# A Mathematical Extension to the General Four-Component Scattering Power Decomposition With Unitary Transformation of Coherency Matrix 

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#### Abstract

As an improvement of the four-component scattering power decomposition with rotation of coherency matrix (Y4R) and extension of volume model (S4R), the general four-component decomposition with unitary transformation (G4U) was devised to make the full use of the polarimetric information in coherency matrix. This article enables an extension to G4U by deriving the scattering balance equation system in G4U to investigate the role of unitary transformation first. Despite self-contained, the scattering balance equation system in Y4R and $S 4 R$ is independent of the $T_{13}$ entry of coherency matrix. To include $T_{13}$ in decomposition, the unitary transformation in G4U adds a $T_{13}$-related but redundant balance equation into the original system. As a result, $T_{13}$ is accounted for by G4U, and we attain no exact solution to the equation system but some approximate ones. By deducing the general expression of the approximate solutions, a generalized G4U (GG4U) is then created and denoted as $G(\psi)$. The decomposition constant $\psi$ determines a GG4U by producing a $\psi$-rotated double-bounce scattering matrix. We treat this as the scattering preference of $\mathcal{G}(\psi)$ to characterize the physical mechanism. By assigning appropriate values to $\psi$, we attain GG4U of different preferences, while $G(0)$ and $G(+\pi / 8)$ just correspond to $S 4 R$ and G4U. A dual G4U $G(-\pi / 8)$ is also achieved. The duality $G( \pm \pi / 8)$ provides us an adaptive improvement to both G4U and S4R by strengthening the double-bounce scattering over urban and building area while enhancing the surface scattering over water and land area. Both theoretical derivation and experiments on ten polarimetric synthetic aperture radar data sets validate the outperformance. Nonetheless, for whatever unitary transformation employed, there is, forever, a $T_{13}$-related residual component in GG4U. Thus, the incorporation of unitary transformation into Y4R and S4R for the full modeling of polarimetric information is impossible in theory only when the canonical scattering model with nonzero $(1,3)$ entry of coherency matrix is used to add the balance equation system an independent $T_{13}$-related equation rather than a redundant one.


[^0]Index Terms-Polarimetric decomposition, radar polarimetry, scattering model, unitary transformation.

## I. Introduction

$\mathbf{P}$OLARIMETRIC incoherent target decomposition plays an important role in the recognition and discrimination of the mixed radar targets [1]-[4]. It pursues the scattering mechanism of the unknown scatterer by extracting the dominant or average target (such as the Huynen-type phenomenological dichotomies [5]-[8] and the eigenvalue/eigenvectorbased decompositions [9]-[12]) or expanding the target on the canonical models (such as the model-based scattering power decompositions [13]-[21]). Among these decompositions, the model-based decompositions have been a hot topic recently because of the influence of target orientation, diversification of scattering models, problem of negative power, and the imperfect utilization of polarimetric information [22]-[45].

The issue of the full use of polarimetric information concentrates on the complete coverage of the nine degrees of freedom (DoF) of coherency or covariance matrix into the accounted scattering models [24]. As the first model-based decomposition, the three-component Freeman-Durden decomposition (FDD) accounts for only five DoF owing to the assumption of symmetric reflection [13]. Yamaguchi et al. [14] rectified this assumption by introducing a fourth helix component and two additional models of volume scattering. The Yamaguchi four-component decomposition (Y4O) then leaves only three DoF unaccounted for: $T_{13}$ and the real part of $T_{23}\left(\operatorname{Re}\left\{T_{23}\right\}\right)$ entry of target coherency matrix $\langle[T]\rangle$. The same target will present differently by a simple rotation about the line of sight of radar [24], [46]. The deorientation should be first conducted on $\langle[T]\rangle$ to eliminate the influences [47]. As a result, $\operatorname{Re}\left\{T_{23}\right\}$ changes to zero, and Y4O with rotation (Y4R) accounts for seven DoF [26]. Based on Y4R, Sato et al. [29] further proposed to add a new model to characterize the volume scattering generated by even-bounce structure. However, Sato's extended Y4R (S4R) still leaves $T_{13}$ unaccounted. To solve this, Singh et al. in 2013 proposed a general four-component decomposition (G4U) based on a special unitary matrix of degree three $(\mathrm{SU}(3))$ [31]. G4U enables $T_{13}$ included in the accounted models by carrying out unitary transformation to the rotated version of $\langle[T]\rangle$. Singh et al. [31] claimed that G4U could make the full
use of polarimetric parameters. As a result, in comparison with the four-component model-based decompositions such as S4R and Y4R, G4U could enhance double-bounce scattering power over urban area while enhancing the surface scattering contribution over area where surface scattering is preferable. All these establish G4U the state-of-the-art four-component scattering power decomposition, which has been widely used in the remote sensing of agriculture, forestry, wetland, snow, glaciated terrain, man-made target, environment, as well as damages caused by earthquake/tsunami and landside, recently. For details on the influence, evaluation, improvement, development, and application of G4U, please refer the literature survey concisely conducted in [48].

This article is dedicated to enable an extension to G4U from a mathematical point of view. The role of unitary transformation in G4U is investigated by deriving the G4U scattering balance equation system. It is indicated that the unitary transformation in G4U adds a $T_{13}$-related but redundant balance equation to the original self-contained equation system in Y4R and S4R. Then $T_{13}$ is accounted for by G4U, and we obtain no exact solution to the system but the approximate ones. The general expression of the approximate solutions is formulated to enable a generalized G4U (GG4U), while G4U and S4R represent two special forms. Information accounted for in modeled part shows the scattering preference of GG4U. A dual G4U (DG4U) is also attained. The general solution indicates that G4U cannot always enhance the double-bounce scattering power over urban area nor strengthen the surface scattering power over area, where surface scattering is preferable unless we adaptively integrate G4U and DG4U for an extended G4U (EG4U). Both the mathematic derivation and experiments on real data demonstrate EG4U outperformance over S4R and G4U. Despite $T_{13}$ is included in the modeled part of GG4U, there is always an unaccounted residue in GG4U for whatever unitary transformation used. Hence, the incorporation of unitary transformation in Y4R and S4R for the full modeling of polarimetric information is not possible. A complete modeling requires an independent $T_{13}$-related equation incorporated into the original balance equation system. The canonical model with nonzero $(1,3)$ entry of coherency matrix should be used.

The remainder of this article is arranged as follows. Section II presents the $\operatorname{SU}(3)$ matrices and transformations used in G4U. G4U is then described in Section III and generalized in Section IV for GG4U. As a special form of GG4U and also an adaptive combination of G4U and DG4U, Section V provides a detailed depiction to EG4U. Its outperformance is validated on polarimetric synthetic aperture radar (PolSAR) images in Section VI by comparing with S4R, G4U, and DG4U. Section VII discuses some important issues on GG4U and G4U further. This article is concluded in Section VIII with the balance equation system in GG4U being derived in Appendix A.

## II. $\mathrm{SU}(3)$ Matrices and Transformations

The scattering matrix [ $S$ ] of a single target is

$$
[S]=\left[\begin{array}{ll}
S_{\mathrm{HH}} & S_{\mathrm{HV}}  \tag{1}\\
S_{\mathrm{VH}} & S_{\mathrm{VV}}
\end{array}\right]
$$

In reciprocal backscattering, we have $S_{\mathrm{HV}}=S_{\mathrm{VH}}$, and matrix [ $S$ ] covers five DoF then. As for a mixed target subjected to spatial and/or temporal variations, we cannot model its scattering with a determined [S], and the coherency matrix $\langle[T]\rangle$ is constructed as the statistical average of the acquired scattering information

$$
\begin{align*}
\langle[T]\rangle & =\left\langle\boldsymbol{k} \boldsymbol{k}^{\dagger}\right\rangle=\left[\begin{array}{lll}
T_{11} & T_{12} & T_{13} \\
T_{21} & T_{22} & T_{23} \\
T_{31} & T_{32} & T_{33}
\end{array}\right] \\
\boldsymbol{k} & =\frac{1}{\sqrt{2}}\left[\begin{array}{c}
S_{\mathrm{HH}}+S_{\mathrm{VV}} \\
S_{\mathrm{HH}}-S_{\mathrm{VV}} \\
2 S_{\mathrm{HV}}
\end{array}\right] \tag{2}
\end{align*}
$$

where $\langle\cdot\rangle$ and superscript $\dagger$ indicate the operations of ensemble average and conjugate transpose, and $\boldsymbol{k}$ is the Pauli vector. The spatial/temporal depolarization pushes the DoF of $\langle[T]\rangle$ to nine.

The unitary transformation of matrix $\langle[T]\rangle$ under an arbitrary $\mathrm{SU}(3)$ matrix $\left[U_{3}\right]$ is defined as [3]

$$
\begin{equation*}
\operatorname{Unitary}(\langle[T]\rangle) \stackrel{\text { def }}{=}\left[U_{3}\right]\langle[T]\rangle\left[U_{3}\right]^{\dagger} \tag{3}
\end{equation*}
$$

Target deorientation is based on the $\mathrm{SU}(3)$ rotation matrix [24]

$$
\left[U_{3}(\theta)\right]=\left[\begin{array}{ccc}
1 & 0 & 0  \tag{4}\\
0 & \cos 2 \theta & \sin 2 \theta \\
0 & -\sin 2 \theta & \cos 2 \theta
\end{array}\right]
$$

where the rotation $2 \theta$ is attained by minimizing the $(3,3)$ entry of coherency matrix [26]

$$
\begin{equation*}
2 \theta=\frac{1}{2} \tan ^{-1}\left(\frac{2 \operatorname{Re}\left\{T_{23}\right\}}{T_{22}-T_{33}}\right) \tag{5}
\end{equation*}
$$

Combine (4) into (3), the deoriented coherency matrix $\left\langle\left[T^{\prime}\right]\right\rangle$ is

$$
\begin{align*}
\left\langle\left[T^{\prime}\right]\right\rangle & =\left[U_{3}(\theta)\right]\langle[T]\rangle\left[U_{3}(\theta)\right]^{\dagger} \\
& =\left[\begin{array}{ccc}
T_{11}^{\prime} & T_{12}^{\prime} & T_{13}^{\prime} \\
T_{21}^{\prime} & T_{22}^{\prime} & j \operatorname{Im}\left\{T_{23}^{\prime}\right\} \\
T_{31}^{\prime} & j \operatorname{Im}\left\{T_{32}^{\prime}\right\} & T_{33}^{\prime}
\end{array}\right] \tag{6}
\end{align*}
$$

The deorientation makes $T_{23}^{\prime}$ become purely imaginary and reduces the DoF from nine to eight. In order to eliminate the imaginary part further, Singh et al. [31] developed another $\mathrm{SU}(3)$ matrix

$$
\left[U_{3}(\varphi)\right]=\left[\begin{array}{ccc}
1 & 0 & 0  \tag{7}\\
0 & \cos 2 \varphi & j \sin 2 \varphi \\
0 & j \sin 2 \varphi & \cos 2 \varphi
\end{array}\right]
$$

A coherency matrix $\left\langle\left[T^{\prime \prime}\right]\right\rangle$ with zero $T_{23}^{\prime \prime}$ entry is then achieved

$$
\left\langle\left[T^{\prime \prime}\right]\right\rangle=\left[U_{3}(\varphi)\right]\left\langle\left[T^{\prime}\right]\right\rangle\left[U_{3}(\varphi)\right]^{\dagger}=\left[\begin{array}{ccc}
T_{11}^{\prime \prime} & T_{12}^{\prime \prime} & T_{13}^{\prime \prime}  \tag{8}\\
T_{21}^{\prime \prime} & T_{22}^{\prime \prime} & 0 \\
T_{31}^{\prime \prime} & 0 & T_{33}^{\prime \prime}
\end{array}\right]
$$

where element $T_{i j}^{\prime \prime}(i, j=1,2,3)$ is shown in (A2) in Appendix A. The parameter $2 \varphi$ is obtained by minimizing $T_{33}^{\prime \prime}$ [31]

$$
\begin{equation*}
2 \varphi=\frac{1}{2} \tan ^{-1}\left(\frac{2 \operatorname{Im}\left\{T_{23}^{\prime}\right\}}{T_{22}^{\prime}-T_{33}^{\prime}}\right) \tag{9}
\end{equation*}
$$

## III. From Y4R and S4R to G4U

## A. $Y 4 R$ and $S 4 R$

Y4R and S4R achieve the decomposition of target by linearly expanding matrix $\left\langle\left[T^{\prime}\right]\right\rangle$ on the canonical model $\left\langle\left[T_{M}^{\prime \mathrm{S} 4 \mathrm{R} / \mathrm{Y} 4 \mathrm{R}}\right]\right\rangle$. As shown in Fig. $1,\left\langle\left[T_{M}^{\prime \mathrm{S} 4 \mathrm{R} / \mathrm{Y} 4 \mathrm{R}}\right]\right\rangle$ further comprises of four components, which are the surface scattering model $\left\langle\left[T_{S}^{\prime}\right]\right\rangle$, the double-bounce scattering model $\left\langle\left[T_{D}^{\prime}\right]\right\rangle$, the volume scattering model $\left\langle\left[T_{V}^{\prime}\right]\right\rangle$, and the helix scattering model $\left\langle\left[T_{C}^{\prime}\right]\right\rangle$

$$
\begin{align*}
\left\langle\left[T_{M}^{\prime \text { S4R } / \mathrm{Y} 4 \mathrm{R}}\right]\right\rangle=f_{S}\left\langle\left[T_{S}^{\prime}\right]\right\rangle+ & f_{D}\left\langle\left[T_{D}^{\prime}\right]\right\rangle \\
& +f_{V}\left\langle\left[T_{V}^{\prime}\right]\right\rangle+f_{C}\left\langle\left[T_{C}^{\prime}\right]\right\rangle \tag{10}
\end{align*}
$$

where parameters $f_{S}, f_{D}, f_{V}$, and $f_{C}$ denote the contributions of the four models; $\beta$ gives the surface scattering mechanism ratio determined by the dielectric constant of soil and local incidence angle; $\alpha$ denotes the double-bounce scattering mechanism ratio, which relates to both the dielectric constant of soil and truck; and $a, b, c$, and $d$ in $\left\langle\left[T_{V}^{\prime}\right]\right\rangle$ are real constants satisfying $a+b+c=1$, which involve in four volume scattering models

$$
\begin{align*}
& \left\langle\left[T_{V 1}^{\prime}\right]\right\rangle=\frac{1}{30}\left[\begin{array}{ccc}
15 & 5 & 0 \\
5 & 7 & 0 \\
0 & 0 & 8
\end{array}\right], \quad\left\langle\left[T_{V 2}^{\prime}\right]\right\rangle=\frac{1}{4}\left[\begin{array}{lll}
2 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{array}\right] \\
& \left\langle\left[T_{V 3}^{\prime}\right]\right\rangle=\frac{1}{30}\left[\begin{array}{ccc}
15 & -5 & 0 \\
-5 & 7 & 0 \\
0 & 0 & 8
\end{array}\right],\left\langle\left[T_{V 4}^{\prime}\right]\right\rangle=\frac{1}{15}\left[\begin{array}{lll}
0 & 0 & 0 \\
0 & 7 & 0 \\
0 & 0 & 8
\end{array}\right] . \tag{11}
\end{align*}
$$

They are adaptively selected according to the branch conditions (BC)

$$
\left\langle\left[T_{V}^{\prime}\right]\right\rangle= \begin{cases}\left\langle\left[T_{V 1}^{\prime}\right]\right\rangle, & \mathrm{BC}_{1}>0 \text { and } \mathrm{BC}_{2} \leq-2  \tag{12}\\ \left\langle\left[T_{V 2}^{\prime}\right]\right\rangle, & \mathrm{BC}_{1}>0 \text { and }-2<\mathrm{BC}_{2} \leq 2 \\ \left\langle\left[T_{V 3}^{\prime}\right]\right\rangle, & \mathrm{BC}_{1}>0 \text { and } \mathrm{BC}_{2}>2 \\ \left\langle\left[T_{V 4}^{\prime}\right]\right\rangle, & \mathrm{BC}_{1} \leq 0\end{cases}
$$

where ${ }^{1}$

$$
\begin{align*}
\mathrm{BC}_{1} & =T_{11}^{\prime}-T_{22}^{\prime}+\frac{7}{8} T_{33}^{\prime}+\frac{1}{16} f_{C}  \tag{13}\\
\mathrm{BC}_{2} & =10 \log \left[\frac{T_{11}^{\prime}+T_{22}^{\prime}-2 \operatorname{Re}\left\{T_{12}^{\prime}\right\}}{T_{11}^{\prime}+T_{22}^{\prime}+2 \operatorname{Re}\left\{T_{12}^{\prime}\right\}}\right] \tag{14}
\end{align*}
$$

Combine the matrices $\left\langle\left[T_{S}^{\prime}\right]\right\rangle,\left\langle\left[T_{D}^{\prime}\right]\right\rangle,\left\langle\left[T_{V}^{\prime}\right]\right\rangle$, and $\left\langle\left[T_{C}^{\prime}\right]\right\rangle$ shown in Fig. 1 into (10), the ( $i, j$ ) element of $\left\langle\left[T_{M}^{\prime \mathrm{S} 4 \mathrm{R} / \mathrm{Y} 4 \mathrm{R}}\right]\right\rangle$, $T_{M i j}^{\mathrm{S} 4 \mathrm{R} / \mathrm{Y} 4 \mathrm{R}}(i, j=1,2,3)$ is obtained and expressed in (A1) in Appendix A. The $\mathrm{S} 4 \mathrm{R} / \mathrm{Y} 4 \mathrm{R}$ scattering balance equation system on the unknown parameters $f_{S}, f_{D}, f_{V}, f_{C}, \alpha$, and $\beta$
${ }^{1} \mathrm{BC}_{1}$ was originally denoted in S 4 R as $\mathrm{BC}_{1}=T_{11}^{\prime}-T_{22}^{\prime}+f_{C} / 2$ [29]. Singh et al. [31] indicated that $\mathrm{BC}_{1}$ should be as precise as ever possible for assigning the volume models, and thus, they proposed to improve $\mathrm{BC}_{1}$ as (13). In view of this, we also extend (13) to S 4 R in this article. In Section IV-A, we will show that $\mathrm{BC}_{1}$ is related to volume model $\left\langle\left[T_{V 4}^{\prime}\right]\right\rangle$ only, which is a special form of the BC in (29). Hence, it is used in (12) to identify $\left\langle\left[T_{V 4}^{\prime}\right]\right\rangle$ from $\left\langle\left[T_{V 1}^{\prime}\right]\right\rangle$ to $\left\langle\left[T_{V 3}^{\prime}\right]\right\rangle$, while the selection among $\left\langle\left[T_{V 1}^{\prime}\right]\right\rangle,\left\langle\left[T_{V 2}^{\prime}\right]\right\rangle$, and $\left\langle\left[T_{V 3}^{\prime}\right]\right\rangle$ is based on $\mathrm{BC}_{2}$.
is then formulated by letting $T_{M i j}^{\prime \text { S4R/Y4R }}=T_{i j}^{\prime}$ [29]

$$
\begin{cases}f_{S}+f_{D}|\alpha|^{2}+f_{V} a=T_{11}^{\prime} & -1)  \tag{15}\\ f_{S} \beta+f_{D} \alpha+f_{V} d=T_{12}^{\prime} & -2) \\ f_{S}|\beta|^{2}+f_{D}+f_{V} b+\frac{f_{C}}{2}=T_{22}^{\prime} & -3) \\ \pm j \frac{f_{C}}{2}=j \operatorname{Im}\left\{T_{23}^{\prime}\right\} & -4) \\ f_{V} c+\frac{f_{C}}{2}=T_{33}^{\prime} & -5)\end{cases}
$$

An exact solution is obtained by fixing $\alpha$ or $\beta$ zero according to $\mathrm{BC}_{1}$ and another parameter $\mathrm{BC}_{0}$ [13], [14]. This will be detailed in Section IV-A. However, we achieve no scattering balance equation on $T_{13}^{\prime}$ in (15) because $T_{M 13}^{\prime \mathrm{S} 4 \mathrm{R} / \mathrm{Y} 4 \mathrm{R}}$ in (A1-3) is always zero. As a result, there always exists a $T_{13}^{\prime}$-related unaccounted residual part $\left\langle\left[T_{R}^{\prime \text { S4R } / \mathrm{Y} 4 \mathrm{R}}\right]\right\rangle$. Thus, the $\mathrm{S} 4 \mathrm{R} / \mathrm{Y} 4 \mathrm{R}$ decomposition can be generally formulated as follows:

$$
\begin{equation*}
\left\langle\left[T^{\prime}\right]\right\rangle=\left\langle\left[T_{M}^{\prime \mathrm{S} 4 \mathrm{R} / \mathrm{Y} 4 \mathrm{R}}\right]\right\rangle+\left\langle\left[T_{R}^{\prime \mathrm{S} 4 \mathrm{R} / \mathrm{Y} 4 \mathrm{R}}\right]\right\rangle \tag{16}
\end{equation*}
$$

with

$$
\begin{gather*}
\left\langle\left[T_{M}^{\prime \mathrm{S} 4 \mathrm{R} / \mathrm{Y} 4 \mathrm{R}}\right]\right\rangle=\left[\begin{array}{ccc}
T_{11}^{\prime} & T_{12}^{\prime} & 0 \\
T_{21}^{\prime} & T_{22}^{\prime} & j \operatorname{Im}\left\{T_{23}^{\prime}\right\} \\
0 & j \operatorname{Im}\left\{T_{32}^{\prime}\right\} & T_{33}^{\prime}
\end{array}\right]  \tag{17}\\
\left\langle\left[T_{R}^{\prime \mathrm{Y} 4 \mathrm{R} / \mathrm{S} 4 \mathrm{R}}\right]\right\rangle=\left[\begin{array}{ccc}
0 & 0 & T_{13}^{\prime} \\
0 & 0 & 0 \\
T_{31}^{\prime} & 0 & 0
\end{array}\right] . \tag{18}
\end{gather*}
$$

## B. $G 4 U$

To account for $T_{13}^{\prime}$ in the modeled part, G4U uses $\left[U_{3}(\varphi)\right]$ to conduct unitary transformation to both $\left\langle\left[T^{\prime}\right]\right\rangle$ and $\left\langle\left[T_{M}^{\prime \mathrm{S} 4 \mathrm{R} / \mathrm{Y} 4 \mathrm{R}}\right]\right\rangle$ for $\left\langle\left[T^{\prime \prime}\right]\right\rangle$ in (8) and for $\left\langle\left[T_{M}^{\prime / G 4 U}\right]\right\rangle$ in (19) first

$$
\begin{equation*}
\left\langle\left[T_{M}^{\prime \prime \mathrm{G} 4 \mathrm{U}}\right]\right\rangle=\left[U_{3}(\varphi)\right]\left\langle\left[T_{M}^{\prime \mathrm{S} 4 \mathrm{R} / \mathrm{Y} 4 \mathrm{R}}\right]\right\rangle\left[U_{3}(\varphi)\right]^{\dagger} \tag{19}
\end{equation*}
$$

where $T_{M i j}^{\prime / G 4 U}$, the $(i, j)$ entry of $\left\langle\left[T_{M}^{\prime / G 4 \mathrm{U}}\right]\right\rangle$, is expressed in (A3) in Appendix A. The decomposition of $\left\langle\left[T^{\prime}\right]\right\rangle$ is then attained by letting $T_{M i j}^{\prime / \mathrm{G} 4 \mathrm{U}}=T_{i j}^{\prime \prime}$ for (A4) and by killing $\varphi$ in (A4) via (A5) to (A12) for (A13) in Appendix A. This is equivalent to further doing inverse unitary transformation to $\left\langle\left[T^{\prime \prime}\right]\right\rangle$ and $\left\langle\left[T_{M}^{\prime / \mathrm{G} 4 \mathrm{U}}\right]\right\rangle$ using $\left[U_{3}(\varphi)\right]$. Nevertheless, the unitary transformation in (19) makes $T_{M 13}^{\prime / G 4 U}$ in (A3-3) no longer zero. As a result, an additional balance equation is brought into G4U, as given in (A13-3). This additional equation, nonetheless, is not independent of the other five equations in (A13), because it is resulted from the mathematical transformation rather than the physical process. Then, we obtain the following scattering balance equation system (the deduction of it is formulated in Appendix A):

$$
\left.\left\{\begin{array}{ll}
f_{S}+f_{D}|\alpha|^{2}+f_{V} a=T_{11}^{\prime} & -1)  \tag{20}\\
f_{S} \beta+f_{D} \alpha+f_{V} d=T_{12}^{\prime}+T_{13}^{\prime} \\
f_{S} \beta+f_{D} \alpha+f_{V} d=T_{12}^{\prime}-T_{13}^{\prime}
\end{array}\right\} \quad-2\right)
$$



Fig. 1. Canonical models involved in the four-component scattering power decompositions.

Comparing (20) and (15), we can realize that (20-2) enables a dichotomy to (15-2). The redundancy makes that (20) has no such exact solution like (15) but some approximate ones. Singh et al. preferred the first equation of (20-2) only (as shown in Fig. 2 of [31] in terms of the expression of parameter $C$ )

$$
\begin{cases}f_{S}+f_{D}|\alpha|^{2}+f_{V} a=T_{11}^{\prime} & -1)  \tag{21}\\ f_{S} \beta+f_{D} \alpha+f_{V} d=T_{12}^{\prime}+T_{13}^{\prime} & -2) \\ f_{S}|\beta|^{2}+f_{D}+f_{V} b+\frac{f_{C}}{2}=T_{22}^{\prime} & -3) \\ \pm j \frac{f_{C}}{2}=j \operatorname{Im}\left\{T_{23}^{\prime}\right\} & -4) \\ f_{V} c+\frac{f_{C}}{2}=T_{33}^{\prime} & -5)\end{cases}
$$

The only difference between (15) and (21) lies in the expression of (15-2) and (21-2): $T_{13}^{\prime}$ is included into the right side of (21-2). By analogy to the S4R/Y4R decomposition in (16), we can also formulate the G4U decomposition as follows:

$$
\begin{equation*}
\left\langle\left[T^{\prime}\right]\right\rangle=\left\langle\left[T_{M}^{\prime \mathrm{G} 4 \mathrm{U}}\right]\right\rangle+\left\langle\left[T_{R}^{\prime \mathrm{G} 4 \mathrm{U}}\right]\right\rangle . \tag{22}
\end{equation*}
$$

In view of the reality that the modeled part $\left\langle\left[T_{M}^{, \mathrm{G4U}}\right]\right\rangle$ in G 4 U is also composed by four components like $\left\langle\left[T_{M}^{\prime \text { S4R } / \mathrm{Y} 4 \mathrm{R}}\right]\right\rangle$ in (10) and (17), we can then simply obtain from (21) that

$$
\left\langle\left[T_{M}^{\prime \mathrm{G} 4 \mathrm{U}}\right]\right\rangle=\left[\begin{array}{ccc}
T_{11}^{\prime} & T_{12}^{\prime}+T_{13}^{\prime} & 0  \tag{23}\\
T_{21}^{\prime}+T_{31}^{\prime} & T_{22}^{\prime} & j \operatorname{Im}\left\{T_{23}^{\prime}\right\} \\
0 & j \operatorname{Im}\left\{T_{32}^{\prime}\right\} & T_{33}^{\prime}
\end{array}\right]
$$

whereas the corresponding residual part $\left\langle\left[T_{R}^{/ \mathrm{G4U}}\right]\right\rangle$ becomes

$$
\left\langle\left[T_{R}^{\prime \mathrm{G} 4 \mathrm{U}}\right]\right\rangle=\left[\begin{array}{ccc}
0 & -T_{13}^{\prime} & T_{13}^{\prime}  \tag{24}\\
-T_{31}^{\prime} & 0 & 0 \\
T_{31}^{\prime} & 0 & 0
\end{array}\right] .
$$

As a result, $T_{13}^{\prime}$ is included in the modeled part, and all the nine parameters in coherency matrix are finally utilized by G4U.

## IV. From G4U to GG4U

## A. General Expression of the Approximate Solutions

The unitary transformation in (19) makes $T_{13}^{\prime}$ modeled at the cost of producing a redundant equation, which brings (20) a lot of approximate solutions, while G4U just indicates one of them. Here, we focus on the general formulation of the solutions to (20) for the unknowns $f_{S}, f_{D}, f_{V}, f_{C}, \alpha$, and $\beta$.


Fig. 2. Flowchart of GG4U. $\mu$ is a real decomposition constant, based on which GG4U can be customized to any preferable form. GG4U changes to S4R when $\mu=0$, to G4U when $\mu=+1$, and to a dual G4U, i.e., DG4U when $\mu=-1$.

Let

$$
\left\{\begin{array}{l}
S=T_{11}^{\prime}-f_{V} a  \tag{25}\\
C_{1}=T_{12}^{\prime}+T_{13}^{\prime}-f_{V} d \\
C_{2}=T_{12}^{\prime}-T_{13}^{\prime}-f_{V} d \\
D=T_{22}^{\prime}-f_{V} b-\frac{f_{C}}{2} .
\end{array}\right.
$$

Based on $C_{1}$ and $C_{2}$, we further define

$$
\begin{equation*}
C=\frac{1+\mu}{2} C_{1}+\frac{1-\mu}{2} C_{2} \tag{26}
\end{equation*}
$$

where $\mu$ is a real constant. ${ }^{2}$ Then, (20) can be rearranged as

$$
\begin{cases}f_{S}+f_{D}|\alpha|^{2}=S & -1)  \tag{27}\\ f_{S} \beta+f_{D} \alpha=C & -2) \\ f_{S}|\beta|^{2}+f_{D}=D & -3) \\ f_{C}=2\left|\operatorname{Im}\left\{T_{23}^{\prime}\right\}\right| & -4) \\ f_{V}=\frac{1}{2 c}\left(2 T_{33}^{\prime}-f_{C}\right) & -5)\end{cases}
$$

Equation (27) comprises of five equations and six unknowns. Following van Zyl [49], Freeman-Durden [13], and Yamaguchi et al. [14], we can fix $\alpha$ or $\beta$ in terms of the sign of $\operatorname{Re}\left\langle S_{\mathrm{HH}} S_{\mathrm{VV}}^{*}\right\rangle$ or the sign of $S-D$ for the superior between surface scattering and double-bounce scattering ${ }^{3}$

$$
\left\{\begin{array}{l}
\mathrm{BC}>0 \Rightarrow \text { dominant surface scattering } \Rightarrow \alpha=0  \tag{28}\\
\mathrm{BC} \leq 0 \Rightarrow \text { dominant double-bounce scattering } \Rightarrow \beta=0
\end{array}\right.
$$

where

$$
\begin{equation*}
\mathrm{BC}=S-D=T_{11}^{\prime}-T_{22}^{\prime}-\frac{a-b}{c}\left(T_{33}^{\prime}-\frac{f_{C}}{2}\right)+\frac{f_{C}}{2} . \tag{29}
\end{equation*}
$$

As expressed in (11), for volume models $\left\langle\left[T_{V 1}^{\prime}\right]\right\rangle$ to $\left\langle\left[T_{V 3}^{\prime}\right]\right\rangle$, we have $(a-b) / c=1$, and BC then just becomes the parameter $\mathrm{BC}_{0}$ used in both S4R [29] and G4U [31]

$$
\begin{equation*}
\mathrm{BC}_{0}=T_{11}^{\prime}-T_{22}^{\prime}-T_{33}^{\prime}+f_{C} . \tag{30}
\end{equation*}
$$

While as for volume model $\left\langle\left[T_{V 4}^{\prime}\right]\right\rangle$, we have $(b-a) / c=7 / 8$, BC then changes to the $\mathrm{BC}_{1}$ in (13). Substitute $\mathrm{BC}_{0}$ and $\mathrm{BC}_{1}$ for BC, we can also formulate (28) as

$$
\left\{\begin{array}{l}
\alpha=0, \quad \mathrm{BC}_{0}>0 \text { and } \mathrm{BC}_{1}>0  \tag{31}\\
\beta=0, \quad \mathrm{BC}_{0} \leq 0 \text { or } \mathrm{BC}_{1} \leq 0
\end{array}\right.
$$

Combine (28) or (31) into (27), we obtain

$$
\left\{\begin{array}{l}
\mathrm{BC}>0 \Rightarrow\left\{\begin{array}{l}
\alpha=0, \beta=\frac{C}{S} \\
f_{S}=S, \quad f_{D}=D-\frac{|C|^{2}}{S} \\
f_{C}=2\left|\operatorname{Im}\left\{T_{23}^{\prime}\right\}\right|, \quad f_{V}=\frac{1}{2 c}\left(2 T_{33}^{\prime}-f_{C}\right)
\end{array}\right.  \tag{32}\\
\mathrm{BC} \leq 0 \Rightarrow\left\{\begin{array}{l}
\alpha=\frac{C}{D}, \quad \beta=0 \\
f_{S}=S-\frac{|C|^{2}}{D}, \quad f_{D}=D \\
f_{C}=2\left|\operatorname{Im}\left\{T_{23}^{\prime}\right\}\right|, \quad f_{V}=\frac{1}{2 c}\left(2 T_{33}^{\prime}-f_{C}\right)
\end{array}\right.
\end{array}\right.
$$

[^1]
## B. Generalized Decomposition

Substitute (25) and (26) into (27), we obtain

$$
\begin{cases}f_{S}+f_{D}|\alpha|^{2}+f_{V} a=T_{11}^{\prime} & -1)  \tag{33}\\ f_{S} \beta+f_{D} \alpha+f_{V} d=T_{12}^{\prime}+\mu T_{13}^{\prime} & -2) \\ f_{S}|\beta|^{2}+f_{D}+f_{V} b+\frac{f_{C}}{2}=T_{22}^{\prime} & -3) \\ \pm j \frac{f_{C}}{2}=j \operatorname{Im}\left\{T_{23}^{\prime}\right\} & -4) \\ f_{V} c+\frac{f_{C}}{2}=T_{33}^{\prime} & -5)\end{cases}
$$

Obviously, (33) actually indicates a generalized G4U (GG4U)

$$
\begin{equation*}
\left\langle\left[T^{\prime}\right]\right\rangle=\left\langle\left[T_{M}^{\prime \mathrm{GG} 4 \mathrm{U}}\right]\right\rangle+\left\langle\left[T_{R}^{\prime \mathrm{GG} 4 \mathrm{U}}\right]\right\rangle \tag{34}
\end{equation*}
$$

where the modeled part also comprises of four components like $\left\langle\left[T_{M}^{\prime \text { S4R } / \mathrm{Y} 4 \mathrm{R}}\right]\right\rangle$ in (10). One can then simply obtain from (33) that

$$
\left\langle\left[T_{M}^{\prime \text { GG4U }}\right]\right\rangle=\left[\begin{array}{ccc}
T_{11}^{\prime} & T_{12}^{\prime}+\mu T_{13}^{\prime} & 0  \tag{35}\\
T_{21}^{\prime}+\mu^{*} T_{31}^{\prime} & T_{22}^{\prime} & j \operatorname{Im}\left\{T_{23}^{\prime}\right\} \\
0 & j \operatorname{Im}\left\{T_{32}^{\prime}\right\} & T_{33}^{\prime}
\end{array}\right]
$$

with the corresponding residual part $\left\langle\left[T_{R}^{\prime \text { GG4U }}\right]\right\rangle$

$$
\left\langle\left[T_{R}^{\prime \mathrm{GG} 4 \mathrm{U}}\right]\right\rangle=\left[\begin{array}{ccc}
0 & -\mu T_{13}^{\prime} & T_{13}^{\prime}  \tag{36}\\
-\mu^{*} T_{31}^{\prime} & 0 & 0 \\
T_{31}^{\prime} & 0 & 0
\end{array}\right] .
$$

We can easily obtain, from (32), the scattering powers of the four components such as $P_{S}, P_{D}, P_{V}$, and $P_{C}$

$$
\begin{cases}P_{S}=f_{S}\left(1+|\beta|^{2}\right)= \begin{cases}S+\frac{|C|^{2}}{S}, & \mathrm{BC}>0 \\ S-\frac{|C|^{2}}{D}, & \mathrm{BC} \leq 0\end{cases}  \tag{37}\\ P_{D}=f_{D}\left(1+|\alpha|^{2}\right)= \begin{cases}D-\frac{|C|^{2}}{S}, & \mathrm{BC}>0 \\ D+\frac{|C|^{2}}{D}, & \mathrm{BC} \leq 0\end{cases} \\ P_{C}=f_{C} H\left(T_{33}^{\prime}-\left|\operatorname{Im}\left\{T_{23}^{\prime}\right\}\right|\right) \\ P_{V}=\frac{1}{2 c}\left(2 T_{33}^{\prime}-P_{C}\right)\end{cases}
$$

where $H(\cdot)$ denotes the Heaviside step function

$$
H(t)= \begin{cases}1, & t>0  \tag{38}\\ 0, & t \leq 0\end{cases}
$$

which is used here to adjust the value of $P_{C}$ for nonnegative $P_{V}$ ruling: $P_{C}=f_{C}$ only when $P_{V}>0$; otherwise, we attain $P_{C}=0$ [22]. It can be easily formulated from (20-5), (25), (37), and the relationship $a+b+c=1$ that

$$
\begin{equation*}
P_{S}+P_{D}+P_{V}+P_{C}=T_{11}^{\prime}+T_{22}^{\prime}+T_{33}^{\prime}=\mathrm{SPAN} \tag{39}
\end{equation*}
$$

where SPAN denotes the total scattering power. Hence, GG4U provides a decomposition of scattering power. The flowchart of GG4U is shown in Fig. 2.

## C. Special Decompositions

$P_{V}$ and $P_{C}$ are independent of the decomposition constant $\mu$ so is the addition of $P_{S}$ and $P_{D}$ because

$$
\begin{equation*}
P_{S}+P_{D}=S+D \tag{40}
\end{equation*}
$$

We can, thus, simply employ $S+D$ for SPAN reservation ruling in GG4U before the calculation of $P_{S}$ and $P_{D}$, as shown in Fig. 2. Nevertheless, $P_{S}$ and $P_{D}$ depend on $\mu$ due to the parameter $C$ in (26). To obtain nonnegative $P_{S}$ and $P_{D}$, the following condition on $\mu$ can be simply extracted from (37):

$$
\begin{equation*}
|C|^{2}=\left|\frac{1+\mu}{2} C_{1}+\frac{1-\mu}{2} C_{2}\right|^{2} \leq S D \tag{41}
\end{equation*}
$$

Solve (41), we obtain that

$$
\begin{equation*}
\mu_{-} \leq \mu \leq \mu_{+} \tag{42}
\end{equation*}
$$

where

$$
\begin{equation*}
\mu_{ \pm}=\frac{\left|C_{2}\right|^{2}-\left|C_{1}\right|^{2} \pm 2 \sqrt{S D\left|C_{1}-C_{2}\right|^{2}-\operatorname{Im}^{2}\left(C_{1} C_{2}^{*}\right)}}{\left|C_{1}-C_{2}\right|^{2}} \tag{43}
\end{equation*}
$$

By taking appropriate value to $\mu$, we can have some different decompositions which are denoted as a function $\mathcal{G}(\mu)$. Here, we are particularly interested to the following special cases of $\mathcal{G}(\mu)$ :

1) $\mathcal{G}(+1):=G 4 U$

$$
\begin{equation*}
C=C_{1}=T_{12}^{\prime}+T_{13}^{\prime}-f_{V} d=C_{\mathrm{G} 4 \mathrm{U}} \tag{44}
\end{equation*}
$$

This is just the parameter $C$ used in G4U. Then, (37) denotes the scattering powers in G4U. GG4U changes to G4U in this case.
2) $\mathcal{G}(-1):=D G 4 U$

$$
\begin{equation*}
C=C_{2}=T_{12}^{\prime}-T_{13}^{\prime}-f_{V} d \tag{45}
\end{equation*}
$$

This acts as the complement of case 1 ), thus, we name it as the dual G4U (DG4U). The duality $\mathcal{G}( \pm 1)$ provides a nice extension to G4U (EG4U). This will be presented in Section V.
3) $\mathcal{G}(0):=S 4 R$

$$
\begin{equation*}
C=\left(C_{1}+C_{2}\right) / 2=T_{12}^{\prime}-f_{V} d=C_{\mathrm{S} 4 \mathrm{R}} \tag{46}
\end{equation*}
$$

This is the parameter $C$ adopted in S 4 R , i.e., S 4 R also signifies a special form of GG4U. Hence, the essential difference between S4R and G4U just lies in the different definition of parameter $C$ in (44) and (46). The unitary transformation is just to enable the $T_{13}^{\prime}$ entry contained in $C_{\mathrm{G} 4 \mathrm{U}}$ and finally in $P_{S}$ and $P_{D}$. Parameter $C$ defined in (26) is the generalization of $C_{\mathrm{G} 4 \mathrm{U}}$ and $C_{\mathrm{S} 4 \mathrm{R}}$.

A general model-based decomposition (GMD) was proposed in [19] to minimize the L2-norm of the residue for model inversion. We adopt this criterion here to identify a minimum GG4U

$$
\begin{equation*}
\left\|\left\langle\left[T_{R}^{\prime \mathrm{GG} 4 \mathrm{U}}\right]\right\rangle\right\|_{2}=\sqrt{2\left(1+|\mu|^{2}\right)}\left|T_{13}^{\prime}\right| \tag{47}
\end{equation*}
$$

Obviously, the minimum is obtained when $\mu=0$

$$
\begin{equation*}
\left\|\left\langle\left[T_{R}^{\prime \mathrm{GG} 4 \mathrm{U}}\right]\right\rangle\right\|_{2}^{\min }=\left\|\left\langle\left[T_{R}^{\prime \mathrm{Y} 4 \mathrm{R} / \mathrm{S} 4 \mathrm{R}}\right]\right\rangle\right\|_{2}=\sqrt{2}\left|T_{13}^{\prime}\right| \tag{48}
\end{equation*}
$$



Fig. 3. Physical mechanism underlying GG4U. The decomposition constant $\mu$ determines $G(\mu)$ by creating a scattering matrix $\left[S_{C}\right]$ first. We can then bridge $\left[S_{C}\right.$ ] to $G(\mu)$ and treat it as the scattering preference of $G(\mu)$. [ $S_{C}$ ] denotes a $\psi$-rotated double-bounce scatterer. Therefore, we can also express $G(\mu)$ as $G(\psi)$ to directly reveal the physical mechanism underlying GG4U.

The minimized residue is achieved in Y4R and S4R, i.e., S4R is the optimal GG4U if the minimizing of the L2-norm of residue is adopted as the optimization criterion. Bring $\mu= \pm 1$ into (47)

$$
\begin{equation*}
\left\|\left\langle\left[T_{R}^{\prime \mathrm{G} 4 \mathrm{U}}\right]\right\rangle\right\|_{2}=\left\|\left\langle\left[T_{R}^{\prime \mathrm{DG} 4 \mathrm{U}}\right]\right\rangle\right\|_{2}=2\left|T_{13}^{\prime}\right| \tag{49}
\end{equation*}
$$

The L2-norm of G4U and DG4U residues is equal and always $\sqrt{2}$ times larger than the residues of Y4R and S4R. ${ }^{4}$

## D. Physical Mechanism

As formulated in (35), parameter $\mu$ determines $G(\mu)$ mainly through the $(1,2)$ element of $\left\langle\left[T_{M}^{\prime \mathrm{GG4U}}\right]\right\rangle: T_{M 12}^{\prime \mathrm{GG} 4 \mathrm{U}}=T_{12}^{\prime}+\mu T_{13}^{\prime}$. From the mathematical point of view, this indicates that a given $\mu$ will preserve certain polarimetric information into $\left\langle\left[T_{M}^{\prime \text { GG4U }}\right]\right\rangle$. Hence, we can have a chance to explore the physical mechanism of the related GG4U. $T_{M 12}^{\prime \text { GG4U }}$ in (35) can be further expressed in terms of the elements of scattering matrix as

$$
\begin{equation*}
T_{M 12}^{\prime \mathrm{GG} 4 \mathrm{U}}=\left\langle\left(S_{\mathrm{HH}}+S_{\mathrm{VV}}\right) \cdot \operatorname{Tr}\left([S]^{\dagger}\left[S_{C}\right]\right)\right\rangle \tag{50}
\end{equation*}
$$

where $\operatorname{Tr}(\cdot)$ denotes the extraction of matrix trace, and $\left[S_{C}\right]$ is a scattering matrix

$$
\left[S_{C}\right]=\frac{1}{2}\left[\begin{array}{cc}
1 & \mu  \tag{51}\\
\mu & -1
\end{array}\right]
$$

Equation (50) indicates that the reserved information $T_{M 12}^{\prime \text { GG4U }}$ is directly related to $\left[S_{C}\right]$, which is controlled by parameter $\mu$ only, i.e., $\mu$ creates a GG4U via [ $S_{C}$ ]. Then, we can relate $\mathcal{G}(\mu)$ to [ $\left.S_{C}\right]$, as shown in Fig. 3. The polarimetric similarity between two scatterers $[S]$ and $\left[S_{C}\right]$ is defined in [51]-[53]

$$
\begin{equation*}
r\left([S],\left[S_{C}\right]\right)=\frac{\left|\operatorname{Tr}\left([S]^{\dagger}\left[S_{C}\right]\right)\right|^{2}}{\left|\operatorname{Tr}\left([S]^{\dagger}[S]\right)\right|^{2}\left|\operatorname{Tr}\left(\left[S_{C}\right]^{\dagger}\left[S_{C}\right]\right)\right|^{2}} \tag{52}
\end{equation*}
$$

Then, (50) indicates that the nonnormalized complex scattering similarity between $[S]$ and $\left[S_{C}\right]$ is reserved in $T_{M 12}^{\prime G G 4 U}$. Thus, the obtained GG4U prefers [ $S_{C}$ ]. As for S4R

$$
\left[S_{C}^{S 4 \mathrm{R}}\right]=\frac{1}{2}\left[\begin{array}{cc}
1 & 0  \tag{53}\\
0 & -1
\end{array}\right]
$$

[^2]This shows the preference of S4R for double-bounce scattering, which is consistent with the proposition of S 4 R , i.e., to account for the double-bounce scattering from urban area [29]. As for $\mu$, we can always define

$$
\begin{equation*}
\mu=\tan 2 \psi \tag{54}
\end{equation*}
$$

where $\psi$ is an angle within the interval $\left[(1 / 2) \tan ^{-1} \mu_{-}\right.$, $\left.(1 / 2) \tan ^{-1} \mu_{+}\right]$. We can then express [ $S_{C}$ ] in (51) as

$$
\begin{equation*}
\left[S_{C}\right]=\sec 2 \psi\left[U_{2}(\psi)\right]\left[S_{C}^{S 4 \mathrm{R}}\right]\left[U_{2}(\psi)\right]^{\mathrm{T}} \tag{55}
\end{equation*}
$$

where superscript T shows the matrix transpose, and $\left[U_{2}(\psi)\right]$ is the $\mathrm{SU}(2)$ counterpoint of the $\mathrm{SU}(3)$ rotation matrix $\left[U_{3}(\theta)\right]$ in (4)

$$
\left[U_{2}(\psi)\right]=\left[\begin{array}{cc}
\cos \psi & -\sin \psi  \tag{56}\\
\sin \psi & \cos \psi
\end{array}\right]
$$

Thus, $\left[S_{C}\right]$ indicates a $\psi$-rotated double-bounce scatterer. From (54), we can easily obtain that

$$
\psi=\frac{1}{2} \tan ^{-1} \mu \Rightarrow\left\{\begin{array}{l}
\psi^{\mathrm{G} 4 \mathrm{U}}=+\frac{\pi}{8}  \tag{57}\\
\psi^{\mathrm{DG4U}}=-\frac{\pi}{8} .
\end{array}\right.
$$

Hence, G4U and DG4U prefer the $\pi / 8$-rotated and $-\pi / 8$-rotated double-bounce scatterers, respectively, and we can denote them as $\mathcal{G}(+\pi / 8)$ and $\mathcal{G}(-\pi / 8)$ to reveal the scattering preferences, respectively. By adjusting $\psi$, we can achieve GG4U of different preferences. The expression of $\mathcal{G}(\mu)$ as $\mathcal{G}(\psi)$ provides a direct indication of the physical mechanism underlying each GG4U. ${ }^{5}$

## V. EG4U

## A. Theoretical Evaluation of $S 4 R$ and $G 4 U$ on $P_{S}$ and $P_{D}$

Despite sharing the same modeled part (17) and residual part (18), S4R introduces the volume model $\left\langle\left[T_{V 4}^{\prime}\right]\right\rangle$ in Y4R so as to improve the double-bounce scattering in urban area [29], [31]. Singh et al. [31] indicated that G4U could further improve S 4 R in this aspect by adding the unitary transformation. As revealed in Tables I and II of [31], compared with $\mathrm{S} 4 \mathrm{R}, \mathrm{G} 4 \mathrm{U}$ strengthens the surface scattering in area where surface scattering is preferable to double-bounce scattering, while increases the double-bounce scattering in urban area where the double-bounce scattering is preferable to surface scattering. By combining the ruling in (28), we can formulate these observations as

$$
\begin{cases}P_{S}^{\mathrm{G} 4 \mathrm{U}} \geq P_{S}^{\mathrm{S} 4 \mathrm{R}}, & \mathrm{BC}>0  \tag{58}\\ P_{D}^{\mathrm{G} 4 \mathrm{U}} \geq P_{D}^{\mathrm{S} 4 \mathrm{R}}, & \mathrm{BC} \leq 0\end{cases}
$$

Singh et al. obtained (58) on image patches extracted from both the ALOS-PALSAR and Radarsat-2 data sets of San Francisco. In terms of the general expression of $P_{S}$ and $P_{D}$ in (37), here, we give a simple validation to (58) by combining

[^3]$\mu=0$ and $\mu=1$ into (26) and (37)
\[

\left\{$$
\begin{array}{l}
\left\{\begin{array}{l}
P_{S}^{\mathrm{G} 4 \mathrm{U}}=S+\frac{\left|C_{1}\right|^{2}}{S} \\
P_{S}^{\mathrm{S} 4 \mathrm{R}}=S+\frac{\left|C_{1}+C_{2}\right|^{2}}{4 S},
\end{array}\right.  \tag{59}\\
\left\{\begin{array}{l}
P_{D}^{\mathrm{G} 4 \mathrm{U}}=D+\frac{\left|C_{1}\right|^{2}}{D} \\
P_{D}^{\mathrm{S} 4 \mathrm{R}}=D+\frac{\left|C_{1}+C_{2}\right|^{2}}{4 D},
\end{array} \quad \mathrm{BC} \leq 0\right.
\end{array}
$$\right.
\]

From (59), we have

$$
\begin{cases}P_{S}^{\mathrm{G} 4 \mathrm{U}}-P_{S}^{\mathrm{S} 4 \mathrm{R}}=\frac{\left|2 C_{1}\right|^{2}-\left|C_{1}+C_{2}\right|^{2}}{4 S}, & \mathrm{BC}>0  \tag{60}\\ P_{D}^{\mathrm{G} 4 \mathrm{U}}-P_{D}^{\mathrm{S} 4 \mathrm{R}}=\frac{\left|2 C_{1}\right|^{2}-\left|C_{1}+C_{2}\right|^{2}}{4 D}, & \mathrm{BC} \leq 0\end{cases}
$$

Then, (58) will hold iff $\left|2 C_{1}\right|^{2}-\left|C_{1}+C_{2}\right|^{2} \geq 0$. Obviously, this condition is not always tenable. This will be further validated in Section VI on Radarsat-2 data of San Francisco. Hence, despite the better performance in some areas, G4U cannot improve S4R in every target area. To tackle with this, the EG4U is developed in the following as an adaptive combination of G4U and DG4U.

## B. EG4U: Adaptive Combination of G4U and DG4U

The preference analysis in Section IV-D shows that G4U and DG4U are just symmetric around S4R. Hence, the duality $\mathcal{G}( \pm 1)$ may provide an improvement to S 4 R . Combine $\mu=-1$ into (26) and (37), DG4U surface and double-bounce scattering powers can be formulated as

$$
\begin{cases}P_{S}^{\mathrm{DG} 4 \mathrm{U}}=S+\frac{\left|C_{2}\right|^{2}}{S}, & \mathrm{BC}>0  \tag{61}\\ P_{D}^{\mathrm{DG} 4 \mathrm{U}}=D+\frac{\left|C_{2}\right|^{2}}{D}, & \mathrm{BC} \leq 0\end{cases}
$$

Combine (59) and (61), after some simple deduction, we obtain

$$
\begin{align*}
& \left\{\begin{array}{l}
\frac{P_{S}^{\mathrm{G} 4 \mathrm{U}}+P_{S}^{\mathrm{DG} 4 \mathrm{U}}}{2}-P_{S}^{\mathrm{S4R}}=\frac{\left|C_{1}-C_{2}\right|^{2}}{4 S} \geq 0, \quad \mathrm{BC}>0 \\
\frac{P_{D}^{\mathrm{G} 4 \mathrm{U}}+P_{D}^{\mathrm{DG4U}}}{2}-P_{D}^{\mathrm{S} 4 \mathrm{R}}=\frac{\left|C_{1}-C_{2}\right|^{2}}{4 D} \geq 0, \quad \mathrm{BC} \leq 0
\end{array}\right.  \tag{62}\\
& \left\{\begin{array}{l}
P_{S}^{\mathrm{G} 4 \mathrm{U}}-P_{S}^{\mathrm{DG} 4 \mathrm{U}}=\frac{\left|C_{1}\right|^{2}-\left|C_{2}\right|^{2}}{S}, \quad \mathrm{BC}>0 \\
P_{D}^{\mathrm{G} 4 \mathrm{U}}-P_{D}^{\mathrm{DG} 4 \mathrm{U}}=\frac{\left|C_{1}\right|^{2}-\left|C_{2}\right|^{2}}{D}, \quad \mathrm{BC} \leq 0 .
\end{array}\right. \tag{63}
\end{align*}
$$

We can immediately obtain from (62) that

$$
\begin{cases}\max \left\{P_{S}^{\mathrm{G} 4 \mathrm{U}}, P_{S}^{\mathrm{DG} 4 \mathrm{U}}\right\} \geq P_{S}^{\mathrm{S} 4 \mathrm{R}}, & \mathrm{BC}>0  \tag{64}\\ \max \left\{P_{D}^{\mathrm{G} 4 \mathrm{U}}, P_{D}^{\mathrm{DG} 4 \mathrm{U}}\right\} \geq P_{D}^{\mathrm{S4R} \mathrm{R}}, & \mathrm{BC} \leq 0\end{cases}
$$

From (63), we obtain

$$
\left\{\left\{\begin{array}{rl}
\max \left\{P_{S}^{\mathrm{G} 4 \mathrm{U}}, P_{S}^{\mathrm{DG} 4 \mathrm{U}}\right\} & =P_{S}^{\mathrm{G} 4 \mathrm{U}}, \mathrm{BC}>0  \tag{65}\\
\max \left\{P_{D}^{\mathrm{G} 4 \mathrm{U}}, P_{D}^{\mathrm{DG} 4 \mathrm{U}}\right\} & =P_{D}^{\mathrm{G} 4 \mathrm{U}}, \mathrm{BC} \leq 0, \\
\max \left\{P_{S}^{\mathrm{G} 4 \mathrm{U}}, P_{S}^{\mathrm{DG} 4 \mathrm{U}}\right\} & =P_{S}^{\mathrm{DG} 4 \mathrm{U}}, \mathrm{BC}>0 \\
\max \left\{P_{D}^{\mathrm{G} 4 \mathrm{U}}, P_{D}^{\mathrm{DG} 4 \mathrm{U}}\right\} & =P_{D}^{\mathrm{DG} 4 \mathrm{U}}, \mathrm{BC} \leq 0
\end{array} \quad \mathrm{BC}_{3} \leq 0\right.\right.
$$

where

$$
\begin{equation*}
\mathrm{BC}_{3}=\left|C_{1}\right|-\left|C_{2}\right| . \tag{66}
\end{equation*}
$$

Equation (65) just lays the foundation for EG4U, which is defined as follows:

$$
\mathrm{EG} 4 \mathrm{U}:=\mathcal{G}( \pm 1)= \begin{cases}\mathcal{G}(+1)=\mathrm{G} 4 \mathrm{U}, & \mathrm{BC}_{3}>0  \tag{67}\\ \mathcal{G}(-1)=\mathrm{DG} 4 \mathrm{U}, & \mathrm{BC}_{3} \leq 0\end{cases}
$$

As the adaptive combination of G4U and DG4U, EG4U is also a special case of GG4U. Hence, we denote it as $\mathcal{G}( \pm 1)$ or $\mathcal{G}( \pm \pi / 8)$. By bringing $\mu=+1$ or $\mu=-1$ into (26) and (37) based on the $\mathrm{BC}_{3}$, we can achieve the scattering powers of four components in EG4U, which also reserve the conservation (39) and (40). Furthermore, from (64), (65), and (67), we have

$$
\left\{\begin{array}{l}
P_{S}^{\mathrm{EG} 4 \mathrm{U}}=\max \left\{P_{S}^{\mathrm{G} 4 \mathrm{U}}, P_{S}^{\mathrm{DG} 4 \mathrm{U}}\right\} \geq\left\{P_{S}^{\mathrm{S} 4 \mathrm{R}}, P_{S}^{\mathrm{G} 4 \mathrm{U}}, P_{S}^{\mathrm{DG} 4 \mathrm{U}}\right\}  \tag{68}\\
\mathrm{BC}>0 \\
P_{D}^{\mathrm{EG} 4 \mathrm{U}}=\max \left\{P_{D}^{\mathrm{G} 4 \mathrm{U}}, P_{D}^{\mathrm{DG} 4 \mathrm{U}}\right\} \geq\left\{P_{D}^{\mathrm{S} 4 \mathrm{R}}, P_{D}^{\mathrm{G} 4 \mathrm{U}}, P_{D}^{\mathrm{DG} 4 \mathrm{U}}\right\}, \\
\mathrm{BC} \leq 0
\end{array}\right.
$$

Compared with S4R, G4U, and DG4U, the EG4U increases the surface scattering in area where surface scattering is superior to double-bounce scattering and strengthens double-bounce scattering in area where double-bounce scattering is preferable to surface scattering. Therefore, EG4U achieves not only a nice improvement to S4R but also an effective extension to G4U. Based on the flowchart of GG4U shown in Fig. 2, the procedure of EG4U is outlined in Algorithm 1.

## VI. Experiments and Validation

We test GG4U by comparing S4R, G4U, DG4U, and EG4U on C-band Radarsat-2 data of San Francisco acquired on April 9, 2008. Fig. 4(a) shows the Pauli image of the data, where $Z 1-Z 5$ denote five selected zones for comparison. The (c)Google Earth optical image of the area and zones are shown in Fig. 4(b). We use a $12 \times 6$ boxcar filtering to suppress the speckles first. Then, S4R, G4U, DG4U, and EG4U are employed for decomposition. The decomposed results of the four are found very consistent with one another as they all observe the conservation of $P_{V}, P_{C}, P_{S}+P_{D}$, and $P_{S}+P_{D}+P_{V}+P_{C}$. Therefore, only the pseudocolor power decomposition result of EG4U is shown in Fig. 5 by encoding $\{R, G, B\}$ with $\left\{P_{D}^{\mathrm{EG4U}}, P_{V}^{\mathrm{EG} 4 \mathrm{U}}, P_{S}^{\mathrm{EG} 4 \mathrm{U}}\right\}$.

## A. Conservation Versus Nonconservation

In fact, as for Y4R, S4R, and G4U, Singh et al. [31] also evaluated the conservation. They indicated that S4R breaks the conservation of $P_{S}+P_{D}$ in the South of Market (SoMa) region of San Francisco ${ }^{6}$ (i.e., Z1 in Fig. 4). As a result, the summation $P_{S}+P_{D}+P_{V}+P_{C}$ is smaller than $S P A N$, and a relative error is then achieved and the relative error order is G4U $<$ Y4R $<$ S4R [31]. To examine this, Fig. 6(a)-(d) shows $P_{S}+P_{D}, P_{V}, P_{C}$, and $P_{S}+P_{D}+P_{V}+P_{C}$ extracted by S4R, G4U, DG4U, and EG4U along the red line in $Z 1$ of

[^4]```
Algorithm 1 EG4U
    Input: \(\langle[T]\rangle\)
    Conduct deorientation to \(\langle[T]\rangle\) for \(\left\langle\left[T^{\prime}\right]\right\rangle\)
    03: Compute helix power \(P_{C}=2\left|\operatorname{Im}\left\{T_{23}^{\prime}\right\}\right| H\left(T_{33}^{\prime}-\right.\)
        \(\left.\left|\operatorname{Im}\left\{T_{23}^{\prime}\right\}\right|\right)\)
    04: Calculate \(\mathrm{BC}_{0}, \mathrm{BC}_{1}\), and \(\mathrm{BC}_{2}\)
    05: Determine volume scattering model based on \(\mathrm{BC}_{1}\) and
        \(B_{2}\)
    06: Obtain volume scattering power \(P_{V}=\left(2 T_{33}^{\prime}-P_{C}\right) / 2 c\)
    07: Calculate decomposition parameters \(S, D, C_{1}\), and \(C_{2}\),
    and \(\mathrm{BC}_{3}\)
    08: Implement \(S P A N\) reservation ruling based on \(S+D\)
    09: if \(S+D>0\)
    10: Adaptively select between G4U and DG4U based on
        \(\mathrm{BC}_{3}\)
        if \(\mathrm{BC}_{3}>0\)
            \(C=C_{1}\)
        else
            \(C=C_{2}\)
        end if
        Calculate surface scattering power \(P_{S}\) and double-
        bounce scattering power \(P_{D}\) according to \(\mathrm{BC}_{0}\) and
        \(\mathrm{BC}_{1}\)
        if \(\mathrm{BC}_{0}>0\) and \(\mathrm{BC}_{1}>0\)
            \(P_{S}=S+|C|^{2} / S, P_{D}=D-|C|^{2} / S\)
        else
            \(P_{S}=S-|C|^{2} / D, P_{D}=D+|C|^{2} / D\)
        end if
        Implement nonnegative \(P_{S}\) and \(P_{D}\) ruling
        else
        \(P_{S}=P_{D}=0, P_{V}=S P A N-P_{C}\)
    end if
    Output: \(P_{S}, P_{D}, P_{V}, P_{C}\)
```

Fig. 4(a), respectively. As shown, the conservation holds well and conforms to the strict formulations in Section IV. Thus, we prefer to attribute the deviation of $P_{S}+P_{D}+P_{V}+P_{C}$ from $S P A N$ observed in SoMa by Singh et al. to the potential approximation in the software implementation of S4R. It is also shown in Fig. 6 that $P_{S}+P_{D}$ is averagely larger than $P_{V}$. This is helpful to the understanding of the complex scattering in SoMa.

## B. GG4U Versus $54 R$

Despite the good coherence and conservation of $P_{S}+P_{D}$, the powers $P_{S}$ and $P_{D}$ depend on constant $\mu$ and vary with different GG4U algorithms. Treating S4R as a reference, we first examine whether G4U, DG4U, and EG4U can adaptive enhance double-bounce scattering or surface scattering in comparison with S4R.

To better show the difference among algorithms, we normalize $P_{S}$ and $P_{D}$ with $P_{S}+P_{D}$ for $\eta_{S}$ and $\eta_{D}$ first

$$
\left\{\begin{array}{l}
\eta_{S}=\frac{P_{S}}{P_{S}+P_{D}} \times 100 \%  \tag{69}\\
\eta_{D}=\frac{P_{D}}{P_{S}+P_{D}} \times 100 \%
\end{array}\right.
$$



Fig. 4. (a) Radarsat-2 polarimetric Pauli image and (b) Google Earth optical image of San Francisco. Z1 to Z5 denote five focused zones for deep analysis.


Fig. 5. Color-coded scattering power decomposition result (red: $P_{D}$, green: $P_{V}$, blue: $P_{S}$ ) obtained by EG4U on Radarsat-2 data of San Francisco.

A pseudocolor difference image between $S 4 R$ and any a GG4U is then obtained by treating $\left\{\eta_{D}^{\mathrm{GG} 4 \mathrm{U}}-\eta_{D}^{\mathrm{S} 4 \mathrm{R}}, 0, \eta_{S}^{\mathrm{GG} 4 \mathrm{U}}-\right.$ $\left.\eta_{S}^{\mathrm{S} 4 \mathrm{R}}\right\}$ as $\{R, G, B\}$, and we denote such difference image as $\mathrm{GG} 4 \mathrm{U}-\mathrm{S} 4 \mathrm{R}$. The conservation of $P_{V}$ indicates that the green component $G$ of $\mathrm{GG} 4 \mathrm{U}-\mathrm{S} 4 \mathrm{R}$ is always zero, whereas the conservation of $P_{S}+P_{D}$ shows $\eta_{D}^{\mathrm{GG} 4 \mathrm{U}}-\eta_{D}^{\mathrm{S} 4 \mathrm{R}}=\eta_{S}^{\mathrm{S} 4 \mathrm{R}}-\eta_{S}^{\mathrm{GG} 4 \mathrm{U}}$. Taking the nonnegative reality of $R$ and $B$ into consideration, then the difference image $\mathrm{GG} 4 \mathrm{U}-\mathrm{S} 4 \mathrm{R}$ will always present itself in an either red or blue pattern: the red pixel denotes $\eta_{D}^{\mathrm{GG} 4 \mathrm{U}} \geq \eta_{D}^{\mathrm{S4R}}$, whereas the blue one indicates $\eta_{S}^{\mathrm{GG4U}}>\eta_{S}^{\mathrm{S4R}}$. This enables an excellent visualization of the change from S4R to GG4U. Fig. 7(a)-(c) shows EG4U - S4R, G4U - S4R, and $D G 4 U-S 4 R$, respectively. It is interesting to observe that the distribution of red and blue pixels in Fig. 7(a) is highly corresponded to the typical targets in the scene: red pixel mainly arises in building and urban area, whereas blue pixel mainly appears in area such as ocean, mountain, and airport. However, this perfect correspondence seems lost in Fig. 7(b), because we can also see blue pixels in urban area and red pixels in ocean and mountain, i.e., G4U cannot always enhance the surface scattering power in ocean and mountain nor strengthen double-bounce scattering power in building and urban area, and DG4U in Fig. 7(c) similarly. To rigorously check these, Fig. 8(a) shows the binary image of BC, where the white pixel denotes $\mathrm{BC}>0$ or $S>D$, i.e., surface scattering is stronger than double-bounce scattering, which occupies area of ocean, airport, and mountain, ${ }^{7}$ whereas the

[^5]black one denotes $\mathrm{BC} \leq 0$ or $D \geq S$, i.e., the stronger double-bounce scattering, which occupies the urban area. BC is widely used in model-based decompositions as a crucial feature to discriminate surface scattering and double-bounce scattering [13]-[45]. Taking BC as a priori scattering truth, we can further classify the pixel in GG4U - S4R into four categories.

1) $S \mid S$ : GG4U correctly improves S 4 R by enhancing the surface scattering contribution over area where surface scattering is stronger than double-bounce scattering. We denote this as $S \mid S$, which is ruled by $\left(\eta_{S}^{\mathrm{GG4U}}>\eta_{S}^{\mathrm{S4R}}\right) \&(S>D)$.
2) $D \mid S$ : GG4U wrongly improves double-bounce scattering power over area dominated by surface scattering rather than double-bounce scattering. We denote this as $D \mid S$ and rule it by $\left(\eta_{D}^{\mathrm{GG4U}} \geq \eta_{D}^{\mathrm{S4R}}\right) \&(S>D)$.
3) $D \mid D$ : GG4U correctly enhances double-bounce scattering power over area dominated by double-bounce scattering rather than surface scattering. We denote this as $D \mid D$ and rule it if $\left(\eta_{D}^{\mathrm{GG4U}} \geq \eta_{D}^{\mathrm{S} 4 \mathrm{R}}\right) \&(D \geq S)$.
4) $S \mid D$ : GG4U wrongly strengthens the surface scattering contribution over a area dominated by double-bounce scattering rather than surface scattering. We denote this as $S \mid D$ and rule it by $\left(\eta_{S}^{\mathrm{GG} 4 \mathrm{U}}>\eta_{S}^{\mathrm{S} 4 \mathrm{R}}\right) \&(D \geq S)$.

We conduct this classification to every pixel of EG4U-S4R and render pixel attributed to $S|S, D| D$, $D \mid S$, or $S \mid D$ in blue, red, purple, or yellow in Fig. 9(a). By relating it to the binary image of BC in Fig. 8(a), we can observe a high consistency: $S \mid S$ aligns with BC $>0$, whereas $D \mid D$ corresponds to $\mathrm{BC} \leq 0$, and we can hardly detect any $D \mid S$ or $S \mid D$ pixels in Fig. 9(a). The similar classification can also be done to G4U - S4R and DG4U -S 4 R , as shown in Fig. 9(b) and (c), which, however, present clear inconsistency with Figs. 8(a) and 9(a). For instance, Fig. 9(b) \& (c) shows many yellow pixels in urban area and purple pixels in ocean, indicating the impropriety of G4U and DG4U. To obtain a quantitative measure of the consistency and inconsistency, we define a pseudoprobability of occurrence $p$ for each of the four categories. The $p(S \mid S)$ and $p(D \mid D)$ measure the consistency

$$
\left\{\begin{array}{l}
p(S \mid S)=\frac{\#\left\{\left(\eta_{S}^{\mathrm{GG} 4 \mathrm{U}}>\eta_{S}^{\mathrm{S} 4 \mathrm{R}}\right) \&(S>D)\right\}}{\#\{S>D\}} \times 100 \%  \tag{70}\\
p(D \mid D)=\frac{\#\left\{\left(\eta_{D}^{\mathrm{GG4U}} \geq \eta_{D}^{\mathrm{S} 4 \mathrm{R}}\right) \&(D \geq S)\right\}}{\#\{D \geq S\}} \times 100 \%
\end{array}\right.
$$



Fig. 6. Decomposition scattering power profiles of (a) $P_{S}+P_{D}$, (b) $P_{V}$, (c) $P_{C}$, and (d) $P_{S}+P_{D}+P_{V}+P_{C}$ achieved by S4R, G4U, DG4U, and EG4U along the red transect line in Z 1 of Fig. 4(a) for various targets.
where $\#\{\cdot\}$ denotes "the number of." Then, the complementary principle tells us $p(D \mid S)=1-p(S \mid S), p(S \mid D)=1-$ $p(D \mid D)$, which measure the inconsistency, and thus, only $p(S \mid S)$ and $p(D \mid D)$ are shown in Table I for comparison. As shown, both $p(D \mid D)$ and $p(S \mid S)$ achieved by EG4U on the Radarsat-2 data set of San Francisco are $100 \%$, i.e., EG4U can completely strengthen S 4 R in area dominated by either surface or double-bounce scattering. The $p(D \mid D)$ attained by G4U is $60.47 \%$, i.e., G4U enhances the double-bounce contribution on about $60 \%$ of urban area of San Francisco, whereas on the other $40 \%$ area, G4U does not enhance but reduces the double-bounce scattering. If we use G4U in this area to extract building and evaluate damage caused by tsunami or earthquake, the reduced double-bounce scattering caused by G4U may result in the underestimation of building scale and the overestimation of the damage level. The $p(S \mid S)$ attained by G 4 U is $65.03 \%$, higher than $p(D \mid D)$; nevertheless, G4U also fails to improve but reduces the surface scattering on the other $34.97 \%$ of the majority of the ocean and mountain area in San Francisco. Such a reduced surface scattering power may lead to the incorrect estimation and mapping of snow density and
wetness if G4U is employed in the quantitative remote sensing of snow. The scale of landslide and deforestation may also be underestimated if we employ the reduced surface scattering for the remote sensing of mountain and forest. The similar situation also occurs in DG4U, as shown in Table I, and $p(S \mid S)$ and $p(D \mid D)$ achieved by DG4U on the same scene are $54.14 \%$ and $60.16 \%$, respectively, even lower than that obtained by G4U. Furthermore, according to the law of total probability, we define a pseudo-probability $p(\mathrm{C} \mid T)$ from $p(D \mid D)$ and $p(S \mid S)$ to show the total consistency between GG4U - S4R and the a priori scattering truth BC

$$
\begin{equation*}
p(\mathrm{C} \mid T)=p(S \mid S) p(S)+p(D \mid D) p(D) \tag{71}
\end{equation*}
$$

where $p(S)$ gives the probability taken up by $\mathrm{BC}>0$ or $S>D$, and $p(D)$ is the probability accounted for by $\mathrm{BC} \leq 0$ or $D \geq S$. Similarly, the total inconsistency $p(\mathrm{I} \mid T)$ is simply derived from the complementary principle by $p(\mathrm{I} \mid T)=1-$ $p(\mathrm{C} \mid T)$. Table I further shows the $p(\mathrm{C} \mid T)$ obtained by EG4U, G4U, and DG4U on San Francisco data set. The highest 100\% consistency indicates EG4U's universal improvement of S4R everywhere. The worst consistency on this data is provided


Fig. 7. Evaluating the performance of EG4U, G4U, and DG4U on the scattering power difference image constructed by taking S4R as reference. (a) EG4U - S4R, (b)G4U - S4R, (c)DG4U - S4R, and (d) $(G 4 U+D G 4 U) / 2-S 4 R$. EG4U - S4R here denotes the normalized difference of the color-coded scattering power images of EG4U and S4R. The meaning of difference images G4U $-S 4 R$, DG4U $-S 4 R$, and ( $G 4 U+$ DG4U) $/ 2-S 4 R$ can be likewise inferred.


Fig. 8. Binary display of (a) BC and (b) $\mathrm{BC}_{3}$. The white pixels in (a) and (b) denote $B C>0$ and $B C_{3}>0$, respectively, while the black pixels correspond to $B C \leq 0$ and $B C_{3} \leq 0$, respectively.
by DG4U with half a little more improvement (54.78\%) and nearly half degradation. G4U improves S4R in $64.55 \%$ of the data area but also degrades the performance in the other $35.45 \%$ area. To remove the potential bias from PolSAR data set, we also evaluate GG4U algorithms on other nine PolSAR data sets acquired by both spaceborne and airborne systems from different places. The result is also shown in Table I. On all the nine data sets, $p(S \mid S), p(D \mid D)$, and $p(\mathrm{C} \mid T)$ attained by GG4U are forever $100 \%$ as ever. However, G4U does not always obtain a better consistency than DG4U. The performance of G4U and DG4U is comparable. None of them can strengthen S4R without any degradation only when we integrate them into EG4U.

## C. EG4U Versus G4U and DG4U

The performance of GG4U is evaluated in Section VI-B by referring EG4U, G4U, and DG4U to S4R for the difference images, respectively. In this section, the underlying relation among EG4U, G4U, and DG4U is investigated on PolSAR data to further demonstrate the outperformance of EG4U over G4U and DG4U. We start from the total consistency $p(\mathrm{C} \mid T)$ shown in Table I. The addition of the $p(\mathrm{C} \mid T)$ attained by G4U and DG4U on each of the ten exemplified PolSAR data sets is higher than $100 \%$, so is the addition of $p(S \mid S)$ and that of $p(D \mid D)$. Hence, there must be data cells where both G4U and DG4U strengthen the performance of S4R. Nevertheless, as the other side of the medal, actually,


Fig. 9. Evaluating the performance of EG4U, G4U, and DG4U by classifying the scattering power difference images (a) EG4U $-S 4 R$, (b)G4U $-S 4 R$, (c)DG4U -S 4 R , and (d) $(\mathrm{G} 4 \mathrm{U}+\mathrm{DG} 4 \mathrm{U}) / 2-\mathrm{S} 4 \mathrm{R}$ based on a priori scattering truth $B C$.

TABLE I
Quantitative Evaluation of the Performance of G4U, DG4U, and EG4U on Ten Typical PolSAR Datasets in Terms of $p(\mathrm{~S} \mid \mathrm{S}), p(\mathrm{D} \mid \mathrm{D})$, AND $p(\mathrm{C} \mid \mathrm{T})$

| PolSAR dataset | $p(\mathrm{~S} \mid \mathrm{S})$ |  |  | $p$ (D\|D) |  |  | $p(\mathrm{C} \mid \mathrm{T})$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | G4U | DG4U | EG4U | G4U | DG4U | EG4U | G4U | DG4U | EG4U |
| RADARSAT-2 San Francisco | 65.0265\% | 54.1449\% | 100\% | 60.4737\% | 60.1550\% | 100\% | 64.5464\% | 54.7788\% | 100\% |
| RADARSAT-2 Altona | 66.3186\% | 64.9273\% | 100\% | 59.2979\% | 55.6926\% | 100\% | 66.3038\% | 64.9078\% | 100\% |
| RADARSAT-2 Flevoland | 65.7755\% | 62.3782\% | 100\% | 66.2451\% | 66.0557\% | 100\% | 65.8133\% | 62.6743\% | 100\% |
| RADARSAT-2 Oberpfaffenhofen | 67.2382\% | 66.5386\% | 100\% | 65.8780\% | 66.8431\% | 100\% | 67.1435\% | 66.5598\% | 100\% |
| RADARSAT-2 Strait of Gibraltar | 67.4298\% | 61.3836\% | 100\% | 69.5546\% | 68.9909\% | 100\% | 67.5348\% | 61.7593\% | 100\% |
| AIRSAR San Francisco | 68.5254\% | 86.5860\% | 100\% | 70.6587\% | 59.0393\% | 100\% | 69.5072\% | 73.9091\% | 100\% |
| ESAR Oberpfaffenhofen | 58.9685\% | 53.8481\% | 100\% | 55.5691\% | 58.4707\% | 100\% | 58.0201\% | 55.1378\% | 100\% |
| EMISAR Foulum | 63.7717\% | 53.2961\% | 100\% | 66.1728\% | 66.1728\% | 100\% | 64.2420\% | 55.2833\% | 100\% |
| ALOS-PALSAR San Francisco | 58.0714\% | 61.2075\% | 100\% | 63.7875\% | 62.2562\% | 100\% | 58.7085\% | 61.3244\% | 100\% |
| ALOS-PALSAR2 Fuji Mountain | 61.4205\% | 65.7991\% | 100\% | 61.9397\% | 65.3332\% | 100\% | 61.6427\% | 65.5996\% | 100\% |

we care more about whether there are data cells where both G4U and DG4U degrade S4R. We investigate this by taking the average of the difference images G4U - S4R and DG4U - S4R for $(G 4 U+D G 4 U) / 2-S 4 R$. Fig. 7(d) shows the average difference image achieved on the Radarsat-2 data set of San Francisco. Although the red color in certain urban area and the blue color in ocean area are somewhat light, Fig. 7(d) shows the similar perfect correspondence to the typical targets as that shown in Fig. 7(a). To illustrate this more clearly, taking BC as a priori scattering truth, a classification is also conducted to the average difference image (G4U + DG4U) $/ 2-\mathrm{S} 4 \mathrm{R}$ for $S|S, D| D, D \mid S$, or $S \mid D$.

The result is shown in Fig. 9(d), which looks the same as Fig. 9(a). We can hardly detect any $D \mid S$ or $S \mid D$, but the good correspondence between $S \mid S$ and BC $>0$, and perfect consistency between $D \mid D$ and $\mathrm{BC} \leq 0$. This is also validated in term of pseudo-probability: all the $p(S \mid S), p(D \mid D)$, and $p(\mathrm{C} \mid T)$ change to the highest $100 \%$ after averaging, and they maintain $100 \%$ also on the other nine PolSAR data sets. Therefore, there never exists a situation that both G4U and DG4U degrade S4R. They either improve S4R separately or achieve this at the same time. We use duality to depict such complementary redundancy. To deeply display the duality, particularly the complementation, S4R, G4U, and DG4U are

TABLE II
Normalized Four-Component Scattering Powers Obtained by S4R, G4U, DG4U, and EG4U in Z2, THE San Andreas Lake of San Francisco

| Method | $P_{S} / S P A N$ | $P_{D} / S P A N$ | $P_{V} / S P A N$ | $P_{C} / S P A N$ |
| :---: | :--- | :--- | :--- | :--- |
| S4R | $81.0908 \%$ | $3.8794 \%$ | $15.0011 \%$ | $0.0287 \%$ |
| G4U | $81.1536 \% \uparrow$ | $3.8166 \% \downarrow$ | $15.0011 \%$ | $0.0287 \%$ |
| DG4U | $81.0301 \% \downarrow$ | $3.9402 \% \uparrow$ | $15.0011 \%$ | $0.0287 \%$ |
| EG4U | $81.1536 \%$ | $3.8166 \%$ | $15.0011 \%$ | $0.0287 \%$ |

TABLE III
Normalized Four-Component Scattering Powers Obtained By S4R, G4U, DG4U, and EG4U in Z3, THE

Naval Air Station Alameda

| Method | $P_{S} / S P A N$ | $P_{D} / S P A N$ | $P_{V} / S P A N$ | $P_{C} / S P A N$ |
| :---: | :--- | :--- | :--- | :--- |
| S4R | $52.9782 \%$ | $17.0772 \%$ | $29.8466 \%$ | $0.0980 \%$ |
| G4U | $52.9593 \% \downarrow$ | $17.0961 \% \uparrow$ | $29.8466 \%$ | $0.0980 \%$ |
| DG4U | $53.0086 \% \uparrow$ | $17.0469 \% \downarrow$ | $29.8466 \%$ | $0.0980 \%$ |
| EG4U | $53.0086 \%$ | $17.0469 \%$ | $29.8466 \%$ | $0.0980 \%$ |

further carried out on the four typical zones from $Z 2$ to $Z 5$ shown in Fig. 4. We average the $\langle[T]\rangle$ matrices in each zone and decompose the mean target for the normalized scattering power of the four components. Tables II-V show the obtained result on each zone. Z2 denotes the San Andreas Lake. The Cloude-Pottier entropy $H$ and $\alpha$ angle here are 0.3889 and $15.3208^{\circ}$, respectively, indicating the preeminent low-entropy Bragg scattering [10]. As shown in Table II, in comparison with S4R, G4U successfully improves the surface scattering power, whereas DG4U improperly strengthens double-bounce scattering but decreases the surface scattering contribution. Nevertheless, as shown in Table III, the performance of G4U and DG4U is exchanged in $Z 3$, which is extracted from the former naval air station Alameda. The parameters $H$ and $\alpha$ here are 0.7300 and $29.8487^{\circ}$, respectively, indicating the medium-entropy random surface scattering. We should resort to DG4U for improvement. Different from $Z 2$ and Z3, Z4 and $Z 5$ are dominated by double-bounce scattering of different scattering randomness. $Z 4$ shows a San Francisco urban area with $\alpha=29.8487^{\circ}$ and $H=0.7300$, corresponding to the dihedral scattering with moderate entropy. Just as that claimed in [31], compared with S4R, G4U improves the double-bounce scattering contribution here and reduces the surface scattering power, whereas DG4U just behaves in precisely the opposite way, as shown in Table IV. Unlike $Z 4, Z 5$ is not a building area but a harbor in Point Richmond of California. The surface scattering here should be dominant just like that occurs in the airport Z3. The scattering powers achieved by S4R, G4U, and DG4U in Table V, however, all exhibit the overwhelmingly dominant double-bounce scattering, which takes up about $83 \%$ of the total scattering power. This interesting finding is further ensured by $H=0.4432$ and $\alpha=68.1058^{\circ}$, which attribute $Z 5$ as the double- or even-bounce scattering of low entropy [10]. To explain this, we also exhibit the optical image of $Z 5$ in Fig. 4(b), which shows that there are always a lot of cars parked neatly in the harbor through the year. Thus, the dominant double-bounce scattering here may mainly be credited to the

TABLE IV
Normalized Four-Component Scattering Powers Obtained by S4R, G4U, DG4U, and EG4U in Z4, An Urban

Area of San Francisco

| Method | $P_{S} / S P A N$ | $P_{D} / S P A N$ | $P_{V} / S P A N$ | $P_{C} / S P A N$ |
| :---: | :--- | :--- | :--- | :--- |
| S4R | $33.8193 \%$ | $60.5359 \%$ | $4.8117 \%$ | $0.8331 \%$ |
| G4U | $33.6173 \% \downarrow$ | $60.7379 \% \uparrow$ | $4.8117 \%$ | $0.8331 \%$ |
| DG4U | $34.0134 \% \uparrow$ | $60.3417 \% \downarrow$ | $4.8117 \%$ | $0.8331 \%$ |
| EG4U | $33.6173 \%$ | $60.7379 \%$ | $4.8117 \%$ | $0.8331 \%$ |

TABLE V
Normalized Four-Component Scattering Powers Obtained by S4R, G4U, DG4U, and EG4U in Z5, A Harbor
in Point Richmond, CA

| Method | $P_{S} / S P A N$ | $P_{D} / S P A N$ | $P_{V} / S P A N$ | $P_{C} / S P A N$ |
| :---: | :--- | :--- | :--- | :--- |
| S4R | $15.5071 \%$ | $82.7559 \%$ | $1.5889 \%$ | $0.1481 \%$ |
| G4U | $15.5913 \% \uparrow$ | $82.6717 \% \downarrow$ | $1.5889 \%$ | $0.1481 \%$ |
| DG4U | $15.4210 \% \downarrow$ | $82.8420 \% \uparrow$ | $1.5889 \%$ | $0.1481 \%$ |
| EG4U | $15.4210 \%$ | $82.8420 \%$ | $1.5889 \%$ | $0.1481 \%$ |

numerous aligned ground-vehicle backscatterers at C-band. As shown in Table V, rather than G4U, DG4U should be selected to improve S 4 R for the enhanced retrieval of the double-bounce scattering here.

The duality of G4U and DG4U indicates that the appropriate combination of them will not only solve the deficiency in G4U and DG4U themselves but also achieve a full improvement of S4R by adaptively strengthening the double-bounce scattering or surface scattering according to the preferable scattering. One simple strategy to obtain this is to conduct G4U and DG4U first and then compare them on scattering power $P_{S}$ or $P_{D}$ according to BC for the larger one. Despite feasible, this requires us to do both decompositions to the same data. EG4U obtains this based on $\mathrm{BC}_{3}$, which enables us an adaptive selection between DG4U and G4U ahead of decomposition: DG4U operates if $\mathrm{BC}_{3} \leq 0$, otherwise, G4U functions. This makes the combination of G4U and DG4U more natural. The binary image in Fig. 8(b) shows $\mathrm{BC}_{3}$ attained on Radarsat-2 data set of San Francisco. The white pixel (i.e., $\mathrm{BC}_{3}>0$ ) indicates the data cell where G4U operates, whereas the black one (i.e., $\mathrm{BC}_{3} \leq 0$ ) signifies the data cell where DG4U works. The white and black pixels account for $54.91 \%$ and $45.09 \%$ of the whole image, respectively. Therefore, G4U improves S4R only on $54.91 \%$ area of the San Francisco scene. As for the rest area, we should resort to DG4U for improvement. Tables II-V also show the normalized scattering power of the four components decomposed by EG4U on the four zones from $Z 2$ to $Z 5$. EG4U achieves the same excellent result as G4U in $Z 2$ and $Z 4$ just because of the positive $\mathrm{BC}_{3}$ here, whereas in $Z 3$ and $Z 5$, it achieves the same excellent scattering power decomposition as DG4U also because of the negative $\mathrm{BC}_{3}$ there. Therefore, $\mathrm{BC}_{3}$ and the G4U-DG4U duality are the key to the success of EG4U. Nevertheless, in comparison with BC , the physical significance of $\mathrm{BC}_{3}$ is still unclear yet. Further investigation is necessary so as to make it as widely-accepted and widely-used as BC.

## VII. Discussion

G4U was developed to incorporate $T_{13}^{\prime}$ into the modeled part $\left\langle\left[T_{M}^{\prime G 4 U}\right]\right\rangle$ based on the $\mathrm{SU}(3)$ matrix $\left[U_{3}(\varphi)\right]$, as shown in (23). Nonetheless, if we turn attention to (24), the equivalent $-T_{13}^{\prime}$ is also observed in the residue $\left\langle\left[T_{R}^{\prime \mathrm{G} 4 \mathrm{U}}\right]\right\rangle$ since $\left\langle\left[T_{M}^{\prime \mathrm{G4U}}\right]\right\rangle$ includes $T_{13}^{\prime}$ in the $(1,2)$ entry rather than $(1,3)$ entry. Such incomplete utilization of $T_{13}^{\prime}$ also exists in GG4U, as given in (35) and (36). Matrix $\left[U_{3}(\varphi)\right]$ devised by Singh et al. looks to account for one more DoF by eliminating $\operatorname{Im}\left\{T_{23}^{\prime}\right\}$ for G4U. However, $\operatorname{Im}\left\{T_{23}^{\prime}\right\}$ has already been accounted for in S4R and Y4R by helix power $f_{C}$, as formulated in (15-4). Hence, although all the elements in $\langle[T]\rangle$ are involved in $\left\langle\left[T_{M}^{\prime \mathrm{G} 4 \mathrm{U}}\right]\right\rangle$, the DoF completely modeled in G4U is still seven, the same as S 4 R , i.e., GG 4 U in the form of $\mathcal{G}(\mu)$ shares the same DoF. Theoretically, G4U and S4R indicate two special solutions of GG4U of equal status only. Therefore, G4U cannot fully improve S4R only if we ascend the status of G4U by combining the duality of G4U, i.e., DG4U and G4U together for EG4U. EG4U can adaptively increase the surface scattering and double-bounce scattering. Hence, it will definitely improve the competence and performance of G4U in the remote sensing of agriculture, forestry, snow, wetland, environment, man-made targets, glaciated terrain, as well as damages caused by landside and earthquake/tsunami. We will investigate these in the future.

Are there any other $\operatorname{SU}(3)$ matrices that can be used to obtain GG4U? The answer is affirmative. For instance, we can replace $\left[U_{3}(\varphi)\right]$ in G 4 U with the following $\mathrm{SU}(3)$ helix matrix [ $\left.U_{3}(\tau)\right]:$

$$
\left[U_{3}(\tau)\right]=\left[\begin{array}{ccc}
\cos 2 \tau & 0 & j \sin 2 \tau  \tag{72}\\
0 & 1 & 0 \\
j \sin 2 \tau & 0 & \cos 2 \tau
\end{array}\right]
$$

[ $\left.U_{3}(\tau)\right]$ has been used to evaluate the symmetry-asymmetry nature of targets in Touzi decomposition [11]. Its performance on the indication of helicity and asymmetry has been proven by comparing with the related indicator in Paladini decomposition [12] and Cameron decomposition [55]. It has also been adopted recently by Bhattacharya et al. [56] to develop an adaptive G4U (AG4U). ${ }^{8}$ Similarly, a new generalized balance equation system different from (33) can be created if we replace $\left[U_{3}(\varphi)\right]$ in G 4 U with $\left[U_{3}(\tau)\right.$ ]

$$
\begin{cases}f_{S}+f_{D}|\alpha|^{2}+f_{V} a=T_{11}^{\prime}+\rho \operatorname{Im}\left\{T_{13}^{\prime}\right\} & -1)  \tag{73}\\ f_{S} \beta+f_{D} \alpha+f_{V} d=T_{12}^{\prime} & -2) \\ f_{S}|\beta|^{2}+f_{D}+f_{V} b+\frac{f_{C}}{2}=T_{22}^{\prime} & -3) \\ \pm j \frac{f_{C}}{2}=j \operatorname{Im}\left\{T_{23}^{\prime}\right\} & -4) \\ f_{V} c+\frac{f_{C}}{2}=T_{33}^{\prime} & -5)\end{cases}
$$

[^6]where
\[

$$
\begin{equation*}
\rho=\tan 4 \tau \tag{74}
\end{equation*}
$$

\]

The redundancy in this case appears in (73-1), which includes $T_{13}^{\prime}$ information in the $(1,1)$ entry of $\left\langle\left[T_{M}^{\prime \text { G4U }}\right]\right\rangle$. This signifies a new type of GG4U, and we denote it as $\mathcal{G}(\rho)$ or $\mathcal{G}(\tau)$. By taking an appropriate value to $\rho$ or $\tau$, we can also have GG4U of different forms, while $\mathcal{G}(0)$ just corresponds to S4R which also bears the minimized residue. The preference analysis conducted on $\mathcal{G}(\psi)$ in Section IV can be similarly extended to $\mathcal{G}(\tau)$. A different type of GG4U and redundancy will be, of course, achieved if we use a different $\mathrm{SU}(3)$ matrix. Nonetheless, the unitary invariance of L2-norm indicates that, for whatever $\mathrm{SU}(3)$ matrix used, there is always a $T_{13}^{\prime}$-related residue in GG4U, while S4R always bears the minimized residue. Therefore, the incorporation of unitary transformation in S4R for the complete modeling of scattering information in coherency matrix is impossible in theory.

To fully reserve $T_{13}^{\prime}$ in the modeled part, we should consider to introduce a $T_{13}^{\prime}$-related independent equation into the balance equation system instead of a redundant one, which indicates that the canonical scattering model with nonzero $(1,3)$ entry should be used, while Y4R, S4R, G4U, and GG4U cannot obtain this just because the $(1,3)$ element of all the models in (10) is zero. One way to achieve this is to adopt the general model of surface and double-bounce scatterings [10], which has been applied by Cui et al. [20] and An and Xie [36] in the complete three-component model-based decomposition and has been further employed by Chen et al. [19] and Xie et al. [40], [43] in GMD to tackle with the limitation of Y4R and S4R on inversion priority, orientation effects, BC, and negative powers. Another way to obtain this is to introduce some new physical scattering models into four-component decomposition. Singh and Yamaguchi [44] recently derived two new models, i.e., the oriented dipole scattering and the oriented quarter-wave reflection, to account for the real and imaginary parts of $T_{13}^{\prime}$, and added them into S4R for a six-component model-based decomposition (6SD), which, recently, has been further extended to a seven-component decomposition (7SD) by Singh et al. [45]. Both G4U/GG4U and 6SD/7SD are dedicated to improve S 4 R to account for $T_{13}^{\prime}$ : the former obtains this based on unitary transformation, whereas the latter depends on the $T_{13}^{\prime}$-related models. Strictly speaking, 6SD/7SD no longer belongs to the series of four-component decomposition because of the newly added scattering components. Nonetheless, EG4U may be also beneficial to 6SD/7SD as an alternative to S 4 R to strengthen the double-bounce scattering and surface scattering. Full information utilization requires an exact fit of all scattering components. Nevertheless, the exact fit is not always necessary because a coherency matrix may have some noise and/or show variations across the mixed targets, and the exact fit will just be fitting noise then. From the minimum residue point of view, the residue of EG4U is $\sqrt{2}$ times larger than that of S4R, but EG4U can adaptively improve S4R based on the preferable scattering. This article just enables a mathematical extension to G4U. We should be scrupulous when evaluating the performance of a decomposition algorithm based on only one criterion.

From the power estimation point of view, the progress from FDD to EG4U is just aimed to better retrieve the contribution of double-bounce and surface components. FDD overestimates $P_{V}$ while underestimates $P_{S}$ and $P_{D}$. Y4O decreases $P_{V}$ to increase $P_{S}$ and $P_{D}$ by introducing a helix component and two additional volume models. Y4R further decreases $P_{V}$ and increases $P_{S}$ and $P_{D}$ by implementing deorientation. S4R continues this progress by introducing an even-bounce structure-related volume model to reduce $P_{V}$ and improve $P_{D}$. G4U adopts an imagery rotation of coherency matrix to strengthen $P_{D}$ in urban and building area while improve $P_{S}$ in land and water area further, but which is not always the case. To enable a full enhancement of G4U and S4R, EG4U reserves the minimized $P_{V}$ obtained in the previous progress and adaptively integrates the G4U-DG4U duality so that $P_{D}$ in building and urban region and $P_{S}$ in land and water region can be always improved. It signifies a state-of-the-art four-component scattering power decomposition.

Model-based decompositions provide a set of parameters of physical significance, $P_{S}, P_{D}, P_{V}, P_{C}, \varphi, \beta$, and $\alpha$. Nevertheless, these parameters have not attracted fair treatment and attention in existing researches. We attribute it mainly to the availability of ground truth. Undoubtedly, among these parameters, $P_{S}, P_{D}, P_{V}$, and $P_{C}$, especially the first three, attracted the most attention because they can establish an intuitive relationship with ground truth by visualizing the decomposition result in terms of pseudocolor. Such the ground truth can be easily achieved from the optical image and/or map of region of interest. Hence, the model-based decomposition is also referred to as the model-based scattering power decomposition, and seeking the best balance among the scattering powers has become the main driving force behind the model-based decompositions. To make all the parameters fairly treated and any new model-based decomposition appropriately evaluated on all the parameters, benchmark data sets with complete ground truth are of crucial importance. We highly acknowledge the initiative made by BallesterBerman et al. [50] recently, "The use of benchmark data sets available to any researcher and agreed by the community for validation purposes is a way to better quantify the potential progress by any new proposal."

Except for few recent progress [50], [40], the existing model-based decompositions are mostly evaluated from the qualitative point of view. The quantitative assessment of the whole set of parameters involved in model-based decompositions has been almost overlooked in the field [50]. The strength of quantitative assessment is to quantify the performance of an algorithm. As a necessary complement to the qualitative evaluation and identification, the quantitative analysis is more qualified to promote model-based parameters to the quantitative remote sensing application. Nonetheless, whether for the qualitative or quantitative assessment, estimation accuracy is always affected by many random factors because of the missing of "entropy control" in radar polarimetry. The combination of polarimetry and interferometry can help us to obtain a better parameter estimation, even in the presence of high entropy and strong depolarization [3].

From a quantitative point of view, the progress from FDD to EG4U can be attributed into two phases. Phase 1 corresponds to the progress from FDD to S4R, including FDD, Y4O, Y4R, and S4R, which is aimed to significantly adjust $P_{S}, P_{D}$, and $P_{V}$ so as to correct their relationship from $P_{V}>P_{S}, P_{D}$ to $P_{D}>P_{S}, P_{V}$ in some urban and building areas and to $P_{S}>P_{D}, P_{V}$ in some land and water areas. The changes in this phase are so significant that the color-coded scattering power images are sufficient to show them. Phase 2 involves in the progress from S4R to EG4U, including S4R, G4U, and EG4U, which is more like a fine tuning process. It conserves the relative relationship among $P_{S}, P_{D}$, and $P_{V}$ but finely revises $P_{D}$ and $P_{S}$ so as to adaptively enhance the surface or double-bounce contribution further. It is then better to reveal the resulted differences with the color-coded differential power images (Fig. 7) or quantify the changes based on the statistical consistencies and normalized scattering powers (Tables I-V).

S4R, G4U, DG4U, and EG4U denote four typical algorithms of GG4U, which can be also expressed as $\mathcal{G}(0)$, $\mathcal{G}(+1), \mathcal{G}(-1)$, and $\mathcal{G}( \pm 1)$, or $\mathcal{G}(0), \mathcal{G}(+\pi / 8), \mathcal{G}(-\pi / 8)$, and $\mathcal{G}( \pm \pi / 8)$. We suggest naming them with $\mathcal{G}(\mu)$ or $\mathcal{G}(\psi)$ to directly reveal their mathematical relationship and underlying physical mechanism. By taking an appropriate value to $\mu$ or $\psi$ even $\rho$ or $\tau$ or integrating any other $\mathrm{SU}(3)$ matrix into S4R, we can, of course, obtain some other forms of GG4U which may provide us as competent or even better target decomposition performance than S4R, G4U, DG4U, and EG4U. We leave all these as the future work.

## VIII. CONCLUSION

The unitary transformation plays an important role in G4U. It accounts for $T_{13}^{\prime}$ by adding a $T_{13}^{\prime}$-related but redundant balance equation into the originally self-contained S4R/Y4R scattering balance equation system. We then have no exact solution to the system but the approximate ones. The general expression of the approximate solutions enables us a GG4U, while G4U and S4R denote two special forms of it. Information accounted for in the modeled components shows the scattering preference of GG4U for the $\psi$-rotated double-bounce scattering. G4U cannot always improve S4R unless we combine G4U and its duality for EG4U. EG4U provides an adaptive improvement to both S4R and G4U by strengthening the surface scattering or double-bounce scattering. A generalized decomposition other than GG4U will be achieved by considering a different unitary transformation. However, for whatever unitary transformation adopted, there is always a $T_{13}^{\prime}$-related unaccounted residue in GG4U. Complete $T_{13}^{\prime}$ modeling is not obtained unless canonical scattering models with nonzero $(1,3)$ entry of coherency matrix are used to add an independent $T_{13}^{\prime}$-related equation to the $\mathrm{S} 4 \mathrm{R} / \mathrm{Y} 4 \mathrm{R}$ balance equation system.

## Appendix A

The balance equation system (20) is deduced in Appendix A. Bring matrices $\left\langle\left[T_{S}^{\prime}\right]\right\rangle,\left\langle\left[T_{D}^{\prime}\right]\right\rangle,\left\langle\left[T_{V}^{\prime}\right]\right\rangle$, and $\left\langle\left[T_{C}^{\prime}\right]\right\rangle$ into (10), entry $T_{M i j}^{\mathrm{S} 4 \mathrm{R} / \mathrm{Y} 4 \mathrm{R}}(i, j=1,2,3)$ of $\left\langle\left[T_{M}^{\prime \mathrm{S} 4 \mathrm{R} / \mathrm{Y} 4 \mathrm{R}}\right]\right\rangle$ is expressed
as

$$
\begin{cases}T_{M 11}^{\prime \mathrm{S} 4 \mathrm{R} / \mathrm{Y} 4 \mathrm{R}}=f_{S}+f_{D}|\alpha|^{2}+f_{V} a & -1)  \tag{-1}\\ T_{M 12}^{\mathrm{S} 4 \mathrm{R} / \mathrm{Y} 4 \mathrm{R}}=T_{M 21}^{\prime \mathrm{S} 4 \mathrm{R} / \mathrm{Y} 4 \mathrm{R} *}=f_{S} \beta+f_{D} \alpha+f_{V} d & -2) \\ T_{M 13}^{\prime \mathrm{S} 4 \mathrm{R} / \mathrm{Y} 4 \mathrm{R}}=T_{M 31}^{\mathrm{S} 4 \mathrm{R} / \mathrm{Y} 4 \mathrm{R} *}=0 & -3) \\ T_{M 22}^{\mathrm{SS4R} / \mathrm{Y} 4 \mathrm{R}}=f_{S}|\beta|^{2}+f_{D}+f_{V} b+\frac{f_{C}}{2} & -4) \\ T_{M 23}^{\mathrm{S} 4 \mathrm{R} / \mathrm{Y} 4 \mathrm{R}}=T_{M 32}^{\prime \mathrm{S} 4 \mathrm{R} / \mathrm{Y} 4 \mathrm{R} *}= \pm j \frac{f_{C}}{2} & -5) \\ T_{M 33}^{\prime \mathrm{S} 4 \mathrm{R} / \mathrm{Y} 4 \mathrm{R}}=f_{V} c+\frac{f_{C}}{2} & -6) .\end{cases}
$$

G4U first adopts $\left[U_{3}(\varphi)\right]$ to carry out unitary transformation to $\left\langle\left[T^{\prime}\right]\right\rangle$ for $\left\langle\left[T^{\prime \prime}\right]\right\rangle$ in (8) and to $\left\langle\left[T_{M}^{\text {'S4R } / \mathrm{Y} 4 \mathrm{R}}\right]\right\rangle$ for $\left\langle\left[T_{M}^{\prime / \mathrm{G} 4 \mathrm{U}}\right]\right\rangle$ in (19). Entries $T_{i j}^{\prime \prime}$ of $\left\langle\left[T^{\prime \prime}\right]\right\rangle$ and $T_{M i j}^{\prime \prime G 4 U}$ of $\left\langle\left[T_{M}^{\prime \prime G 4 U}\right]\right\rangle$ are obtained in (A2) and (A3), shown at the bottom of the page. By forcing $T_{M i j}^{\prime / G 4 U}=T_{i j}^{\prime \prime}$, G4U then associates the unknowns $f_{S}, f_{D}, f_{V}, f_{C}, \alpha$, and $\beta$ with the entry $T_{i j}^{\prime}$ of $\left\langle\left[T^{\prime}\right]\right\rangle$ so as to combine (A2) and (A3) for (A4), shown at the bottom of this page. (A4-2) $-j \times(\mathrm{A} 4-3)$, we have

$$
\begin{equation*}
f_{S} \beta+f_{D} \alpha+f_{V} d=T_{12}^{\prime}+T_{13}^{\prime} . \tag{A5}
\end{equation*}
$$

Similarly, (A4-2) $+j \times$ (A4-3), we obtain

$$
\begin{gather*}
f_{S} \beta+f_{D} \alpha+f_{V} d=T_{12}^{\prime}-T_{13}^{\prime}  \tag{A6}\\
(\mathrm{A} 4-4)+(\mathrm{A} 4-6)  \tag{A12}\\
f_{S}|\beta|^{2}+f_{D}+f_{V}(b+c)+f_{C}=T_{22}^{\prime}+T_{33}^{\prime} \tag{A7}
\end{gather*}
$$

(A4-4) - (A4-6)

$$
\begin{align*}
& \left(f_{S}|\beta|^{2}+f_{D}+f_{V}(b-c)\right) \cos 4 \varphi \pm f_{C} \sin 4 \varphi \\
& \quad=\left(T_{22}^{\prime}-T_{33}^{\prime}\right) \cos 4 \varphi+2 \operatorname{Im}\left\{T_{23}^{\prime}\right\} \sin 4 \varphi \tag{-3}
\end{align*}
$$

$\sin 4 \varphi \times(\mathrm{A} 4-5)+\cos 4 \varphi \times(\mathrm{A} 8)$

$$
\begin{equation*}
f_{S}|\beta|^{2}+f_{D}+f_{V}(b-c)=T_{22}^{\prime}-T_{33}^{\prime} \tag{-5}
\end{equation*}
$$

$\sin 4 \varphi \times(\mathrm{A} 8)-\cos 4 \varphi \times(\mathrm{A} 4-5)$

$$
\begin{equation*}
\pm j \frac{f_{C}}{2}=j \operatorname{Im}\left\{T_{23}^{\prime}\right\} \tag{A10}
\end{equation*}
$$

$(1 / 2) \times((\mathrm{A} 7)+(\mathrm{A} 9))$

$$
\begin{equation*}
f_{S}|\beta|^{2}+f_{D}+f_{V} b+\frac{f_{C}}{2}=T_{22}^{\prime} \tag{A11}
\end{equation*}
$$

$(1 / 2) \times((\mathrm{A} 7)-(\mathrm{A} 9))$

$$
f_{V} c+\frac{f_{C}}{2}=T_{33}^{\prime}
$$

$\begin{cases}T_{11}^{\prime \prime}=T_{11}^{\prime} & -1) \\ T_{12}^{\prime \prime}=T_{21}^{\prime \prime *}=T_{12}^{\prime} \cos 2 \varphi-j T_{13}^{\prime} \sin 2 \varphi & -2) \\ T_{13}^{\prime \prime}=T_{31}^{\prime \prime *}=T_{13}^{\prime} \cos 2 \varphi-j T_{12}^{\prime} \sin 2 \varphi & -3) \\ T_{22}^{\prime \prime}=T_{22}^{\prime} \cos ^{2} 2 \varphi+T_{33}^{\prime} \sin ^{2} 2 \varphi+\operatorname{Im}\left\{T_{23}^{\prime}\right\} \sin 4 \varphi & -4) \\ T_{23}^{\prime \prime}=T_{32}^{\prime \prime *}=-\frac{j}{2}\left(\left(T_{22}^{\prime}-T_{33}^{\prime}\right) \sin 4 \varphi-2 \operatorname{Im}\left\{T_{23}^{\prime}\right\} \cos 4 \varphi\right) & -5) \\ T_{33}^{\prime \prime}=T_{22}^{\prime} \sin ^{2} 2 \varphi+T_{33}^{\prime} \cos ^{2} 2 \varphi-\operatorname{Im}\left\{T_{23}^{\prime}\right\} \sin 4 \varphi & -6)\end{cases}$

$$
\begin{cases}T_{M 11}^{\prime \mathrm{G} 4 \mathrm{U}}=f_{S}+f_{D}|\alpha|^{2}+f_{V} a & -1)  \tag{A3}\\ T_{M 12}^{\prime \prime \mathrm{G} 4 \mathrm{U}}=T_{M 21}^{\prime \prime \mathrm{G} 4 \mathrm{U}^{*}}=\left(f_{S} \beta+f_{D} \alpha+f_{V} d\right) \cos 2 \varphi & -2) \\ T_{M 13}^{\prime \prime \mathrm{G} 4 \mathrm{U}}=T_{M 31}^{\prime \prime \mathrm{G} 4 \mathrm{U}^{*}}=-j\left(f_{S} \beta+f_{D} \alpha+f_{V} d\right) \sin 2 \varphi \\ T_{M 22}^{\prime \prime \mathrm{G} 4 \mathrm{U}}=\left(f_{S}|\beta|^{2}+f_{D}\right) \cos ^{2} 2 \varphi+\frac{f_{C}}{2}(1 \pm \sin 4 \varphi)+f_{V}\left(b \cos ^{2} 2 \varphi+c \sin ^{2} 2 \varphi\right) & -4) \\ T_{M 23}^{\prime \prime \mathrm{G} 4 \mathrm{U}}=T_{M 32}^{\prime \prime \mathrm{G4U}}=-\frac{j}{2}\left(\left(f_{S}|\beta|^{2}+f_{D}+f_{V}(b-c)\right) \sin 4 \varphi \mp f_{C} \cos 4 \varphi\right) & -5) \\ T_{M 33}^{\prime \prime \mathrm{G} 4 \mathrm{U}}=\left(f_{S}|\beta|^{2}+f_{D}\right) \sin ^{2} 2 \varphi+\frac{f_{C}}{2}(1 \mp \sin 4 \varphi)+f_{V}\left(b \sin ^{2} 2 \varphi+c \cos ^{2} 2 \varphi\right) & -6)\end{cases}
$$

$$
\left\{\begin{array}{l}
f_{S}+f_{D}|\alpha|^{2}+f_{V} a=T_{11}^{\prime} \\
\left(f_{S} \beta+f_{D} \alpha+f_{V} d\right) \cos 2 \varphi=T_{12}^{\prime} \cos 2 \varphi-j T_{13}^{\prime} \sin 2 \varphi \\
\left(f_{S} \beta+f_{D} \alpha+f_{V} d\right) \sin 2 \varphi=T_{12}^{\prime} \cos 2 \varphi+j T_{13}^{\prime} \cos 2 \varphi \\
\left(f_{S}|\beta|^{2}+f_{D}\right) \cos ^{2} 2 \varphi+\frac{f_{C}}{2}(1 \pm \sin 4 \varphi)+f_{V}\left(b \cos ^{2} 2 \varphi+c \sin ^{2} 2 \varphi\right) \\
=T_{22}^{\prime} \cos ^{2} 2 \varphi+T_{33}^{\prime} \sin ^{2} 2 \varphi+\operatorname{Im}\left\{T_{23}^{\prime}\right\} \sin 4 \varphi \\
\left(f_{S}|\beta|^{2}+f_{D}+f_{V}(b-c)\right) \sin 4 \varphi \mp f_{C} \cos 4 \varphi=\left(T_{22}^{\prime}-T_{33}^{\prime}\right) \sin 4 \varphi-2 \operatorname{Im}\left\{T_{23}^{\prime}\right\} \cos 4 \varphi \\
\left(f_{S}|\beta|^{2}+f_{D}\right) \sin ^{2} 2 \varphi+\frac{f_{C}}{2}(1 \mp \sin 4 \varphi)+f_{V}\left(b \sin ^{2} 2 \varphi+c \cos ^{2} 2 \varphi\right) \\
=T_{22}^{\prime} \sin ^{2} 2 \varphi+T_{33}^{\prime} \cos ^{2} 2 \varphi-\operatorname{Im}\left\{T_{23}^{\prime}\right\} \sin 4 \varphi
\end{array}\right.
$$

(A4)

Replace (A4-2) with (A5), (A4-3) with (A6), (A4-4) with (A11), (A4-5) with (A10), and (A4-6) with (A12), we obtain

$$
\begin{cases}f_{S}+f_{D}|\alpha|^{2}+f_{V} a=T_{11}^{\prime} & -1)  \tag{A13}\\ f_{S} \beta+f_{D} \alpha+f_{V} d=T_{12}^{\prime}+T_{13}^{\prime} & -2) \\ f_{S} \beta+f_{D} \alpha+f_{V} d=T_{12}^{\prime}-T_{13}^{\prime} & -3) \\ f_{S}|\beta|^{2}+f_{D}+f_{V} b+\frac{f_{C}}{2}=T_{22}^{\prime} & -4) \\ \pm j \frac{f_{C}}{2}=j \operatorname{Im}\left\{T_{23}^{\prime}\right\} & -5) \\ f_{V} c+\frac{f_{C}}{2}=T_{33}^{\prime} & -6)\end{cases}
$$

Obviously, (A13-2) and (A13-3) are redundant. They cannot provide independent information on decomposition and should be merged into one equation; then, (A13) just becomes (20). The general balance equation system (73) under $\operatorname{SU}(3)$ helix matrix (72) can be derived likewise.

## AcKnowledgment

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[^1]:    ${ }^{2} \mu$ is defined real in this article. Nonetheless, the related findings here can be likewise extended to $\mu$ with imagery or complex value.
    ${ }^{3}$ In view of the fact that $T_{11}^{\prime}-T_{22}^{\prime}=2 \operatorname{Re}\left\langle S_{\mathrm{HH}} S_{\mathrm{VV}}^{*}\right\rangle$, we can also use the sign of $T_{11}^{\prime}-T_{22}^{\prime}$ as a criterion here. Since the surface scattering and double-bounce scattering are identified after the volume scattering and helix scattering in fourcomponent scattering power decomposition, we should remove the volume and helix contributions from $T_{11}^{\prime}$ and $T_{22}^{\prime}$ before the ruling, and then $T_{11}^{\prime}$ and $T_{22}^{\prime}$ change to $S$ and $D$, respectively, as formulated in (25). Hence, the sign of $S-D$ can be used as an alternative to the sign of $\operatorname{Re}\left\langle S_{\mathrm{HH}} S_{\mathrm{VV}}^{*}\right\rangle$ here.

[^2]:    ${ }^{4}$ We should note that the residual analysis is only a mathematical evaluation. It cannot be a proof to say G4U is not better than Y4R and S4R. Singh et al. [31] compared among G4U, S4R, and Y4R by validating whether the decomposition can strengthen double-bounce scattering in urban area while strengthen the surface scattering in land and water area. This idea will be trailed in Section V for EG4U. Moreover, the quantitative evaluation of Y4O, Y4R, S4R, G4U, and GMD was also contributed by Xie et al. [40] and Ballester-Berman et al. [50], recently.

[^3]:    ${ }^{5}$ It should be noted that, however, scattering preference just provides a physical description of the reservation of GG4U, which cannot replace the modeled part to determine the scattering mechanism of targets.

[^4]:    ${ }^{6} \mathrm{SoMa}$ is a dense urban district in San Francisco. The streets there are about $40^{\circ}$ tilted. As a result, the normal of vertical wall of buildings will be no longer within the radar incidence plane and orientation is created. Such the misalignment aggravates the scattering complexity further, which is hard to be compensated by any unitary deorientation method [54].

[^5]:    ${ }^{7}$ Since the surface and double-bounce components are attained after volume and helix components, BC only determines the dominance between surface and double-bounce components not that among all the four components. Therefore, although volume scattering usually dominates the mountain and forest area, we can also have $\mathrm{BC}>0$ or even $\mathrm{BC} \leq 0$ there.

[^6]:    ${ }^{8}$ Unlike G4U which transforms both $\left\langle\left[T^{\prime}\right]\right\rangle$ and $\left\langle\left[T_{M}^{\prime}{ }^{\mathrm{S} 4 \mathrm{R} / \mathrm{Y} 4 \mathrm{R}}\right]\right\rangle$ in (8) and (19) for the decomposition of $\left\langle\left[T^{\prime}\right]\right\rangle$ in (22), AG4U is essentially a S4R dedicated to adaptively adopt S 4 R for the decomposition of $\left\langle\left[T^{\prime \prime}\right]\right\rangle$ by unitarily transforming $\left\langle\left[T^{\prime}\right]\right\rangle$ only. The only difference between S4R and AG4U is to carry out S4R to $\left\langle\left[T^{\prime}\right]\right\rangle$ or to $\left\langle\left[T^{\prime \prime}\right]\right\rangle .\left\langle\left[T^{\prime \prime}\right]\right\rangle$ is created in AG4U by adaptively transforming $\left\langle\left[T^{\prime}\right]\right\rangle$ based on matrix $\left[U_{3}(\varphi)\right]$ or $\left[U_{3}(\tau)\right]$ according to the degree of polarization $m$. Please refer to [56] for details.

