Fuzzy SMC for Quantized Nonlinear Stochastic Switching Systems With Semi-Markovian Process and Application

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***Abstract*—This article is concerned with the issue of quantized sliding-mode control (SMC) design methodology for nonlinear stochastic switching systems subject to semi-Markovian switching parameters, T–S fuzzy strategy, uncertainty, signal quantization, and nonlinearity. Compared with the previous literature, the quantized control input is first considered in studying T–S fuzzy stochastic switching systems with a semi-Markovian process. A mode-independent sliding surface is adopted to avoid the potential repetitive jumping effects. Then, by means of the Lyapunov function, stochastic stability criteria are proposed to be dependent of sojourn time for the corresponding sliding-mode dynamics. Furthermore, the fuzzy-model-based SMC law is proposed to ensure the finite-time reachability of the sliding-mode dynamics. Finally, an application example of a modified series dc motor model is provided to demonstrate the effectiveness of the theoretical findings.**

***Index Terms*—Semi-Markovian process (SMP), semi-Markovian switching parameters, signal quantization, T–S fuzzy strategy.**

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I. Introduction

It is well known that most of the practical systems are characterized by nonlinear models, which may lead to serious difficulties in stability analysis and control synthesis. In such a case, control for nonlinear systems faces many intractable problems and the T–S fuzzy model introduces an efficient way to represent complex nonlinear systems by fuzzy blending [1]. By the use of the T–S fuzzy model, nonlinear systems can be regarded as a convex combination of some linear subsystems through the membership functions. Based on this approach, many well-established results for linear systems, such as linear matrix inequalities, can be extended to deal with nonlinear systems. Due to its great importance, the T–S fuzzy model finds a wide utilization in electrical systems, mass–spring systems, and mechanical systems. During the past years, numerous excellent results have been presented for T–S fuzzy systems [2]–[7].

Besides, stochastic switching systems are known as a class of hybrid systems [8]–[10] that may experience some structural and parametrical changes [11]–[13]. In practical applications, this special kind of system can be found in economic systems, flight systems, power systems, communication networks, etc., and contains multimodal subsystems and mode evolution governed by the stochastic switching law. To model the switching behaviors, the Markovian process has been widely adopted and accordingly Markovian switching systems (MSSs) have attracted considerable attention as a special class of stochastic switching system (see [14]–[21]). In fact, the transition rate (TR) in MSSs is independent of the history information of the past switching sequence, which leads to memoryless TR. However, the Markovian process may not be suitable in many practical applications and the corresponding theories cannot solve complex stochastic switching systems. In order to relax this strict restriction, the concept of semi-MSSs (S-MSSs) has been introduced into the control field [22]–[24]. It should be noted that the probability distribution of sojourn time (ST) in continuous-time S-MSSs is not confined to an exponential distribution, which leads to the "memory" property of the TR. Over the past few years, lots of remarkable results on the stability analysis and control issues have been proposed for S-MSSs [25]–[34] and some results are on fuzzy S-MSSs systems [35]–[38].

As one robust control strategy, it is noted that the sliding-mode control (SMC) law drives the state signals onto an artificially specified hypersurface (i.e., the sliding-mode surface) and ensures that the origin of the state space is the desired asymptotically stable equilibrium point with the help of suitable controller design conditions [39]–[45]. In a finite-time interval, the convergence of the sliding-mode surface can be guaranteed when a large enough control signal is designed to suppress the adverse effects of the uncertainty and nonlinearity. When the evolutions of the state variables reach the expected sliding manifold, they become insensitive to the uncertainties and nonlinearities. Very recently, quite a few applications of the SMC approach for T–S fuzzy systems and stochastic jump systems have been witnessed; for details, see [46]–[48].

At the same time, with the rapid development of digital computers and digital communication facilities, the issue of control systems over quantized measurements has attracted increasing attention from the research field. In practical digital network communication systems, the output signals of the controller must be quantized to be transmitted to the plant via a finite rate network. The quantization error seriously affects the stability, accuracy, and reliability of control systems, which has become one of the most challenging projects in modern control engineering [49], [50]. In order to save system resources and achieve desired goals, it is necessary to consider the impact of quantization error. Recently, the dynamical behaviors of quantized control systems have attracted a great amount of attention and many significant results have been reported (see [29] and [51]–[55]).

Furthermore, for stochastic switching systems, there are still some obvious limitations. One typical constraint is that the ST in the Markovian process follows the memoryless exponential distribution [7], [14]–[21], [39]–[42], [53], [55] whereas many dynamical systems do not always meet the rigorous requirement. Next, the controller design is based on the tacit assumption that the quantized constraint is not considered in [25]–[35]. For practical network communication systems, the controller signals must be quantized to be transmitted to the plant. In such a case, the influence of quantization error cannot be ignored. If the quantization error is neglected, it is impossible to achieve the desired performance and even makes the system unstable. In addition, although fuzzy SMC [46]–[48]; quantization [33], [51]–[55]; and S-MSSs [25]–[38] have been extensively studied, due to the existence of the semi-Markovian process (SMP), uncertainty, quantized constraint, and nonlinearity, it comes with challenges to investigate this kind of system. Moreover, many factors, such as SMP, uncertainty, quantized constraint, and nonlinearity, play an important part in describing practical complex stochastic switching systems. Therefore, a critical issue about quantized SMC design methodology for nonlinear S-MSSs is whether there exists a fuzzy SMC law to suppress the influences of uncertainty, nonlinear term, and quantization error. However, up to now, there are no theoretical results, which motivate our study. Especially, there are two innovations to be addressed during the quantized SMC design.

Q1: In comparison with exponential distribution [7], [14]–[21], [39]–[42], [53], [55], how to obtain the weak infinitesimal operator in the presence of complex stochastic SMP?

Q2: How to design an updated SMC law in order to ensure the finite-time attractiveness of the sliding surface?

In this article, we aim to put forward the T–S fuzzy method to describe the quantized nonlinear S-MSSs. And then, a fuzzy SMC law is designed to obtain better performance of dynamical systems. The main contributions are highlighted as follows.

1) In contrast with [7], [14]–[21], [39]–[42], [53], and [55], one unrealistic assumption, that is, the ST in stochastic switching systems is subject to an exponential distribution, is removed, which are more suitable to describe practical systems in the presence of a sudden change of the parameters or structures. A mode-independent sliding surface is proposed to avoid the potential repetitive jumping effects.
2) By the semi-Markovian Lyapunov function and logarithmic quantizer, it is our first attempt to construct ST-dependent sufficient conditions for stochastic stability in standard linear matrix inequalities.
3) Furthermore, by defining a bounded area around the sliding surface, the SMC law depending on the quantizer level is constructed to drive the state responses onto the sliding surface within a finite-time region.

Notations:

1) $\Im$*:* Weak infinitesimal operator.
2) $\mathcal{V}_\sigma(\ell)$*:* Cumulative distribution functions of ST when the system remains in σ.
3) $\lambda_{\sigma\rho}$*:* Probability intensity from σ to ρ.
4) $\chi_\sigma(\ell)$*:* TR of system jump from σ.

II. Preliminaries

Consider the nonlinear T–S fuzzy S-MSSs as follows.

Plant Rule θ*:* IF $\zeta_1(t)$ is $\mathcal{M}_1^\theta$, $\zeta_2(t)$ is $\mathcal{M}_2^\theta$, and $\cdots$ and $\zeta_l(t)$ is $\mathcal{M}_l^\theta$, THEN

$$\dot{z}(t) = (\mathcal{A}_\theta(\delta_t) + \Delta\mathcal{A}_\theta(\delta_t))z(t) + \mathcal{B}_\theta(\delta_t)(u(t) + f_\theta(t, \delta_t, z(t))) \tag{1}$$

where $z(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$ are the state vector and input vector. $\mathcal{M}_{j_1}^\theta$ ($\theta = 1, 2, \ldots, r$, $j_1 = 1, 2, \ldots, l$), $\zeta_1(t)$, $\zeta_2(t)$, $\ldots$, $\zeta_l(t)$, and l are fuzzy sets, premise variables, and the number of premise variables. $\{\delta_t, t \geq 0\}$ means the SMP in $\Theta = \{1, 2, \ldots, \wp\}$ with probability transitions

$$\Pr\{\delta_{t+\bar{\Delta}} = \rho | \delta_t = \sigma\} = \begin{cases} \chi_{\sigma\rho}(\ell)\bar{\Delta} + o(\bar{\Delta}), & \sigma \neq \rho \\ 1 + \chi_{\sigma\sigma}(\ell)\bar{\Delta} + o(\bar{\Delta}), & \sigma = \rho \end{cases}$$

where ℓ means the ST, $\chi_{\sigma\rho}(\ell) \geq 0$ stands for the TR from σ to ρ for $\sigma \neq \rho$, and $\sum_{\rho=1,\rho\neq\sigma}^{\wp} \chi_{\sigma\rho}(\ell) = -\chi_{\sigma\sigma}(\ell)$. The TR is given as $\underline{\chi}_{\sigma\rho} \leq \chi_{\sigma\rho}(\ell) \leq \bar{\chi}_{\sigma\rho}$ with real constant scalars $\underline{\chi}_{\sigma\rho}$ and $\bar{\chi}_{\sigma\rho}$.

For $\delta_t = \sigma \in \Theta$, $\mathcal{A}_\theta(\delta_t)$, $\Delta\mathcal{A}_\theta(\delta_t)$, $\mathcal{B}_\theta(\delta_t)$, and $f_\theta(t, \delta_t, z(t))$ are, respectively, denoted as $\mathcal{A}_{\theta\sigma}$, $\Delta\mathcal{A}_{\theta\sigma}(t)$, $\mathcal{B}_{\theta\sigma}$, and $f_{\theta\sigma}(t, z(t))$. Here, it is assumed that the input matrices $\mathcal{B}_{\theta\sigma}$ ($\theta = 1, 2, \ldots, r$, $\sigma = 1, 2, \ldots, \wp$) satisfy $\mathcal{B}_{1\sigma} = \mathcal{B}_{2\sigma} =$

$\cdots = \mathcal{B}_{r\sigma} = \mathcal{B}_\sigma$. The nonlinear function $f_{\theta\sigma}(t, z(t))$ and the uncertainty $\Delta\mathcal{A}_{\theta\sigma}$ are unknown and satisfy

$$\|f_{\theta\sigma}(t, z(t))\| \leq \rho_{\theta\sigma}\|z(t)\| \tag{2}$$

$$\Delta\mathcal{A}_{\theta\sigma}(t) = \mathcal{M}_{\theta\sigma}\mathcal{F}_{\theta\sigma}(t)\mathcal{N}_{\theta\sigma} \tag{3}$$

where $\rho_{\theta\sigma}$ denotes known scalar, $\mathcal{M}_{\theta\sigma}$ and $\mathcal{N}_{\theta\sigma}$ stand for known matrices, and $\mathcal{F}_{\theta\sigma}(t)$ satisfies $\mathcal{F}^T_{\theta\sigma}(t)\mathcal{F}_{\theta\sigma}(t) \leq \mathcal{I}$.

Therefore, by fuzzy blending, the overall fuzzy model is deduced as

$$\dot{z}(t) = \sum_{\theta=1}^{r} p_\theta(\zeta(t))\big[(\mathcal{A}_{\theta\sigma} + \Delta\mathcal{A}_{\theta\sigma}(t))z(t) + \mathcal{B}_\sigma \times (u(t) + f_{\theta\sigma}(t, z(t)))\big] \tag{4}$$

where $\zeta(t) = \begin{bmatrix}\zeta_1(t) & \zeta_2(t) & \cdots & \zeta_l(t)\end{bmatrix}^T$, and $p_\theta(\zeta(t))$ is the membership function given as

$$p_\theta(\zeta(t)) = \frac{\prod_{j_1=1}^{l}\mathcal{M}^\theta_{j_1}\big(\zeta_{j_1}(t)\big)}{\sum_{\theta=1}^{r}\prod_{j_1=1}^{l}\mathcal{M}^\theta_{j_1}\big(\zeta_{j_1}(t)\big)} \tag{5}$$

$\mathcal{M}^\theta_{j_1}(\zeta_{j_1}(t)) \in [0, 1]$ represents the grade of the membership of $\zeta_{j_1}(t)$ in $\mathcal{M}^\theta_{j_1}$. In fact, since $\mathcal{M}^\theta_{j_1}(\zeta_{j_1}(t)) \geq 0$, one has

$$\sum_{\theta=1}^{r} p_\theta(\zeta(t)) = 1, p_\theta(\zeta(t)) \geq 0. \tag{6}$$

Before entering the plant, the control signal $u(t)$ is quantized via a logarithmic quantizer given as

$$\mathcal{Q}(\cdot) = \begin{bmatrix}\mathcal{Q}_1(\cdot) & \mathcal{Q}_2(\cdot) & \cdots & \mathcal{Q}_m(\cdot)\end{bmatrix} \tag{7}$$

where $\mathcal{Q}(\cdot)$ is assumed to be symmetric, that is

$$\mathcal{Q}_\varpi(-u_\varpi(t)) = -\mathcal{Q}_\varpi(u_\varpi(t)),\ 1 \leq \varpi \leq m. \tag{8}$$

The set of quantized levels of $\mathcal{Q}_\varpi(\cdot)$ takes

$$\Phi_\varpi = \Big\{\pm\vartheta^{(\iota)}_\varpi, |\vartheta^{(\iota)}_\varpi = (\phi_\varpi)^\iota\vartheta^{(0)}_\varpi, \iota = \pm1, \pm2, \ldots\Big\} \bigcup\Big\{\pm\vartheta^{(0)}_\varpi\Big\}\bigcup\{0\},\ 0 < \phi_\varpi < 1,\ \vartheta^{(0)}_\varpi > 0 \tag{9}$$

where ϕ_ϖ and $\vartheta^{(0)}_\varpi$ stand for the quantizer density and the initial quantization values of the subquantizer $\mathcal{Q}_\varpi(\cdot)$ given as

$$\mathcal{Q}_\varpi(u_\varpi(t)) = \begin{cases}\vartheta^{(\iota)}_\varpi, & \text{if } \frac{\vartheta^{(\iota)}_\varpi}{1+\lambda_\varpi} < u_\varpi(t) < \frac{\vartheta^{(\iota)}_\varpi}{1-\lambda_\varpi}\\ 0, & \text{if } u_\varpi(t) = 0\\ -\mathcal{Q}_\varpi(-u_\varpi(t)), & \text{if } u_\varpi(t) < 0\end{cases} \tag{10}$$

with $\bar{\lambda} = \max_{1\leq\varpi\leq m}\{\lambda_\varpi\}$, $\lambda_\varpi = ([1-\phi_\varpi]/[1+\phi_\varpi])$, $1 \leq \varpi \leq m$, $\iota = \pm1, \pm2, \ldots,$, which means that $0 < \lambda_\varpi < 1$, $0 < \bar{\lambda} < 1$.

Then, one has

$$\mathcal{Q}(u(t)) = (\mathcal{I} + \Lambda)u(t) \tag{11}$$

where $\Lambda = \mathrm{diag}\{\Lambda_1, \Lambda_2, \ldots, \Lambda_m\}$ and $\Lambda_\varpi \in [-\lambda_\varpi, \lambda_\varpi]$, $1 \leq \varpi \leq m$. Also, one has $-1 < \Lambda_\varpi < 1$ and nonsingular matrix $\mathcal{I} + \Lambda$.

Replacing $u(t)$ with $(\mathcal{I} + \Lambda)u(t)$ yields

$$\dot{z}(t) = \sum_{\theta=1}^{r} p_\theta(\zeta(t))\big[(\mathcal{A}_{\theta\sigma} + \Delta\mathcal{A}_{\theta\sigma}(t))z(t) + \mathcal{B}_\sigma((\mathcal{I} + \Lambda)u(t) + f_{\theta\sigma}(t, z(t)))\big]. \tag{12}$$

Definition 1 [13]: System (1) is said to be stochastically stable if for $\delta_0 \in \Theta$ and $z_0 \in \mathbb{R}^n$, $\mathcal{E}\{\int_0^\infty ||z(t)||^2dt|\} < \infty$ holds.

Lemma 1 [33]: For any matrices $\mathscr{D} \in \mathbb{R}^{n\times n_f}$, $\mathscr{E} \in \mathbb{R}^{n\times n_f}$, and $\mathscr{F}(t) \in \mathbb{R}^{n_f\times n_f}$ with $\mathscr{F}^T(t)\mathscr{F}(t) \leq \mathscr{I}$, there holds $\mathscr{D}\mathscr{F}(t)\mathscr{E} + \mathscr{E}^T\mathscr{F}^T(t)\mathscr{D}^T \leq \varepsilon\mathscr{D}\mathscr{D}^T + \varepsilon^{-1}\mathscr{E}^T\mathscr{E}$, where ε is any positive scalar.

III. Main Results

A. SMC Law Design

It is noted that many results have been reported for SMC of MSSs, while the proposed sliding surface is mostly mode-dependent. Since the repetitive jumps exist in the sliding surface, it will cause potential instability of sliding-mode motion. In order to avoid this problem, a mode-independent sliding surface is adopted as follows:

$$s(t) = \mathcal{D}z(t) \tag{13}$$

where $\mathcal{D}$ is a sum weighted matrix of $\mathcal{B}_\sigma$ with $\mathcal{D} \triangleq \sum_{\rho=1}^{\wp} h_\rho\mathcal{B}^T_\rho$ defined in [6].

Next, the SMC law is designed as

$$u(t) = \mathcal{B}^T_\sigma\mathcal{P}_\sigma z(t) - \eta_\sigma\,\mathrm{sgn}\big((\mathcal{X}_\sigma\mathcal{D}\mathcal{B}_\sigma)^T s(t)\big) \tag{14}$$

where nonsingular matrix $\mathcal{P}_\sigma$, matrix $\mathcal{X}_\sigma$, and positive scalar η_σ will be given later.

Substituting the equivalent controller (14) into (12) yields

$$\dot{z}(t) = \sum_{\theta=1}^{r} p_\theta(\zeta(t))\Big[\big(\mathcal{A}_{\theta\sigma} + \Delta\mathcal{A}_{\theta\sigma}(t) + \mathcal{B}_\sigma(\mathcal{I}+\Lambda)\mathcal{B}^T_\sigma\mathcal{P}_\sigma\big)z(t) + \mathcal{B}_\sigma\big(-(\mathcal{I}+\Lambda)\eta_\sigma\,\mathrm{sgn}\big((\mathcal{X}_\sigma\mathcal{D}\mathcal{B}_\sigma)^T s(t)\big) + f_{\theta\sigma}(t, z(t))\big)\Big]. \tag{15}$$

B. Stochastic Stability Analysis

Stochastic stability criteria will be proposed in Theorem 1. By the Lyapunov function and probability theory, sufficient conditions are constructed for the corresponding system with quantization.

Theorem 1: If there exist symmetric matrix $\mathcal{P}_\sigma > 0$, matrix $\mathcal{X}_\sigma$, and scalars $\varepsilon_{1\theta\sigma} > 0$, $\varepsilon_{2\theta\sigma} > 0\ \forall\sigma \in \Theta$, $\theta = 1, 2, \ldots, r$, such that

$$\mathcal{P}_\sigma\mathcal{B}_\sigma = \mathcal{D}^T\mathcal{X}_\sigma\mathcal{D}\mathcal{B}_\sigma \tag{16}$$

$$\Pi^\sigma_{1\theta} < 0 \tag{17}$$

where

$$\Pi^\sigma_{1\theta} = \begin{bmatrix}\Pi^\sigma_{11\theta} & \Pi^\sigma_{21\theta}\\ * & \Pi^\sigma_{31\theta}\end{bmatrix}$$

$$\Pi^\sigma_{11\theta} = \mathcal{P}_\sigma\mathcal{A}_{\theta\sigma} + \mathcal{A}^T_{\theta\sigma}\mathcal{P}_\sigma + \sum_{\rho=1}^{\wp}\chi_{\sigma\rho}(\ell)\mathcal{P}_\rho$$

$$\Pi_{21\theta}^{\sigma} = \left[\mathcal{P}_\sigma\mathcal{B}_\sigma, \mathcal{P}_\sigma\mathcal{M}_{\theta\sigma}, \varepsilon_{1\theta\sigma}\mathcal{N}_{\theta\sigma}^T, \mathcal{P}_\sigma\mathcal{B}_\sigma, \varepsilon_{2\theta\sigma}\rho_{\theta\sigma}\mathcal{I}\right]$$
$$\Pi_{31\theta}^{\sigma} = -\mathrm{diag}\left\{\frac{1}{2}\left(\mathcal{I}+\bar{\lambda}\mathcal{I}\right)^{-1}, \varepsilon_{1\theta\sigma}\mathcal{I}, \varepsilon_{1\theta\sigma}\mathcal{I}, \varepsilon_{2\theta\sigma}\mathcal{I}, \varepsilon_{2\theta\sigma}\mathcal{I}\right\}$$

then system (15) achieves robust stochastic stability.

Proof: For the Lyapunov function

$$\mathcal{S}_1(z(t),\sigma) = z^T(t)\mathcal{P}_\sigma z(t) \tag{18}$$

one has

$$\begin{aligned}
&\Im\mathcal{S}_1(z(t),\sigma)\\
&= \lim_{\bar{\Delta}\to 0}\frac{1}{\bar{\Delta}}\left[\mathcal{E}\{\mathcal{S}\left(z\left(t+\bar{\Delta}\right),\delta_{t+\bar{\Delta}}\right)|\delta_t=\sigma\} - \mathcal{S}_1(z(t),\sigma)\right]\\
&= \lim_{\bar{\Delta}\to 0}\frac{1}{\bar{\Delta}}\Bigg[\sum_{\rho=1,\rho\neq\sigma}^{\wp}\Pr\{\delta_{t+\bar{\Delta}}=\rho|\delta_t=\sigma\}z^T\left(t+\bar{\Delta}\right)\mathcal{P}_\rho\\
&\qquad\times z\left(t+\bar{\Delta}\right) + \Pr\{\delta_{t+\bar{\Delta}}=\sigma|\delta_t=\sigma\}z^T\\
&\qquad\times \left(t+\bar{\Delta}\right)\mathcal{P}_\sigma z\left(t+\bar{\Delta}\right) - z^T(t)\mathcal{P}_\sigma z(t)\Bigg]\\
&= \lim_{\bar{\Delta}\to 0}\frac{1}{\bar{\Delta}}\Bigg[\sum_{\rho=1,\rho\neq\sigma}^{\wp}\frac{\Pr\{\delta_{t+\bar{\Delta}}=\rho,\delta_t=\sigma\}}{\Pr\{\delta_t=\sigma\}}z^T\left(t+\bar{\Delta}\right)\mathcal{P}_\rho\\
&\qquad\times z\left(t+\bar{\Delta}\right) + \frac{\Pr\{\delta_{t+\bar{\Delta}}=\sigma,\delta_t=\sigma\}}{\Pr\{\delta_t=\sigma\}}z^T\\
&\qquad\times \left(t+\bar{\Delta}\right)\mathcal{P}_\sigma z\left(t+\bar{\Delta}\right) - z^T(t)\mathcal{P}_\sigma z(t)\Bigg]\\
&= \lim_{\bar{\Delta}\to 0}\frac{1}{\bar{\Delta}}\Bigg[\sum_{\rho=1,\rho\neq\sigma}^{\wp}\frac{\lambda_{\sigma\rho}\left(\mathcal{V}_\sigma\left(\ell+\bar{\Delta}\right)-\mathcal{V}_\sigma(\ell)\right)}{1-\mathcal{V}_\sigma(\ell)}z^T\left(t+\bar{\Delta}\right)\\
&\qquad\times \mathcal{P}_\rho z\left(t+\bar{\Delta}\right) + \frac{1-\mathcal{V}_\sigma\left(\ell+\bar{\Delta}\right)}{1-\mathcal{V}_\sigma(\ell)}z^T\\
&\qquad\times \left(t+\bar{\Delta}\right)\mathcal{P}_\sigma z\left(t+\bar{\Delta}\right) - z^T(t)\mathcal{P}_\sigma z(t)\Bigg].
\end{aligned}\tag{19}$$

Considering the expansion of the Taylor formula leads to

$$z\left(t+\bar{\Delta}\right) = z(t) + \dot{z}(t)\bar{\Delta} + o\left(\bar{\Delta}\right) \tag{20}$$

where $\bar{\Delta}\to 0$.

Then, one has

$$\begin{aligned}
\Im\mathcal{S}_1(z(t),\sigma) = \lim_{\bar{\Delta}\to 0}\frac{1}{\bar{\Delta}}\Bigg[&\sum_{\rho=1,\rho\neq\sigma}^{\wp}\frac{\lambda_{\sigma\rho}\left(\mathcal{V}_\sigma\left(\ell+\bar{\Delta}\right)-\mathcal{V}_\sigma(\ell)\right)}{1-\mathcal{V}_\sigma(\ell)}\\
&\times\left[z(t)+\dot{z}(t)\bar{\Delta}+o\left(\bar{\Delta}\right)\right]^T\mathcal{P}_\rho\\
&\times\left[z(t)+\dot{z}(t)\bar{\Delta}+o\left(\bar{\Delta}\right)\right]\\
&+\frac{1-\mathcal{V}_\sigma\left(\ell+\bar{\Delta}\right)}{1-\mathcal{V}_\sigma(\ell)}\\
&\times\left[z(t)+\dot{z}(t)\bar{\Delta}+o\left(\bar{\Delta}\right)\right]^T\\
&\times\mathcal{P}_\sigma\left[z(t)+\dot{z}(t)\bar{\Delta}+o\left(\bar{\Delta}\right)\right]\\
&- z^T(t)\mathcal{P}_\sigma z(t)\Bigg].
\end{aligned}\tag{21}$$

From [19], we have

$$\begin{aligned}
&\lim_{\bar{\Delta}\to 0}\frac{1-\mathcal{V}_\sigma\left(\ell+\bar{\Delta}\right)}{1-\mathcal{V}_\sigma(\ell)} = 1,\ \lim_{\bar{\Delta}\to 0}\frac{\mathcal{V}_\sigma(\ell)-\mathcal{V}_\sigma\left(\ell+\bar{\Delta}\right)}{1-\mathcal{V}_\sigma(\ell)} = 0\\
&\lim_{\bar{\Delta}\to 0}\frac{\mathcal{V}_\sigma\left(\ell+\bar{\Delta}\right)-\mathcal{V}_\sigma(\ell)}{\bar{\Delta}(1-\mathcal{V}_\sigma(\ell))} = \chi_\sigma(\ell).
\end{aligned}\tag{22}$$

With $\chi_{\sigma\rho}(\ell) = \lambda_{\sigma\rho}\chi_\rho(\ell), \sigma\neq\rho$, we can obtain

$$\begin{aligned}
&\lim_{\bar{\Delta}\to 0}\frac{1}{\bar{\Delta}}\sum_{\rho=1,\rho\neq\sigma}^{\wp}\frac{\lambda_{\sigma\rho}\left(\mathcal{V}_\sigma\left(\ell+\bar{\Delta}\right)-\mathcal{V}_\sigma(\ell)\right)}{1-\mathcal{V}_\sigma(\ell)}z^T(t)\mathcal{P}_\rho z(t)\\
&= \sum_{\rho=1,\rho\neq\sigma}^{\wp}\lambda_{\sigma\rho}\chi_\sigma(\ell)z^T(t)\mathcal{P}_\rho z(t)\\
&= \sum_{\rho=1,\rho\neq\sigma}^{\wp}\chi_{\sigma\rho}(\ell)z^T(t)\mathcal{P}_\rho z(t).
\end{aligned}\tag{23}$$

According to (19)–(23), we have

$$\begin{aligned}
\mathcal{E}\{\Im\mathcal{S}_1(z(t),\sigma)\} = 2&\sum_{\theta=1}^{r}p_\theta(\zeta(t))z^T(t)\mathcal{P}_\sigma\\
&\times\left[\left(\mathcal{A}_{\theta\sigma}+\Delta\mathcal{A}_{\theta\sigma}(t)+\mathcal{B}_\sigma(\mathcal{I}+\Lambda)\mathcal{B}_\sigma^T\mathcal{P}_\sigma\right)z(t)\right.\\
&\quad+\mathcal{B}_\sigma\left(-(\mathcal{I}+\Lambda)\eta_\sigma\,\mathrm{sgn}\left((\mathcal{X}_\sigma\mathcal{D}\mathcal{B}_\sigma)^Ts(t)\right)\right.\\
&\quad\left.\left.+f_{\theta\sigma}(t,z(t))\right)\right] + z^T(t)\sum_{\rho=1}^{\wp}\chi_{\sigma\rho}(\ell)\mathcal{P}_\rho z(t).
\end{aligned}\tag{24}$$

According to Lemma 1, we have

$$\begin{aligned}
&2z^T(t)\mathcal{P}_\sigma\,\triangle\,\mathcal{A}_{\theta\sigma}(t)z(t)\\
&\quad\le \varepsilon_{1\theta\sigma}^{-1}z^T(t)\mathcal{P}_\sigma\mathcal{M}_{\theta\sigma}\mathcal{M}_{\theta\sigma}^T\mathcal{P}_\sigma z(t) + \varepsilon_{1\theta\sigma}z^T(t)\mathcal{N}_{\theta\sigma}^T\mathcal{N}_{\theta\sigma}z(t)\\
&2z^T(t)\mathcal{P}_\sigma\mathcal{B}_\sigma f_{\theta\sigma}(t,z(t))\\
&\quad\le \varepsilon_{2\theta\sigma}^{-1}z^T(t)\mathcal{P}_\sigma\mathcal{B}_\sigma\mathcal{B}_\sigma^T\mathcal{P}_\sigma z(t) + \varepsilon_{2\theta\sigma}\rho_{\theta\sigma}^2z^T(t)z(t).
\end{aligned}\tag{25}$$

Under condition (16), it is obtained that

$$\begin{aligned}
&-2z^T(t)\mathcal{P}_\sigma\mathcal{B}_\sigma(\mathcal{I}+\Lambda)\eta_\sigma\,\mathrm{sgn}\left((\mathcal{X}_\sigma\mathcal{D}\mathcal{B}_\sigma)^Ts(t)\right)\\
&\quad\le -2(\mathcal{I}+\Lambda)\eta_\sigma\|\mathcal{P}_\sigma\mathcal{B}_\sigma z(t)\| < 0.
\end{aligned}\tag{26}$$

Thus, it holds

$$\mathcal{E}\{\Im\mathcal{S}_1(z(t),\sigma)\} \le \sum_{\theta=1}^{r}p_\theta(\zeta(t))z^T(t)\Xi_\theta^\sigma z(t) \tag{27}$$

where $\Xi_\theta^\sigma = \mathcal{P}_\sigma\mathcal{A}_{\theta\sigma} + \mathcal{A}_{\theta\sigma}^T\mathcal{P}_\sigma + 2\mathcal{P}_\sigma\mathcal{B}_\sigma(\mathcal{I}+\bar{\lambda}\mathcal{I})\mathcal{B}_\sigma^T\mathcal{P}_\sigma + \varepsilon_{1\theta\sigma}^{-1}\mathcal{P}_\sigma\mathcal{M}_{\theta\sigma}\mathcal{M}_{\theta\sigma}^T\mathcal{P}_\sigma + \varepsilon_{1\theta\sigma}\mathcal{N}_{\theta\sigma}^T\mathcal{N}_{\theta\sigma} + \varepsilon_{2\theta\sigma}^{-1}\mathcal{P}_\sigma\mathcal{B}_\sigma\mathcal{B}_\sigma^T\mathcal{P}_\sigma + \varepsilon_{2\theta\sigma}\rho_{\theta\sigma}^2\mathcal{I} + \sum_{\rho=1}^{\wp}\chi_{\sigma\rho}(\ell)\mathcal{P}_\rho$.

Applying the Schur complement lemma to (17) leads to

$$\mathcal{E}\{\Im\mathcal{S}_1(z(t),\sigma)\} \le 0. \tag{28}$$

Furthermore, system (15) realizes stochastic stability. ■

Remark 1: In this article, the system jump is a stochastic SMP related to nonexponential distribution with a time-varying TR matrix. When the ST obeys exponential distribution, S-MSSs are reduced to ordinary MSSs [39]–[42].

Remark 2: For Q1, compared with ST obeying exponential distribution in general MSSs [7], [14]–[21], [39]–[42], [53], [55], it needs to reconstruct the weak infinitesimal operator under SMP constraints [see (19)–(27)]. Then, by means of the Taylor-series formula (20) and $\lim_{\bar{\Delta}\to 0}([1-\mathcal{V}_\sigma(\ell+\bar{\Delta})]/[1-\mathcal{V}_\sigma(\ell)]) = 1$, $\lim_{\bar{\Delta}\to 0}([\mathcal{V}_\sigma(\ell)-\mathcal{V}_\sigma(\ell+\bar{\Delta})]/[1-\mathcal{V}_\sigma(\ell)]) = 0$, $\lim_{\bar{\Delta}\to 0}([\mathcal{V}_\sigma(\ell+\bar{\Delta})-\mathcal{V}_\sigma(\ell)]/[\bar{\Delta}(1-\mathcal{V}_\sigma(\ell))]) = \chi_\sigma(\ell)$, $\chi_{\sigma\rho}(\ell) = \lambda_{\sigma\rho}\chi_\rho(\ell)$, $\sigma \neq \rho$, one has the weak infinitesimal operator.

In Theorem 1, sufficient conditions are proposed for robust stochastic stability of the system (15). However, the nonlinear element $\sum_{\rho=1}^{\wp}\chi_{\sigma\rho}(\ell)\mathcal{P}_\rho$ and equality constraint (16) are not solvable in standard linear matrix inequalities. Next, strict linear matrix inequalities conditions are provided to determine $\mathcal{P}_\sigma$ and $\mathcal{X}_\sigma$ for stochastic stability purposes as mentioned aforehand.

Theorem 2: If there exist symmetric matrix $\mathcal{P}_\sigma > 0$, matrix $\mathcal{X}_\sigma$, and scalars $\gamma > 0$, $\varepsilon_{1\theta\sigma} > 0$, $\varepsilon_{2\theta\sigma} > 0$ $\forall\sigma \in \Theta$, $\theta = 1, 2, \ldots, r$, such that

$$\begin{bmatrix} -\gamma\mathcal{I} & \left(\mathcal{P}_\sigma\mathcal{B}_\sigma - \mathcal{D}^T\mathcal{X}_\sigma\mathcal{D}\mathcal{B}_\sigma\right)^T \\ * & -\mathcal{I}\end{bmatrix} < 0 \tag{29}$$

$$\bar{\Pi}_{1\theta}^{\sigma} < 0 \tag{30}$$

$$\underline{\Pi}_{1\theta}^{\sigma} < 0 \tag{31}$$

where

$$\bar{\Pi}_{1\theta}^{\sigma} = \begin{bmatrix}\bar{\Pi}_{11\theta}^{\sigma} & \Pi_{21\theta}^{\sigma} \\ * & \Pi_{31\theta}^{\sigma}\end{bmatrix}$$

$$\underline{\Pi}_{1\theta}^{\sigma} = \begin{bmatrix}\underline{\Pi}_{11\theta}^{\sigma} & \Pi_{21\theta}^{\sigma} \\ * & \Pi_{31\theta}^{\sigma}\end{bmatrix}$$

$$\bar{\Pi}_{11\theta}^{\sigma} = \mathcal{P}_\sigma\mathcal{A}_{\theta\sigma} + \mathcal{A}_{\theta\sigma}^T\mathcal{P}_\sigma + \sum_{\rho=1}^{\wp}\bar{\chi}_{\sigma\rho}\mathcal{P}_\rho$$

$$\underline{\Pi}_{11\theta}^{\sigma} = \mathcal{P}_\sigma\mathcal{A}_{\theta\sigma} + \mathcal{A}_{\theta\sigma}^T\mathcal{P}_\sigma + \sum_{\rho=1}^{\wp}\underline{\chi}_{\sigma\rho}\mathcal{P}_\rho$$

with $\Pi_{21\theta}^{\sigma}$ and $\Pi_{31\theta}^{\sigma}$ described in Theorem 1, then system (15) achieves robust stochastic stability.

Proof: The TR $\chi_{\sigma\rho}(\ell)$ can be represented by $\chi_{\sigma\rho}(\ell) = \theta_1\underline{\chi}_{\sigma\rho} + \theta_2\bar{\chi}_{\sigma\rho}$, where $\theta_1 + \theta_2 = 1$ and $\theta_1 > 0$, $\theta_2 > 0$. Multiplying (30) by θ_1 and (31) by θ_2 and using the Schur complement lemma lead to

$$\Sigma_{1\theta}^{\sigma} < 0 \tag{32}$$

where

$$\Sigma_{1\theta}^{\sigma} = \begin{bmatrix}\Sigma_{11\theta}^{\sigma} & \Pi_{21\theta}^{\sigma} \\ * & \Pi_{31\theta}^{\sigma}\end{bmatrix}$$

$$\Sigma_{11\theta}^{\sigma} = \mathcal{P}_\sigma\mathcal{A}_{\theta\sigma} + \mathcal{A}_{\theta\sigma}^T\mathcal{P}_\sigma + \sum_{\rho=1}^{\wp}\left(\theta_1\underline{\chi}_{\sigma\rho} + \theta_2\bar{\chi}_{\sigma\rho}\right)\mathcal{P}_\rho$$

with $\Pi_{21\theta}^{\sigma}$ and $\Pi_{31\theta}^{\sigma}$ described in Theorem 1.

By tuning θ_1 and θ_2, all possible $\chi_{\sigma\rho}(\ell) \in [\underline{\chi}_{\sigma\rho}, \bar{\chi}_{\sigma\rho}]$ can be obtained. Then, inequality (17) holds.

Since $\mathcal{P}_\sigma\mathcal{B}_\sigma = \mathcal{D}^T\mathcal{X}_\sigma\mathcal{D}\mathcal{B}_\sigma$, one has

$$\mathrm{Trace}\left[\left(\mathcal{P}_\sigma\mathcal{B}_\sigma - \mathcal{D}^T\mathcal{X}_\sigma\mathcal{D}\mathcal{B}_\sigma\right)^T \times \left(\mathcal{P}_\sigma\mathcal{B}_\sigma - \mathcal{D}^T\mathcal{X}_\sigma\mathcal{D}\mathcal{B}_\sigma\right)\right] = 0 \tag{33}$$

which implies that there exists a positive scalar γ such that

$$\left(\mathcal{P}_\sigma\mathcal{B}_\sigma - \mathcal{D}^T\mathcal{X}_\sigma\mathcal{D}\mathcal{B}_\sigma\right)^T\left(\mathcal{P}_\sigma\mathcal{B}_\sigma - \mathcal{D}^T\mathcal{X}_\sigma\mathcal{D}\mathcal{B}_\sigma\right) < \gamma\mathcal{I}. \tag{34}$$

Applying the Schur complement lemma, (34) is equivalent to (29).

Therefore, the feasible problem of Theorem 1 is changed into the following minimization problem:

$$\begin{aligned} &\min_{\mathcal{P}_\sigma,\, \mathcal{X}_\sigma,\, \varepsilon_{1\sigma},\, \varepsilon_{2\sigma}} \ \gamma \\ &\text{s.t. Inequalities (29)–(31).}\end{aligned} \tag{35}$$

■

C. Reachability Analysis

This part will deal with reachability of the sliding surface $s(t) = 0$ that is determined by the SMC law (14). Therefore, the state trajectories can be driven onto the sliding surface within the finite-time region.

Theorem 3: Consider the system (1) and sliding surface (13). Then, the finite-time attractiveness of the sliding surface can be realized by the SMC law (14), in which η_σ satisfies

$$\left(1+\bar{\lambda}\right)\eta_\sigma\|\mathcal{X}_\sigma\mathcal{D}\mathcal{B}_\sigma\| - \varrho > 0 \tag{36}$$

with positive constant ϱ and $\bar{\lambda} = \max_{1\le\varpi\le m}\{\lambda_\varpi\}$, $\lambda_\varpi = ([1-\phi_\varpi]/[1+\phi_\varpi])$.

Proof: For the Lyapunov function

$$\mathcal{S}_2(s(t),\sigma) = \frac{1}{2}s^T(t)\mathcal{X}_\sigma s(t) \tag{37}$$

one has

$$\begin{aligned}
&\Im\mathcal{S}_2(s(t),\sigma)\\
&\quad= \lim_{\bar{\Delta}\to 0}\frac{1}{\bar{\Delta}}\left[\mathcal{E}\left\{\mathcal{S}\left(s\left(t+\bar{\Delta}\right),\delta_{t+\bar{\Delta}}\right)|\delta_t=\sigma\right\} - \mathcal{S}_2(s(t),\sigma)\right]\\
&\quad= \lim_{\bar{\Delta}\to 0}\frac{1}{\bar{\Delta}}\Bigg[\frac{1}{2}\sum_{\rho=1,\rho\neq\sigma}^{\wp}\Pr\{\delta_{t+\bar{\Delta}}=\rho|\delta_t=\sigma\}s^T\left(t+\bar{\Delta}\right)\\
&\qquad\times\mathcal{X}_\rho s\left(t+\bar{\Delta}\right) + \frac{1}{2}\Pr\{\delta_{t+\bar{\Delta}}=\sigma|\delta_t=\sigma\}s^T\\
&\qquad\times\left(t+\bar{\Delta}\right)\mathcal{X}_\sigma s\left(t+\bar{\Delta}\right) - \frac{1}{2}s^T(t)\mathcal{X}_\sigma s(t)\Bigg]\\
&\quad= \lim_{\bar{\Delta}\to 0}\frac{1}{\bar{\Delta}}\Bigg[\frac{1}{2}\sum_{\rho=1,\rho\neq\sigma}^{\wp}\frac{\Pr\{\delta_{t+\bar{\Delta}}=\rho,\delta_t=\sigma\}}{\Pr\{\delta_t=\sigma\}}s^T\left(t+\bar{\Delta}\right)\\
&\qquad\times\mathcal{X}_\rho s\left(t+\bar{\Delta}\right) + \frac{1}{2}\frac{\Pr\{\delta_{t+\bar{\Delta}}=\sigma,\delta_t=\sigma\}}{\Pr\{\delta_t=\sigma\}}s^T\\
&\qquad\times\left(t+\bar{\Delta}\right)\mathcal{X}_\sigma s\left(t+\bar{\Delta}\right) - \frac{1}{2}s^T(t)\mathcal{X}_\sigma s(t)\Bigg]\\
&\quad= \lim_{\bar{\Delta}\to 0}\frac{1}{\bar{\Delta}}\Bigg[\frac{1}{2}\sum_{\rho=1,\rho\neq\sigma}^{\wp}\frac{\chi_{\sigma\rho}\left(\mathcal{V}_\sigma\left(\ell+\bar{\Delta}\right)-\mathcal{V}_\sigma(\ell)\right)}{1-\mathcal{V}_\sigma(\ell)}s^T\left(t+\bar{\Delta}\right)\\
&\qquad\times\mathcal{X}_\rho s\left(t+\bar{\Delta}\right) + \frac{1}{2}\frac{1-\mathcal{V}_\sigma\left(\ell+\bar{\Delta}\right)}{1-\mathcal{V}_\sigma(\ell)}s^T\left(t+\bar{\Delta}\right)\\
&\qquad\times\mathcal{X}_\sigma s\left(t+\bar{\Delta}\right) - \frac{1}{2}s^T(t)\mathcal{X}_\sigma s(t)\Bigg].
\end{aligned} \tag{38}$$

Considering the expansion of the Taylor formula leads to

$$s\big(t+\bar{\Delta}\big) = s(t) + \dot{s}(t)\bar{\Delta} + o\big(\bar{\Delta}\big) \tag{39}$$

where $\bar{\Delta} \to 0$.

Following the proof of Theorem 1, one has

$$\begin{aligned}
&\mathcal{E}\{\Im\mathcal{S}_2(s(t),\sigma)\}\\
&= s^T(t)\mathcal{X}_\sigma\mathcal{D}\dot{z}(t) + \frac{1}{2}s^T(t)\sum_{\rho=1}^{\wp}\chi_{\sigma\rho}(\ell)\mathcal{X}_\sigma s(t)\\
&= s^T(t)\sum_{\theta=1}^{r}p_\theta(\zeta(t))\mathcal{X}_\sigma\mathcal{D}\\
&\quad\times\Big[\big(\mathcal{A}_{\theta\sigma} + \Delta\mathcal{A}_{\theta\sigma}(t) + \mathcal{B}_\sigma(\mathcal{I}+\Lambda)\mathcal{B}_\sigma^T\mathcal{P}_\sigma\big)z(t)\\
&\qquad + \mathcal{B}_\sigma\big(-(\mathcal{I}+\Lambda)\eta_\sigma\,\mathrm{sgn}\big((\mathcal{X}_\sigma\mathcal{D}\mathcal{B}_\sigma)^T s(t)\big)\\
&\qquad\quad + f_{\theta\sigma}(t,z(t))\big)\Big]\\
&\quad + \frac{1}{2}s^T(t)\sum_{\rho=1}^{\wp}\chi_{\sigma\rho}(\ell)\mathcal{X}_\sigma s(t)\\
&\le \|s(t)\|\big[\max_{1\le\theta\le r}(\|\mathcal{X}_\sigma\mathcal{D}\mathcal{A}_{\theta\sigma}\|)\\
&\qquad + \max_{1\le\theta\le r}(\|\mathcal{X}_\sigma\mathcal{D}\mathcal{M}_{\theta\sigma}\|\|\mathcal{N}_{\theta\sigma}\|)\\
&\qquad + \big(1+\bar{\lambda}\big)\|\mathcal{X}_\sigma\mathcal{D}\mathcal{B}_\sigma\mathcal{B}_\sigma^T\mathcal{P}_\sigma\|\\
&\qquad + \max_{1\le\theta\le r}(\rho_{\theta\sigma})\big]\|z(t)\|\\
&\quad - \|s(t)\|\big(1+\bar{\lambda}\big)\eta_\sigma\|\mathcal{X}_\sigma\mathcal{D}\mathcal{B}_\sigma\|\\
&\quad + \frac{1}{2}\|s(t)\|\left\|\sum_{\rho=1,\rho\neq\sigma}^{\wp}\bar{\chi}_{\sigma\rho}\mathcal{X}_\sigma\mathcal{D}\right\|\|z(t)\|\\
&= \|s(t)\|\big[\varsigma_\sigma\|z(t)\| - \big(\big(1+\bar{\lambda}\big)\eta_\sigma\|\mathcal{X}_\sigma\mathcal{D}\mathcal{B}_\sigma\| - \varrho\big)\big]\\
&\quad - \varrho\|s(t)\|
\end{aligned} \tag{40}$$

where $\varsigma_\sigma = \max_{1\le\theta\le r}(\|\mathcal{X}_\sigma\mathcal{D}\mathcal{A}_{\theta\sigma}\|) + \max_{1\le\theta\le r}(\|\mathcal{X}_\sigma\mathcal{D}\mathcal{M}_{\theta\sigma}\|\|\mathcal{N}_{\theta\sigma}\|) + (1+\bar{\lambda})\|\mathcal{X}_\sigma\mathcal{D}\mathcal{B}_\sigma\mathcal{B}_\sigma^T\mathcal{P}_\sigma\| + \max_{1\le\theta\le r}(\rho_{\theta\sigma}) + [1/2]\|\sum_{\rho=1,\rho\neq\sigma}^{\wp}\bar{\chi}_{\sigma\rho}\mathcal{X}_\sigma\mathcal{D}\|$.

Defining the following domain:

$$\Omega \triangleq \big\{z(t) : \varsigma_\sigma\|z(t)\| \le \big(1+\bar{\lambda}\big)\eta_\sigma\|\mathcal{X}_\sigma\mathcal{D}\mathcal{B}_\sigma\| - \varrho\big\} \tag{41}$$

gives rise to

$$\|s(t)\|\big[\varsigma_\sigma\|z(t)\| - \big(\big(1+\bar{\lambda}\big)\eta_\sigma\|\mathcal{X}_\sigma\mathcal{D}\mathcal{B}_\sigma\| - \varrho\big)\big] \le 0. \tag{42}$$

Thus, we can obtain

$$\mathcal{E}\{\Im\mathcal{S}_2(s(t),\sigma)\} \le -\varrho\|s(t)\| \le -\frac{\varrho}{\zeta}\sqrt{\mathcal{E}\{\mathcal{S}_2(s(t),\sigma)\}} \tag{43}$$

where $\zeta = \max_{\sigma\in\Theta}\sqrt{[(\lambda_{\max}[\mathcal{X}_\sigma])/2]} > 0$. It is shown from (43) that there exists an instant $t' = 2\zeta\sqrt{\mathcal{E}\{\mathcal{S}_2(s(t_0),\delta_{t_0})\}}/\varrho$ such that $\mathcal{S}_2(s(t_0),\delta_{t_0}) = 0$ (equivalently, $s(t_0) = 0$) when $t \ge t'$. Therefore, finite-time attractiveness can be realized. ■

Remark 3: Due to the discontinuous property of the SMC law, chattering may appear in the control input. In such case, the discontinuous term $\mathrm{sgn}((\mathcal{X}_\sigma\mathcal{D}\mathcal{B}_\sigma)^T s(t))$ can be replaced by a smooth term $([(\mathcal{X}_\sigma\mathcal{D}\mathcal{B}_\sigma)^T s(t)]/[\|(\mathcal{X}_\sigma\mathcal{D}\mathcal{B}_\sigma)^T s(t)\| + \epsilon])$ with a small positive constant ϵ. This means that the sliding surface will produce some changes within a small neighborhood and its size depends on the value of ϵ.

Remark 4: For Q2, the mode-independent sliding surface (13) is proposed to avoid the potential repetitive jumping effects. Next, due to the multimodal characteristic of S-MSSs, a mode-dependent Lyapunov function (37) is naturally considered as an alternative, which makes full use of semi-Markovian switching information. In addition, the SMC law (14) depends on the quantizer level $\bar{\lambda}$ whose accurate information is easy to be obtained in practical systems. Moreover, when the system trajectories arrive at the defining domain Ω, the finite-time reachability of the predefined sliding surface can be guaranteed. Meantime, due to the unavoidable chattering effects, the sliding motion is not always staying on the predefined sliding surface all the time in practical systems. In fact, the state trajectories will stay in a bounded neighborhood around the predefined sliding surface. Thus, it is reasonable to define the domain Ω. Hence, under the effects of semi-Markovian switching, T–S fuzzy rules, and quantization constraint, the SMC law (14) can realize the finite-time reachability of the predefined sliding surface and improve the system performance.

TABLE I
SYSTEM PARAMETERS

Parameters	Meanings
J	Moment of inertia
K_m	Torque/back emf constant
D	Viscous friction coefficient
R_a	Armature resistance
R_f	Field winding resistance
L_a	Armature inductance
L_f	Field winding inductance

TABLE II
SYSTEM TERMINOLOGY

Mode σ	Moment of inertia	$J_\sigma(kg\ m^2)$
1	Small	0.0005
2	Normal	0.005
3	Large	0.05

IV. Case Study

Consider a modified series dc motor model in Fig. 1, taken from [45], described by

$$\begin{aligned}
J\frac{d\tilde{w}(t)}{dt} &= K_m L_f \tilde{i}^2(t) - D\tilde{w}(t)\\
L\frac{d\tilde{i}(t)}{dt} &= -R\tilde{i}(t) - K_m L_f\tilde{i}(t)\tilde{w}(t) + \tilde{V}(t)
\end{aligned} \tag{44}$$

where $\tilde{w}(t) = w(t) - w_{\mathrm{ref}}(t)$, $\tilde{i}(t) = i(t) - i_{\mathrm{ref}}(t)$, and $\tilde{V}(t) = V(t) - V_{\mathrm{ref}}(t)$ are the deviations of actual angular velocity from desired angular velocity, actual current from desired current, and actual input voltage from desired input voltage. The meanings of the parameters J, K_m, D, R_a, R_f, L_a, and L_f are given in Table I. When one has a modified series dc motor model, we have $i(t) = i_a(t) = i_f(t)$. The parameter J has three different modes shown in Table II. The transformation between different speeds obeys the SMP $\{\delta_t, t \ge 0\}$ in $\Theta = \{1, 2, 3\}$ with the TR matrices as

$$\underline{\chi} = \begin{bmatrix} -1.0 & 0.5 & 0.5\\ 0.8 & -1.2 & 0.4\\ 0.5 & 0.6 & -1.1 \end{bmatrix},\ \bar{\chi} = \begin{bmatrix} -1.5 & 0.6 & 0.9\\ 1.7 & -2.5 & 0.8\\ 1.2 & 0.8 & -2.0 \end{bmatrix}.$$

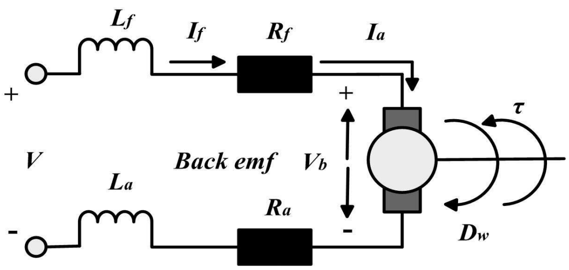

Fig. 1. Modified series dc motor model.

Defining $z_1(t) = \tilde{w}(t)$, $z_2(t) = \tilde{i}(t)$, and $u(t) = \tilde{V}(t)$, (44) can be represented as

$$\begin{bmatrix} \dot{z}_1(t) \\ \dot{z}_2(t) \end{bmatrix} = \begin{bmatrix} -\frac{D}{J_\sigma} & \frac{K_m L_f}{J_\sigma} z_2(t) \\ -\frac{K_m L_f}{L} z_2(t) & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} u(t) \tag{45}$$

where $L = L_f + L_a$ and $R = R_f + R_a$. The parameters are given as $R_a = 5\ \Omega$, $R_b = 5\ \Omega$, $L_f = 0.005$ H, $L_a = 0.995$ H, $D = 0.05$ Nm/rad/s, and $K_m = 1$ Nm/A.

The state variable $z_2(t)$ belongs to the range $[N_1, N_2]$. Then, one has the membership functions as

$$z_1(z_2(t)) = \frac{z_2(t) - N_1}{N_2 - N_1}, z_2(z_2(t)) = \frac{-z_2(t) + N_2}{N_2 - N_1}. \tag{46}$$

When $z_2(t)$ is about N_1, then $z_1(z_2(t)) = 0$ and $z_2(z_2(t)) = 1$ and when $z_2(t)$ is about N_2, then $z_1(z_2(t)) = 1$ and $z_2(z_2(t)) = 0$.

In addition, the external disturbance factors, such as air resistance and friction, always exist in the modified series dc motor model, which can be described as nonlinearity. Furthermore, when taking parametric uncertainty and quantized constraint into account, the modified series dc motor model can be described by the following.

Plant Rule 1: IF $z_2(t)$ is "N_1,"
THEN

$$\begin{aligned}\dot{z}(t) = (\mathcal{A}_{1\sigma} + \Delta\mathcal{A}_{1\sigma}(t))z(t) + \mathcal{B}_\sigma((\mathcal{I} + \Lambda)u(t) \\ + f_{1\sigma}(t, z(t))).\end{aligned}$$

Plant Rule 2: IF $z_2(t)$ is "N_2,"
THEN

$$\begin{aligned}\dot{z}(t) = (\mathcal{A}_{2\sigma} + \Delta\mathcal{A}_{2\sigma}(t))z(t) + \mathcal{B}_\sigma((\mathcal{I} + \Lambda)u(t) \\ + f_{2\sigma}(t, z(t)))\end{aligned}$$

where

$$\mathcal{A}_{11} = \begin{bmatrix} -100 & 10N_1 \\ -0.005N_1 & -10 \end{bmatrix}, \mathcal{B}_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
$$\mathcal{A}_{12} = \begin{bmatrix} -10 & N_1 \\ -0.005N_1 & -10 \end{bmatrix}, \mathcal{B}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
$$\mathcal{A}_{13} = \begin{bmatrix} -1 & 0.1N_1 \\ -0.005N_1 & -10 \end{bmatrix}, \mathcal{B}_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
$$\mathcal{A}_{21} = \begin{bmatrix} -100 & 10N_2 \\ -0.005N_2 & -10 \end{bmatrix}, \mathcal{A}_{22} = \begin{bmatrix} -10 & N_2 \\ -0.005N_2 & -10 \end{bmatrix}$$
$$\mathcal{A}_{23} = \begin{bmatrix} -1 & 0.1N_2 \\ -0.005N_2 & -10 \end{bmatrix}, \mathcal{M}_{11} = \mathcal{M}_{21} = \begin{bmatrix} 0 \\ 0.3 \end{bmatrix}$$

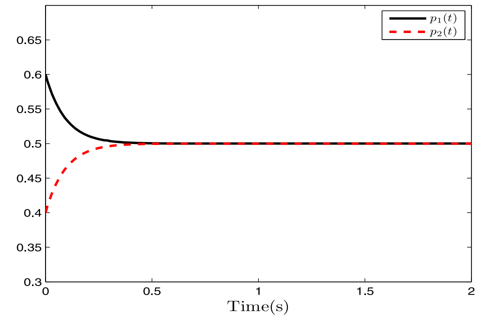

Fig. 2. Membership functions.

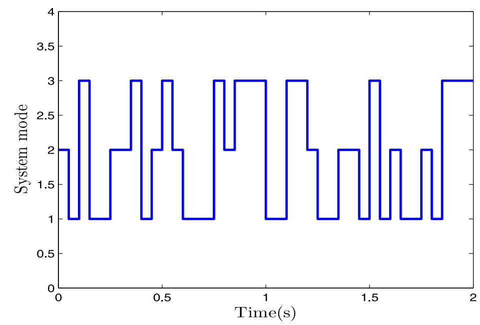

Fig. 3. System mode.

$$\mathcal{M}_{12} = \mathcal{M}_{22} = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}, \mathcal{M}_{13} = \mathcal{M}_{23} = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}$$
$$\mathcal{N}_{11} = \mathcal{N}_{21} = \begin{bmatrix} 0.1 & -0.1 \end{bmatrix}, \mathcal{N}_{12} = \mathcal{N}_{22} = \begin{bmatrix} -0.1 & 0.2 \end{bmatrix}$$
$$\mathcal{N}_{13} = \mathcal{N}_{23} = \begin{bmatrix} 0 & 0.1 \end{bmatrix}.$$

The other parameters are given as $N_1 = -10$, $N_2 = 10$, $\rho_{\theta\sigma} = 0.5$, and $h_\sigma = 1/3$, $\varrho = 0.1$, $\theta = 1, 2$, $\sigma = 1, 2, 3$. The quantizer density is given as $\phi = 0.4$.

In this model, the SMC law is constructed to ensure the robust stochastic stability and dispel the adverse effects of quantization error, nonlinearity, and parametric uncertainty. Solving Theorem 2 results in

$$\gamma = 2.3120 * 10^{-5}, \mathcal{P}_1 = \begin{bmatrix} 6.0587 & -0.0001 \\ -0.0001 & 0.0404 \end{bmatrix}$$
$$\mathcal{P}_2 = \begin{bmatrix} 44.0045 & 0.0000 \\ 0.0000 & 0.0411 \end{bmatrix}, \mathcal{P}_3 = \begin{bmatrix} 107.8242 & 0.0000 \\ 0.0000 & 0.0413 \end{bmatrix}$$
$$\mathcal{X}_1 = 0.0020, \mathcal{X}_2 = 0.0018, \mathcal{X}_3 = 0.0017.$$

Meantime, the sliding surface is given as $s(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} z(t)$. By the solutions, we can choose $\eta_1 = \eta_2 = \eta_3 = 0.42$ with $\varrho = 0.001$. Then, the SMC law (14) can be computed as when $\delta_t = 1$, $u(t) = \begin{bmatrix} -0.0001 & 0.0404 \end{bmatrix} - 0.42\text{sgn}(0.002s(t))$; when $\delta_t = 2$, $u(t) = \begin{bmatrix} 0 & 0.0411 \end{bmatrix} - 0.42\text{sgn}(0.0018s(t))$; and when $\delta_t = 3$, $u(t) = \begin{bmatrix} 0 & 0.0413 \end{bmatrix} - 0.42\text{sgn}(0.0017s(t))$.

For given $\delta_0 = 2$ and $z_0 = \begin{bmatrix} -1.5 & 2.0 \end{bmatrix}^T$, replacing $\text{sgn}((\mathcal{X}_\sigma \mathcal{D}\mathcal{B}_\sigma)^T s(t))$ with a smooth term $([(\mathcal{X}_\sigma \mathcal{D}\mathcal{B}_\sigma)^T s(t)]/[\|(\mathcal{X}_\sigma \mathcal{D}\mathcal{B}_\sigma)^T s(t)\| + 0.01])$, Fig. 2 plots the membership functions. Fig. 3 stands for the system mode. The state responses $z(t)$ are shown in Figs. 4 and 5, from which, we can see that $z(t)$ satisfies robust stochastic stability. Fig. 6 depicts the finite-time reachability of the predefined sliding surface. Fig. 7 plots the control input $u(t)$. Therefore,

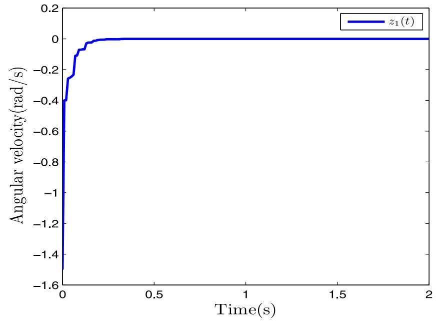

Fig. 4. State response $z_1(t)$.

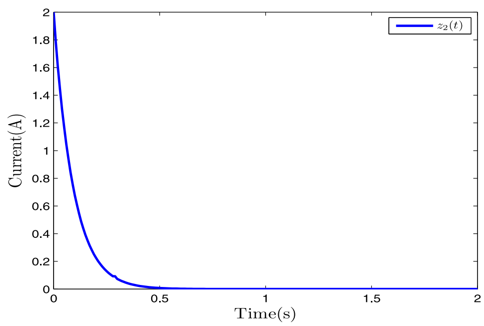

Fig. 5. State response $z_2(t)$.

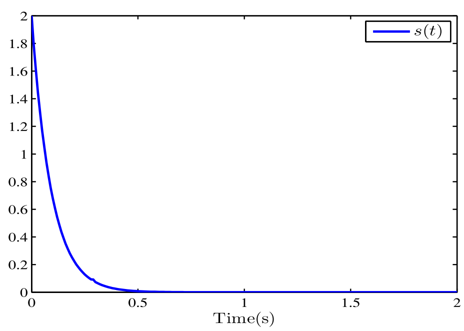

Fig. 6. Sliding surface $s(t)$.

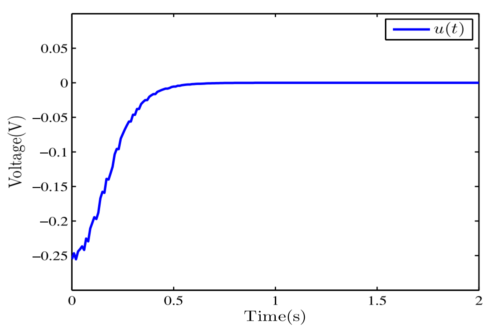

Fig. 7. SMC law $u(t)$.

the SMC law can ensure robust stochastic stability and reachability of nonlinear S-MSSs with quantization.

Remark 5: There exist some obvious limitations for MSSs, that is, the ST obeys an exponential distribution. This strict condition greatly limits the practical applications of MSSs, which may lead to intrinsic conservativeness of many results obtained for MSSs [7], [14]–[21], [39]–[42], [53], [55]. In fact, MSSs can be seen as a special kind of S-MSSs with the advantage in describing practical dynamical systems subject to sudden change of the parameters or structures than MSSs. Considering complex factors, including parametric uncertainty, input quantization, and nonlinearity, a modified series dc motor model is described by nonlinear uncertain S-MSSs. For given system parameters, by solving Theorem 2, we can obtain the SMC law $u(t)$ that could realize finite-time reachability.

V. CONCLUSION

In this article, the fuzzy SMC problem has been addressed for T–S fuzzy S-MSSs in the absence of quantization. First, robust stochastic stability criteria have been given for the corresponding system. Then, the desired SMC is constructed to depend on the quantizer level. A modified series dc motor model is provided to illustrate the advantages of the proposed method. Moreover, for reducing the occupancy of network bandwidth resources, SMC for event-triggered fuzzy S-MSSs is significant in future work.

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