Comments on "Finite-Time H_∞ Fuzzy Control of Nonlinear Jump Systems With Time Delays Via Dynamic Observer-Based State Feedback"

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THE resulting closed-loop error dynamic Markov jump system (MJS) in this comment in paper [1] is described by

$$\begin{cases} \dot{\tilde{x}}(t) = G_{ij}(r)\tilde{x}(t) + M_{ij}(r)\tilde{x}(t-H) + U_i(r)d(t) \\ z(t) = H_{ij}(r)\tilde{x}(t) + N_{ij}(r)\tilde{x}(t-H) + V_i(r)d(t) \\ \tilde{x}(t) = [\lambda^T(t), \lambda^T(t) - \varphi^T(t)]^T, r = r_0, t \in [-H, 0] \end{cases} \tag{1}$$

where coefficient matrices $G_{ij}(r), M_{ij}(r), U_i(r), H_{ij}(r), N_{ij}(r)$, and $V_i(r)$ are the same as [1, eq. (7)], and the external disturbance $d(t)$ is time-varying and satisfies the constraint condition

$$\int_0^T d^T(t)d(t)dt \le d,\ d \ge 0. \tag{2}$$

Furthermore, the output of this closed-loop error dynamic MJS (1) satisfies the following H_∞ performance:

$$\mathrm{E}\big[\int_0^T z^T(t)z(t)dt\big] \le \gamma^2 \mathrm{E}\big[\int_0^T d^T(t)d(t)dt\big]. \tag{3}$$

Paper [1] presented sufficient conditions ensuring stochastic finite-time boundedness of the error dynamic closed-loop MJS (1), and designed an observer-based state feedback controller ensuring finite-time boundedness and satisfying a prescribed level of H_∞ disturbance attenuation of the fuzzy MJS. However, it is not right that the nonlinear term $X(r)QX(r)$ is replaced by a mode-dependent variable $Q(r)$ since both $X(r)$ and Q are matrix variables [1, Th. 3]. Therefore, both $\Lambda_{1ij}(r)$ and $\Lambda_{4ij}(r)$ are nonlinear, which result that conditions (17) and (18) are not linear matrix inequalities (LMIs) in the key Theorem 3. From [1, proof of Th. 2], one could see that the disturbance attention γ should be replaced with $\gamma e^{-\eta T/2}$. In the following, we give the right statement for Theorem 3. To maintain the integrity of this paper, we also give the presentation of [1, Ths. 1 and 2]. For a more detailed proof, see [2, Th. 3]. Then, the improved optimal algorithms and new simulation results are provided.

Theorem 1: For given $T > 0, \eta > 0, c_1 > 0, d > 0,$ and $\tilde{R}(r) > 0$, the closed-loop error dynamic MJS (1) is stochastically finite-time bounded with respect to $(c_1, c_2, T, \tilde{R}(r), d)$, if there exist scalars $\gamma > 0, c_2 > 0$, a set of mode-dependent symmetric positive-definite matrices $\{\tilde{P}(r), r \in \Psi\}$, and a positive-definite matrix $\tilde{Q}$, for all $r \in \Psi$, such that the following inequalities hold:

$$\begin{bmatrix} \Xi_{ij}(r) & \tilde{P}(r)M_{ij}(r) & \tilde{P}(r)U_i(r) \\ * & -\tilde{Q} & 0 \\ * & * & -\gamma^2 e^{-\eta T} I \end{bmatrix} < 0 \tag{4}$$

$$c_1(\bar{\sigma}_{\tilde{P}} + H\sigma_{\tilde{Q}}) + \frac{\gamma^2 d e^{-\eta T}}{\eta}(1 - e^{-\eta T}) < c_2 e^{-\eta T} \underline{\sigma}_{\tilde{P}} \tag{5}$$

where $\Xi_{ij}(r), \tilde{P}(r), \bar{\sigma}_{\tilde{P}}, \underline{\sigma}_{\tilde{P}}$ and $\sigma_{\tilde{Q}}$ are the same as [1, Th. 1].

Theorem 2: For given $T > 0, \eta > 0, c_1 > 0, d > 0,$ and $\tilde{R}(r) > 0$, the closed-loop error dynamic MJS (1) is stochastically finite-time bounded with respect to $(c_1, c_2, T, \tilde{R}(r), d)$ and satisfies the cost function inequality (3) for all admissible $d(t)$ with the constraint condition (2), if there exist scalars $\gamma > 0, c_2 > 0$, a set of mode-dependent symmetric positive-definite matrices $\{\tilde{P}(r), r \in \Psi\}$, and a symmetric positive-definite matrix $\tilde{Q}$, for all $r \in \Psi$, such that condition (5) and the following inequalities hold:

$$\begin{bmatrix} \Xi_{ij}(r) & \tilde{P}(r)M_{ij}(r) & \tilde{P}(r)U_i(r) & H_{ij}^T(r) \\ * & -\tilde{Q} & 0 & N_{ij}^T(r) \\ * & * & -\gamma^2 e^{-\eta T} I & V_i^T(r) \\ * & * & * & -I \end{bmatrix} < 0. \tag{6}$$

Hence, applying the Schur complement property, [1, Th. 3] should be corrected as follows.

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Theorem 3: For given $T > 0, \eta > 0, c_1 > 0, d > 0$, and $R(r) > 0$, there exist a state feedback control gain $K_i(r) = Y_i(r)X^{-1}(r)$ and an observer gain $F_i(r) = -X(r)C_{1i}^T(r)$ such that the closed-loop error dynamic MJS (1) is stochastically finite-time bounded with respect to $(c_1, c_2, T, \tilde{R}(r), d)$ and satisfies the cost function inequality (3) for all admissible $d(t)$ with the constraint condition (2), if there exist scalars $\gamma > 0, c_2 > 0, \sigma_1 > 0, \sigma_2 > 0$, a set of mode-dependent symmetric positive-definite matrices $\{X(r), r \in \Psi\}$, and a symmetric positive-definite matrix Q, for all $r \in \Psi$ and $i, j = 1, 2, \ldots, S$, such that the following inequalities hold:

$$\Sigma_{ii}(r) = \begin{bmatrix} \Sigma_{1ii}(r) & \Sigma_{2ii}(r) & \Sigma_{4ii}(r) \\ * & \Sigma_{3ii}(r) & 0 \\ * & * & \Sigma_{5ii}(r) \end{bmatrix} < 0 \tag{7}$$

$$\Sigma_{ij}(r) < 0, i < j \tag{8}$$

$$\sigma_1 R^{-1}(r) < X(r) < R^{-1}(r) \tag{9}$$

$$0 < Q < \sigma_2^{-1} R(r) \tag{10}$$

$$\begin{bmatrix} -\frac{c_2}{e^{\eta T}} + \frac{\gamma^2 d}{\eta e^{\eta T}}(1 - e^{-\eta T}) & \sqrt{c_1} & \sqrt{c_1 H} \\ * & -\sigma_1 & 0 \\ * & * & -\sigma_2 \end{bmatrix} < 0 \tag{11}$$

where

$$\Sigma_{ij}(r) = \begin{bmatrix} \Sigma_{1ij}(r) + \Sigma_{1ji}(r) & \Sigma_{2ij}(r) & \Sigma_{4ij}(r) \\ * & \frac{1}{2}\Sigma_{3ij}(r) & 0 \\ * & * & \frac{1}{2}\Sigma_{5ij}(r) \end{bmatrix}$$

$$\Sigma_{1ij}(r) = [\, \Sigma_{1ij}^{11}(r) \quad \Sigma_{1ij}^{12}(r) \,]$$

$$\Sigma_{1ij}^{11}(r) = \begin{bmatrix} \Lambda_{1ij}(r) & \Lambda_{2ij}(r) & A_{di}(r) \\ * & \Lambda_{4ij}(r) & 0 \\ * & * & -Q \\ * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}$$

$$\Sigma_{1ij}^{12}(r) = \begin{bmatrix} 0 & B_{di}(r) & \Lambda_{3ij}(r) \\ \Lambda_{5ij}(r) & B_{di}(r) & -Y_j^T(r)D_i^T(r) \\ 0 & 0 & C_{d2i}^T(r) \\ -Q & 0 & 0 \\ * & -\gamma^2 e^{-\eta T} I & D_{di}^T(r) \\ * & * & -I \end{bmatrix}$$

$$\Sigma_{2ij}(r) = \begin{bmatrix} 0 & 0 & \Lambda_{6ij}(r) & 0 \\ X(r)C_{1i}^T(r) & X(r)C_{1j}^T(r) & 0 & \Lambda_{6ij}(r) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Sigma_{3ij}(r) = -\mathrm{diag}\{I, I, \Lambda_{7ij}(r), \Lambda_{7ij}(r)\}$$

$$\Sigma_{4ij}(r) = \begin{bmatrix} X(r) & 0 \\ 0 & X(r) \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Sigma_{5ij}(r) = -\mathrm{diag}\{\sigma_2 R^{-1}(r), \sigma_2 R^{-1}(r)\}$$

$$\begin{aligned} \Lambda_{1ij}(r) &= A_i(r)X(r) + X(r)A_i^T(r) + B_i(r)Y_j(r) \\ &\quad + Y_j^T(r)B_i^T(r) + (\pi_{rr} - \eta)X(r) \\ \Lambda_{2ij}(r) &= -B_i(r)Y_j(r) \\ \Lambda_{3ij}(r) &= X(r)C_{2i}^T(r) + Y_j^T(r)D_i^T(r) \\ \Lambda_{4ij}(r) &= A_i(r)X(r) + X(r)A_i^T(r) + (\pi_{rr} - \eta)X(r) \\ \Lambda_{5ij}(r) &= A_{di}(r) + X(r)C_{1i}^T(r)C_{d1j}(r) \\ \Lambda_{6ij}(r) &= [\sqrt{\pi_{r1}}X(r), \ldots, \sqrt{\pi_{r(r-1)}}X(r) \\ &\quad \sqrt{\pi_{r(r+1)}}X(r), \ldots, \sqrt{\pi_{rN}}X(r)] \\ \Lambda_{7ij}(r) &= \mathrm{diag}\{X(1), \ldots, X(r-1) \\ &\quad X(r+1), \ldots, X(N)\}. \end{aligned}$$

Proof: Notice that [1, condition (22)] is equivalent to the following inequality:

$$\sum_{i=1}^{S}\sum_{j>i}^{S} h_i(\mu(t))h_j(\mu(t))[\Sigma_{ij}(r) + \Sigma_{ji}(r)] + \sum_{i=1}^{S} h_i^2(\mu(t))\Sigma_{ii}(r) < 0. \tag{12}$$

Applying the Schur complement lemma, we can obtain from (7) and (8) that condition (12) holds. Then, the remaining proof is similar to the proof of [1, Th. 3] and is thus omitted.

Remark 1: From the aforementioned theorem, we can observe that conditions (7), (8), and (11) in Theorem 3 are not strict in an LMI form; however, once we fix the parameter η, then the feasibility of conditions stated in Theorem 3 can be turned into the following LMI-based feasibility problem with a fixed parameter η, respectively:

$$\begin{aligned} &\min \quad (c_2 + \delta) \\ &X(r), Y_i(r), Q, \sigma_1, \sigma_2 \\ &\text{s.t. (7)–(11) with } \delta = \gamma^2. \end{aligned} \tag{13}$$

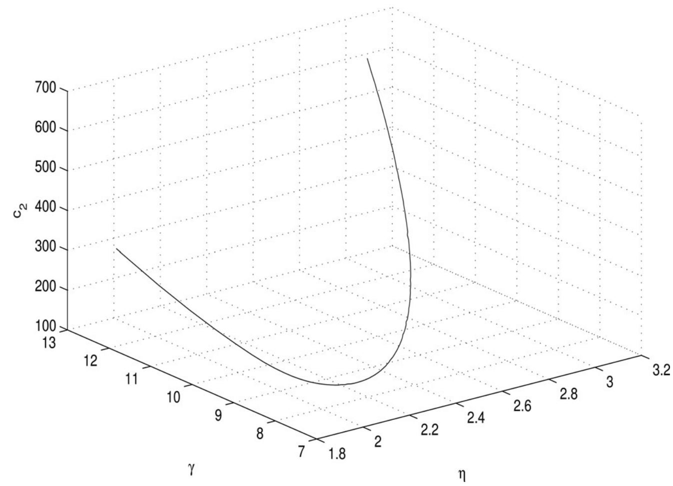

Fig. 1. Local optimal bound of γ and c_2.

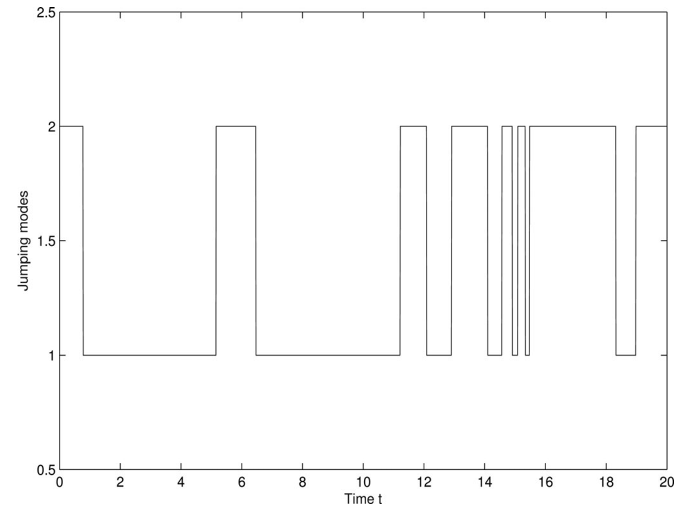

Fig. 2. Estimation of changing between modes during the simulation with the initial mode 2.

Furthermore, one can find the parameter η by an unconstrained nonlinear optimization approach, from which a locally convergent solution can be obtained by using the program *fminsearch* in the optimization toolbox of MATLAB.

Consider the numerical example in [1]; we can obtain different simulation results that are based on the aforementioned corrected results.

Let $c_1 = 0.5, T = 2, R(r) = I_2, d = 4,$ and $H = 0.9$, by Theorem 3; the optimal bound with minimum value of $c_2 + \delta$ relies on the parameter η. We can find a feasible solution when $1.80 \leq \eta \leq 10.63$. Fig. 1 shows the optimal value with different values of η. Noting that when $\eta = 3$, it yields the optimal values $\gamma = 12.4570$ and $c_2 = 618.8601$. Furthermore, by using the program *fminsearch* in the optimization toolbox of MATLAB starting at $\eta = 2$, the locally convergent solution can be derived as

$$K_1(1) = [-0.0079 \quad -0.0434]$$

$$K_2(1) = [-0.0609 \quad -0.0562]$$

$$K_1(2) = [0.0050 \quad -0.0362]$$

$$K_2(2) = [-0.0645 \quad -0.3237]$$

$$F_1(1) = F_2(1) = \begin{bmatrix} -0.0031 \\ -0.1999 \end{bmatrix}$$

$$F_1(2) = F_2(2) = \begin{bmatrix} -0.0120 \\ -0.1987 \end{bmatrix}$$

with $\eta = 2.0248$ and the optimal values $\gamma = 7.5773$, and $c_2 = 176.6233$. Now, we assume that the unknown inputs are white noise power 0.01 over a finite-time interval $t \in [0, 20]$. With the initial mode $r_0 = 2$ and the initial states $\lambda^T(t) = [0.5 \quad -0.3]$ and $\varphi^T(t) = [0.8 \quad -0.5]$ for all $t \in [-0.9, 0]$, the jumping modes r_t and system states $x(t)$ are depicted in Figs. 2–4, respectively. It can be seen from Figs. 2–4 that the MJS is finite-time stabilized by the designed finite-time H_∞ controller and observer, which demonstrates the effectiveness of the theoretical results.

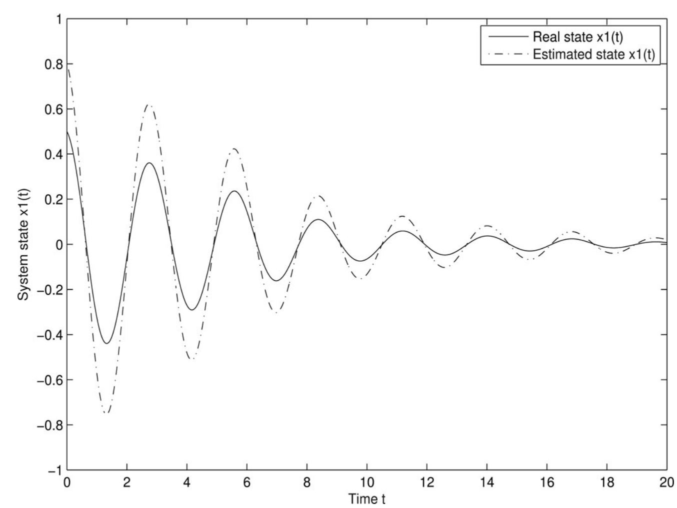

Fig. 3. Response of the system $x_1(t)$.

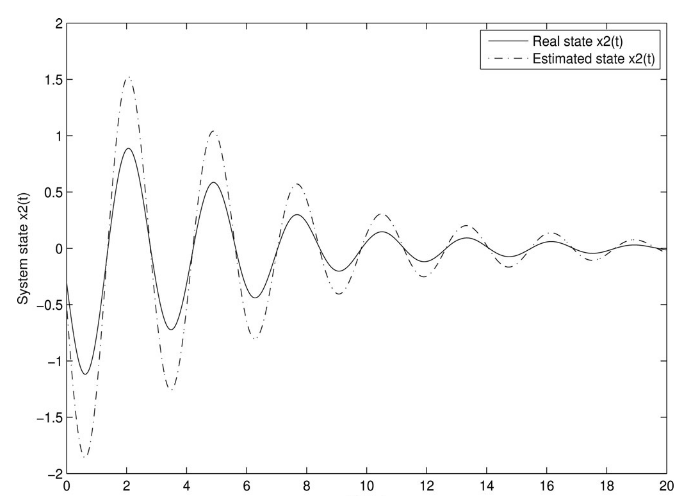

Fig. 4. Response of the system $x_2(t)$.

Remark 2: In order to get the minimum optimal value of $c_2 + \delta$, by Theorem 3, one can first find the range of the feasible solution of the parameter η, and then the locally convergent solution could be obtained according to Remark 1.

Remark 3: It is necessary to point out that the problem of H_∞ filtering or control has been investigated for discrete-time T–S fuzzy systems with time delays. The problem of H_∞ filtering was studied for a class of discrete-time T–S fuzzy time-varying delay systems in [3]. By applying the delay partitioning approach, the authors in [4] and [5] addressed the problem of H_∞ control for discrete-time T–S fuzzy systems with time delays. We can also discuss the problem of observer-based finite-time H_∞ control for discrete-time fuzzy jump nonlinear systems with time delays and obtain the corresponding similar results.

Remark 4: Recently, the topic on fault detection of T–S fuzzy systems has attracted considerable attention [6]–[9]. It is noted that the results that are proposed in [8] and [9] can be extended to the problem of finite-time H_∞ control for discrete-time T–S fuzzy systems with fault estimation, and more results on stochastic finite-time control for T–S fuzzy jump systems with fault control will be studied in our future work.

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