An Improved Fuzzy Event-Triggered Asynchronous Dissipative Control to T–S FMJSs With Nonperiodic Sampled Data

Xin Wang, Ju H. Park, Huilan Yang, and Shouming Zhong

***Abstract*—In this article, an investigation about the issue of fuzzy event-triggered asynchronous dissipative control for T–S fuzzy Markov jump systems (FMJSs) with unknown transition probabilities and nonuniform sampling is conducted. First of all, a mode-dependent looped Lyapunov–Krasovskii functional (LKF) is proposed, which takes full use of the available information not only from sawtooth structure characteristics but also from the inner sampling dynamics. Meanwhile, a hidden Markov chain is employed to depict the mismatch between the original system and the state-dependent fuzzy event-triggered controller. Then, based on the LKF methodology, matrix inequality techniques, and the reciprocally convex combination protocol, some relaxed criteria with respect to the stochastic stable of the considered system and the desired dissipative performance are derived, simultaneously. A numerical experiment is given to illustrate the significance of the theoretical results.**

***Index Terms*—Asynchronous control, fuzzy event triggered control, Fuzzy Markov systems, reciprocally convex technique.**

I. Introduction

RECOGNIZED as one of the hybrid dynamic systems, Markov jump systems (MJSs) have been successfully applied to various fields of science and engineering, including but not limited to industrial manufacturing, aviation systems, communication system and transportation [1]–[5]. The crucial feature of MJS is usually utilized to model many physical plants with abrupt changes in parameters and structures caused by component failures, environmental noises, or actuators failures. Accordingly, it comes to no surprise that the research of dynamical behavior in MJSs has harvested fruitful results. Please refer to [6]–[10] and the relevant references therein. On the other side, the T–S fuzzy model proposed by Takagi and Sugeno [11] has acquired increasing attention from various fields in the last decade. The reason why we study T–S fuzzy systems (FSs) is that they can be utilized to approximate complex nonlinear systems by employing a quantity of IF–THEN rules on fuzzy sets [11]–[16]. Recently, stability analysis of fuzzy MJS (FMJSs) has been hotly debated on account of its mode evolution in T–S FSs, thus many excellent achievements have emerged in [17]–[19] and references therein. For example, by taking external noise and matched uncertainties into account, the authors in [20] concerned the dissipativity-based asynchronous stability of FMJSs via fuzzy integral sliding mode control. Xue *et al.* [21] discussed the imperfect premise matching $\mathcal{H}_\infty$ output tracking control for FMJSs, some new fuzzy-basis-dependent stability criteria were derived under a mode-dependent LKF approach. In light of asynchronous switching resulting from the sampling intervals, the dissipative sampled-data control scheme with respect to FMJSs has been designed in [22], and the developed results can be utilized to $\mathcal{H}_\infty$ and passivity issues. Meanwhile, a hidden Markov model (HMM) is introduced to address the fuzzy jumping genetic regulatory networks in [23], where the filters cannot directly be applied to the mode information of the plant.

In addition, with the development of digital technology, digital control has an important status in speed, accuracy, and low cost. Particularly, sampled-data control, an important digital control strategy, has attracted much more attention on account of its efficiency, high reliability, and easy installation. Ulteriorly, in the respect of the event-triggered scheme [29]-[31], the sampled data will be delivered when it meets a certain event-triggered condition. Different from the ordinary sampled-data control protocol [24]–[28], the event-triggered control (ETC) mechanism can effectively improve limited network bandwidth by reducing data transmission rate. Naturally, it is significant to research stabilization for T–S FSs via event-triggered control. In this regard, by considering both the available information of threshold error and sampling behavior, Wang *et al.* [32] improved the communication efficiency of T–S FSs by an adaptive ETC. Further, Cheng *et al.* [33] proposed a novel ETC to study the stabilization of FMJSs with general switching rates, the new ETC is characterized by state-independent and state-dependent indexes, which is utilized to derive low transmission rates.

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Finally, for the sake of saving network bandwidth, Xue *et al.* [34] concerned the problem of the ETC for delayed FMJSs with general transition probability matrix (TPM), where the TPM is partly available.

Despite the unremitting efforts devoted by many researchers to the study of FMJSs, many aspects are still worthy of attention and need to be further improved. First, these works in [10] and [17]–[20] were grounded on an ideal assumption such that the TPM is available for the related elements. Actually, this is an unrealistic expectation. Because it is difficult to estimate or measure the value of all elements, thus how to achieve the stabilization of FMJSs with unknown measurable transition probabilities has become a tricky and crucial point of the theoretical study. Besides, the fuzzy controllers designed in [33] shared the same modes with the considered systems. However, because of environmental noises and communication constraints, the controllers cannot receive the accurate modes information of plants. Accordingly, it is necessary to improve the existing traditional controllers for FMJSs. On the other hand, it is of great necessity to further explore and study the FETC based on nonperiodic sampled state with respect to the FMJSs, for the reason that it is not capable of improving the communication efficiency, but also reducing the data transmission rate. However, this is seldom mentioned in FMJSs. What's more, the aforementioned results on LKF methodology can be applied to determine the stability criteria for many FMJSs, but their conditions seem to be restrictive. For example, most of the above literature requires that the LKF is both positive definite and fixed on each subinterval [20], [33], [34]. All these motivate us to conduct research works in this article.

As a result of the above analysis, the issue of asynchronous fuzzy event-triggered control (AFETC) for FMJSs with unknown transition probabilities and nonuniform sampling is elaborated on. The main advantages of this article can be highlighted as follows:

1) The studied FMJSs are subject to the unknown transition probabilities, which may be partly unknown or even completely unknown. So, the criteria can loosen some constraints as compared to existing works [18], [20], [21], [33], [34].
2) A new AFETC based on the HMM and nonperiodic sampled data is constructed to save the limited communication resource as compared to the published results [30]–[33], where the network-induced delays randomly occur.
3) Unlike earlier works, the restrictions with respect to the LKF is very loose. By using LKF methodology, matrix inequality techniques, as well as the reciprocally convex protocol, some new criteria with respect to the stochastic stable of the considered system and the desired dissipative performance are derived, simultaneously.

Notations: Throughout this article, $\mathbb{R}^n$ denotes n-dimensional Euclidean space, and $\mathbb{R}^{n\times m}$ stands for $n \times m$ real matrices. Sym$\{\mathcal{X}\}$ represents $\mathcal{X}+\mathcal{X}^T$. The superscript T and -1 stand for the transpose and inverse, respectively. The symbol $*$ is used to represent the transposed element in symmetric matrix. diag$\{...\}$ denotes a diagonal matrix.

II. Preliminaries

Consider a fuzzy HMM-based MJS, the ith rule of the FMJS is presented as follows:

$Plant\ Rule\ i$: If $\theta_1(t)$ is ω_{i1}, $\theta_2(t)$ is ω_{i2}, …, and $\theta_p(t)$ is ω_{ip},

THEN

$$\begin{cases} \dot{x}(t) = \mathcal{A}_{\sigma(t),i}x(t) + \mathcal{G}_{\sigma(t),i}\omega(t) + \mathcal{B}_{\sigma(t),i}u(t) \\ z(t) = \mathcal{C}_{\sigma(t),i}x(t) + \mathcal{F}_{\sigma(t),i}\omega(t) + \mathcal{D}_{\sigma(t),i}u(t) \end{cases} \tag{1}$$

where $i \in \mathcal{S} = \{1, 2, \ldots, r\}$ is the number of IF–THEN rules; $\theta(t) = (\theta_1(t), \theta_2(t), \ldots \theta_p(t))$ is the premise variable vector; $\{\omega_{ij}\}(j = 1, 2, \ldots, p)$ denotes the fuzzy sets; $x(t) \in \mathbb{R}^n$ represents state vector, $u(t) \in \mathbb{R}^{n_u}$ and $z(t) \in \mathbb{R}^{n_z}$ imply control input and output vector. $\omega(t) \in \mathbb{R}^{\omega}$ represents external disturbance belonging to $l_2[0, +\infty)$. $\mathcal{A}_{\sigma(t),i}$, $\mathcal{B}_{\sigma(t),i}$, $\mathcal{C}_{\sigma(t),i}$, $\mathcal{D}_{\sigma(t),i}$, $\mathcal{F}_{\sigma(t),i}$, and $\mathcal{G}_{\sigma(t),i}$ are known constant matrices. $\sigma(t) \in \mathcal{N} = \{1, 2, \ldots, N\}$ stands for the system mode, which describes the jump phenomenon. The transition rate matrix (TRM) $\Lambda = \{\rho_{mn}\}$ is given as follows:

$$\begin{aligned} &\text{Prob}\{\sigma(t+\Delta t) = n|\sigma(t) = m\} \\ &\quad = \begin{cases} \rho_{mn}\Delta t + o(\Delta t), & n \neq m \\ 1 + \rho_{mm}\Delta t + o(\Delta t), & n = m \end{cases} \end{aligned}$$

where $m,\ n \in \mathcal{N}$, $\Delta t > 0$, $\lim_{\Delta t\to 0} \frac{o(\Delta t)}{\Delta t} = 0$, ρ_{mn} is the transition rate from mode m at time t to mode n at time $t + \Delta t$, which is subject to $0 \leq \rho_{mn} \leq 1$, $\rho_{mm} = -\sum_{n=1,n\neq m}^{N} \rho_{mn}$.

In this article, the TRM Λ may be expressed as

$$\Lambda = \begin{bmatrix} \rho_{11} & ? & ? & \cdots & ? \\ \rho_{21} & \rho_{22} & \rho_{23} & \cdots & \rho_{2N} \\ ? & \rho_{32} & ? & \cdots & ? \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ ? & ? & ? & \cdots & ? \end{bmatrix}$$

where ? represents the unknown transition rate.

For convenience, $\Lambda = \Lambda_k^m \bigcup \Lambda_{uk}^m$ with $\Lambda_k^m = \{n :$ Transition rate ρ_{mn} is known$\}$ and $\Lambda_{uk}^m = \{n :$ Transition rate ρ_{mn} is unknown$\}$, $\mathcal{A}_{m,i}$ denotes $\mathcal{A}_{\sigma(t),i}$. By using the fuzzy method in [11], the system (1) can be inferred as follows:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\theta(t)) \{\mathcal{A}_{m,i}x(t) + \mathcal{G}_{m,i}\omega(t) + \mathcal{B}_{m,i}u(t)\} \\ z(t) = \sum_{i=1}^{r} \mu_i(\theta(t)) \{\mathcal{C}_{m,i}x(t) + \mathcal{F}_{m,i}\omega(t) + \mathcal{D}_{m,i}u(t)\} \end{cases} \tag{2}$$

where $\mu_i(\theta(t)) = \frac{\prod_{j=1}^{p} \omega_{ij}(\theta_j(t))}{\sum_{\rho=1}^{r}\prod_{j=1}^{p} \omega_{\rho j}(\theta_j(t))} \geq 0$, and $\omega_{ij}(\theta_j(t))$ is the membership value of $\theta_j(t)$ in ω_{ij}. In addition, $\mu_i(\theta(t)) \in [0, 1]$ and $\sum_{i=1}^{r} \mu_i(\theta(t)) = 1$.

In order to save network bandwidth, an improved state-dependent ETC mechanism is developed. The configuration of event-triggered FMJS is shown in Fig. 1. We use t_{s_k} represent the last triggered sampling instant, then the next triggered instant

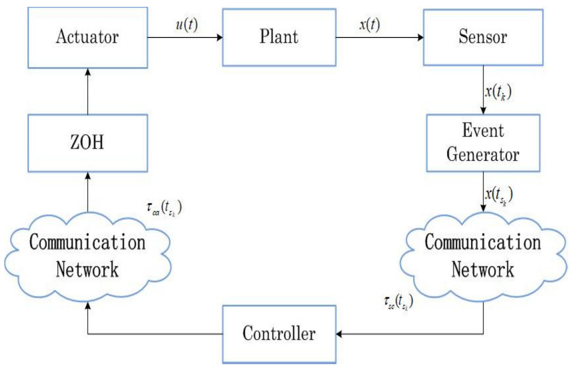

Fig. 1. Configuration of event triggered FMJS.

$t_{s_{k+1}}$ is determined as follows:

$$t_{s_{k+1}} = t_{s_k} + \min_{\ell}\left\{ \sum_{i=s_k}^{\ell} \eta_i \middle| \hat{x}^T(t_{s_k,\ell})\Phi(\sigma(t_{s_k,\ell}))\hat{x}(t_{s_k,\ell}) > \zeta(\sigma(t_{s_k,\ell}))x^T(t_{s_k})\Phi(\sigma(t_{s_k,\ell}))x(t_{s_k}) \right\}$$
$$= t_{s_k} + \sum_{i=s_k}^{s_{k+1}-1} \eta_i \tag{3}$$

where η_k is the nonperiodic sampling period of sensors and satisfies $0 \le \eta_k = t_{k+1} - t_k \le \eta$, η is a known constant. The scalar parameters $\zeta(\sigma(t_{s_k,\ell})) \in [0,1)$ is the threshold, and $\Phi(\sigma(t_{s_k,\ell})) > 0$ is a weighting matrix to be determined. $t_{s_k,\ell} = t_{s_k} + \sum_{i=s_k}^{\ell}\eta_k$, $s_{k+1} - 1 = \min\{\ell | \hat{x}^T(t_{s_k,\ell})\Phi(\sigma(t_{s_k,\ell}))\hat{x}(t_{s_k,\ell}) > \zeta(\sigma(t_{s_k,\ell}))x^T(t_{s_k})\Phi(\sigma(t_{s_k,\ell}))x(t_{s_k})\}$, $k \in \mathbb{Z}^+$, $k \le s_k$. $\hat{x}(t_{s_k,\ell}) = x(t_{s_k,\ell}) - x(t_{s_k})$ denotes the error with respect to the current sampling instant and the last triggered sampling. Moreover, the current signal data $x(t_{s_k,\ell})$ can be delivered if the state-dependent event-triggered condition $\hat{x}^T(t_{s_k,\ell})\Phi(\sigma(t_{s_k,\ell}))\hat{x}(t_{s_k,\ell}) > \zeta(\sigma(t_{s_k,\ell}))x^T(t_{s_k})\Phi(\sigma(t_{s_k,\ell}))x(t_{s_k})$ is satisfied. Otherwise, the signal data would be discarded.

On one hand, as pointed out in [29], two communication delays are unavoidable, i.e., sensor-to-controller delay $\tau_{sc}(t)$, controller-to-actuator delay $\tau_{ca}(t)$. Hence, the network-induced delays can be denoted as follows:

$$\tau(t) = \tau_{sc}(t) + \tau_{ac}(t)$$

where $\tau(t) \in [0, \tau]$ and $\tau(0) = 0$.

On the other side, in view of the fact that there exist communication networks during the event generator to actuator, the considered AFETC can be designed as follows:

Controller rule j: If $\theta_1(t_{s_k})$ is ω_{j1}, $\theta_2(t_{s_k})$ is ω_{j2}, …, and $\theta_p(t_{s_k})$ is ω_{jp}

THEN

$$u(t) = K_{\vartheta(t),j}x(t_{s_k}) \tag{4}$$

where $j \in \mathcal{S}$, and the control signal is characterized by a zero-order-hold function under holding time $t \in [r_k, r_{k+1})$ with $r_k = t_{s_k} + \tau(s_k)$. Moreover, $K_{\vartheta(t),j}$ is the jth fuzzy control parameter to be determined. $\vartheta(t)$ describes the HMM, which denotes the mode of the controller and takes value in $\mathcal{N}^\dagger = \{1, 2, \ldots, N^\dagger\}$. Similarly, the corresponding conditional probability matrix (CPM) $\Delta = \{\delta_{mn}\}$ is satisfying the following condition:

$$\text{Prob}\{\vartheta(t) = n^\dagger | \delta(t) = m\} = \delta_{mn^\dagger},\ m \in \mathcal{N},\ n^\dagger \in \mathcal{N}^\dagger$$

where $0 \le \delta_{mn^\dagger} \le 1$, $\sum_{n^\dagger=1}^{N^\dagger}\delta_{mn^\dagger} = 1$.

Then, the HMM-based AFETC (4) with $\vartheta(t) = n^\dagger$ can be inferred by

$$u(t) = \sum_{j=1}^{r}\mu_j(\theta(t_{s_k}))K_{n^\dagger,j}x(t_{s_k}). \tag{5}$$

For the technical convenience, the holding interval $[r_k, r_{k+1})$ can be divided as

$$[r_k, r_{k+1}) = \bigcup_{\ell=s_k}^{s_{k+1}-1}\Omega_{k,\ell}$$

where $\Omega_{k,\ell} = [r_{k,\ell}, r_{k,\ell+1})$ with $r_{k,\ell} = t_{s_k} + \sum_{i=s_k}^{\ell}\eta_{i-1}\text{sgn}(i - s_k) + \tau(t_{s_k} + \sum_{i=s_k}^{\ell}\eta_{i-1}\text{sgn}(i - s_k))(\ell = s_k, s_k + 1, \ldots, s_{k+1} - 1)$.

In addition, we denote

$$h_k(t) = t - r_{k,\ell},\ h_k = r_{k,\ell+1} - r_{k,\ell}$$
$$\hbar_k(t) = t - \left(t_{s_k} + \sum_{i=s_k}^{\ell}\eta_{i-1}\text{sgn}(i - s_k)\right),\ t \in \Omega_{k,\ell}.$$

Then, we have $0 < h_k(t) \le \hbar_k(t) \le \hbar$, where $\hbar = \eta + \tau$ is a known constant.

Based on the above discussions, the final controller can be described by

$$u(t) = \sum_{j=1}^{r}\mu_j(\theta(t_{s_k}))K_{n^\dagger,j}\{x(t - \hbar_k(t)) - \hat{x}(t - \hbar_k(t))\}. \tag{6}$$

In conclusion, one has

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j\{\mathcal{A}_{m,i}x(t) + \mathcal{G}_{m,i}\omega(t) \\ \qquad +\mathcal{B}_{m,i}K_{n^\dagger,j}x(t - \hbar_k(t)) \\ \qquad -\mathcal{B}_{m,i}K_{n^\dagger,j}\hat{x}(t - \hbar_k(t))\} \\ z(t) = \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j\{\mathcal{C}_{m,i}x(t) + \mathcal{F}_{m,i}\omega(t) \\ \qquad +\mathcal{D}_{m,i}K_{n^\dagger,j}x(t - \hbar_k(t)) \\ \qquad -\mathcal{D}_{m,i}K_{n^\dagger,j}\hat{x}(t - \hbar_k(t))\} \end{cases} \tag{7}$$

where $\mu_i = \mu_i(\theta(t))$, $\mu_j = \mu_j(\theta(t_{s_k}))$ for $\forall i,\ j \in \mathcal{S}$.

Remark 1: Since there exist the network-induced delays, a wide variety of approaches have been adopted to analyze the considered system (7) by the well-developed delay system theory studied in [29]–[31], [33], [34]. However, different from these existing works, by denoting $\hbar_k(t) = t - (t_{s_k} + \sum_{i=s_k}^{\ell}\eta_{i-1}\text{sgn}(i - s_k))$, we make the analysis process of the system (7) clean, simple, and intuitive as possible in this article.

Definition 1: The system (7) is stochastically stable with $\omega(t) = 0$, if

$$\mathbb{E}\left\{\int_0^{+\infty} \|x(t)\|^2 dt\right\} < \infty.$$

Definition 2: For a given constant $\alpha > 0$, any matrices $Q = Q^T = -\tilde{Q}^T\tilde{Q} \leq 0,\ S,\ R = R^T > 0$, the system (7) is said to be strictly $(Q, S, R)-\alpha-$dissipative, if the following two requirements are satisfied: 1) The system (7) is stochastically stable when $\omega(t) = 0$ in Definition 1; 2) under the zero initial condition, for nonzero $\omega(t)$ and $\mathcal{T} > 0$, the inequality

$$\mathbb{E}\left\{\int_0^{\mathcal{T}} [z^T(t)Qz(t) + 2z^T(t)S\omega(t) + \omega^T(t)R\omega(t)]dt\right\} > \alpha\int_0^{\mathcal{T}} \omega^T(t)\omega(t)dt.$$

Lemma 1: (see [35]) For a given matrix $\mathcal{M} \in \mathbb{R}^{n\times n}$, $\mathcal{M} = \mathcal{M}^T > 0$, and a continuously differentiable function $\omega : [\alpha, \beta] \to \mathbb{R}^n$, the inequality:

$$\int_\alpha^\beta \dot{\omega}^T(s)\mathcal{M}\dot{\omega}(s)ds \geq \frac{1}{\beta-\alpha}(\chi_1^T\mathcal{M}\chi_1 + 3\chi_2^T\mathcal{M}\chi_2)$$

where $\chi_1 = \omega(\beta) - \omega(\alpha),\ \chi_2 = \omega(\beta) + \omega(\alpha) - \int_\alpha^\beta (2\omega(s)/(\beta-\alpha))ds$.

Lemma 2: (see [36]) Let $\varpi_1, \varpi_2, \ldots, \varpi_\aleph : \mathbb{R}^m \to \mathbb{R}$ have positive values in an open subset $\mathfrak{D}$ of $\mathbb{R}^m$. Then, the following reciprocally convex combination of ϖ_i over $\mathfrak{D}$ is satisfied:

$$\min_{\{\rho_i|\rho_i>0,\sum_i \rho_i=1\}} \sum_i \frac{1}{\rho_i}\varpi_i(t) = \sum_i \varpi_i(t) + \max_{\varphi_{i,j}(t)} \sum_{i\neq j} \varphi_{i,j}(t)$$

subject to

$$\left\{\varphi_{i,j} : \mathbb{R}^m \to \mathbb{R}, \varphi_{j,i}(t) \cong \varphi_{i,j}(t), \begin{bmatrix} \varpi_i(t) & \varphi_{i,j}(t) \\ \varphi_{j,i}(t) & \varpi_j(t) \end{bmatrix} \geq 0\right\}.$$

Lemma 3: (see [37]) If the following inequalities hold:

$$\mathcal{W}_{ii} < 0,\ \ 1 \leq i \leq r$$

$$\frac{2}{r-1}\mathcal{W}_{ii} + \mathcal{W}_{ij} + \mathcal{W}_{ji} < 0,\ \ 1 \leq i \neq j \leq r$$

then, the inequality $\sum_{i=1}^r \sum_{j=1}^r \mu_i(t)\mu_j(t)\mathcal{W}_{ij} < 0$ is satisfied, where $\mu_i(t) \in [0, 1]$, $\sum_{i=1}^r \mu_i(t) = 1$.

Remark 2: Based on the HMM, a new AFETC is constructed to save the limited communication resource as compared to the published results [30]–[33]. It is noted that if $\mathcal{N}^\dagger = \{1\}$, the AFETC is converted to the case investigated by [31], [32], while when $\mathcal{N}^\dagger = \{1, 2, \ldots, N\}$, the AFETC turns into the case addressed by [30], [33].

III. MAIN RESULTS

Based on a new AFETC with a hidden topology, some new conditions with respect to FMJS (7) will be derived. For the sake of simplicity, $e_\ell = [0_{n,(\ell-1)n}\ \ I_n\ \ 0_{n,(10-\ell)n}\ \ 0_{n,\omega}]^T (\ell = 1, 2, \ldots, 10)$, $e_{11} = [0_{\omega,10n}\ \ I_\omega]^T$ are defined as block entry matrices and the other notations are defined in Appendix A.

Theorem 1: For given scalars η, τ and ζ_m, matrices $Q = Q^T = -\tilde{Q}^T\tilde{Q} \leq 0,\ S,\ R = R^T > 0$ and controller gain matrices $K_{n^\dagger,j}$, the T–S FMJS (7) is said to stochastic stable and strictly $(Q, S, R)-\alpha-$dissipative, if there exist positive definite matrices Φ_m, $\mathscr{P}_m$, $\mathscr{Q}_\ell(\ell = 1, 2)$, symmetric matrices $\mathscr{X} = \begin{bmatrix} \mathscr{X}_{11} & \mathscr{X}_{12} \\ * & \mathscr{X}_{22}\end{bmatrix}$, $\mathscr{R} = \begin{bmatrix} \mathscr{R}_{11} & \mathscr{R}_{12} \\ * & \mathscr{R}_{22}\end{bmatrix}$, $\mathscr{M} = \begin{bmatrix} \mathscr{M}_{11} & \mathscr{M}_{12} & \mathscr{M}_{13} \\ * & \mathscr{M}_{22} & \mathscr{M}_{23} \\ * & * & \mathscr{M}_{33}\end{bmatrix}$, any appropriate dimensional matrices $\mathcal{Y}$, $\mathcal{Y}^\dagger$, $\mathscr{S}_\ell$ $(\ell = 1, 2, 3)$ and $\mathcal{N}_\ell$ $(\ell = 1, 2)$, for $\forall h_k \in (0, \hbar]$, $i, j \in \mathcal{S}$, $m \in \mathcal{N}$, $n^\dagger \in \mathcal{N}^\dagger$, the following conditions hold:

Case 1: if $\Lambda_k^m \neq \emptyset, \Lambda_{uk}^m \neq \emptyset, \rho_{mm} \in \Lambda_k^m$ and for any $\ell \in \Lambda_{uk}^m$

$$\tilde{\mathscr{M}} = \begin{bmatrix} \mathscr{M}_{22} & \mathscr{M}_{23} \\ * & \mathscr{M}_{33} + \mathscr{R}_{22}\end{bmatrix} > 0 \tag{8}$$

$$\begin{bmatrix} \overline{\mathscr{Q}}_2 & \mathscr{S}_1 & \mathscr{S}_2 \\ * & \overline{\mathscr{Q}}_2 & \mathscr{S}_3 \\ * & * & \overline{\mathscr{Q}}_2\end{bmatrix} > 0 \tag{9}$$

$$\Sigma_{mn^\dagger ii}(h_k, s) < 0,\ \ 1 \leq i \leq r \tag{10}$$

$$\frac{2}{r-1}\Sigma_{mn^\dagger ii}(h_k, s) + \Sigma_{mn^\dagger ij}(h_k, s) + \Sigma_{mn^\dagger ji}(h_k, s) < 0,\ \ 1 \leq i \neq j \leq r \tag{11}$$

with $s = \{0, h_k\}$, and

$$\Sigma_{mn^\dagger ij}(h_k, 0) = \begin{bmatrix} \overline{\Psi}_{mn^\dagger ij}(h_k, 0) & \Theta_m^T \\ * & -I\end{bmatrix}$$

$$\Sigma_{mn^\dagger ij}(h_k, h_k) = \begin{bmatrix} \overline{\Psi}_{mn^\dagger ij}(h_k, h_k) & \Theta_m^T & \sqrt{h_k}\Gamma_1\mathcal{Y} \\ * & -I & 0 \\ * & * & -\tilde{\mathscr{M}}\end{bmatrix}$$

in addition, $\overline{\Psi}_{mn^\dagger ij}(h_k, 0)$ and $\overline{\Psi}_{mn^\dagger ij}(h_k, h_k)$ and other parameters are given in Appendix B.

Case 2: if $\Lambda_k^m \neq \emptyset$, $\Lambda_{uk}^m \neq \emptyset$, $\rho_{mm} \in \Lambda_{uk}^m$ and there exists $\mathscr{P}_m - \mathscr{P}_n > 0 (n \in \Lambda_{uk}^m, n \neq m)$, (8), (9) hold and

$$\Sigma^*_{mn^\dagger ii}(h_k, s) < 0,\ \ 1 \leq i \leq r \tag{12}$$

$$\frac{2}{r-1}\Sigma^*_{mn^\dagger ii}(h_k, s) + \Sigma^*_{mn^\dagger ij}(h_k, s) + \Sigma^*_{mn^\dagger ji}(h_k, s) < 0,\ \ 1 \leq i \neq j \leq r \tag{13}$$

with $s = \{0, h_k\}$, and

$$\Sigma^*_{mn^\dagger ii}(h_k, 0) = \begin{bmatrix} \Psi^*_{mn^\dagger ij}(h_k, 0) & \Theta_m^T \\ * & -I\end{bmatrix}$$

$$\Sigma^*_{mn^\dagger ii}(h_k, h_k) = \begin{bmatrix} \Psi^*_{mn^\dagger ij}(h_k, h_k) & \Theta_m^T & \sqrt{h_k}\Gamma_1\mathcal{Y} \\ * & -I & 0 \\ * & * & -\tilde{\mathscr{M}}\end{bmatrix}$$

where $\Psi^*_{mn^\dagger ij}(h_k, 0)$ and $\Psi^*_{mn^\dagger ij}(h_k, h_k)$ and other parameters are given in Appendix B.

Case 3: if there exists a matrix $\mathscr{P}_m^\dagger$ such that $\mathscr{P}_m^\dagger + \mathscr{P}_n \leq 0$ for $n \neq m$, and $\mathscr{P}_m^\dagger + \mathscr{P}_m \geq 0$ for $n = m$, (8), (9) hold and

$$\Sigma^\dagger_{mn^\dagger ii}(h_k, s) < 0,\ \ 1 \leq i \leq r \tag{14}$$

$$\frac{2}{r-1}\Sigma^{\dagger}_{mn^{\dagger}ii}(h_k,s)+\Sigma^{\dagger}_{mn^{\dagger}ij}(h_k,s) + \Sigma^{\dagger}_{mn^{\dagger}ji}(h_k,s)<0,\ \ 1\le i\ne j\le r \tag{15}$$

with $s=\{0,h_k\}$, and

$$\Sigma^{\dagger}_{mn^{\dagger}ii}(h_k,0)=\begin{bmatrix}\Psi_{mn^{\dagger}ij}(h_k,0) & \Theta_m^T\\ * & -I\end{bmatrix}$$

$$\Sigma^{\dagger}_{mn^{\dagger}ii}(h_k,h_k)=\begin{bmatrix}\Psi_{mn^{\dagger}ij}(h_k,h_k) & \Theta_m^T & \sqrt{h_k}\Gamma_1\mathcal{Y}\\ * & -I & 0\\ * & * & -\tilde{\mathscr{M}}\end{bmatrix}$$

where $\Psi_{mn^{\dagger}ij}(h_k,0)$ and $\Psi_{mn^{\dagger}ij}(h_k,h_k)$ and other parameters are given in Appendix B.

Proof: Consider a looped LKF:

$$V(t)=\sum_{j=1}^{5}V_j(t),\ t\in\Omega_{k,\ell} \tag{16}$$

where

$$V_1(t)=x^T(t)\mathscr{P}_{\sigma(t)}x(t)$$
$$V_2(t)=(h_k-h_k(t))\beta_1^T(t)\mathscr{X}\beta_1(t)$$
$$V_3(t)=(h_k-h_k(t))\int_{t-h_k(t)}^{t}\beta_2^T(s)\mathscr{R}\beta_2(s)ds$$
$$V_4(t)=(h_k-h_k(t))\int_{t-h_k(t)}^{t}\beta_3^T(s)\mathscr{M}\beta_3(s)ds$$
$$V_5(t)=\int_{t-\hbar}^{t}x^T(s)\mathscr{Q}_1x(s)ds+\hbar\int_{-\hbar}^{0}\int_{t+\varphi}^{t}\dot{x}^T(s)\mathscr{Q}_2\dot{x}(s)dsd\varphi.$$

First of all, for $t\in\Omega_{k,\ell}$, we prove that the mode-dependent looped LKF $V(t)$ is continuous. It is clear that $\lim_{t\to r^-_{k,\ell}}V_j(t)=\lim_{t\to r^+_{k,\ell}}V_j(t)>0,\ (j=1,5)$. In addition, when $t\to r^-_{k,\ell}$, we have $\lim_{t\to r^-_{k,\ell}}V_j(t)=\lim_{t\to r^+_{k,\ell}}V_j(t)=0,\ (j=2,3,4)$, Thus, one has $\lim_{t\to r_{k,\ell}}V(t)=V(r_{k,\ell})$.

Let $\mathscr{L}$ be the infinitesimal operator along system (7). Differentiating the time derivative of $V(t)$, we then have

$$\mathscr{L}V_1(t)=2x^T(t)\mathscr{P}_m\dot{x}(t)+\sum_{n=1}^{N}\rho_{mn}x^T(t)\mathscr{P}_nx(t) \tag{17}$$

$$\mathscr{L}V_2(t)=2(h_k-h_k(t))\beta_1^T(t)\mathscr{X}\beta_4(t)-\beta_1^T(t)\mathscr{X}\beta_1(t) \tag{18}$$

$$\begin{aligned}\mathscr{L}V_3(t)&=(h_k-h_k(t))\beta_2^T(t)\mathscr{R}\beta_2(t)\\&\quad-\int_{t-h_k(t)}^{t}\beta_2^T(s)\mathscr{R}\beta_2(s)ds\\&=(h_k-h_k(t))\beta_2^T(t)\mathscr{R}\beta_2(t)\\&\quad-h_k(t)x^T(t-h_k(t))\mathscr{R}_{11}x(t-h_k(t))\\&\quad-2x^T(t-h_k(t))\mathscr{R}_{12}\int_{t-h_k(t)}^{t}x(s)ds\\&\quad-\int_{t-h_k(t)}^{t}x^T(s)\mathscr{R}_{22}x(s)ds\end{aligned} \tag{19}$$

$$\begin{aligned}\mathscr{L}V_4(t)&=(h_k-h_k(t))\beta_3^T(t)\mathscr{M}\beta_3(t)\\&\quad-\int_{t-h_k(t)}^{t}\beta_3^T(s)\mathscr{M}\beta_3(s)ds\\&=(h_k-h_k(t))\beta_3^T(t)\mathscr{M}\beta_3(t)\\&\quad-h_k(t)x^T(t-h_k(t))\mathscr{M}_{11}x(t-h_k(t))\\&\quad-2x^T(t-h_k(t))\Big[\mathscr{M}_{12}(x(t)-x(t-h_k(t)))\\&\qquad+\mathscr{M}_{13}\int_{t-h_k(t)}^{t}x(s)ds\Big]\\&\quad-\int_{t-h_k(t)}^{t}\beta_5^T(s)\begin{bmatrix}\mathscr{M}_{22} & \mathscr{M}_{23}\\ & \mathscr{M}_{33}\end{bmatrix}\beta_5(s)ds.\end{aligned} \tag{20}$$

For any matrix $\mathcal{Y}$ with appropriate dimensional, if the (8) holds, the following inequality can be derived:

$$\begin{aligned}&-\int_{t-h_k(t)}^{t}\beta_5^T(s)\tilde{\mathscr{M}}\beta_5(s)ds\\&\le h_k(t)\gamma^T(t)\mathcal{Y}\tilde{\mathscr{M}}^{-1}\mathcal{Y}^T\gamma(t)\\&\quad+2\gamma^T(t)\mathcal{Y}\begin{bmatrix}x(t)-x(t-h_k(t))\\ \int_{t-h_k(t)}^{t}x(s)ds\end{bmatrix}.\end{aligned} \tag{21}$$

Furthermore, for any matrix $\mathcal{Y}^{\dagger}$ with appropriate dimension, one has

$$2\gamma^T(t)\mathcal{Y}^{\dagger}\left[\int_{t-h_k(t)}^{t}x(s)ds-h_k(t)\alpha(t)\right]=0. \tag{22}$$

On the other side,

$$\begin{aligned}\mathscr{L}V_5(t)&=x^T(t)\mathscr{Q}_1x(t)-x^T(t-\hbar)\mathscr{Q}_1x(t-\hbar)\\&\quad+\hbar^2\dot{x}^T(t)\mathscr{Q}_2\dot{x}(t)-\hbar\int_{t-\hbar}^{t}\dot{x}^T(s)\mathscr{Q}_2\dot{x}(s)ds.\end{aligned} \tag{23}$$

Further, by Lemmas 1 and 2, we have

$$\begin{aligned}&-\hbar\int_{t-\hbar}^{t}\dot{x}^T(s)\mathscr{Q}_2\dot{x}(s)ds\\&=-\hbar\int_{t-h_k(t)}^{t}\dot{x}^T(s)\mathscr{Q}_2\dot{x}(s)ds-\hbar\int_{t-\hbar_k(t)}^{t-h_k(t)}\dot{x}^T(s)\mathscr{Q}_2\dot{x}(s)ds\\&\quad-\hbar\int_{t-\hbar}^{t-\hbar_k(t)}\dot{x}^T(s)\mathscr{Q}_2\dot{x}(s)ds\\&\le-\frac{1}{\frac{h_k(t)}{\hbar}}\{(x(t)-x(t-h_k(t)))^T\mathscr{Q}_2(x(t)-x(t-h_k(t)))\\&\quad+(x(t)+x(t-h_k(t))-\frac{2}{h_k(t)}\int_{t-h_k(t)}^{t}x(s)ds)^T3\mathscr{Q}_2(x(t)\\&\quad+x(t-h_k(t))-\frac{2}{h_k(t)}\int_{t-h_k(t)}^{t}x(s)ds)\}\end{aligned}$$

$$
\begin{aligned}
&- \frac{1}{\frac{\hbar_k(t)-h_k(t)}{\hbar}}\{(x(t-h_k(t))-x(t-\hbar_k(t)))^T \\
&\times \mathscr{Q}_2(x(t-h_k(t))-x(t-\hbar_k(t))) \\
&+ (x(t-h_k(t))+x(t-\hbar_k(t)) \\
&- \frac{2}{\hbar_k(t)-h_k(t)}\int_{t-\hbar_k(t)}^{t-h_k(t)} x(s)ds)^T 3\mathscr{Q}_2(x(t-h_k(t)) \\
&+ x(t-\hbar_k(t)) - \frac{2}{\hbar_k(t)-h_k(t)}\int_{t-\hbar_k(t)}^{t-h_k(t)} x(s)ds)\} \\
&- \frac{1}{\frac{\hbar-\hbar_k(t)}{\hbar}}\{(x(t-\hbar_k(t))-x(t-\hbar))^T \\
&\times \mathscr{Q}_2(x(t-\hbar_k(t))-x(t-\hbar)) \\
&+ (x(t-\hbar_k(t))+x(t-\hbar) \\
&- \frac{2}{\hbar-\hbar_k(t)}\int_{t-\hbar}^{t-\hbar_k(t)} x(s)ds)^T 3\mathscr{Q}_2(x(t-\hbar_k(t)) \\
&+ x(t-\hbar) - \frac{2}{\hbar-\hbar_k(t)}\int_{t-\hbar}^{t-\hbar_k(t)} x(s)ds)\} \\
\leq &- \begin{bmatrix}\beta_6(t)\\ \beta_7(t)\\ \beta_8(t)\end{bmatrix}^T \begin{bmatrix}\overline{\mathscr{Q}}_2 & \mathscr{S}_1 & \mathscr{S}_2\\ & \overline{\mathscr{Q}}_2 & \mathscr{S}_3\\ & * & \overline{\mathscr{Q}}_2\end{bmatrix} \begin{bmatrix}\beta_6(t)\\ \beta_7(t)\\ \beta_8(t)\end{bmatrix}. \qquad (24)
\end{aligned}
$$

From (7), for any matrices $\mathcal{N}_\ell(\ell=1,2)$ with appropriate dimension, it yields

$$
\begin{aligned}
&2\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j(x^T(t)\mathcal{N}_1^T+\dot{x}^T(t)\mathcal{N}_2^T) \\
&\times[-\dot{x}(t)+\mathcal{A}_{m,i}x(t)+\mathcal{B}_{m,i}K_{n^\dagger,j}x(t-\hbar_k(t)) \\
&-\mathcal{B}_{m,i}K_{n^\dagger,j}\hat{x}(t-\hbar_k(t))+\mathcal{G}_{m,i}\omega(t)]=0. \qquad (25)
\end{aligned}
$$

In addition, when the current sampled-data is not transmitted, it follows from (3) that

$$
\begin{aligned}
0\leq &-\hat{x}^T(t-\hbar_k(t))\Phi_m\hat{x}(t-\hbar_k(t))+\zeta_m(x(t-\hbar_k(t)) \\
&-\hat{x}(t-\hbar_k(t)))^T\Phi_m(x(t-\hbar_k(t))-\hat{x}(t-\hbar_k(t))). \qquad (26)
\end{aligned}
$$

Next, define $\mathcal{J}(t)=\alpha\omega^T(t)\omega(t)-z^T(t)Qz(t)-2z^T(t)S\omega(t)-\omega^T(t)R\omega(t)$. Then, according to (17)–(26), we then get

$$
\begin{aligned}
\mathbb{E}\{\mathscr{L}V(t)+\mathcal{J}(t)\}&\leq\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j\xi^T(t)\Xi_{mn^\dagger ij}(h_k,h_k(t))\xi(t) \\
&=\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j\xi^T(t)\left[\frac{h_k-h_k(t)}{h_k}\Xi_{mn^\dagger ij}(h_k,0)\right. \\
&\qquad\left.+\frac{h_k(t)}{h_k}\Xi_{mn^\dagger ij}(h_k,h_k)\right]\xi(t) \qquad (27)
\end{aligned}
$$

where $\Xi_{mn^\dagger ij}(h_k,h_k(t))=\Psi_{mn^\dagger ij}(h_k,h_k(t))+h_k(t)\Gamma_1\mathcal{Y}\tilde{\mathscr{M}}^{-1}\mathcal{Y}^T\Gamma_1^T+\Theta_m^T\Theta_m+\sum_{n=1}^{N}\rho_{mn}e_1\mathscr{P}_ne_1^T$, $\Psi_{mn^\dagger ij}(h_k,h_k(t))$ is given in Appendix B.

When $\Lambda_k^m\neq\emptyset$ and $\Lambda_{uk}^m\neq\emptyset$, we have the following two cases:

Case 1: if $\rho_{mm}\in\Lambda_k^m$, then for any $\ell\in\Lambda_{uk}^m$, one has

$$
\begin{aligned}
\sum_{n=1}^{N}\rho_{mn}\mathscr{P}_n &= \sum_{n\in\Lambda_k^m}\rho_{mn}\mathscr{P}_n+\sum_{\ell\in\Lambda_{uk}^m}\rho_{m\ell}\mathscr{P}_\ell \\
&= \sum_{\ell\in\Lambda_{uk}^m}\frac{\rho_{m\ell}}{\sum_{n\in\Lambda_{uk}^m}\rho_{mn}}\left\{\sum_{n\in\Lambda_k^m}\rho_{mn}\mathscr{P}_n\right. \\
&\qquad\left.+\sum_{n\in\Lambda_{uk}^m}\rho_{mn}\mathscr{P}_\ell\right\} \\
&= \sum_{\ell\in\Lambda_{uk}^m}\frac{\rho_{m\ell}}{\sum_{n\in\Lambda_{uk}^m}\rho_{mn}}\left\{\sum_{n\in\Lambda_k^m}\rho_{mn}(\mathscr{P}_n-\mathscr{P}_\ell)\right\} \\
&= \sum_{n\in\Lambda_k^m}\rho_{mn}(\mathscr{P}_n-\mathscr{P}_\ell).
\end{aligned}
$$

Case 2: if $\rho_{mm}\in\Lambda_{uk}^m$ and there exists $\mathscr{P}_m-\mathscr{P}_n>0(n\in\Lambda_{uk}^m,n\neq m)$, then we have

$$
\begin{aligned}
\sum_{n=1}^{N}\rho_{mn}\mathscr{P}_n &= \sum_{n\in\Lambda_k^m}\rho_{mn}\mathscr{P}_n+\rho_{mm}\mathscr{P}_m+\sum_{n\in\Lambda_{uk}^m,n\neq m}\rho_{mn}\mathscr{P}_n \\
&\leq \sum_{n\in\Lambda_k^m}\rho_{mn}\mathscr{P}_n+\rho_{mm}\mathscr{P}_m+\sum_{n\in\Lambda_{uk}^m,n\neq m}\rho_{mn}\mathscr{P}_m \\
&= \sum_{n\in\Lambda_k^m}\rho_{mn}\mathscr{P}_n+\rho_{mm}\mathscr{P}_m \\
&\quad -\left\{\rho_{mm}+\sum_{n\in\Lambda_k^m}\rho_{mn}\mathscr{P}_m\right\} \\
&= \sum_{n\in\Lambda_k^m}\rho_{mn}(\mathscr{P}_n-\mathscr{P}_m).
\end{aligned}
$$

When $\Lambda_k^m=\emptyset$ and $\Lambda_{uk}^m=\Lambda$, we have the following case:

Case 3: if there exists a matrix $\mathscr{P}_m^\dagger$ such that $\mathscr{P}_m^\dagger+\mathscr{P}_n\leq 0$ for $n\neq m$, and $\mathscr{P}_m^\dagger+\mathscr{P}_m\geq 0$ for $n=m$. From the property of TRM, we have

$$
\sum_{n=1}^{N}\rho_{mn}\mathscr{P}_n=\sum_{n=1}^{N}\rho_{mn}(\mathscr{P}_n+\mathscr{P}_m^\dagger)\leq 0.
$$

In conclusion, based on Lemma 3 and Schur complement to the conditions of Theorem 1, it yields

$$
\mathbb{E}\{\mathscr{L}V(t)+\mathcal{J}(t)\}<-\varepsilon\mathbb{E}\{\|x(t)\|^2\},\ t\in\Omega_{k,\ell} \qquad (28)
$$

where $\varepsilon=\min_{i,j\in\mathcal{S}}\{\lambda_{min}(-\Xi_{mn^\dagger ij}(0,0)),\lambda_{min}(-\Xi_{mn^\dagger ij}(\hbar,0))\lambda_{min}(-\Xi_{mn^\dagger ij}(\hbar,\hbar))\}$.

When $\omega(t)=0$, one has $\mathcal{J}(t)=-z^T(t)Qz(t)\geq 0$, thus we have

$$
\mathbb{E}\{\mathscr{L}V(t)\}<-\varepsilon\mathbb{E}\{\|x(t)\|^2\},\ t\in\Omega_{k,\ell}. \qquad (29)
$$

Then, for $\xi(t) \neq 0$, it yields $V(t) > \lim_{t\to} r^-_{k,\ell+1} V(t) = V(r_{k,\ell+1}) > 0$, which means that $V(t)$ is positive definite.

In addition, by integral transformation for $\ell = s_k, s_k + 1, \ldots, s_{k+1} - 1$, it follows from (29) that

$$\mathbb{E}\{V(r^-_{k,s_{k+1}-1})\} - \mathbb{E}\{V(r_{k,s_k})\} < -\varepsilon \sum_{\ell=s_k}^{s_{k+1}-1} \mathbb{E}\left\{\int_{r_{k,\ell}}^{r_{k,\ell+1}} \|x(s)\|^2 ds\right\}. \quad (30)$$

Thus, we have

$$\sum_{k=0}^{+\infty} \mathbb{E}\left\{\int_{t_k+\tau(t_k)}^{t_{k+1}+\tau(t_{k+1})} \|x(s)\|^2 ds\right\} < \varepsilon^{-1}\mathbb{E}\{V(0)\}. \quad (31)$$

That is to say, the system (7) is stochastically stable.

When $\omega(t) \neq 0$, denote $\mathcal{T}_\wp = t_\wp + \tau(t_\wp)(\wp = 0, 1, \ldots, k, k+1, \ldots)$, we assume $\mathcal{T} \in [\mathcal{T}_{s_{k+1}-1}, \mathcal{T}_{s_{k+1}})$, it follows from (28) that

$$\mathbb{E}\{V(\mathcal{T})\} - \mathbb{E}\{V(\mathcal{T}_{s_{k+1}-1})\} + \mathbb{E}\left\{\int_{\mathcal{T}_{s_{k+1}-1}}^{\mathcal{T}} \mathcal{J}(s)ds\right\} < 0 \quad (32)$$

and

$$\mathbb{E}\{V(\mathcal{T}^-_{s_{k+1}-1})\} - \mathbb{E}\{V(0)\} + \sum_{\wp=0}^{s_{k+1}-2} \mathbb{E}\left\{\int_{\mathcal{T}_\wp}^{\mathcal{T}_{\wp+1}} \mathcal{J}(s)ds\right\} < 0. \quad (33)$$

Under the zero initial condition $V(0) = 0$, from the above inequalities, we can obtain

$$\mathbb{E}\left\{\int_{\mathcal{T}_{s_{k+1}-1}}^{\mathcal{T}} \mathcal{J}(s)ds\right\} + \sum_{\wp=0}^{s_{k+1}-2} \mathbb{E}\left\{\int_{\mathcal{T}_\wp}^{\mathcal{T}_{\wp+1}} \mathcal{J}(s)ds\right\} < 0. \quad (34)$$

Based on Definition 2, system (7) is strictly dissipative. This completes the proof. ■

Remark 3: Given some uncertain factors including data dropouts and quantization, not only the considered FMJS is subjected to the unknown transition probabilities, but also weighting matrices and the thresholds are mode-dependent in this article, thus the derived results are more practical as compared to existing works [18], [20], [21], [33], [34] for describing real-world scenario.

Remark 4: In comparison with the aforementioned literature require that the LKF is both positive definite and fixed on each subinterval [20], [33], [34], we established a new mode-dependent looped LKF, which takes full use of the available information on sawtooth structure characteristics. Meanwhile, by employing the reciprocally convex combination approaches together, some new conditions concerning the stochastic stable and the desired dissipative performance are derived for the related FMJSs under the state-dependent AFETC.

Next, the controller design synthesis results are provided by the theorem as follows.

Theorem 2: For given scalars ∂, η, τ and ζ_m, matrices $Q = Q^T = -\tilde{Q}^T\tilde{Q} \leq 0, S, R = R^T > 0$, the T–S FMJS (7) is said to stochastic stable and strictly $(Q, S, R)-\alpha-$dissipative, if there exist positive definite matrices $\hat{\Phi}_m$, $\hat{\mathscr{P}}_m$, $\hat{\mathscr{Q}}_\ell(\ell = 1, 2)$, symmetric matrices $\hat{\mathscr{X}} = \begin{bmatrix} \hat{\mathscr{X}}_{11} & \hat{\mathscr{X}}_{12} \\ * & \hat{\mathscr{X}}_{22} \end{bmatrix}$, $\hat{\mathscr{R}} = \begin{bmatrix} \hat{\mathscr{R}}_{11} & \hat{\mathscr{R}}_{12} \\ * & \hat{\mathscr{R}}_{22} \end{bmatrix}$, $\hat{\mathscr{M}} = \begin{bmatrix} \hat{\mathscr{M}}_{11} & \hat{\mathscr{M}}_{12} & \hat{\mathscr{M}}_{13} \\ * & \hat{\mathscr{M}}_{22} & \hat{\mathscr{M}}_{23} \\ * & * & \hat{\mathscr{M}}_{33} \end{bmatrix}$, any appropriate dimensional matrices $\hat{\mathcal{Y}}$, $\hat{\mathcal{Y}}^\dagger$, $\hat{\mathscr{S}}_\ell$ $(\ell = 1, 2, 3)$, $\mathcal{K}_{n^\dagger,j}$, and any invertible matrix $\hat{\mathcal{N}}$, for $\forall h_k \in (0, \hbar]$, $i, j \in \mathcal{S}$, $m \in \mathcal{N}$, $n^\dagger \in \mathcal{N}^\dagger$, the following conditions hold:

Case 1: if $\Lambda^m_k \neq \emptyset, \Lambda^m_{uk} \neq \emptyset, \rho_{mm} \in \Lambda^m_k$ and for any $\ell \in \Lambda^m_{uk}$

$$\tilde{\mathscr{M}}^* = \begin{bmatrix} \hat{\mathscr{M}}_{22} & \hat{\mathscr{M}}_{23} \\ * & \hat{\mathscr{M}}_{33} + \hat{\mathscr{R}}_{22} \end{bmatrix} > 0 \quad (35)$$

$$\begin{bmatrix} \hat{\bar{\mathscr{Q}}}_2 & \hat{\mathscr{S}}_1 & \hat{\mathscr{S}}_2 \\ * & \hat{\bar{\mathscr{Q}}}_2 & \hat{\mathscr{S}}_3 \\ * & * & \hat{\bar{\mathscr{Q}}}_2 \end{bmatrix} > 0 \quad (36)$$

$$\hat{\Sigma}_{mn^\dagger ii}(h_k, s) < 0, \ \ 1 \leq i \leq r \quad (37)$$

$$\frac{2}{r-1}\hat{\Sigma}_{mn^\dagger ii}(h_k, s) + \hat{\Sigma}_{mn^\dagger ij}(h_k, s) + \hat{\Sigma}_{mn^\dagger ji}(h_k, s) < 0, \ \ 1 \leq i \neq j \leq r \quad (38)$$

with $s = \{0, h_k\}$, and

$$\hat{\Sigma}_{mn^\dagger ij}(h_k, 0) = \begin{bmatrix} \hat{\bar{\Psi}}_{mn^\dagger ij}(h_k, 0) & \hat{\Theta}^T_m \\ * & -I \end{bmatrix}$$

$$\hat{\Sigma}_{mn^\dagger ij}(h_k, h_k) = \begin{bmatrix} \hat{\bar{\Psi}}_{mn^\dagger ij}(h_k, h_k) & \hat{\Theta}^T_m & \sqrt{h_k}\Gamma_1\hat{\mathcal{Y}} \\ * & -I & 0 \\ * & * & -\tilde{\mathscr{M}}^* \end{bmatrix}$$

where $\hat{\bar{\Psi}}_{mn^\dagger ij}(h_k, 0)$ and $\hat{\bar{\Psi}}_{mn^\dagger ij}(h_k, h_k)$ and other parameters are given in Appendix C.

Case 2: if $\Lambda^m_k \neq \emptyset$, $\Lambda^m_{uk} \neq \emptyset$, $\rho_{mm} \in \Lambda^m_{uk}$ and there exists $\hat{\mathscr{P}}_m - \hat{\mathscr{P}}_n > 0(n \in \Lambda^m_{uk}, n \neq m)$, (35), (36) hold and

$$\hat{\Sigma}^*_{mn^\dagger ii}(h_k, s) < 0, \ \ 1 \leq i \leq r \quad (39)$$

$$\frac{2}{r-1}\hat{\Sigma}^*_{mn^\dagger ii}(h_k, s) + \hat{\Sigma}^*_{mn^\dagger ij}(h_k, s) + \hat{\Sigma}^*_{mn^\dagger ji}(h_k, s) < 0, \ \ 1 \leq i \neq j \leq r \quad (40)$$

with $s = \{0, h_k\}$, and

$$\hat{\Sigma}^*_{mn^\dagger ii}(h_k, 0) = \begin{bmatrix} \hat{\Psi}^*_{mn^\dagger ij}(h_k, 0) & \hat{\Theta}^T_m \\ * & -I \end{bmatrix}$$

$$\hat{\Sigma}^*_{mn^\dagger ii}(h_k, h_k) = \begin{bmatrix} \hat{\Psi}^*_{mn^\dagger ij}(h_k, h_k) & \hat{\Theta}^T_m & \sqrt{h_k}\Gamma_1\hat{\mathcal{Y}} \\ * & -I & 0 \\ * & * & -\tilde{\mathscr{M}}^* \end{bmatrix}$$

where $\hat{\Psi}^*_{mn^\dagger ij}(h_k, 0)$ and $\hat{\Psi}^*_{mn^\dagger ij}(h_k, h_k)$ and other parameters are given in Appendix C.

Case 3: if there exists a matrix $\hat{\mathscr{P}}_m^\dagger$ such that $\hat{\mathscr{P}}_m^\dagger + \hat{\mathscr{P}}_n \le 0$ for $n \neq m$, and $\hat{\mathscr{P}}_m^\dagger + \hat{\mathscr{P}}_m \ge 0$ for $n = m$, (35), (36) hold and

$$\hat{\Sigma}_{mn^\dagger ii}^\dagger(h_k, s) < 0, \;\; 1 \le i \le r \tag{41}$$

$$\frac{2}{r-1}\hat{\Sigma}_{mn^\dagger ii}^\dagger(h_k, s) + \hat{\Sigma}_{mn^\dagger ij}^\dagger(h_k, s) + \hat{\Sigma}_{mn^\dagger ji}^\dagger(h_k, s) < 0, \;\; 1 \le i \neq j \le r \tag{42}$$

with $s = \{0, h_k\}$, and

$$\hat{\Sigma}_{mn^\dagger ii}^\dagger(h_k, 0) = \begin{bmatrix} \hat{\Psi}_{mn^\dagger ij}(h_k, 0) & \hat{\Theta}_m^T \\ * & -I \end{bmatrix}$$

$$\hat{\Sigma}_{mn^\dagger ii}^\dagger(h_k, h_k) = \begin{bmatrix} \hat{\Psi}_{mn^\dagger ij}(h_k, h_k) & \hat{\Theta}_m^T & \sqrt{h_k}\Gamma_1\hat{\mathcal{Y}} \\ * & -I & 0 \\ * & * & -\tilde{\mathscr{M}}^* \end{bmatrix}$$

where $\hat{\Psi}_{mn^\dagger ij}(h_k, 0)$ and $\hat{\Psi}_{mn^\dagger ij}(h_k, h_k)$ and other parameters are given in Appendix C.

In addition, the controller gain matrix $K_{n^\dagger, j} = \mathcal{K}_{n^\dagger, j}\hat{\mathcal{N}}^{-1}$.

Proof: Define

$$\mathcal{N}_1 = \hat{\mathcal{N}}^{-1}, \; \mathcal{N}_2 = \partial\hat{\mathcal{N}}^{-1}, \; \hat{\mathscr{P}}_m = \hat{\mathcal{N}}^T\mathscr{P}_m\hat{\mathcal{N}}$$

$$\hat{\Phi}_m = \hat{\mathcal{N}}^T\Phi_m\hat{\mathcal{N}}, \; \mathcal{K}_{n^\dagger, j} = K_{n^\dagger, j}\hat{\mathcal{N}}$$

$$\hat{\mathscr{Q}}_\ell = \hat{\mathcal{N}}^T\mathscr{Q}_\ell\hat{\mathcal{N}}(\ell = 1, 2), \; \mathcal{I}_\ell = diag\{\overbrace{\hat{\mathcal{N}}, \, \hat{\mathcal{N}}, \, \ldots, \, \hat{\mathcal{N}}}^{\ell}\}$$

$$\hat{\mathscr{X}} = \mathcal{I}_2^T\mathscr{X}\mathcal{I}_2, \; \hat{\mathscr{R}} = \mathcal{I}_2^T\mathscr{R}\mathcal{I}_2, \; \hat{\mathscr{M}} = \mathcal{I}_3^T\mathscr{M}\mathcal{I}_3$$

$$\hat{\mathcal{Y}} = \mathcal{I}_4^T\mathcal{Y}\mathcal{I}_2, \; \hat{\mathcal{Y}}^\dagger = \mathcal{I}_4^T\mathcal{Y}^\dagger\mathcal{I}_1, \; \hat{\mathscr{S}} = \mathcal{I}_2^T\mathscr{S}\mathcal{I}_2(\ell = 1, 2, 3)$$

$$\hat{\mathcal{I}}_1 = diag\{\mathcal{I}_{10}, \; I, \; \underbrace{I, \ldots, I}_{\mathcal{N}^\dagger}\}$$

$$\hat{\mathcal{I}}_2 = diag\{\mathcal{I}_{10}, \; I, \; \underbrace{I, \ldots, I}_{\mathcal{N}^\dagger}, \; \hat{\mathcal{N}}, \; \hat{\mathcal{N}}\}.$$

Pre- and post-multiplying (8) by $\mathcal{I}_2^T$ and $\mathcal{I}_2$, it yields (35). Pre- and post-multiplying (9) by $\mathcal{I}_6^T$ and $\mathcal{I}_6$, it yields (36). In addition, when $s = 0$, pre- and post-multiplying (10)–(15) by $\hat{\mathcal{I}}_1^T$ and $\hat{\mathcal{I}}_1$, while $s = h_k$, pre- and post-multiplying (10)–(15) by $\hat{\mathcal{I}}_2^T$ and $\hat{\mathcal{I}}_2$, we have the inequalities (37)–(42) hold. This completes the proof. ■

IV. Illustrative Example

This section will use a practical example to verify the validity of the proposed approach in this article.

Consider a Hénon system [38], which can be modeled by the following T–S FMJSs:

$Plant\ rule$ 1:

IF $x_1(t)$ is $-m_{\sigma(t)}$, **THEN**

$$\begin{cases} \dot{x}(t) = \mathcal{A}_{m,1}x(t) + \mathcal{G}_{m,1}\omega(t) + \mathcal{B}_{m,1}u(t) \\ z(t) = \mathcal{C}_{m,1}x(t) + \mathcal{F}_{m,1}\omega(t) + \mathcal{D}_{m,1}u(t). \end{cases}$$

TABLE I
Control Gain Matrices

$K_{n^\dagger,j}$ \ j / $n^\dagger$	1	2
1	$[-0.7045 \;\; -0.6513]$	$[-0.6574 \;\; -0.6407]$
2	$[-0.8497 \;\; -0.7293]$	$[-0.5351 \;\; -0.5195]$
3	$[-0.6554 \;\; -0.6245]$	$[-0.6976 \;\; -0.6805]$

$Plant\ rule$ 2:

IF $x_1(t)$ is $m_{\sigma(t)}$, **THEN**

$$\begin{cases} \dot{x}(t) = \mathcal{A}_{m,2}x(t) + \mathcal{G}_{m,2}\omega(t) + \mathcal{B}_{m,2}u(t) \\ z(t) = \mathcal{C}_{m,2}x(t) + \mathcal{F}_{m,2}\omega(t) + \mathcal{D}_{m,2}u(t). \end{cases}$$

where

$$\left[\begin{array}{c|c} \mathcal{A}_{11} & \mathcal{A}_{12} \\ \hline \mathcal{A}_{21} & \mathcal{A}_{22} \end{array}\right] = \left[\begin{array}{cc|cc} 0.14 & 0.3 & -0.14 & 0.3 \\ 0.7 & 0 & 0.7 & 0 \\ \hline 0.14 & 0.5 & -0.14 & 0.5 \\ 0.7 & 0 & 0.7 & 0 \end{array}\right]$$

$$\left[\begin{array}{c|c} \mathcal{B}_{11} & \mathcal{B}_{21} \\ \hline \mathcal{G}_{12} & \mathcal{G}_{22} \end{array}\right] = \left[\begin{array}{c|c} \mathcal{B}_{12} & \mathcal{B}_{22} \\ \hline \mathcal{G}_{11} & \mathcal{G}_{21} \end{array}\right] = \left[\begin{array}{c|c} 0.5 & 0.4 \\ 0.3 & 0.4 \\ \hline 0 & 0 \\ 0.1 & 0.2 \end{array}\right]$$

$$\mathcal{C}_{11} = \mathcal{C}_{21} = \mathcal{C}_{12} = \mathcal{C}_{22} = \begin{bmatrix} 0.7 & 0.7 \end{bmatrix}, \; \mathcal{F}_{11} = \mathcal{F}_{12} = 0.5$$

$$\mathcal{D}_{11} = \mathcal{D}_{12} = \mathcal{D}_{21} = \mathcal{D}_{22} = 0.5, \;\; \mathcal{F}_{21} = \mathcal{F}_{22} = 0.2.$$

The TRM: $\Lambda = \begin{bmatrix} -3 & 3 \\ ? & ? \end{bmatrix}$, and the modes of controller obey the CPM: $\Delta = \begin{bmatrix} 0.4 & 0.2 & 0.4 \\ 0.4 & 0.1 & 0.5 \end{bmatrix}$.

In simulation, $\omega(t) = 0.9^t\sin(t)$, the membership functions are assumed as $\mu_1(x_1(t)) = 0.5(1 - x_1(t)/m_{\sigma(t)})$, $\mu_2(x_1(t)) = 1 - \mu_1(x_1(t))$ with $m_1 = m_2 = 3$. $\tilde{Q} = 0.7$, $S = 0.5$, $R = 1.5$, For $\partial = 1.5$, $\tau = 0.01$, $\alpha = 0.1$, $\zeta_1 = 0.1$, and $\zeta_2 = 0.15$, by solving conditions in Theorem 2, when the sampling interval $\eta = 0.057$, the corresponding control gains are shown in Table I, and the weighting matrices are given as follows:

$$\Phi_1 = \begin{bmatrix} 35.8970 & 34.0706 \\ * & 32.5644 \end{bmatrix}, \; \Phi_2 = \begin{bmatrix} 31.1725 & 29.5787 \\ * & 28.2619 \end{bmatrix}.$$

Under the initial condition $x(0) = [0.5, \;\; -0.2]^T$ and controller gains, the modes evolution of the system and controller are shown in Fig. 2, the control input $u(t)$ is shown in Fig. 3, respectively. Besides, from Fig. 4, it is easy to see that the network bandwidth is greatly saved by AFETC. The state trajectories of the FMJS are plotted in Fig. 5, which implies the controlled system is stable. Besides, when $u(t) = 0$, the state response of the FMJS is unstable, which is plotted in Fig. 6.

Remark 5: To make a comparison with other ETC approaches, if $\zeta_1 = \zeta_2 = \zeta$, $\Phi_1 = \Phi_2 = \Phi$ the mode-dependent AFETC will turn into common ETC algorithm studied in [31], [32], and [3]. Furthermore, if nonperiodic sampling period $\eta_k = \eta$, then the

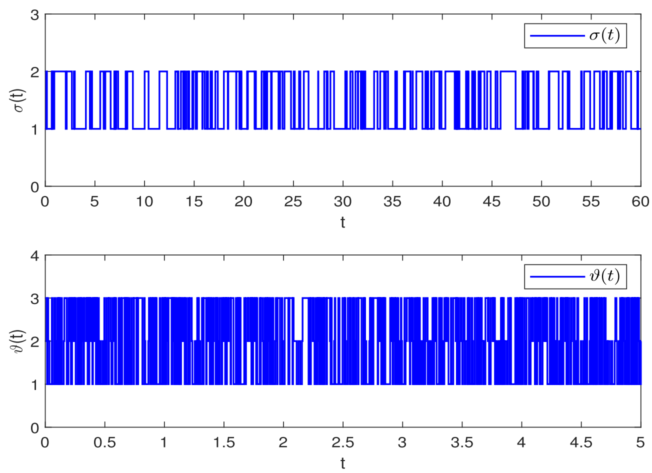

Fig. 2. Curves of the switching signal.

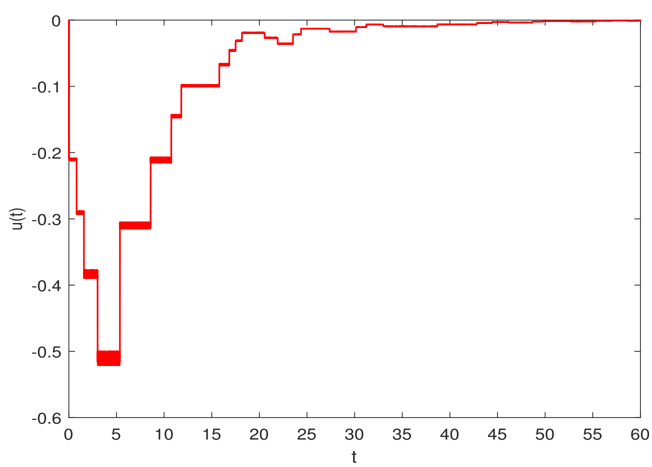

Fig. 3. Fuzzy sampled-data control input $u(t)$.

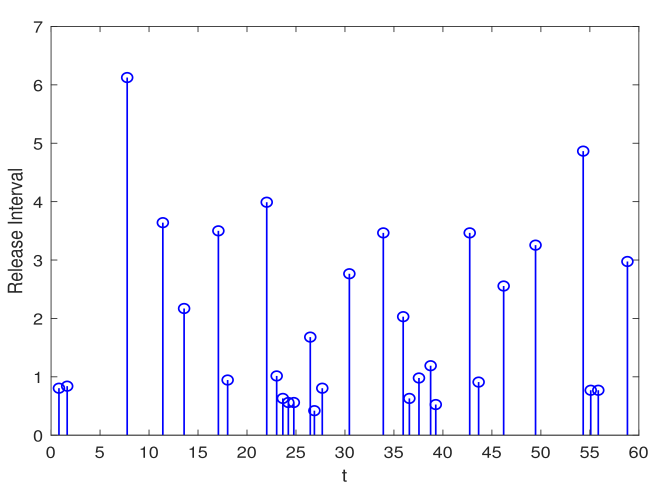

Fig. 4. Release time and release interval.

proposed ETC mechanism in this article will reduce to the ETC ones with constant sampling period [31], [39]. On the other side, if the threshold parameter $\zeta(\sigma(t_{s_k,\ell})) = 0$, then the proposed ETC strategy will reduce to aperiodic time-triggered ones [14],

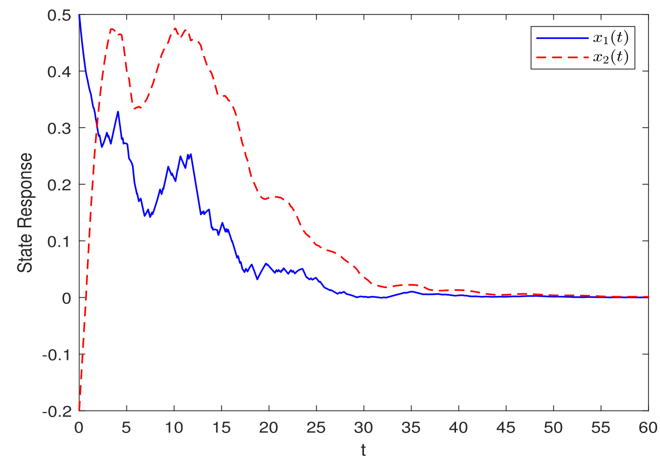

Fig. 5. State curve of FMJS.

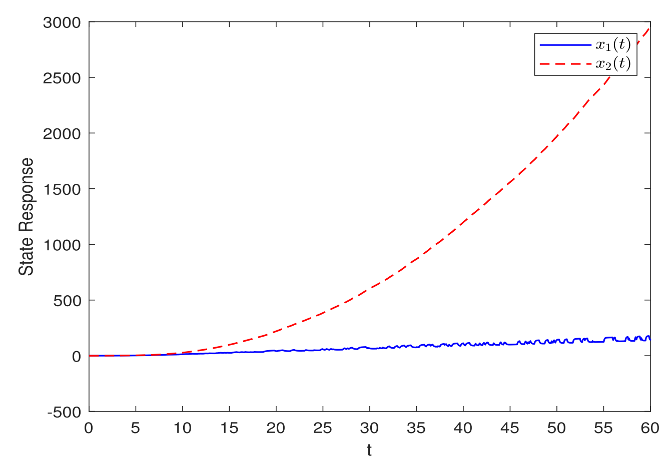

Fig. 6. State curve of FMJS without controller.

[34]. Thus, the proposed ETC covers the existing ones as special cases.

V. Conclusion

In this article, we studied the issue of fuzzy event-triggered asynchronous disspative control for T–S FMJSs with unknown TRs and nonuniform sampling. First, a mode-dependent looped LKF was proposed, which takes full use of the available information not only on both sawtooth structure characteristic but also on the inner sampling dynamics. Moreover, an HMM was employed to depict the mismatch between the original system and the state-dependent FETC. Then, based on the LKF methodology, matrix inequality technique, and the reciprocally convex protocol, some relaxed criteria with respect to the stochastic stable and the desired dissipative performance were derived for the related FMJSs, simultaneously. One numerical experiment was given to illustrate the usefulness of the theoretical results. Further research works would be considered to extend the proposed approach to FMJSs with asynchronous premise constraints.

Appendix A

$$\xi(t) = [x^T(t),\ \dot{x}^T(t),\ x^T(t-h_k(t)),\ x^T(t-\hbar_k(t))$$
$$x^T(t-\hbar),\ \hat{x}^T(t-\hbar_k(t)),\ \int_{t-h_k(t)}^{t} x^T(s)ds$$
$$\alpha^T(t),\ \frac{1}{\hbar_k(t)-h_k(t)}\int_{t-\hbar_k(t)}^{t-h_k(t)} x^T(s)ds$$
$$\frac{1}{\hbar-\hbar_k(t)}\int_{t-\hbar}^{t-\hbar_k(t)} x^T(s)ds,\ \omega^T(t)]^T$$

$$\beta_1(t) = [x^T(t)-x^T(t-h_k(t)),\ \int_{t-h_k(t)}^{t} x^T(s)ds]^T$$
$$\beta_2(t) = [x^T(t-h_k(t)),\ x^T(t)]^T$$
$$\beta_3(t) = [x^T(t-h_k(t)),\ \dot{x}^T(t),\ x^T(t)]^T$$
$$\beta_4(t) = [\dot{x}^T(t),\ x^T(t)-x^T(t-h_k(t))]^T$$
$$\beta_5(t) = [\dot{x}^T(t)\ \ x^T(t)]^T$$
$$\beta_6(t) = [x^T(t)-x^T(t-h_k(t))$$
$$x^T(t)+x^T(t-h_k(t))$$
$$-(2/h_k(t))\int_{t-h_k(t)}^{t} x^T(s)ds]^T$$
$$\beta_7(t) = [x^T(t-h_k(t))-x^T(t-\hbar_k(t))$$
$$x^T(t-h_k(t))+x^T(t-\hbar_k(t))$$
$$-2/(\hbar_k(t)-h_k(t))\int_{t-\hbar_k(t)}^{t-h_k(t)} x^T(s)ds]^T$$
$$\beta_8(t) = [x^T(t-\hbar_k(t))-x^T(t-\hbar)$$
$$x^T(t-\hbar_k(t))+x^T(t-\hbar)$$
$$-2/(\hbar-\hbar_k(t))\int_{t-\hbar}^{t-\hbar_k(t)} x^T(s)ds]^T$$
$$\gamma(t) = [x^T(t),\ x^T(t-h_k(t)),\ \alpha^T(t)$$
$$\int_{t-h_k(t)}^{t} x^T(s)ds]^T$$
$$\alpha(t) = \frac{1}{h_k(t)}\int_{t-h_k(t)}^{t} x^T(s)ds$$

$$\Pi_1 = [e_1-e_3,\ e_7],\ \ \Pi_2 = [e_2,\ e_1-e_3]$$
$$\Pi_3 = [e_3,\ e_1],\quad \Pi_4 = [e_3,\ e_2,\ e_1]$$
$$\Pi_5 = [e_1-e_3,\ e_7],\ \Pi_6 = [e_1-e_3,\ e_1+e_3-2e_8]$$
$$\Pi_7 = [e_3-e_4,\ e_3+e_4-2e_9]$$
$$\Pi_8 = [e_4-e_5,\ e_4+e_5-2e_{10}]$$
$$\Pi = [\Pi_6,\ \Pi_7,\ \Pi_8],\quad \Gamma_1 = [e_1,\ e_3,\ e_8,\ e_7].$$

Appendix B

$$\theta_{1,m} = \begin{bmatrix} \sqrt{\delta_{m1}}\tilde{Q}\mathcal{C}_{m,i} \\ \sqrt{\delta_{m2}}\tilde{Q}\mathcal{C}_{m,i} \\ \vdots \\ \sqrt{\delta_{m\mathcal{N}^\dagger}}\tilde{Q}\mathcal{C}_{m,i} \end{bmatrix},\quad \theta_{2,m} = \begin{bmatrix} \sqrt{\delta_{m1}}\tilde{Q}\mathcal{F}_{m,i} \\ \sqrt{\delta_{m2}}\tilde{Q}\mathcal{F}_{m,i} \\ \vdots \\ \sqrt{\delta_{m\mathcal{N}^\dagger}}\tilde{Q}\mathcal{F}_{m,i} \end{bmatrix}$$

$$\theta_{3,m} = \begin{bmatrix} \sqrt{\delta_{m1}}\tilde{Q}\mathcal{D}_{m,i}K_{1,j} \\ \sqrt{\delta_{m2}}\tilde{Q}\mathcal{D}_{m,i}K_{2,j} \\ \vdots \\ \sqrt{\delta_{m\mathcal{N}^\dagger}}\tilde{Q}\mathcal{D}_{m,i}K_{n^\dagger,j} \end{bmatrix},\quad \overline{\mathscr{Q}}_2 = \begin{bmatrix} \mathscr{Q}_2 & 0 \\ * & 3\mathscr{Q}_2 \end{bmatrix}$$

$$\Gamma_2 = e_1\mathcal{C}_{m,i}^T + e_4(\mathcal{D}_{m,i}K_{n^\dagger,j})^T - e_6(\mathcal{D}_{m,i}K_{n^\dagger,j})^T + e_{11}\mathcal{F}_{m,i}^T$$

$$\tilde{\mathscr{M}} = \begin{bmatrix} \mathscr{M}_{22} & \mathscr{M}_{23} \\ * & \mathscr{M}_{33}+\mathscr{R}_{22} \end{bmatrix},\quad \overline{\mathcal{Q}} = \begin{bmatrix} \overline{\mathscr{Q}}_2 & \mathscr{S}_1 & \mathscr{S}_2 \\ * & \overline{\mathscr{Q}}_2 & \mathscr{S}_3 \\ * & * & \overline{\mathscr{Q}}_2 \end{bmatrix}$$

$$\Theta_m = \begin{bmatrix} \theta_{1,m} & 0 & 0 & \theta_{3,m} & 0 & -\theta_{3,m} & 0 & 0 & 0 & 0 & \theta_{2,m} \end{bmatrix}.$$

$$\overline{\Psi}_{mn^\dagger ij}(h_k,0) = \Psi_{mn^\dagger ij}(h_k,0) + \sum_{n\in\Lambda_k^m}\rho_{mn}e_1(\mathscr{P}_n-\mathscr{P}_\ell)e_1^T,$$
$$\overline{\Psi}_{mn^\dagger ij}(h_k,h_k) = \Psi_{mn^\dagger ij}(h_k,h_k) + \sum_{n\in\Lambda_k^m}\rho_{mn}e_1(\mathscr{P}_n-\mathscr{P}_\ell)e_1^T$$
$$\Psi^*_{mn^\dagger ij}(h_k,0) = \Psi_{mn^\dagger ij}(h_k,0) + \sum_{n\in\Lambda_k^m}\rho_{mn}e_1(\mathscr{P}_n-\mathscr{P}_m)e_1^T$$
$$\Psi^*_{mn^\dagger ij}(h_k,h_k) = \Psi_{mn^\dagger ij}(h_k,h_k) + \sum_{n\in\Lambda_k^m}\rho_{mn}e_1(\mathscr{P}_n-\mathscr{P}_m)e_1^T.$$

$$\Psi_{mn^\dagger ij}(h_k,h_k(t))$$
$$= Sym\{e_1\mathscr{P}_m e_2^T\} - \Pi_1\mathscr{X}\Pi_1^T - Sym\{e_3\mathscr{R}_{12}e_7^T\}$$
$$+ Sym\{e_3[\mathscr{M}_{12}(e_1^T-e_3^T)+\mathscr{M}_{13}e_7^T]\} + Sym\{\Gamma_1\mathcal{Y}\Pi_5^T\}$$
$$+ e_1\mathscr{Q}_1e_1^T - e_5\mathscr{Q}_1e_5^T + \hbar^2e_2\mathscr{Q}_2e_2^T - \Pi\overline{\mathcal{Q}}\Pi^T$$
$$+ Sym\{\sum_{n^\dagger=1}^{N^\dagger}\delta_{mn^\dagger}(e_1\mathcal{N}_1^T+e_2\mathcal{N}_2^T)(\mathcal{A}_{m,i}e_1^T$$
$$+ \mathcal{B}_{m,i}K_{n^\dagger,j}e_4^T - \mathcal{B}_{m,i}K_{n^\dagger,j}e_6^T + \mathcal{G}_{m,i}e_{11}^T - e_2^T)\}$$
$$- e_6\Phi_me_6^T + \zeta_m(e_4-e_6)\Phi_m(e_4^T-e_6^T)$$
$$+ \alpha e_{11}e_{11}^T - Sym\{\sum_{n^\dagger=1}^{N^\dagger}\delta_{mn^\dagger}\Gamma_2Se_{11}^T\} - e_{11}Re_{11}^T$$
$$+ (h_k-h_k(t))\{Sym\{\Pi_1\mathscr{X}\Pi_2^T\} + \Pi_3\mathscr{R}\Pi_3^T$$

$$+\Pi_4\mathscr{M}\Pi_4^T\}+h_k(t)\{-e_3(\mathscr{R}_{11}+\mathscr{M}_{11})e_3^T\}$$
$$+Sym\{\Gamma_1\mathcal{Y}^\dagger(e_7^T-h_k(t)e_8^T)\}.$$

Appendix C

$$\hat{\theta}_{1,m}=\begin{bmatrix}\sqrt{\delta_{m1}}\tilde{Q}\mathcal{C}_{m,i}\hat{\mathcal{N}}\\ \sqrt{\delta_{m2}}\tilde{Q}\mathcal{C}_{m,i}\hat{\mathcal{N}}\\ \vdots\\ \sqrt{\delta_{m\mathcal{N}^\dagger}}\tilde{Q}\mathcal{C}_{m,i}\hat{\mathcal{N}}\end{bmatrix},\quad \hat{\theta}_{2,m}=\begin{bmatrix}\sqrt{\delta_{m1}}\tilde{Q}\mathcal{F}_{m,i}\\ \sqrt{\delta_{m2}}\tilde{Q}\mathcal{F}_{m,i}\\ \vdots\\ \sqrt{\delta_{m\mathcal{N}^\dagger}}\tilde{Q}\mathcal{F}_{m,i}\end{bmatrix}$$

$$\hat{\theta}_{3,m}=\begin{bmatrix}\sqrt{\delta_{m1}}\tilde{Q}\mathcal{D}_{m,i}\mathcal{K}_{1,j}\\ \sqrt{\delta_{m2}}\tilde{Q}\mathcal{D}_{m,i}\mathcal{K}_{2,j}\\ \vdots\\ \sqrt{\delta_{m\mathcal{N}^\dagger}}\tilde{Q}\mathcal{D}_{m,i}\mathcal{K}_{n^\dagger,j}\end{bmatrix},\quad \hat{\hat{\mathscr{Q}}}_2=\begin{bmatrix}\hat{\mathscr{Q}}_2 & 0\\ & 3\hat{\mathscr{Q}}_2\end{bmatrix}$$

$$\hat{\Gamma}_2=e_1(\mathcal{C}_{m,i}\hat{\mathcal{N}})^T+e_4(\mathcal{D}_{m,i}\mathcal{K}_{n^\dagger,j})^T-e_6(\mathcal{D}_{m,i}\mathcal{K}_{n^\dagger,j})^T+e_{11}\mathcal{F}_{m,i}^T,$$

$$\hat{\mathscr{M}}^*=\begin{bmatrix}\hat{\mathscr{M}}_{22} & \hat{\mathscr{M}}_{23}\\ & \hat{\mathscr{M}}_{33}+\hat{\mathscr{R}}_{22}\end{bmatrix},\quad \hat{\hat{\mathscr{Q}}}=\begin{bmatrix}\hat{\hat{\mathscr{Q}}}_2 & \hat{\mathscr{S}}_1 & \hat{\mathscr{S}}_2\\ & \hat{\hat{\mathscr{Q}}}_2 & \hat{\mathscr{S}}_3\\ & * & \hat{\hat{\mathscr{Q}}}_2\end{bmatrix}$$

$$\hat{\Theta}_m=\begin{bmatrix}\hat{\theta}_{1,m} & 0 & 0 & \hat{\theta}_{3,m} & 0 & -\hat{\theta}_{3,m} & 0 & 0 & 0 & 0 & \hat{\theta}_{2,m}\end{bmatrix}$$

$$\hat{\hat{\Psi}}_{mn^\dagger ij}(h_k,0)=\hat{\Psi}_{mn^\dagger ij}(h_k,0)+\sum_{n\in\Lambda_k^m}\rho_{mn}e_1(\hat{\mathscr{P}}_n-\hat{\mathscr{P}}_\ell)e_1^T$$

$$\hat{\hat{\Psi}}_{mn^\dagger ij}(h_k,h_k)=\hat{\Psi}_{mn^\dagger ij}(h_k,h_k)+\sum_{n\in\Lambda_k^m}\rho_{mn}e_1(\hat{\mathscr{P}}_n-\hat{\mathscr{P}}_\ell)e_1^T$$

$$\hat{\Psi}^*_{mn^\dagger ij}(h_k,0)=\hat{\Psi}_{mn^\dagger ij}(h_k,0)+\sum_{n\in\Lambda_k^m}\rho_{mn}e_1(\hat{\mathscr{P}}_n-\hat{\mathscr{P}}_m)e_1^T$$

$$\hat{\Psi}^*_{mn^\dagger ij}(h_k,h_k)=\hat{\Psi}_{mn^\dagger ij}(h_k,h_k)+\sum_{n\in\Lambda_k^m}\rho_{mn}e_1(\hat{\mathscr{P}}_n-\hat{\mathscr{P}}_m)e_1^T.$$

$$\begin{aligned}&\hat{\Psi}_{mn^\dagger ij}(h_k,h_k(t))\\ &=Sym\{e_1\hat{\mathscr{P}}_me_2^T\}-\Pi_1\hat{\mathscr{X}}\Pi_1^T-Sym\{e_3\hat{\mathscr{R}}_{12}e_7^T\}\\ &\quad+Sym\{e_3[\hat{\mathscr{M}}_{12}(e_1^T-e_3^T)+\hat{\mathscr{M}}_{13}e_7^T]\}+Sym\{\Gamma_1\hat{\mathcal{Y}}\Pi_5^T\}\\ &\quad+e_1\hat{\mathscr{Q}}_1e_1^T-e_5\hat{\mathscr{Q}}_1e_5^T+\hbar^2e_2\hat{\mathscr{Q}}_2e_2^T-\Pi\hat{\hat{\mathscr{Q}}}\Pi^T\\ &\quad+Sym\{\sum_{n^\dagger=1}^{N^\dagger}\delta_{mn^\dagger}(e_1+\partial e_2)(\mathcal{A}_{m,i}\hat{\mathcal{N}}e_1^T\\ &\quad+\mathcal{B}_{m,i}\mathcal{K}_{n^\dagger,j}e_4^T-\mathcal{B}_{m,i}\mathcal{K}_{n^\dagger,j}e_6^T+\mathcal{G}_{m,i}e_{11}^T-\hat{\mathcal{N}}e_2^T)\}\\ &\quad-e_6\hat{\Phi}_me_6^T+\zeta_m(e_4-e_6)\hat{\Phi}_m(e_4^T-e_6^T)\\ &\quad+\alpha e_{11}e_{11}^T-Sym\{\sum_{n^\dagger=1}^{N^\dagger}\delta_{mn^\dagger}\hat{\Gamma}_2Se_{11}^T\}-e_{11}Re_{11}^T\\ &\quad+(h_k-h_k(t))\{Sym\{\Pi_1\hat{\mathscr{X}}\Pi_2^T\}+\Pi_3\hat{\mathscr{R}}\Pi_3^T\\ &\quad+\Pi_4\hat{\mathscr{M}}\Pi_4^T\}+h_k(t)\{-e_3(\hat{\mathscr{R}}_{11}+\hat{\mathscr{M}}_{11})e_3^T\}\\ &\quad+Sym\{\Gamma_1\hat{\mathcal{Y}}^\dagger(e_7^T-h_k(t)e_8^T)\}.\end{aligned}$$

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