Delay-Dependent Stability Analysis for Switched Stochastic Networks With Proportional Delay

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***Abstract*—In this article, the issue of exponential stability (ES) is investigated for a class of switched stochastic neural networks (SSNNs) with proportional delay (PD). The key feature of PD is an unbounded time-varying delay. By considering the comparison principle and combining the extended formula for the variation of parameters, we conquer the difficulty in consideration of PD effects for such networks for the first time, where the subsystems addressed may be stable or unstable. New delay-dependent conditions with respect to the mean-square ES of systems are established by employing the average dwell-time (ADT) technique, stochastic analysis theory, and Lyapunov approach. It is shown that the acquired minimum average dwell time (MADT) is not only relevant to the stable subsystems (SSs) and unstable subsystems (USs) but also dependent on the decay ratio (DR), increasing ratio (IR), as well as PD. Finally, the availability of the derived results under an average dwell-time-switched regulation (ADTSR) is illustrated through two numerical simulation examples.**

***Index Terms*—Comparison principle, proportional delay (PD), switched neural networks (NNs), unstable subsystems (USs).**

I. Introduction

In RECENT years, much attention has been paid to researching the state estimation, dissipative analysis, as well as synchronization and stability issues of neural networks (NNs), since they have great application value in various fields related to science and engineering, such as image processing, artificial intelligence, secure communication, associative memories [1]–[12], etc. Moreover, in many real-life and natural processes, the actual signal transmission between networks is inevitably subject to stochastic disturbances from various uncertainties [13], [14], such as the synaptic transmission in NNs, which may lead to the package loss or the transmitted signal loss [15]. Up to now, a lot of literature regarding the study of stability for NNs concerning stochastic disturbances has been introduced and achieved many remarkable results [16]–[20], and references cited therein.

It is worth reminding that the switched system, consisting of a series of continuous or discrete subsystems in accordance with a rule that orchestrates the switching law between these subsystems, can be more effectively applied to establish a physical or man-made system compared to NNs for the various jumping system parameters and changing environmental factors. Thereafter, as a central and fundamental problem, corresponding stability analysis of switched NNs has drawn wide attention, and a large number of interesting results have been harvested in [21]–[26] as well as references cited therein. For example, Luo *et al.* [27] studied the uncertain switched NNs based on the Lyapunov functional (LF) and some stochastic analysis techniques (SATs) to ensure that the addressed systems could be asymptotically stable. By utilizing the mathematical induction strategy, and the piecewise LF and average dwell-time (ADT) method, the mean square stability conditions of switched stochastic NNs (SSNNs) were obtained in [28]. Also, Wu *et al.* [29] focused on the finite-time stabilization issue of switched NNs by using the LF method and ADT technique. However, it should be noted that the switched NNs mentioned above mainly consisted of stable subsystems (SSs). Recently, in order to reduce the conservatism of derived results [28]–[31], the stability of SSNNs composed of unstable subsystems (USs) has been achieved in [32] on the basis of a state-dependent average dwell-time-switched regulation (ADTSR).

On another research front, time delay is unavoidable [33], [34], which usually results in various problems, including poor performance, oscillation, or even instability [35] etc. Presently, many scholars and researchers have attracted great importance to the bounded time-varying delay and infinite-time distributed delay [36]. Nevertheless, there exists another type of delay, called proportional delay (PD), in many practical systems. For instance, Li *et al.* [37] introduced the PD into the human brain model, and the historical information would make an impact on the present. Generally speaking, the state of PD is often considered as $z(qt)$, where the parameter $q \in (0, 1)$. If we define $\tau(t) = t - qt$, one has $z(qt) = z(t - \tau(t))$, which implies $\tau(t) \to +\infty$ as time goes to the infinity. Based on the fact, the PD is an unbounded

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time-varying delay different from the traditional infinite-time-distributed delay. Hence, how to achieve the stability of NNs with the PD has become a hot point of the theoretical study in recent years [38]–[42]. For example, in [40], by utilizing the Wirtinger inequality and reciprocally convex technique, the synchronization of coupled NNs with PD was discussed, and some less conservative criteria were established to guarantee that the studied networks were asymptotically synchronized. Tang *et al.* [41] researched the exponential synchronization of nonidentically coupled Lur'e networks with PD by means of the distributed impulsive pinning control protocol. Meanwhile, by employing the finite-time stability theory and LF method, the synchronization issue of fuzzy NNs with PD and time-varying coefficients was investigated in [42].

Although many scholars and researchers have devoted themselves to the investigation of SSNNs, many aspects are still worthy of attention and need to be further improved. On one hand, many researchers mainly focus on the stability issue of SSNNs with respect to the bounded time-varying delay because the PD will tremendously increase the difficulty in dealing with such systems, which is completely different from the bounded time-varying delay. Up to now, there is litter literature regarding the research of SSNNs with PD. On the other hand, if the PD indeed exists, it will cause the effect that many existing techniques that can be used to handle SSNNs cannot be applied to deal with the switched system with PD effects, especially when the state of subsystems is unstable. Thus, how to tolerate the existence of USs for the switched system with PD has become a difficult and important point of the theoretical study. In this case, we are inspired to carry out this research and conclude this article.

According to the above analysis, we have studied the exponential stability (ES) of SSNNs with PD in this article. The main contributions of this article are listed as follows.

1) By considering the comparison principle and combining the extended formula for the variation of parameters, we manage to conquer the difficulty in consideration of PD effects for SSNNs. This is a breakthrough in this article when compared with the studied works in [16], [18], [28], [29], and [32].
2) Compared with the published works in [28], [29], [32], [40], and [42], the SSs, USs, stochastic disturbances, as well as PD, are taken into full consideration in this article which includes some existing models as special cases.
3) By considering the problem of PD, we acquire some novel stability criteria for SSNNs. In addition to that, we further work out how to tolerate the existence of USs under an ADTSR, which is the first result regarding such systems.

Notations: Let Λ stand for the set $\{1, 2, \ldots, \eta\}$, $\mathcal{Z}$ denotes the set of positive integers, $\mathbb{R}$ represents the set of real numbers, and $\mathbb{R}^n$ is the n-dimensional real space. The symbol $\|\cdot\|$ denotes the Euclid norm of the matrix or the vector. For $\tau > 0$, let $\wp^{1,2}$ denote the set of all non-negative functions $V(t, i)$ on $[t_0 - \tau, \infty) \times \Lambda$ that are continuous except at some finite number of points t_p. For function $\vartheta(\cdot) : \mathbb{R} \to \mathbb{R}$, denote $\vartheta(t^-) = \lim_{s\to 0^-} \vartheta(t + s)$ and the Dini derivative of $\vartheta(t)$ is defined as $\mathcal{D}^+\vartheta(t) = \limsup_{s\to 0^+}([\vartheta(t+s) - \vartheta(t)]/s)$. In addition, denote $g(z(t), z(qt), \sigma(t))$ by $g(t, \sigma(t))$.

II. Networks Model and Preliminaries

Consider the following SSNN with PD:

$$\begin{aligned}\mathcal{D}z(t) = \big[&-\mathcal{A}_{\sigma(t)}z(t) + \mathcal{B}_{\sigma(t)}f(z(t)) + \mathcal{C}_{\sigma(t)}f(z(qt))\big]\mathcal{D}t\\ &+ g(z(t), z(qt), \sigma(t))\mathcal{D}\omega(t) \end{aligned} \tag{1}$$

where $z(t) = (z_1(t), z_2(t), \ldots, z_n(t))^T \in \mathbb{R}^n$ stands for the state vector of neurons, $\sigma(t) : [0, \infty) \to \Lambda$ denotes the switched signal, and $\mathcal{A}_i$, $\mathcal{B}_i$, and $\mathcal{C}_i$ are known constant matrices, $i \in \Lambda$. $f(\cdot) : \mathbb{R}^n \to \mathbb{R}^n$ is an activation function, $g(\cdot, \cdot, \cdot) : \mathbb{R}^n \times \mathbb{R}^n \times \Lambda \to \mathbb{R}^{n\times m}$ is a Borel function, which denotes the noise disturbance, and $\omega(t)$ is an m-dimension Brown motion defined on a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$. The PD rate (PDR) $q \in (0, 1)$. In addition, the nonlinear functions $f(\cdot)$ and $g(\cdot, \cdot, \cdot)$ satisfy the following conditions.

Assumption 1: For $\forall x, y \in \mathbb{R}, x \neq y$, the activation function $f_i(\cdot)$ satisfies

$$l_i^- \le \frac{f_i(x) - f_i(y)}{x - y} \le l_i^+$$

where $f_i(0) = 0$, l_i^- and l_i^+ are constants.

Assumption 2: There exist matrices $\Xi_{1i} > 0$ and $\Xi_{2i} > 0$ such that

$$\begin{aligned}&\mathrm{Trace}\big[g^T(z(t), z(qt), i)g(z(t), z(qt), i)\big]\\ &\quad\le z^T(t)\Xi_{1i}z(t) + z^T(qt)\Xi_{2i}z(qt),\ \ i \in \Lambda.\end{aligned}$$

Definition 1: The SSNN (1) with PD is said to be globally ES with respect to switching signal $\sigma(t)$, if for any initial condition φ_0, there exists two positive constants λ and M such that the following condition holds:

$$\mathbb{E}\|z(t)\|^2 < M\mathbb{E}\|\varphi_0\|^2 e^{-\lambda(t-t_0)}$$

where $\mathbb{E}\|\varphi_0\|^2 = \mathbb{E}[\sup_{-\alpha\le s\le t_0}\|\varphi(s)\|^2]$ and $\alpha \in [0, \infty]$.

Definition 2 [43]: Let $\mathcal{N}(t, t_0)$ be the switching number of $\sigma(t)$ in the time interval (t_0, t), and $\mathcal{T}_a$ and $\mathcal{N}_0$ be the ADT and chatter bound, respectively. We assume that the following condition holds:

$$\mathcal{N}(t, t_0) \le \frac{t - t_0}{\mathcal{T}_a} + \mathcal{N}_0,\ \ t \ge t_0. \tag{2}$$

Lemma 1 [44]: Let $\mathcal{G}(t, u, \bar{u}_1) : \mathbb{R}^+ \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ be the nondecreasing functional in $\bar{u}_1$ for any fixed (t, u), time delay $\tau(t)$ satisfies $0 \le \tau(t) \le \tau$, and $\mathcal{I}_p : \mathbb{R} \to \mathbb{R}$ is nondecreasing functional in u. Suppose that functional $\vartheta(t)$ is continuous except at some finite number of points t_p and $V(t, \sigma(t)) \in \wp^{1,2}$, such that

$$\begin{cases}\mathcal{D}^+\mathbb{E}V(t, \sigma(t)) \le \mathcal{G}(t, \mathbb{E}V(t, \sigma(t)), \quad \mathbb{E}V(t - \tau(t), \sigma(t - \tau(t)))\\ t \neq t_p,\ t \ge t_0\\ \mathbb{E}V\big(t_p, \sigma\big(t_p\big)\big) \le \mathcal{I}_p\Big(\mathbb{E}V\Big(t_p^-, \sigma\Big(t_p^-\Big)\Big)\Big),\ \ p \in \mathcal{Z}\end{cases}$$

and

$$\begin{cases}\mathcal{D}^+\vartheta(t) > \mathcal{G} \quad (t, \vartheta(t), \vartheta(t - \tau(t)))\\ t \neq t_p,\ t \ge t_0\\ \vartheta\big(t_p\big) > \mathcal{I}_p\Big(\vartheta\Big(t_p^-\Big)\Big),\ \ p \in \mathcal{Z}\end{cases}$$

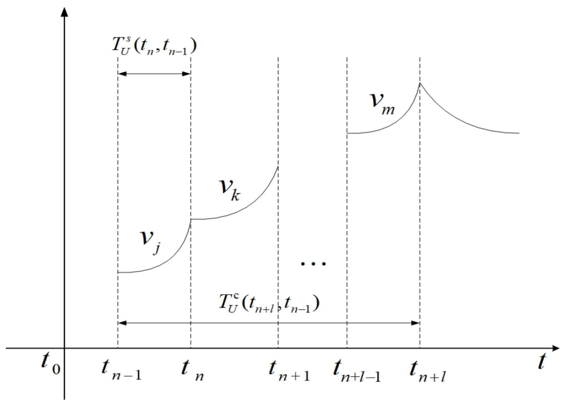

Fig. 1. $\mathcal{T}_U^s(t_n, t_{n-1})$ and $\mathcal{T}_U^c(t_{n+\ell}, t_{n-1})$, $n, \ell \in \mathcal{Z}$.

where $\sigma(t) \in \Lambda$, and t_p is the pth switching time instant. Then, $\mathbb{E}V(t, \sigma(t)) \leq \vartheta(t)$ for any $t \in [-\tau, t_0]$ implies that $\mathbb{E}V(t, \sigma(t)) \leq \vartheta(t)$ for $t \geq t_0$.

Lemma 2 [45]: For the symmetric matrices $\mathcal{X} \in \mathbb{R}^{n\times n}$ and $\mathcal{Y} \in \mathbb{R}^{n\times n}$, if $\mathcal{X} > 0$, then the following inequalities hold:

$$\lambda_{\min}\Big(\mathcal{X}^{-1}\mathcal{Y}\Big)z^T\mathcal{X}z \leq z^T\mathcal{Y}z \leq \lambda_{\max}\Big(\mathcal{X}^{-1}\mathcal{Y}\Big)z^T\mathcal{X}z,\ \ z \in \mathbb{R}^n.$$

Remark 1: It shall be pointed out that the aforementioned results in [16], [18], [28], [29], and [32] have only concentrated on the stability issue of SSNNs with bounded time-varying delay since the PD will tremendously increase the difficulty for the related systems, which is completely different from the bounded time-varying delay.

III. Main Results

In this section, by using the comparison principle, ADTT, stochastic analysis theory, and Lyapunov method, we will derive the new criteria with respect to the SSNN with PD effects consisting of both SSs and USs.

Without loss of generality, first, when $\sigma(t) \in \Lambda_S = \{1, 2, \ldots, m\}$, suppose that the subsystems are stable. While when $\sigma(t) \in \Lambda_U = \{m+1, m+2, \ldots, \eta\}$, the subsystems are unstable, and $\Lambda_U \neq \emptyset$. Second, let $\mathcal{T}_S(t, s)$ denote the total activation time of the SSs in the time interval $[s, t)$, while $\mathcal{T}_U(t, s)$ denotes the total activation time of the USs on $[s, t)$. Third, assume that $\mathcal{T}_U^s(t_n, t_{n-1}) = \{t_n - t_{n-1}{:}\sigma(t) \in \Lambda_U, t \in [t_{n-1}, t_n), n \in \mathcal{Z}\}$ stands for the run time of the switched system on a single US, and $\mathcal{T}_U^c(t_{n+\ell}, t_{n-1}) = \{t_{n+\ell} - t_{n-1} : \sigma(t) \in \Lambda_U, \sigma(t_{n+\ell}) \in \Lambda_S, t \in [t_{n-1}, t_{n+\ell}), n, \ell \in \mathcal{Z}\}$ stands for the time for switched system persistent active on USs. Subsequently, the corresponding $\mathcal{T}_U^s(t_n, t_{n-1})$ and $\mathcal{T}_U^c(t_{n+\ell}, t_{n-1})$ are elucidated in Fig. 1, and for switching instant j, the corresponding energy function is denoted by V_j. In the meanwhile, let $\tau_s = \mathcal{T}_U^s(t_n, t_{n-1})$ denote the maximal length of a single US runs time, while $\tau_c = \mathcal{T}_U^c(t_{n+\ell}, t_{n-1})$ denotes the maximal length of USs runtime.

In the following, we present the following results.

Theorem 1: Suppose that Assumptions 1 and 2 hold, and the ADT of switching signal is defined in Definition 2, for any $i \in \Lambda_S$ (resp., $i \in \Lambda_U$), if there exist positive-definite matrices P_i and scalars $\alpha_i > 0$, (resp., $\beta_i > 0$), $\delta_i > 0$, $\mu > 1$, and $\lambda \in (0, \beta^\dagger)$ such that

$$\begin{bmatrix} \Omega_i & P_i\mathcal{C}_i \\ * & -I \end{bmatrix} < 0,\ \ i \in \Lambda_S \tag{3}$$

$$\begin{bmatrix} \Sigma_i & P_i\mathcal{C}_i \\ * & -I \end{bmatrix} < 0,\ \ i \in \Lambda_U \tag{4}$$

$$P_i \leq \delta_i I \tag{5}$$

$$P_i \leq \mu P_j \tag{6}$$

$$\begin{cases} \lambda q - \beta^\dagger + \alpha^\dagger\theta e^{\lambda(1-q)t_0} < 0 \\ \lambda\big(\lambda q - \beta^\dagger + q\alpha^\dagger\theta e^{\lambda(1-q)t_0}\big) > \beta^\dagger\big(\lambda q - \beta^\dagger \\ \quad + \alpha^\dagger\theta e^{\lambda(1-q)t_0}\big) \\ \beta^{\dagger 2}\big(\beta^\dagger - \lambda q - \alpha^\dagger\theta e^{\lambda(1-q)t_0}\big) > \lambda^2\big(\beta^\dagger - \lambda q \\ \quad - q\alpha^\dagger\theta e^{\lambda(1-q)t_0}\big) \end{cases} \tag{7}$$

where $\Omega_i = -P_i\mathcal{A}_i - \mathcal{A}_i^T P_i + P_i + \lambda_{\max}(P_i)\|\mathcal{B}_i\|^2 L^T L + \delta_i\Xi_{1i} + \alpha_i P_i$, $\alpha^\dagger = \mu^{\mathcal{N}_0} e^{(\alpha+\beta)(1+\mathcal{N}_0)\tau_s}$, $\Sigma_i = -P_i\mathcal{A}_i - \mathcal{A}_i^T P_i + P_i + \lambda_{\max}(P_i)\|\mathcal{B}_i\|^2 L^T L + \delta_i\Xi_{1i} - \beta_i P_i$, $\beta^\dagger = \alpha - ([(\alpha + \beta)\tau_s + ln\mu]/[\mathcal{T}_a])$, $\alpha = \min\{\alpha_i\}$, $\beta = \max\{\beta_i\}$, $\theta = \mu\theta^\dagger$, $\theta^\dagger = \max_i\{\theta_i\} = \max_i\{\lambda_{\max}(P_i^{-1}L^T L) + \delta_i\lambda_{\max}(P_i^{-1}\Xi_{2i})\}$, $\delta = \lambda_{\max}(\delta_i)$, and $\rho^\dagger = \min\{\lambda_{\min}(P_i)\}$.

Then, the SSNN with PD in (1) is globally exponentially stable.

Proof: Construct the following LF:

$$V(t, \sigma(t)) = z^T(t)P_{\sigma(t)}z(t) \tag{8}$$

where $\sigma(t) = \gamma_n \in \Lambda$ for $t \in [t_{n-1}, t_n)$, $n \in \mathcal{Z}$.

By Itô's formula

$$\mathcal{D}V(t, \gamma_n) = \mathscr{L}V(t, \gamma_n) + V_z(t, \gamma_n) \times g(z(t), z(qt), \gamma_n)\mathcal{D}\omega(t).$$

For $t \in [t_{n-1}, t_n)$, we have

$$\begin{aligned} &\mathscr{L}V(t, \gamma_n) \\ &= -2z^T(t)P_{\gamma_n}\mathcal{A}_{\gamma_n}z(t) + 2z^T(t)P_{\gamma_n}\mathcal{B}_{\gamma_n}f(z(t)) \\ &\quad + 2z^T(t)P_{\gamma_n}\mathcal{C}_{\gamma_n}f(z(qt)) \\ &\quad + \frac{1}{2}\mathrm{Trace}\big[g^T(z(t), z(qt), \gamma_n)V_{zz}(t, z(t), \gamma_n) \\ &\qquad \times g(z(t), z(qt), \gamma_n)\big] \\ &\leq -2z^T(t)P_{\gamma_n}\mathcal{A}_{\gamma_n}z(t) + z^T(t)P_{\gamma_n}z(t) \\ &\quad + f^T(z(t))\mathcal{B}_{\gamma_n}^T P_{\gamma_n}\mathcal{B}_{\gamma_n}f(z(t)) \\ &\quad + z^T(t)P_{\gamma_n}\mathcal{C}_{\gamma_n}\mathcal{C}_{\gamma_n}^T P_{\gamma_n}z(t) + f^T(z(qt))f(z(qt)) \\ &\quad + \mathrm{Trace}\big[g^T(z(t), z(qt), \gamma_n)P_{\gamma_n}g(z(t), z(qt), \gamma_n)\big]. \end{aligned} \tag{9}$$

By introducing Assumption 1, then

$$\begin{aligned} &\mathscr{L}V(t, \gamma_n) \\ &\leq -2z^T(t)P_{\gamma_n}\mathcal{A}_{\gamma_n}z(t) + z^T(t)P_{\gamma_n}z(t) \\ &\quad + \lambda_{\max}\big(P_{\gamma_n}\big)\|\mathcal{B}_{\gamma_n}\|^2 z^T(t)L^T Lz(t) \\ &\quad + \lambda_{\max}\Big(P_{\gamma_n}^{-1}L^T L\Big)z^T(qt)P_{\gamma_n}z(qt) \\ &\quad + z^T(t)P_{\gamma_n}\mathcal{C}_{\gamma_n}\mathcal{C}_{\gamma_n}^T P_{\gamma_n}z(t) \\ &\quad + \mathrm{Trace}\big[g^T(z(t), z(qt), \gamma_n)P_{\gamma_n} \\ &\qquad \times g(z(t), z(qt), \gamma_n)\big] \end{aligned} \tag{10}$$

where $L = \mathrm{diag}\{l_1, l_2, \ldots, l_n\}$ and $l_i = \max\{|\ell_i^+|, |\ell_i^-|\}$.

Case 1: First, consider the SSs as follows, from the conditions (3) and (5) of Theorem 1 and Assumption 2, we can obtain:

$$\begin{aligned}&\mathscr{L}V(t,\gamma_n)\\&\le -2z^T(t)P_{\gamma_n}\mathcal{A}_{\gamma_n}z(t)+z^T(t)P_{\gamma_n}z(t)\\&\quad+\lambda_{\max}\left(P_{\gamma_n}\right)\|\mathcal{B}_{\gamma_n}\|^2z^T(t)L^TLz(t)\\&\quad+z^T(t)P_{\gamma_n}\mathcal{C}_{\gamma_n}\mathcal{C}_{\gamma_n}^TP_{\gamma_n}z(t)\\&\quad+\lambda_{\max}\left(P_{\gamma_n}^{-1}L^TL\right)z^T(qt)P_{\gamma_n}z(qt)\\&\quad+\delta_{\gamma_n}\Big[z^T(t)\Xi_{1\gamma_n}z(t)\\&\qquad+\lambda_{\max}\left(P_{\gamma_n}^{-1}\Xi_{2\gamma_n}\right)z^T(qt)P_{\gamma_n}z(qt)\Big]\\&\le -\alpha_{\gamma_n}V(t,\gamma_n)\\&\quad+\theta_{\gamma_n}V(qt,\gamma_n),\ \ \gamma_n\in\Lambda_S.\end{aligned}\tag{11}$$

Also, in view of (6), it is easy to see that

$$\begin{aligned}\mathscr{L}V(t,\gamma_n)\le&-\alpha_{\gamma_n}V(t,\gamma_n)\\&+\mu\theta_{\gamma_n}V(qt,\sigma(qt)),\ \ \gamma_n\in\Lambda_S.\end{aligned}\tag{12}$$

Then, applying the Itô formula, integrating both sides of (12), and taking the mathematical expectation for a small enough $\varepsilon>0$ such that

$$\begin{aligned}&\mathbb{E}V(t+\varepsilon,\gamma_n)-\mathbb{E}V(t,\gamma_n)\\&=\int_t^{t+\varepsilon}\mathbb{E}\mathscr{L}V(t,\gamma_n)+\int_t^{t+\varepsilon}\mathbb{E}V_z(t,\gamma_n)\\&\quad\times g(z(t),z(qt),\gamma_n)\mathcal{D}\omega(t)\\&=\int_t^{t+\varepsilon}\mathbb{E}\mathscr{L}V(t,\gamma_n).\end{aligned}$$

Let $\varepsilon\to 0$, then we have

$$\begin{aligned}\mathcal{D}^+\mathbb{E}V(t,\gamma_n)&=\mathbb{E}\mathscr{L}V(t,\gamma_n)\\&\le-\alpha_{\gamma_n}\mathbb{E}V(t,\gamma_n)\\&\quad+\mu\theta_{\gamma_n}\mathbb{E}V(qt,\sigma(qt)),\ \ \gamma_n\in\Lambda_S.\end{aligned}\tag{13}$$

Case 2: On the other hand, consider the USs case, in view of the conditions (4)–(6) of Theorem 1 and similarly to the proof of the SSs, we obtain

$$\begin{aligned}\mathcal{D}^+\mathbb{E}V(t,\gamma_n)\le&\,\beta_{\gamma_n}\mathbb{E}V(t,\gamma_n)\\&+\mu\theta_{\gamma_n}\mathbb{E}V(qt,\sigma(qt)),\ \ \gamma_n\in\Lambda_U.\end{aligned}\tag{14}$$

Based on the aforementioned discussions, one has

$$\begin{cases}\mathcal{D}^+\mathbb{E}V(t,\sigma(t))\le\xi(t)\mathbb{E}V(t,\sigma(t))\\+\theta\mathbb{E}V(qt,\sigma(qt)),\ \ t\ne t_n,\ \ t>t_0\\\mathbb{E}V(t_n,\gamma_n)\le\mu\mathbb{E}V\left(t_n^-,\gamma_{n-1}\right),\ \ n\in\mathcal{Z}\\\mathbb{E}V(t,\sigma(t))\le\delta\mathbb{E}\|\varphi_0\|^2,\ \ t\le t_0\end{cases}$$

where

$$\xi(t)=\begin{cases}-\alpha,&\sigma(t)\in\Lambda_S\\\beta,&\sigma(t)\in\Lambda_U.\end{cases}$$

Next, for any $\varepsilon>0$, we consider the following delayed system with a unique solution $\chi(t)$:

$$\begin{cases}\dot{\chi}(t)=\xi(t)\chi(t)+\theta\chi(qt)+\varepsilon,\ \ t\ne t_n,\ \ t\ge t_0\\\chi(t_n)=\mu\chi\left(t_n^-\right),\ \ n\in\mathcal{Z}\\\chi(t)=\delta\mathbb{E}\|\varphi_0\|^2.\end{cases}\tag{15}$$

Then, it follows from Lemma 1 that:

$$\mathbb{E}V(t,\sigma(t))\le\chi(t),\ \ t\ge t_0.$$

On basis of the extended formula for the variation of parameters [46], it is evident that

$$\chi(t)=\Pi(t,t_0)\chi(t_0)+\int_{t_0}^t\Pi(t,s)(\theta\chi(qs)+\varepsilon)ds\tag{16}$$

where $\Pi(t,s)$, $s\in[t_0,t]$ is the Cauchy matrix of the following system:

$$\begin{cases}\dot{\zeta}(t)=\xi(t)\zeta(t),\ \ t\ne t_n\\\zeta(t_n)=\mu\zeta\left(t_n^-\right),\ \ n\in\mathcal{Z}.\end{cases}\tag{17}$$

In view of Definition 2, the Cauchy matrix $\Pi(t,s)$ can be estimated as follows:

$$\begin{aligned}\Pi(t,s)&=e^{\int_s^t\xi(u)du}\prod_{s\le t_n\le t}\mu\\&\le e^{-\alpha\mathcal{T}_S(t,s)+\beta\mathcal{T}_U(t,s)}\mu^{\mathcal{N}(t,s)}\\&=e^{-\alpha(t-s)+(\alpha+\beta)\mathcal{T}_U(t,s)}\mu^{\mathcal{N}(t,s)}\\&\le e^{-\alpha(t-s)+(\alpha+\beta)\tau_s(1+\mathcal{N}(t,s))}\mu^{\mathcal{N}(t,s)}\\&\le\alpha^\dagger e^{-\beta^\dagger(t-s)}.\end{aligned}\tag{18}$$

Then, it is calculated that

$$\begin{aligned}\chi(t)&=\Pi(t,t_0)\chi(t_0)+\int_{t_0}^t\Pi(t,s)(\theta\chi(qs)+\varepsilon)ds\\&\le\alpha^\dagger\delta\mathbb{E}\|\varphi_0\|^2e^{-\beta^\dagger(t-t_0)}+\int_{t_0}^t\alpha^\dagger e^{-\beta^\dagger(t-s)}(\theta\chi(qs)+\varepsilon)ds\\&=\mathscr{M}e^{-\beta^\dagger(t-t_0)}+\int_{t_0}^t\alpha^\dagger e^{-\beta^\dagger(t-s)}(\theta\chi(qs)+\varepsilon)ds\end{aligned}\tag{19}$$

where $\mathscr{M}=\alpha^\dagger\delta\mathbb{E}\|\varphi_0\|^2$.

Based on the aforementioned discussions, if there exists λ satisfying the condition (7), we can further prove that the following inequality holds:

$$\chi(t)<\mathscr{M}e^{-\lambda(t-t_0)}+\frac{\alpha^\dagger\varepsilon}{\beta^\dagger-\theta\alpha^\dagger},\ \ t>t_0.\tag{20}$$

Specifically, when $t=t_0$, one has $\chi(t_0)<\mathscr{M}+\alpha^\dagger\varepsilon/(\beta^\dagger-\theta\alpha^\dagger)$.

To show the inequality (20) is true, we will use the mathematical method, that is, proof by contradiction.

In what follows, if the condition (20) does not hold, then there exists $t^\dagger>t_0$ such that:

$$\chi\left(t^\dagger\right)\ge\mathscr{M}e^{-\lambda\left(t^\dagger-t_0\right)}+\frac{\alpha^\dagger\varepsilon}{\beta^\dagger-\theta\alpha^\dagger}\tag{21}$$

and

$$\chi(t)\le\mathscr{M}e^{-\lambda(t-t_0)}+\frac{\alpha^\dagger\varepsilon}{\beta^\dagger-\theta\alpha^\dagger},\ \ \text{for } t<t^\dagger.\tag{22}$$

Combining (19) and (22), it yields

$$\begin{aligned}\chi\left(t^\dagger\right) &\le \mathscr{M}e^{-\beta^\dagger(t^\dagger-t_0)}+\int_{t_0}^{t^\dagger}e^{-\beta^\dagger(t^\dagger-s)}\\&\quad\times\left(\alpha^\dagger\theta\chi(qs)+\alpha^\dagger\varepsilon\right)ds\\&\le \mathscr{M}e^{-\beta^\dagger(t^\dagger-t_0)}+\int_{t_0}^{t^\dagger}e^{-\beta^\dagger(t^\dagger-s)}\alpha^\dagger\theta\\&\quad\times\left(\mathscr{M}e^{-\lambda(qs-t_0)}+\frac{\alpha^\dagger\varepsilon}{\beta^\dagger-\theta\alpha^\dagger}\right)ds\\&\quad+\int_{t_0}^{t^\dagger}e^{-\beta^\dagger(t^\dagger-s)}\alpha^\dagger\varepsilon ds\\&= \mathscr{M}e^{-\beta^\dagger(t^\dagger-t_0)}+\alpha^\dagger\theta\mathscr{M}\int_{t_0}^{t^\dagger}e^{-\beta^\dagger(t^\dagger-s)}e^{-\lambda(qs-t_0)}ds\\&\quad+\left(\alpha^\dagger\theta\frac{\alpha^\dagger\varepsilon}{\beta^\dagger-\theta\alpha^\dagger}+\alpha^\dagger\varepsilon\right)\int_{t_0}^{t^\dagger}e^{-\beta^\dagger(t^\dagger-s)}ds\\&= \mathscr{M}e^{-\beta^\dagger(t^\dagger-t_0)}\\&\quad+\frac{\alpha^\dagger\theta\mathscr{M}}{\beta^\dagger-\lambda q}e^{-\beta^\dagger t^\dagger+\lambda t_0}\left(e^{(\beta^\dagger-\lambda q)t^\dagger}-e^{(\beta^\dagger-\lambda q)t_0}\right)\\&\quad+\frac{\alpha^\dagger\varepsilon\left(\frac{\alpha^\dagger\theta}{\beta^\dagger-\theta\alpha^\dagger}+1\right)e^{-\beta^\dagger t^\dagger}}{\beta^\dagger}\left(e^{\beta^\dagger t^\dagger}-e^{\beta^\dagger t_0}\right)\\&= \mathscr{M}e^{-\beta^\dagger(t^\dagger-t_0)}\\&\quad-\frac{\alpha^\dagger\theta\mathscr{M}}{\lambda q-\beta^\dagger}e^{\lambda(1-q)t_0}\left(e^{-\lambda q(t^\dagger-t_0)}-e^{-\beta^\dagger(t^\dagger-t_0)}\right)\\&\quad+\frac{\alpha^\dagger\varepsilon}{\beta^\dagger-\theta\alpha^\dagger}\left(1-e^{-\beta^\dagger(t^\dagger-t_0)}\right).\end{aligned}\tag{23}$$

Let us define a continuous parameter function $\psi(t)$ as follows:

$$\begin{aligned}\psi(t)&=-\beta^\dagger\left(\lambda q-\beta^\dagger+\alpha^\dagger\theta e^{\lambda(1-q)t_0}\right)e^{-\beta^\dagger(t-t_0)}\\&\quad+\lambda\left(\lambda q-\beta^\dagger+q\alpha^\dagger\theta e^{\lambda(1-q)t_0}\right)e^{-\lambda(t-t_0)}.\end{aligned}\tag{24}$$

Calculating the derivative of $\psi(t)$ yields

$$\begin{aligned}\dot\psi(t)&=\beta^{\dagger 2}\left(\lambda q-\beta^\dagger+\alpha^\dagger\theta e^{\lambda(1-q)t_0}\right)e^{-\beta^\dagger(t-t_0)}\\&\quad-\lambda^2\left(\lambda q-\beta^\dagger+q\alpha^\dagger\theta e^{\lambda(1-q)t_0}\right)e^{-\lambda(t-t_0)}.\end{aligned}\tag{25}$$

Moreover, it is not difficult to see that $\dot\psi(t)=0$ iff the time t is equivalent to the following condition:

$$t=t^*=\frac{1}{\beta^\dagger-\lambda}ln\frac{\beta^{\dagger 2}\left(\lambda q-\beta^\dagger+\alpha^\dagger\theta e^{\lambda(1-q)t_0}\right)}{\lambda^2\left(\lambda q-\beta^\dagger+q\alpha^\dagger\theta e^{\lambda(1-q)t_0}\right)}+t_0.\tag{26}$$

Hence, according to the condition (7) of Theorem 1, when $t\in(t_0,t^*)$, one has $\dot\psi(t)>0$, which implies that $\psi(t)$ is increasing in the time interval (t_0,t^*). While when $t>t^*$, one has $\dot\psi(t)<0$, it indicates that $\psi(t)$ is decreasing on $(t^*,+\infty)$.

Meanwhile, from the condition (7), it is noted that $\psi(t_0)=\lambda(\lambda q-\beta^\dagger+q\alpha^\dagger\theta e^{\lambda(1-q)t_0})-\beta^\dagger(\lambda q-\beta^\dagger+\alpha^\dagger\theta e^{\lambda(1-q)t_0})>0$, and from (24), $\lim_{t\to+\infty}\psi(t)=0$. Therefore, we can conclude that $\psi(t)\ge 0$ for any $t\ge t_0$, and then we have

$$\begin{aligned}&-\beta^\dagger\left(1+\frac{\alpha^\dagger\theta e^{\lambda(1-q)t_0}}{\lambda q-\beta^\dagger}\right)e^{-\beta^\dagger(t-t_0)}\\&\quad+\lambda\left(1+\frac{q\alpha^\dagger\theta e^{\lambda(1-q)t_0}}{\lambda q-\beta^\dagger}\right)e^{-\lambda(t-t_0)}\le 0.\end{aligned}\tag{27}$$

In view of $q\in(0,1)$, it is easy to see that $e^{-\lambda q(t-t_0)}>e^{-\lambda(t-t_0)}$, then by simple induction, we obtain

$$\begin{aligned}\mathcal{R}(t)&=-\beta^\dagger\left(1+\frac{\alpha^\dagger\theta e^{\lambda(1-q)t_0}}{\lambda q-\beta^\dagger}\right)e^{-\beta^\dagger(t-t_0)}\\&\quad+\lambda\frac{q\alpha^\dagger\theta e^{\lambda(1-q)t_0}}{\lambda q-\beta^\dagger}e^{-\lambda q(t-t_0)}+\lambda e^{-\lambda(t-t_0)}\\&\le 0.\end{aligned}\tag{28}$$

Next, we define another parameter function $\mathcal{S}(t)$ as follows:

$$\begin{aligned}\mathcal{S}(t)&=\left(1+\frac{\alpha^\dagger\theta e^{\lambda(1-q)t_0}}{\lambda q-\beta^\dagger}\right)e^{-\beta^\dagger(t-t_0)}\\&\quad-\frac{\alpha^\dagger\theta e^{\lambda(1-q)t_0}}{\lambda q-\beta^\dagger}e^{-\lambda q(t-t_0)}-e^{-\lambda(t-t_0)}.\end{aligned}\tag{29}$$

It is calculated that $\mathcal{S}(t_0)=0$ and $\dot{\mathcal{S}}(t)=\mathcal{R}(t)$, which means that $\dot{\mathcal{S}}(t)\le 0$. That is to say, $\mathcal{S}(t)$ is a monotone decreasing function under the initial condition $\mathcal{S}(t_0)=0$. Thus, it follows from (28) that $\mathcal{S}(t)\le\mathcal{S}(t_0)$, we have:

$$\begin{aligned}&\left(1+\frac{\alpha^\dagger\theta e^{\lambda(1-q)t_0}}{\lambda q-\beta^\dagger}\right)e^{-\beta^\dagger(t-t_0)}-\frac{\alpha^\dagger\theta e^{\lambda(1-q)t_0}}{\lambda q-\beta^\dagger}e^{-\lambda q(t-t_0)}\\&\le e^{-\lambda(t-t_0)},\ \ t\ge t_0.\end{aligned}\tag{30}$$

Then, combining (23) and (30) yields

$$\begin{aligned}\chi\left(t^\dagger\right)&\le\left(1+\frac{\alpha^\dagger\theta e^{\lambda(1-q)t_0}}{\lambda q-\beta^\dagger}\right)\mathscr{M}e^{-\beta^\dagger(t^\dagger-t_0)}\\&\quad-\frac{\alpha^\dagger\theta e^{\lambda(1-q)t_0}}{\lambda q-\beta^\dagger}\mathscr{M}e^{-\lambda q(t^\dagger-t_0)}\\&\quad+\frac{\alpha^\dagger\varepsilon}{\beta^\dagger-\theta\alpha^\dagger}\left(1-e^{-\beta^\dagger(t^\dagger-t_0)}\right)\\&\le\mathscr{M}e^{-\lambda(t^\dagger-t_0)}+\frac{\alpha^\dagger\varepsilon}{\beta^\dagger-\theta\alpha^\dagger}\left(1-e^{-\beta^\dagger(t^\dagger-t_0)}\right)\\&<\mathscr{M}e^{-\lambda(t^\dagger-t_0)}+\frac{\alpha^\dagger\varepsilon}{\beta^\dagger-\theta\alpha^\dagger}.\end{aligned}\tag{31}$$

It is clear that inequality (31) is a contradiction with assumption (21). Thus, the condition (20) is true for all $t\ge t_0$.

Let $\varepsilon\to 0$ and we can obtain

$$\chi(t)\le\mathscr{M}e^{-\lambda(t-t_0)}.$$

Then, we have

$$\mathbb{E}V(t,\sigma(t))\le\chi(t)\le\mathscr{M}e^{-\lambda(t-t_0)},\ \ t\ge t_0$$

that is

$$\mathbb{E}\|z(t)\|^2\le\frac{\mathscr{M}}{\rho^\dagger}e^{-\lambda(t-t_0)},\ \ t\ge t_0$$

so based on the above discussions, the ES of the SSNN with PD in (1) can be achieved. The proof is completed. ■

Remark 2: It shall be noted that the only related result is that Wu *et al.* [28] established an effective stability condition for SSNNs with time-varying delay. However, the effects of PD are ignored. Thus, our results can be applied more widely than that in [28]. In addition, as shown above, some new auxiliary functions are adopted, which is completely different from the traditional strategies in the switched systems.

In particular, if there are only two switched states in Λ, that is $\Lambda_S = \{1\}$, $\Lambda_U = \{2\}$, then we present the following criterion to determine the ES of the SSNN with the effects of PD.

Corollary 1: Suppose that Assumptions 1 and 2 hold, and the ADT of switching signal is defined in Definition 2, for any $i \in \Lambda_S$ (resp., $i \in \Lambda_U$), if there exist positive-definite matrices P_i and scalars $\alpha_i > 0$, (resp., $\beta_i > 0$), $\delta_i > 0$, $\mu > 1$, and $\tilde{\lambda} \in (0, \tilde{\beta})$ such that the conditions (3)–(6), and the following inequalities hold:

$$\begin{cases} \tilde{\lambda} q - \tilde{\beta} + \tilde{\alpha}\theta e^{\tilde{\lambda}(1-q)t_0} < 0 \\ \tilde{\lambda}\Big(\tilde{\lambda} q - \tilde{\beta} + q\tilde{\alpha}\theta e^{\tilde{\lambda}(1-q)t_0}\Big) > \tilde{\beta}\Big(\tilde{\lambda} q - \tilde{\beta} \\ \qquad\qquad + \tilde{\alpha}\theta e^{\tilde{\lambda}(1-q)t_0}\Big) \\ \tilde{\beta}^2\Big(\tilde{\beta} - \tilde{\lambda} q - \tilde{\alpha}\theta e^{\tilde{\lambda}(1-q)t_0}\Big) > \tilde{\lambda}^2\Big(\tilde{\beta} - \tilde{\lambda} q \\ \qquad\qquad - q\tilde{\alpha}\theta e^{\tilde{\lambda}(1-q)t_0}\Big) \end{cases} \tag{32}$$

where $\tilde{\alpha} = \mu^{\mathcal{N}_0} e^{(\alpha+\beta)(1+[\mathcal{N}_0/2])\tau_s}$, $\tilde{\beta} = \alpha - ([(\alpha+\beta)\tau_s + 2ln\mu]/[2\mathcal{T}_a])$, $\alpha = \min\{\alpha_i\}$, $\beta = \max\{\beta_i\}$, $\theta = \mu\theta^\dagger$, $\theta^\dagger = \max_i\{\theta_i\} = \max_i\{\lambda_{\max}(P_i^{-1}L^TL) + \delta_i\lambda_{\max}(P_i^{-1}\Xi_{2i})\}$, $\delta = \lambda_{\max}(\delta_i)$, and $\rho^\dagger = \min\{\lambda_{\min}(P_i)\}$.

Then, the SSNN (1) with PD is exponentially stable.

Proof: Since there exist only two states $\Lambda_S = \{1\}$, $\Lambda_U = \{2\}$, thus the occurrence number of activation on state 2 is less than $1 + ([\mathcal{N}(t,s)]/2)$. So in view of Definition 2, $\Pi(t,s)$ can be estimated as follows:

$$\begin{aligned} \Pi(t,s) &= e^{\int_s^t \xi(u)du} \prod_{s\le t_n\le t} \mu \\ &\le e^{-\alpha\mathcal{T}_S(t,s)+\beta\mathcal{T}_U(t,s)}\mu^{\mathcal{N}(t,s)} \\ &= e^{-\alpha(t-s)+(\alpha+\beta)\mathcal{T}_U(t,s)}\mu^{\mathcal{N}(t,s)} \\ &\le e^{-\alpha(t-s)+(\alpha+\beta)\tau_s\left(1+\frac{\mathcal{N}(t,s)}{2}\right)}\mu^{\mathcal{N}(t,s)} \\ &\le \tilde{\alpha} e^{-\tilde{\beta}(t-s)}. \end{aligned} \tag{33}$$

Then, based on the aforementioned discussions and using the same method as the proof in Theorem 1, we have Corollary 1, immediately. ■

Remark 3: In contrast to the existing works in [28], [29], [38], and [42], by considering the effects of PD, we conduct a research on ES for SSNNs. Moreover, we further figure out how to tolerate the existence of USs under an ADTSR for the related systems.

Next, different from the schemes therein, if the total running time with respect to SSs and USs satisfies some constraints, we can present the following Theorem 2.

Theorem 2: Suppose that Assumptions 1 and 2 hold, and the ADT of switching signal is defined in Definition 2, for any $i \in \Lambda_S$ (resp., $i \in \Lambda_U$), if there exist positive-definite matrices P_i and scalars $\alpha_i > 0$, (resp., $\beta_i > 0$), $\delta_i > 0$, $\pi > 0$, $\mu > 1$, and $\lambda^* \in (0, \beta^*)$ such that the conditions (3)–(6), and the following inequalities hold:

$$\begin{aligned} &-\alpha\mathcal{T}_S(t,s) + \beta\mathcal{T}_U(t,s) \\ &\le -\pi(t-s) + (\pi+\beta)\tau_c,\ t > s > t_0 \end{aligned} \tag{34}$$

$$\begin{cases} \lambda^* q - \beta^* + \alpha^*\theta e^{\lambda^*(1-q)t_0} < 0 \\ \lambda^*\Big(\lambda^* q - \beta^* + q\alpha^*\theta e^{\lambda^*(1-q)t_0}\Big) > \beta^*(\lambda^* q - \beta^* \\ + \alpha^*\theta e^{\lambda^*(1-q)t_0}\Big) \\ \beta^{*2}\Big(\beta^* - \lambda^* q - \alpha^*\theta e^{\lambda^*(1-q)t_0}\Big) > \lambda^{*2}(\beta^* - \lambda^* q \\ - q\alpha^*\theta e^{\lambda^*(1-q)t_0}\Big) \end{cases} \tag{35}$$

where $\alpha^* = \mu^{\mathcal{N}_0} e^{(\pi+\beta)\tau_c}$, $\beta^* = \pi - (ln\mu/\mathcal{T}_a)$, $\alpha = \min\{\alpha_i\}$, $\beta = \max\{\beta_i\}$, $\theta = \mu\theta^\dagger$, $\theta^\dagger = \max_i\{\theta_i\} = \max_i\{\lambda_{\max}(P_i^{-1}L^TL) + \delta_i\lambda_{\max}(P_i^{-1}\Xi_{2i})\}$, $\delta = \lambda_{\max}(\delta_i)$, and $\rho^\dagger = \min\{\lambda_{\min}(P_i)\}$.

Then, the SSNN (1) with the effects of PD is exponentially stable.

Proof: In view of (34), the Cauchy matrix $\Pi(t,s)$ can be estimated as follows:

$$\begin{aligned} \Pi(t,s) &= e^{\int_s^t \xi(u)du} \prod_{s\le t_n\le t} \mu \\ &\le e^{-\alpha\mathcal{T}_S(t,s)+\beta\mathcal{T}_U(t,s)}\mu^{\mathcal{N}(t,s)} \\ &= e^{-\alpha(t-s)+(\alpha+\beta)\mathcal{T}_U(t,s)}\mu^{\mathcal{N}(t,s)} \\ &\le e^{-\pi(t-s)+(\pi+\beta)k\tau_c}\mu^{\mathcal{N}(t,s)} \\ &\le \alpha^* e^{-\beta^*(t-s)}. \end{aligned} \tag{36}$$

Similarly, based on the aforementioned discussions and using the same method as the proof in Theorem 1, we have the Theorem 2, immediately. The proof is considered complete. ■

Remark 4: The results of this article are novel and different from the traditional approaches for PD. In fact, there mainly exist two traditional approaches to deal with the tricky PD term (PDT) $z(qt)$. For example, Wang [42] needed to design an appropriate controller to eliminate the PDT $z(qt)$, and then just obtains the delay-independent criteria, which is more conservative. Besides, it is also required to construct an integral term $\int_{qt}^t z^T(s)\mathcal{R}z(s)ds(\mathcal{R} > 0)$ first, for example, in [38], and then derive the derivative estimation for $V(t, z(t))$ by using LMI techniques. However, without using these strategies: 1) we can obtain the exponential estimation for $\mathbb{E}V(t, \sigma(t))$ directly by considering the comparison principle and combining the extended formula for the variation of parameters and 2) we can take into consideration of the effects of USs. Therefore, derived results are less conservative.

Remark 5: Considering the limitations, the PD can be applied more extensively when compared with the invariant delay or time-varying delay. Moreover, due to the effects of PD, many existing techniques cannot be directly applied to the switched system, especially when the state of subsystems are unstable. In conclusion, the results of this article are interesting when compared with these published results in [28], [29], [32], [38], and [42].

IV. Illustrative Examples

In this section, we will use two numerical examples to illustrate the feasibility and effectiveness of the obtained results.

Example 1: First, consider the 2-D SSNNs (1) with PD consisting of both SSs and USs, the corresponding parameters are given as follows:

$$\left[\begin{array}{c|c}\mathcal{A}_1 & \mathcal{B}_1\\ \hline \mathcal{C}_1 & g(t,1)\end{array}\right]=\left[\begin{array}{cc|cc}-2 & 0 & 0.2 & 0.1\\ 0 & -2 & 0.1 & 0.2\\ \hline 0.2 & 0.1 & 0.2z_1(t) & 0.3z_1(qt)\\ 0.1 & 0.3 & 0.1z_1(qt) & 0.3z_2(qt)\end{array}\right]$$

$$\left[\begin{array}{c|c}\mathcal{A}_2 & \mathcal{B}_2\\ \hline \mathcal{C}_2 & g(t,2)\end{array}\right]=\left[\begin{array}{cc|cc}6 & 0 & 0.1 & 0.2\\ 0 & 6 & 0.2 & 0.1\\ \hline 0.3 & 0.1 & 0.2z_2(t) & 0.3z_2(qt)\\ 0.2 & 0.3 & 0.1z_2(qt) & 0.3z_1(qt)\end{array}\right]$$

$$\left[\begin{array}{c|c}\mathcal{A}_3 & \mathcal{B}_3\\ \hline \mathcal{C}_3 & g(t,3)\end{array}\right]=\left[\begin{array}{cc|cc}-4 & 0 & 0.3 & 0.2\\ 0 & -4 & 0.2 & 0.3\\ \hline 0.2 & 0.4 & 0.2z_1(t) & 0.3z_1(qt)\\ 0.1 & 0.2 & 0.1z_1(qt) & 0.3z_2(qt)\end{array}\right]$$

$$\left[\begin{array}{c|c}\mathcal{A}_4 & \mathcal{B}_4\\ \hline \mathcal{C}_4 & g(t,4)\end{array}\right]=\left[\begin{array}{cc|cc}8 & 0 & 0.4 & 0.3\\ 0 & 8 & 0.2 & 0.4\\ \hline 0.3 & 0.2 & 0.2z_2(t) & 0.3z_2(qt)\\ 0.2 & 0.3 & 0.1z_2(qt) & 0.3z_1(qt)\end{array}\right].$$

The nonlinear activation function is considered as $f(x) = 0.4\tanh(x)$, and the PDR $q = 0.8$. It is obvious that the matrices $\Xi_{11} = \Xi_{13} = 0.04I$, $\Xi_{21} = \Xi_{23} = 0.1I$, $\Xi_{12} = \Xi_{14} = 0.04I$, and $\Xi_{22} = \Xi_{24} = 0.1I$, which satisfy Assumption 2. For $\mu = 1.01$, let $\alpha = 10$, $\beta = 9.5$, $\tau_s = 0.025$, and the free adjust constant $\mathcal{N}_0 = 1$. After some calculations, we have found the following feasible solutions for Corollary 1:

$$\left[\begin{array}{c|c}\mathcal{P}_1 & \mathcal{P}_2\\ \hline \mathcal{P}_3 & \mathcal{P}_4\end{array}\right]=\left[\begin{array}{cc|cc}1.4842 & -0.3562 & 1.4756 & -0.3552\\ * & 1.9613 & * & 1.9530\\ \hline 1.4765 & -0.3549 & 1.4870 & -0.3566\\ * & 1.9510 & * & 1.9638\end{array}\right]$$

$$\left[\begin{array}{c|c}\delta_1 & \delta_2\\ \hline \delta_3 & \delta_4\end{array}\right]=\left[\begin{array}{c|c}5.6253 & 5.2612\\ \hline 3.8079 & 5.6336\end{array}\right].$$

Thus, by Corollary 1, when the estimation of ADT $\mathcal{T}_a$ is no less than 0.0290, the ES of the SSNN with PD in (1) can be achieved. In what follows, we assume that the SSNN in the numerical simulation runs on each subsystem in the light of sequence $1 \to 2 \to 3 \to 4 \to 1$. It is not difficult to see that subsystems 1 and 3 are USs, while subsystems 2 and 4 are SSs. At the same time, we suppose that subsystems 1 and 3 run for 0.03 s, while subsystems 2 and 4 run for 0.04 and 0.02 s in one period, respectively. Here, the period for one round is 0.12 s. The corresponding state and the switching sequence used are illustrated in Figs. 2 and 3, which indicate that the ES of SSNN (1) using the ADTSR can be achieved. Moreover, the relationship between ADT and the decay ratio (DR) is illustrated in Fig. 4. It is shown that with the increase of ADT, a wider range of DR can be acquired.

Example 2: Consider the 3-D SSNNs (1) with PD consisting of both SSs and USs, the other parameters are shown as follows:

$$\left[\begin{array}{c|c|c}\mathcal{A}_1 & \mathcal{B}_1 & \mathcal{C}_1\\ \hline \mathcal{B}_2 & \mathcal{A}_2 & \mathcal{C}_2\\ \hline \mathcal{C}_3 & \mathcal{B}_3 & \mathcal{A}_3\end{array}\right]=\left[\begin{array}{ccc|ccc|ccc}-1 & 0 & 0 & 0.1 & 0 & 0 & & & \\ 0 & -1 & 0 & 0 & 0.1 & 0.2 & & \mathcal{C}_3 & \\ 0 & 0 & -0.5 & 0.2 & 0.1 & 0 & & & \\ \hline & & & 5 & 0.1 & 0 & 0.2 & 0.1 & 0.1\\ & \mathcal{B}_1 & & 0.1 & 5 & 0 & 0.1 & 0.2 & 0\\ & & & 0 & 0 & 5 & 0 & 0.1 & 0.1\\ \hline 0.1 & 0 & 0.1 & & & & -2 & 0.1 & 0\\ 0 & 0.3 & 0.1 & & \mathcal{C}_2 & & 0.1 & -2 & 0\\ 0.1 & 0.2 & 0 & & & & 0 & 0.1 & -1\end{array}\right]$$

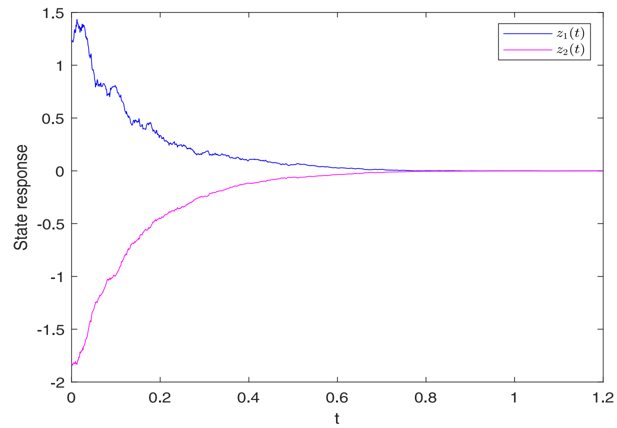

Fig. 2. State trajectories of 2D-SSNN with SSs and USs.

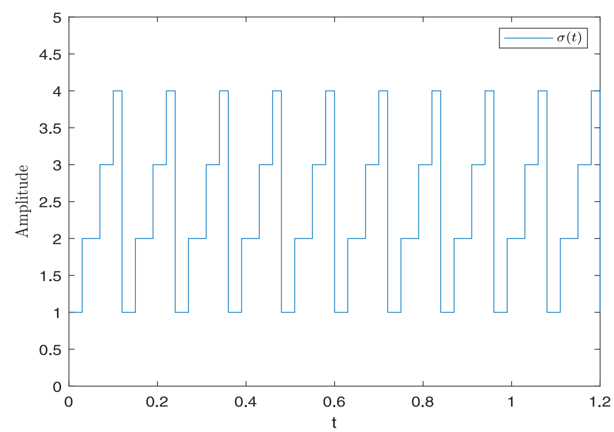

Fig. 3. Switching signals $\sigma(t)$.

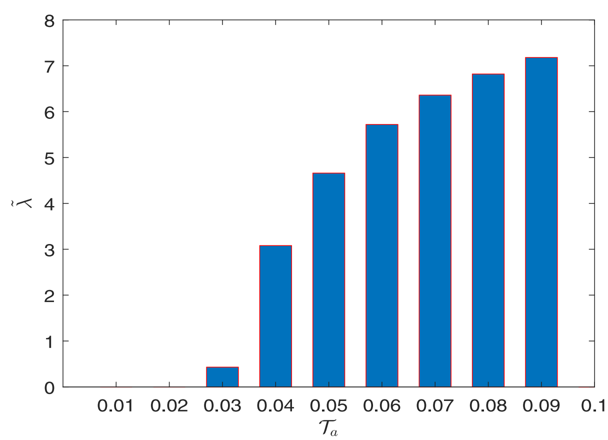

Fig. 4. Relationships between $\mathcal{T}_a$ and $\tilde{\lambda}$ for $(\mu = 1.01, q = 0.8)$.

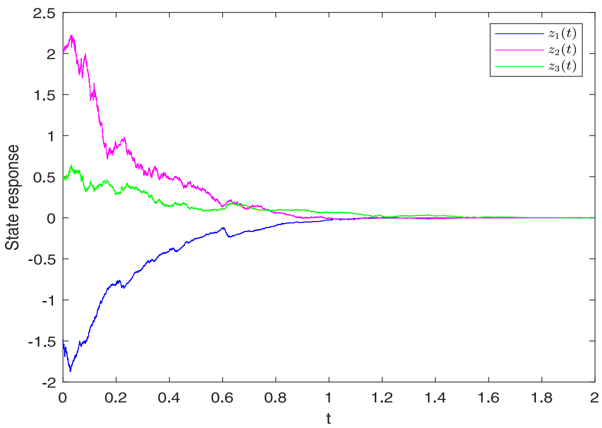

Fig. 5. State trajectories of 3-D SSNN with SSs and USs.

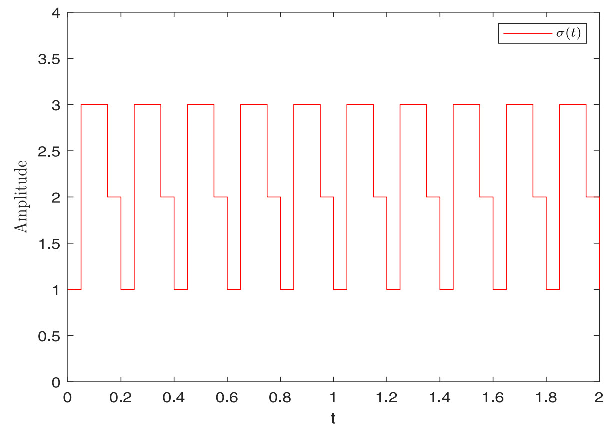

Fig. 6. Switching signals $\sigma(t)$.

$$g(t,1) = \begin{bmatrix} 0.1z_2(t) & 0.3z_2(qt) & 0 \\ 0.2z_2(qt) & 0.3z_1(qt) & 0 \\ 0 & 0 & 0.3z_3(t) \end{bmatrix}$$

$$g(t,2) = \begin{bmatrix} 0.2z_2(qt) & 0 & 0 \\ 0 & 0.2z_1(t) & 0.3z_1(qt) \\ 0 & 0.1z_2(t) & 0.3z_3(qt) \end{bmatrix}$$

$$g(t,3) = \begin{bmatrix} 0.2z_1(t) & 0.3z_2(qt) & 0.1z_3(t) \\ 0.1z_2(qt) & 0.1z_2(t) & 0.3z_1(qt) \\ 0.3z_3(t) & 0 & 0.3z_1(qt) \end{bmatrix}.$$

Suppose that $f(x) = 0.4\tanh(x)$ and the PDR $q = 0.7$. It is calculated that the matrices $\Xi_{11} = \Xi_{13} = \Xi_{22} = 0.1I$, $\Xi_{21} = 0.15I$, $\Xi_{12} = 0.05I$, and $\Xi_{23} = 0.2I$ satisfy Assumption 2.

For $\mu = 1.01$, let $\alpha = 7$, $\beta = 5.5$, $\tau_s = 0.05$, and the free adjust constant $\mathcal{N}_0 = 1$. By some simple calculations, we have found the following feasible solutions for Corollary 1:

$$\mathcal{P}_1 = \begin{bmatrix} 2.2278 & -0.1529 & -0.0174 \\ * & 1.9687 & -0.0165 \\ * & * & 1.6983 \end{bmatrix}$$

$$\mathcal{P}_2 = \begin{bmatrix} 2.2269 & -0.1527 & -0.0174 \\ * & 1.9677 & -0.0165 \\ * & * & 1.6961 \end{bmatrix}$$

$$\mathcal{P}_3 = \begin{bmatrix} 2.2262 & -0.1522 & -0.0171 \\ * & 1.9672 & -0.0168 \\ * & * & 1.6964 \end{bmatrix}$$

$$\begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix} = \begin{bmatrix} 4.5410 \\ 4.4713 \\ 3.1454 \end{bmatrix}.$$

Thus, by Corollary 1, when the estimation of ADT $\mathcal{T}_a$ is no less than 0.0695, the exponential stable of the SSNN (1) with PD can be achieved.

On the other hand, let $\tau_c = 0.15$ and $\pi = 5$ in Theorem 2. By employing Theorem 2, when the estimation of ADT $\mathcal{T}_a > 0.0085$, the conditions in Theorem 2 are satisfied, that is, the ES of the SSNN (1) with both SSs and USs can be achieved. But all the criteria derived in [28], [29], [38], and [42] do not determine the stability of system (1).

In order to verify the above conclusion, in what follows, we assume that the SSNN in the numerical simulation runs on each subsystem in accordance with sequence $1 \to 3 \to 2$.

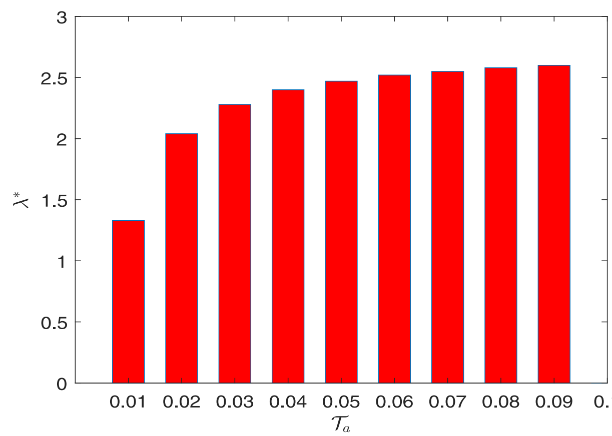

Fig. 7. Relationships between $\mathcal{T}_a$ and λ^* for $(\mu = 1.01, q = 0.7)$.

It is not difficult to see that the subsystems 1 and 3 are USs, while subsystem 2 is SS. At the same time, we suppose that 1 and 3 run for 0.05 and 0.1 s, respectively, while 2 runs for 0.05 s in one period, respectively. Here, the period for one round is 0.2 s. The corresponding state and the switching sequence used are illustrated in Figs. 5 and 6, which imply that the exponential stable of SSNN (1) under the ADTSR can be achieved. Moreover, the relationship between ADT and the DR is illustrated in Fig. 7. It is shown that with the increase of ADT, a wider range of DR can be acquired.

V. Conclusion

In this article, we have focused on the ES issue for a class of SSNNs with PD. The considered PD is a kind of unbounded time-varying delay compared to general time delay. By considering the comparison principle and combining the extended formula for the variation of parameters, we conquer the difficulty in the consideration of PD effects for the switched systems with both SSs and USs for the first time. New delay-dependent conditions with respect to the mean-square ES of systems are established by employing the ADT technique, stochastic analysis theory, and Lyapunov method. It is shown that the acquired minimum average dwell time (MADT) is not only relevant to the SSs and USs but also dependent on the

DR, increasing ratio (IR), as well as PD. Finally, the availability of the derived results under the ADTSR is illustrated through two numerical simulation examples.

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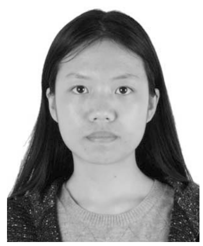

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