Design of Energy-efficient EPON: a Novel Protocol Proposal and its Performance Analysis

Sourav Dutta and Goutam Das

Abstract-Economical and environmental concerns necessitate network engineers to focus on energy-efficient access network design. The optical network units (ONUs), being predominantly responsible for the energy consumption of Ethernet Passive Optical Network (EPON), motivates us towards designing a novel protocol for saving energy at ONU. The proposed protocol exploits different low power modes (LPM) and opts for the suitable one using traffic prediction. This scheme provides a significant improvement of energy-efficiency especially at high load ($\sim 40\%$) over existing protocols. A better understanding of the performance and a deeper insight into several design aspects can only be addressed through a detailed mathematical analysis. The proposed protocol involves traffic prediction which infringes Markovian property. However, some pragmatic assumptions along with a proper selection of observation instances and state descriptions allow us to form a Discrete Time Markov Chain (DTMC) of the proposed algorithm. Thus, the primary objective of this paper is to propose a novel scheme for achieving energyefficiency at ONU and mathematically analyze the performance of it with the help of a DTMC. The analysis reveals that the energy-efficiency is more sensitive to the power consumption of doze mode as compared to other LPM while the effect of sleepto-wake-up time is minor.

Index Terms-Energy-efficiency EPON, ONU-assisted.

I. INTRODUCTION

The persistent enhancement of Internet traffic promotes research on diminishing the Internet power consumption. Owing to the primary source (80 - 90%) of Internet power consumption [1], research on energy-efficient Internet network design is predominantly targeted from the perspective of the access network. An Ethernet Passive Optical Network (EPON), one of the most widely accepted and deployed access technology, consists of an Optical Line Terminal (OLT), multiple Optical Network Units (ONUs), and Remote Nodes (RNs) [2]. As ONUs are responsible for consuming the substantial fraction ($\sim 70\%$) of the overall EPON power consumption, energy-efficient ONU design has already emerged to be a well-established research field. Energy-efficiency is achieved at ONU by switching-off some of its active components (low power mode). However, a certain time is required to switch-on those inactive components and this time is also known as sleep-to-wake-up time. It is widely understood that the power consumption figures of ONUs can be reduced by switching off more components whereas it increases the sleepto-wake-up time leading to a trade-off between the power consumption and sleep-to-wake-up time. This trade-off allows designing different low power modes: power-shedding, doze mode, fast sleep, deep sleep, cyclic sleep [3]. In all those low power modes, it is essential to diminish the waste due to wake-up time in order to maximize energy-efficiency which requires elongation of the time duration over which low power mode is employed, termed as sleep duration. However, the limitation on the buffer size of ONUs or the presence of delaysensitive traffic in an EPON network imposes an upper bound on sleep durations. Due to the trade-off between the power consumptions and the wake-up time durations of different low power modes, the enhancement of energy-efficiency calls for an efficient protocol design for the proper selection of a low power mode as a function of the calculated upper-bound of the sleep duration. In addition, energy-efficiency at ONUs can be further enhanced by designing advanced circuitries that reduce power consumptions [4], [5] or wake-up time durations of different low power modes [5], [6]. Among them, it is essential to look for the parameters that deserve more attention in order to achieve maximum energy benefit. This requires the knowledge about the dependency of energy-efficiency on all of the concerned parameters which can solely be obtained by a rigorous mathematical analysis of the underlying protocol. Thus, after designing an efficient protocol for saving energy at ONU, the performance analysis of it is of utmost importance. In this paper, we focus on both of these aspects.

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The protocols for energy-efficient ONU design in EPON are classified as OLT-assisted and ONU-assisted. In case of the OLT-assisted protocols, OLT decides sleep durations of ONUs and informs them through extra information within the GATE message [7]. Several OLT-assisted protocols have been proposed in [8]-[15], [17]. In [8], authors have proposed an OLT-assisted protocol, termed as Sleep Mode Aware (SMA) protocol, where OLT send GATE message to every ONU in every polling cycle like traditional Dynamic Bandwidth Allocation (DBA) [2] protocol. However, before sending the GATE message, the OLT calculates the minimum time instant up to which the next GATE message will not be sent to that ONU and this time instant is informed to the ONU through the GATE message. During this time period, ONUs employ a power saving mode. The effect of employing different power saving modes in this time period for different technologies is demonstrated in [9]. In SMA protocol, the Down-Stream (DS) traffic is queued and it is transmitted to an ONU only during its allocated transmission slot. Different schemes for calculating this transmission slot has been proposed in [10]. In the proposed protocol of [11], ONUs observe the DS traffic of some x number of cycles and if no packets arrive over this period then it sleeps for some y number of cycles. The same

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authors have extended the protocol for both Down-Stream (DS) and Up-stream traffic (US) in [12]. Authors of [11] and [12] have mathematically analyzed the energy-efficiency figures of their proposed schemes with the aid of Discrete Time Markov Chain (DTMC). The authors of [13] have shown that a significant improvement in energy efficiency can be achieved by selecting between two predefined sleep times (one short and one long) instead of using a single long sleep time. The proposed Green Bandwidth Allocation (GBA) protocol of [14] is analyzed by modeling buffer of each ONU as an M/G/1 queue with vacation [15]. In [17], the authors have unitized the advantage of both doze mode and cyclic sleep and proposed a new sleep mode. In this unified sleep mode, three states have been introduced: Aware state (ONU is fully active), Listen state (ONU is in doze mode), and Sleep state (ONU is in cyclic sleep). An ONU cyclically alters between Aware state and Sleep state and during sleep period it periodically enters into Listen state when the handshaking with the OLT is performed. The performance analysis of this sleep mode has been proposed in [18]. The major drawback of all OLT-assisted protocols is that they require changing the MAC of both OLT and ONUs [16]. Thus, a prodigious amount of expenditure is required to employ an OLT-assisted protocol in a legacy network [16].

The drawbacks of OLT-assisted protocols can be eradicated by ONU-assisted protocols where an ONU itself decides its own sleep duration. Few ONU-assisted protocols have also been proposed in [16], [19], [20]. However, to the best of our knowledge, mathematical analysis of none of the ONUassisted protocols is present in the literature. The authors of [19] have employed only doze mode for saving energy. However, it is well known that the power consumption figure of doze mode is quite high [3]. In [16], [20], we have proposed a novel protocol where ONUs dynamically swerve between the active mode and different low power modes where ONUs select the most suitable low power mode based on the sleep duration (time interval over which low power mode is employed) before entering into a low power mode. Once an ONU enters into a low power mode, a certain time interval is required to wake-up from the low power mode and to initiate the US [2] transmission. Traffic arrival over this time period is predicted by Autoregressive-Moving Average (ARMA) model in order to minimize the packet drop probability.

In all existing ONU-assisted protocols, an ONU remains in a low power mode for a certain time interval and then wakes-up from it and transmits US data for multiple cycles before entering into a low power mode again. After waking up from sleep, energy-efficient ONUs behave like traditional ONUs (i.e. both transmitter and receiver are active) till it sleeps once again and no energy saving is achieved over this time period (active period). It is well known that, in Multi-Point-Control-Protocol (MPCP) [7], standardized for realizing statistical multiplexing in EPON, the transmitter of an ONU remains idle during other ONUs US transmissions. This creates an opportunity to employ the doze mode during these periods without losing any possibility of transmission or reception. This process provides a significant improvement in energy-efficiency, especially at high load scenarios when 2

ONUs are needed to be active in all cycles to avoid packet drop. However, it is essential for ONUs to initiate the wakeup process from the doze mode at least T_{sw}^{dz} (T_{sw}^{dz} - sleep-towake-up time from doze mode) duration before the beginning of its allocated US transmission slot and hence, an ONU must know its transmission slot at least T_{sw}^{dz} beforehand. In traditional MPCP, this provision is absent as OLT informs an ONU about its transmission slot through GATE message and the slot stats immediately after receiving its GATE message. Thus, it appears to be impossible to achieve this mechanism in a complete ONU-assisted manner.

In this paper, we prove that this process can easily be realized in a complete ONU-assisted manner by modifying the traditional ranging process [7]. By adjoining our proposed mechanism of employing the doze mode during active periods with our previous proposal [16], [20], in this paper, we propose a novel ONU-assisted sleep mode protocol for energy-efficient ONU design in EPON (OSMP-EO) where all necessary modifications are provided. It can be noted that modifications are required only for US traffic since receivers remain active during the doze mode. We demonstrate that a significant energy benefit is achieved by employing OSMP-EO especially at high load ($\sim 40\%$) as compared to other existing ONU-assisted protocols.

In this paper, we also analyze the proposed OSMP-EO protocol. The presence of traffic prediction in OSMP-EO infringes the memoryless property making the analysis a hard problem to solve. Some realistic assumptions along with an intelligent selection of discrete observation points and state descriptions aid to eliminate the dependency on the past and allow to formulate a DTMC of OSMP-EO for analyzing it. The analysis reveals that if the doze mode power consumption can be reduced by efficient circuitry design, a drastic enhancement in energy-efficiency figures of OSMP-EO is achieved. On the other hand, the energy-efficiency figures of OSMP-EO are less sensitive to the power consumption of the deep sleep and the fast sleep. Further, the decrement of sleep-to-wake-up times has a minor impact on energy-efficiency.

The rest of the paper is organized as follows. In Section II, the proposed OSMP-EO algorithm is described. The Markov model, used for analyzing the proposed OSMP-EO algorithm, is formulated in Section III. State transition probabilities and state probabilities are calculated in Section IV and Section V respectively. The energy-efficiency figure of OSMP-EO is analyzed mathematically in Section VI. Model validation and few design insights are described in Section VII. In Section VIII, we provide concluding statements.

II. PROTOCOL DESCRIPTION

In this section, we first propose a simple mechanism for exploiting the doze mode during active periods in a complete ONU-assisted manner. This is followed by a detail discussion on the proposed OSMP-EO protocol.

A. Mechanism to employ doze mode during active period (MDA)

As discussed before (refer Section I), in all existing ONUassisted protocols, no low power mode is employed during

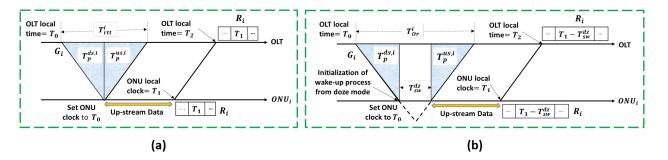


Fig. 1: (a) Measurement of round-trip time in MPCP (b) mechanism employed for exporting the doze mode in active periods.

active periods of ONUs. The transmitter of an ONU being idle during US transmission of other ONUs, an opportunity of saving energy during the active periods by employing doze mode is created. However, to facilitate this, an ONU must ensure that OLT transmits its GATE message at least T_{sw}^{dz} duration beforehand so that it gets enough time to wake-up from doze mode and initiate its US transmission exactly at the beginning of its allocated transmission slot. We now explain that this can easily be achieved by an ONU by modifying the ranging process employed at the OLT to measure the roundtrip time $(T_{rtt}^i$ - round-trip time of $ONU_i)$ of ONUs. In general, the following steps are used to measure T_{rtt}^i at the OLT [7]. At the time of sending the GATE message for ONU_i , denoted by G_i , the OLT sets the time-stamp field of G_i by its local time (say T_0). ONU_i resets its clock to T_0 immediately after receiving G_i . When ONU_i sends the REPORT message (R_i) , the time-tamp field of R_i is set to its local time (say T_1). After receiving R_i (say at T_2), the OLT calculates T_{rtt}^i by (refer Fig. 1(a)):

$$T_{rtt}^{i} = T_{pn}^{ds,i} + T_{pn}^{us,i} = (T_2 - T_0) - (T_1 - T_0) = T_2 - T_1$$
(1)

Instead of setting the time-stamp field by T_1 , if ONU_i sets it by $T_1 - T_{sw}^{dz}$ then the OLT measures round-trip time of ONU_i as; $T_{OLT}^i = T_{rtt}^i + T_{sw}^{dz}$ (refer Fig. 1(b)). While scheduling ONU_i , OLT considers round-trip time of ONU_i as T_{OLT}^i , whereas its actual value is $T_{OLT}^i - T_{sw}^{dz}$. Thus, ONU_i receives the GATE message exactly T_{sw}^{dz} duration before its US transmission slot while the OLT remains oblivious of this fact. We now calculate the average power consumption of an ONU during the active periods when MDA is employed.

Calculation of the average power consumption during active states: Let us denote the duration of the k^{th} cycle and the US transmission slot of ONU_i as T_c^k and $T_s^{i,k}$ respectively. In our proposed mechanism, an ONU starts waking up from doze mode exactly T_{sw}^{dz} duration before the initialization of its allocated transmission slot and again enters into doze mode immediately at the end of it. Thus, ONU_i remains in active mode in the k^{th} cycle for $T_s^{i,k} + T_{sw}^{dz}$ duration, when it consumes P_{on} amount of power. For the rest of the k^{th} cycle (i.e. $T_c^k - T_s^{i,k} - T_{sw}^{dz}$), it is in doze mode and the power consumption is P_{dz} . Hence, the average power consumption of ONU_i during the active mode $(P_{on}^{avg,i})$ is given by eq. (2).

$$P_{on}^{avg,i} = \frac{\mathbb{E}_k \left[(T_s^{i,k} + T_{sw}^{dz}) P_{on} + (T_c^k - T_s^{i,k} - T_{sw}^{dz}) P_{dz} \right]}{\mathbb{E}_k [T_c^k]}$$
(2)

where \mathbb{E}_k denotes expectation over k. Let the average arrival rate of ONU_i and the bandwidth of the feeder fiber be λ_i and λ_d respectively. The average duration that is occupied by the US data of ONU_i in a cycle is $\lambda_i T_c^{avg}/\lambda_d$ where $T_c^{avg} = \mathbb{E}_k[T_c^k]$. In MPCP, each transmission slot includes a REPORT message of duration T_R and guard duration T_G . Thus, $\mathbb{E}_k[T_s^{i,k}] = \frac{\lambda_i}{\lambda_d}T_c^{avg} + T_R + T_G$. Substituting the values of $E_k[T_s^{i,k}]$ in eq. (2), we get:

$$P_{on}^{avg,i} = P_{dz} + \left(\frac{\lambda_i}{\lambda_d} + \frac{T_R + T_G + T_{sw}^{dz}}{T_c^{avg}}\right) (P_{on} - P_{dz}) \quad (3)$$

Now, we describe our proposed OSMP-EO protocol in detail.

B. OSMP-EO

Here, we describe our proposed OSMP-EO protocol for delay insensitive traffic where the buffers of ONUs have limitations. The protocol can be easily extended for delay sensitive traffic as well. In OSMP-EO, ONUs dynamically alter between three modes: deep sleep (ds), fast sleep (fs), and active mode (on). During active mode, ONUs employ MDA (refer Section II-A). We denote the mode of ONU_i at the current and next decision instants (say t and t_1 respectively) by S_c^i and S_{nx}^i respectively. Further, the time interval between the current and the next decision instants is denoted by T_{nd} (i.e. $t_1 = t + T_{nd}$). Now, we describe the value of T_{nd} and the rules that are followed by ONU_i to decide its mode at time t_1 (i.e. S_{nx}^i) with the information of the buffer fill-up time, predicted at t_1 ($T_{bf,n}^i$), for all values of S_c^i .

Case $\mathbf{S}_{\mathbf{c}}^{i} = \mathbf{S}_{\mathbf{m}} \in \{\mathbf{ds}, \mathbf{fs}\}$: In case of $S_{c}^{i} = S_{m}$, the next decision is taken after a fixed time interval T_{m} (i.e. $T_{nd} = T_{m}$) when ONU_{i} decides whether to retain the same sleep mode S_{m} (i.e. $S_{nx}^{i} = S_{m}$) or wake up from it (i.e. $S_{nx}^{i} = on$), same as our previous proposal. Thus, if ONU_{i} decides $S_{nx}^{i} = S_{m}$, it must remain in S_{m} for T_{m} duration and then only the wake-up process can be initiated which takes $T_{sw}^{S_{m}}$ duration ($T_{sw}^{S_{m}}$ -Sleep-to-wake-up time for sleep mode S_{m}). The OSMP-EO being an ONU-assisted protocol, in the worst possible case, an ONU may wake-up from sleep mode immediately after

Notation	Description
N_m^i	Maximum granted bandwidth of ONU_i in one cycle (Packets)
P_s	Power consumption of ONUs when they are in mode $s \in \{ds, fs, dz, on\}$ (Watts)
T_m	Time interval between two consecutive decision instances when ONUs are in sleep mode (seconds)
$T_{sw}^{S_m}$	Sleep-to-wake-up time of sleep mode $S_m \in \{ds, fs, dz\}$ (seconds)
$\begin{bmatrix} T_{sw}^{S_m} \\ T_{bf}^i \\ \end{bmatrix}$	Predicted buffer fill-up time of ONU_i at the current observation instant (seconds)
$\begin{bmatrix} T_{bf,n}^i \\ T_{lb}^{S_m} \\ T_{no} \end{bmatrix}$	Predicted buffer fill-up time of ONU_i at the next observation instant (seconds)
$T_{lb}^{S_m}$	Minimum buffer fill-up time to enter sleep mode S_m (seconds)
T_{no}	Time interval between the current and the next observation instants (seconds)
T_{nd}	Time interval between the current and the next decision instants (seconds)
T_{pc}	Time interval over which prediction is performed at the current observation instant (seconds)
T_{pn}	Time interval over which prediction is performed at the next observation instant (seconds)
S_c^i	Mode of ONU_i in the current observation instant $(S_c^i \in \{ds, fs, on\})$
S_p^i	Mode of ONU_i at the previous observation instant
$S_n^{\tilde{i}}$	Mode of ONU_i at the next observation instant
S_{nx}^i	Mode of ONU_i at the next decision instant
N_{sz}^i	Buffer size of ONU_i (Packets)
N_{th}^i	Buffer threshold of ONU_i (Packets)
$\begin{bmatrix} T_{pn} \\ S_{c}^{i} \\ S_{p}^{i} \\ S_{nx}^{i} \\ N_{sz}^{i} \\ N_{sz}^{i} \\ b_{c}^{i} \\ b_{nn}^{i} \\ b_{nn}^{i} \\ A^{i,T} \end{bmatrix}$	Current buffer state of ONU_i (Packets)
b_n^i	Buffer state of ONU_i at the next observation instant (Packets)
$A^{i,T}$	Number of packet arrivals to ONU_i over the time interval T (Packets)
$\begin{bmatrix} A_p^{i,T} \\ A_p^{i} \\ T_{th}^{i} \\ T_{mw}^{S_m} \end{bmatrix}$	Number of packet that is predicted to arrive at ONU_i over the time interval T (Packets)
T_{th}^i	Time requires to up-stream N_{th}^i number of packets (seconds)
$T_{mw}^{S_m}$	The minimum value of the time interval between the time instants when the buffer fills up and the ONU initiates the wake-up process from sleep mode S_m which is required to eliminate the possibility of packet drop.

the arrival of its GATE message requiring a complete cycle to receive the next one when the desired bandwidth will be reported. The US transmission can only be initiated after receiving the next GATE message requiring another complete cycle. Thus, ONU_i requires at most $2T_{cm}$ (T_{cm} - Maximum cycle time) duration to initiate the US transmission. Therefore, if $T_{mw}^{S_m}$ denotes the minimum value of the time interval between the time instants when the buffer fills up and the ONU initiates the wake-up process from sleep mode S_m which is required to eliminate the possibility of packet drop, then $T_{mw}^{S_m} = T_{sw}^{S_m} + 2T_{cm} + T_m$. The rules for deciding S_{nx}^i are the following:

- If $T_{bf,n}^i > T_{mw}^{S_m}$ then $S_{nx}^i = S_m$ Otherwise, it initiates the wake-up process from sleep mode S_m ($S_{nx}^i = on$)

The OSMP-EO concentrates only on the worst case scenario in order to avoid the packet-drop possibility. However, in practical scenarios, the prediction error may cause a non-zero probability of packet-drop. This can easily be managed by setting a threshold (N_{th}^i) , smaller than the actual buffer-size (N_{sz}^i) . Hereafter, buffer-fill up time indicates the time taken to fill-up N_{th}^i packets.

Case $\mathbf{S}_{\mathbf{c}}^{i} = \mathbf{on}$: If $S_{c}^{i} = on$ then in OSMP-EO, T_{nd} depends on the mode of ONU_i in the previous decision instant, denoted as S_{pr}^i . If $S_{pr}^i = S_m \in \{ds, fs\}$ then the next decision is taken after up-streaming N_{th}^i number of packets. On the other hand, in case of $S_{pr}^i = on$, the next decision is taken after upstreaming all packets that are currently stored in the buffer, denoted by b_c^i , which is same as our previous proposal [16]. We show in Section II-C that due to the trade-off between the sleep-to-wake-up time and power consumption [3], a sleep mode provides improvement in energy-efficiency as compared to other modes with higher power consumption figures if $T_{bf,n}^{i}$ is longer than a threshold. This threshold for ds and fs is denoted as T_{lb}^{ds} and T_{lb}^{fs} . The following conditions are followed for deciding S_{nx}^i :

- If Tⁱ_{bf,n} > T^{ds}_{lb} then select the mode in the next decision point as deep sleep (Sⁱ_{nx} = ds).
 If T^{fs}_{lb} ≤ Tⁱ_{bf,n} < T^{ds}_{lb} then select the next mode as fast
- sleep $(S_{nx}^i = fs)$.
- Otherwise, the active mode is retained $(S_{nx}^i = on)$.

Now, we illustrate the OSMP-EO protocol with the aid of Fig. 2(a). Let, at an arbitrary decision time t, ONU_i is in sleep mode S_m . Thus, the next decision is taken after T_m duration by following the rules, discussed for the case: $S_c^i = S_m$. Let, at time t_1 (= $t + T_m$), the mode is decided as on. Since, in the previous decision point (i.e. at t) mode was S_m , the next decision is taken after up-streaming N_{th}^i number of packets which requires T_{th}^i duration. The mode, at $t_2 (= t_1 + T_{th}^i)$, is decided by following the rules, mentioned for the case of $S_c^i = on$. Let, at t_2 , mode is again decided as on. As, in this case, $S_{pr}^i = on$, the mode will be decided after up-streaming b_c^i number of packets (requires $T_{b_c^i}$ duration) by following the rules same as t_1 .

C. Calculation of T_{lb}^{ds} and T_{lb}^{fs}

Let, at the current decision instant, the buffer fill-up time is predicted as T_{bf}^{i} . We now explain the energy consumption over this T_{bf}^{i} time interval with the help of Fig. 2(b) if ONU_{i} decides its mode as sleep mode S_m . In OSMP-EO, ONU_i wakes up from sleep mode S_m , $T_{sw}^{S_m} + 2T_{cm} + T_m = T_{mw}^{S_m}$ duration before the buffer gets filled up. If the buffer fillup time is correctly predicted then ONU_i remains in sleep mode S_m for $T_{bf}^i - T_{mw}^{S_m}$ duration when it consumes P_{S_m} amount of power. Thereafter, ONU_i requires $T_{sw}^{S_m}$ duration to

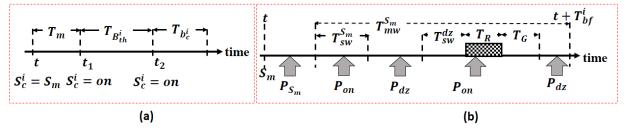


Fig. 2: Illustration of (a) OSMP-EO protocol (b) energy consumption during sleep mode S_m .

wake-up from sleep mode S_m when the power consumption is P_{on} . During the rest of the period (i.e. $2T_{cm} + T_m$) only one REPORT message will be transmitted. Since, in OSMP-EO, ONUs enter into doze mode immediately after waking up from sleep mode S_m , ONU_i needs to wake-up from doze mode before sending the REPORT which takes T_{sw}^{dz} duration. Thus, the power consumption over $T_{sw}^{dz} + T_R + T_G$ time interval is P_{on} and during the remaining period power consumption is P_{dz} . Thus, the energy consumption of ONU_i during sleep mode S_m (E_{S_m}) is:

$$E_{S_m} = T_{bf}^i P_{S_m} + T_{sw}^{S_m} (P_{on} - P_{S_m}) + (2T_{cm} + T_m) \\ \times (P_{dz} - P_{S_m}) + (T_R + T_G + T_{sw}^{dz})(P_{on} - P_{dz})$$

The achieved energy-efficiency of ds, is better than fs if $E_{ds} > E_{fs}$ and it requires:

$$T_{bf}^{i} \geq \frac{T_{sw}^{fs} P_{fs} - T_{sw}^{ds} P_{ds} + (T_{sw}^{ds} - T_{sw}^{fs}) P_{on}}{P_{fs} - P_{ds}} + 2T_{cm} + T_m = T_{lb}^{ds}$$
(4)

If ONU_i remain active then energy consumption is $E_{on} = T_{bf}^{i} P_{on}^{avg,i}$. The fast sleep provides better energy savings as compared to active mode if $E_{fs} > E_{on}$ which requires:

$$T_{bf}^{i} \geq \frac{T_{sw}^{fs}(P_{on} - P_{fs}) + (2T_{cm} + T_{m})(P_{dz} - P_{fs})}{P_{on}^{avg,i} - P_{fs}} + \frac{(T_{R} + T_{G} + T_{sw}^{dz})(P_{on} - P_{dz})}{P_{om}^{avg,i} - P_{fs}} = T_{lb}^{fs} \quad (5)$$

Next, we prove that energy-efficiency of the OSMP-EO protocol can be analyzed with the help of a Discrete Time Markov Chain (DTMC).

III. MODEL FORMULATION AND DESCRIPTION

In this section, we formulate a Markov model of the proposed OSMP-EO protocol which is used for analyzing the achieved energy-efficiency figure. The OSMP-EO protocol, being an ONU-assisted protocol, the Markov analysis can be performed for each ONU separately. Without loss of generality, in this paper, we formulate the Markov model for ONU_i . Since, in this paper, we are focused on analyzing the performance of the OSMP-EO protocol, the grant-sizing scheme is taken to be the simplest one: the fixed grant-sizing scheme [2]. We also explain how this analysis can be extended for other grant-sizing schemes (refer section IV). The following assumptions are taken in this analysis.

- The traffic arrival to ONU_i follows Poisson process with mean λ which has already been assumed in all existing queuing analysis in this area (EPON) [11], [12], [15], [21]–[24].
- The predicted buffer fill-up time always replicates the actual values. Thus, our analysis provides the best possible performance and it is independent of traffic prediction mechanism. As the prediction mechanism is improved, the actual performance of the OSMP-EO protocol asymptotically follows our analytical results.

With these assumptions, we now try to formulate a Discrete Time Markov Model (DTMC) of OSMP-EO. Let us define the mode of ONU_i in the previous, current and the next observation points as S_p^i , S_c^i and S_n^i . If the observation points are identical to the decision points then $S_p^i = S_{pr}^i$ and $S_n^i = S_{nx}^i$; otherwise they are different. Since, in each decision point (refer II-B), ONU_i decides its mode, an obvious choice for observation points of the DTMC are the decision instants. With this observation instant, we first try to find a state description that obeys the Memoryless property [25], termed as valid state description. Firstly, we claim that both S_c^i and b_c^i must be included in the state description.

Claim 1. The state description of DTMC must comprise of S_c^i and b_c^i .

Proof. We know the energy consumption figures for different modes ($\{ds, fs, on\}$) are dissimilar. Thus, evaluation of energy-efficiency requires calculating probabilities of being in all modes, needing S_a^i to be included into the state description. If S_c^i individually hold the memoryless property then mode of ONU_i of the next observation instant i.e. S_n^i can be decided only by S_c^i . However, we now show that the decision about S_n^i requires b_c^i for the case of $S_c^i = S_m \in \{ds, fs\}$, which proves our claim. In case of $S_c^i = S_m$, ONU_i decide S_n^i by comparing $T_{bf,n}^i$ with $T_{mw}^{S_m}$ (refer Section II-B). S_n^i is decided as S_m if $T_{mw}^{S_m} < T_{bf,n}^i$ or in other words, $N_{th}^i > b_n^i + A_p^{i,T_{mw}^{S_m}}$ where $A_p^{i,T}$ is a random process that denotes the number of packets, predicted to arrive to ONU_i over the time interval T. Thus, S_n^i depends on b_n^i , and b_n^i can be calculated as: $b_n^i = b_c^i + A^{i, \overline{T_{no}}}$ where $\overline{T_{no}}$ and $A^{i, \overline{T_{no}}}$ denotes the time interval between the current and next observation instants and the traffic arrival over the time interval T_{no} respectively. Hence, S_n^i is dependent on b_c^i necessitating b_c^i to be included in the state description.

Claim 1 proves the necessity of including both S_c^i and

 b_c^i into the state description. Our next step would then be to verify the sufficiency condition that is whether S_c^i and b_c^i jointly ({ S_c^i, b_c^i }) hold the Markovian property so that $\{S_c^i, b_c^i\}$ qualifies as a valid state description for the DTMC. Our assumption of ideal traffic prediction causes the predicted arrival process exactly same as actual one. In case of $S_c^i = S_m$, as $A^{i,T_{mw}^{S_m}}$ follows Poisson process, $A_p^{i,T_{mw}^{S_m}}$ is also Poisson distributed with mean $\lambda T_{mw}^{S_m}$. Since $T_{mw}^{S_m}$ depends only on $S_c^i = S_m, S_n^i$ dependents only on S_c^i and b_n^i (refer Claim 1). Further, in this case, we know that $T_{no} = T_m$ (refer Section II-B) and hence, b_n^i can be calculated only by the information of b_c^i . Thus, $\{S_c^i, b_c^i\}$ seems to be a valid state description of the DTMC for the case $S_c^i = S_m$. Similar argument can further be provided for the case of $S_c^i = on$ as well. Thus, at the first glance, $\{S_c^i, b_c^i\}$ appears to be qualified as a valid state description with observation instants exactly same as decision instants. However, a closer look into the protocol proves that $\{S_c^i, b_c^i\}$ violates Memoryless property. Not only that, we now claim that if the observation instances are identical to decision points then there doesn't exist any set of parameters at the current decision point that summarizes the complete history for the next instant which proves the non-Markovian nature of the model.

Claim 2. The modeling of OSMP-EO by choosing observation points identical to the decision instances is non-Markovian.

Proof. At current time t, prediction is performed over T_{pc} duration and next decision is taken after T_{no} duration. Thus, there always exists an overlap region between T_{pc} and T_{no} . Owing to the assumption of ideal traffic prediction, $A_p^{i,T_{pc}}$ imposes an additional condition on deciding the next state (i.e. both b_n^i and S_n^i). In OSMP-EO, if $S_p^i = S_m$ then $T_{pc} = T_{mw}^{S_m}$ while if $S_p^i = on$ then $T_{pc} = T_{lb}^{ds}$ or T_{lb}^{fs} , depending on S_c^i (refer Section II-B) and therefore, T_{pc} depends on S_p^i . Hence, mode of ONU_i in the next observation point (S_n^i) depends not only on S_c^i but also on S_p^i . We now prove with the aid of Fig. 3 that S_n^i depends not only on S_p^i but also on the mode of multiple previous decision instants. Let, at time t, ONU_i decides to change its state from active mode (on) to sleep mode S_m (refer Fig. 3). This requires satisfying the condition $N_{th}^i > b_c^i + A_p^{i,T_{lb}^{S_m}}$ (refer Section II-B). Owing to the assumption of ideal traffic prediction, buffer of ONU_i contains lesser than N_{th}^i number of packets till $t_6 (= t + T_{lb}^{S_m})$. Since, at t, mode of ONU_i is S_m , the next decision is taken after $T_{no} = T_m$ duration (i.e. at $t_1 = t + T_m$) by comparing N_{th}^i with $b_c^i + A_p^{i,T_{mw}^{S_m}}$ (refer Section II-B). As $T_{lb}^{S_m} > T_{mw}^{S_m} + T_m$ (refer eq. (4) and eq. (5)), the condition $N_{th}^i > b_c^i + A_p^{i,T_{mw}^{Sm}}$ is surely be satisfied which ensures $S_c^i = S_m$ at t_1 . By following the same argument, it can be proved that $S_c^i = S_m$ for the next $N_{po} = \left\lfloor \frac{T_{lb}^{S_m} - T_{mw}^{S_m}}{T_m} \right\rfloor$ number of observation instants (refer Fig. 3). Thus, $S_c^{i^{T_m}}$ of an observation instant depends on S_c^i of the previous N_{po} number of points which proves the non-Markovian nature of OSMP-EO.

Claim 2 proves that the proposed OSMP-EO protocol violates the Memoryless property. We now reason that intelligent selection of state description and discrete observation points, different from the actual decision points, allows converting this non-Markovian system to a Markovian one firstly for the case of $S_c^i = S_m \in \{ds, fs\}$ and then for $S_c^i = on$.

Case $\mathbf{S}_{\mathbf{c}}^{\mathbf{i}} = \mathbf{S}_{\mathbf{m}}$: From the afore-mentioned discussion (refer Claim 2), we understand that once ONU_i enters into sleep mode S_m from active mode, it retains the same state for N_{po} observation instants which introduces the dependency on the past. This dependency will be absent if $T_{no} > T_{lb}^{S_m} - T_{mw}^{S_m}$. Thus, we have to choose a value of T_{no} such that T_{no} > $T_{lb}^{S_m} - T_{mw}^{S_m}$ is satisfied without loosing any information. Since, in OSMP-EO, ONU_i decides its mode only at some discrete decision points (separated by T_m in this case), observation instants must coincide with some decision points causing $T_{no} \geq (N_{po} + 1)T_m$. At the time instant when $T_{no} =$ $(N_{po}+1)T_m$, ONU_i has a non-zero probability of entering into active mode. Thus, this information cannot be captured if $T_{no} > (N_{po} + 1)T_m$. Consequently, $T_{no} = (N_{po} + 1)T_m$ is the only possible option which is selected in the analysis for the case of $S_p^i = on$ and $S_c^i = S_m$. However, if $S_p^i = S_m$ and $S_c^i = S_m$ then the possibility of choosing $S_n^i = on$ is present in all decision points. Thus, in this case, we choose $T_{no} = T_m$. As a result, T_{no} depends on S_n^i resulting in violation of Markovian property. Nevertheless, the system can be converted to a Markovian process simply by including S_n^i into the state description. It is now clear that b_c^i and S_c^i at the next observation point can be summarized by b_c^i , S_c^i and S_p^i of the current observation instant (refer Section IV for further discussion). S_n^i of the next state is identical to S_c^i of the current state. Hence, S_p^i , S_c^i and b_c^i jointly holds Markovian property allowing $\{S_p^i, \hat{S}_c^i, b_c^i\}$ as a valid state of the DTMC.

Case $S_{c}^{i} = on$: Our next step is to prove that $\{S_{p}^{i}, S_{c}^{i}, b_{c}^{i}\}$ is a valid state description even for $S_c^i = on$. If $S_p^i = S_m \in$ $\{ds, fs\}$ and $S_c^i = on$, ONU_i requires $T_{sw}^{S_m}$ duration to wakeup from sleep mode S_m . The OSMP-EO being an ONUassisted protocol, ONU_i takes the decision of waking up from sleep mode S_m without any prior information of the GATE message arrival time. Consequently, it is possible for ONU_i to wake-up anywhere in between two consecutive GATE messages. The fixed grant sizing scheme ensures a constant time interval between two consecutive GATE messages: T_{cm} . Thus, reception of next GATE message after waking up from sleep mode S_m requires $T_{cm}/2$ duration on an average. Thereafter, the US transmission is initiated and it continues until N_{th}^{i} number of packets get up-streamed which requires $\left[\frac{N_{th}^{*}}{N^{i}}\right]T_{cm}$ time interval where N_m^i is the maximum granted bandwidth of ONU_i in one cycle. As a result, the next decision about its state is taken after $T_{sw}^{S_m} + \left[\frac{N_{th}^i}{N_m^i}\right]T_{cm} + 0.5T_{cm} = T_{mo}^{S_m}$ duration. On the other hand, if $S_p^i = S_c^i = on$ then ONU_i up-stream b_i^c number of packets before taking further decision about S_c^i . Since, in this case, no time gets wasted for waking up, up-streaming b_i^c number of packets requires $\left\lceil \frac{b_i^c}{N_m^i} \right\rceil T_{cm}$ duration which is the next decision point. In this case, if observation instants of the DTMC are considered to be same as decision points of the OSMP-EO protocol, by following the similar argument as provided for the case of $S_c^i = S_m$ (discussed above), it is evident that $\{S_p^i, S_c^i, b_c^i\}$ is a valid state description even for $S_c^i = on$.

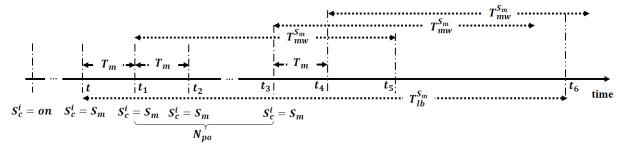


Fig. 3: Illustration of Claim 2.

From the above discussion it is now clear that the OSMP-EO protocol can be modeled by selecting observation instances for different cases as discussed above, and the states as $\{S_n^i, S_c^i, b_c^i\}$. The proposed DTMC of the OSMP-EO algorithm is presented in Fig. 4(a) where all state transitions from states with $b_c^i = k$ is shown. In OSMP-EO, transition from one sleep mode to other is not possible, and therefore, the states $\{ds, fs, b_c^i \ (\forall b_c^i)\}$ and $\{fs, ds, b_c^i \ (\forall b_c^i)\}$ are unfeasible. N_{sz}^i being the buffer size of ONU_i , states with $b_c^i > N_{sz}^i$ are also invalid. ONU_i decides $S_c^i = S_m \in \{ds, fs\}$ only if the buffer of it contains lesser than N_{th}^i number of packets till the next $T_{mw}^{S_m}$ duration and the next observation instant is after $T_m < T_{mw}^{S_m}$ duration. Thus, at the current as well as next observation instants, the buffer occupancy of ONU_i is lesser than N_{th}^i . Consequently, the states $\{S_m, S_c^i, b_c^i \geq N_{th}^i\}$ and $\{on, S_m, b_c^i \geq N_{th}^i\}$ are invalid. As S_c^i of the current state becomes S_p^i of the next state, state transitions from states with S_c^i to states with $S_p^i \neq S_c^i$ are impossible. If in the current observation instant $S_c^i = S_m \in \{ds, fs\}$, the up-stream transmission of ONU_i is terminated making state transition from states with $b_c^i = k$ to states with $b_c^i < k$ is impractical (refer Fig. 4(a)). From the above discussion it is now clear that the OSMP-EO protocol can be modeled by observing the system in some discrete time instants and the state space is finite. Hence, the proposed Markov model is a DTMC. State transition probabilities for all other cases are calculated next.

IV. CALCULATION OF STATE TRANSITION PROBABILITIES

Here, state transition probabilities are calculated for all cases segregated based on all feasible combinations of S_p^i and S_c^i .

A. Transition from states with $S_c^i = S_m \in \{ds, fs\}$

Let, at the current observation time t, the state of ONU_i be $\{S_p^i \ (= S_m \text{ or } on), S_c^i = S_m, b_c^i = k\}$. At time t, for deciding $S_c^i = S_m$, ONU_i must satisfy the condition $A_p^{i,T_{pc}} \le N_{th}^i - k - 1$ where $T_{pc} = T_{mw}^{S_m}$ if $S_p^i = S_m$ and otherwise, $T_{pc} = T_{lb}^{S_m}$ (refer Section II-B). The next observation instant is after $T_{no} = T_m$ duration if $S_p^i = S_m$ while if $S_p^i = on$, $T_{no} = T_{no} = \left(\lfloor \frac{T_{lb}^{S_m} - T_{mw}^{S_m}}{T_m} \rfloor + 1 \right) T_m$ (refer Section III). As, during sleep mode, no US transmission is possible, at the next observation instant (i.e. at $t_1 = t + T_{no}$), queue length will be j(>k) ($b_c^i = j$) if during T_{no} duration j - k number of packets arrive (i.e. $A^{i,T_{no}} = j - k$). The rules, followed by ONU_i for deciding S_c^i at t_1 , are: $S_c^i = S_m$ if $A_p^{i,T_{pn}} \leq N_{th}^i - j - 1$ and otherwise, $S_c^i = on$ where $T_{pn} = T_{mw}^{S_m}$ Thus, at the next observation point the state of ONU_i will be $\{S_p^i = S_m, S_c^i = S_m, b_c^i = j\}$ if the events $A^{i,T_{no}} = j - k$ and $A_p^{i,T_{pn}} \leq N_{th}^i - j - 1$ occurs simultaneously whereas, the state will be $\{S_p^i = S_m, S_c^i = on, b_i = j\}$ if both $A^{i,T_{no}} = j - k$ and $A_p^{i,T_{pn}} \geq N_{th}^i - j$ happens. However, due to the assumption of ideal prediction mechanism, the condition that is satisfied at time t (i.e. $A_p^{i,T_{pc}} \leq N_{th}^i - k - 1$) must hold true. Let the events $A^{i,T_{no}} = j - k$, $A_p^{i,T_{pn}} \leq N_{th}^i - j - 1$, $A_p^{i,T_{pn}} \geq N_{th}^i - j$ and $A_p^{i,T_{pc}} \leq N_{th}^i - k - 1$ is denoted by Y_{j-k} , E_1 , E_1 and E_2 respectively. Probability of state transition from $\{S_p^i, S_m, k\}$ to $\{S_m, S_m, j\}$ and $\{S_m, on, j\}$, denoted by $P_{S_p^i, S_m, k}^{S_m, on, j}$ and $P_{S_p^i, S_m, k}^{S_m, on, j}$ respectively, are given by:

$$p_{S_{p}^{i},S_{m},k}^{S_{m},S_{m},j} = Pr\{Y_{j-k}, E_{1}|E_{2}\} \text{ and} p_{S_{p}^{i},S_{m},k}^{S_{m},on,j} = Pr\{Y_{j-k}, \overline{E}_{1}|E_{2}\}$$
(6)

where $Pr\{E\}$ denotes probability of occurrence of event E. By using Bayes' theorem, eq. (6) can be rewritten by eq. (7).

$$p_{S_{p},S_{m},k}^{S_{m},S_{m},j} = \frac{Pr\{E_{1}, E_{2}|Y_{j-k}\}}{Pr\{E_{2}\}} Pr\{Y_{j-k}\} \quad \text{and} \\ p_{S_{p}^{i},S_{m},k}^{S_{m},on,j} = \frac{Pr\{\overline{E}_{1}, E_{2}|Y_{j-k}\}}{Pr\{E_{2}\}} Pr\{Y_{j-k}\} \quad (7)$$

As the arrival process to ONU_i follows the Poisson distribution with mean λ , $Pr\{Y_{j-k}\}$ is evaluated as the probability of arrival of j - k packets from a Poisson traffic source with mean λT_{no} , denoted by $\alpha_{j-k}^{\lambda T_{no}}$. Thus, $Pr\{Y_{j-k}\}$ is given by eq. (8).

$$Pr\{Y_{j-k}\} = \alpha_{j-k}^{\lambda T_{no}} \quad \text{where} \quad \alpha_l^{\mu} = \frac{e^{-\mu}\mu^l}{(l)!} \tag{8}$$

We now calculate all other probability terms of eq. (7) in Lemma 1, Lemma 2 and Lemma 3.

Lemma 1.
$$Pr\{E_2\} = \sum_{w=0}^{N_{th}^i - k - 1} \alpha_w^{\lambda T_{pc}}$$

Proof. Due to the assumption of ideal traffic prediction, the random process $A_p^{i,T_{pc}}$ become identical to $A^{i,T_{pc}}$ which follows Poisson distribution with mean λT_{pc} . Thus, the event E_2 occurs if this Poisson process generates lesser than $N_{th}^i - k$ packets which proves this lemma.

Lemma 2. If $T_1 = T_{pc} - T_{no}$ and $T_2 = T_{no} + T_{pn} - T_{pc}$ then $Pr\{E_1, E_2|Y_{j-k}\} = \sum_{l=0}^{N_{th}^i - j - l} \sum_{m=0}^{N_{th}^i - j - l - 1} \alpha_l^{\lambda T_1} \alpha_m^{\lambda T_2}.$

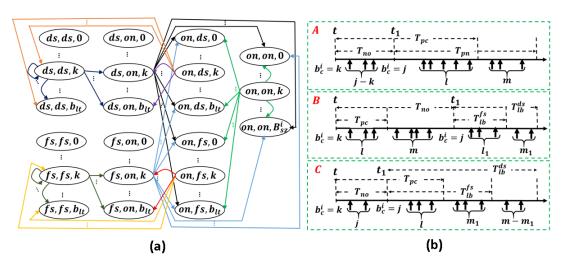


Fig. 4: (a) DTMC of OSMP-EO (b) Calculation of state transition probabilities. $b_{lt} = N_{th}^i - 1$

Proof. Let l and m number of packets arrive over the time interval T_1 and T_2 respectively as shown in A of Fig 4(b). Since, at $t, b_c^i = k$, the event Y_{j-k} results in $b_c^i = j$ at t_1 and hence, $A_p^{i,T_{pc}} = j - k + l$ and $A_p^{i,T_{pn}} = l + m$. The event E_2 and E_1 restricts l and m by $N_{th}^i - j - 1$ and $N_{th}^{i} - j - l - 1$ respectively. Hence, $Pr\{E_1, E_2|Y_{j-k}\}$ is evaluated as the probability of arrival of l and m number of packets over the time interval T_1 and T_2 by a Poisson source with mean arrival rate λ where $l \leq N_{th}^i - j - 1$ and $m \leq N_{th}^i - j - l - 1$. This proves Lemma 3.

Lemma 3. If
$$T_1 = T_{pc} - T_{no}$$
 and $T_2 = T_{no} + T_{pn} - T_{pc}$ then $Pr\{\overline{E}_1, E_2|Y_{j-k}\} = \sum_{l=0}^{N_{th}^i - j - 1} \sum_{m=N_{th}^i - j - l}^{\infty} \alpha_l^{\lambda T_1} \alpha_m^{\lambda T_2}.$

Proof. Lemma 3 can be easily proved by following similar argument as Lemma 2.

B. Transition from states with $S_p^i = S_m \in \{ds, fs\}$ and $S_c^i = on$

Let, at current observation instant t, the state of ONU_i is $\{S_m, on, k\}$. For changing the mode from S_m to on, at t, ONU_i must ensures $A_p^{i,T_{pc}} \ge N_{th}^i - k$ where $T_{pc} = T_{mw}^{S_m}$. The next observation instant is after $T_{no} = T_{sw}^{S_m} + \lceil \frac{N_{th}^i}{N_m^i} \rceil T_{cm} + \frac{T_{cm}}{2} = T_{mo}^{S_m}$ duration (i.e. at $t_1 = t + T_{no}$) when N_{th}^i number of packets depart. Thus, at t_1 , $b_c^i = j$ requires $A^{i,T_{no}} = j - k + N_{th}^i$. An approximate analysis of OSMP-EO protocol for grant sizing schemes different from fixed one can be carried out in the similar manner only by replacing $T_{no} = T_{sw}^{S_m} + \lceil \frac{N_{th}^i}{N_m^i} \rceil T_c^{avg} + 0.5T_c^{avg}$ where T_c^{avg} is the average cycle time of the OLT, calculated by analyzing the underlying Dynamic Bandwidth Allocation (DBA) scheme. At t_1 , $S_c^i = ds$ if $A_p^{i,T_{tb}^a} \le N_{th}^i - j$ (refer Section II-B). We denote the events: $A_p^{i,T_{pc}} \ge N_{th}^i - k$, $A_p^{i,T_{tb}^a} < N_{th}^i - j$ by \overline{E}_2 , E_{ds} respectively. Hence, the transition probability from state $\{S_m, on, k\}$ to $\{on, ds, j\}$, denoted by $p_{S_m, on, k}^{on, ds, j}$, is given by eq. (9).

$$p_{S_m,on,k}^{on,ds,j} = Pr\{Y_{j-k+N_{th}^i}, E_{ds}|\bar{E}_2\}$$
(9)

Since, in this case, $T_{pc} < T_{no}$, the time interval over which the prediction is perform to decide S_c^i at the next observation instant (i.e. T_{pn}) does not intersects with both T_{pc} and T_{no} (refer B of Fig. 4(b)). Thus, E_{ds} is independent of both $Y_{j-k+N_{th}^i}$ and \overline{E}_2 . By using Bayes' Theorem and then this independence condition, eq. (9) turns out to be eq. (10).

$$p_{S_c,on,k}^{on,ds,j} = \frac{Pr\{Y_{j-k+N_{th}^i}, E_2\}}{Pr\{\bar{E}_2\}} Pr\{E_{ds}\}$$
(10)

We know, if $A_p^{i,T_{lb}^{fs}} < N_{th}^i - j$ and $A_p^{i,T_{lb}^{ds}} \ge N_{th}^i - j$ occur simultaneously then at $t_1, S_c^i = fs$ and otherwise, $S_c^i = on$. Let the events: $A_p^{i,T_{lb}^{ds}} \ge N_{th}^i - j$, $A_p^{i,T_{lb}^{fs}} < N_{th}^i - j$ and $A_p^{i,T_{lb}^{fs}} \ge N_{th}^i - j$ is denoted by $\overline{E_{ds}}, E_{fs}, \overline{E_{fs}}$ respectively. Thus, transition probabilities from state $\{S_m, on, k\}$ to $\{on, fs, j\}$ and $\{on, on, j\}$, denoted by $p_{S_m, on, k}^{on, on, j}$ and $p_{S_m, on, k}^{on, on, j}$ respectively, are given by eq. (11).

$$p_{S_{c},on,k}^{on,fs,j} = \frac{Pr\{Y_{j-k+N_{th}^{i}}, \bar{E}_{2}\}}{Pr\{\bar{E}_{2}\}} Pr\{\bar{E}_{ds}E_{fs}\} \text{ and}$$

$$p_{S_{c},on,k}^{on,on,j} = \frac{Pr\{Y_{j-k+N_{th}^{i}}, \bar{E}_{2}\}}{Pr\{\bar{E}_{2}\}} Pr\{\bar{E}_{fs}\}$$
(11)

It is evident that if $j \ge N_{th}^i$ then $Pr\{E_{ds}\} = Pr\{\overline{E_{ds}}E_{fs}\} = 0$ and $Pr\{\overline{E_{fs}}\} = 1$. Since N_{sz}^i is the buffer size, if $A^{i,T_{no}} > N_{sz}^i + N_{th}^i - k$ then the buffer remains full (i.e. $b_c^i = N_{sz}^i$) and the excess packets will drop causing state transition from state with $b_c^i = k$ to state with $b_c^i = N_{sz}^i$. Hence, $p_{S_m,on,k}^{on,on,N_{sz}^i}$ is given by eq. (12).

$$p_{S_m,on,k}^{on,on,N_{s_z}^i} = \sum_{j=N_{s_z}^i}^{\infty} Pr\{Y_{j-k+N_{th}^i} | \bar{E}_2\} \\ = \frac{\sum_{j=N_{s_z}^i}^{\infty} Pr\{Y_{j-k+N_{th}^i}, \bar{E}_2\}}{Pr\{\bar{E}_2\}}$$
(12)

Now, we calculate all probability terms of eq. (10), eq. (11) and eq. (12) in Lemma 4–Lemma 7.

Lemma 4.
$$Pr{\overline{E}_2} = \sum_{w=N_{th}^i-k}^{\infty} \alpha_w^{\lambda T_{mu}^{S_n}}$$

Proof. We know, $Pr\{\overline{E}_2\} = 1 - Pr\{E_2\}$ where $Pr\{E_2\}$ is given by Lemma 1.

Lemma 5.
$$Pr\{E_{S_m}\} = \sum_{w=0}^{N_{th}^i - j - 1} \alpha_w^{\lambda T_{lb}^{S_m}}$$
 and $Pr\{\overline{E_{S_m}}\} = \sum_{w=N_{th}^i - j}^{\infty} \alpha_w^{\lambda T_{lb}^{S_m}}$, $S_m \in \{ds, fs\}$

Proof. $Pr\{\overline{E_{S_m}}\} = 1 - Pr\{E_{S_m}\}$ where $Pr\{E_{S_m}\}$ is calculated similar to Lemma 1.

 $\begin{array}{c} \textbf{Lemma} \qquad \textbf{6.} \quad Pr\{Y_{j-k+N_{th}^{i}}, \overline{E}_{2}\} \\ \sum_{l=N_{th}^{i}-k}^{N_{th}^{i}-k+j} \alpha_{l}^{\lambda T_{pc}} \alpha_{j-k-l+N_{th}^{i}}^{\lambda (T_{no}-T_{pc})} \text{ where } T_{pc} = T_{mw}^{S_{m}}. \end{array}$

Proof. Let l and m number of packets arrive over T_{pc} and next $T_{no} - T_{pc}$ duration respectively (refer B of Fig. 4(b)) resulting in $A^{i,T_{no}} = l + m$. The event \overline{E}_2 imposes a lower limit of $l: l \ge N_{th}^i - k$. Further, the event $Y_{j-k+N_{th}^i}$ ensures $m = j - k - l + N_{th}^i$. $m \ge 0$ restricts the value of l by; $l \le N_{th}^i - k + j$. This proves Lemma 6.

$$\begin{array}{l} \text{Lemma} & \textbf{7.} & Pr\{\overline{E}_{ds}E_{fs}\} \\ \sum_{l_1=0}^{N_{th}^i - j - 1} \sum_{m_1=N_{th}^i - j - l_1}^{\infty} \alpha_{l_1}^{\lambda T_{lb}^{fs}} \alpha_{m_1}^{\lambda (T_{lb}^{ds} - T_{lb}^{fs})} \end{array} =$$

Proof. Let l_1 and m_1 number of packets arrive over time interval T_{lb}^{fs} and next $T_{lb}^{ds} - T_{lb}^{fs}$ (refer B of Fig. 4(b)) causing $A_p^{i,T_{lb}^{ds}} = l_1 + m_1$. The events E_{fs} and $\overline{E_{ds}}$ restricts l_1 and m_1 by; $l_1 \leq N_{th}^i - j - 1$ and $m_1 \geq N_{th}^i - j - l_1$ respectively which proves Lemma 7.

C. Transition from states with $S_p^i = S_c^i = on$

Let, at current time t, ONU_i retain the active state which requires satisfying the condition $A_p^{i,T_{lb}^{fs}} \ge N_{th}^i - j$ (i.e. \overline{E}_2) if $b_c^i < N_{th}^i$ whereas if $b_c^i \ge N_{th}^i$ then no condition is needed to be satisfied. In this case, $T_{no} = \lceil b_c^i/N_m^i \rceil T_{cm}$ (refer Section III) when all $b_c^i = k$ number of packets get upstreamed. Thus, at $t_1 = t + T_{no}$, $b_c^i = j$ requires $A^{i,T_{no}} = j$. Extension of this analysis for other grant sizing schemes can be performed by modifying T_{no} in the similar manner as discussed above. Further, at t_1, S_c^i is decided by verifying the following conditions: $S_c^i = ds$ if E_{ds} occurs, while if both \overline{E}_{ds} and E_{fs} happens then $S_c^i = fs$ and otherwise, $S_c^i = on$. Now, we calculate the state transition probability for three different cases: $T_{no} < T_{lb}^{fs}, b_c^i < N_{th}^i; T_{no} > T_{lb}^{fs}, b_c^i < N_{th}^i; b_c^i \ge N_{th}^i$.

Case 1 ($T_{no} < T_{lb}^{fs}, b_c^i = k < N_{th}^i$)

By following similar steps as used for calculating eq. (7), $p_{on,on,k}^{on,ds,j}$, $p_{on,on,k}^{on,fs,j}$, and $p_{on,on,k}^{on,on,j}$ is given by eq. (13).

$$p_{on,on,k}^{on,ds,j} = \frac{Pr\{\bar{E}_{2}, E_{ds}|Y_{j}\}}{Pr\{\bar{E}_{2}\}} \alpha_{j}^{\lambda T_{no}},$$

$$p_{on,on,k}^{on,fs,j} = \frac{Pr\{\bar{E}_{2}, \bar{E}_{ds}E_{fs}|Y_{j}\}}{Pr\{\bar{E}_{2}\}} \alpha_{j}^{\lambda T_{no}},$$

$$p_{on,on,k}^{on,on,j} = \frac{Pr\{\bar{E}_{2}, \bar{E}_{fs}|Y_{j}\}}{Pr\{\bar{E}_{2}\}} \alpha_{j}^{\lambda T_{no}}$$
(13)

 $Pr{\overline{E}_2}$ is calculated by using Lemma 4. All other probability terms of eq. (13) are calculated by using Lemma 8–Lemma 10. For proving Lemma 8–Lemma 10, we assume that l, m_1 and

 $m-m_1$ number of packets arrives over three consecutive time intervals $T_1 = T_{pc} - T_{no}$, $T_2 = T_{no} + T_{lb}^{fs} - T_{pc}$ and $T_3 = T_{lb}^{ds} - T_{lb}^{fs}$ respectively as shown in C of Fig. 4(b). The event Y_j ensures $A^{i,T_{pc}} = j+l$, $A_p^{i,T_{lb}^{fs}} = l+m_1$ and $A_p^{i,T_{lb}^{ds}} = l+m$.

Proof. The events \overline{E}_2 and E_{ds} requires $l \ge N_{th}^i - k - j$ and $m \le N_{th}^i - j - l - 1$. We know both l and m are positive. m > 0 upper bounds the values of l by $N_{th}^i - j - 1$. Thus, the event $\overline{E}_2, E_{ds}|Y_j$ requires arrival of l and m number of packets over the time interval T_1 and $T_2 + T_3$ with all restrictions on l and m as discussed above which proves Lemma 8.

Lemma 9. $Pr\{\overline{E}_2, \overline{E}_{ds}E_{fs}|Y_j\}$

$$=\sum_{l=\max(0,N_{th}^{i}-k-j)}^{N_{th}^{i}-j-1}\sum_{m_{1}=0}^{N_{th}^{i}-j-l-1}\sum_{m=N_{th}^{i}-j-l}^{\infty}\alpha_{l}^{\lambda T_{1}}\alpha_{m_{1}}^{\lambda T_{2}}\alpha_{m-m_{1}}^{\lambda T_{3}}$$

Proof. The events \overline{E}_2 , E_{fs} and \overline{E}_{ds} requires $l \ge N_{th}^i - k - j$, $m_1 \le N_{th}^i - j - l - 1$ and $m \ge N_{th}^i - j - l$ respectively. Thus, for all m and m_1 , $m \ge m_1$. Further, both l and m_1 are non-negative. $m_1 \ge 0$ necessitates $l \le N_{th}^i - j - 1$. Thus, the event \overline{E}_2 , $\overline{E}_{ds}E_{fs}|Y_j$ occurs if l, m_1 and $m - m_1$ number of packets over the time interval T_1 , T_2 and T_3 respectively with all restrictions on l, m_1 and m as discussed above which proves Lemma 9.

$$\underbrace{ \text{Lemma}}_{\sum_{l=\max(0,N_{th}^{i}-k-j)}^{\infty} \sum_{m_{1}=\max(0,N_{th}^{i}-l-j)}^{\infty} \alpha_{l}^{\lambda T_{1}} \alpha_{m_{1}}^{\lambda T_{2}} } =$$

Proof. The events \overline{E}_2 and \overline{E}_{fs} ensures $l \ge N_{th}^i - k - j$ and $m_1 \ge N_{th}^i - l - j$ respectively. Further, both l and m_1 are positive. By using this, Lemma 10 can be easily proved.

Case 2 ($\mathbf{t_{no}} \ge \mathbf{t_{pp}}, \mathbf{b_c^i} < \mathbf{N_{th}^i}$): This case is exactly same to the case $S_p^i = S_m \in \{ds, fs\}, S_c^i = on$ (refer Section IV-B).

Case 3 ($\mathbf{b}_{\mathbf{c}}^{i} = \mathbf{k} \geq \mathbf{N}_{\mathbf{th}}^{i}$): In this case, ONU_{i} enters into the ON state without any prediction. Further, it can be easily noticed that in this case, the arrival process over T_{no} duration is independent of the arrival process over T_{lb}^{ds} or T_{lb}^{fs} . Hence, in this case, state transition probabilities $p_{on,on,k}^{on,ds,j}$, $p_{on,on,k}^{on,fs,j}$, and $p_{on,on,k}^{on,on,N_{sz}^{i}}$ is given by 14.

$$p_{on,on,k}^{on,ds,j} = \alpha_j^{\lambda T_{no}} \sum_{l=0}^{N_{th}^i - j - 1} \alpha_l^{\lambda T_{lb}^{ds}},$$

$$p_{on,on,k}^{on,fs,j} = \alpha_j^{\lambda T_{no}} \sum_{l=0}^{N_{th}^i - j - 1} \alpha_l^{\lambda T_{lb}^{fs}} \sum_{m=N_{th}^i - j - l}^{\infty} \alpha_m^{\lambda T_3},$$

$$p_{on,on,k}^{on,on,j} = \alpha_j^{\lambda T_{no}} \sum_{l=\max(0,N_{th}^i - j)}^{\infty} \alpha_l^{\lambda T_{lb}^{fs}},$$

$$p_{on,on,k}^{on,ds,N_{sz}^i} = \sum_{l=N_{sz}^i}^{\infty} \alpha_l^{\lambda T_{no}}$$
(14)

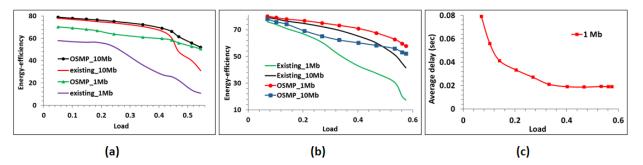


Fig. 5: Comparison with existing protocols for scenarios: (a) only one ONU (say ONU_i) (b) all ONUs follows OSMP-EO protocol, (c) delay characteristic of OSMP-EO.

V. CALCULATION OF STATE PROBABILITIES

Here, we calculate all state probabilities. In OSMP-EO, transition between two sleep modes is not possible. Further, S_p^i of the next state is exactly same as S_c^i of the current state (refer Section III). If $S_c^i = S_m \in \{ds, fs\}$ then the upstream transmission gets terminated, eliminating the possibility of state transition between states with $b_c^i = k$ to states with $b_c^i < k$. By considering these restrictions on state transitions, the probabilities of being in states $\{S_m, S_c^i, j\}$ ($\pi_{S_m, S_c^i, j}$) and $\{on, S_c^i, j\}$ ($\pi_{on, S_c^i, j}$) are given by eq. (15) and eq. (16) respectively (refer Fig. 4(a)).

$$\pi_{S_m,S_c^i,j} = \sum_{k=0}^{j} \pi_{S_m,S_m,k} p_{S_m,S_m,k}^{S_m,S_c^i,j} + \sum_{k=0}^{j} \pi_{on,S_m,k} p_{on,S_m,k}^{S_m,S_c^i,j}$$
(15)

$$\pi_{on,S_{c}^{i},j} = \sum_{k=0}^{N_{th}^{i}-1} \pi_{ds,on,j} p_{ds,on,j}^{on,S_{c}^{i},j} + \sum_{k=0}^{N_{th}^{i}-1} \pi_{fs,on,j} p_{fs,on,j}^{on,S_{c}^{i},j} + \sum_{k=0}^{N_{sz}^{i}} \pi_{on,on,j} p_{on,on,j}^{on,S_{c}^{i},j}$$
(16)

VI. CALCULATION OF AVERAGE ENERGY-EFFICIENCY

In this Section, we analyze the average energy savings figure of the OSMP-EO protocol. ONU_i retains the mode of the current observation instant (S_c^i) till the next observation instant i.e. for a duration of T_{no} . We denote T_{no} and the energy consumption of ONU_i over T_{no} for state $\{S_p^i, S_c^i, b_c^i\}$ as $T^{S_p^i, S_c^i, b_c^i}$ and $E^{S_p^i, S_c^i, b_c^i}$ respectively. If $S_c^i = S_m \in \{ds, fs\}$ then $T_{no} = T_m$ and power consumption is P_{S_m} (refer Section III) and hence, $E^{S_p^i, S_m, b_c^i} = P_{S_m} T_m$. In case of $S_p^i = S_m$ and $S_c^i = on$, $T_{no} = T^{S_m, on, b_c^i} = T^{S_m}_{sw} + \left\lceil \frac{N_{th}^i}{N_m^i} \right\rceil T_{cm} + \frac{T_{cm}}{2}$. Among this period ONU_i takes $T^{S_m}_{sw}$ duration to wake-up from sleep mode when power consumption is P_{on} . After waking up, ONU_i requires $1.5T_{cm}$ duration to start the upstream data transmission when only one REPORT message is up-streamed. The rest of the period $(t_{rp} = \lceil N_{th}^i/N_m^i \rceil - 1)$ is used for up-stream data transmission when the power consumption is $P_{on}^{avg,i}$ (refer eq. (3)). Thus, in this case, $E^{S_m, on, b_c^i} = T^{S_m}_{sw} P_{on} + (1.5T_{cm} - T_{sw}^{dz} - T_R - T_G)P_{dz} + (T^{dz}_{sw} - T_R - T_G)P_{on} + t_{rp}P_{on}^{avg,i}$. In case of $S_p^i = S_c^i = on$,

over entire $T_{no} = T^{on,on,b_c^i} = \left\lceil \frac{b_c^i}{N_m^i} \right\rceil T_{cm}$ duration power consumption is $P_{on}^{avg,i}$ and hence, $E^{on,on,b_c^i} = T_{no}P_{on}^{avg,i}$. Therefore, the average power consumption (P_{avg}) of the OSMP-EO algorithm is given by eq. (17).

$$P_{avg} = \frac{\sum_{S_p^i} \sum_{S_c^i} \sum_j E^{S_p^i, S_c^i, j} \pi_{S_p^i, S_c^i, j}}{\sum_{S_p^i} \sum_{S_c^i} \sum_j T^{S_p^i, S_c^i, j} \pi_{S_p^i, S_c^i, j}}$$
(17)

However, if no energy-efficient mode is employed then power consumption is always P_{on} . Hence, the average energy-efficiency of OSMP-EO algorithm is given by: $\eta_{avg} = 1 - \frac{P_{avg}}{P_{avg}}$

P_{on} .

VII. RESULT AND DISCUSSION

In this Section, firstly, the average energy-efficiency figures of the OSMP-EO protocol are compared with the same of other existing schemes. We then validate the analytical results with the results obtained from simulations. We next compare the results obtained by using our analytical model with that generated by simulations for self-similar traffic. All simulations are performed in OMNET++ for a network runtime of 50s and the results are plotted with 95% confidence interval. The link rate of the feeder fiber, the maximum traffic arrival rate at each ONU, the packet size and T_m are assumed to be 1Gbps, 100Mbps, 1500Bytes and 0.5ms respectively [16]. Sleep-to-wake-up time of ds, fs, and dz are considered to be 5.125ms, $125\mu s$ and $1\mu s$ respectively [16]. Whereas power consumption of ds, fs, dz and on are 0.75W, 1.28W, 2.39Wand 3.984W respectively [16].

A. Comparison with existing literature

We have already demonstrated in [16] that our previous proposal outperforms all existing protocols. Here, we compare the OSMP-EO protocol with our previous proposal where no sleep mode was employed during active periods. In OSMP-EO, we introduce a mechanism of employing doze mode during the active periods which provides additional energy savings. In order to quantify this improvement, in Fig. 5(a) and Fig. 5(b), the average energy-efficiency figures are plotted as a function of traffic load for $N_{th}^i = N_{sz}^i = 1Mb$ and 10Mb for two different sceneries: (i) only one ONU (say ONU_i) follows OSMP-EO protocol while all other ONUs are traditional ONUs and (ii) all ONUs follows OSMP-EO protocol

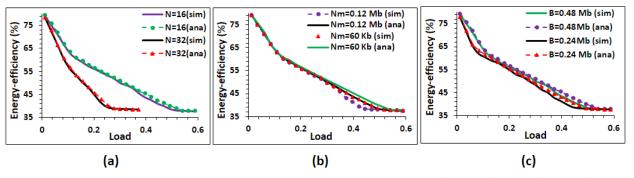


Fig. 6: Validation of analytical model for different values of (a) N (b) N_m^i (c) N_{th}^i . Nm- N_m^i , B- N_{th}^i

respectively. The average delay of OSMP-EO protocol for $N_{th}^i = N_{sz}^i = 1$ Mb is plotted if Fig. 5(c). Here, traffic load of an ONU is defined with respect to the maximum arrival rate to that ONU. The traffic arrivals are considered to be selfsimilar with Hurst parameter (H) 0.8 and it is generated by aggregating 16 ON-OFF Pareto sources. Traffic prediction is performed by using ARMA(2,2) model in a similar manner as discussed in our previous proposal [16]. Fig. 5(a) and Fig. 5(b) depict that at low load, especially for the case of $N_{th}^i = 10$ Mb, the improvement in energy efficiency, as compared to our previous proposal [16] is very small whereas at high load a significant energy benefit (~ 40% for $N_{th}^i = 1Mb$) can be achieved. This is due to the following fact. Both the reduction of traffic load and the increment of N_{th}^{i} enhances the buffer fill-up time. As a result, the sleep duration increases and hence, ONUs wake-up from sleep mode less frequently. In OSMP-EO, a certain time interval is wasted when an ONU wakes-up from sleep mode and this wastage can be reduced by increasing of N_{th}^i or by reducing traffic load. Further, the reduction of traffic load decreases the time interval over which an ONU up-streams. As a result of these two facts, the total active period gets reduced with the reduction of traffic load and increment of N_{th}^{i} . Since OSMP-EO enhances energy efficiency by introducing doze mode during active periods; at a low load, for s higher value of N_{th}^{i} (when the active period is very small), the increment is negligible. Whereas, at a high load with low value of N_{th}^i , the improvement is significant. Further, since the increment of traffic load reduces the sleep duration, an increase of traffic load results in a decrement of average delay as seen in 5(c).

B. Validation of analytical model with simulations

Here, the proposed Markov model of the OSMP-EO protocol is validated with simulation results for Poisson traffic. It is well known that in case of Poisson traffic, the current arrival is independent of the previous arrivals, and hence, traffic prediction with the knowledge of previous arrivals is impossible. In simulations, we consider that ONU_i predicts the number of packet arrivals over T_{od} duration by $\lambda_i T_{od}$ where λ_i is average packet arrival rate to ONU_i . We assume the buffer size N_{sz}^i as 1.2Mb while all other parameters are same as they are taken in Section VII-A. The achieved energyefficiency figures as a function of traffic load along with the same obtained from our analytical model for different values

of N, N_m^i and N_{th}^i are plotted in Fig. 6(a), Fig. 6(b) and Fig. 6(c) respectively. In Fig. 6(a), we plot the energy-efficiency figures for N = 16, 32 by considering $N_m^i = 60Kb$ and $N_{th}^i = 0.48Mb$. Whereas in Fig. 6(b) and Fig. 6(c), the same is plotted for $N_m^i = 0.12Mb$, 60Kb with N = 16, $N_{th}^i = 0.48Mb$ and for $N_{th}^i = 0.48Mb$, 0.24Mb with $N = 16, N_m^i = 0.12Mb$. It can be observed from the figures that the analytical results match closely with simulation results for all considered scenarios which validate our analytical model. However, at higher load, the simulation results deviate from the analytical results which are due to prediction error. It is interesting to note that this deviation increases with an increase in traffic load. This is due to the following facts. In simulations, Poisson traffic arrivals are predicted by its mean and hence, the prediction error is proportional to the variance of the Poisson distribution. An increment of traffic load enhances the variance of the Poisson distribution and, hence the prediction error.

1) Effect of N: It can be noticed from Fig. 6(a) that the achieved energy-efficiency figures reduce drastically with an increase in N from N = 16 to N = 32. The reason can be explained as follows. In OSMP-EO, if ONU_i enters into the active mode from a sleep mode, it remains active until it upstreams N_{th}^i number of packets. Since in each cycle maximum of N_m^i number of packets get cleared, $\lceil \frac{N_{th}^i}{N_m^i} \rceil$ number of cycles are required to up-stream N_{th}^i number of packets and over this entire period, ONU_i remains active. If the value of N is doubled then the duration of each cycle almost become double for fixed allocation scheme and hence, the duration of active periods of ONU_i almost become double resulting in a drastic reduction of energy-efficiency.

2) Effect of N_m^i : It can be observed from Fig. 6(b) that the energy-efficiency figures remain almost same with the variation N_m^i at low load. However, at higher load, a slight decrement in energy-efficiency occurs if N_m^i is incremented. The reason can be explained as follows. If the value of N_m^i is doubled, then the number of cycles over which ONU_i remains in active mode before taking the next decision become half. Whereas, the duration of each cycle become almost double. Thus, the duration of active periods over which the US data is transmitted remains unchanged. However, in OSMP-EO, once an ONU decides to be active from sleep mode, on an average half of a cycle is wasted to receive the next GATE message (refer Section II-B). An increment of N_m^i causes an increase in

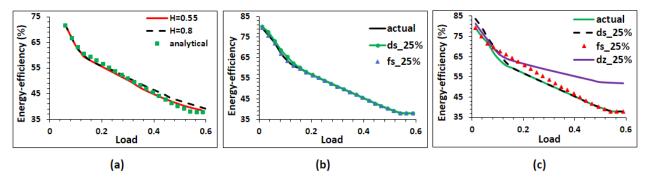


Fig. 7: (a) Applicability to self-similar traffic (b) Effect of reducing sleep-to-wake-up time of sleep mode (c) Effect of reducing power consumption of sleep modes.

cycle time and hence, this waste of half of a cycle which results in a reduction of energy-efficiency. Since this waste is present in every occurrence of state transitions from sleep mode to active mode, at low load, when the probability of waking up is very small, the effect of variation of N_m^i is negligible.

3) Effect of N_{th}^i : A decrement of N_{th}^i results in a reduction of buffer fill-up time and hence, the probability of state transition from sleep mode to active mode increases. In every occurrence of waking up from S_m , $T_{sw}^{S_m} + T_{cm}/2$ duration is wasted (refer Section IV-B). Thus, the decrement of N_{th}^i causes a reduction in energy-efficiency.

C. Applicability to self similar traffic

Here, we compare our analytical results with the results, obtained from simulations with self-similar traffic. In order to do so, in Fig. 7(a), we plot the energy-efficiency figures that are obtained from simulations with self-similar traffic for H =0.55, 0.8 and from the analytical model with Poisson traffic as a function of traffic load. Self-similar traffic is generated and predicted in the same way as it is discussed in Section VII-A. We consider N = 16, $N_m^i = 60Kb$ and $N_{th}^i = 0.48Mb$. It can be observed that the analytical results match closely with simulation results even for self-similar traffic. However, at high load, the energy-efficiency figures for self-similar traffic is little higher than the analytical results. This is due to the fact that the bursty nature of self-similar traffic increases the packet drop probability and hence, reduces the effective traffic load. The decrement of H reduces the burstiness and wherefore, the packets drop probability. Thus, the results for self-similar traffic approach to the analytical result with the reduction of H which can also be observed from Fig.

D. Design Insights

Here, we observe the effect of reducing the sleep-to-wakeup times and the power consumption figures of all sleep modes on the energy-efficiency which can be achieved by designing advance circuitries. For this purpose, energy-efficiency figures that are obtained from the analytical model if the sleep-towake-up time and power consumption figures of all sleep modes are reduced individually by 25% are plotted in Fig. 7(b) and Fig. 7(c) respectively. It can be noticed from Fig. 7(b) that the effect of diminishing the sleep-to-wake-up time of all modes are insignificant and hence, makes no sense for a circuit designer to focus on it. Reduction in power consumption of deep sleep and fast sleep increase energy-efficiency at low and moderate load respectively (refer Fig. 7(c)). This is because, ONU_i enter into the deep sleep if the buffer fill-up time is more than T_{lb}^{ds} which has a significantly high value (refer eq. (4)) and hence, exists only at low load. Similarly, fast sleep exists at moderate load where the effect of reducing P_{fs} is present. However, in OSMP-EO, doze mode reduces the energy consumption of active periods and therefore, the effect of reducing p_{dz} exists in all loads. An increment of traffic load increases the duration of active periods and hence, the enhances the improvement of energy-efficiency. Thus, from the point of view of a design engineer, he should mostly focus on reducing the power consumption figures of the doze mode. 7(a).

VIII. CONCLUSION

In this paper, we propose an ONU-assisted sleep mode protocol (OSMP-EO) for energy-efficient ONU design in EPON where ONUs are allowed to save energy even during the active cycles. This provides a significant improvement in energy savings especially at high load ($\sim 40\%$) when the energy-efficiency approaches zero in all existing ONU-assisted protocols. A mathematical analysis of the OSMP-EO protocol is also performed. We first prove that this protocol infringes the memoryless property. We then explain that the entire history can be easily captured by an intelligent selection of discrete observation instants and associated state descriptions which allow formulating a DTMC for analyzing energy-efficiency figures. The analytical model is verified with simulations for both Poisson and self-similar traffic. The analysis reveals that the protocol is almost insensitive to sleep-to-wake-up times of all sleep modes. An improvement in energy-efficiency can be achieved at low and moderate load if the power consumption figures can be reduced by designing better circuitries. Further, the energy-efficiency figure of OSMP-EO is highly sensitive to doze mode power consumption figure and hence, circuit designer should focus on reducing it.

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