beta(1-(q/p)))]\} := \mathbb{N} < \infty$, that is, $T^\dagger < \sigma^{\mathbb{N}-1} * 2.1879$. Choose $q_k = 1.05$, and the number of the DI points $\mathbb{N} = 4$. The state trajectories of drive system (35) and response system (36) with DI sequences $\{t_k\}^4 = \{0.5, 0.7, 1, 2.55\}$ are given in Figs. 6 and 7. Fig. 8 shows the trajectories of SESs with respect to the systems (35) and (36) with impulse sequences $\{t_k\}^0$ and DI sequences $\{t_k\}^3 = \{0.3, 0.5, 2.45\}$, $\{t_k\}^4 = \{0.5, 0.7, 1, 2.55\}$, respectively.

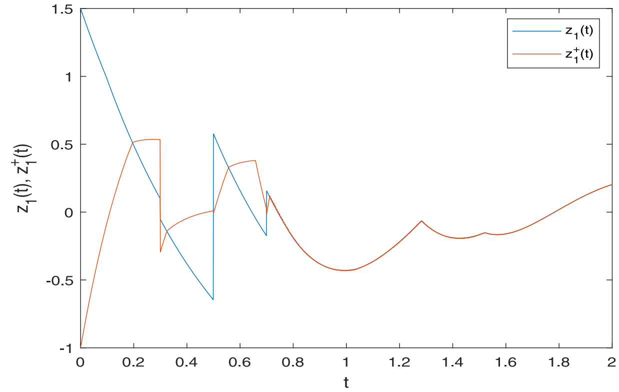

Fig. 3. Synchronization curves of $z_1(t)$ and $z_1^\dagger(t)$.

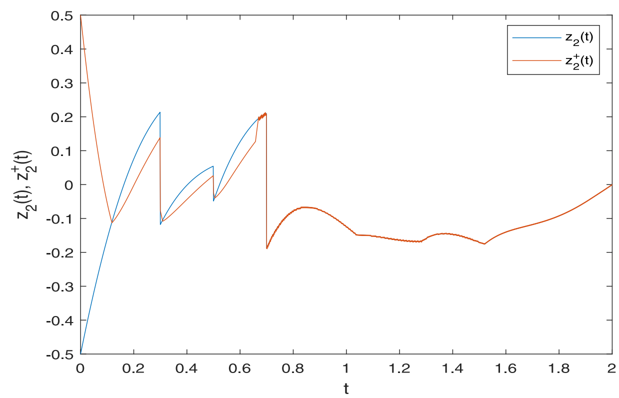

Fig. 4. Synchronization curves of $z_2(t)$ and $z_2^\dagger(t)$.

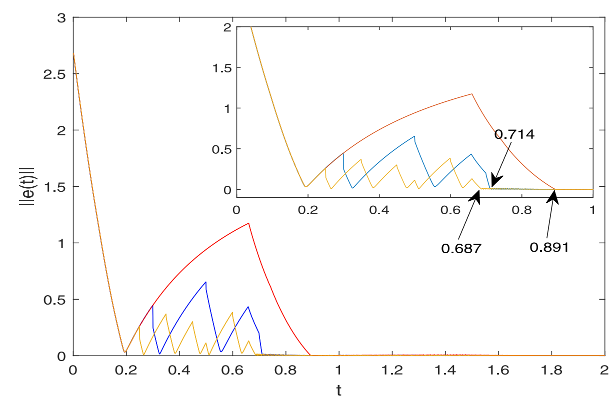

Fig. 5. Synchronization curves of SESs.

However, the conditions of [35], [37, Ths. 1 and 2], and [38] fail in this case because σ does not hold for all $\sigma > 1$. Thus, our results greatly improve existing results.

Example 2: Consider the following 3-D IMNN:

$$\begin{cases} \mathcal{D}z_i(t) = -d_i z_i(t) + \sum_{j=1}^{3} b_{ij}(z_i(t))f_j\big(z_j(t)\big) \\ \qquad\qquad + \sum_{j=1}^{3} c_{ij}(z_i(t))g_j\big(z_j(t-\tau_j(t))\big), \ t \neq t_k \\ \Delta z_i(t) = q_k z_i\big(t_k^-\big), \ t = t_k, \ k \in \mathcal{Z}^+, \ i = 1, 2, 3 \end{cases} \tag{37}$$

where $d_i = 0.8$, $\tau_i(t) = e^t/(1+e^t)$, $i = 1, 2, 3$

$$b_{11}(z_1(t)) = \begin{cases} -0.2, & |z_1(t)| \leq 0.5 \\ -0.3, & |z_1(t)| > 0.5 \end{cases}$$

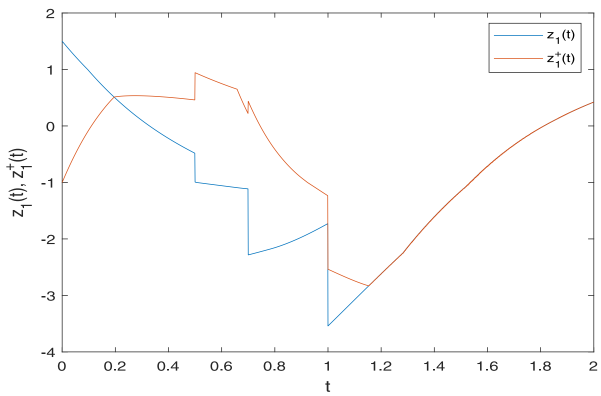

Fig. 6. Synchronization curves of $z_1(t)$ and $z_1^{\dagger}(t)$.

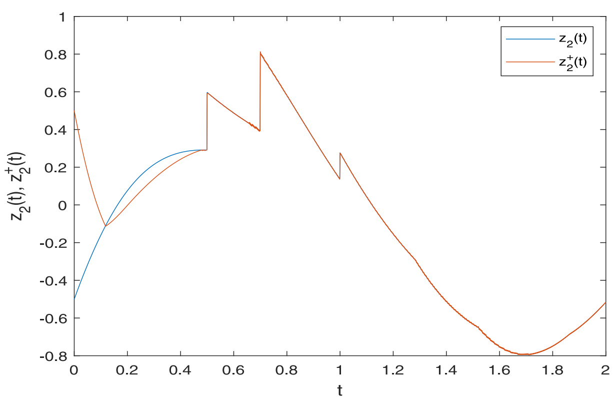

Fig. 7. Synchronization curves of $z_2(t)$ and $z_2^{\dagger}(t)$.

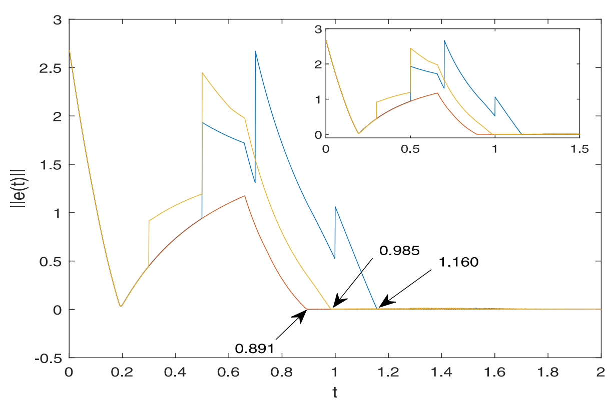

Fig. 8. Synchronization curves of SESs.

$$b_{12}(z_1(t)) = \begin{cases} 1.1, & |z_1(t)| \leq 0.5 \\ 1.3, & |z_1(t)| > 0.5 \end{cases}$$

$$b_{13}(z_1(t)) = \begin{cases} 0.8, & |z_1(t)| \leq 0.5 \\ 0.7, & |z_1(t)| > 0.5 \end{cases}$$

$$b_{22}(z_2(t)) = \begin{cases} -0.5, & |z_2(t)| \leq 0.5 \\ -0.9, & |z_2(t)| > 0.5 \end{cases}$$

$$b_{21}(z_2(t)) = \begin{cases} 1.5, & |z_2(t)| \leq 0.5 \\ 1.3, & |z_2(t)| > 0.5 \end{cases}$$

$$b_{23}(z_2(t)) = \begin{cases} 0.45, & |z_2(t)| \leq 0.5 \\ 0.3, & |z_2(t)| > 0.5 \end{cases}$$

$$b_{33}(z_3(t)) = \begin{cases} -0.3, & |z_3(t)| \leq 0.5 \\ -0.4, & |z_3(t)| > 0.5 \end{cases}$$

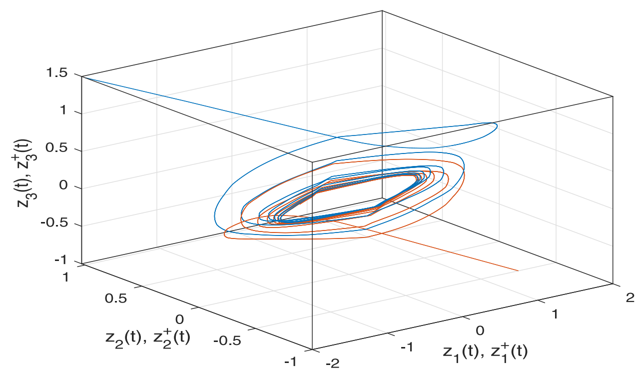

Fig. 9. Phase Trajectories of the drive system and response system.

$$b_{31}(z_3(t)) = \begin{cases} 2, & |z_3(t)| \leq 0.5 \\ 1.5, & |z_3(t)| > 0.5 \end{cases}$$

$$c_{11}(z_1(t)) = \begin{cases} -2.5, & |z_1(t)| \leq 0.5 \\ -2, & |z_1(t)| > 0.5 \end{cases}$$

$$c_{12}(z_1(t)) = \begin{cases} 0.8, & |z_1(t)| \leq 0.5 \\ 0.5, & |z_1(t)| > 0.5 \end{cases}$$

$$c_{21}(z_2(t)) = \begin{cases} 1.3, & |z_2(t)| \leq 0.5 \\ 1.2, & |z_2(t)| > 0.5 \end{cases}$$

$$c_{22}(z_2(t)) = \begin{cases} -2.5, & |z_2(t)| \leq 0.5 \\ -1.8, & |z_2(t)| > 0.5. \end{cases}$$

$$c_{33}(z_3(t)) = \begin{cases} -0.5, & |z_3(t)| \leq 0.5 \\ -0.6, & |z_3(t)| > 0.5. \end{cases}$$

$$c_{32}(z_3(t)) = \begin{cases} 3, & |z_3(t)| \leq 0.5 \\ 2.5, & |z_3(t)| > 0.5. \end{cases}$$

$$b_{32}(z_3(t)) = c_{13}(z_1(t)) = c_{23}(z_2(t)) = c_{31}(z_3(t)) = 0.$$

The DAFs are considered as $f_j(x) = g_j(x) = \tanh(x) + 0.05\mathrm{sign}(x)$, $j = 1, 2, 3$. The phase plot of (37) is shown in Fig. 9 with the initial conditions $z(s) = (-2, 1, 1.5)^T$ $\forall s \in [-1, 0)$.

Meanwhile, the corresponding response system with initial values $z^{\dagger}(s) = (1.5, -0.5, -1)$ is shown as follows:

$$\begin{cases} \mathcal{D}z_i^{\dagger}(t) = -d_i z_i^{\dagger}(t) + \sum_{j=1}^{3} b_{ij}\Big(z_i^{\dagger}(t)\Big) f_j\Big(z_j^{\dagger}(t)\Big) \\ \qquad + \sum_{j=1}^{3} c_{ij}\Big(z_i^{\dagger}(t)\Big) g_j\Big(z_j^{\dagger}\big(t - \tau_j(t)\big)\Big) + u_i(t) \\ \qquad t \neq t_k \\ z_i^{\dagger}(t_k) \ = (1 + q_k) z_i^{\dagger}\big(t_k^-\big), \ k \in \mathcal{Z}^+, \ i = 1, 2, 3. \end{cases} \tag{38}$$

As shown in Fig. 10, the SES of (37) and (38) without controller does not converge to 0 as time goes on.

Now, we consider the design of controller (8) under case 2.

Case 1 [Adaptive Synchronization Between (37) and (38) of IMNNs Using SIs]: Let $p = 2$, $\gamma = 0.5$, $\eta = 0.5$, $\rho = 0.7$, when we take $\pi_i = 0.05$, $\mu_i = 0.02$, $\vartheta_{ij} = 0.5$, and $\alpha_i(0) = \delta_i(0) = 0.5$, $\xi_{ij}(0) = 0.3$, $i = 1, 2, 3$, $j = 1, 2, 3$, by using Theorem 3, we have found the impulse sequences $\{t_k\}^{\mathcal{N}}$ satisfy $t_{\mathcal{N}} < 0.7^{\mathcal{N}-1} * [(V^{\dagger 1-q/p}(t_0))/(\beta(1 - (q/p)))]$. Meanwhile, by using Theorem 4, we can also take another DIs into account.

Case 2 [Adaptive Synchronization Between (37) and (38) of IMNNs Using DIs]: The tunable parameters are considered the same as the case 1 except for $q_k = 1.05$, Fig. 11 shows the trajectories of SESs with respect to the systems (37) and

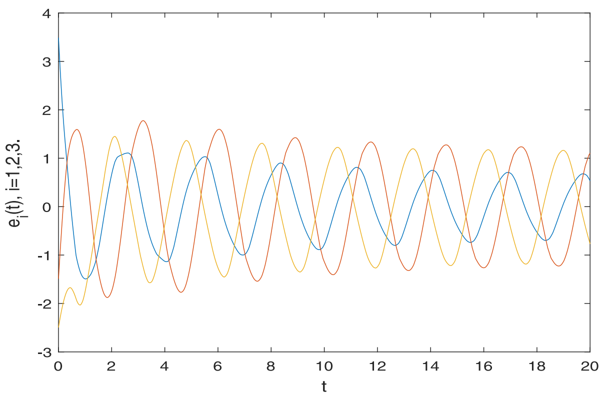

Fig. 10. State trajectories of SES without controller.

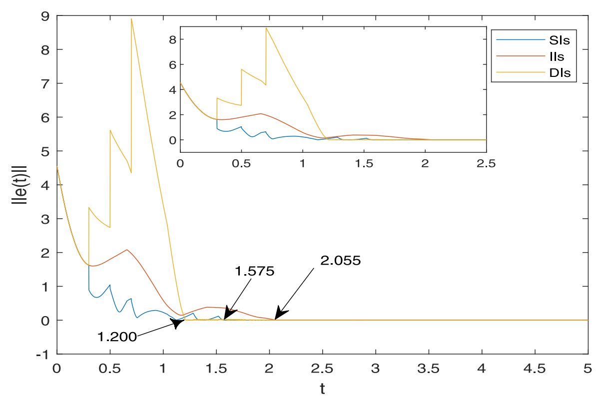

Fig. 11. Synchronization curves of SESs.

(38) for the impulse sequences $\{t_k\}^0$, SI sequences $\{t_k\}^3 = \{0.3, 0.5, 0.7\}$, and DI sequences $\{t_k\}^4 = \{0.3, 0.5, 0.7, 2.75\}$, respectively.

V. Conclusion

In this work, the problems of FTS and FTAS for a class of IMNNs with DAFs were analyzed by taking the SIs, IIs, and the DIs into account. Compared with earlier works, a wider range of impulses in the context of impulsive effects has been analyzed. Without introducing the known AIIS and the comparison principle, by employing the theories of differential inclusions and set-valued map, as well as impulsive control, new sufficient conditions with respect to the estimated settling time for synchronization of the related IMNNs are obtained using two switching control strategies, which sufficiently utilizes information from not only the SIs, DIs, and DAFs but also from the impulse sequences. Finally, numerical experiments are given to validate the efficiency of the theoretical results.

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