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Distributed Set-Membership Fusion Filtering for Nonlinear Two-Dimensional Systems Over Sensor Networks: An Encoding-Decoding Scheme

Kaiqun Zhu, Zidong Wang, Qing-Long Han, and Guoliang Wei

***Abstract*—In this paper, the distributed set-membership fusion filtering problem is investigated for a class of nonlinear two-dimensional shift-varying systems subject to unknown-but-bounded noises over sensor networks. The sensors are communicated with their neighbors according to a given topology through wireless networks of limited bandwidth. With the purpose of relieving the communication burden as well as enhancing the transmission security, a logarithmic-type encoding-decoding mechanism is introduced for each sensor node so as to encode the transmitted data with a finite number of bits. A distributed set-membership filter is designed to determine the local ellipsoidal set that contains the system state by only utilizing the data from the local sensor node and its neighbors, where the proposed filter scheme is truly distributed with desirable scalability. Then, a new ellipsoid-based fusion rule is developed for the designed set-membership filters in order to form the fused ellipsoidal set that has a globally smaller volume than all local ellipsoidal sets. With the aid of the mathematical induction technique, the set theory and the convex optimization approach, sufficient conditions are derived for the existence of the desired distributed set-membership filters and the fusion weights. Then, the filter parameters and the fusion weights are acquired by solving a set of constrained optimization problems. Finally, an illustrative example is given to demonstrate the effectiveness of the proposed fusion filtering algorithm.**

***Index Terms*—Two-dimensional systems, sensor networks, distributed set-membership filtering, fusion filtering, encoding-decoding mechanism.**

I. Introduction

Since Fornasini-Marchesini (F-M) model was proposed in 1976 by using the Nerode equivalence method, the two-dimensional (2-D) systems have been gaining steadily growing research attention owing to their practical significance and theoretical importance [8], [9]. Specifically, 2-D systems are suitable for modeling a variety of practical processes ranging from grid sensor networks, sheet forming, stream heating, to image processing [35], [36]. Moreover, the 2-D system theory serves as a theoretical basis for batch process control, iterative learning control, and so on [38]. It is worth noting that the system states in 2-D systems evolve along two independent directions and, due to this distinguishing feature, existing theories for one-dimensional (1-D) systems may become inapplicable in the 2-D settings. Accordingly, a host of research interest has been aroused in the dynamics analysis problem for 2-D systems from both industry and academia, see, e.g., [1], [14], [17], [18], [24], [33].

In the past decades, sensor networks have played an increasingly important role in a wide variety of practical situations, for instance, remote system monitoring, information collection, and target tracking [31], [41], [48]. A sensor network is normally made up of a multitude of smart sensor nodes that are distributed spatially in a certain region via communicating with their neighboring nodes over typically wireless networks, and this kind of features brings several advantages, e.g., easy installation, satisfactory fault tolerance and self-organizing flexibility [20], [28]. It is well acknowledged that the *distributed* filtering/estimation issue is one of the fundamental research topics for signal processing over sensor networks, whose main concern is to improve the filtering performance in a collaborative manner [15], [25].

In response to the popularity of sensor networks, the theory of distributed filtering has recently undergone a rapid development and the available results are centered on two general categories, namely, distributed *pointwise* filtering and the distributed *set-membership* filtering (SMF, also known as set-valued filtering). For the former, each distributed filter computes a point estimation of the system state at each instant based on the neighboring sensors' information or the distributed fusion strategy, and some of the representative algorithms include distributed H_∞ filtering methods and distributed Kalman filtering methods [2], [10], [13], [27], [30]. For the distributed SMF, each distributed filter determines an ellipsoidal set that contains the system state with 100% confidence by using information from neighboring sensors according to the topology [21], [39], [42].

When the sensor networks are exposed to unknown-but-bounded (UBB) noises, the distributed H_∞/Kalman filters might not provide the satisfactory performance. In this case, the distributed SMF method serves as an ideal candidate. In most existing SMF-related literature, the *local* ellipsoidal set containing the system state is determined for each sensor

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node by designing a suitable distributed set-membership filter structure [23], [40], [46]. Compared with the local ellipsoidal set, an adequately *fused* ellipsoidal set contains the intersection of all local ellipsoidal sets and could have a comparatively small region that contains the system state, thereby ensuring better filtering performance. Unfortunately, such a seemingly interesting ellipsoid-based *fusion* filtering issue has received very little attention for 2-D systems over sensor networks, let alone the case where nonlinearities and shift-varying parameters are also taken into account, and this situation motivates our current investigation.

When it comes to the applications of sensor networks, two challenges we have to face are the communication constraints and the transmission security that are unavoidable due to the inherently limited communication capacity and the nowadays increasing demands on cybersecurity. These two challenges, if not handled appropriately in the sensor networks, would inevitably deteriorate the system performance [4], [5], [11], [16], [19], [26], [43]. Fortunately, the so-called *encoding-decoding* mechanism (EDM) provides a rather promising countermeasure to the challenges. A typical EDM consists of two parts, namely, the encoder and the decoder, where the encoder is capable of encrypting the system signals and then mapping them into codewords with finite bits before transmission, and the decoder can reconstruct the codewords into the original data as accurately as possible according to certain rules.

Recently, the EDM has become a prevailing research topic from both signal processing and control communities [29], [32], [37], [45]. For example, an EDM-based iterative learning control strategy has been designed in [32] to investigate the tracking control problem for linear discrete-time systems. Moreover, the recursive filtering issue has been studied in [37] for 1-D nonlinear systems with a coding scheme. Nevertheless, to the best of the authors' knowledge, the *EDM-based* distributed set-membership fusion filtering (SMFF) problem has not been fully examined yet, not to mention the simultaneous consideration of the 2-D shift-varying systems, sensor networks and nonlinearities, and this constitutes another motivation of our current research.

In connection with the discussions made so far, in this paper, our focus is on the EDM-based distributed SMFF problem for nonlinear 2-D shift-varying systems over sensor networks. The *research questions* we are confronted with are summarized as follows: 1) how to cope with the nonlinear functions in 2-D shift-varying systems over sensor networks in the framework of distributed SMF? 2) how to construct an appropriate EDM for each sensor node under the 2-D setting with a focus on the inherent characteristics of information propagation along with two independent directions? and 3) how to develop a distributed SMFF mechanism under the 2-D setting to achieve the efficient distributed processing of system information gathered from sensor nodes? The main objective of this paper is, therefore, to answer the above questions by initializing a systematic investigation.

The main *contributions* of this paper can be highlighted as follows.

- With the help of the Taylor series expansion formula and the interval analysis technique, the nonlinear function in the 2-D shift-varying system is tackled and the Lagrange remainder is bounded by a minimized ellipsoidal set.
- A logarithmic-type EDM is, for the first time, proposed for a class of nonlinear 2-D shift-varying systems over sensor networks with aim to reduce the burden of network communication and strengthen the security of signal transmission, where the developed logarithmic-type EDM is dependent on the bidirectional evolution of the system dynamics.
- A truly distributed set-membership filter is constructed for nonlinear 2-D systems over sensor networks, where the proposed distributed filter scheme is scalable as it only uses the information from the local sensor and its neighboring sensors.
- An ellipsoid-based fusion scheme is proposed for the filtering problem in the 2-D setting, and the fused filtering performance is shown to be better than that of any local sensor by means of matrix trace.

The rest of this paper is organized as follows. In Section II, the distributed set-membership filter is formulated for nonlinear 2-D shift-varying systems over sensor networks under purposely introduced EDMs. In Section III, both the distributed SMF algorithm and the ellipsoid-based fusion filtering rule are developed, and the filter parameters and the fusion weights are then obtained by solving a set of optimization problems. Section IV utilizes an illustrative example to demonstrate the effectiveness of the proposed fusion filtering algorithm. Conclusions are drawn in Section V.

Notations: $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ denote, respectively, the n-dimensional Euclidean space and the set of all $m\times n$ real matrices. For a symmetric matrix X, $X>0$ means that X is a positive definite matrix. $\{X_{i,j}\}_{i,j\in[0,\mathcal{T}]}$ denotes the set of matrices $\{X_{i^\star,j^\star} \mid 0\le i^\star\le\mathcal{T}, 0\le j^\star\le\mathcal{T}\}$. $[X]_{k,l}$ represents the (k,l) element of the matrix X. $\mathrm{col}\{x_1,x_2,\ldots,x_M\}$ means $\begin{bmatrix} x_1^{\mathrm{T}} & x_2^{\mathrm{T}} & \cdots & x_M^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}$ and $\mathrm{col}_M\{\cdot\}$ is a column vector with M blocks. $\mathrm{diag}\{\cdot\}$ stands for a block-diagonal matrix and $\mathrm{diag}_M\{\cdot\}$ is a block-diagonal matrix with M blocks. In symmetric block matrices, "*" is used as an ellipsis for terms induced by symmetry. I and 0 denote the identity matrix and zero matrix with appropriate dimensions, respectively. The superscript "T" stands for the transpose of a matrix.

II. Problem Formulation and Preliminaries

A. System Model

Consider a nonlinear 2-D shift-varying system described by the general F-M second model:

$$\begin{aligned} x_{i+1,j+1} = {} & f^{(1)}(x_{i,j+1}) + f^{(2)}(x_{i+1,j}) \\ & + B^{(1)}_{i,j+1}w_{i,j+1} + B^{(2)}_{i+1,j}w_{i+1,j} \end{aligned} \tag{1}$$

where $i,j\in[0,\mathcal{T}]$ are horizontal and vertical coordinates with $\mathcal{T}\in\mathbb{N}$; $x_{i,j}\in\mathbb{R}^{n_x}$ is the state vector; $f^{(1)}(x_{i,j})$ and $f^{(2)}(x_{i,j})$ are known continuously differentiable nonlinear functions; $B^{(1)}_{i,j}$ and $B^{(2)}_{i,j}$ are known shift-varying matrices with appropriate dimensions; and $w_{i,j}\in\mathbb{R}^{n_w}$ is the UBB process noise that is confined to the following ellipsoidal set:

$$\mathscr{E}(0,W_{i,j}) \triangleq \left\{w_{i,j} \mid w_{i,j}^{\mathrm{T}}W_{i,j}^{-1}w_{i,j}\le 1\right\} \tag{2}$$

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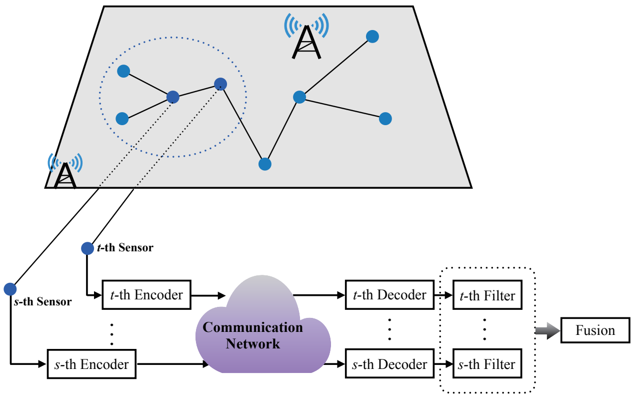

Fig. 1. Block diagram for 2-D systems with EDM

with $W_{i,j}$ being a known positive definite matrix.

In this paper, the sensor network has M sensor nodes whose topology is represented by a directed graph $\mathbf{G} = (\mathbf{V}, \mathbf{E}, \mathbf{A})$. Here, $\mathbf{V} = \{1, 2, \ldots, M\}$ is the set of sensing nodes; $\mathbf{E} \subseteq \mathbf{V} \times \mathbf{V}$ is the set of edges; $\mathbf{A} = [\alpha_{st}]_{M\times M}$ is the weighted adjacency matrix with $\alpha_{st} > 0$ if $(s,t) \in \mathbf{E}$; and the set of neighbors for node $s \in \mathbf{V}$ is denoted by $\mathbf{N}_s = \{t \in \mathbf{V} \mid (s,t) \in \mathbf{E}\}$.

The measurement of the sth sensor node is described as

$$y_{i,j}^{(s)} = C_{i,j}^{(s)} x_{i,j} + D_{i,j}^{(s)} v_{i,j}^{(s)}, \quad s = 1, 2, \ldots, M \tag{3}$$

where $y_{i,j}^{(s)} \in \mathbb{R}^{n_y}$ is the measurement output of the sth sensor node; $C_{i,j}^{(s)}$ and $D_{i,j}^{(s)}$ are known shift-varying matrices with appropriate dimensions; and $v_{i,j}^{(s)} \in \mathbb{R}^{n_v}$ is the UBB measurement noise that is confined to the following ellipsoidal set:

$$\mathscr{E}\left(0, V_{i,j}^{(s)}\right) \triangleq \left\{ v_{i,j}^{(s)} \mid (v_{i,j}^{(s)})^{\mathrm{T}} (V_{i,j}^{(s)})^{-1} v_{i,j}^{(s)} \leq 1 \right\} \tag{4}$$

with $V_{i,j}^{(s)}$ being a known positive definite matrix.

B. Encoding-Decoding Mechanism

During data transmissions, the signals are processed by the logarithmic-type EDM as shown in Fig. 1, where the main principles are described as follows.

Encoding:

The encoding rule for the sth sensor node is given by

$$\begin{cases} \chi_{0,j}^{(s)} = \chi_{i,0}^{(s)} = 0, \quad \forall\, i,j \in \mathbb{N} \\ \chi_{i,j}^{(s)} = \delta_{i,j}^{(s)} \psi_{i,j}^{(s)} + \daleth_{i-1,j}^{(s,1)} \chi_{i-1,j}^{(s)} + \daleth_{i,j-1}^{(s,2)} \chi_{i,j-1}^{(s)} \\ \psi_{i,j}^{(s)} = \mathfrak{Q}\left\{ \dfrac{1}{\delta_{i,j}^{(s)}} \left(y_{i,j}^{(s)} - \daleth_{i-1,j}^{(s,1)} \chi_{i-1,j}^{(s)} - \daleth_{i,j-1}^{(s,2)} \chi_{i,j-1}^{(s)} \right) \right\} \end{cases} \tag{5}$$

where $\chi_{i,j}^{(s)} \in \mathbb{R}^{n_\chi}$ and $\psi_{i,j}^{(s)} \in \mathbb{R}^{n_\psi}$ are the internal state and the output of the encoder, respectively; $\delta_{i,j}^{(s)}$ is the known scaling parameter; and $\daleth_{i,j}^{(s,1)}$, $\daleth_{i,j}^{(s,2)}$ are known shift-varying matrices with appropriate dimensions. Here, the logarithmic quantizer $\mathfrak{Q}$ is characterized by

$$\mathfrak{Q}(\zeta) \triangleq \begin{bmatrix} \mathscr{Q}(\zeta_1) \\ \mathscr{Q}(\zeta_2) \\ \vdots \\ \mathscr{Q}(\zeta_{n_\zeta}) \end{bmatrix} \tag{6}$$

where, for $\hbar = 1, 2, \ldots, n_\zeta$,

$$\mathscr{Q}(\zeta_\hbar) \triangleq \begin{cases} \ell_\kappa, & \dfrac{\ell_\kappa}{1+\tau} \leq \zeta_\hbar < \dfrac{\ell_\kappa}{1-\tau} \\ 0, & \zeta_\hbar = 0 \\ -\mathscr{Q}(-\zeta_\hbar), & \zeta_\hbar < 0 \end{cases} \tag{7}$$

with the set of quantization levels being

$$\mathscr{U} = \{\pm \ell_\kappa \mid \ell_\kappa = \wp^\kappa \ell_0, \kappa = 0, \pm 1, \pm 2, \ldots, \pm \mathcal{R}\} \cup \{0\}. \tag{8}$$

Here, $\tau \triangleq \frac{1-\wp}{1+\wp}$, $0 < \wp \leq 1$, $\ell_0 > 0$, and $\mathcal{R}$ is a positive integer.

Decoding:

The decoding rule for the sth filter is given as

$$\begin{cases} \breve{y}_{0,j}^{(s)} = \breve{y}_{i,0}^{(s)} = 0, \quad \forall\, i,j \in \mathbb{N} \\ \breve{y}_{i,j}^{(s)} = \delta_{i,j}^{(s)} \psi_{i,j}^{(s)} + \daleth_{i-1,j}^{(s,1)} \breve{y}_{i-1,j}^{(s)} + \daleth_{i,j-1}^{(s,2)} \breve{y}_{i,j-1}^{(s)} \end{cases} \tag{9}$$

where $\breve{y}_{i,j}^{(s)} \in \mathbb{R}^{n_y}$ is the output of the decoder.

By denoting the decoding error as $e_{i,j}^{(s)} \triangleq \breve{y}_{i,j}^{(s)} - y_{i,j}^{(s)}$, we have

$$\begin{aligned} & e_{i,j}^{(s)} \triangleq \breve{y}_{i,j}^{(s)} - y_{i,j}^{(s)} \\ & = \delta_{i,j}^{(s)} \psi_{i,j}^{(s)} + \daleth_{i-1,j}^{(s,1)} \breve{y}_{i-1,j}^{(s)} + \daleth_{i,j-1}^{(s,2)} \breve{y}_{i,j-1}^{(s)} - y_{i,j}^{(s)} \\ & = \delta_{i,j}^{(s)} \left\{ \mathfrak{Q}\left\{ \frac{1}{\delta_{i,j}^{(s)}} \left(y_{i,j}^{(s)} - \daleth_{i-1,j}^{(s,1)} \chi_{i-1,j}^{(s)} - \daleth_{i,j-1}^{(s,2)} \chi_{i,j-1}^{(s)} \right) \right\} \right. \\ & \qquad \left. - \frac{1}{\delta_{i,j}^{(s)}} \left(y_{i,j}^{(s)} - \daleth_{i-1,j}^{(s,1)} \breve{y}_{i-1,j}^{(s)} - \daleth_{i,j-1}^{(s,2)} \breve{y}_{i,j-1}^{(s)} \right) \right\} \\ & = \delta_{i,j}^{(s)} \left\{ \mathfrak{Q}\left\{ \frac{1}{\delta_{i,j}^{(s)}} \left(y_{i,j}^{(s)} - \daleth_{i-1,j}^{(s,1)} \breve{y}_{i-1,j}^{(s)} - \daleth_{i,j-1}^{(s,2)} \breve{y}_{i,j-1}^{(s)} \right) \right\} \right. \\ & \qquad \left. - \frac{1}{\delta_{i,j}^{(s)}} \left(y_{i,j}^{(s)} - \daleth_{i-1,j}^{(s,1)} \breve{y}_{i-1,j}^{(s)} - \daleth_{i,j-1}^{(s,2)} \breve{y}_{i,j-1}^{(s)} \right) \right\} \\ & = \delta_{i,j}^{(s)} q_{i,j}^{(s)}. \end{aligned} \tag{10}$$

Here, $q_{i,j}^{(s)}$ is the quantization error and satisfies the following condition:

$$(q_{i,j}^{(s)})^{\mathrm{T}} (q_{i,j}^{(s)} - 2\tau \varsigma_{i,j}^{(s)}) \leq 0 \tag{11}$$

where

$$\varsigma_{i,j}^{(s)} \triangleq \frac{1}{\delta_{i,j}^{(s)}} \left(y_{i,j}^{(s)} - \daleth_{i-1,j}^{(s,1)} \breve{y}_{i-1,j}^{(s)} - \daleth_{i,j-1}^{(s,2)} \breve{y}_{i,j-1}^{(s)} \right).$$

Remark 1: For 1-D systems, *logarithmic*-type EDM has been quite popular in analog-digital signal conversion. The EDM characterized in (5)–(9) is, to the best of the authors' knowledge, the first one of the kind for *2-D systems* that takes into account two time-indexes representing bidirectional dynamics evolution. Note that the introduction of the 2-D version of EDM (5)–(9) makes it possible to relieve the communication burden (because of the reduced data size) and

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enhance the communication security (because of the encoding-encoding scheme) at the cost of dealing with added complexity in system analysis/synthesis. More specifically, such added complexity stems from the strong coupling effects between the decoding error $e_{i,j}^{(s)}$ and the system measurement output $y_{i,j}^{(s)}$, the decoder output $\breve{y}_{i,j}^{(s)}$, as well as the scaling parameter $\delta_{i,j}^{(s)}$, and one of the technical challenge is therefore to handle this coupling issue in the subsequent investigation.

Remark 2: The proposed 2-D version of logarithmic-type EDM possesses the following *characteristics*: 1) it facilitates the distributed implementation and is therefore suitable for sensor networks; 2) the parameter $\delta_{i,j}^{(s)}$ can be dynamically adjusted, which provides extra flexibilities in the subsequent distributed filter design for a better performance; and 3) the security of the transmitted data can be further improved due to the introduction of coefficients $\daleth_{i,j}^{(s,1)}$ and $\daleth_{i,j}^{(s,2)}$.

C. Distributed Set-Membership Filter

In this paper, a distributed encoding-decoding-based set-membership filter is constructed for the nonlinear 2-D shift-varying system over sensor networks as follows:

$$\begin{aligned}\hat{x}_{i+1,j+1}^{(s)} &= f^{(1)}(\hat{x}_{i,j+1}^{(s)}) + f^{(2)}(\hat{x}_{i+1,j}^{(s)}) \\ &\quad + K_{i,j+1}^{(s,1)}\nu_{i,j+1}^{(s)} + K_{i+1,j}^{(s,2)}\nu_{i+1,j}^{(s)} \\ &\quad + \sum_{t\in\mathbf{N}_s}\alpha_{st}G_{i,j+1}^{(st,1)}(\hat{x}_{i,j+1}^{(t)} - \hat{x}_{i,j+1}^{(s)}) \\ &\quad + \sum_{t\in\mathbf{N}_s}\alpha_{st}G_{i+1,j}^{(st,2)}(\hat{x}_{i+1,j}^{(t)} - \hat{x}_{i+1,j}^{(s)}) \end{aligned} \tag{12}$$

where $\hat{x}_{i,j}^{(s)} \in \mathbb{R}^{n_x}$ is the local estimate of $x_{i,j}$ on sensor s, $K_{i,j}^{(s,1)}$, $K_{i,j}^{(s,2)}$, $G_{i,j}^{(st,1)}$ and $G_{i,j}^{(st,2)}$ are filter parameters to be calculated, and

$$\nu_{i,j}^{(s)} \triangleq \breve{y}_{i,j}^{(s)} - C_{i,j}^{(s)}\hat{x}_{i,j}^{(s)}.$$

By applying the Taylor series expansion formula, the nonlinear functions $f^{(1)}(x_{i,j})$ and $f^{(2)}(x_{i,j})$ are linearized as

$$f^{(1)}(x_{i,j}) = f^{(1)}(\hat{x}_{i,j}^{(s)}) + \Psi_{i,j}^{(s,1)}(x_{i,j} - \hat{x}_{i,j}^{(s)}) + r_{i,j}^{(s,1)} \tag{13a}$$
$$f^{(2)}(x_{i,j}) = f^{(2)}(\hat{x}_{i,j}^{(s)}) + \Psi_{i,j}^{(s,2)}(x_{i,j} - \hat{x}_{i,j}^{(s)}) + r_{i,j}^{(s,2)} \tag{13b}$$

where $\Psi_{i,j}^{(s,1)}$, $\Psi_{i,j}^{(s,2)}$ are Jacobian matrices; and $r_{i,j}^{(s,1)}$, $r_{i,j}^{(s,2)}$ are high order Lagrange remainders. Here,

$$\begin{aligned}&\Psi_{i,j}^{(s,1)} \triangleq \frac{\partial f^{(1)}}{\partial x}\Big|_{x=\hat{x}_{i,j}^{(s)}}, \quad \Psi_{i,j}^{(s,2)} \triangleq \frac{\partial f^{(2)}}{\partial x}\Big|_{x=\hat{x}_{i,j}^{(s)}} \\ &r_{i,j}^{(s,1)} \triangleq \frac{1}{2}\mathrm{diag}_M\{(x_{i,j} - \hat{x}_{i,j}^{(s)})^{\mathrm{T}}\}\frac{\partial^2 f^{(1)}}{\partial x^2}\Big|_{x=\rho_{i,j}^{(s)}}(x_{i,j} - \hat{x}_{i,j}^{(s)}) \\ &r_{i,j}^{(s,2)} \triangleq \frac{1}{2}\mathrm{diag}_M\{(x_{i,j} - \hat{x}_{i,j}^{(s)})^{\mathrm{T}}\}\frac{\partial^2 f^{(2)}}{\partial x^2}\Big|_{x=\rho_{i,j}^{(s)}}(x_{i,j} - \hat{x}_{i,j}^{(s)}) \\ &\rho_{i,j}^{(s)} \triangleq \lambda_{i,j}^{(s)}x_{i,j} + (1-\lambda_{i,j}^{(s)})\hat{x}_{i,j}^{(s)} \\ &\lambda_{i,j}^{(s)} \triangleq \mathrm{diag}\left\{\lambda_{i,j}^{(s,1)}, \lambda_{i,j}^{(s,2)}, \ldots, \lambda_{i,j}^{(s,n_x)}\right\} \\ &\lambda_{i,j}^{(s,k)} \in [0,1] \quad (k = 1,2,\ldots,n_x).\end{aligned}$$

Denoting the estimation error as $\eta_{i,j}^{(s)} \triangleq x_{i,j} - \hat{x}_{i,j}^{(s)}$, we obtain

$$\begin{aligned}\eta_{i+1,j+1}^{(s)} &= \mathscr{A}_{i,j+1}^{(s,1)}\eta_{i,j+1}^{(s)} + \mathscr{A}_{i+1,j}^{(s,2)}\eta_{i+1,j}^{(s)} + r_{i,j+1}^{(s,1)} \\ &\quad + r_{i+1,j}^{(s,2)} + B_{i,j+1}^{(1)}w_{i,j+1} + B_{i+1,j}^{(2)}w_{i+1,j} \\ &\quad - K_{i,j+1}^{(s,1)}(D_{i,j+1}^{(s)}v_{i,j+1}^{(s)} + e_{i,j+1}^{(s)}) \\ &\quad - K_{i+1,j}^{(s,2)}(D_{i+1,j}^{(s)}v_{i+1,j}^{(s)} + e_{i+1,j}^{(s)}) \\ &\quad - \sum_{t\in\mathbf{N}_s}\alpha_{st}G_{i,j+1}^{(st,1)}(\hat{x}_{i,j+1}^{(t)} - \hat{x}_{i,j+1}^{(s)}) \\ &\quad - \sum_{t\in\mathbf{N}_s}\alpha_{st}G_{i+1,j}^{(st,2)}(\hat{x}_{i+1,j}^{(t)} - \hat{x}_{i+1,j}^{(s)}) \end{aligned} \tag{14}$$

where

$$\begin{aligned}\mathscr{A}_{i,j}^{(s,1)} &\triangleq \Psi_{i,j}^{(s,1)} - K_{i,j}^{(s,1)}C_{i,j}^{(s)} \\ \mathscr{A}_{i,j}^{(s,2)} &\triangleq \Psi_{i,j}^{(s,2)} - K_{i,j}^{(s,2)}C_{i,j}^{(s)}.\end{aligned}$$

Assumption 1: The initial conditions of (1) are given by

$$\begin{cases} x_{0,j} \in \mathscr{E}\left(\hat{x}_{0,j}^{(s)}, Q_{0,j}^{(s)}\right) \triangleq \left\{x_{0,j} \,\middle|\, h^{(s)}(x_{0,j}) \le 1\right\} \\ x_{i,0} \in \mathscr{E}\left(\hat{x}_{i,0}^{(s)}, Q_{i,0}^{(s)}\right) \triangleq \left\{x_{i,0} \,\middle|\, h^{(s)}(x_{i,0}) \le 1\right\} \end{cases} \tag{15}$$

for $s = 1,2,\ldots,M$, where

$$\begin{aligned}h^{(s)}(x_{0,j}) &\triangleq (x_{0,j} - \hat{x}_{0,j}^{(s)})^{\mathrm{T}}(Q_{0,j}^{(s)})^{-1}(x_{0,j} - \hat{x}_{0,j}^{(s)}) \le 1 \\ h^{(s)}(x_{i,0}) &\triangleq (x_{i,0} - \hat{x}_{i,0}^{(s)})^{\mathrm{T}}(Q_{i,0}^{(s)})^{-1}(x_{i,0} - \hat{x}_{i,0}^{(s)}) \le 1\end{aligned}$$

with $Q_{0,j}^{(s)}$ and $Q_{i,0}^{(s)}$ being known positive definite matrices.

The main purpose of this paper is highlighted in threefold as follows.

- ***First***, we aim to design distributed set-membership filter gains $K_{i,j}^{(s,1)}$, $K_{i,j}^{(s,2)}$, $G_{i,j}^{(st,1)}$ and $G_{i,j}^{(st,2)}$ such that, for $s = 1,2,\ldots,M$, the system state $x_{i,j}$ is confined to the local ellipsoidal set $\mathscr{X}_{i,j}^{(s)}$ as

$$\mathscr{X}_{i,j}^{(s)} \triangleq \mathscr{E}\left(\hat{x}_{i,j}^{(s)}, Q_{i,j}^{(s)}\right) = \left\{x_{i,j} \,\middle|\, h^{(s)}(x_{i,j}) \le 1\right\} \tag{16}$$

  where

$$h^{(s)}(x_{i,j}) \triangleq (x_{i,j} - \hat{x}_{i,j}^{(s)})^{\mathrm{T}}(Q_{i,j}^{(s)})^{-1}(x_{i,j} - \hat{x}_{i,j}^{(s)})$$

  with $Q_{i,j}^{(s)}$ being the positive definite matrix.
- ***Second***, based on the obtained results in the first step, we shall determine the fusion weights such that

$$\left(\mathscr{X}_{i,j}^{(1)} \bigcap \mathscr{X}_{i,j}^{(2)} \bigcap \cdots \bigcap \mathscr{X}_{i,j}^{(M)}\right) \subseteq \mathscr{X}_{i,j}^{(f)} \tag{17}$$

  where $\mathscr{X}_{i,j}^{(f)}$ is the fused ellipsoidal set with

$$\begin{aligned}\mathscr{X}_{i,j}^{(f)} &\triangleq \mathscr{E}\left(\hat{x}_{i,j}^{(f)}, Q_{i,j}^{(f)}\right) = \left\{x_{i,j} \,\middle|\, h^{(f)}(x_{i,j}) \le 1\right\} \\ h^{(f)}(x_{i,j}) &\triangleq (x_{i,j} - \hat{x}_{i,j}^{(f)})^{\mathrm{T}}(Q_{i,j}^{(f)})^{-1}(x_{i,j} - \hat{x}_{i,j}^{(f)}).\end{aligned}$$

  Here, $\hat{x}_{i,j}^{(f)}$ is the fused estimate and $Q_{i,j}^{(f)}$ is the positive definite matrix.
- ***Third***, we intend to establish a sufficient condition such that

$$\mathrm{Tr}(Q_{i,j}^{(f)}) \le \mathrm{Tr}(Q_{i,j}^{(s)}), \quad s = 1,2,\ldots,M \tag{18}$$

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and calculate the optimal values of the parameters $\hat{x}^{(f)}_{i,j}$ and the positive definite matrix $Q^{(f)}_{i,j}$ by solving a constrained optimization problem.

III. Main Results

In this section, an encoding-decoding-based distributed SMFF scheme is designed for shift-varying nonlinear 2-D systems over sensor networks. Sufficient conditions are established for the existence of the desired parameters which guarantee that the required performance constraints are satisfied. Then, the design parameters are obtained by solving certain optimization problems.

Before proceeding further, the following lemmas are recalled to facilitate the derivation of our main results.

Lemma 1: (Two-dimensional mathematical induction [34]) Let $\mathcal{S}_{i,j}$ denote a proposition with $i,j\in\mathbb{N}$. Suppose that

1) (*initial step*) $\mathcal{S}_{i,j}$ is true for all $(i,j)\in\{(i,j)|i,j\in\mathbb{N}, i=0 \text{ or } j=0\}$;
2) (*inductive step*) if $\mathcal{S}_{i^\diamond,j^\diamond}$ is true for all $(i^\diamond,j^\diamond)\in\Delta_{i,j}\triangleq\{(i-1,j),(i,j-1)\}$, then $\mathcal{S}_{i,j}$ is true.

Then, $\mathcal{S}_{i,j}$ is true for all $i,j\in\mathbb{N}$.

Lemma 2: Consider the vectors $\chi^{(s)}_{i,j}$ and $\breve{y}^{(s)}_{i,j}$ in (5) and (9). For $s=1,2,\ldots,M$, we have

$$\chi^{(s)}_{i,j}=\breve{y}^{(s)}_{i,j}. \tag{19}$$

Proof: This lemma is proved by two-dimensional mathematical induction, which is conducted via the following two steps.

1) *Initial step*. It is known from (5) and (9) that $\chi^{(s)}_{i,j}=\breve{y}^{(s)}_{i,j}$ is true for all $(i,j)\in\{(i,j)|i,j\in\mathbb{N}, i=0 \text{ or } j=0\}$.

2) *Inductive step*. Suppose that $\chi^{(s)}_{i^\diamond,j^\diamond}=\breve{y}^{(s)}_{i^\diamond,j^\diamond}$ is true for all $(i^\diamond,j^\diamond)\in\Delta_{i,j}\triangleq\{(i-1,j),(i,j-1)\}$. Then, it remains to prove that $\chi^{(s)}_{i,j}=\breve{y}^{(s)}_{i,j}$ is true. In fact, one has

$$\begin{aligned}
&\chi^{(s)}_{i,j}-\breve{y}^{(s)}_{i,j}\\
=&\delta^{(s)}_{i,j}\psi^{(s)}_{i,j}+\daleth^{(s,1)}_{i-1,j}\breve{y}^{(s)}_{i-1,j}+\daleth^{(s,2)}_{i,j-1}\breve{y}^{(s)}_{i,j-1}\\
&-\delta^{(s)}_{i,j}\psi^{(s)}_{i,j}-\daleth^{(s,1)}_{i-1,j}\chi^{(s)}_{i-1,j}-\daleth^{(s,2)}_{i,j-1}\chi^{(s)}_{i,j-1}\\
=&\daleth^{(s,1)}_{i-1,j}(\breve{y}^{(s)}_{i-1,j}-\chi^{(s)}_{i-1,j})+\daleth^{(s,2)}_{i,j-1}(\breve{y}^{(s)}_{i,j-1}-\chi^{(s)}_{i,j-1})\\
=&0,
\end{aligned}$$

which ends the proof. ■

Lemma 3: Consider the high-order Lagrange remainders $r^{(s,1)}_{i,j}$ and $r^{(s,2)}_{i,j}$ in (13). For $s=1,2,\ldots,M$, suppose that

$$x_{i,j}\in\mathscr{E}\left(\hat{x}^{(s)}_{i,j},Q^{(s)}_{i,j}\right). \tag{20}$$

Then, we have

$$\begin{cases} r^{(s,1)}_{i,j}\in\mathscr{E}\left(0,R^{(s,1)}_{i,j}\right)\triangleq\left\{r^{(s,1)}_{i,j}\,\middle|\,h(r^{(s,1)}_{i,j})\le 1\right\}\\ r^{(s,2)}_{i,j}\in\mathscr{E}\left(0,R^{(s,2)}_{i,j}\right)\triangleq\left\{r^{(s,2)}_{i,j}\,\middle|\,h(r^{(s,2)}_{i,j})\le 1\right\}\end{cases} \tag{21}$$

where, for $\varrho=1,2$,

$$\begin{aligned}
&h(r^{(s,\varrho)}_{i,j})\triangleq(r^{(s,\varrho)}_{i,j})^{\mathrm{T}}(R^{(s,\varrho)}_{i,j})^{-1}r^{(s,\varrho)}_{i,j}\\
&\left[R^{(s,\varrho)}_{i,j}\right]_{k,l}\triangleq\begin{cases}2\left[\bar{R}^{(s,\varrho)}_{i,j}\right]_k, & k=l\\ 0, & k\neq l\end{cases}\\
&\left[\bar{R}^{(s,\varrho)}_{i,j}\right]_k\triangleq\left(\left[\left(\vec{\mathcal{X}}^{(s,\varrho)}_{i,j}\right)^+\right]_k-\left[\left(\vec{\mathcal{X}}^{(s,\varrho)}_{i,j}\right)^-\right]_k\right)^2\\
&\vec{\mathcal{X}}^{(s,\varrho)}_{i,j}\triangleq\frac{1}{2}\mathrm{diag}_M\{(\mathcal{X}^{(s)}_{i,j})^{\mathrm{T}}\}\begin{bmatrix}\mathcal{H}^{(\varrho)}_1(\hat{x}^{(s)}_{i,j}+\mathcal{X}^{(s)}_{i,j})\\ \mathcal{H}^{(\varrho)}_2(\hat{x}^{(s)}_{i,j}+\mathcal{X}^{(s)}_{i,j})\\ \vdots\\ \mathcal{H}^{(\varrho)}_{n_x}(\hat{x}^{(s)}_{i,j}+\mathcal{X}^{(s)}_{i,j})\end{bmatrix}\mathcal{X}^{(s)}_{i,j}\\
&\mathcal{X}^{(s)}_{i,j}\triangleq\left[-\mathcal{Q}^{(s)}_{i,j},\ \mathcal{Q}^{(s)}_{i,j}\right]\\
&\mathcal{Q}^{(s)}_{i,j}\triangleq\left[\left(\sqrt{[Q^{(s)}_{i,j}]_{1,1}}\right)^{\mathrm{T}}\ \ \ldots\ \ \left(\sqrt{[Q^{(s)}_{i,j}]_{n_x,n_x}}\right)^{\mathrm{T}}\right]^{\mathrm{T}}.
\end{aligned}$$

Here, $\left[R^{(s,\varrho)}_{i,j}\right]_{k,l}$ and $\left[Q^{(s)}_{i,j}\right]_{k,l}$ are, respectively, the (k,l) element of the matrices $R^{(s,\varrho)}_{i,j}$ and $Q^{(s)}_{i,j}$; $\left[\bar{R}^{(s,\varrho)}_{i,j}\right]_k$ is the kth element of the vector $\bar{R}^{(s,\varrho)}_{i,j}$; $\left(\vec{\mathcal{X}}^{(s,\varrho)}_{i,j}\right)^+$ and $\left(\vec{\mathcal{X}}^{(s,\varrho)}_{i,j}\right)^-$ denote the maximum and minimum values in the interval vector $\vec{\mathcal{X}}^{(s,\varrho)}_{i,j}$; and $\mathcal{H}^{(\varrho)}_k(\cdot)$ is the Hessian matrix of the nonlinear function $f^{(\varrho)}_k(\cdot)$ that is the kth entry of the vector $f^{(\varrho)}(\cdot)$ $(k=1,2,\ldots,n_x)$.

Proof: It follows from (20) that

$$(x_{i,j}-\hat{x}^{(s)}_{i,j})^{\mathrm{T}}(Q^{(s)}_{i,j})^{-1}(x_{i,j}-\hat{x}^{(s)}_{i,j})\le 1. \tag{22}$$

Then, it is easy to verify that, for

$$\rho^{(s)}_{i,j}\triangleq\lambda^{(s)}_{i,j}x_{i,j}+(1-\lambda^{(s)}_{i,j})\hat{x}^{(s)}_{i,j},$$

one obtains

$$(\rho^{(s)}_{i,j}-\hat{x}^{(s)}_{i,j})^{\mathrm{T}}(Q^{(s)}_{i,j})^{-1}(\rho^{(s)}_{i,j}-\hat{x}^{(s)}_{i,j})\le 1. \tag{23}$$

According to (23), we have

$$\rho^{(s)}_{i,j}\in\bar{\mathcal{X}}^{(s)}_{i,j}\triangleq\left[-\mathcal{Q}^{(s)}_{i,j}+\hat{x}^{(s)}_{i,j},\ \mathcal{Q}^{(s)}_{i,j}+\hat{x}^{(s)}_{i,j}\right]. \tag{24}$$

Then, with the help of the interval analysis technique and according to the high-order Lagrange remainders $r^{(s,1)}_{i,j}$ and $r^{(s,2)}_{i,j}$ in (13), we obtain the following interval vector:

$$\vec{\mathcal{X}}^{(s,\varrho)}_{i,j}\triangleq\frac{1}{2}\mathrm{diag}_M\{(\mathcal{X}^{(s)}_{i,j})^{\mathrm{T}}\}\begin{bmatrix}\mathcal{H}^{(\varrho)}_1(\hat{x}^{(s)}_{i,j}+\mathcal{X}^{(s)}_{i,j})\\ \mathcal{H}^{(\varrho)}_2(\hat{x}^{(s)}_{i,j}+\mathcal{X}^{(s)}_{i,j})\\ \vdots\\ \mathcal{H}^{(\varrho)}_{n_x}(\hat{x}^{(s)}_{i,j}+\mathcal{X}^{(s)}_{i,j})\end{bmatrix}\mathcal{X}^{(s)}_{i,j}. \tag{25}$$

By recurring to (25), the high-order Lagrange remainders $r^{(s,1)}_{i,j}$ and $r^{(s,2)}_{i,j}$ can be bounded by ellipsoidal sets (21) with minimized volume [7], which completes the proof. ■

Remark 3: The ellipsoidal sets $\mathscr{E}\left(0,R^{(s,1)}_{i,j}\right)$ and $\mathscr{E}\left(0,R^{(s,2)}_{i,j}\right)$ containing the high order Lagrange remainders $r^{(s,1)}_{i,j}$ and $r^{(s,2)}_{i,j}$ are determined in Lemma 3 by applying the interval analysis technique. It should be noted that this method represents one of the first few attempts to deal with the distributed set-membership filter design issue for nonlinear 2-D shift-varying systems over sensor networks.

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A. Design of the Distributed Set-Membership Filter

To simplify notations, we denote

$$
\begin{aligned}
&\Pi^{(0)} \triangleq \mathrm{diag}\{1,0,0,0,0,0,0,0,0,0,0\}\\
&\Pi^{(s,1)}_{i^\diamond,j^\diamond} \triangleq \mathrm{diag}\{-1,I,0,0,0,0,0,0,0,0,0\}\\
&\Pi^{(s,2)}_{i^\diamond,j^\diamond} \triangleq \mathrm{diag}\{-1,0,I,0,0,0,0,0,0,0,0\}\\
&\Pi^{(s,3)}_{i^\diamond,j^\diamond} \triangleq \mathrm{diag}\left\{-1,0,0,R^{(s,1)}_{i,j+1},0,0,0,0,0,0,0\right\}\\
&\Pi^{(s,4)}_{i^\diamond,j^\diamond} \triangleq \mathrm{diag}\left\{-1,0,0,0,R^{(s,2)}_{i+1,j},0,0,0,0,0,0\right\}\\
&\Pi^{(s,5)}_{i^\diamond,j^\diamond} \triangleq \mathrm{diag}\{-1,0,0,0,0,W_{i,j+1},0,0,0,0,0\}\\
&\Pi^{(s,6)}_{i^\diamond,j^\diamond} \triangleq \mathrm{diag}\{-1,0,0,0,0,0,W_{i+1,j},0,0,0,0\}\\
&\Pi^{(s,7)}_{i^\diamond,j^\diamond} \triangleq \mathrm{diag}\left\{-1,0,0,0,0,0,0,V^{(s)}_{i,j+1},0,0,0\right\}\\
&\Pi^{(s,8)}_{i^\diamond,j^\diamond} \triangleq \mathrm{diag}\left\{-1,0,0,0,0,0,0,0,V^{(s)}_{i+1,j},0,0\right\}\\
&\Pi^{(s,9)}_{i^\diamond,j^\diamond} \triangleq \begin{bmatrix} 0 & * & * \\ -\tau\bar{\delta}^{(s)}_{i,j}\Omega^{(s,1)}_{i,j+1} & I & * \\ 0 & 0 & 0 \end{bmatrix}\\
&\Pi^{(s,10)}_{i^\diamond,j^\diamond} \triangleq \begin{bmatrix} 0 & * \\ -\tau\bar{\delta}^{(s)}_{i,j}\Omega^{(s,2)}_{i+1,j} & I \end{bmatrix}\\
&\Omega^{(s,1)}_{i,j} \triangleq \begin{bmatrix}\Omega^{(s,1,1)}_{i,j} & \Omega^{(s,1,2)}_{i,j} & \Omega^{(s,1,3)} & D^{(s)}_{i,j} & 0\end{bmatrix}\\
&\Omega^{(s,2)}_{i,j} \triangleq \begin{bmatrix}\Omega^{(s,1,1)}_{i,j} & 0 & \Omega^{(s,1,2)}_{i,j} & \Omega^{(s,1,3)} & D^{(s)}_{i,j} & 0\end{bmatrix}\\
&\Omega^{(s,1,1)}_{i,j} \triangleq C^{(s)}_{i,j}\hat{x}^{(s)}_{i,j} - \tilde{y}^{(s)}_{i,j}, \quad \Omega^{(s,1,2)}_{i,j} \triangleq C^{(s)}_{i,j}\Xi^{(s)}_{i,j}\\
&\Omega^{(s,1,3)} \triangleq \begin{bmatrix}0 & 0 & 0 & 0 & 0\end{bmatrix}, \quad \bar{\delta}^{(s)}_{i,j} \triangleq (\delta^{(s)}_{i,j})^{-1}\\
&\tilde{y}^{(s)}_{i,j} \triangleq \urcorner^{(s,1)}_{i-1,j}\breve{y}^{(s)}_{i-1,j} + \urcorner^{(s,2)}_{i,j-1}\breve{y}^{(s)}_{i,j-1}\\
&(i^\diamond,j^\diamond) \in \Delta_{i+1,j+1} \triangleq \{(i,j+1),(i+1,j)\}.
\end{aligned}
$$

The following theorem is given to provide a sufficient condition that guarantees that the system state satisfies the performance constraint (16).

Theorem 1: Consider the system (1), the logarithmic-type EDM (5)–(9), and the distributed set-membership filter (12). For $s = 1,2,\ldots,M$, let the sequences of positive definite matrices $\{Q^{(s)}_{i,j+1}\}_{i,j\in\mathbb{N}}$ and $\{Q^{(s)}_{i+1,j}\}_{i,j\in\mathbb{N}}$ be given. The system state $x_{i+1,j+1}$ belongs to the local ellipsoidal set $\mathscr{E}\left(\hat{x}^{(s)}_{i+1,j+1}, Q^{(s)}_{i+1,j+1}\right)$ if there exist filter parameters $K^{(s,1)}_{i,j+1}$, $K^{(s,2)}_{i+1,j}$, $G^{(st,1)}_{i,j+1}$ and $G^{(st,2)}_{i+1,j}$, positive scalars $\epsilon^{(s,\theta)}_{i^\diamond,j^\diamond}$ ($\theta = 1,2,\ldots,10$), and a positive definite matrix $Q^{(s)}_{i+1,j+1}$ satisfying

$$\begin{bmatrix} -\vec{\Pi}^{(s)}_{i^\diamond,j^\diamond} & * \\ \Phi^{(s)}_{i^\diamond,j^\diamond} & -Q^{(s)}_{i+1,j+1}\end{bmatrix} \leq 0 \tag{26}$$

where

$$
\begin{aligned}
\vec{\Pi}^{(s)}_{i^\diamond,j^\diamond} &\triangleq \Pi^{(0)} + \sum_{\theta=1}^{10}\epsilon^{(s,\theta)}_{i^\diamond,j^\diamond}\Pi^{(s,\theta)}_{i^\diamond,j^\diamond}\\
\Phi^{(s)}_{i^\diamond,j^\diamond} &\triangleq \begin{bmatrix}\Phi^{(s,1)}_{i^\diamond,j^\diamond} & \Phi^{(s,2)}_{i^\diamond,j^\diamond} & \Phi^{(s,3)}_{i^\diamond,j^\diamond} & \Phi^{(s,4)}_{i^\diamond,j^\diamond}\end{bmatrix}\\
\Phi^{(s,1)}_{i^\diamond,j^\diamond} &\triangleq -\sum_{t\in\mathbf{N}_s}\alpha_{st}G^{(st,1)}_{i,j+1}(\hat{x}^{(t)}_{i,j+1} - \hat{x}^{(s)}_{i,j+1})\\
&\quad -\sum_{t\in\mathbf{N}_s}\alpha_{st}G^{(st,2)}_{i+1,j}(\hat{x}^{(t)}_{i+1,j} - \hat{x}^{(s)}_{i+1,j})\\
\Phi^{(s,2)}_{i^\diamond,j^\diamond} &\triangleq \begin{bmatrix}\mathscr{A}^{(s,1)}_{i,j+1}\Xi^{(s)}_{i,j+1} & \mathscr{A}^{(s,2)}_{i+1,j}\Xi^{(s)}_{i+1,j} & I & I & B^{(1)}_{i,j+1}\end{bmatrix}\\
\Phi^{(s,3)}_{i^\diamond,j^\diamond} &\triangleq \begin{bmatrix}B^{(2)}_{i+1,j} & -K^{(s,1)}_{i,j+1}D^{(s)}_{i,j+1} & -K^{(s,2)}_{i+1,j}D^{(s)}_{i+1,j}\end{bmatrix}\\
\Phi^{(s,4)}_{i^\diamond,j^\diamond} &\triangleq \begin{bmatrix}-\delta^{(s)}_{i,j+1}K^{(s,1)}_{i,j+1} & -\delta^{(s)}_{i+1,j}K^{(s,2)}_{i+1,j}\end{bmatrix}.
\end{aligned}
$$

Proof: This theorem is proved by two-dimensional mathematical induction, which is conducted via the following two steps.

1) *Initial step*. It is known immediately from Assumption 1 that

$$x_{i,j} \in \mathscr{E}\left(\hat{x}^{(s)}_{i,j}, Q^{(s)}_{i,j}\right), \quad s = 1,2,\ldots,M \tag{27}$$

is true for all $(i,j) \in \{(i,j)|i,j\in\mathbb{N}, i=0 \text{ or } j=0\}$.

2) *Inductive step*. Let

$$x_{i^\diamond,j^\diamond} \in \mathscr{E}\left(\hat{x}^{(s)}_{i^\diamond,j^\diamond}, Q^{(s)}_{i^\diamond,j^\diamond}\right), \quad s = 1,2,\ldots,M \tag{28}$$

be true for all $(i^\diamond,j^\diamond) \in \Delta_{i+1,j+1} \triangleq \{(i,j+1),(i+1,j)\}$. Then, it remains to prove that

$$x_{i+1,j+1} \in \mathscr{E}\left(\hat{x}^{(s)}_{i+1,j+1}, Q^{(s)}_{i+1,j+1}\right), \quad s = 1,2,\ldots,M \tag{29}$$

is also true.

First, it is easy to verify from (14) and (28) that there exist $\vartheta^{(s)}_{i,j+1}$ and $\vartheta^{(s)}_{i+1,j}$ (with $\|\vartheta^{(s)}_{i,j+1}\| \leq 1$ and $\|\vartheta^{(s)}_{i+1,j}\| \leq 1$) such that

$$\begin{cases} x_{i,j+1} = \hat{x}^{(s)}_{i,j+1} + \Xi^{(s)}_{i,j+1}\vartheta^{(s)}_{i,j+1} \\ x_{i+1,j} = \hat{x}^{(s)}_{i+1,j} + \Xi^{(s)}_{i+1,j}\vartheta^{(s)}_{i+1,j} \end{cases} \tag{30}$$

where $\Xi^{(s)}_{i,j+1}$ and $\Xi^{(s)}_{i+1,j}$ are factorizations of $Q^{(s)}_{i,j+1}$ and $Q^{(s)}_{i+1,j}$, respectively, i.e., $Q^{(s)}_{i,j+1} = \Xi^{(s)}_{i,j+1}(\Xi^{(s)}_{i,j+1})^{\mathrm{T}}$ and $Q^{(s)}_{i+1,j} = \Xi^{(s)}_{i+1,j}(\Xi^{(s)}_{i+1,j})^{\mathrm{T}}$.

Letting

$$\xi^{(s)}_{i^\diamond,j^\diamond} \triangleq \begin{bmatrix}1 & (\xi^{(s,1)}_{i^\diamond,j^\diamond})^{\mathrm{T}} & (\xi^{(s,2)}_{i^\diamond,j^\diamond})^{\mathrm{T}} & (\xi^{(s,3)}_{i^\diamond,j^\diamond})^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}},$$

in view of (30), system (14) is rewritten as

$$\eta^{(s)}_{i+1,j+1} = \Phi^{(s)}_{i^\diamond,j^\diamond}\xi^{(s)}_{i^\diamond,j^\diamond} \tag{31}$$

where

$$\xi^{(s,1)}_{i^\diamond,j^\diamond} \triangleq \begin{bmatrix}\vartheta^{(s)}_{i,j+1} \\ \vartheta^{(s)}_{i+1,j}\end{bmatrix}, \quad \xi^{(s,2)}_{i^\diamond,j^\diamond} \triangleq \begin{bmatrix} r^{(s,1)}_{i,j+1} \\ r^{(s,2)}_{i+1,j} \\ w_{i,j+1} \\ w_{i+1,j}\end{bmatrix}, \quad \xi^{(s,3)}_{i^\diamond,j^\diamond} \triangleq \begin{bmatrix} v^{(s)}_{i,j+1} \\ v^{(s)}_{i+1,j} \\ q^{(s)}_{i,j+1} \\ q^{(s)}_{i+1,j}\end{bmatrix}.$$

According to (2), (4), (21) and (30), the following conditions are satisfied:

$$\begin{cases} \|\vartheta^{(s)}_{i,j+1}\| \leq 1, \quad \|\vartheta^{(s)}_{i+1,j}\| \leq 1 \\ r^{(s,1)}_{i,j+1} \in \mathscr{E}\left(0, R^{(s,1)}_{i,j+1}\right), \quad r^{(s,2)}_{i+1,j} \in \mathscr{E}\left(0, R^{(s,2)}_{i+1,j}\right) \\ w_{i,j+1} \in \mathscr{E}\left(0, W_{i,j+1}\right), \quad w_{i+1,j} \in \mathscr{E}\left(0, W_{i+1,j}\right) \\ v^{(s)}_{i,j+1} \in \mathscr{E}\left(0, V^{(s)}_{i,j+1}\right), \quad v^{(s)}_{i+1,j} \in \mathscr{E}\left(0, V^{(s)}_{i+1,j}\right) \end{cases} \tag{32}$$

which can be rearranged in terms of $\xi^{(s)}_{i^\diamond,j^\diamond}$ as follows:

$$(\xi^{(s)}_{i^\diamond,j^\diamond})^{\mathrm{T}}\Pi^{(s,\bar{\theta})}_{i^\diamond,j^\diamond}\xi^{(s)}_{i^\diamond,j^\diamond} \leq 0, \quad \bar{\theta} = 1,2,\ldots,8. \tag{33}$$

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Next, we proceed to cope with the decoding errors $e^{(s)}_{i,j+1}$ and $e^{(s)}_{i+1,j}$ in system (14). According to (10)–(11), we have

$$\begin{cases} (e^{(s)}_{i,j+1})^{\mathrm{T}} e^{(s)}_{i,j+1} - 2\tau\delta^{(s)}_{i,j+1}(e^{(s)}_{i,j+1})^{\mathrm{T}}\varsigma^{(s)}_{i,j+1} \le 0 \\ (e^{(s)}_{i+1,j})^{\mathrm{T}} e^{(s)}_{i+1,j} - 2\tau\delta^{(s)}_{i+1,j}(e^{(s)}_{i+1,j})^{\mathrm{T}}\varsigma^{(s)}_{i+1,j} \le 0 \end{cases} \tag{34}$$

which, in terms of $\xi^{(s)}_{i^\diamond,j^\diamond}$, are expressed as

$$\begin{cases} (\xi^{(s)}_{i^\diamond,j^\diamond})^{\mathrm{T}}\Pi^{(s,9)}_{i^\diamond,j^\diamond}\xi^{(s)}_{i^\diamond,j^\diamond} \le 0 \\ (\xi^{(s)}_{i^\diamond,j^\diamond})^{\mathrm{T}}\Pi^{(s,10)}_{i^\diamond,j^\diamond}\xi^{(s)}_{i^\diamond,j^\diamond} \le 0. \end{cases} \tag{35}$$

By applying Schur Complement Lemma [3], it follows from (26) that

$$-\vec{\Pi}^{(s)}_{i^\diamond,j^\diamond} + (\Phi^{(s)}_{i^\diamond,j^\diamond})^{\mathrm{T}}(Q^{(s)}_{i+1,j+1})^{-1}\Phi^{(s)}_{i^\diamond,j^\diamond} \le 0. \tag{36}$$

It is deduced from (36) that

$$\begin{aligned} &(\Phi^{(s)}_{i^\diamond,j^\diamond}\xi^{(s)}_{i^\diamond,j^\diamond})^{\mathrm{T}}(Q^{(s)}_{i+1,j+1})^{-1}\Phi^{(s)}_{i^\diamond,j^\diamond}\xi^{(s)}_{i^\diamond,j^\diamond} \\ &\le (\xi^{(s)}_{i^\diamond,j^\diamond})^{\mathrm{T}}\vec{\Pi}^{(s)}_{i^\diamond,j^\diamond}\xi^{(s)}_{i^\diamond,j^\diamond}. \end{aligned} \tag{37}$$

By further resorting to $\mathcal{S}$-procedure [3], it can be derived from (31), (33), (35) and (37) that

$$(\eta^{(s)}_{i+1,j+1})^{\mathrm{T}}(Q^{(s)}_{i+1,j+1})^{-1}\eta^{(s)}_{i+1,j+1} \le 1 \tag{38}$$

is true, which means that the system state $x_{i+1,j+1}$ belongs to the local ellipsoidal set $\mathscr{E}\left(\hat{x}^{(s)}_{i+1,j+1}, Q^{(s)}_{i+1,j+1}\right)$. Therefore, according to the principle of mathematical induction, the proof is now complete. ■

B. Design of the Fusion Rule for Ellipsoid-Based Filtering

It follows from Theorem 1 that the system state $x_{i,j}$ belongs to the intersection of all local ellipsoidal sets $\mathscr{X}^{(s)}_{i,j}$ $(s = 1, 2, \dots, M)$. Obviously, the intersection set has a smaller volume/region than all local ellipsoidal sets (see Fig. 2), thereby ensuring a better filtering performance. Based on this fact, we shall first determine a fused ellipsoidal set $\mathscr{X}^{(f)}_{i,j}$ that contains the intersection set. Then, we proceed to calculate corresponding fusion parameters such that the fused ellipsoidal set guarantees the satisfactory filtering performance.

Theorem 2: Consider the system (1), the logarithmic-type EDM (5)–(9), and the distributed set-membership filter (12). For $s = 1, 2, \dots, M$, let the local estimate $\hat{x}^{(s)}_{i,j}$ and the positive definite matrix $Q^{(s)}_{i,j}$ be given. Then, $\vec{\mathscr{X}}_{i,j} \subseteq \mathscr{X}^{(f)}_{i,j}$ if the fused estimate $\hat{x}^{(f)}_{i,j}$ and the positive definite matrix $Q^{(f)}_{i,j}$ are given as

$$\hat{x}^{(f)}_{i,j} \triangleq \bar{Q}^{(f)}_{i,j}\sum_{s=1}^{M}\varepsilon^{(s)}_{i,j}(Q^{(s)}_{i,j})^{-1}\hat{x}^{(s)}_{i,j} \tag{39}$$

$$Q^{(f)}_{i,j} \triangleq (1-\varpi_{i,j})\bar{Q}^{(f)}_{i,j} \tag{40}$$

where

$$\varepsilon^{(s)}_{i,j} \ge 0, \quad \sum_{s=1}^{M}\varepsilon^{(s)}_{i,j} = 1$$

$$(\bar{Q}^{(f)}_{i,j})^{-1} \triangleq \sum_{s=1}^{M}\varepsilon^{(s)}_{i,j}(Q^{(s)}_{i,j})^{-1}$$

$$\begin{aligned} \varpi_{i,j} \triangleq& \sum_{s=1}^{M}\varepsilon^{(s)}_{i,j}(\hat{x}^{(s)}_{i,j})^{\mathrm{T}}(Q^{(s)}_{i,j})^{-1}\hat{x}^{(s)}_{i,j} \\ &- (\hat{x}^{(f)}_{i,j})^{\mathrm{T}}(\bar{Q}^{(f)}_{i,j})^{-1}\hat{x}^{(f)}_{i,j} \end{aligned}$$

$$\vec{\mathscr{X}}_{i,j} \triangleq \mathscr{X}^{(1)}_{i,j}\bigcap\mathscr{X}^{(2)}_{i,j}\bigcap\cdots\bigcap\mathscr{X}^{(M)}_{i,j}.$$

Proof: First, for any $x_{i,j}$, it follows from Theorem 1 that

$$x_{i,j} \in \mathscr{X}^{(s)}_{i,j} \triangleq \mathscr{E}\left(\hat{x}^{(s)}_{i,j}, Q^{(s)}_{i,j}\right), \quad s = 1, 2, \dots, M \tag{41}$$

which means

$$x_{i,j} \in \vec{\mathscr{X}}_{i,j} \triangleq \mathscr{X}^{(1)}_{i,j}\bigcap\mathscr{X}^{(2)}_{i,j}\bigcap\cdots\bigcap\mathscr{X}^{(M)}_{i,j}. \tag{42}$$

Next, combining (16) and (41), it is easy to see that there exist positive scalars $\varepsilon^{(s)}_{i,j}$ with $\sum_{s=1}^{M}\varepsilon^{(s)}_{i,j} = 1$ such that the following inequality holds:

$$\sum_{s=1}^{M}\varepsilon^{(s)}_{i,j}(x_{i,j}-\hat{x}^{(s)}_{i,j})^{\mathrm{T}}(Q^{(s)}_{i,j})^{-1}(x_{i,j}-\hat{x}^{(s)}_{i,j}) \le 1. \tag{43}$$

Then, by some straightforward algebraic manipulations, it follows from (43) that

$$\begin{aligned} &\left(x_{i,j} - \bar{Q}^{(f)}_{i,j}\sum_{s=1}^{M}\varepsilon^{(s)}_{i,j}(Q^{(s)}_{i,j})^{-1}\hat{x}^{(s)}_{i,j}\right)^{\mathrm{T}}(\bar{Q}^{(f)}_{i,j})^{-1} \\ &\times\left(x_{i,j} - \bar{Q}^{(f)}_{i,j}\sum_{s=1}^{M}\varepsilon^{(s)}_{i,j}(Q^{(s)}_{i,j})^{-1}\hat{x}^{(s)}_{i,j}\right) \\ &+\sum_{s=1}^{M}\varepsilon^{(s)}_{i,j}(\hat{x}^{(s)}_{i,j})^{\mathrm{T}}(Q^{(s)}_{i,j})^{-1}\hat{x}^{(s)}_{i,j} \\ &-(\hat{x}^{(f)}_{i,j})^{\mathrm{T}}(\bar{Q}^{(f)}_{i,j})^{-1}\hat{x}^{(f)}_{i,j} \le 1. \end{aligned} \tag{44}$$

Substituting (39)–(40) into (44), we have

$$(x_{i,j}-\hat{x}^{(f)}_{i,j})^{\mathrm{T}}(Q^{(f)}_{i,j})^{-1}(x_{i,j}-\hat{x}^{(f)}_{i,j}) \le 1 \tag{45}$$

which implies that the following relationship is satisfied:

$$x_{i,j} \in \mathscr{X}^{(f)}_{i,j} \triangleq \mathscr{E}\left(\hat{x}^{(f)}_{i,j}, Q^{(f)}_{i,j}\right) \tag{46}$$

and, therefore, it is easy to deduce that $\vec{\mathscr{X}}_{i,j} \subseteq \mathscr{X}^{(f)}_{i,j}$. The proof is now complete. ■

From Theorem 2, it is easily seen that the volume of the fused ellipsoidal set $\mathscr{X}^{(f)}_{i,j}$ is dependent on parameters $\varepsilon^{(s)}_{i,j}$ $(s = 1, 2, \dots, M)$. In order to derive a fused ellipsoidal set that has a smaller volume than all local ellipsoidal sets $\mathscr{X}^{(s)}_{i,j}$ $(s = 1, 2, \dots, M)$ in the sense of matrix trace, we present the following theorem.

Theorem 3: Consider the system (1), the logarithmic-type EDM (5)–(9), and the distributed set-membership filter (12). For $s = 1, 2, \dots, M$, let the local estimate $\hat{x}^{(s)}_{i,j}$ and the positive definite matrix $Q^{(s)}_{i,j}$ be given. Then, $\mathrm{Tr}(Q^{(f)}_{i,j}) \le \mathrm{Tr}(Q^{(s)}_{i,j})$ $(s = 1, 2, \dots, M)$ if there exist positive scalars $\varepsilon^{(s)}_{i,j}$ with $\sum_{s=1}^{M}\varepsilon^{(s)}_{i,j} = 1$ satisfying

$$\sum_{s=1}^{M}\varepsilon^{(s)}_{i,j}(Q^{(s)}_{i,j})^{-1} \ge (1-\varpi_{i,j})(Q^{(t)}_{i,j})^{-1}, \quad t = 1, 2, \dots, M. \tag{47}$$

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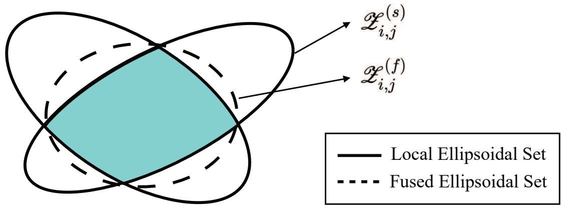

Fig. 2. The scheme for ellipsoid-based filtering fusion

Proof: It is evident that the following equivalences hold:

$$\begin{aligned}&\sum_{s=1}^{M}\varepsilon_{i,j}^{(s)}(Q_{i,j}^{(s)})^{-1}\geq(1-\varpi_{i,j})(Q_{i,j}^{(t)})^{-1}\\ \Longleftrightarrow\ &(1-\varpi_{i,j})^{-1}\sum_{s=1}^{M}\varepsilon_{i,j}^{(s)}(Q_{i,j}^{(s)})^{-1}\geq(Q_{i,j}^{(t)})^{-1}\\ \Longleftrightarrow\ &(1-\varpi_{i,j})^{-1}(\bar{Q}_{i,j}^{(f)})^{-1}\geq(Q_{i,j}^{(t)})^{-1}\\ \Longleftrightarrow\ &Q_{i,j}^{(f)}\leq Q_{i,j}^{(t)},\quad t=1,2,\ldots,M. \end{aligned}\tag{48}$$

We can conclude from (48) that $\mathrm{Tr}(Q_{i,j}^{(f)})\leq\mathrm{Tr}(Q_{i,j}^{(s)})$, which completes the proof. ■

Remark 4: In accordance with Theorems 1–3, an ellipsoid-based fusion filtering scheme is, for the first time, formulated to handle the distributed SMFF problem for 2-D systems over sensor networks. The proposed scheme has the following *advantages*: 1) the distributed SMF algorithm proposed in Theorems 1 only utilizes the information from the local sensor node and its neighboring nodes, which avoids the computational complexity issue associated with the increased number of sensor nodes; and 2) the ellipsoid-based fusion filtering rule proposed in Theorems 2–3 provides a better filtering performance than the local SMF algorithm in the sense of matrix trace.

C. Optimization Problem

Theorems 1–3 outline the procedure of seeking filter parameters and fusion parameters. It should be noted that this procedure does not provide an optimal solution. In what follows, some corollaries are presented to determine the filter parameters and the fusion parameters via optimizing the constraint sets in the sense of matrix trace.

Corollary 1: Consider the system (1), the logarithmic-type EDM (5)–(9), and the distributed set-membership filter (12). For $s=1,2,\ldots,M$, let the sequences of positive definite matrices $\{Q_{i,j+1}^{(s)}\}_{i,j\in\mathbb{N}}$ and $\{Q_{i+1,j}^{(s)}\}_{i,j\in\mathbb{N}}$ be given. The local ellipsoidal set $\mathscr{X}_{i+1,j+1}^{(s)}$ is minimized in the sense of matrix trace if there exist filter gains $K_{i,j+1}^{(s,1)}$, $K_{i+1,j}^{(s,2)}$, $G_{i,j+1}^{(st,1)}$ and $G_{i+1,j}^{(st,2)}$ such that the following optimization problem (**OP 1**) is feasible:

$$\textbf{OP 1}:\min_{\substack{K_{i,j+1}^{(s,1)},K_{i+1,j}^{(s,2)},\\ G_{i,j+1}^{(st,1)},G_{i+1,j}^{(st,2)}}}\ \mathrm{Tr}\left(Q_{i+1,j+1}^{(s)}\right)\quad\text{subject to }(26).\tag{49}$$

Corollary 2: Consider the system (1), the logarithmic-type EDM (5)–(9), and the distributed set-membership filter (12). For $s=1,2,\ldots,M$, let the local estimate $\hat{x}_{i,j}^{(s)}$ and the positive definite matrix $Q_{i,j}^{(s)}$ be given. Then, $\vec{\mathscr{X}}_{i,j}\subseteq\mathscr{X}_{i,j}^{(f)}$, $\mathrm{Tr}(Q_{i,j}^{(f)})\leq\mathrm{Tr}(Q_{i,j}^{(s)})$, and the fused ellipsoidal set $Q_{i,j}^{(f)}$ is minimized in the sense of matrix trace if there exist positive scalars $\varepsilon_{i,j}^{(s)}$ such that the following optimization problem (**OP 2**) is feasible:

$$\textbf{OP 2}:\min_{\varepsilon_{i,j}^{(s)},s=1,2,\ldots,M}\ \mathrm{Tr}\left(Q_{i,j}^{(f)}\right)\quad\text{subject to }(47)\text{ and }\sum_{s=1}^{M}\varepsilon_{i,j}^{(s)}=1.\tag{50}$$

For the purpose of numerical calculation, we describe the filter design procedure in Algorithm 1, which is based on the recursive linear matrix inequality (RLMI) approach.

Algorithm 1: Encoding-decoding-based distributed SMFF algorithm

Input: System initial conditions $x_{0,j}$, $x_{i,0}$, $\hat{x}_{0,j}^{(s)}$, $\hat{x}_{i,0}^{(s)}$, $\delta_{i,j}^{(s)}$, $\daleth_{i,j}^{(s,1)}$, $\daleth_{i,j}^{(s,2)}$ $(s=1,2,\ldots,M)$.
Output: $K_{i,j}^{(s,1)}$, $K_{i,j}^{(s,2)}$, $G_{i,j}^{(st,1)}$, $G_{i,j}^{(st,2)}$, $Q_{i,j}^{(f)}$, $\hat{x}_{i,j}^{(f)}$.

```
for i=1:𝒯 do
    for j=1:𝒯 do
        Compute the filter parameters K_{i,j}^{(s,1)}, K_{i,j}^{(s,2)},
        G_{i,j}^{(st,1)} and G_{i,j}^{(st,2)} by solving the OP 1 from
        Corollary 1;
        Compute the local estimate \hat{x}_{i,j}^{(s)} from sth sensor
        node by system (12);
        Compute the fused estimate x_{i,j}^{(f)} by Theorem 2
        and Corollary 2;
return K_{i,j}^{(s,1)}, K_{i,j}^{(s,2)}, G_{i,j}^{(st,1)}, G_{i,j}^{(st,2)}, Q_{i,j}^{(f)}, \hat{x}_{i,j}^{(f)};
```

Remark 5: So far, the distributed SMFF problem has been solved for the addressed nonlinear 2-D shift-varying system over sensor networks. Note that, in comparison to the rich body of existing literature on set-membership filtering problems, our results exhibit the following distinguishing features: 1) the addressed distributed SMFF problem is new that represents one of the first few attempts to cope with both the distributed SMF and the fusion filtering problems for nonlinear 2-D shift-varying systems over sensor networks; 2) the proposed logarithmic-type EDM is new, which is designed based on the logarithmic-type zooming-in/out encoder and decoder under the 2-D setting and is capable of dealing with dynamics evolving along both horizontal and vertical coordinates; and 3) the designed distributed SMFF algorithm is new, scalable, and efficient.

Remark 6: This paper launches a systematic investigation on the distributed SMFF issue for a class of nonlinear 2-D shift-varying systems over sensor networks in the context of networked systems with certain engineering-oriented complexities (i.e., EDMs and UBB noises). By exploiting a

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combination of several up-to-date approaches such as set-membership filtering method, interval analysis technique, two-dimensional mathematical induction approach, and set theory, the addressed problem has been thoroughly examined and the desired parameters have been formulated in terms of the solutions to a set of optimization problems. Within the established framework, it is not difficult to extend our results to more general systems with more complicated dynamics with more complex network-induced phenomena.

IV. Illustrative Example

In this section, the effectiveness of the proposed encoding-decoding-based distributed SMFF algorithm is verified by a simulation example. The system parameters are given as

$$
\begin{aligned}
f^{(1)}(x_{i,j}) &= \begin{bmatrix} 0.3x_{i,j}^1 + 0.05\sin(x_{i,j}^2) \\ 0.25x_{i,j}^2 + 0.1\cos(x_{i,j}^1) \end{bmatrix} \\
f^{(2)}(x_{i,j}) &= \begin{bmatrix} 0.45x_{i,j}^1 + 0.1\sin(x_{i,j}^2) \\ 0.2x_{i,j}^2 + 0.15\sin(x_{i,j}^1) \end{bmatrix} \\
B_{i,j}^{(1)} &= \begin{bmatrix} 0.55 + 0.3\mathrm{e}^{-3i} \\ 0.05 \end{bmatrix}, \quad B_{i,j}^{(2)} = \begin{bmatrix} 0.45 \\ 0.1\cos(i) \end{bmatrix} \\
C_{i,j}^{(s)} &= \begin{cases} \begin{bmatrix} 0.8 & 0.2 + \mathrm{e}^{-2i} \end{bmatrix}, & s = 1 \\ \begin{bmatrix} 1.1 & 0.25 + \mathrm{e}^{-j} \end{bmatrix}, & s = 2 \\ \begin{bmatrix} -0.1 & 0.5 + \sin(5j) \end{bmatrix}, & s = 3 \\ \begin{bmatrix} -0.15 & 0.35 + \sin(3j) \end{bmatrix}, & s = 4 \\ \begin{bmatrix} 0.25 - \mathrm{e}^{-2j} & 0.75 \end{bmatrix}, & s = 5 \\ \begin{bmatrix} -0.85 & -0.25 + \mathrm{e}^{-2i} \end{bmatrix}, & s = 6 \end{cases} \\
D_{i,j}^{(1)} &= 0.65, \quad D_{i,j}^{(2)} = 0.3, \quad D_{i,j}^{(3)} = 0.5 \\
D_{i,j}^{(4)} &= 0.45, \quad D_{i,j}^{(5)} = 0.35, \quad D_{i,j}^{(6)} = 0.55.
\end{aligned}
$$

The process noise $w_{i,j}$ and the measurement noise $v_{i,j}^{(s)}$ are selected as

$$
\begin{aligned}
w_{i,j} &= 0.15\sin(0.3(i+j)) \\
v_{i,j}^{(1)} &= 0.1\sin(0.25(i+j)) \\
v_{i,j}^{(2)} &= 0.18\sin(0.5(i+j)) \\
v_{i,j}^{(3)} &= 0.05\cos(0.4i), \quad v_{i,j}^{(4)} = 0.2\cos(0.3i) \\
v_{i,j}^{(5)} &= 0.3\sin(0.2j), \quad v_{i,j}^{(6)} = 0.25\sin(0.25j)
\end{aligned}
$$

whose weighting matrices are chosen as $W_{i,j} = 0.03I$, $V_{i,j}^{(1)} = 0.02I$, $V_{i,j}^{(2)} = 0.03I$, $V_{i,j}^{(3)} = 0.01I$, $V_{i,j}^{(4)} = 0.05I$, $V_{i,j}^{(5)} = 0.1I$ and $V_{i,j}^{(6)} = 0.1I$. The quantization density is set as $\wp = 0.7$. The scaling parameters are selected as $\delta_{i,j}^{(s)} = 0.9$ and $\daleth_{i,j}^{(s,1)} = \daleth_{i,j}^{(s,2)} = 0.45I$ $(s = 1, 2, \ldots, 6)$. The topology structure of the sensor networks is shown in Fig. 3. Let the initial conditions be given as

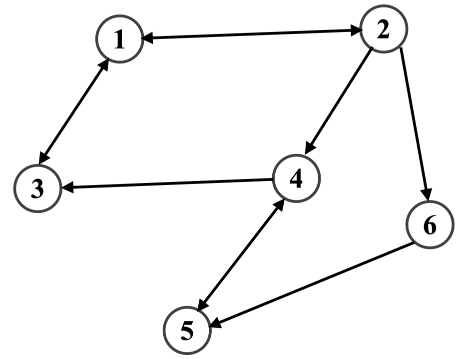

Fig. 3. The topology structure of the sensor networks

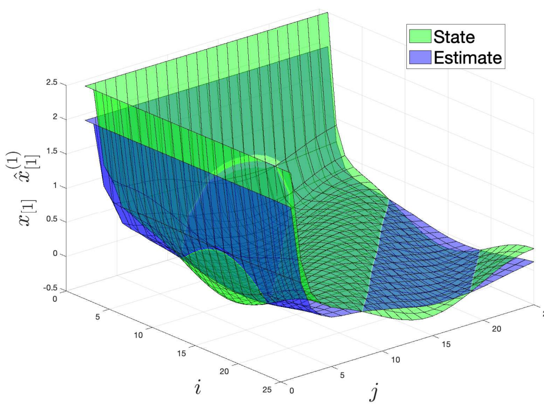

Fig. 4. The system state $x_{[1]}$ and local estimate $\hat{x}_{[1]}^{(1)}$ of sensor node 1

$$
\begin{cases}
x_{0,j} = x_{i,0} = \begin{bmatrix} 2.5 & 1.8 \end{bmatrix}^{\mathrm{T}}, & \forall i, j \in [0 \ 25] \\
\hat{x}_{0,j}^{(1)} = \hat{x}_{i,0}^{(1)} = \begin{bmatrix} 2 & 1.5 \end{bmatrix}^{\mathrm{T}}, & \forall i, j \in [0 \ 25] \\
\hat{x}_{0,j}^{(2)} = \hat{x}_{i,0}^{(2)} = \begin{bmatrix} 3 & 2 \end{bmatrix}^{\mathrm{T}}, & \forall i, j \in [0 \ 25] \\
\hat{x}_{0,j}^{(3)} = \hat{x}_{i,0}^{(3)} = \begin{bmatrix} 2.2 & 1.45 \end{bmatrix}^{\mathrm{T}}, & \forall i, j \in [0 \ 25] \\
\hat{x}_{0,j}^{(4)} = \hat{x}_{i,0}^{(4)} = \begin{bmatrix} 1.9 & 1.9 \end{bmatrix}^{\mathrm{T}}, & \forall i, j \in [0 \ 25] \\
\hat{x}_{0,j}^{(5)} = \hat{x}_{i,0}^{(5)} = \begin{bmatrix} 2.5 & 1.7 \end{bmatrix}^{\mathrm{T}}, & \forall i, j \in [0 \ 25] \\
\hat{x}_{0,j}^{(6)} = \hat{x}_{i,0}^{(6)} = \begin{bmatrix} 2.8 & 2 \end{bmatrix}^{\mathrm{T}}, & \forall i, j \in [0 \ 25].
\end{cases}
$$

The simulation results are presented in Figs. 4–12. Among them, Figs. 4–9 plot the system state $x_{i,j}$ and the local estimates $\hat{x}_{i,j}^{(s)}$ of sensor nodes 1, 3, and 6, respectively. Fig. 10–11 depicts the system state $x_{i,j}$ and the fused estimate $\hat{x}_{i,j}^{(f)}$. The trace evolution of matrices $Q_{i,j}^{(f)}$ and $Q_{i,j}^{(s)}$ $(s = 1, 2, \ldots, 6)$ are shown in Fig. 12, from which we can see that the $\mathrm{Tr}(Q_{i,j}^{(f)})$ is the smallest of all. Thus, Figs. 4–12 show the effectiveness of the proposed encoding-decoding-based distributed SMFF algorithm.

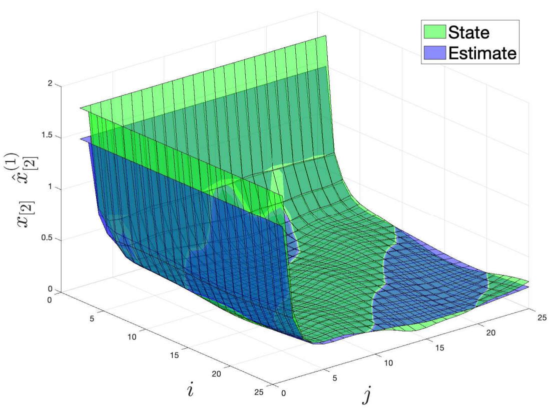

Fig. 5. The system state $x_{[2]}$ and local estimate $\hat{x}^{(1)}_{[2]}$ of sensor node 1

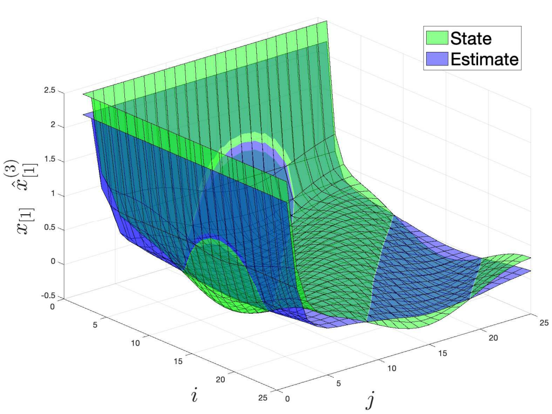

Fig. 6. The system state $x_{[1]}$ and local estimate $\hat{x}^{(1)}_{[1]}$ of sensor node 3

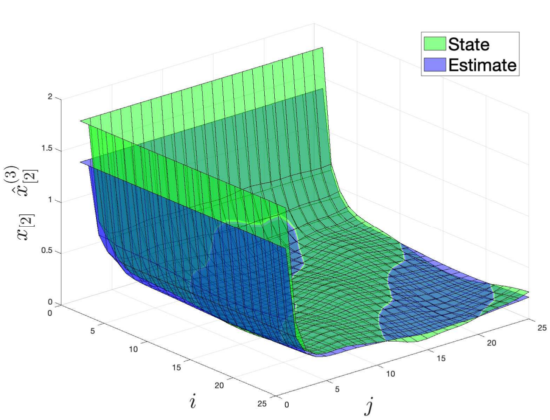

Fig. 7. The system state $x_{[2]}$ and local estimate $\hat{x}^{(1)}_{[2]}$ of sensor node 3

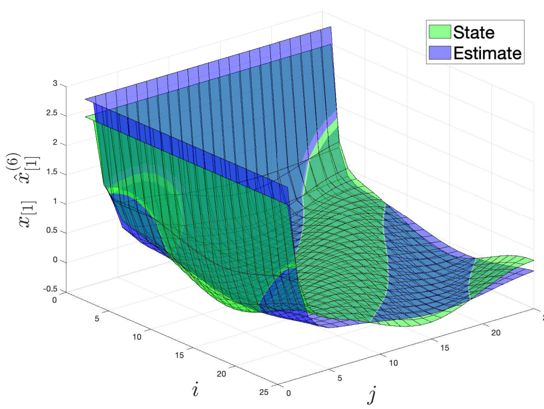

Fig. 8. The system state $x_{[1]}$ and local estimate $\hat{x}^{(1)}_{[1]}$ of sensor node 6

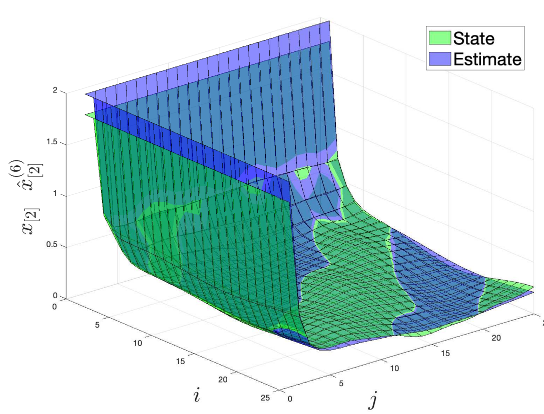

Fig. 9. The system state $x_{[2]}$ and local estimate $\hat{x}^{(1)}_{[2]}$ of sensor node 6

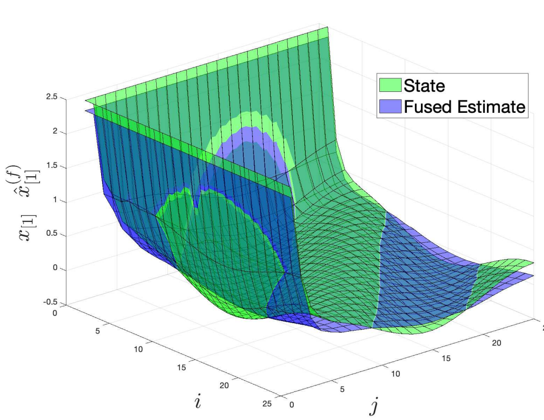

Fig. 10. The system state $x_{[1]}$ and fused estimate $\hat{x}^{(f)}_{[1]}$

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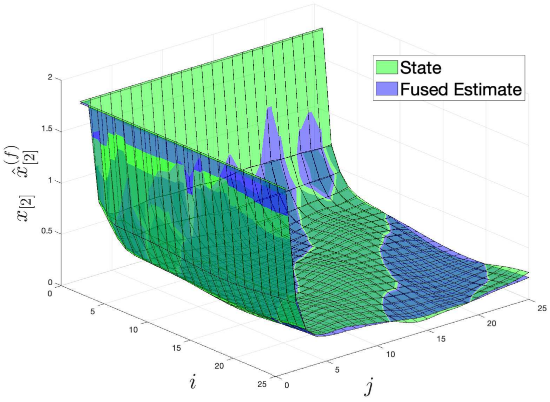

Fig. 11. The system state $x_{[2]}$ and fused estimate $\hat{x}^{(f)}_{[2]}$

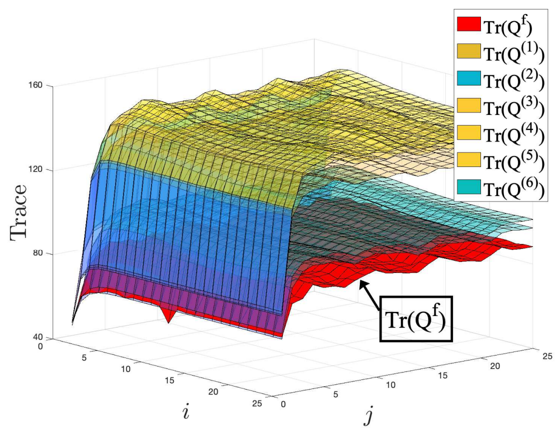

Fig. 12. The trace evolution of $Q^{(f)}$ and $Q^{(s)}$ $(s = 1, 2, \ldots, 6)$

V. Conclusion

This paper has addressed the distributed SMFF problem for a class of nonlinear 2-D shift-varying systems subject to UBB noises over sensor networks. A new logarithmic-type EDM has been designed for 2-D systems, where the zooming-in/out-based encoder and decoder have been utilized to strengthen communication security and efficiency. An ellipsoid-based fusion filtering rule has been developed for 2-D systems over sensor networks to confine the system state to a fused ellipsoidal set in a global view. It has been shown that the fused filtering performance is better than the local filtering performance in the sense of matrix trace. By resorting to the two-dimensional mathematical induction approach and the set theory, the feasibility of the proposed fusion filtering algorithm has been examined, and the parameters can be computed by solving a series of optimization problems. Finally, a simulation example has been provided to verify the usefulness of the proposed fusion filtering scheme. One of the future research topics would be to extend the main results in this paper to systems with network-induced phenomena [6], [12], [22], [44], [47]. It is worth mentioning that the proposed distributed fusion filtering scheme in this paper can be applied to some practical applications such as chemical process and the specific applications are our future work.

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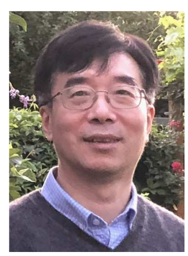

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