Quantized Interval Type-2 Fuzzy Control for Persistent Dwell-Time Switched Nonlinear Systems With Singular Perturbations

Jing Wang, Xinmiao Liu, Jianwei Xia, *Member, IEEE*, Hao Shen, *Member, IEEE*, and Ju H. Park, *Senior Member, IEEE*

***Abstract*—This article investigates the problem of quantized fuzzy control for discrete-time switched nonlinear singularly perturbed systems, where the singularly perturbed parameter (SPP) is employed to represent the degree of separation between the fast and slow states. Taking a full account of features in such switched nonlinear systems, the persistent dwell-time switching rule, the technique of singular perturbation and the interval type-2 Takagi–Sugeno fuzzy model are introduced. Then, by means of constructing SPP-dependent multiple Lyapunov-like functions, some sufficient conditions with the ability to ensure the stability and an expected H_∞ performance of the closed-loop system are deduced. Afterward, through solving a convex optimization problem, the gains of the controller are obtained. Finally, the correctness of the proposed method and the effectiveness of the designed controller are demonstrated by an explained example.**

***Index Terms*—Interval type-2 Takagi–Sugeno (IT2 T–S) fuzzy model, persistent dwell-time (PDT) switching rule, quantized control, singularly perturbed nonlinear systems.**

I. Introduction

SOME parasitic parameters, such as small time constants, capacitances, and inductances, may increase the order of dynamic equations and lead to the numerical ill-conditioning problem. In order to overcome these obstacles, the singular perturbation technique is usually employed. Examples of singularly perturbed systems (SPSs) are power systems, airplanes, and so on [1]–[3]. When discretizing continuous-time SPSs, the sampling period influences the discrete-time model, which results in the fast-sampling and slow-sampling models [4]. For the first time, the stable conditions were deduced in [5] for the fast-sampling model of SPSs on the strength of linear matrix inequalities (LMIs). Since then, many results about SPSs have sprung up. To name a few, Shen *et al.* [6] investigated the control problem for nonlinear SPSs (NSPSs) under the slow-sampling model. The fault-tolerant control problem for NSPSs based on slow-state feedback was studied in [7]. From the mentioned literature and relative references, one can see that the control problem of NSPSs has became a hot topic.

As an excellent way to tackle the complex nonlinear part in systems, the fuzzy theory vastly promotes the research of NSPSs [6]–[10]. For example, because the fact that premise variables usually rely on unmeasurable states, the observer-based controller has been studied in many published papers to obtain premise variables [11]–[16]. Tong and Li [17], [18] investigated the adaptive fuzzy backstepping output-feedback control problem for uncertain nonlinear systems with unmeasured states, which are challenging and pioneering works. Considering that parameter uncertainties have a significant effect on the performance of nonlinear systems [19]–[21], the interval type-2 Takagi–Sugeno (IT2 T–S) fuzzy model with uncertain membership functions is more suitable than the type-1 T–S fuzzy model when parameter uncertainties of practical systems cannot be ignored [22]–[27]. In this regard, Lam and Seneviratne [28] applied the IT2 T–S fuzzy theory to the control community, where the technique of dividing the footprint of uncertainty (FOU) and some necessary slack matrices with the information of uncertain membership functions were used. Furthermore, when the two-time-scale characteristic of IT2 T–S fuzzy systems is taken into account, how to employ the singular perturbation technique and the IT2 T–S fuzzy theory to address the control problem at the same time is interesting, and this article focuses on settling this issue.

In fact, most of the practical systems work in different cases which are characterized by different operating parameters or topological structures [29]–[32]. Usually, when focusing on the interval between two contiguous changing instants, the changes among modes obey a law, called the switching rule [33]–[35]. Over the past few years, in time-dependent switching rules, the dwell-time (DT), the average DT (ADT), and the persistent DT (PDT) switching rules have attracted much attention [36]–[38]. The DT switching rule requires that every switching interval should be not less than a positive

Manuscript received 7 March 2020; revised 14 July 2020, 5 October 2020, and 2 December 2020; accepted 3 January 2021. Date of publication 10 February 2021; date of current version 1 July 2022. This work was supported in part by NNSFC under Grant 61873002, Grant 61703004, Grant 61833005, and Grant 61503002; and in part by the Major Natural Science Foundation of Higher Education Institutions of Anhui Province under Grant KJ2020ZD28. The work of Ju H. Park was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea Government (MSIT) under Grant 2020R1A2B5B02002002. This article was recommended by Associate Editor F. Wu. *(Corresponding authors: Ju H. Park; Hao Shen.)*

Jing Wang, Xinmiao Liu, and Hao Shen are with the Anhui Province Key Laboratory of Special Heavy Load Robot, Anhui University of Technology, Ma'anshan 243002, China, and also with the School of Electrical and Information Engineering, Anhui University of Technology, Ma'anshan 243002, China (e-mail: jingwang08@126.com; liuxinmiao19@gmail.com; haoshen10@gmail.com).

Jianwei Xia is with the School of Mathematical Sciences, Liaocheng University, Liaocheng 252059, China (e-mail: njustxjw@126.com).

Ju H. Park is with the Department of Electrical Engineering, Yeungnam University, Kyongsan 38541, South Korea (e-mail: jessie@ynu.ac.kr).

Color versions of one or more figures in this article are available at https://doi.org/10.1109/TCYB.2021.3049459.

Digital Object Identifier 10.1109/TCYB.2021.3049459

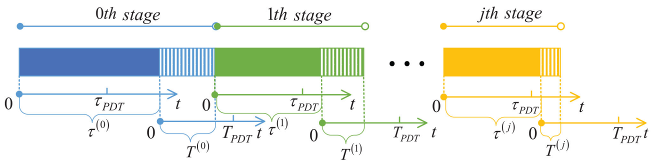


Fig. 1. Sketch map of the PDT switching rule.

Notations	Representations
t	The discrete-time
$\tau^{(j)}$	The during time of τ-portion in the jth stage
$T^{(j)}$	The persistent time of T-portion in the jth stage
$t_{m_j}, t_{m_{j-1}}, \cdots$	The start instants of the jth stage, the $(j-1)$th stage, $\cdots$
$t_{m_j}, t_{m_j+1}, \cdots$	The start instants of the m_jth switching, the (m_j+1)th switching, $\cdots$, in the jth stage
$NUM(t, t_{m_j-1})$	The total number of switchings in the interval $[t_{m_j-1}, t)$

scalar τ_{DT}. The ADT switching rule demands that not all running times of the acting systems should be not less than a positive scalar τ_{ADT}. The time sequence generated by the PDT switching rule includes infinite stages, and each stage is composed of a τ-portion and a T-portion [39], [40], whose sketch map is shown in Fig. 1. It should be stressed that the during time of each τ-portion is not less than a positive scalar τ_{PDT}, and the persistent time of each T-portion is not more than a positive scalar T_{PDT}. In addition, only one subsystem can be triggered in a τ-portion, called the slow switching, while the T-portion allows the arbitrary fast switching, which means that the running time of the acting system is less than τ_{PDT}. As a consequence, the PDT switching rule is more suitable than the other two rules in depicting systems where the fast switching and slow switching occur in turn. Furthermore, the study in [37] showed that the PDT switching rule could be transformed into the DT and the ADT switching rules through setting appropriate parameters. As far as the authors' knowledge, there are few papers about nonlinear switched SPSs (NSSPSs), to say nothing of NSSPSs based on the IT2 T–S fuzzy model and the PDT switching rule. To narrow this gap, this article is carried out.

Illuminated by the above investigation, this article aims to propose a new method for designing a quantized controller for NSSPSs. In summary, contributions of this work are mainly three aspects as follows.

1) A quantized controller is designed for NSSPSs with the PDT switching rule and the IT2 fuzzy model at the first attempt. Compared with the existing results of fuzzy switched systems in [39]–[41], the two-time-scale characteristic is fully taken into account, and the singularly perturbed parameter (SPP) may bring the difficulty of solving the gains of the controller.
2) Unlike the crisp fuzzy membership functions in the type-1 T–S fuzzy-switched model, the IT2 T–S fuzzy switched model has the uncertain membership functions that endow the designed controller with a more powerful ability to represent and capture parameter uncertainties.
3) By fully considering the features of NSSPSs, proper Lyapunov functions, the SPP-dependent multiple Lyapunov-like functions (MLFs), are established in order to obtain some less conservative conditions that can guarantee the stability and the performance of the resulting closed-loop system. Finally, the effectiveness of the designed controller is verified by a numerical example.

The notations used are standard and one can refer to [42] for more details. In addition, other important notations about the PDT switching rule are highlighted below.

II. System Description

In this section, we construct NSSPSs based on the IT2 T–S fuzzy theory. It is worth mentioning that in this article, the type reduction of the IT2 T–S fuzzy model is realized by dividing the state space of interesting and FOU. Then, some necessary preliminaries are given, including a brief expression of the PDT switching rule, some definitions, and lemmas. Finally, the objective of this article is provided.

A. System Establishment

Consider the vth rule of NSSPSs as follows.

Plant Rule v: IF $l_1(x(t))$ is S_{v1}, and …, and $l_i(x(t))$ is S_{vi}, THEN

$$x(t+1) = A^{\epsilon}_{v,\iota(t)}x(t) + B^{\epsilon}_{v,\iota(t)}\omega(t) + C^{\epsilon}_{v,\iota(t)}U(t) \quad (1)$$

$$z(t) = D_{v,\iota(t)}x(t) + E_{v,\iota(t)}\omega(t) + F_{v,\iota(t)}U(t) \quad (2)$$

with

$$x(t) \triangleq \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, A^{\epsilon}_{v,\iota(t)} \triangleq \begin{bmatrix} A^{(1,1)}_{v,\iota(t)} & A^{(1,2)}_{v,\iota(t)} \\ \epsilon A^{(2,1)}_{v,\iota(t)} & \epsilon A^{(2,2)}_{v,\iota(t)} \end{bmatrix}$$

$$B^{\epsilon}_{v,l(t)} \triangleq \begin{bmatrix} B^{(1,1)}_{v,\iota(t)} \\ \epsilon B^{(2,1)}_{v,\iota(t)} \end{bmatrix}, C^{\epsilon}_{v,l(t)} \triangleq \begin{bmatrix} C^{(1,1)}_{v,\iota(t)} \\ \epsilon C^{(2,1)}_{v,\iota(t)} \end{bmatrix}$$

$$D_{v,l(t)} \triangleq \begin{bmatrix} D^{(1,1)}_{v,\iota(t)} & D^{(1,2)}_{v,\iota(t)} \end{bmatrix}$$

where ϵ expresses SPP meeting $\epsilon > 0$, for $t \in \mathbb{N}$, $x_1(t) \in \mathbb{R}^{n_{x_1}}$, $x_2(t) \in \mathbb{R}^{n_{x_2}}$, and $x(t) \in \mathbb{R}^{n_x}$ are the fast state vector, the slow state vector and the state vector, respectively, where $n_{x_1} + n_{x_2} = n_x$ holds, $z(t) \in \mathbb{R}^{n_z}$ stands for the output signal and $\omega(t) \in l_2[0, \infty)$ symbolizes the disturbance signal, $U(t)$ denotes the controlled input, $\iota(t) \in \mathcal{T} \triangleq \{1, 2, \ldots, \hat{\iota}\}$ represents the switching signal subject to the PDT switching rule, in which $\hat{\iota} \in \mathbb{N}^+$ means the number of nonlinear subsystems, $l_1(x(t)), \ldots,$ and $l_i(x(t))$ imply the premise variables of the plant, in which $i \in \mathbb{N}^+$ is the number of the premise variables, $S_{v1}, \ldots,$ and S_{vi} indicate the IT2 fuzzy sets of the plant, in which $v \in \mathcal{V} \triangleq \{1, 2, \ldots, \hat{v}\}$ and $\hat{v}$ expresses the number of the rules, and $A^{\epsilon}_{v,\iota(t)}$, $B^{\epsilon}_{v,\iota(t)}$, $C^{\epsilon}_{v,\iota(t)}$, $D_{v,\iota(t)}$, $E_{v,\iota(t)}$, and $F_{v,\iota(t)}$ are the system matrices with appropriate dimensions.

An interval set is employed as the firing strength of the vth plant rule, and it is represented as follows:

$$O_v(x(t)) \triangleq \begin{bmatrix} \underline{g}_v(x(t)) & \overline{g}_v(x(t)) \end{bmatrix}$$

where

$$\underline{g}_v(x(t)) \triangleq \prod_{\varepsilon=1}^{i} \underline{\mu}_{S_{v\varepsilon}}(l_\varepsilon(x(t)))$$
$$\overline{g}_v(x(t)) \triangleq \prod_{\varepsilon=1}^{i} \overline{\mu}_{S_{v\varepsilon}}(l_\varepsilon(x(t)))$$

satisfying

$$0 \le \underline{g}_v(x(t)) \le \overline{g}_v(x(t)) \le 1$$
$$0 \le \underline{\mu}_{S_{v\varepsilon}}(l_\varepsilon(x(t))) \le \overline{\mu}_{S_{v\varepsilon}}(l_\varepsilon(x(t))) \le 1$$

in which $\underline{\mu}_{S_{v\varepsilon}}(l_\varepsilon(x(t)))$ and $\overline{\mu}_{S_{v\varepsilon}}(l_\varepsilon(x(t)))$ imply the lower membership function (LMF) and the upper membership function (UMF) of the plant, separately. $\underline{g}_v(x(t))$ and $\overline{g}_v(x(t))$ indicate the lower grade of membership (LGM) and the upper grade of membership (UGM) of the plant, respectively. Then, the weighted model of the plant is depicted as follows for $\theta \triangleq \iota(t)$:

$$x(t+1) = \sum_{v=1}^{\hat{v}} g_v(x(t))\big[A^{\epsilon}_{v\theta}x(t) + B^{\epsilon}_{v\theta}\omega(t) + C^{\epsilon}_{v\theta}U(t)\big] \quad (3)$$

$$z(t) = \sum_{v=1}^{\hat{v}} g_v(x(t))[D_{v\theta}x(t) + E_{v\theta}\omega(t) + F_{v\theta}U(t)] \quad (4)$$

where

$$g_v(x(t)) \triangleq \frac{\tilde{g}_v(x(t))}{\sum_{\tilde{v}=1}^{\hat{v}} \tilde{g}_{\tilde{v}}(x(t))}$$
$$\tilde{g}_v(x(t)) \triangleq \underline{\varsigma}_v(x(t))\underline{g}_v(x(t)) + \overline{\varsigma}_v(x(t))\overline{g}_v(x(t))$$

meeting

$$\sum_{\tilde{v}=1}^{\hat{v}} g_{\tilde{v}}(x(t)) = 1, 0 \le g_v(x(t)) \le 1$$
$$0 \le \underline{\varsigma}_v(x(t)) \le 1, 0 \le \overline{\varsigma}_v(x(t)) \le 1$$
$$\underline{\varsigma}_v(x(t)) + \overline{\varsigma}_v(x(t)) = 1$$

in which $g_v(x(t))$ is the grade of membership to the plant.

In this article, the communication network including a quantizer is employed to connect the plant and the controller. In terms of the IT2 T–S fuzzy theory, the form of the controller is as follows.

Control Rule w: IF $\tilde{l}_1(x(t))$ is $\tilde{S}_{w1}$, and $\cdots$, and $\tilde{l}_{\tilde{\imath}}(x(t))$ is $\tilde{S}_{w\tilde{\imath}}$, THEN

$$\tilde{U}(t) = K_{w\theta}x(t) \quad (5)$$

where $\tilde{l}_1(x(t)), \ldots,$ and $\tilde{l}_{\tilde{\imath}}(x(t))$ imply the premise variables of the controller, for $w \in \mathcal{W} \triangleq \{1, 2, \ldots, \hat{w}\}$, $\tilde{S}_{w1}, \ldots,$ and $\tilde{S}_{w\tilde{\imath}}$ indicate the IT2 fuzzy sets of the controller, $\tilde{\imath}$ denotes the number of the premise variables, $\hat{w}$ is the number of rules, and $K_{w\theta} \in \mathbb{R}^{n_k \times n_x}$ symbolizes the gain of the controller to be determined.

Remark 1: It is observed from (1), (2), and (5) that the number of premise variables in the plant is i and the number of the plant rules is $\hat{v}$, while the number of the premise variables in the controller is $\tilde{\imath}$ and the number of the controller rules is $\hat{w}$. Contrasted with designing controller under the parallel distributed compensation approach, those different values are the features of the application of the IT2 T–S fuzzy set theory in the control community, which can relax the design of the controller via freely choosing the number of premise variables, the number of rules and the membership functions of the controller.

The input signal from the controller to the system is transmitted through a communication network with the logarithmic quantizer. For revealing the error between the input signal and the quantized signal, the form and the function of the quantizer are briefly depicted as follows. In this article, the matrix function $F(\cdot)$ denotes the mathematical model of the logarithmic quantizer, and the quantization levels are shown as follows:

$$\mathcal{F} \triangleq \Big\{\pm f_{(\wp)}, f_{(\wp)} \triangleq \beta f_{(\wp-1)}, \wp = \pm 1, \pm 2, \ldots, \Big\} \cup \big\{\pm f_{(0)}\big\} \cup \{0\}$$

with

$$0 < f_{(0)},\ 0 < \beta < 1$$

where β represents the quantization density [43], [44]. Therefore, the quantizer embodies as follows:

$$F(\alpha) \triangleq \begin{cases} f_{(\wp)}, & \frac{f_{(\wp)}}{1+\vartheta} < \alpha \le \frac{f_{(\wp)}}{1-\vartheta} \\ 0, & \alpha = 0 \\ -F(-\alpha), & \alpha < 0 \end{cases}$$

with

$$\vartheta \triangleq \frac{1-\beta}{1+\beta}.$$

It should be noticed that this logarithmic quantizer means $F([a_{ij}]_{i\times j}) \triangleq [F(a_{ij})]_{i\times j}$ for the matrix $[a_{ij}]_{i\times j}$. Therefore, the actual controlled input is $U(t) \triangleq F(\tilde{U}(t))$. Based on the above discussion, the quantization error of the controller can be deduced as

$$e(t) \triangleq U(t) - \tilde{U}(t) \triangleq \Delta(t)\tilde{U}(t)$$

where

$$\Delta(t) \triangleq \mathrm{diag}\big\{\hbar_1(t), \ldots, \hbar_{n_k}(t)\big\}$$

satisfying

$$|\hbar_{\eth}(t)| \le \vartheta \le 1,\ \text{for } \eth \in \{1, 2, \ldots, n_k\}.$$

Remark 2: With the enlargement of system scale and complexity, the communication network is usually adopted to transmit data. Considering the limited transmission capacity, the quantizer, as an effective way to alleviate communication burden, is frequently used. Mapping a real-valued function into a piecewise constant function is the essence of the quantizer, which may decrease the transmission pressure at the cost of the accuracy of the data transmitted to some extent. It is worth noting that the larger the quantization error, the less valid information the receiver can obtain, while if the error is too small, the data package is so large that the transmission burden of the communication network could be not efficiently reduced. Therefore, choosing a proper quantizer with the reasonable quantization error plays a significant part in the work of the controller.

Then, the quantized state-feedback controller can be written as

$$U(t) = (I + \Delta(t)) \sum_{w=1}^{\hat{w}} h_w(x(t))[K_{w\theta} x(t)] \tag{6}$$

in which $h_w(x(t))$ is the grade of membership to the controller. An interval set is employed as the firing strength of the wth controller rule and it is represented as follows:

$$\tilde{O}_w(x(t)) \triangleq \left[\underline{h}_w(x(t)) \quad \overline{h}_w(x(t))\right]$$

where

$$\underline{h}_w(x(t)) \triangleq \prod_{\tilde{\varepsilon}=1}^{\tilde{\iota}} \underline{\mu}_{\tilde{S}_{w\tilde{\varepsilon}}}\left(\tilde{l}_{\tilde{\varepsilon}}(x(t))\right)$$
$$\overline{h}_w(x(t)) \triangleq \prod_{\tilde{\varepsilon}=1}^{\tilde{\iota}} \overline{\mu}_{\tilde{S}_{w\tilde{\varepsilon}}}\left(\tilde{l}_{\tilde{\varepsilon}}(x(t))\right)$$

satisfying

$$0 \le \underline{h}_w(x(t)) \le \overline{h}_w(x(t)) \le 1$$
$$0 \le \underline{\mu}_{\tilde{S}_{w\tilde{\varepsilon}}}\left(\tilde{l}_{\tilde{\varepsilon}}(x(t))\right) \le \overline{\mu}_{\tilde{S}_{w\tilde{\varepsilon}}}\left(\tilde{l}_{\tilde{\varepsilon}}(x(t))\right) \le 1$$

in which $\underline{\mu}_{\tilde{S}_{w\tilde{\varepsilon}}}(\tilde{l}_{\tilde{\varepsilon}}(x(t)))$ and $\overline{\mu}_{\tilde{S}_{w\tilde{\varepsilon}}}(\tilde{l}_{\tilde{\varepsilon}}(x(t)))$ imply LMF and UMF of the controller, separately. $\underline{h}_w(x(t))$ and $\overline{h}_w(x(t))$ indicate LGM and UGM of the controller, respectively. In the following, define

$$h_w(x(t)) \triangleq \frac{\tilde{h}_w(x(t))}{\sum_{\tilde{w}=1}^{\hat{w}} \tilde{h}_{\tilde{w}}(x(t))}$$
$$\tilde{h}_w(x(t)) \triangleq \underline{\xi}_w(x(t))\underline{h}_w(x(t)) + \overline{\xi}_w(x(t))\overline{h}_w(x(t))$$

satisfying

$$\sum_{w=1}^{\hat{w}} h_w(x(t)) = 1, 0 \le h_w(x(t)) \le 1$$
$$0 \le \underline{\xi}_w(x(t)) \le 1, 0 \le \overline{\xi}_w(x(t)) \le 1$$
$$\underline{\xi}_w(x(t)) + \overline{\xi}_w(x(t)) = 1.$$

So, the fuzzy closed-loop system is deduced as

$$x(t+1) = \sum_{v=1}^{\hat{v}} \sum_{w=1}^{\hat{w}} g_v h_w \mathcal{A}^{\epsilon}_{vw\theta} \eta(t) \tag{7}$$
$$z(t) = \sum_{v=1}^{\hat{v}} \sum_{w=1}^{\hat{w}} g_v h_w \mathcal{B}_{vw\theta} \eta(t) \tag{8}$$

with

$$\mathcal{A}^{\epsilon}_{vw\theta} \triangleq \left[\tilde{\mathcal{A}}^{\epsilon}_{vw\theta} \quad B^{\epsilon}_{v\theta}\right],\ \mathcal{B}_{vw\theta} \triangleq \left[\tilde{\mathcal{B}}_{vw\theta} \quad E_{v\theta}\right]$$
$$\tilde{\mathcal{A}}^{\epsilon}_{vw\theta} \triangleq A^{\epsilon}_{v\theta} + C^{\epsilon}_{v\theta}(I + \Delta(t))K_{w\theta}$$
$$\tilde{\mathcal{B}}_{vw\theta} \triangleq D_{v\theta} + F_{v\theta}(I + \Delta(t))K_{w\theta}$$
$$\eta(t) \triangleq \left[x^T(t) \quad \omega^T(t)\right]^T$$

where for simplification, g_v and h_w denote $g_v(x(t))$ and $h_w(x(t))$, respectively.

The method of type-reduction in this article is similar to the one in [45], which is shown as follows:

$$x(t+1) = \sum_{v=1}^{\hat{v}} \sum_{w=1}^{\hat{w}} \delta_{vw} \mathcal{A}^{\epsilon}_{vw\theta} \eta(t) \tag{9}$$
$$z(t) = \sum_{v=1}^{\hat{v}} \sum_{w=1}^{\hat{w}} \delta_{vw} \mathcal{B}_{vw\theta} \eta(t) \tag{10}$$

where

$$\sum_{v=1}^{\hat{v}} \sum_{w=1}^{\hat{w}} \delta_{vw} = \sum_{v=1}^{\hat{v}} \sum_{w=1}^{\hat{w}} g_v h_w = 1$$
$$= \sum_{v=1}^{\hat{v}} g_v = \sum_{w=1}^{\hat{w}} h_w. \tag{11}$$

Then, the state space of interesting Γ and FOU Ξ are, respectively, separated into $\hat{k}$ disjoint state subspaces and $\hat{\ell}+1$ disjoint FOU subspaces such that

$$\Gamma = \bigcup_{k=1}^{\hat{k}} \Gamma_k, \quad \Xi = \bigcup_{\ell=1}^{\hat{\ell}+1} \Xi_{\ell}.$$

Therefore, the grade of membership of the closed-loop system has an expression as follows:

$$\delta_{vw} \triangleq \sum_{\ell=1}^{\hat{\ell}+1} \varphi_{vw\ell}\left(\underline{\rho}_{vw\ell}\underline{k}_{vw\ell} + \overline{\rho}_{vw\ell}\overline{k}_{vw\ell}\right) \tag{12}$$

where

$$\varphi_{vw\ell} \triangleq \begin{cases} 1, & \delta_{vw} \in \text{ the } \ell\text{th subspace of the FOU} \\ 0, & \text{otherwise} \end{cases}$$
$$\underline{k}_{vw\ell} \triangleq \sum_{k=1}^{\hat{k}} \sum_{\varpi_1=1}^{2} \cdots \sum_{\varpi_{n_x}=1}^{2} \prod_{\sigma=1}^{n_x} \times \chi_{\sigma\varpi_\sigma k\ell}(x_\sigma(t))\underline{\psi}_{vw\varpi_1\cdots\varpi_{n_x}k\ell}$$
$$\overline{k}_{vw\ell} \triangleq \sum_{k=1}^{\hat{k}} \sum_{\varpi_1=1}^{2} \cdots \sum_{\varpi_{n_x}=1}^{2} \prod_{\sigma=1}^{n_x} \times \chi_{\sigma\varpi_\sigma k\ell}(x_\sigma(t))\overline{\psi}_{vw\varpi_1\cdots\varpi_{n_x}k\ell} \tag{13}$$

with

$$0 \le \chi_{\sigma\varpi_\sigma k\ell}(x_\sigma(t)) \le 1$$
$$0 \le \delta_{vw} \le 1, 0 \le \underline{k}_{vw\ell} \le \overline{k}_{vw\ell} \le 1$$
$$0 \le \underline{\psi}_{vw\varpi_1\cdots\varpi_\iota k\ell} \le \overline{\psi}_{vw\varpi_1\cdots\varpi_\iota k\ell} \le 1$$
$$0 \le \underline{\rho}_{vw\ell} \le 1, 0 \le \overline{\rho}_{vw\ell} \le 1, \underline{\rho}_{vw\ell} + \overline{\rho}_{vw\ell} = 1$$
$$\sum_{k=1}^{\hat{k}} \sum_{\varpi_1=1}^{2} \cdots \sum_{\varpi_{n_x}=1}^{2} \prod_{\sigma=1}^{n_x} \chi_{\sigma\varpi_\sigma k\ell}(x_\sigma(t)) = 1.$$

In the above, $\underline{\rho}_{vw\ell}$ and $\overline{\rho}_{vw\ell}$ are two nonlinear functions, which are not necessarily known but exist. $\underline{\psi}_{vw\varpi_1\cdots\varpi_{n_x}k\ell}$ and $\overline{\psi}_{vw\varpi_1\cdots\varpi_{n_x}k\ell}$ are constant scalars to be determined. When $x(t)$ belongs to the kth subspace of the state space Γ, the equality $\chi_{\sigma 1k\ell}(x_\sigma(t)) + \chi_{\sigma 2k\ell}(x_\sigma(t)) = 1$ holds; otherwise, $\chi_{\sigma\varpi_\sigma k\ell}(x_\sigma(t)) = 0$ for $\varpi_\sigma = 1$ and $\varpi_\sigma = 2$.

B. Preliminaries

Definition 1 [37]: The switching signal subject to the PDT switching rule with respect to $(\tau_{\mathrm{PDT}}, T_{\mathrm{PDT}})$ includes an infinite number of disjoint intervals of length no less than τ_{PDT} on which $\iota(t)$ is a constant, and consecutive intervals with this feature are separated by intervals no large than T_{PDT}, where τ_{PDT} and T_{PDT} are two positive scalars.

Definition 2 [42]: When $\omega(t) \equiv 0$, if there exist $0 < \tilde{\varrho} < 1$ and $0 < \zeta$ such that for any $t_0 \leq t$

$$\|x(t)\|^2 \leq \zeta \tilde{\varrho}^{t-t_0} \|x(t_0)\|^2$$

system (9) is globally uniform exponential stable (GUES).

Definition 3 [5]: If system (9) is GUES and under zero-initial state conditions, the following inequality holds with $0 < \tilde{r}$:

$$\sum_{p=t_0}^{\infty} z^T(p)z(p) < \tilde{r}^2 \sum_{p=t_0}^{\infty} \omega^T(p)\omega(p)$$

then, this system is GUES with an H_∞ performance index $\tilde{r}$.

Lemma 1 [46]: For scalars $\delta_{vw} \in [0,1]$ satisfying $\sum_{v=1}^{\hat{v}} \sum_{w=1}^{\hat{w}} \delta_{vw} = 1$ and real matrices $\mathcal{H}_{vw\theta}$, $\mathcal{L}_\theta > 0$, and $\mathcal{K}_{vw\theta}$ with appropriate dimensions, the following inequality holds:

$$\left[\sum_{v=1}^{\hat{v}} \sum_{w=1}^{\hat{w}} \delta_{vw} \mathcal{H}_{vw\theta}\right]^T \mathcal{L}_\theta \left[\sum_{v=1}^{\hat{v}} \sum_{w=1}^{\hat{w}} \delta_{vw} \mathcal{K}_{vw\theta}\right] \leq \frac{1}{2} \sum_{v=1}^{\hat{v}} \sum_{w=1}^{\hat{w}} \delta_{vw} \big[\mathcal{H}_{vw\theta}^T \mathcal{L}_\theta \mathcal{K}_{vw\theta} + \mathcal{K}_{vw\theta}^T \mathcal{L}_\theta \mathcal{H}_{vw\theta}\big].$$

Lemma 2 [47]: For scalar $\varepsilon \in (0, \bar{\varepsilon}]$ and two symmetric matrices Q_1 and Q_2 with appropriate dimensions, the below necessary and sufficient condition exists

$$\begin{cases} Q_1 < 0 \\ Q_1 + \bar{\varepsilon} Q_2 \leq 0 \end{cases} \iff Q_1 + \varepsilon Q_2 \leq 0.$$

The problem to be tackled is summed up as follows: design an attractive quantized controller to ensure the stability and an expected H_∞ performance of closed-loop system (9), (10). In the next section, some sufficient conditions are given to obtain the desired controller under the consideration of the logarithmic quantizer.

III. Main Results

Theorem 1: Presetting $0 < \hat{k}$, $0 \leq \hat{\ell}$, $0 < \hat{v}$, $0 < \hat{w}$, $0 < r$, $0 < T_{\mathrm{PDT}}$, $0 < \tau_{\mathrm{PDT}}$, $0 < \lambda < 1$, $1 < \phi$, and $\lambda\phi \neq 1$ and considering the state space of interesting and FOU are partitioned into $\hat{k}$ and $\hat{\ell}+1$ disjoint subspaces, separately, if there exist symmetric matrices P_θ^ϵ, $\mathcal{X}_\theta$, and $\mathcal{Y}_{vw\ell\theta}$ for $\theta \neq \tilde{\theta}$ $\forall \theta \in \mathcal{T}$ and $\forall \tilde{\theta} \in \mathcal{T}$, such that the following inequalities hold:

$$0 < \mathcal{Y}_{vw\ell\theta} \tag{14}$$

$$0 < P_\theta^\epsilon < \phi P_{\tilde{\theta}}^\epsilon \tag{15}$$

$$\Theta_{1vw\theta}^\epsilon + \mathcal{X}_\theta - \mathcal{Y}_{vw\ell\theta} < 0 \tag{16}$$

$$\sum_{v=1}^{\hat{v}} \sum_{w=1}^{\hat{w}} \Theta_{0vw\theta}^\epsilon - \mathcal{X}_\theta < 0 \tag{17}$$

$$(T_{\mathrm{PDT}}+1)\ln(\phi) + (T_{\mathrm{PDT}}+\tau_{\mathrm{PDT}})\ln(\lambda) < 0 \tag{18}$$

where

$$\Theta_{0vw\theta}^\epsilon \triangleq \sum_{\ell=1}^{\hat{\ell}+1} \varphi_{vw\ell}\big[\underline{k}_{vw\ell}\Theta_{1vw\theta}^\epsilon + \underline{k}_{vw\ell}\mathcal{X}_\theta - \big(\underline{k}_{vw\ell} - \overline{k}_{vw\ell}\big)\mathcal{Y}_{vw\ell\theta}\big]$$

$$\Theta_{1vw\theta}^\epsilon \triangleq \begin{bmatrix} \Theta_{1vw\theta}^{\epsilon(1,1)} & * \\ \Theta_{1vw\theta}^{\epsilon(2,1)} & \Theta_{1vw\theta}^{\epsilon(2,2)} \end{bmatrix}$$

$$\Theta_{1vw\theta}^{\epsilon(1,1)} \triangleq \left(\tilde{\mathcal{A}}_{vw\theta}^\epsilon\right)^T P_\theta^\epsilon \tilde{\mathcal{A}}_{vw\theta}^\epsilon - \lambda P_\theta^\epsilon + \tilde{\mathcal{B}}_{vw\theta}^T \tilde{\mathcal{B}}_{vw\theta}$$

$$\Theta_{1vw\theta}^{\epsilon(2,1)} \triangleq B_{v\theta}^{\epsilon T} P_\theta^\epsilon \tilde{\mathcal{A}}_{vw\theta}^\epsilon + E_{v\theta}^T \tilde{\mathcal{B}}_{vw\theta}$$

$$\Theta_{1vw\theta}^{\epsilon(2,2)} \triangleq B_{v\theta}^{\epsilon T} P_\theta^\epsilon B_{v\theta}^\epsilon + E_{v\theta}^T E_{v\theta} - r^2 I$$

then, systems (1) and (2) are GUES with an H_∞ performance index $\tilde{r}$ with

$$\tilde{r} \triangleq r\sqrt{\frac{r_1(1-\lambda)}{1-r_2}}$$

where

$$r_1 \triangleq \phi^{(T_{\mathrm{PDT}}+1)\left(1+\frac{1}{T_{\mathrm{PDT}}+\tau_{\mathrm{PDT}}}\right)},\ r_2 \triangleq \lambda\phi^{\frac{T_{\mathrm{PDT}}+1}{T_{\mathrm{PDT}}+\tau_{\mathrm{PDT}}}}.$$

Proof: See Appendix A. ■

Remark 3: In Theorem 1, based on the classical Lyapunov stability theory and the merits of switched systems, MLFs are formed via constructing a Lyapunov function for each subsystem. It is worth mentioning that the MLFs approach is more efficient at offering greater freedom for demonstrating the stability of switched systems than the common Lyapunov function method. Some sufficient conditions for ensuring the stability and performance of the closed-loop system are established in terms of the technology of singular perturbations and some slack matrices together with the parameter matrix inequality techniques, which imposes restrictions on the continuous dynamics and the discrete dynamics of the switched system by the sampling instant decay rate λ and the switching instant variation rate ϕ.

Remark 4: The value of ϱ_j is dependent on $T^{(j)}$ and independent of $\tau^{(j)}$. It should be noted that (27) is established whatever which portion t belongs to. Considering the last stage, the jth stage, if t belongs to the T-portion, $T^{(j)}$ means the time from the start time of the T-portion to t, while $T^{(j)} = 0$ if t belongs to the τ-portion. For t_0, there is not any restriction on which portion it belongs to. Therefore, this point makes the proof about the stability more general.

Theorem 2: Presetting $0 < \bar{\epsilon}$, $0 < \hat{k}$, $0 \leq \hat{\ell}$, $0 < \hat{v}$, $0 < \hat{w}$, $0 < r$, $0 < T_{\mathrm{PDT}}$, $0 < \tau_{\mathrm{PDT}}$, $0 < \lambda < 1$, $1 < \phi$, and $\lambda\phi \neq 1$ and considering the state space of interesting and FOU is partitioned into $\hat{k}$ and $\hat{\ell}+1$ disjoint subspaces, separately, if there exist scalars $0 < \gamma_1$, $0 < \gamma_2$, diagonal matrix $L_{w\theta}$, and matrices R_θ, $Q_\theta^{\bar{\epsilon}} = (Q_\theta^{\bar{\epsilon}})^T$, $\mathcal{X}_{1\theta} = \mathcal{X}_{1\theta}^T$, $\mathcal{X}_{2\theta}$, $\mathcal{X}_{3\theta} = \mathcal{X}_{3\theta}^T$, $\mathcal{Y}_{1vw\ell\theta} = \mathcal{Y}_{1vw\ell\theta}^T$, $\mathcal{Y}_{2vw\ell\theta}$, and $\mathcal{Y}_{3vw\ell\theta} = \mathcal{Y}_{3vw\ell\theta}^T$ for $\kappa \in \{1,2,3,4\}$, $\theta \neq \tilde{\theta}$ $\forall \theta \in \mathcal{T}$ $\forall \tilde{\theta} \in \mathcal{T}$ $\forall v \in \mathcal{V}$ $\forall w \in \mathcal{W}$, and $\forall \ell \in \{1,2,\ldots,\hat{\ell}+1\}$, such that inequality (18) and the below conditions hold

$$\Upsilon_{vw\theta}^\kappa < 0 \tag{19}$$

$$\phi Q_{\tilde{\theta}}^{\bar{\epsilon}} > Q_\theta^{\bar{\epsilon}} > 0 \tag{20}$$

$$\phi Q_{1\tilde{\theta}} > Q_{1\theta} > 0 \tag{21}$$

$$\begin{bmatrix} \mathcal{Y}_{1vw\ell\theta} & * \\ \mathcal{Y}_{2vw\ell\theta} & \mathcal{Y}_{3vw\ell\theta} \end{bmatrix} > 0 \tag{22}$$

where

$$\Upsilon^{1}_{vw\theta} \triangleq \begin{bmatrix} \Upsilon^{1(1,1)}_{vw\theta} & * & * \\ \Upsilon^{1(2,1)}_{vw\theta} & \Upsilon^{1(2,2)}_{vw\theta} & * \\ \Upsilon^{1(3,1)}_{vw\theta} & \Upsilon^{1(3,2)}_{vw\theta} & \Upsilon^{1(3,3)}_{vw\theta} \end{bmatrix}$$

$$\Upsilon^{2}_{vw\theta} \triangleq \begin{bmatrix} \Upsilon^{2(1,1)}_{vw\theta} & * & * \\ \Upsilon^{2(2,1)}_{vw\theta} & \Upsilon^{2(2,2)}_{vw\theta} & * \\ \Upsilon^{1(3,1)}_{vw\theta} & \Upsilon^{2(3,2)}_{vw\theta} & \Upsilon^{1(3,3)}_{vw\theta} \end{bmatrix}$$

$$\Upsilon^{3}_{vw\theta} \triangleq \begin{bmatrix} \Upsilon^{3(1,1)}_{vw\theta} & * & * \\ \Upsilon^{3(2,1)}_{vw\theta} & \Upsilon^{1(2,2)}_{vw\theta} & * \\ \Upsilon^{3(3,1)}_{vw\theta} & \Upsilon^{3(3,2)}_{vw\theta} & \Upsilon^{3(3,3)}_{vw\theta} \end{bmatrix}$$

$$\Upsilon^{4}_{vw\theta} \triangleq \begin{bmatrix} \Upsilon^{4(1,1)}_{vw\theta} & * & * \\ \Upsilon^{4(2,1)}_{vw\theta} & \Upsilon^{2(2,2)}_{vw\theta} & * \\ \Upsilon^{3(3,1)}_{vw\theta} & \Upsilon^{4(3,2)}_{vw\theta} & \Upsilon^{3(3,3)}_{vw\theta} \end{bmatrix}$$

$$\Upsilon^{1(1,1)}_{vw\theta} \triangleq \begin{bmatrix} \Upsilon_{11} & * \\ \Upsilon_{21} & \Upsilon_{22} \end{bmatrix}$$

$$\Upsilon^{2(1,1)}_{vw\theta} \triangleq \begin{bmatrix} \Upsilon^{\bar{\epsilon}}_{11} & * \\ \Upsilon_{21} & \Upsilon_{22} \end{bmatrix}$$

$$\Upsilon^{3(1,1)}_{vw\theta} \triangleq \begin{bmatrix} \tilde{\Upsilon}_{11} & * \\ \tilde{\Upsilon}_{21} & \tilde{\Upsilon}_{22} \end{bmatrix}$$

$$\Upsilon^{4(1,1)}_{vw\theta} \triangleq \begin{bmatrix} \tilde{\Upsilon}^{\bar{\epsilon}}_{11} & * \\ \tilde{\Upsilon}_{21} & \tilde{\Upsilon}_{22} \end{bmatrix}$$

$$\Upsilon^{3(2,1)}_{vw\theta} \triangleq \begin{bmatrix} A_{v\theta}R_\theta + C_{v\theta}L_{w\theta} & B_{v\theta} \\ D_{v\theta}R_\theta + F_{v\theta}L_{w\theta} & E_{v\theta} \end{bmatrix}$$

$$\mathcal{G} \triangleq \operatorname{diag}\left\{\underbrace{\vartheta, \ldots, \vartheta}_{n_k}\right\}$$

$$\Upsilon^{4(2,1)}_{vw\theta} \triangleq \begin{bmatrix} A^{\bar{\epsilon}}_{v\theta}R_\theta + C^{\bar{\epsilon}}_{v\theta}L_{w\theta} & B^{\bar{\epsilon}}_{v\theta} \\ D_{v\theta}R_\theta + F_{v\theta}L_{w\theta} & E_{v\theta} \end{bmatrix}$$

$$\Upsilon^{3(3,1)}_{vw\theta} \triangleq \begin{bmatrix} 0 & 0 \\ L_{w\theta} & 0 \end{bmatrix}$$

$$\Upsilon^{1(3,2)}_{vw\theta} \triangleq \begin{bmatrix} \psi_5\gamma_1\mathcal{G}C^T_{v\theta} & \psi_5\gamma_1\mathcal{G}F^T_{v\theta} \\ 0 & 0 \end{bmatrix}$$

$$\Upsilon^{2(3,2)}_{vw\theta} \triangleq \begin{bmatrix} \psi_5\gamma_1\mathcal{G}C^{\bar{\epsilon}T}_{v\theta} & \psi_5\gamma_1\mathcal{G}F^T_{v\theta} \\ 0 & 0 \end{bmatrix}$$

$$\Upsilon^{3(3,2)}_{vw\theta} \triangleq \begin{bmatrix} \gamma_2\mathcal{G}C^T_{v\theta} & \gamma_2\mathcal{G}F^T_{v\theta} \\ 0 & 0 \end{bmatrix}, \quad C_{v\theta} \triangleq \begin{bmatrix} C^{(1,1)}_{v\theta} \\ 0 \end{bmatrix}$$

$$\Upsilon^{4(3,2)}_{vw\theta} \triangleq \begin{bmatrix} \gamma_2\mathcal{G}C^{\bar{\epsilon}T}_{v\theta} & \gamma_2\mathcal{G}F^T_{v\theta} \\ 0 & 0 \end{bmatrix}, \quad C^{\bar{\epsilon}}_{v\theta} \triangleq \begin{bmatrix} C^{(1,1)}_{v\theta} \\ \bar{\epsilon}C^{(2,1)}_{v\theta} \end{bmatrix}$$

$$A_{v\theta} \triangleq \begin{bmatrix} A^{(1,1)}_{v\theta} & A^{(1,2)}_{v\theta} \\ 0 & 0 \end{bmatrix}, \quad B_{v\theta} \triangleq \begin{bmatrix} B^{(1,1)}_{v\theta} \\ 0 \end{bmatrix}$$

$$A^{\bar{\epsilon}}_{v\theta} \triangleq \begin{bmatrix} A^{(1,1)}_{v\theta} & A^{(1,2)}_{v\theta} \\ \bar{\epsilon}A^{(2,1)}_{v\theta} & \bar{\epsilon}A^{(2,2)}_{v\theta} \end{bmatrix}, \quad B^{\bar{\epsilon}}_{v\theta} \triangleq \begin{bmatrix} B^{(1,1)}_{v\theta} \\ \bar{\epsilon}B^{(2,1)}_{v\theta} \end{bmatrix}$$

$$\Upsilon^{1(2,1)}_{vw\theta} \triangleq \psi_4\Upsilon^{3(2,1)}_{vw\theta}, \quad \Upsilon^{1(3,1)}_{vw\theta} \triangleq \psi_5\Upsilon^{3(3,1)}_{vw\theta}$$

$$\Upsilon^{2(2,1)}_{vw\theta} \triangleq \psi_4\Upsilon^{4(2,1)}_{vw\theta}$$

$$\Upsilon^{1(2,2)}_{vw\theta} \triangleq \operatorname{diag}\{-Q_{1\theta}, -I\}, \quad \Upsilon^{2(2,2)}_{vw\theta} \triangleq \operatorname{diag}\left\{-Q^{\bar{\epsilon}}_{\theta}, -I\right\}$$

$$\Upsilon^{1(3,3)}_{vw\theta} \triangleq \operatorname{diag}\{-\gamma_1 I, -\gamma_1 I\}, \quad Q^{\bar{\epsilon}}_{\theta} \triangleq Q_{1\theta} + \bar{\epsilon}Q_{2\theta}$$

$$\Upsilon^{3(3,3)}_{vw\theta} \triangleq \operatorname{diag}\{-\gamma_2 I, -\gamma_2 I\}, \quad \psi_1 \triangleq \underline{\psi}_{vw\varpi_1\cdots\varpi_{n_x}k\ell}$$

$$\psi_2 \triangleq \underline{\psi}_{vw\varpi_1\cdots\varpi_{n_x}k\ell} - 1/(\hat{v}\hat{w}), \quad \psi_4 \triangleq \sqrt{\psi_1}$$

$$\psi_3 \triangleq \underline{\psi}_{vw\varpi_1\cdots\varpi_{n_x}k\ell} - \overline{\psi}_{vw\varpi_1\cdots\varpi_{n_x}k\ell}, \quad \psi_5 \triangleq \sqrt{\psi_4}$$

$$\mathcal{F} \triangleq Q_{1\theta} - R_\theta - R^T_\theta, \quad \mathcal{F}^{\bar{\epsilon}} \triangleq Q^{\bar{\epsilon}}_{\theta} - R_\theta - R^T_\theta$$

$$\Upsilon_{11} \triangleq \lambda\psi_1\mathcal{F} + \psi_2\mathcal{X}_{1\theta} - \psi_3\mathcal{Y}_{1vw\ell\theta}$$

$$\tilde{\Upsilon}_{11} \triangleq \lambda\mathcal{F} + \mathcal{X}_{1\theta} - \mathcal{Y}_{1vw\ell\theta}$$

$$\Upsilon^{\bar{\epsilon}}_{11} \triangleq \lambda\psi_1\mathcal{F}^{\bar{\epsilon}} + \psi_2\mathcal{X}_{1\theta} - \psi_3\mathcal{Y}_{1vw\ell\theta}$$

$$\tilde{\Upsilon}^{\bar{\epsilon}}_{11} \triangleq \lambda\mathcal{F}^{\bar{\epsilon}} + \mathcal{X}_{1\theta} - \mathcal{Y}_{1vw\ell\theta}$$

$$\Upsilon_{21} \triangleq \psi_2\mathcal{X}_{2\theta} - \psi_3\mathcal{Y}_{2vw\ell\theta}, \quad \tilde{\Upsilon}_{21} \triangleq \mathcal{X}_{2\theta} - \mathcal{Y}_{2vw\ell\theta}$$

$$\Upsilon_{22} \triangleq \psi_2\mathcal{X}_{3\theta} - \psi_3\mathcal{Y}_{3vw\ell\theta} - r^2\psi_1 I$$

$$\tilde{\Upsilon}_{22} \triangleq \mathcal{X}_{3\theta} - \mathcal{Y}_{3vw\ell\theta} - r^2 I$$

then, for $\epsilon \in (0, \bar{\epsilon}]$, system (1) and (2) is GUES with an H_∞ performance index $\tilde{r}$ and the designed controller gains are as follows:

$$K_{w\theta} = L_{w\theta}(R_\theta)^{-1}.$$

Proof: See Appendix B. ■

Remark 5: It is easy to see that equality (12) includes the function in (13). The main purpose of introducing equality (12) in our article is to deduce another presentation of grades of membership of the closed-loop system to make the following process easy. For a practical system under a certain case, the actual uncertainty is determined and belongs to a certain subspace of FOU, which is called the ℓth subsystem. Since the entire nonlinear system is divided by IF-THEN rules and FOU is also separated, locating this actual uncertainty needs three values, v, w, and ℓ, which means that when the three values are fixed, the location of this actual uncertainty is determined. In other words, for the fixed values of v and w, the value of this certain uncertainty is nonzero in only one subspace and this subspace is the spacial ℓ. Therefore, if the actual uncertainty belongs to the FOU subspace ℓ, the membership functions of the closed-loop system can be expressed by the minimum and the maximum of this FOU subspace as well as two weighted nonlinear functions, $\underline{\rho}_{vw\ell}$ and $\overline{\rho}_{vw\ell}$. Furthermore, this special function helps the deduction from (17) to (33) via $\sum_{\ell=1}^{\hat{\ell}+1}\varphi_{vw\ell} = 1$.

Remark 6: The existence of FOU in membership functions is a main difference between the type-1 T–S fuzzy model and the type-2 T–S fuzzy model. Therefore, many papers about FOU were published. For example, Zhou *et al.* [48] studied the effect of the partition about FOU on the work of the controller. In [28], the maximum and minimum values of the subspace of FOU are used to express the uncertainty by the average partition of FOU. Furthermore, in future work, some advanced techniques of tacking FOU will be researched, such as the research of adaptive nonsingleton type-2 fuzzy systems in [49], the design of a feedback error learning control for the type-2 fuzzy neural networks in [50], and the study of adaptive dynamic surface for uncertain nonlinear systems based on the IT2 fuzzy set theory [51].

IV. Simulation

In this section, a numerical example is adopted to verify the correctness and effectiveness of our method.

System parameters are given as follows:

$$A_{11}^{\epsilon} = \begin{bmatrix} 1.1000 & 1.1840 \\ -0.1814\epsilon & 0.8179\epsilon \end{bmatrix},\ C_{11}^{\epsilon} = \begin{bmatrix} -0.1874 \\ 0.1812\epsilon \end{bmatrix}$$
$$A_{12}^{\epsilon} = \begin{bmatrix} 1.2000 & 1.7526 \\ -0.1237\epsilon & 1.2534\epsilon \end{bmatrix},\ C_{12}^{\epsilon} = \begin{bmatrix} 0.1874 \\ 0.1812\epsilon \end{bmatrix}$$
$$A_{21}^{\epsilon} = \begin{bmatrix} 1.0300 & 1.8148 \\ -1.1815\epsilon & 0.8179\epsilon \end{bmatrix},\ C_{21}^{\epsilon} = \begin{bmatrix} 1.1875 \\ 1.1812\epsilon \end{bmatrix}$$
$$A_{22}^{\epsilon} = \begin{bmatrix} 1.0200 & 1.5816 \\ -0.1635\epsilon & 1.2563\epsilon \end{bmatrix},\ C_{22}^{\epsilon} = \begin{bmatrix} 1.2700 \\ -1.8120\epsilon \end{bmatrix}$$
$$B_{v,l(t)}^{\epsilon} = \begin{bmatrix} 0.5 \\ 0.1\epsilon \end{bmatrix},\ D_{11}^{T} = \begin{bmatrix} -1 \\ 0.5 \end{bmatrix},\ D_{12}^{T} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$
$$D_{21} = \begin{bmatrix} -1 & 1 \end{bmatrix},\ D_{22} = \begin{bmatrix} -1 & 1 \end{bmatrix},\ \epsilon = 0.005$$
$$F_{11} = 1,\ F_{12} = 3,\ F_{21} = -3,\ F_{22} = 1,\ E_{v\vartheta} = 0.1.$$

LMF and UMF of the plant and the controller are as follows (where the symbol $\mathcal{U} \in [\underline{\mathcal{U}}, \overline{\mathcal{U}}]$ represents the uncertainty of systems):

$$\underline{\mu}_{S_{11}}(x_1(t)) = 1 - \frac{1}{1 + \exp\left(-x_1(t)/2 - \overline{\mathcal{U}}\right)}$$
$$\overline{\mu}_{S_{11}}(x_1(t)) = 1 - \frac{1}{1 + \exp\left(-x_1(t)/2 - \underline{\mathcal{U}}\right)}$$
$$\underline{\mu}_{S_{21}}(x_1(t)) = 1 - \overline{\mu}_{S_{11}}(x_1(t))$$
$$\overline{\mu}_{S_{21}}(x_1(t)) = 1 - \underline{\mu}_{S_{11}}(x_1(t))$$

and

$$\underline{\mu}_{\tilde{S}_{11}}(x_1(t)) = 0.8\exp\left(-x_1^2(t)\right)$$
$$\overline{\mu}_{\tilde{S}_{11}}(x_1(t)) = \underline{\mu}_{\tilde{S}_{11}}(x_1(t))$$
$$\underline{\mu}_{\tilde{S}_{21}}(x_1(t)) = 1 - \underline{\mu}_{\tilde{S}_{11}}(x_1(t))$$
$$\overline{\mu}_{\tilde{S}_{21}}(x_1(t)) = \underline{\mu}_{\tilde{S}_{21}}(x_1(t)).$$

The state space Γ and the FOU Ξ are divided into ten and two regions, respectively, which means that $\Gamma = \bigcup_{k=1}^{10} \Gamma_k$ and $\Xi = \bigcup_{\ell=1}^{2} \Xi_\ell$. The minimum and maximum values of Γ_k are $\underline{x}_{1k\ell} = 0.2(k-6)$, $\overline{x}_{1k\ell} = 0.2(k-5)$, as well as the minimum and maximum values of Ξ_ℓ are $\underline{\mathcal{U}}_\ell = 1.5\ell + 0.5$ and $\overline{\mathcal{U}}_\ell = 1.5\ell + 2$, respectively. Other functions about the IT2 T–S fuzzy set are selected as follows:

$$\underline{\varsigma}_v(x_1(t)) = \sin^2(x_1(t))$$
$$\overline{\varsigma}_v(x_1(t)) = 1 - \underline{\varsigma}_v(x_1(t))$$
$$\underline{\xi}_w(x_1(t)) = \cos^2(x_1(t))$$
$$\overline{\xi}_w(x_1(t)) = 1 - \underline{\xi}_w(x_1(t))$$
$$\chi_{11k\ell}(x_1(t)) = 1 - \frac{x_1(t) - \underline{x}_{1k\ell}}{\overline{x}_{1k\ell} - \underline{x}_{1k\ell}}$$
$$\chi_{12k\ell}(x_1(t)) = 1 - \chi_{11k\ell}(x_1(t))$$
$$\underline{\psi}_{vw1k\ell} = \underline{g}_v\left(\underline{x}_{1k\ell}\right)\underline{h}_w\left(\underline{x}_{1k\ell}\right)$$
$$\underline{\psi}_{vw2k\ell} = \underline{g}_v(\overline{x}_{1k\ell})\underline{h}_w(\overline{x}_{1k\ell})$$
$$\overline{\psi}_{vw1k\ell} = \overline{g}_v\left(\underline{x}_{1k\ell}\right)\overline{h}_w\left(\underline{x}_{1k\ell}\right)$$
$$\overline{\psi}_{vw2k\ell} = \overline{g}_v(\overline{x}_{1k\ell})\overline{h}_w(\overline{x}_{1k\ell}).$$

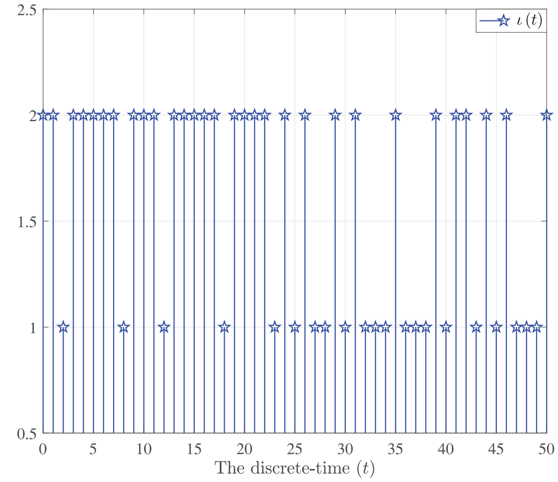


Fig. 2. Switching sequence subject to the PDT switching rule.

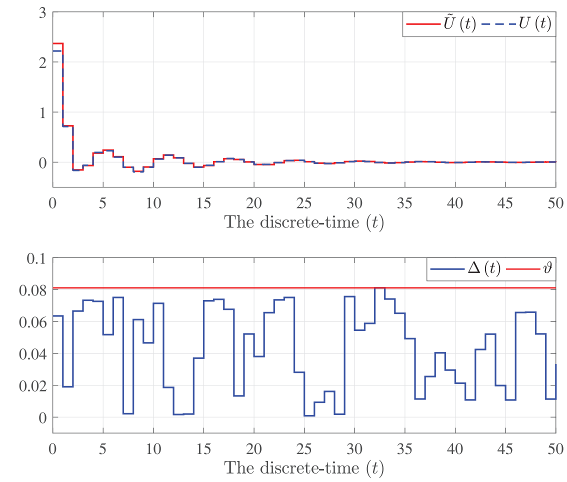


Fig. 3. Trajectories of quantization inputs, outputs, and errors.

Furthermore, choosing $r = 5$, $\phi = 1.1$, $T_{\mathrm{PDT}} = 4$, $\tau_{\mathrm{PDT}} = 2$, $\lambda = 0.9$, $\beta = 0.85$, $f_{(0)} = 5$, $x(0) = [-5\ \ 1]^T$, and $\omega(t) = \sin(t)\exp(-0.1t)$ and solving the conditions in Theorem 2, the controller gains are obtained as follows:

$$K_{11} = \begin{bmatrix} -0.7714 & -1.1443 \end{bmatrix}$$
$$K_{12} = \begin{bmatrix} -0.7851 & -1.2957 \end{bmatrix}$$
$$K_{21} = \begin{bmatrix} -0.7027 & -1.0444 \end{bmatrix}$$
$$K_{22} = \begin{bmatrix} -0.7126 & -1.1942 \end{bmatrix}.$$

Under the switching sequence in Fig. 2, the data from the system to the controller are disposed by a quantizer, which is demonstrated in Fig. 3. Then, the state responses of the open-loop system and the closed-loop system are drawn in Fig. 4. It is clear that Fig. 4 shows the effectiveness of the designed controller.

Besides, Table I is provided to illustrate the relationship between r and the upper bound of ϵ under ϵ-dependent MLFs and ϵ-independent MLFs. When setting $Q_{2\theta} = 0$ in Theorem 2, we can obtain sufficient conditions guaranteeing the stability and the performance of the considered system in terms of ϵ-independent MLFs. It is observed from this table that

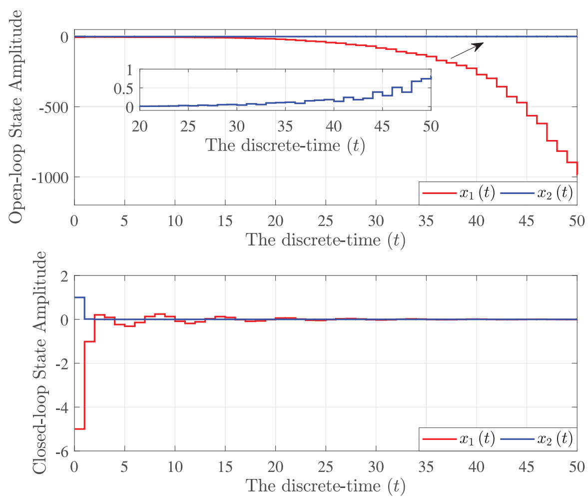


Fig. 4. State responses of open-loop system and closed-loop system.

TABLE I
OPTIMIZED UPPER BOUNDS OF ϵ FOR DIFFERENT r AND MLFS

$\bar{\epsilon}$	$r = 5$	$r = 5.5$	$r = 6$
ϵ-dependent MLFs	0.0352	0.0603	0.0704
ϵ-independent MLFs	0.0062	0.0209	0.0507

compared with ϵ-independent MLFs, the method based on SPP-dependent MLFs can increase the upper bound of ϵ, which means that the conservativeness of sufficient conditions derived by ϵ-dependent MLFs is less than that derived by ϵ-independent MLFs. Furthermore, this table also reveals the fact that the increase of r enlarges the upper bound of ϵ.

V. CONCLUSION

In the study, we have researched the quantized control issue for NSSPSs subject to the uncertainty. The interval type-2 fuzzy set theory has been utilized to cope with the nonlinearity and the uncertainty of the system. Furthermore, the switching feature of the considered system has been managed by the persistent dwell-time switching rule. By solving ϵ-independent and membership-function-dependent conditions, the specific form of the controller satisfying prescribed requirements has been deduced. Ultimately, a numerical example has proved the effectiveness of the designed controller and the less conservativeness of the obtained sufficient conditions. In addition, how to study NSSPSs on the basis of the adaptive control scheme will be our future work [52]–[57].

APPENDIX A
PROOF OF THEOREM 1

The ϵ-dependent MLFs for the closed-loop system (9) and (10) are constructed as

$$V_{\iota(t)}(x(t)) = x^T(t)P^{\epsilon}_{\iota(t)}x(t)$$

and the target function is defined as

$$\begin{aligned}\mathcal{J}(t) \triangleq\ & V_\theta(x(t+1)) - \lambda V_\theta(x(t)) \\ & + z^T(t)z(t) - r^2\omega^T(t)\omega(t).\end{aligned}$$

By introducing the slack matrices $\mathcal{X}_\theta$, $0 < \mathcal{Y}_{vw\ell\theta}$, we have the following results from (11) and (12):

$$\left[\sum_{v=1}^{\hat{v}}\sum_{w=1}^{\hat{w}}\sum_{\ell=1}^{\hat{\ell}+1}\varphi_{vw\ell}\left(\underline{\rho}_{vw\ell}\underline{k}_{vw\ell} + \overline{\rho}_{vw\ell}\overline{k}_{vw\ell}\right) - 1\right]\mathcal{X}_\theta = 0 \quad (23)$$

$$\sum_{v=1}^{\hat{v}}\sum_{w=1}^{\hat{w}}\sum_{\ell=1}^{\hat{\ell}+1}\varphi_{vw\ell}(1-\overline{\rho}_{vw\ell})(\overline{k}_{vw\ell} - \underline{k}_{vw\ell})\mathcal{Y}_{vw\ell\theta} > 0. \quad (24)$$

It follows from Lemma 1 that:

$$\begin{aligned}\mathcal{J}(t) =\ & x^T(t+1)P^{\epsilon}_\theta x(t+1) - \lambda x^T(t)P^{\epsilon}_\theta x(t) \\ & + z^T(t)z(t) - r^2\omega^T(t)\omega(t) \\ =\ & \left[\sum_{v=1}^{\hat{v}}\sum_{w=1}^{\hat{w}}\delta_{vw}\mathcal{A}^{\epsilon}_{vw\theta}\eta(t)\right]^T P^{\epsilon}_\theta\left[\sum_{v=1}^{\hat{v}}\sum_{w=1}^{\hat{w}}\delta_{vw}\mathcal{A}^{\epsilon}_{vw\theta}\eta(t)\right] \\ & + \left[\sum_{v=1}^{\hat{v}}\sum_{w=1}^{\hat{w}}\delta_{vw}\mathcal{B}_{vw\theta}\eta(t)\right]^T\left[\sum_{v=1}^{\hat{v}}\sum_{w=1}^{\hat{w}}\delta_{vw}\mathcal{B}_{vw\theta}\eta(t)\right] \\ & - \lambda x^T(t)P^{\epsilon}_\theta x(t) - r^2\omega^T(t)\omega(t) \\ <\ & \eta^T(t)\left[\left(\sum_{v=1}^{\hat{v}}\sum_{w=1}^{\hat{w}}\Theta^{\epsilon}_{0vw\theta} - \mathcal{X}_\theta\right) + \sum_{v=1}^{\hat{v}}\sum_{w=1}^{\hat{w}}\sum_{\ell=1}^{\hat{\ell}+1}\varphi_{vw\ell}\right. \\ & \left. \times\ \overline{\rho}_{vw\ell}(\overline{k}_{vw\ell} - \underline{k}_{vw\ell})(\Theta^{\epsilon}_{1vw\theta} + \mathcal{X}_\theta - \mathcal{Y}_{vw\theta\ell})\right] \\ & \times\ \eta(t).\end{aligned}$$

From (16) and (17) in Theorem 1, $\mathcal{J}(t) < 0$ is easily obtained.

In what follows, the stability of system (9) is testified, and then the rest part is used to further demonstrate that system (9) is GUES with an H_∞ performance index $\tilde{r}$.

When $\omega(t) \equiv 0$, from $\mathcal{J}(t) < 0$ and (15), we can know that

$$V_\theta(x(t+1)) < \lambda V_\theta(x(t)) \quad (25)$$

$$V_\theta(x(t)) < \phi V_{\tilde{\theta}}(x(t)). \quad (26)$$

Therefore, for $t \in [t_{m_j}, t_{m_{j+1}})$, it can be derived that

$$V_{\iota(t)}(x(t)) < \varrho_j V_{\iota\left(t_{m_j}\right)}\left(x\left(t_{m_j}\right)\right) \quad (27)$$

where

$$\varrho_j \triangleq (\phi\lambda)^{T^{(j)}}\phi\lambda^{\tau_{\mathrm{PDT}}}.$$

Thereafter, take the following two cases into consideration.

1) When $\phi\lambda < 1$, it can be achieved that $\varrho_j < 1$.
2) When $\phi\lambda > 1$, $\varrho_j \leq (\phi\lambda)^{T_{\mathrm{PDT}}}\phi\lambda^{\tau_{\mathrm{PDT}}} < 1$ can be observed based on (18).

In conclusion, it is easy to obtain that

$$V_{\iota\left(t_{m_j}\right)}\left(x\left(t_{m_j}\right)\right) < \varrho V_{\iota\left(t_{m_{j-1}}\right)}\left(x\left(t_{m_{j-1}}\right)\right)$$

where

$$\varrho \triangleq \begin{cases}\max_{0\leq\tilde{j}\leq j}\varrho_{\tilde{j}}, & \phi\lambda < 1\\ (\phi\lambda)^{T_{\mathrm{PDT}}}\phi\lambda^{\tau_{\mathrm{PDT}}}, & \phi\lambda > 1\end{cases}$$

which implies

$$\varrho < 1.$$

Hence, denoting the initial time as $t_0 \triangleq t_{m_0}$, it can be inferred that

$$\|x(t)\|^2 < \zeta \tilde{\varrho}^{t-t_0} \|x(t_0)\|^2$$

with

$$\tilde{\varrho} \triangleq \max_{t_0 \le t} \left(\varrho^{\frac{j+1}{t-t_0+1}} \right), \ \zeta \triangleq \frac{\tilde{\varrho} \max_{\theta \in \mathcal{S}} \lambda_{\max}\left(P_\theta^\varepsilon\right)}{\min_{\theta \in \mathcal{S}} \lambda_{\min}\left(P_\theta^\varepsilon\right)}$$

which is consistent with Definition 1.

On the other hand, from $\mathcal{J}(t) < 0$ and (26), we can know that

$$V_{\iota\left(t_{m_j}\right)}(x(t)) < \lambda^{t-t_0} \phi^{NUM(t,t_0)} V_{\iota(t_0)}(x(t_0)) - \sum_{p=t_0}^{t-1} \varkappa(t, p) \tag{28}$$

where

$$\varkappa(t, p) \triangleq \phi^{NUM(t,p)} \lambda^{t-1-p} \left(z^T(p) z(p) - r^2 \omega^T(p) \omega(p) \right).$$

On account of $x(t_0) = 0$, one obtains from (28) that

$$-\sum_{p=t_0}^{t-1} \varkappa(\lambda, p) > V_{\iota\left(t_{m_j}\right)}(x(t)) > 0. \tag{29}$$

The total number of switching in the interval $[p, t)$ satisfies the following conditions:

$$0 \le NUM(t, p) \le \left(\frac{t-p}{T_{\text{PDT}} + \tau_{\text{PDT}}} + 1 \right) (T_{\text{PDT}} + 1).$$

Based on the above discussion, we have

$$\sum_{p=t_0}^{\infty} z^T(p) z(p) < \tilde{r}^2 \sum_{p=t_0}^{\infty} \omega^T(p) \omega(p).$$

That ends the proof.

Appendix B
Proof of Theorem 2

On account of (19) and (21) for $\kappa \in \{3, 4\}$ and by utilizing Lemma 2, we have

$$0 < Q_\theta^\epsilon < \phi Q_{\tilde{\theta}}^\epsilon \tag{30}$$

$$\Upsilon_{vw\theta}^\epsilon < 0 \tag{31}$$

where

$$Q_\theta^\epsilon \triangleq Q_{1\theta} + \epsilon Q_{2\theta}$$

$$\Upsilon_{vw\theta}^\epsilon \triangleq \begin{bmatrix} \Upsilon_{vw\theta}^{\epsilon(1.1)} & * & * \\ \Upsilon_{vw\theta}^{\epsilon(2.1)} & \Upsilon_{vw\theta}^{\epsilon(2.2)} & * \\ \Upsilon_{vw\theta}^{3(3.1)} & \Upsilon_{vw\theta}^{\epsilon(3.2)} & \Upsilon_{vw\theta}^{3(3.3)} \end{bmatrix}$$

with

$$F^\epsilon \triangleq Q_\theta^\epsilon - R_\theta - R_\theta^T$$

$$\tilde{\Upsilon}_{11}^\epsilon \triangleq \lambda F^\epsilon + \mathcal{X}_{1\theta} - \mathcal{Y}_{1vw\ell\theta}$$

$$\Upsilon_{vw\theta}^{\epsilon(2.2)} \triangleq \operatorname{diag}\left\{-Q_\theta^\epsilon, -I\right\}$$

$$\Upsilon_{vw\theta}^{\epsilon(1.1)} \triangleq \begin{bmatrix} \tilde{\Upsilon}_{11}^\epsilon & * \\ \tilde{\Upsilon}_{21} & \tilde{\Upsilon}_{22} \end{bmatrix}$$

$$\Upsilon_{vw\theta}^{\epsilon(3.2)} \triangleq \begin{bmatrix} \gamma_2 \mathcal{G} C_{v\theta}^{\epsilon T} & \gamma_2 \mathcal{G} F_{v\theta}^T \\ 0 & 0 \end{bmatrix}$$

$$\Upsilon_{vw\theta}^{\epsilon(2.1)} \triangleq \begin{bmatrix} A_{v\theta}^\epsilon R_\theta + C_{v\theta}^\epsilon L_{w\theta} & B_{v\theta}^\epsilon \\ D_{v\theta} R_\theta + F_{v\theta} L_{w\theta} & E_{v\theta} \end{bmatrix}.$$

Defining the matrix Q_θ^ϵ as $Q_\theta^\epsilon \triangleq (P_\theta^\epsilon)^{-1}$, inequality (30) implies the establishment of (15). On the other hand, we denote

$$\mathcal{X}_\theta \triangleq \begin{bmatrix} \tilde{\mathcal{X}}_{1\theta} & * \\ \tilde{\mathcal{X}}_{2\theta} & \mathcal{X}_{3\theta} \end{bmatrix}, \ \mathcal{Y}_{vw\ell\theta} \triangleq \begin{bmatrix} \tilde{\mathcal{Y}}_{1vw\ell\theta} & * \\ \tilde{\mathcal{Y}}_{2vw\ell\theta} & \mathcal{Y}_{3vw\ell\theta} \end{bmatrix}.$$

By utilizing the Schur complement to condition (16) in Theorem 1, the result is as follows:

$$\begin{bmatrix} \tilde{\Phi}_{vw\theta}^{\epsilon(1.1)} & * \\ \tilde{\Phi}_{vw\theta}^{\epsilon(2.1)} & \Upsilon_{vw\theta}^{\epsilon(2.2)} \end{bmatrix} < 0 \tag{32}$$

where

$$\tilde{\Phi}_{vw\theta}^{\epsilon(1.1)} \triangleq \begin{bmatrix} -\lambda\left(Q_\theta^\epsilon\right)^{-1} + \tilde{\mathcal{X}}_{1\theta} - \tilde{\mathcal{Y}}_{1vw\ell\theta} & * \\ \tilde{\mathcal{X}}_{2\theta} - \tilde{\mathcal{Y}}_{2vw\ell\theta} & \mathcal{X}_{3\theta} - \mathcal{Y}_{3vw\ell\theta} - r^2 I \end{bmatrix}$$

$$\tilde{\Phi}_{vw\theta}^{\epsilon(2.1)} \triangleq \begin{bmatrix} A_{v\theta}^\epsilon + C_{v\theta}^\epsilon (I + \Delta(t)) K_{w\theta} & B_{v\theta}^\epsilon \\ D_{v\theta} + F_{v\theta} (I + \Delta(t)) K_{w\theta} & E_{v\theta} \end{bmatrix}.$$

Then, employ

$$Q_\theta^\epsilon - R_\theta - R_\theta^T \ge -R_\theta^T \left(Q_\theta^\epsilon\right)^{-1} R_\theta$$

and define

$$\tilde{R}_\theta \triangleq \operatorname{diag}\{R_\theta, I\}, \ \mathcal{X}_{2\theta} \triangleq \tilde{\mathcal{X}}_{2\theta} R_\theta, \ \mathcal{Y}_{2vw\ell\theta} \triangleq \tilde{\mathcal{Y}}_{2vw\ell\theta}$$
$$\mathcal{X}_{1\theta} \triangleq R_\theta^T \tilde{\mathcal{X}}_{1\theta} R_\theta, \ \mathcal{Y}_{1vw\ell\theta} \triangleq R_\theta^T \tilde{\mathcal{Y}}_{1vw\ell\theta} R_\theta.$$

Meanwhile, by performing congruent transformations to (32) with $\operatorname{diag}\{\tilde{R}_\theta, I\}$ and the Schur complement, it ensures the establishment of (31).

Similarly, employing the same method to (19)–(21) for $\kappa \in \{1, 2\}$, it yields

$$\psi_1 \Theta_{1vw\theta}^\epsilon + \psi_2 \mathcal{X}_\theta - \psi_3 \mathcal{Y}_{vw\ell\theta} < 0$$

which implies

$$\sum_{k=1}^{\hat{k}} \sum_{\varpi_1=1}^{2} \cdots \sum_{\varpi_{n_x}=1}^{2} \prod_{\sigma=1}^{n_x} \chi_{\sigma\varpi_\sigma k\ell} \left(\psi_1 \Theta_{1vw\theta}^\epsilon + \psi_2 \mathcal{X}_\theta - \psi_3 \mathcal{Y}_{vw\ell\theta} \right) < 0. \tag{33}$$

Therefore, based on (12), inequality (17) holds. That ends the proof.

References

[1] H. Xin, D. Gan, M. Huang, and K. Wang, "Estimating the stability region of singular perturbation power systems with saturation nonlinearities: An linear matrix inequality-based method," *IET Control Theory Appl.*, vol. 4, no. 3, pp. 351–361, 2010.

[2] T. Sun, D. Liang, and Y. Song, "Singular-perturbation-based nonlinear hybrid control of redundant parallel robot," *IEEE Trans. Ind. Electron.*, vol. 65, no. 4, pp. 3326–3336, Apr. 2018.

[3] L. Ma, Z. Wang, C. Cai, and F. E. Alsaadi, "Dynamic event-triggered state estimation for discrete-time singularly perturbed systems with distributed time-delays," *IEEE Trans. Syst.*, vol. 50, no. 9, pp. 3258–3268, Sep. 2020.

[4] D. S. Naidu, "Singular perturbations and time scales in control theory and applications: An overview," *Dyn. Continous Discr. Impulsive Syst. B Appl. Algorithm*, vol. 9, no. 2, pp. 233–278, 2002.

[5] J. Dong and G.-H. Yang, "H_∞ control for fast sampling discrete-time singularly perturbed systems," *Automatica*, vol. 44, no. 5, pp. 1385–1393, 2008.

[6] H. Shen, F. Li, Z.-G. Wu, J. H. Park, and V. Sreeram, "Fuzzy-model-based nonfragile control for nonlinear singularly perturbed systems with semi-Markov jump parameters," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 6, pp. 3428–3439, Dec. 2018.

[7] H. Shen, Y. Men, Z.-G. Wu, J. Cao, and G. Lu, "Network-based quantized control for fuzzy singularly perturbed semi-Markov jump systems and its application," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 66, no. 3, pp. 1130–1140, Mar. 2019.

[8] C. Peng, Q.-L. Han, and D. Yue, "To transmit or not to transmit: A discrete event-triggered communication scheme for networked Takagi–Sugeno fuzzy systems," *IEEE Trans. Fuzzy Syst.*, vol. 21, no. 1, pp. 164–170, Feb. 2013.

[9] H. Zhang and D. Liu, *Fuzzy Modeling and Fuzzy Control*. Boston, MA, USA: Birkhäuser, 2006.

[10] Y. Liu, F. Wu, and X. Ban, "Dynamic output feedback control for continuous-time T–S fuzzy systems using fuzzy Lyapunov functions," *IEEE Trans. Fuzzy Syst.*, vol. 25, no. 5, pp. 1155–1167, Oct. 2017.

[11] H. Hassani, J. Zarei, M. Chadli, and J. Qiu, "Unknown input observer design for interval type-2 T–S fuzzy systems with immeasurable premise variables," *IEEE Trans. Cybern.*, vol. 47, no. 9, pp. 2639–2650, Sep. 2017.

[12] J. Dong and G.-H. Yang, "Observer-based output feedback control for discrete-time T–S fuzzy systems with partly immeasurable premise variables," *IEEE Trans. Syst.*, vol. 47, no. 1, pp. 98–110, Jan. 2017.

[13] E. S. Tognetti and T. M. Linhares, "Dynamic output feedback controller design for uncertain Takagi–Sugeno fuzzy systems: A premise variable selection approach," *IEEE Trans. Fuzzy Syst.*, early access, Mar. 19, 2020, doi: 10.1109/TFUZZ.2020.2981931.

[14] S. H. Esfahani, "Optimal output feedback H_∞-tracking control design for Takagi and Sugeno fuzzy systems with immeasurable premise variables," *IET Control Theory Appl.*, vol. 7, no. 9, pp. 1300–1308, 2013.

[15] B. Jiang, H. R. Karimi, Y. Kao, and C. Gao, "Adaptive control of nonlinear semi-Markovian jump T–S fuzzy systems with immeasurable premise variables via sliding mode observer," *IEEE Trans. Cybern.*, vol. 50, no. 2, pp. 810–820, Feb. 2020.

[16] M. H. Asemani and V. J. Majd, "A robust H_∞ Non-PDC design scheme for singularly perturbed T–S fuzzy systems with immeasurable state variables," *IEEE Trans. Fuzzy Syst.*, vol. 23, no. 3, pp. 525–541, Jun. 2015.

[17] S. Tong and Y. Li, "Observer-based adaptive fuzzy backstepping control of uncertain nonlinear pure-feedback systems," *Sci. China Inf. Sci.*, vol. 57, no. 1, pp. 1–14, 2014.

[18] S. Tong and Y. Li, "Robust adaptive fuzzy backstepping output feedback tracking control for nonlinear system with dynamic uncertainties," *Sci. China Inf. Sci.*, vol. 53, no. 2, pp. 307–324, 2010.

[19] X. Zhao, P. Shi, and X. Zheng, "Fuzzy adaptive control design and discretization for a class of nonlinear uncertain systems," *IEEE Trans. Cybern.*, vol. 46, no. 6, pp. 1476–1483, Jun. 2016.

[20] M. Wang, J. Qiu, and G. Feng, "A novel piecewise affine filtering design for T–S fuzzy affine systems using past output measurements," *IEEE Trans. Cybern.*, vol. 50, no. 4, pp. 1509–1518, Apr. 2020.

[21] W. Ji, J. Qiu, and H. R. Karimi, "Fuzzy-model-based output feedback sliding mode control for discrete-time uncertain nonlinear systems," *IEEE Trans. Fuzzy Syst.*, vol. 28, no. 8, pp. 1519–1530, Aug. 2020.

[22] M. Xue, Y. Tang, L. Wu, W. Zhong, and F. Qian, "Switching stabilization for type-2 fuzzy systems with network-induced packet losses," *IEEE Trans. Cybern.*, vol. 49, no. 7, pp. 2591–2604, Jul. 2019.

[23] Y. Shen, W. Pedrycz, and X. Wang, "Approximation of fuzzy sets by interval type-2 trapezoidal fuzzy sets," *IEEE Trans. Cybern.*, vol. 50, no. 11, pp. 4722–4734, Nov. 2020.

[24] B. Chakraborty, L. Ghosh, and A. Konar, "Optimal selection of EEG electrodes using interval type-2 fuzzy-logic-based semiseparating signaling game," *IEEE Trans. Cybern.*, early access, Feb. 20, 2020, doi: 10.1109/TCYB.2020.2968625.

[25] Y. Zeng, H.-K. Lam, and L. Wu, "Hankel-norm-based model reduction for stochastic discrete-time nonlinear systems in interval type-2 T–S fuzzy framework," *IEEE Trans. Cybern.*, early access, Jan. 17, 2020, doi: 10.1109/TCYB.2019.2950565.

[26] C.-F. Juang and C.-Y. Chen, "Data-driven interval type-2 neural fuzzy system with high learning accuracy and improved model interpretability," *IEEE Trans. Cybern.*, vol. 43, no. 6, pp. 1781–1795, Dec. 2013.

[27] X. Sun and Q. Zhang, "Admissibility analysis for interval type-2 fuzzy descriptor systems based on sliding mode control," *IEEE Trans. Cybern.*, vol. 49, no. 8, pp. 3032–3040, Aug. 2019.

[28] H.-K. Lam and L. D. Seneviratne, "Stability analysis of interval type-2 fuzzy-model-based control systems," *IEEE Trans. Syst. Man, Cybern. B, Cybern.*, vol. 38, no. 3, pp. 617–628, Jun. 2008.

[29] Y.-E. Wang, X.-M. Sun, P. Shi, and J. Zhao, "Input-to-state stability of switched nonlinear systems with time delays under asynchronous switching," *IEEE Trans. Cybern.*, vol. 43, no. 6, pp. 2261–2265, Dec. 2013.

[30] Y. Li, S. Sui, and S. Tong, "Adaptive fuzzy control design for stochastic nonlinear switched systems with arbitrary switchings and unmodeled dynamics," *IEEE Trans. Cybern.*, vol. 47, no. 2, pp. 403–414, Feb. 2017.

[31] S. Kuppusamy and Y. H. Joo, "Memory-based integral sliding-mode control for T–S fuzzy systems with PMSM via disturbance observer," *IEEE Trans. Cybern.*, early access, Nov. 28, 2019, doi: 10.1109/TCYB.2019.2953567.

[32] K. Liu, A. Selivanov, and E. Fridman, "Survey on time-delay approach to networked control," *Annu. Rev. Control*, vol. 48, pp. 57–79, Jul. 2019.

[33] J. Lian, P. Shi, and Z. Feng, "Passivity and passification for a class of uncertain switched stochastic time-delay systems," *IEEE Trans. Cybern.*, vol. 43, no. 1, pp. 3–13, Feb. 2013.

[34] H. Shen, Z. Huang, J. Cao, and J. H. Park, "Exponential $\mathcal{H}_\infty$ filtering for continuous-time switched neural networks under persistent dwell-time switching regularity," *IEEE Trans. Cybern.*, vol. 50, no. 6, pp. 2440–2449, Jun. 2020.

[35] Z. Fei, S. Shi, C. Zhao, and L. Wu, "Asynchronous control for 2-D switched systems with mode-dependent average dwell time," *Automatica*, vol. 79, pp. 198–206, May 2017.

[36] H. Furuichi, J. Huang, T. Fukuda, and T. Matsuno, "Switching dynamic modeling and driving stability analysis of three-wheeled narrow tilting vehicle," *IEEE/ASME Trans. Mechatron.*, vol. 19, no. 4, pp. 1309–1322, Aug. 2014.

[37] J. P. Hespanha, "Uniform stability of switched linear systems: Extensions of LaSalle's invariance principle," *IEEE Trans. Autom. Control*, vol. 49, no. 4, pp. 470–482, Apr. 2004.

[38] Z. Fei, S. Shi, Z. Wang, and L. Wu, "Quasi-time-dependent output control for discrete-time switched system with mode-dependent average dwell time," *IEEE Trans. Autom. Control*, vol. 63, no. 8, pp. 2647–2653, Aug. 2018.

[39] Y. Zhu, W. X. Zheng, and D. Zhou, "Quasi-synchronization of discrete-time Lur'e-type switched systems with parameter mismatches and relaxed PDT constraints," *IEEE Trans. Cybern.*, vol. 50, no. 5, pp. 2026–2037, May 2020.

[40] S. Shi, Z. Fei, T. Wang, and Y. Xu, "Filtering for switched T–S fuzzy systems with persistent dwell time," *IEEE Trans. Cybern.*, vol. 49, no. 5, pp. 1923–1931, May 2019.

[41] X. Ma, Q. Wang, J. Cheng, Y. Guo, C. Yang, and L. Zhou, "Controller design for T–S fuzzy singularly perturbed switched systems," in *Proc. IEEE Int. Conf. Fuzzy Syst.*, 2016, pp. 1159–1166.

[42] H. Shen, M. Xing, Z.-G. Wu, S. Xu, and J. Cao, "Multi-objective fault-tolerant control for fuzzy switched systems with persistent dwell-time and its application in electric circuits," *IEEE Trans. Fuzzy Syst.*, vol. 28, no. 10, pp. 2335–2347, Oct. 2020.

[43] N. Elia and S. K. Mitter, "Stabilization of linear systems with limited information," *IEEE Trans. Autom. Control*, vol. 46, no. 9, pp. 1384–1400, Sep. 2001.

[44] M. Fu and L. Xie, "The sector bound approach to quantized feedback control," *IEEE Trans. Autom. Control*, vol. 50, no. 11, pp. 1698–1711, Nov. 2005.

[45] Y. Gao, H. Li, L. Wu, H. R. Karimi, and H.-K. Lam, "Optimal control of discrete-time interval type-2 fuzzy-model-based systems with D-stability constraint and control saturation," *Signal Process.*, vol. 120, pp. 409–421, Mar. 2016.

[46] H. Li, L. Wu, H.-K. Lam, and Y. Gao, *Analysis and Synthesis for Interval Type-2 Fuzzy-Model-Based Systems*. Singapore: Springer, 2016.

[47] X. Wan, Z. Wang, M. Wu, and X. Liu, "H_∞ state estimation for discrete-time nonlinear singularly perturbed complex networks under the round-robin protocol," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 30, no. 2, pp. 415–426, Feb. 2019.

[48] H. Zhou, H. Ying, and C. Zhang, "Effects of increasing the footprints of uncertainty on analytical structure of the classes of interval type-2 Mamdani and TS fuzzy controllers," *IEEE Trans. Fuzzy Syst.*, vol. 27, no. 9, pp. 1881–1890, Sep. 2019.

[49] N. Sahab and H. Hagras, "Adaptive non-singleton type-2 fuzzy logic systems: A way forward for handling numerical uncertainties in real world applications," *Int. J. Comput. Commun. Control*, vol. 6, no. 3, pp. 503–529, 2011.

[50] M. A. Khanesar, E. Kayacan, M. Reyhanoglu, and O. Kaynak, "Feedback error learning control of magnetic satellites using type-2 fuzzy neural networks with elliptic membership functions," *IEEE Trans. Cybern.*, vol. 45, no. 4, pp. 858–868, Apr. 2015.

[51] Y.-H. Chang and W.-S. Chan, "Adaptive dynamic surface control for uncertain nonlinear systems with interval type-2 fuzzy neural networks," *IEEE Trans. Cybern.*, vol. 44, no. 2, pp. 293–304, Feb. 2014.

[52] C. L. P. Chen, Y.-J. Liu, and G.-X. Wen, "Fuzzy neural network-based adaptive control for a class of uncertain nonlinear stochastic systems," *IEEE Trans. Cybern.*, vol. 44, no. 5, pp. 583–593, May 2014.

[53] C. P. Chen, C.-E. Ren, and T. Du, "Fuzzy observed-based adaptive consensus tracking control for second-order multiagent systems with heterogeneous nonlinear dynamics," *IEEE Trans. Fuzzy Syst.*, vol. 24, no. 4, pp. 906–915, Aug. 2016.

[54] N. Wang, S.-F. Su, J. Yin, Z. Zheng, and M. J. Er, "Global asymptotic model-free trajectory-independent tracking control of an uncertain marine vehicle: An adaptive universe-based fuzzy control approach," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 3, pp. 1613–1625, Jun. 2018.

[55] S.-M. Chen and D. Barman, "Adaptive weighted fuzzy interpolative reasoning based on representative values and similarity measures of interval type-2 fuzzy sets," *Inf. Sci.*, vol. 478, pp. 167–185, Apr. 2019.

[56] C.-M. Lin, R. Ramarao, and S. H. Gopalai, "Self-organizing adaptive fuzzy brain emotional learning control for nonlinear systems," *Int. J. Fuzzy. Syst.*, vol. 21, no. 7, pp. 1989–2007, 2019.

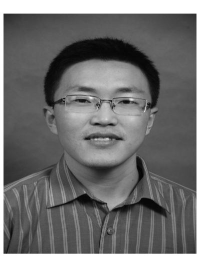
[57] J. Qiu, K. Sun, T. Wang, and H. Gao, "Observer-based fuzzy adaptive event-triggered control for pure-feedback nonlinear systems with prescribed performance," *IEEE Trans. Fuzzy Syst.*, vol. 27, no. 11, pp. 2152–2162, Nov. 2019.

Jing Wang received the Ph.D. degree in power systems and automation from Hohai University, Nanjing, China, in 2019.

She is currently an Associate Professor with the School of Electrical and Information Engineering, Anhui University of Technology, Ma'anshan, China. Her current research interests include Markov jump nonlinear systems, singularly perturbed systems, power systems, and nonlinear control.

Xinmiao Liu received the B.Sc. degree in electrical engineering from North China Electric Power University, Beijing, China, in 2017. She is currently pursuing the M.S. degree with the School of Electrical and Information Engineering, Anhui University of Technology, Ma'anshan, China.

Her current research interests include switched systems, fuzzy systems, and singularly perturbed systems robust control.



Jianwei Xia (Member, IEEE) received the Ph.D. degree in automatic control from the Nanjing University of Science and Technology, Nanjing, China, in 2007.

From 2010 to 2012, he worked as a Postdoctoral Research Associate with the School of Automation, Southeast University, Nanjing. From 2013 to 2014, he was a Postdoctoral Research Associate with the Department of Electrical Engineering, Yeungnam University, Kyongsan, South Korea. He is currently a Professor with the School of Mathematics Science, Liaocheng University, Liaocheng, China. His research topics are robust control, stochastic systems, delay systems, and neural networks.

Hao Shen (Member, IEEE) received the Ph.D. degree in control theory and control engineering from the Nanjing University of Science and Technology, Nanjing, China, in 2011.

From 2013 to 2014, he was a Postdoctoral Fellow with the Department of Electrical Engineering, Yeungnam University, Kyongsan, South Korea. Since 2011, he has been with the Anhui University of Technology, Ma'anshan, China, where he is currently a Professor. He is a Postdoctoral Fellow with the School of Mathematics, Southeast University, Nanjing. His current research interests include stochastic hybrid systems, complex networks, fuzzy systems and control, and nonlinear control.

Prof. Shen was a recipient of the Highly Cited Researcher Award by Clarivate Analytics (formerly, Thomson Reuters) in 2019 and 2020. He has served on the technical program committee for several international conferences. He is an Associate Editor/Guest Editor for several international journals, including *Journal of The Franklin Institute*, *Applied Mathematics and Computation*, IEEE ACCESS, *Neural Processing Letters*, and *Transactions of the Institute Measurement and Control*.

Ju H. Park (Senior Member, IEEE) received the Ph.D. degree in electronics and electrical engineering from the Pohang University of Science and Technology (POSTECH), Pohang, South Korea, in 1997.

From 1997 to 2000, he was a Research Associate with ERC-ARC, POSTECH. In 2000, he joined Yeungnam University, Kyongsan, South Korea, where he is currently the Chuma Chair Professor. From 2006 to 2007, he was a Visiting Professor with the Department of Mechanical Engineering, Georgia Institute of Technology, Atlanta, GA, USA. He has coauthored the monographs *Recent Advances in Control and Filtering of Dynamic Systems With Constrained Signals* (New York, NY, USA: Springer-Nature, 2018) and *Dynamic Systems With Time Delays: Stability and Control* (New York, NY, USA: Springer-Nature, 2019), and is an Editor of an edited volume *Recent Advances in Control Problems of Dynamical Systems and Networks* (New York: Springer-Nature, 2020). His research interests include robust control and filtering, neural/complex networks, fuzzy systems, multiagent systems, and chaotic systems. He has published a number of papers in the above areas.

Prof. Park has been a recipient of Highly Cited Researcher Award listed by Clarivate Analytics (formerly, Thomson Reuters) since 2015, and listed in three fields, engineering, computer sciences, and mathematics, in 2019 and 2020. He serves as an Editor for *International Journal of Control, Automation, and Systems*. He is also an Subject Editor/Associate Editor/Editorial Board Member for several international journals, including IEEE TRANSACTIONS ON FUZZY SYSTEMS, IEEE TRANSACTIONS ON NEURAL NETWORKS AND LEARNING SYSTEMS, and IEEE TRANSACTIONS ON CYBERNETICS, *IET Control Theory and Applications*, *Applied Mathematics and Computation*, *Journal of The Franklin Institute*, *Nonlinear Dynamics*, *Journal of Applied Mathematics and Computing*, and *Cogent Engineering*. He is a Fellow of the Korean Academy of Science and Technology.