Incremental Import Vector Machines for Classifying Hyperspectral Data

Ribana Roscher, Björn Waske, Member, IEEE, Wolfgang Förstner, Member, IEEE

Abstract—In this paper we propose an incremental learning strategy for import vector machines (IVM), which is a sparse kernel logistic regression approach. We use the procedure for the concept of self-training for sequential classification of hyperspectral data. The strategy comprises the inclusion of new training samples to increase the classification accuracy and the deletion of non-informative samples to be memory- and runtime-efficient. Moreover, we update the parameters in the incremental IVM model without re-training from scratch. Therefore, the incremental classifier is able to deal with large data sets. The performance of the IVM in comparison to support vector machines (SVM) is evaluated in terms of accuracy and experiments are conducted to assess the potential of the probabilistic outputs of the IVM.

Experimental results demonstrate that the IVM and SVM perform similar in terms of classification accuracy. However, the number of import vectors is significantly lower when compared to the number of support vectors and thus, the computation time during classification can be decreased. Moreover, the probabilities provided by IVM are more reliable, when compared to the probabilistic information, derived from an SVM's output. In addition, the proposed self-training strategy can increase the classification accuracy. Overall, the IVM and the its incremental version is worthwhile for the classification of hyperspectral data.

Index Terms—Import vector machines, incremental learning, hyperspectral data, self-training.

I. Introduction

Hyperspectral imaging, also known as imaging spectroscopy is used for more than two decades for monitoring the earth [1]. The spectrally continuous data ranges from visible to the short-wave infrared region of the electromagnetic spectrum and thus, enables a detailed separation of similar surface materials. Therefore hyperspectral imagery is used for classification problems that require a precise differentiation in spectral feature space [2]–[4]. Hyperspectral applications become even more attractive, regarding the increased availability of hyperspectral imagery through future space-borne missions, such as the German EnMAP (Environmental Mapping and Analysis Program) [5] and the Italian PRISMA (Hyperspectral Precursor of the Application Mission).

Nevertheless, the special properties of hyperspectral imagery demand more sophisticated image (pre-)processing and

The authors are with the Institute of Geodesy and Geoinformation, Faculty of Agriculture, University of Bonn, 53115 Bonn, Germany (e-mail: rroscher@uni-bonn.de, bwaske@uni-bonn.de, wfoerstn@uni-bonn.de). This work was supported by CROP.SENSe.net project, funded by the German Federal Ministry of Education and Research (BMBF) within the scope of the competitive grants program Networks of excellence in agricultural and

nutrition research (FKZ: 0315529). Manuscript received May 4, 2011 analysis [6], [7]. Conventional methods, such as the maximum likelihood classifier, can be limited when applied to hyperspectral imagery, due to the high-dimensional feature space and a finite number of training samples. Consequently, the classification accuracy often decreases with an increasing number of bands (i. e., the well-known Hughes phenomena). Thus, more flexible classifiers, such as spectral angle mapper, neural networks and support vector machines (SVM), are applied on hyperspectral imagery [2], [4], [8], [9].

Among the various developments in the field of pattern recognition, SVM [10] are perhaps the most popular approach in recent hyperspectral applications [7]. SVM can outperform other methods in terms of the classification accuracy [11], [12] and still exhibit further modification and improvement, e. g., in context of modifying the kernel functions [13] and semi-supervised learning [14]. Whereas other classifiers can directly solve multi-class problems, the binary nature of SVM requires an adequate multi-class strategy (e. g., [15]). In contrast to other classifiers, which directly provide class labels or probabilities, SVM provide the distance of each pixel to the hyperplane of the binary classification problem. This information is used to determine the final class membership. Although the output of SVM can be transferred to probabilities (e. g., [16]), the reliability of these values could be inadequate [17], [18]. Nevertheless, probabilities are of interest and can be used e. g., as input in a markov random field model or for uncertainty analysis [19]-[21].

Logistic regression as an alternative probabilistic discriminative classification model was already used in context of classification and feature selection of hyperspectral imagery [22], [23]. The approach was extended to kernel logistic regression, e. g., [24], [25], showing a better accuracy but a higher complexity. To overcome the limitations in context of efficiency and computation time several sparse realizations of (kernel) logistic regression have been developed, including the explicit usage of a sparsity enforcing prior [26]–[29], an implicit prior used in the relevance vector machines (RVM) [17], [18], [30] or a greedy subset selection for the concept of import vector machines (IVM) [31].

Recent classifier developments, such as SVM, usually perform well on high-dimensional data sets. Nevertheless, the classification accuracy can be affected when a limited number of training samples is available [2], [32]. One possibility to overcome this problem is active learning, which has been studied extensively in the literature, e. g., [27], [33]–[35]. In this paper we use the specialized concept of self-training [36]. Self-training is based on the sequentially training of a classifier with new training samples. I. e., starting with a initial classifier

model learned from a few labeled training samples, an iterative classification procedure is performed. After each classification, new relevant training samples are selected and usually the classifier is re-trained from scratch. That means, the classifier is learned by using old and new training samples without taking into account the previous learned classifier model. For the acquisition of new training samples we use spectral as well as spatial information by using a discriminative random field (DRF) [37]. The gain of integrating spatial information was already discussed in several studies in context of classifying hyperspectral data [20], [21], [26], [34], [38]. Moreover, we

However, to be time- and memory-efficient we need an incremental learning strategy within the self-training approach. Several incremental learning methods have been proposed, extending classical and state-of-the art off-line methods. Incremental generative models, e. g. [39], provide probabilities but tend to become very complex. Discriminative models, on the other hand, like incremental linear discriminant analysis [40] or incremental support vector machines, e. g. [41], show a good performance in classification tasks.

consider the probabilistic outputs by the IVM, to assess the

reliability of the classification result.

Kernel-based algorithms have achieved considerable success in incremental and off-line learning settings [42], [43]. However, incremental kernel-based learning settings need strategies for dealing with existing and new samples. The challenge is to perform efficient, incremental update steps without suffering a loss in performance.

The main objective is to propose an incremental learning strategy to update the trained IVM model. The classifier is called incremental IVM. Therefore, the IVM model is not re-trained from scratch, as usually done in context of active and self-training approaches. The incremental IVM consists of the selection of new training samples, the deletion of irrelevant training samples, and the update of the IVM model. Therefore, the classifier is able to deal with infinitely new training samples.

The potential of the IVM and the its incremental version in context of classifying hyperspectral data is evaluated. The performance is compared to SVM in terms of accuracy. In addition, the reliability of the probabilities outputs provided by IVM and SVM are evaluated by using a discriminative random field and the effect is further investigate by neglecting uncertain test samples. Our study is aiming on the classification of three different hyperspectral data sets, i. e., two urban areas from the city of Pavia and an agricultural area from Indiana, USA, using SVM and IVM.

The paper is organized as follows. Section II discusses the logistic regression, kernel logistic regression and the IVM algorithm. Moreover, the concept of SVM and related classifiers is briefly introduced and compared to IVM. Section III introduces the proposed strategy for self-training, including the DRF model. Also the incremental IVM are explained. The experimental setup is given in Section IV. The results are presented and discussed in Section V. We conclude in Section VI.

II. THEORETICAL BACKGROUND

In this section IVM model is introduced. Starting from the logistic regression, we discuss kernels and sparsity and finally the sparse kernel logistic regression model, i.e., IVM. Also a brief introduction to SVM and sparse multinomial logistic regression is given. In II-D the discriminative random field is introduced, which incorporates spatial information.

A. Logistic Regression and Kernel Logistic Regression

a) Logistic Regression: We assume to have a training set (x_n, y_n) , n = 1, ..., N of N labeled samples with feature vectors $x_n \in \mathbb{R}^M$ and class labels $y_n \in \mathcal{C} = \{\mathcal{C}_1, ..., \mathcal{C}_K\}$. The observations are collected in a matrix $X = [x_1, ..., x_N]$, while the corresponding labels are summarized in the vector $y = [y_1, ..., y_N]$.

In the two-class case the posterior probability p_n of a feature vector x_n is assumed to follow the Logistic Regression model

$$p_n = p(y_n = C_1 | \mathbf{x}_n; \mathbf{w}) = \frac{1}{1 + \exp(-\mathbf{w}^\mathsf{T} \mathbf{x}_n)}$$
 (1)

with the extended feature vector $\mathbf{x}_n^\mathsf{T} = [1, \boldsymbol{x}_n^\mathsf{T}] \in \mathbb{R}^{M+1}$ and the extended parameters $\mathbf{w}^\mathsf{T} = [w_0, \boldsymbol{w}^\mathsf{T}] \in \mathbb{R}^{M+1}$ containing the bias w_0 and the weight vector \boldsymbol{w} .

b) Kernel Logistic Regression: In linear non-separable cases, the original observations X are implicitly mapped from the input space to a higher-dimensional kernel space with the kernel matrix $K = [k_{nm}]$ via the kernel function $k_{nm} = k(\mathbf{x}_n, \mathbf{x}_m)$. The kernel matrix K consists of affinities between the points depending on the distance measure defined by the kernel function.

The parameters, in the kernel-based approach referred to α , are determined in an iterative way with

$$\boldsymbol{\alpha}_{(i)} = \left(\frac{1}{N} \boldsymbol{K}^\mathsf{T} \boldsymbol{R} \boldsymbol{K} + \lambda \boldsymbol{K}\right)^{-1} \boldsymbol{K}^\mathsf{T} \boldsymbol{R} \boldsymbol{z} \tag{2}$$

$$z = \frac{1}{N} \left(K\alpha_{(i-1)} + R^{-1} \left(p - t \right) \right)$$
 (3)

by optimizing the objective function

$$Q_{(i)} = -\frac{1}{N} \sum_{n} \left[t_n \log p_n + (1 - t_n) \log (1 - p_n) \right]$$

$$+ \frac{\lambda}{2} \boldsymbol{\alpha}_{(i)}^\mathsf{T} \boldsymbol{K} \boldsymbol{\alpha}_{(i)}$$

$$(4)$$

using the Newton-Raphson procedure. The $(N \times N)$ -dimensional diagonal matrix R has the elements $r_{nn} = p_n (1-p_n)$ with $p_n = 1/(1+\exp(-k_n\alpha))$ and k_n as the nth row of the kernel matrix K. The binary target vector $t \in \{0,1\}$ of length N codes the labels with $t_n = 0$ for $y_n = \mathcal{C}_1$ and $t_n = 1$ for $y_n = \mathcal{C}_2$. Additionally, we add an L_2 -norm regularization term with parameter λ to prevent overfitting.

B. Import Vector Machines

The kernel logistic regression includes all training samples to train the classifier, which is computationally expensive and memory intensive for data sets with many training samples. Similar to the SVM the IVM algorithm [31] chooses a subset V

of feature vectors out of the training set with $V = |\mathcal{V}|$ samples $X_{\mathcal{V}} = [\mathbf{x}_{\mathcal{V},m}], \ m = 1, \dots, V$, obtaining a sparse solution of the kernel logistic regression. These feature vectors are called import vectors.

Following (2) and (3) the parameters in iteration i are determined by

$$\boldsymbol{\alpha}_{(i)} = \left(\frac{1}{N} \boldsymbol{K}_{\mathcal{V}}^{\mathsf{T}} \boldsymbol{R} \boldsymbol{K}_{\mathcal{V}} + \lambda \boldsymbol{K}_{R}\right)^{-1} \boldsymbol{K}_{\mathcal{V}}^{\mathsf{T}} \boldsymbol{R} \boldsymbol{z} \tag{5}$$

$$z = \frac{1}{N} \left(K_{\mathcal{V}} \alpha_{(i-1)} + R^{-1} \left(p - t \right) \right). \tag{6}$$

The $(N \times V)$ -dimensional kernel matrix is given by $K_{\mathcal{V}} = [k(\mathbf{x}_n, \mathbf{x}_{\mathcal{V},m})]$ and the $(V \times V)$ -dimensional regularization matrix by $K_R = [k(\mathbf{x}_{\mathcal{V},l}, \mathbf{x}_{\mathcal{V},m})], \{l,m\} = 1,\ldots,V$.

The IVM is illustrated in Algorithm 1. The convergence

```
Initialize \mathcal{V}_0 := \{\}, \mathcal{X}_0 := \{\boldsymbol{x}_1, \dots, \boldsymbol{x}_N\}, i := 0; repeat

Compute \boldsymbol{z}_{(i)} from the current set \mathcal{V}_{(i)}; foreach \boldsymbol{x}_n \in \mathcal{X}_{(i)} do

Let \mathcal{V}_{(i)n} := \mathcal{V}_{(i)} \cup \boldsymbol{x}_n;
Compute \alpha_{(i)n} from \mathcal{V}_{(i)n} in a one-step iteration;
Evaluate error function \mathcal{Q}_{(i)n};
end
Find best point \boldsymbol{x}^* = \boldsymbol{x}_n with n = \operatorname{argmin}_n \mathcal{Q}_{(i)n};
Update \mathcal{V}_{(i+1)} := \mathcal{V}_{(i)} \cup \boldsymbol{x}^*, \mathcal{X}_{(i+1)} := \mathcal{X}_{(i)} \setminus \boldsymbol{x}^*, i := i+1;
until \mathcal{Q} converged;
```

Algorithm 1: IVM: In every iteration i each point $x_n \in \mathcal{X}_{(i)}$ from the current training set $\mathcal{X}_{(i)}$ is tested to be in the set of import vectors $\mathcal{V}_{(i)}$. The point x^* yielding the lowest error $\mathcal{Q}_{(i)n}$ is included. The algorithm stops as soon as \mathcal{Q} converged.

criterion is proposed by the ratio $\epsilon = |\mathcal{Q}_{(i)} - \mathcal{Q}_{(i-\Delta i)}|/|\mathcal{Q}_{(i)}|$ with a small integer Δi .

The original algorithm selects the import vectors in a greedy forward selection procedure. The approach is extended to a forward stepwise selection, which allows forward and backward steps. The advantage of this procedure is, that import vectors, which once entered can be dropped if they are no longer relevant. In all experiments an improvement of the results could be observed. Furthermore, an incremental update procedure to compute the inverse in (5) depending on the last iteration is used, which makes the algorithm more efficient. The incremental update is described in a detailed way in Section III-C.

The two class model can be generalized to the multi-class model. Then the objective function is

$$Q = -\frac{1}{N} \sum_{n} \boldsymbol{t}_{n}^{\mathsf{T}} \log \boldsymbol{p}_{n} + \frac{\lambda}{2} \sum_{k} \boldsymbol{\alpha}_{k}^{\mathsf{T}} \boldsymbol{K}_{R} \boldsymbol{\alpha}_{k}$$
 (7)

with the probabilities $P = [p_1, \dots, p_N]$ obtained by

$$p_{nk} = \frac{\exp(\mathbf{k}_{\mathcal{V},n}\alpha_k)}{\sum_{l} \exp(\mathbf{k}_{\mathcal{V},n}\alpha_l)}.$$
 (8)

The binary target vector t_n of length K uses the 1-of-K coding scheme so that all components but t_{nk} are 0 if the point x_n

is from class C_k . In the Newton-Raphson procedure in (5) and (6) we have to use one R_k and one z_k for each class.

3

C. Related Classifiers

Recently several algorithms have been developed, which enforce sparseness to control both the generalization capability of the learned classifier model and the complexity, as e. g., [44]–[47]. In these algorithms the model consists of a sparse weighted linear combination of basis functions, which are the input features themselves, nonlinear transformations of them or kernels centered on them. In this section we restrict ourselves to the review of realizations of sparse (kernel) logistic regression and SVM, whereby the latter one is used for comparison in the experiments.

1) Realizations of Sparse (Kernel) Logistic Regression: Using logistic regression or its kernel realization can be prohibitive regarding memory and time requirements if the dimension of the features or the number of training samples is large. Several sparse algorithms have been developed in the last years to overcome this problem.

The relevance vector machine [18] uses the same model as the kernel logistic regression in combination with an implicit prior as regularization term, the so-called ARD (automatic relevance determination) [48] prior, to induce sparseness. The prior includes several regularization parameters, also called hyperparameters, which are determined during the optimization process. The algorithm have shown to be very sparse, but also tends to underfit, leading to a non well-generalized model [29]. Additionally, the RVM uses an expectation-maximization (EM)-like learning method and therefore, can suffer from local minima leading to non-optimal classification results.

Alternatively, [28], [29] use a Laplace prior enforcing sparseness, which assigned regularization parameter is determined via cross-validation. These approaches propose sparse multinomial (kernel) logistic regression (SMLR) using different methods for a fast computation. In the field of hyperspectral image classification these approaches have been applied and further developed in, e. g., [26], [27].

2) Support Vector Machines: The SVM find an optimal nonlinear decision boundary by minimizing the objective function

$$Q_{SVM} = \frac{1}{N} \sum_{n} [1 - y_n f(\boldsymbol{x}_n)]_+ + \frac{\lambda}{2} ||f||^2$$
 (9)

with $f(\boldsymbol{x}_n) = \sum_n \alpha_n K(\boldsymbol{X}, \boldsymbol{x}_n)$.

Contrary to IVM, which maximize the posterior probabilities, SVM aim to maximize the margin between the hyperplane and the closest training samples, the so-called support vectors. SVM are a binary classifier, with the decision rule given by the sign of $f(x_n)$.

3) Comparison with Import Vector Machines: The main properties of SVM, IVM, SMLR and RVM are summarized in Table I and briefly compared below.

The objective function of the SVM is quadratic and therefore convex and can be efficiently solved with sequential minimal optimization algorithm (SMO) [49]. The objective function of the IVM is convex, but non-quadratic. The function

TABLE I
SUMMARY OF THE SVM, RVM AND IVM ALGORITHM REGARDING SEVERAL CHARACTERISTICS. HERE "-/0/+" MEANS THAT THE ALGORITHM
SATISFIES A CERTAIN PROPERTY "BARELY/PARTIALLY/COMPLETELY".

Algorithm	Objective function, optimization procedure	Sparse	Training	Testing	Probabilistic	Multi-class	
			time	time			ĺ
SVM	convex, greedy with SMO algorihtm	0	+	0	0	0	
RVM	nonconvex, IRLS, EM-like	+	_	+	+	+	ĺ
SMLR	convex, see Sec. II-C1	0/+	0/+	0/+	+	+	ĺ
IVM	convex, IRLS with greedy forward stepwise	+	0/+	+	+	+	
	selection (see Sec. II-B and III-C)						ĺ

can be solved with iterated re-weighted least squares (IRLS) and a greedy forward selection of import vectors.

All models are sparse, i.e., the model parameters are primarily zero. The sparseness arises from different techniques, namely the usage of a prior in the SMLR models and the RVM or the greedy selection of a fraction of the training samples in the IVM model. However, IVM and RVM have shown to be sparser when compared to SVM [30], [31], and thus, require less computation time during classification. The training time of the IVM, on the other hand, can be slower than for the SVM, because of the non-quadratic objective function. Nonetheless, the training time depends on the number of training samples and the attainable sparseness, that therefore also SMLR and IVM can reach a faster training time.

In comparison to RVM and SMLR approaches, which use the whole kernel during the optimization procedure, the IVM algorithm is suitable for large data sets, since only a subset of the training samples is used for computations during the optimization. Consequently, also only a fraction of the whole kernel have to be computed and stored. In contrast to this, e. g., the standard RVM algorithm has to solve a large matrix inversion of the size of the number of training samples during the optimization process and therefore can be slow and intractable for large datasets.

While IVM, SMLR and RVM directly provide a probabilistic output, the SVM algorithm has the opportunity to transform its output to probabilities after some post-processing steps [16]. Though, as Tipping [18] already shows the transformed output is not necessarily statistical interpretable.

Both IVM, SMLR and RVM are introduced as multi-class classifiers. The standard SVM have the opportunity to multi-class classification but they need a coupling strategy to get multi-class classification results. Moreover, these multi-class approaches are more suitable for practical use than direct multi-class approaches for SVM [50].

D. Discriminative Random Fields

A discriminative random field is employed to model prior knowledge about the neighborhood relations within the image. The final classification is assumed to be smooth, i. e., neighboring pixels are more likely to belong to the same class than to different classes.

With this, the best classification C_{DRF} is given by the argument of the minimum of the energy

$$E(\boldsymbol{y}) = -\sum_{j \in \mathcal{I}} \log p(y_j | \boldsymbol{x}_j) - \beta \sum_{\{m,j\} \in \mathcal{N}} \delta(y_j, y_m), \quad (10)$$

where x_j is the observed feature vector from the jth pixel, \mathcal{I} being the set of all pixels and δ being the Kronecker delta function. The weighting parameter is given by β , which can be determined via cross-validation. The first term in (10) models the probability of a class assignment y_j of the jth pixel, defined by the probabilistic output of the IVM. The second term describes the interaction potential as a Potts model over a 2D lattice penalizing every dissimilar pair of labels and therefore heterogeneous regions. The set of all neighboring pixels is given by \mathcal{N} .

III. INCREMENTAL LEARNING STRATEGY FOR SELF-TRAINING

In this section we introduce the self-training concept and the learning strategy for the incremental IVM. Self-training refers to the sequential selection of new training data and the adaption of the previous classifier model. The previous classifier model can (i) be neglected and re-trained from scratch or (ii) incrementally be updated. Following (i), a regular classifier training is performed, using the whole training samples set (i. e., previous + new training samples). The latter approach "simply" updates the previous model, using the newly selected training samples.

A. Self-training Concept

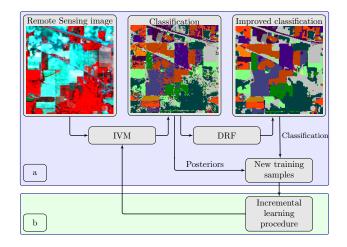


Fig. 1. Self-training scheme consisting of two steps: In the first step (Fig. 1a) the image is classified with the learned incremental IVM model and the DRF. The probabilistic output of the IVM and the classification result of the DRF are used to acquire new training samples. In the second step (Fig. 1b) the classifier model is incrementally updated.

In the first step (Fig. 1a) we identify potential new training samples. For this selection step, we use the classification result provided by the DRF and the probabilistic outputs of the incremental IVM. In the second step (Fig. 1b) we use the incremental learning strategy to update the classifier model, without re-training from scratch. The procedure is repeated until no additional training samples can be selected. The latter step consists of the inclusion of new training samples, the deletion of irrelevant training samples and finally, the update of the IVM model. Therefore the approach can handle large data sets or even infinitive data streams. However, this step is independent of the used self-training strategy and other approaches can be used to identify new training samples. Both steps are explained in detail in the next paragraphs.

B. Acquisition of New Training Samples for Self-training

This acquisition step illustrated in Fig. 2 is subdivided in 3 parts: First, we use a DRF as expert to identify worthwhile samples by evaluating the disagreement between the classification C yielded by the IVM classifier and the DRF result C_{DRF} . The aspect implies, that the samples with estimated label $y_{\mathrm{DRF},j}$ derived from the DRF are sufficient uncertain with $\max(p_j) < 0.5$ obtained from the IVM classifier. Using such samples we ensure progress in self-training.

Second, we exclude from the selected samples these ones, whose probability is too small. I. e., the influence of the new samples is restricted, to ensure that the model is gradually changed and stable.

Finally, we sort the chosen samples by their potentially influence on the model, to enable the flexibility of the model during self-training. The potential influence uses the concept of leverage points in regression [51]. The leverage values in a weighted regression are contained in the vector

$$l = \operatorname{diag}\left(K_{\operatorname{pot}}\left(K_{\operatorname{pot}}^{\mathsf{T}}R_{\operatorname{pot}}K_{\operatorname{pot}}\right)^{-1}K_{\operatorname{pot}}^{\mathsf{T}}R_{\operatorname{pot}}\right). \tag{11}$$

The kernel matrix obtained from the potential new training samples X_{pot} is given by $K_{\text{pot}} = [k(\mathbf{x}_{\text{pot},j},\mathbf{x}_{\mathcal{V},m})]$ and R_{pot} is the weight matrix of the class the training sample belongs to. Training samples with a high leverage value first are considered first, since they causes large effects in the learned model. The self-training is stopped if no more training samples can be acquired.

To prevent an acquisition, which leads to an imbalanced number of training samples per class, we ensure sampling an equal number of training samples for each class. If not enough new training samples can be acquired with the proposed self-training approach, we also consider training samples with a high probability and whose label in C and $C_{\rm DRF}$ are the same. These samples have a small influence onto the model and should not change the result too much, but balance the number of samples of each class. If there are still not enough samples for a class, we follow the over-sampling approach by duplicating existing samples from the concerned class at random and add a small noise to them [52].

C. Incremental Learning

To update the learned classifier, we consider the following aspects:

- New training samples acquired with the proposed selftraining approach (see Section III-B) are included.
- Non-informative samples are deleted.
- The set of import vectors is updated.
- a) Update Training Vectors: For the two-class case the incremental learning procedure is stated as follows. We add training vectors X_{Δ} with targets t_{Δ} so that $N_{(s)} := N_{(s-1)} + \Delta N$ with ΔN as the number of new training samples. At each self-training iteration s we extend the matrices and vectors

$$m{\mathcal{K}}_{(s)} = \left[egin{array}{c} m{\mathcal{K}}_{(s-1)} \ m{\mathcal{K}}_{\Delta} \end{array}
ight], m{t}_{(s)} = \left[egin{array}{c} m{t}_{(s-1)} \ m{t}_{\Delta} \end{array}
ight]$$

to obtain the updated parameters $\alpha_{(s)}^+$ given by (12) and (13) yielding

$$oldsymbol{z}_{(s)} = \left[egin{array}{c} oldsymbol{z}_{(s-1)} \ oldsymbol{z}_{\Delta} \end{array}
ight], \, oldsymbol{p}_{(s)} = \left[egin{array}{c} oldsymbol{p}_{(s-1)} \ oldsymbol{p}_{\Delta} \end{array}
ight].$$

The Sherman-Morrison-Woodbury (SMW) [53] formula is used to compute the inverse in (12) yielding (14) with $A=1/N_{(s)}K_{(s-1)}^\mathsf{T}R_{(s-1)}K_{(s-1)}+\lambda K_{R,(s-1)}$. Note that the update (14) only incorporates an inverse of size $\Delta N \times \Delta N$, since the inverse A^{-1} was computed in time step s-1. With these steps the parameters can be updated in an incremental way without re-training from scratch.

The update (12) can also be formulated for a decreasing number of training vectors in a similar manner, leading to an efficient update rule as in (14), only with the sign of R_{Δ} changed. Training vectors can be removed based on their "age", so that they follow the first-in-first-out strategy. This can lead to instable results. Therefore, we identify training vectors, which can be removed using Cook's distance [54]. Cook's distance measures the effect of deleting a training vector as the higher the value the more informative is the training vector:

$$d_n = \frac{(\boldsymbol{p}_n - \boldsymbol{t}_n)^\mathsf{T} (\boldsymbol{p}_n - \boldsymbol{t}_n)}{a \text{ MSE}} \frac{l_n}{(1 - l_n)^2}$$
(15)

with a as the number of parameters and MSE as the mean squared error summed over all classes given by the difference between 1 and the mean probability of all training samples belonging to class \mathcal{C}_k . The leverage value of each training vector is given by l_n . The training vectors with the lowest distance are removed in a greedy backward selection until the value of the optimization function increases more than of 5%.

b) Update the Set of Import Vectors: To add and remove import vectors we use the forward stepwise selection as described in Section II. We also make use of the SMW formula and proceed in the same way as described in Algorithm 1 until a convergence criterion is reached.

To generalize the two class model to the multi-class model we use the class-specific values $R_{k,(s-1)}$ and $z_{k,(s-1)}$.

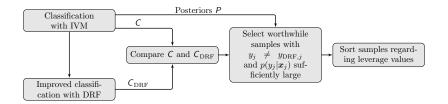


Fig. 2. Schematic diagram of the selection of new training samples: Worthwhile samples are identified by comparing the classification C and C_{DRF} . Samples with a relatively small posteriori probability are excluded. Remaining samples are sorted regarding their leverage value, i. e., samples with high influence are preferred.

$$\boldsymbol{\alpha}_{(s)}^{+} = \left(1/N_{(s)}\left(\boldsymbol{K}_{(s-1)}^{\mathsf{T}}\boldsymbol{R}_{(s-1)}\boldsymbol{K}_{(s-1)} + \boldsymbol{K}_{\Delta}^{\mathsf{T}}\boldsymbol{R}_{\Delta}\boldsymbol{K}_{\Delta}\right) + \lambda \boldsymbol{K}_{R,(s-1)}\right)^{-1}\left(\boldsymbol{K}_{(s-1)}^{\mathsf{T}}\boldsymbol{R}_{(s-1)}\boldsymbol{z}_{(s-1)} + \boldsymbol{K}_{\Delta}^{\mathsf{T}}\boldsymbol{R}_{\Delta}\boldsymbol{z}_{\Delta}\right)$$
(12)

$$z_{(s)} = \frac{1}{N_{(s)}} \begin{bmatrix} K_{(s-1)}\alpha_{(s-1)} + R_{(s-1)}(p_{(s-1)} - t_{(s-1)}) \\ K_{\Delta}\alpha_{(s-1)} + R_{\Delta}(p_{\Delta} - t_{\Delta}) \end{bmatrix}$$
(13)

$$\alpha_{(s)}^{+} = A^{-1} - 1/N_{(s)}A^{-1}K_{\Delta}^{\mathsf{T}}\left(R_{\Delta}^{-1} + 1/N_{(s)}K_{\Delta}A^{-1}K_{\Delta}^{\mathsf{T}}\right)^{-1}K_{\Delta}A^{-1}\left(K_{(s-1)}^{\mathsf{T}}R_{(s-1)}z_{(s-1)} + K_{\Delta}^{\mathsf{T}}R_{\Delta}z_{\Delta}\right)$$
(14)

IV. EXPERIMENTAL SETUP

A. Data Sets

We use three hyperspectral data sets – CENTER OF PAVIA, UNIVERSITY OF PAVIA and INDIAN PINES— from study sites with different environmental setting. The data sets have been used in a multitude of studies, e. g., [7], [13], [32], [55], [56].

The CENTER OF PAVIA image was acquired by ROSIS-3 sensor in 2003. The spatial resolution of the image is 1.3 meter per pixel. The data cover the range from 0.43 μ m to 0.86 μ m of the electromagnetic spectrum. However, some bands have been removed due to noise and finally 102 channels have been used in the classification. The image strip, with 1096×492 pixels in size, lies around the center of Pavia. The classification is aiming on 9 land cover classes. The UNIVERSITY OF PAVIA data set was also acquired by ROSIS-3 sensor with 610×340 pixels in size and 103 channels. The classification is aiming on 9 land cover classes. The INDIAN PINES data set was acquired by the AVIRIS instrument in 1992. The study site lies in a predominately agricultural region in NW Indiana, USA. AVIRIS operates from the visible to the short-wave infrared region of the electromagnetic spectrum, ranging from 0.4 μ m to 2.4 μ m. The data set covers 145×145 pixels, with a spatial resolution of 20 m per pixel. The experiments are aiming on the classification of 16 classes (Table II).

B. Methods

In the experiments the IVM for classifying hyperspectral data is analyzed. In addition SVM were applied on the data sets. SVM are perhaps the most popular approach in more recent applications and seems particularly advantageous when classifying high-dimensional data sets. Thus, the method is regarded as a kind of benchmark classifier for comparison with new approaches.

Moreover, a DRF is applied on the respective probabilistic output. Besides as input for the DRF, we use the probabilistic

outputs to analyze the uncertainty of the classification result. We assess the reliability of the probabilities by rejecting uncertain test samples and deriving the classification accuracy on the non-rejected test points [19]. The rejection rate is given by a threshold on the posterior probability, whereby the accuracy provided by SVM and IVM is reported as a function of the rejection rate in discrete intervals.

In addition, the incremental IVM is evaluated by applying the self-training approach on the three data sets in terms of the classification accuracy and sparsity.

To investigate the impact of the number of training samples on the performance of the model (e.g., in terms of sparsity and accuracy) we use two different training sets, containing (i) all initial training samples and (ii) 10% of each class (with a minimum of at least 10 samples per class). For (ii), we performed a stratified random sampling, selecting 10% of the samples of each class from the initial training set. The final results were averaged.

For the SVM and the (incremental) IVM we use a radial basis function kernel. The kernel parameters are determined by a 5-fold cross-validation. Also the DRF parameter β is determined by by 5-fold cross-validation. The result provided by the common IVM is used for the initialization of the self-training. The self-training procedure is repeated until no more training samples can be selected.

The IVM algorithm¹ is implemented in MATLAB and C++. The SVM classification is performed in MATLAB, using the LIBSVM approach by Chang and Lin [57]. To compute the result in (10) we use the graph-cut algorithm² [58]. Besides the standard SVM classification, we use the method of [16] to convert the output of the SVM to probabilities (from now on referred to as probabilistic SVM).

CENTER OF PAVIA			University of Pavia				Indian Pines				
	class	# training	# test	С	lass	# training	# test	st class		# training	# test
	Asphalt	678	6907		Asphalt	548	6304		Alalfa	27	27
	Bare Soil	820	5729		Bare Soil	532	4572		Corn-no till	778	656
	Bitumen	808	6479		Bitumen	375	981		Corn-min till	408	426
	Meadows	797	2108		Gravel	392	1815		Corn	110	124
	Bricks	485	1667		Meadows	540	18146		Grass/Pasture	208	289
	Shadow	195	1970		Metal Sheets	265	1113		Pasture/Trees	339	408
	Tiles	223	2899		Bricks	514	3364		Hay	228	261
	Trees	785	5723		Shadow	231	795		Soybeans-no till	506	462
	Water	745	64533		Trees	524	2912		Soybeans-mid till	975	1493
	-	-	-	-	_	-	-		Soybeans-clean till	265	349
	-	-	-	-		-	-		Wheat	100	112
	-	-	-	-		_	-		Woods	637	657
	-	-	-	-		-	-		Bldg-Grass	163	217
	-	-	-	-		-	-		Stone	45	50
	_	_	-	_		_	_		Grass/Pasture-mowed	12	14

TABLE II
NUMBER OF TRAINING AND TEST SAMPLES.

TABLE III

OVERALL ACCURACY (OA), AVERAGE ACCURACY (AA) AND KAPPA COEFFICIENT (KAPPA) OF SVM, SVM WITH TRANSFORMED PROBABILITIES (PROB. SVM), IVM, INCREMENTAL IVM (IIVM) WITH SELF-TRAINING (ST) AND ADDITIONAL DRF. FOR THE SMALLER DATA SETS (10%) WE REPORT THE MEAN AND STANDARD DEVIATION IN BRACKETS OVER 20 RUNS.

		CENTER OF PAVIA			Univ	ERSITY OF	PAVIA	Indian Pines		
Train data	Algorithm	OA[%]	AA[%]	Kappa	OA[%]	AA[%]	Kappa	OA[%]	AA[%]	Kappa
100%	SVM	97.1	94.6	0.95	79.0	87.9	0.73	61.6	69.6	0.57
100%	Prob. SVM	97.0	94.2	0.95	78.4	87.7	0.73	60.7	68.9	0.56
100%	IVM	97.2	94.2	0.95	78.3	87.4	0.73	63.7	67.8	0.59
100%	iIVM+ST	97.3	95.2	0.95	86.6	90.0	0.83	63.7	67.8	0.59
100%	Prob. SVM+DRF	97.8	96.4	0.96	82.6	90.5	0.78	66.3	71.6	0.62
100%	IVM+DRF	98.4	96.5	0.97	85.9	91.4	0.82	70.6	80.0	0.67
100%	iIVM+ST+DRF	98.6	97.1	0.98	94.2	93.8	0.92	70.6	80.2	0.67
10%	SVM	92.1 (0.6)	85.1 (1.0)	0.86 (0.01)	70.2 (3.8)	77.5 (2.4)	0.62 (0.04)	54.7 (2.3)	63.0 (1.6)	0.49 (0.03)
10%	Prob. SVM	93.0 (1.3)	86.2 (2.1)	0.87 (0.02)	69.4 (3.8)	77.3 (2.4)	0.61 (0.04)	54.9 (2.5)	63.2 (2.3)	0.50 (0.03)
10%	IVM	92.2 (1.1)	85.4 (1.6)	0.86 (0.02)	70.3 (3.5)	77.3 (0.8)	0.62 (0.04)	58.2 (0.1)	65.2 (0.8)	0.53 (<0.01)
10%	iIVM+ST	94.3 (1.0)	88.2 (1.7)	0.90 (0.02)	74.3 (2.4)	77.9 (1.1)	0.66 (0.02)	64.8 (1.1)	69.2 (1.1)	0.61 (0.01)
10%	Prob. SVM+DRF	93.7 (1.5)	88.9 (2.5)	0.89 (0.03)	76.3 (4.9)	79.2 (4.5)	0.70 (0.06)	60.1 (2.8)	65.9 (1.3)	0.55 (0.03)
10%	IVM+ DRF	92.8 (1.3)	87.8 (2.0)	0.87 (0.02)	78.5 (4.3)	79.9 (1.5)	0.72 (0.05)	64.5 (1.3)	73.5 (2.0)	0.60 (0.02)
10%	iIVM+ST+DRF	95.8 (0.8)	92.3 (2.1)	0.92 (2.2)	80.9 (2.9)	81.0 (1.9)	0.74 (0.03)	70.1 (1.2)	75.8 (2.5)	0.66 (0.02)

V. EXPERIMENTAL RESULTS

Fig. 4 shows the ground truth and the classification results for all three data sets. As shown in Table III, the IVM is competitive to SVM in terms of accuracy and result in almost similar overall accuracies and kappa coefficients, irrespectively from the number of training samples. However, the SVM outperforms the IVM in terms of the averaged class accuracy (AA) for the INDIAN PINES data set, when using all training samples. As expected the accuracies are increased by increasing the number of training samples, independently from the classifier method.

It is interesting to underline that in many cases, the probabilistic SVM provide (slightly) lower accuracies (i.e., OA and AA) than a standard SVM. Behind this fact, the reliability of the probabilistic output of the SVM can be questioned. This assumption is confirmed by the results provided by the DRF. The accuracies of both methods, the probabilistic SVM and the IVM, are increased by the DRF. This is in accordance with other studies that have successfully integrated spatial

information when classifying hyperspectral imagery [20], [21], [26], [34], [38]. However, the improvement of the IVM is usually higher, sometimes to a degree that the combination of IVM and DRF outperforms the accuracy provided by the probabilistic SVM in combination with a DRF.

10

4811

10

Table IV shows the class-specific accuracies achieved in the UNIVERSITY OF PAVIA data set. The results confirm the previous findings, e. g., SVM and IVM show similar overall accuracies. While some classes are more accurately classified by SVM, IVM are more adequate for the separation of other classes.

To underline this finding, an analysis of the probabilistic output is shown in Fig. 3. With an increasing rejection threshold, IVM provide higher OA on the INDIAN PINES data and in most cases on the CENTER OF PAVIA and UNIVERSITY OF PAVIA data set. Consequently, it can be assumed that samples with high class probabilities are more accurately classified by IVM than by SVM, whereas relatively low class probabilities by the IVM are more likely referred to misclassified samples.

Table V shows, that the number of import vectors is lower when compared to the number of support vectors. This is in

¹http://www.ipb.uni-bonn.de/ivm/

²http://vision.csd.uwo.ca/code/

TABLE IV

CLASS-SPECIFIC ACCURACIES OF THE UNIVERSITY OF PAVIA DATA SET OF SVM, SVM WITH TRANSFORMED PROBABILITIES (PROB. SVM), IVM, INCREMENTAL IVM (IIVM) WITH SELF-TRAINING (ST) AND ADDITIONAL DRF. THE RESULTS ARE GIVEN IN PERCENT ([%]).

Class	SVM	Prob.SVM	IVM	iIVM+ST	Prob.SVM + DRF	IVM+DRF	iIVM+ST+DRF
Asphalt	85.4	83.3	83.8	84.3	94.7	95.5	94.1
Bare Soil	93.7	93.6	89.1	93.9	97.7	98.5	98.2
Bitumen	90.5	90.5	90.3	90.7	94.1	95.7	94.5
Gravel	68.8	71.2	69.8	68.3	65.5	60.7	67.2
Meadows	65.9	65.2	66.2	82.8	68.6	66.6	93.7
Metal Sheets	99.4	99.5	99.6	99.4	99.8	99.9	99.7
Bricks	92.5	91.8	92.7	93.7	98.7	99.3	98.8
Shadow	97.5	97.0	98.7	99.5	97.7	99.6	100.0
Trees	97.0	97.4	95.9	97.0	97.9	98.3	98.0

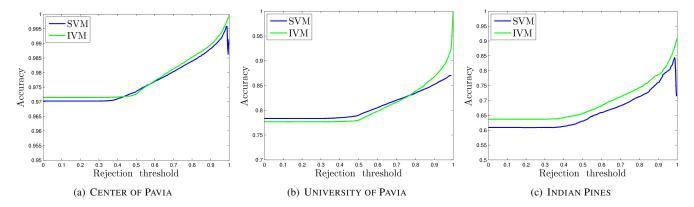


Fig. 3. The overall accuracy of SVM and IVM as a function of rejected test points on the CENTER OF PAVIA data set (left), the UNIVERSITY OF PAVIA data set (middle) and the INDIAN PINES data set (right). The overall accuracy is computed on the non-rejected test points.

TABLE V Number of support/import vectors of SVM and IVM. For the smaller data sets (10%) we report the mean and standard deviation in brackets over 20 runs.

Data	Algorithm	CENTER OF PAVIA	University of Pavia	Indian Pines
100%	SVM	648 SV	1111 SV	2076 SV
100%	IVM	30 IV	109 IV	59 IV
10%	SVM	158.3 (11.8) SV	209.9 (22.7) SV	273.8 (10.1) SV
10%	IVM	27.8 (4.9) IV	35.9 (6.0) IV	75.0 (7.4) IV

accordance with the results of a previous study in context of machine learning data sets [31]. Comparing the number of support vectors and import vectors, respectively, the results confirm that the number of support vectors clearly increases with an increasing number of training samples, whereas the number of import vector increases slowly or almost remains constant. Only for the UNIVERSITY OF PAVIA data set the number increases when all training points are used, because a smaller kernel parameter was chosen and more import vectors are necessary to train the classifier. Consequently, the computation time of the IVM during the classification is much faster when compared to SVM (see Table VI), since the number of required mathematical operations depends on the number of support and import vectors. This is particularly important in context of high-dimensional hyperspectral data sets, which are usually classified with a large number of training data.

Table VI reports the training and testing time of SVM and IVM on an Intel(R) Dual Core with 3.0 GHz. In contrast to the

SVM implementation, the current Matlab/C++ implementation of the IVM is not optimized, so there is still potential for acceleration.

Finally, Table III also shows the positive impact of self-training on the classification accuracy. The classification result was improved in all cases. As expected the improvement on the training sets with 10% of all training samples is higher when compared to the classification results, which were generated by the whole training samples set. Table VII shows, that the number of import vectors remains nearly the same during the self-training procedure. Moreover, the proposed self-training strategy deletes irrelevant training samples. Therefore, the final number of training samples is significantly lower, when compared to initial number of training samples and the samples added during the self-training procedure. This fact is particularly obvious in case of the CENTER OF PAVIA data using the whole training sample set.

VI. CONCLUSION AND OUTLOOK

We proposed the incremental IVM classifier, which includes the addition and deletion of training samples as well as the update of the set of import vectors. The incremental learning strategy updates efficiently the classifier model without retraining from scratch, which makes it capable for large data sets. To evaluate the incremental IVM, we have introduced a self-training strategy, which uses the probabilistic output of the classifier and a DRF.

TABLE VI
Training and test time of SVM, probabilistic SVM and IVM. For the smaller data sets (10%) we report the mean and standard deviation in Brackets over 20 runs.

		CENTER OF PAVIA		UNIVERSIT	Y OF PAVIA	INDIAN PINES	
Data	Algorithm	Train[sec]	Test[sec]	Train[sec]	Test[sec]	Train[sec]	Test[sec]
100%	SVM	3.8	187.9	4.3	126.4	20.0	68.9
100%	Prob. SVM	106.1	0.1	41.7	0.1	12.45	0.1
100%	IVM	225.2	2.3	484.9	1.8	344.3	0.1
10%	SVM	0.1 (< 0.1)	48.2 (3.4)	0.1 (< 0.1)	24.4 (2.5)	0.3 (< 0.1)	6.2 (0.2)
10%	Prob. SVM	105.0 (0.5)	0.1 (< 0.1)	40.5 (0.25)	0.1 (< 0.1)	12.4 (0.1)	0.1 (< 0.1)
10%	IVM	11.4 (6.6)	2.2 (0.2)	5.3 (1.9)	0.9 (0.1)	47.8 (10.0)	0.1 (< 0.1)

TABLE VII

Number of added training samples in the self-training (ST) procedure and number of training and import vectors before and after self-training. The number of training samples after self-training is given by the number of training samples before self-training plus the number of added training samples minus the removed irrelevant training samples. For the smaller data sets (10%) we report the mean and standard deviation in brackets over 20 runs.

	CENT	TER OF PAVIA	Univi	ERSITY OF PAVIA	Indian Pines		
	100%	10%	100%	10%	100%	10%	
# Training samples	5536	556	3921	392	4811	563	
# Import vectors	30	27.8 (4.9)	109	35.9 (6.0)	59	75.0 (7.4)	
# Added training samples	2610	563.0 (158.7)	2340	1390.0 (463.5)	16	584.0 (267.4)	
# Training samples (ST)	538	493.8 (108.2)	4282	1208.6 (439.1)	4719	911.4 (139.9)	
# Import vectors (ST)	21	29.7 (2.6)	49	42.7 (5.7)	61	67.1 (14.7)	

We evaluated the performance of IVM in the context of classifying hyperspectral imagery. IVM constitute a feasible approach and an useful alternative for the classification of remote sensing data, particularly when probabilities are of interest. The experimental results underline that SVM and IVM perform almost similar in terms of the classification accuracy. In addition, the results show the strong dependency of the number of support vectors on the number of available training samples. In contrast to this, the number of import vectors is significantly lower when compared to the number of support vectors and remains constant or only slightly increases with an increasing number of training samples. As confirmed by the experimental results, the probabilities provided by IVM are more reliably, when compared to the probabilistic outputs provided by SVM. This fact is particularly interesting, because the probabilities are useful for further image analysis, e.g., (i) as input in a DRF that increases the classification accuracy, (ii) to detect mislabeled samples by a uncertainty analysis, and (iii) to identity relevant training samples for a self-training strategy. Particularly for hyperspectral data sets, which require a sufficient large number of training samples to ensure an adequate accuracy, the self-training strategy including the incremental IVM is interesting and can further increase the classification result. Moreover, the computation time is reduced by the incremental learning approach rather than re-training the classifier with all training samples. The incremental IVM can further be incorporated into other active learning approaches or more sophisticated models for DRFs. Therefore, the approach seems attractive as well as feasible for operational applications.

Overall, the IVM and its incremental version appears worthwhile for the classification of remote sensing data, especially when the user is interested in reliable class probabilities and a fast classification. More efficient implementation strategies and further modifications will be investigated in the future.

ACKNOWLEDGMENT

The authors would like to thank D. Landgrebe and L. Biehl (Purdue University, USA) for providing the Indian Pines data (available on: http://cobweb.ecn.purdue.edu/~biehl/MultiSpec/) and P. Gamba (University of Pavia, Italy) for providing the Pavia dataset.

REFERENCES

- [1] A. Goetz, "Three Decades of Hyperspectral Remote Sensing of the Earth: A Personal View," *Remote Sens. Environ.*, vol. 113, pp. 5–16, 2009.
- [2] B. Waske, S. van der Linden, J. Benediktsson, A. Rabe, and P. Hostert, "Sensitivity of Support Vector Machines to Random Feature Selection in Classification of Hyperspectral Data," *IEEE Trans. Geosci. Remote Sens.*, vol. 48, no. 7, pp. 2880–2889, 2010.
- [3] G. Mitri and I. Gitas, "Mapping Postfire Vegetation Recovery Using EO-1 Hyperion Imagery," *IEEE Trans. Geosci. Remote Sens.*, vol. 48, no. 3, pp. 1613–1618, 2010.
- [4] J. Benediktsson, J. Palmason, and J. Sveinsson, "Classification of Hyperspectral Data from Urban Areas based on Extended Morphological Profiles," *IEEE Trans. Geosci. Remote Sens.*, vol. 43, no. 3, pp. 480–491, 2005.
- [5] L. Guanter, K. Segl, and H. Kaufmann, "Simulation of Optical Remote-Sensing Scenes With Application to the EnMAP Hyperspectral Mission," *IEEE Trans. Geosci. Remote Sens.*, vol. 47, no. 7, pp. 2340–2351, 2009.
- [6] J. Richards, "Analysis of Remotely Sensed Data: The Formative Decades and the Future," *IEEE Trans. Geosci. Remote Sens.*, vol. 43, no. 3, pp. 422–432, 2005.
- [7] A. Plaza, J. Benediktsson, J. Boardman, J. Brazile, L. Bruzzone, G. Camps-Valls, J. Chanussot, M. Fauvel, P. Gamba, A. Gualtieri, M. Marconcini, J. Tilton, and G. Trianni, "Recent Advances in Techniques for Hyperspectral Image Processing," *Remote Sens. Environ.*, vol. 113, pp. 110–122, 2009.
- [8] X. Chen, T. Warner, and D. Campagna, "Integrating Visible, Near-Infrared and Short-Wave Infrared Hyperspectral and Multispectral Thermal Imagery for Geological Mapping at Cuprite, Nevada," *Remote Sens. Environ.*, vol. 110, no. 3, pp. 344–56, 2007.
- [9] S. Van der Linden, A. Janz, B. Waske, M. Eiden, and P. Hostert, "Classifying Segmented Hyperspectral Data from a Heterogeneous Urban Environment using Support Vector Machines," *J. Appl. Remote Sens.*, vol. 1, no. 1, 2007.
- [10] V. Vapnik, The Nature of Statistical Learning Theory. Springer, 2000.
- [11] M. Pal and P. Mather, "Some Issues in the Classification of DAIS Hyperspectral Data," *Int. J. Remote Sens.*, vol. 27, no. 14, pp. 2895–2916, 2006.

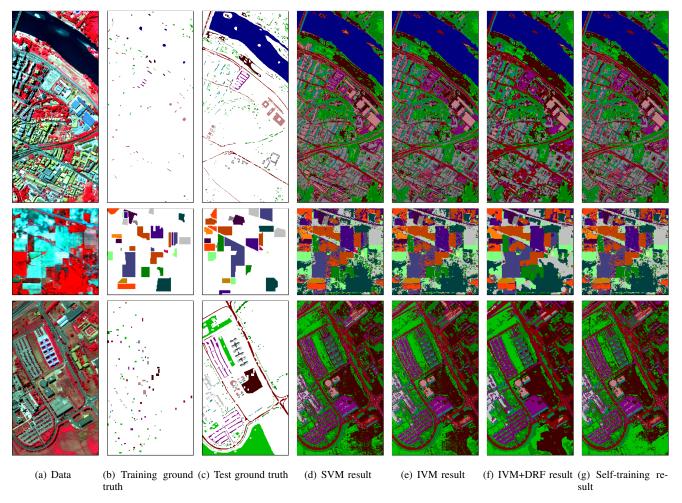


Fig. 4. F.l.t.r.: (a) Data, (b) training ground truth, (c) test ground truth, (d) classification result of SVM, (e) IVM, (f) IVM + DRF and (g) self-training procedure with incremental IVM. The upper row shows the CENTER OF PAVIA data set, the middle row the INDIAN PINES data set and the bottom row the UNIVERSITY OF PAVIA data set.

- [12] B. Waske, J. Benediktsson, K. Arnason, and J. Sveinsson, "Mapping of Hyperspectral AVIRIS Data using Machine-Learning Algorithms," *Can. J. Remote Sensing*, vol. 35, pp. 106–116, 2009.
- [13] G. Camps-Valls, N. Shervashidze, and K. M. Borgwardt, "Spatio-Spectral Remote Sensing Image Classification with Graph Kernels," *IEEE Geosci. Remote Sens. Lett.*, vol. 7, no. 4, pp. 741–745, 2010.
- [14] J. Muñoz-Marí, F. Bovolo, L. Gómez-Chova, L. Bruzzone, and G. Camp-Valls, "Semisupervised One-Class Support Vector Machines for Classification of Remote Sensing Data," *IEEE Trans. Geosci. Remote Sens.*, vol. 48, no. 8, pp. 3188–3197, 2010.
- [15] A. Mathur and G. Foody, "Multiclass and binary SVM classification: implications for training and classification users," *IEEE Geosci. Remote Sens. Lett.*, vol. 5, no. 2, pp. 241–245, 2008.
- [16] J. Platt, N. Cristianini, and J. Shawe-Taylor, "Large Margin DAGs for Multiclass Classification," Adv. Neural Inf. Process. Syst., vol. 12, no. 3, pp. 547–553, 2000.
- [17] G. Foody, "RVM-Based Multi-Class Classification of Remotely Sensed Data," Int. J. Remote Sens., vol. 29, no. 6, pp. 1817–1823, 2008.
- [18] M. Tipping, "Sparse Bayesian Learning and the Relevance Vector Machine," J. of Mach. Learn. Research, vol. 1, pp. 211–244, 2001.
- [19] F. Giacco, C. Thiel, L. Pugliese, S. Scarpetta, and M. Marinaro, "Uncertainty Analysis for the Classification of Multispectral Satellite Images Using SVMs and SOMs," *IEEE Trans. Geosci. Remote Sens.*, no. 99, pp. 1–11, 2010.
- [20] P. Zhong and R. Wang, "Learning Conditional Random Fields for Classification of Hyperspectral Images," *IEEE Trans. Image Process*, vol. 19, pp. 1890–1907, 2010.
- [21] Y. Tarabalka, M. Fauvel, J. Chanussot, and J. Benediktsson, "SVM-and MRF-Based Method for Accurate Classification of Hyperspectral Images," *IEEE Geosci. Remote Sens. Lett.*, vol. 7, pp. 736–740, 2010.

- [22] P. Zhong, P. Zhang, and R. Wang, "Dynamic Learning of SMLR for Feature Selection and Classification of Hyperspectral Data," *IEEE Geosci. Remote Sens. Lett.*, vol. 5, no. 2, pp. 280–284, APR 2008.
- [23] Q. Cheng, P. Varshney, and M. Arora, "Logistic regression for feature selection and soft classification of remote sensing data," *IEEE Geosci. Remote Sens. Lett.*, vol. 3, no. 4, pp. 491–494, 2006.
- [24] S. Keerthi, K. Duan, S. Shevade, and A. Poo, "A Fast Dual Algorithm for Kernel Logistic Regression," *Mach. Learn.*, vol. 61, no. 1, pp. 151– 165, 2005.
- [25] G. Cawley and N. Talbot, "Efficient Model Selection for Kernel Logistic Regression," *Pattern Recogn.*, vol. 2, pp. 439–442, 2004.
- [26] J. Borges, J. Bioucas-Dias, and A. Marcal, "Bayesian Hyperspectral Image Segmentation With Discriminative Class Learning," *IEEE Trans. Geosci. Remote Sens.*, vol. 49, pp. 2151–2164, 2011.
- [27] J. Li, J. Bioucas-Dias, and A. Plaza, "Hyperspectral Image Segmentation Using a New Bayesian Approach With Active Learning," *IEEE Trans. Geosci. Remote Sens.*, pp. 1–14, 2010.
- [28] G. Cawley, N. Talbot, and M. Girolami, "Sparse Multinomial Logistic Regression via Bayesian L1 Regularisation," in Adv. Neural Inf. Process. Syst., 2007.
- [29] B. Krishnapuram, L. Carin, M. Figueiredo, and A. Hartemink, "Sparse Multinomial Logistic Regression: Fast Algorithms and Generalization Bounds," *IEEE Trans. Pattern Anal. Mach. Intell.*, pp. 957–968, 2005.
- [30] B. Demir and S. Erturk, "Hyperspectral Image Classification Using Relevance Vector Machines," *IEEE Geosci. Remote Sens. Lett.*, vol. 4, pp. 586–590, 2007.
- [31] J. Zhu and T. Hastie, "Kernel Logistic Regression and the Import Vector Machine," J. Comput. Graph. Stat., vol. 14, no. 1, pp. 185–205, 2005.
- [32] M. Pal and G. Foody, "Feature Selection for Classification of Hyper-

- spectral Data by SVM," *IEEE Trans. Geosci. Remote Sens.*, vol. 48, no. 5, pp. 2297–2307, 2010.
- [33] D. Tuia, M. Volpi, L. Copa, M. Kanevski, and J. Munoz-Mari, "A survey of active learning algorithms for supervised remote sensing image classification," *IEEE J. Sel. Topics Signal Process.*, vol. 5, pp. 606–617, 2011
- [34] J. Li, J. Bioucas-Dias, and A. Plaza, "Semisupervised Hyperspectral Image Segmentation Using Multinomial Logistic Regression with Active Learning," *IEEE Trans. Geosci. Remote Sens.*, vol. 48, pp. 4085–4098, 2010.
- [35] S. Rajan, J. Ghosh, and M. Crawford, "An Active Learning Approach to Hyperspectral Data Classification," *IEEE Trans. Geosci. Remote Sens.*, vol. 46, pp. 1231–1242, 2008.
- [36] V. Ng and C. Cardie, "Weakly Supervised Natural Language Learning without Redundant Views," in NAACL, 2003, pp. 94–101.
- [37] S. Kumar and M. Hebert, "Discriminative Random Fields," *Int. J. Comput. Vision*, vol. 68, no. 2, pp. 179–201, 2006.
- [38] P. Zhong and R. Wang, "Learning Sparse CRFs for Feature Selection and Classification of Hyperspectral Imagery," *Geoscience and Remote Sensing, IEEE Transactions on*, vol. 46, pp. 4186–4197, 2008.
- [39] L. Fei-Fei, R. Fergus, and P. Perona, "Learning generative visual models from few training examples: An incremental bayesian approach tested on 101 object categories," in CVIU, 2007, pp. 59–70.
- [40] S. Pang, S. Ozawa, and N. Kasabov, "Incremental Linear Discriminant Analysis for Classification of Data Streams," *Trans. Systems, Man, and Cybernetics*, vol. 35, pp. 905–914, 2005.
- [41] G. Cauwenberghs and T. Poggio, "Incremental and Decremental Support Vector Machine Learning," in Adv. Neural Inf. Process. Syst., 2001, pp. 409–415.
- [42] M. Karasuyama and I. Takeuchi, "Multiple Incremental Decremental Learning of Support Vector Machines," *Trans. Neural Netw.*, vol. 21, no. 7, pp. 1048–1059, 2010.
- [43] G. Fung and O. Mangasarian, "Incremental Support Vector Machine Classification," in *SIAM*, 2002, pp. 247–260.
- [44] M. Figueiredo, "Adaptive Sparseness for Supervised Learning," IEEE Trans. Pattern Anal. Mach. Intell., pp. 1050–1159, 2003.
- [45] M. Figueiredo and A. Jain, "Bayesian Learning of Sparse Classifiers," in Proc. IEEE Conf. Computer Vision and Pattern Recognition, 2001.
- [46] L. Csató and M. Opper, "Sparse On-line Gaussian Processes," *Neural Computation*, vol. 14, pp. 641–668, 2002.
- [47] N. Lawrence, M. Seeger, and R. Herbrich, "Fast Sparse Gaussian Process Methods: The Informative Vector Machine," in Adv. Neural Inf. Process. Syst., 2003.
- [48] R. Neal, Bayesian Learning for Neural Networks. Springer Verlag, 1996, vol. 118.
- [49] J. Platt, Advances in Kernel Methods Support Vector Learning. MIT Press, 1999, ch. Fast Training of Support Vector Machines using Sequential Minimal Optimization, pp. 185–208.
- [50] C. Hsu and C. Lin, "A comparison of methods for multiclass support vector machines," *Neural Networks, IEEE Transactions on*, vol. 13, no. 2, pp. 415–425, 2002.
- [51] P. Rousseeuw, A. Leroy, and J. Wiley, Robust Regression and Outlier Detection. Wiley Online Library, 1987, vol. 3.
- [52] N. Japkowicz, "The class imbalance problem: Significance and strategies," in *Int. Conf. Artificial Intelligence*, vol. 1. Citeseer, 2000, pp. 111–117.
- [53] N. Higham, Accuracy and Stability of Numerical Algorithms. Society for Industrial Mathematics, 2002.
- [54] R. Cook and S. Weisberg, Residuals and Influence in Regression. Chapman and Hall New York, 1982.
- [55] J.-M. Yang, B.-C. Kuo, P.-T. Yu, and C.-H. Chuang, "A Dynamic Subspace Method for Hyperspectral Image Classification," *IEEE Trans. Geosci. Remote Sens.*, vol. 48, no. 7, pp. 2840–2853, 2010.
- [56] F. Melgani and L. Bruzzone, "Classification of Hyperspectral Remote Sensing Images with Support Vector Machines," *IEEE Trans. Geosci. Remote Sens.*, vol. 42, no. 8, pp. 1778–1790, 2004.
- [57] C. Chang and C. Lin, "LIBSVM: A Library for Support Vector Machines," 2001.
- [58] Y. Boykov, O. Veksler, and R. Zabih, "Fast Approximate Energy Minimization via Graph Cuts," *IEEE Trans. Pattern Anal. Mach. Intell.*, pp. 1222–1239, 2001.



Ribana Roscher received her Dipl.-Ing. degree in Geodesy from University of Bonn, Germany, in 2008. She is currently a PhD at the Institute of Geodesy and Geoinformation at the University of Bonn, Germany. Her current research activities concentrate on sequential learning and discriminative models for semantic segmentation, especially import vector machines. She is reviewer for IEEE Transactions on Geoscience and Remote Sensing and IEEE Transactions on Pattern Analysis and Machine Intelligence.



Björn Waske (S06-M08) received his degree in Applied Environmental Sciences with a major in Remote Sensing from Trier University, Germany, in 2002. Until mid 2004 he was research assistant the Department of Geosciences at the Munich University, Germany. From 2004 until end of 2007 he pursued a PhD at the Center for Remote Sensing of Land Surfaces (ZFL) at the University of Bonn, Germany and received the PhD degree in Geography. From beginning of 2008 until August 2009 he was a Postdoctoral researcher at the Faculty of Electrical

and Computer Engineering, University of Iceland. Since September 2009 he is a (Junior)professor for Remote Sensing in Agriculture at the University of Bonn, Germany. His current research activities concentrate on advanced concepts for image classification and data fusion. Currently he is Associate Editor IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing (J-STARS). He is reviewer for different international journals, including IEEE Transactions on Geoscience and Remote Sensing, IEEE Geoscience and Remote Sensing Letters, and IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing.



Wolfgang Förstner, born 1946, received his Dipl.-Ing. degree in Geodesy at the University of Stuttgart, Germany, in 1971. He received the PhD degree in 1976 at the University of Stuttgart, Germany. Since 1990 he chairs the Department of Photogrammetry at the University of Bonn, Germany. His fields of interest are digital photogrammetry, statistical methods of image analysis, analysis of image sequences, semantic modeling, machine learning and geoinformation systems. He published more than 130 scientific papers, supervised appr. 70 Bachelor

and Master Theses and more than 30 PhD Theses. From 1994-2001 he was vice president of the German Association for Pattern Recognition (DAGM). He currently is associated editor of IEEE Transactions on Pattern Analysis and Machine Intelligence.