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***Abstract*—This article provides an online time-varying delay estimation method by using an external signal sent along with control inputs and output measurements. Since the external signal is isolated from the system, the linearity and delay-identifiability of the system are no longer required. Moreover, the sliding mode method guarantees the finite-time convergence of the delay estimation. By comparing with the standard sliding mode method, the super-twisting algorithm reduces the chattering and provides better performances. Furthermore, the super-twisting algorithm based delay estimator is implemented on a real remote data transmission system and its performances are illustrated by experimental results.**

***Index Terms*—Delay estimation, time-varying delay, super-twisting.**

I. Introduction

TIME-DELAY systems (TDS) have been widely studied for the last decades given that time-delays can be found in many systems (*i.e.* systems with communication lags or sensor measurements) and can destabilize them [1, p.vii]. Several control techniques are proposed to compensate TDS with known constant or time-varying delay (*e.g.* Smith predictor [2], prediction-based controller [3]–[5]). Therefore, time-delay estimation (TDE) is an effective way to achieve the stabilization of TDS with unknown time-delay. Moreover, large numbers of time-delays in real systems are time-varying or even discontinuous [6]. To stabilize these systems, TDE technique for constant time-delays is helpless. Consequently, TDE techniques for TDS with time-varying delays play an important role in the control of such systems.

A vast literature is available on TDE techniques. An overview of the existing methods is given in the sequel.

(i) Optimization-based approaches: these methods optimize the cost function based on the estimation error. The cost function is usually optimized with least squares method [7], [8]; mean-square method [9]; linear approximation [10] or gradient descent method [11]. The drawback of these approaches is that they cannot always accurately estimate the time-varying delays, especially fast-varying delays.

(ii) Convolution-based algebraic approaches [12]–[14]: these approaches use convolution methods to identify the unknown parameters and the constant delay of a linear TDS with high convergence speeds. The delay-identifiability of the system must be ensured. However, these approaches are still not applied to time-varying delay estimation problems.

(iii) Adaptive backstepping approach [3], [15]: the main conception of this approach is the use of partial differential equation (PDE) transformation. The TDS is stabilized and the constant delay can be estimated if the initial condition of the delay estimator is well chosen. This approach can also deal with a class of nonlinear systems. The main drawback is that the estimation algorithm is sensitive to the initial condition of the adaptation law.

(iv) Sliding mode method based approaches: sliding mode approach has been successfully used for control [16] and observation [17] of TDS, it is also an effective method to solve TDE problems. Sliding mode method is firstly introduced in [18] to estimate the constant or time-varying delays of linear system. Another sliding mode based delay estimator is proposed in [19] to estimate the constant state delay of nonlinear systems. However, only local convergence can be ensured by using these two methods.

(v) Some other approaches: in [20], an adaptive observer is proposed to estimate the constant or slow-varying delay (with slight fluctuating estimation error) of nonlinear systems. However, if the delay is fast-varying, the convergence no longer holds. The authors of [21] have proposed a neural network-based TDE technique, but the estimation error cannot accurately tend towards zero, and the algorithm has large computational load if the neural network is not well trained. A delay measurement approach is introduced in [22], messages are transmitted between a clock-driven sensor and a controller, and the time-varying delay is estimated as the difference between the sending and the receiving instants of the message. However, this approach has the following drawbacks:

- Message rejection: if two messages arrive at the same sampling period, then the first one must be discarded;
- This approach is sensitive to noise and perturbation in

Manuscript received Dec 20, 2018; revised May 13, 2019, and Sep 18, 2019; accepted Dec 03, 2019. The work of Yang Deng is financed by China Scholarship Council (CSC).

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the communication channel.

A comparison between the proposed method and this approach is made in Section IV-E.

In this paper, an external signal is sent along with the control input and the system output by using the same communication channel. It is supposed that the communication between the controller and the system is available. For instance, the components (actuators, sensors) of the Internet of Things technology [23] are connected via the wireless network to reach common goals. Therefore, it is possible to use an external signal instead of system input/output or state information to estimate the time-varying delays. The main contribution of this paper is the use of super-twisting (STW) algorithm [24], [25], it is a very simple approach with finite-time convergence and chattering limitation.

The paper is organized as follows. Notations, problem statements and discussions on the practical implementation are addressed in Section II. The main results of this paper are given in Section III. The performances of the proposed delay estimator are illustrated by hardware-in-the-loop tests in Section IV. Some conclusions and future work are drawn in Section V.

II. Preliminaries

Some mathematical notations are introduced in subsection II-A; then the problem statements are given in subsection II-B; the experimental set-up is introduced in subsection II-C.

A. Notations

In this paper, the following notations are used. A function with k-times continuous derivative is called a $\mathbf{C}^k$ function. The right-hand time derivative of a function f at instant $t-h(t)$ is denoted by

$$\dot{f}(t-h(t)) = \frac{df}{dt}\Big|_{t-h(t)} \tag{1}$$

where $h(t)$ is a time-varying delay. Besides, the time derivative of function $t \mapsto f(t-h(t))$ reads as $\frac{d}{dt}f(t-h(t))$. By using the chain rule given in [26, Theorem 5.5], the relation between $\frac{d}{dt}f(t-h(t))$ and $\dot{f}(t-h(t))$ is given as

$$\frac{d}{dt}f(t-h(t)) = \frac{df(v(t))}{dv(t)} \times \frac{dv(t)}{dt} = \dot{f}(t-h(t))(1-\dot{h}(t)) \tag{2}$$

with $v(t) = t-h(t)$. The sign-function [25, equations (1.13)-(1.14)] is defined by

$$\mathrm{sign}(x) = \begin{cases} 1, & \text{if} \quad x > 0 \\ -1, & \text{if} \quad x < 0 \end{cases} \tag{3}$$

with $\mathrm{sign}(0) \in [-1,1]$. The definition of $x_t(\theta)$ reads as $x_t(\theta) = x(t+\theta)$ with θ satisfies $-h \leq \theta \leq 0$, this notation is given in [27, p.38]. The readers may refer to [27, Chapter 2.1] and [25, Chapter 1] to obtain more detailed definitions and notations of TDS and sliding mode control respectively.

B. Problem statements

Time-varying delays can arise from communication lags which are frequently found in remote data transmission (RDT) processes. Consider an RDT process between two nodes, one node sends the initial signal $s(t)$ through a communication channel that is subject to a time-varying delay $h_i(t)$, then the delayed signal $s(t-h_i(t))$ is received at the other node. Next, the delayed signal $s(t-h_i(t))$ is sent back to the first node, due to the other transmission delay $h_o(t)$, one receives $s(t-h(t))$ at the first node with $h(t)$ defined as

$$h(t) = h_o(t) + h_i(t-h_o(t)). \tag{4}$$

The time-varying delays $h_i(t)$, $h_o(t)$ and $h(t)$ are respectively named as the input delay, the output delay and the round-trip delay.

Remote control system [28, Chapter 3.4] is one of the possible RDT processes mentioned above. For instance, one considers a LTI remote control system with input-output time-varying delays

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t-h_i(t)) \\ y(t) &= Cx(t-h_o(t)) \end{aligned} \tag{5}$$

where $h_i(t)$, $h_o(t)$ represent the transmission delays and A, B, C have appropriate dimensions. The estimation scheme of (5) is shown in Figure 1. The delay estimator sends the external signal (green arrows) to the plant's receiver through the same channel as the one used for the control signal (black arrows). After that, the delayed control input $u(t-h_i(t))$ is injected to the plant and the signal $s(t-h_i(t))$ is transferred to the plant's transmitter. Next, the transmitter sends the output $Cx(t)$ and the delayed signal $s(t-h_i(t))$ back to the controller side. Finally, the controller side receives $y(t)$ and $s(t-h(t))$ at the same time. Consequently, the time-delay of the received signal $s(t-h(t))$ equals to the round-trip delay introduced by the remote control system (4). Therefore, one can effectively estimate the round-trip delay with the inner loop of $s(t)$ (green loop in Figure 1) that is independent of the control system (black loop in Figure 1). Finally, the remote control system can be stabilized[1] by using the delay estimation $\hat{h}(t)$ and a predictor-based controller [29].

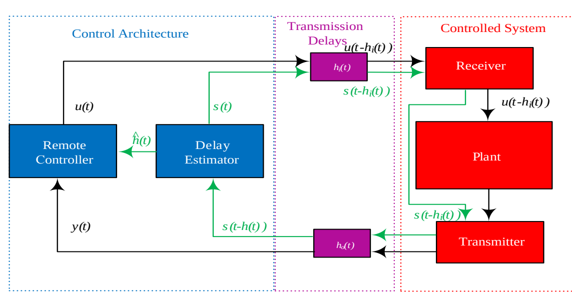


Fig. 1: Delay estimation scheme of remote control system by using external signal.

[1]The statements in [29, p.27] confirm that one can use the round-trip delay (4) to design a predictor-based controller in order to stabilize the system with input-output time-varying delays.

The main objective of this paper is the design of an online update law $\dot{\hat{h}}(t)$ that ensures the global finite-time convergence of $\hat{h}(t)$ to $h(t)$ by using $s(t)$ and $s(t-h(t))$.

C. Test bench description

The experimental set-up is composed of two different computers connected through a Wi-Fi network. Two computers run Robot Operating System (ROS) platform [30] simultaneously in order to actualize an RDT process with input and output time-varying delays (see Figure 2).

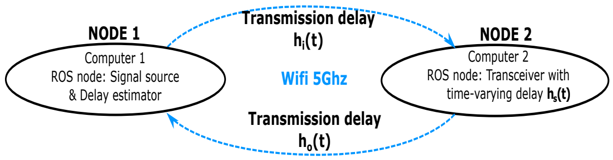


Fig. 2: Application scheme of time-varying delay estimation in an RDT.

In Figure 2, $h_i(t)$ and $h_o(t)$ are similar to the ones defined in subsection II-B. Computer 1 generates the signal $s(t)$, receives the delayed signal $s(t-h(t))$, and estimates the time-delay online. Computer 2 is used to receive the delayed signal $s(t-h_i(t))$ and to send it back to Computer 1. The synchronization between the two computers is not required since all the calculations are done on Computer 1. Moreover, an additional artificial time-varying delay $h_s(t)$ can be introduced by Computer 2 in order to create an arbitrarily long round-trip delays. Thus, the round-trip delay of the RDT process presented in Figure 2 is defined as

$$h(t) = h_o(t) + h_s(t-h_o(t)) + h_i(t-h_o(t)-h_s(t-h_o(t))). \quad (6)$$

The configurations of the test bench are introduced in the sequel. Computer 1 is a Dell Precision 5520 with an Intel i7-6820HQ processor whereas Computer 2 is a Dell Latitude E6410 with an Intel i7-M640 processor. The operating systems (OS) of the two computers are Ubuntu 16.04 with ROS kinetic (Computer 1) and Ubuntu 14.04 with ROS Jade (Computer 2). The two computers communicate with each other through a Wi-Fi hotspot (standard: IEEE 802.11ac) created by a router D-Link DIR-880L ($5GHz$) with communication frequency $f_s = 200Hz$. In other words, the communication between the two computers has a sampling period $T_s = 1/f_s = 0.005$s.

III. Super-twisting algorithm based delay estimator

In this section, a super-twisting algorithm based delay estimator is introduced to estimate the unknown time-varying delay. Some assumptions are given in subsection III-A. The main results of this paper are stated in subsection III-B.

A. Assumptions

Consider an external signal $s(t)$ and a time-varying round-trip delay $h(t)$ that is bounded by $[0, h_{max}]$ with known $h_{max} > 0$. Remind that h_{max} can be larger than the sampling period T_s. The following assumptions are fulfilled.

Assumption 1. *The external signal $s(t)$ satisfies that $s(t) \in \mathbf{C}^1$ and $\dot{s}(t) \neq 0$ for all $t \geq -h_{max}$.*

The statements of Assumption 1 guarantee that the signal $s(t)$ is strictly monotonic.

Assumption 2. *The first and second derivatives of the signal $s(t)$ and the time-varying delay $h(t)$ are bounded for all $t \geq -h_{max}$.*

Assumption 2 ensures that the super-twisting algorithm can be designed. The bounds mentioned in Assumption 2 read as:

$$|\dot{s}(t)| \leq \varepsilon, \quad |\ddot{s}(t)| \leq \varepsilon' \quad (7)$$

and

$$|\dot{h}(t)| \leq \delta, \quad |\ddot{h}(t)| \leq \delta' \quad (8)$$

for all $t \geq -h_{max}$. The boundedness (7) can be easily ensured since the signal $s(t)$ is generated by the user (*e.g.* if one sets $s(t) = kt$, then one has $\varepsilon = k$ and $\varepsilon' = 0$). In real applications, since the round-trip delay $h(t)$ is upper bounded by h_{max}, then one can use h_{max} and the sampling period T_s to find the upper bounds of δ and δ' with

$$\delta \leq \frac{h_{max}}{T_s}, \quad (9)$$

and

$$\delta' \leq \frac{\frac{h_{max}}{T_s} - (-\frac{h_{max}}{T_s})}{T_s} = \frac{2h_{max}}{T_s^2}. \quad (10)$$

Indeed, (9) is obtained by considering the worst case *i.e.* $h(t)$ changes from 0 to h_{max} in a sampling period, and one can also get (10) in the same way. Thus, the boundedness presented by (8) can be ensured, and then Assumption 2 is satisfied.

B. Main results

The main results of this paper are given in Theorem 1.

Theorem 1. *Consider that the external signal $s(t)$ satisfies Assumptions 1 and 2, and the time-varying delay $h(t)$ satisfies Assumption 2. Define the delay estimator dynamics*

$$\dot{\hat{h}}(t) = 1 - \frac{1}{\dot{s}(t-\hat{h}(t))} w(t) \quad (11)$$

with $w(t)$ defined as

$$w(t) = -\lambda |\sigma(t)|^{1/2} sign(\sigma(t)) + w_1(t) \quad (12)$$

where

$$\sigma(t) = s(t-\hat{h}(t)) - s(t-h(t)) \quad (13)$$

and the w_1-dynamics reads as

$$\dot{w}_1(t) = -\alpha \cdot sign(\sigma(t)). \quad (14)$$

If the parameters α and λ are well tuned[2]*, then the delay estimator (11) globally converges to $h(t)$ in a finite-time.*

Proof. The proof is divided into two steps. Step 1 shows that the finite-time convergence of $\sigma(t)$ to zero induces the finite-time convergence of $\hat{h}(t)$ to $h(t)$. Step 2 provides the finite-time convergence of the error term $\sigma(t)$ to zero.

[2]The choice of α and λ will be detailed in the sequel by (24) and (25).

Step 1. In this step, the relation between the convergences of $\sigma(t)$ and $\hat{h}(t)$ is analyzed. As stated after Assumption 1, the signal $s(t)$ is strictly monotonic for all $t \geq -h_{max}$. Therefore, $s(t)$ is bijective for all $t \geq -h_{max}$ by using the results given in [31, p.165, Corollary 4.9]. Assume that $\sigma(t)$ converges to zero in a finite-time T, it implies that

$$s(t-\hat{h}(t)) = s(t-h(t)), \quad \text{for all} \quad t \geq T. \tag{15}$$

Since $s(t)$ is bijective, (15) is equivalent to

$$t-\hat{h}(t) = t-h(t), \quad \text{for all} \quad t \geq T \tag{16}$$

which implies that

$$\hat{h}(t) = h(t), \quad \text{for all} \quad t \geq T. \tag{17}$$

Finally, one proves that the delay estimator (11)-(12)-(13)-(14) converges in finite-time if the error term $\sigma(t)$ converges in finite-time.

Step 2. Taking the time-derivative of the error term $\sigma(t)$, the error dynamics reads as

$$\dot{\sigma}(t) = \dot{s}(t-\hat{h}(t)) - \frac{d}{dt}s(t-h(t)) - \dot{s}(t-\hat{h}(t))\dot{\hat{h}}(t). \tag{18}$$

Indeed, as proven in Step 1, assertions $\sigma(t) = 0$ and $\hat{h}(t) = h(t)$ are equivalent since the signal $s(t)$ is bijective. Thus, the error term $\sigma(t)$ is a sliding variable with respect to the estimation error $\hat{h}(t) - h(t)$.

Assumption 1 implies that $\dot{s}(t-\hat{h}(t)) \neq 0$ for all $t \geq 0$. Then, there is no singularity in delay estimator (11). Substituting (11) into the error dynamics (18) leads to

$$\dot{\sigma}(t) = a(t) + w(t) \tag{19}$$

where $a(t) = -\frac{d}{dt}s(t-h(t))$. Moreover, (19) equals to the following representation

$$\ddot{\sigma}(t) = \dot{a}(t) + \dot{w}(t) \tag{20}$$

that satisfies the form [32, equation (3.30)]. Next, by using the transformation (2), it leads to

$$a(t) = -\dot{s}(t-h(t))(1-\dot{h}(t)). \tag{21}$$

Differentiating (21) yields that

$$\dot{a}(t) = \dot{s}(t-h(t))\ddot{h}(t) - \ddot{s}(t-h(t))(1-\dot{h}(t))^2. \tag{22}$$

The bound of $\dot{a}(t)$ is obtained by using (22), (7), and (8) such that

$$|\dot{a}(t)| \leq C \tag{23}$$

with $C = \varepsilon\delta' + \varepsilon'(1+\delta)^2$. Consider the second-order system (20) that is linearly dependent on the correction term $w(t)$ and consider the "control law" (12)-(14). If the gains α and λ are sufficiently large such that

$$\alpha \geq C \tag{24}$$

and

$$\lambda^2 \geq 4C\frac{\alpha + C}{\alpha - C} \tag{25}$$

then $\sigma(t)$ converges to zero in finite-time according to the simplified super-twisting algorithm and its convergence condition [32, Chapter 3.6.4, equation (3.42)]. The convergence conditions (24)-(25) can always be satisfied due to Assumption 2. This latter ensures the existence of the parameter C; then there always exist sufficiently large α and λ such that (24)-(25) are satisfied. Thus, the proof of Step 2 is finished.

Consequently, Theorem 1 is proven by combining the results stated in Step 1 and Step 2. □

The proposed method has the following advantages:

- it uses an external signal $s(t)$ (the inner loop of Figure 1) to estimate the round-trip delay: as a consequence, this technique has no restriction on the control system as linearity and delay identifiability;
- the existing sliding mode based delay estimators [18], [19] can only ensure the local convergence, but the convergence of the proposed method is global thanks to the use of an external signal;
- one uses the super-twisting algorithm to improve the performance rather than the standard sliding mode one; these two approaches are compared in Sections IV-A-IV-C;
- the hardware configuration of the proposed method is similar to [22], but the simulation results given in Section IV-E show that the proposed method appears to be more robust than [22].

Remark 1. *The convergence conditions* (24)-(25) *are derived from the bounds on* $\dot{h}(t)$ *and* $\ddot{h}(t)$. *If the bounds* δ, δ' *are smaller than the overestimations* h_{max}/T_s *and* $2h_{max}/T_s^2$ *given in* (9)-(10), *then the gains* α, λ *are also smaller than the over-estimated gains derived from these two overestimated bounds. However, conditions* (24)-(25) *are only sufficient conditions; in applications, smaller gains can also ensure the finite-time convergence* (17).

Remark 2. *Note that system* (19) *can also be stabilized in finite-time by using a standard sliding mode "controller"*

$$w(t) = -Ksign(\sigma(t)) \tag{26}$$

with $K > \varepsilon(1+\delta)$. *The control law described by* (26) *is discontinuous due to the use of sign-function* (3). *In practice, the imperfection in the sign-function implementation results in a high frequency oscillation named chattering [25, p.8]. If the sampling period is small, the proposed method* (11)-(14) *reduces the chattering since the term* $|\sigma|^{1/2}sign(\sigma)$ *is continuous and the discontinuous term* (14) *lies in the integral [25, p.35]. If the sampling period is large, the performances of the two approaches are degraded [33]. A comparison between the two sliding mode based delay estimator will be given in Section IV with hardware-in-the-loop tests. Moreover, the chattering effect is undesirable in applications [34, p.283]. Thus, one prefers to use the super-twisting algorithm rather than the standard sliding mode method.*

Remark 3. *Notice that the external signal has to be strictly monotonic because of Assumption 1. As a result of the monotonicity, the signal could tend towards infinity and exceed the calculation limit of the computer. A possible solution in practice is to add a standard projector of* $\hat{h}(t)$ *on* $[0, h_{max}]$ *and periodically reinitialize the signal with a period* $T > h_{max}$. *A* *similar discussion on the choice of the period T is given in [19, p.268] in the sense of the delay identifiability. When the difference between $t-\hat{h}(t)$ and kT is less than a sufficiently small threshold, one sets $\dot{\hat{h}}(t)$ to zero in order to avoid divergence. With the proposed techniques, the condition on the strict monotonicity is relaxed and the delay estimation error is always bounded in a small interval around zero.*

IV. Hardware-in-the-loop tests and simulations

In this section, five tests are given to illustrate the performances of delay estimator (11)-(12)-(13)-(14), the first four of them are hardware-in-the-loop tests (HIL tests) that are based on the WiFi communication (see Figure 2) between the two computers, and the last one is a simulation. The first three HIL tests illustrate that the performance of the proposed method deals with arbitrarily long and fast-varying delays. The fourth HIL test shows that the proposed method is able to estimate discrete-time stochastic delay. Finally, the last simulation confirms that the proposed method is robust with respect to the channel inherent noise.

A. Transmission delay estimation

The aim of this HIL test is to evaluate the performances of the proposed delay estimator on estimating transmission delays of an RDT process. The artificial time-delay $h_s(t)$ (see Figure 2) is set to zero, so the unknown round-trip delay is defined as (4). The performances of the standard sliding mode (SM) estimator (11)-(13)-(26) and the super-twisting (STW) algorithm based estimator (11)-(12)-(13)-(14) are compared in this HIL test. The external signal is set to $s(t)=t$. The standard sliding mode estimator is defined by $K=1.5$; the super-twisting algorithm based estimator is defined by $\alpha=10$ and $\lambda=5$. The initial conditions of two delay estimators are set to $\hat{h}(0)=0.5$s.

The results are presented in Figure 3. Remind that the reference time-delay $h(t)$ is obtained by a ping test[3]. Figures 3a and 3c show that the standard sliding mode estimator has estimation biases, and the chattering amplitude is much higher than the level of the transmission delay $h(t)$. This result implies that the standard sliding mode estimator (11)-(13)-(26) cannot estimate the transmission delay accurately. However, by using the proposed method (11)-(12)-(13)-(14), it shows in Figures 3b-3d that the transmission delay is estimated without estimation bias, and the chattering effect is reduced. Consequently, Figure 3 highlights the benefit of the super-twisting algorithm (12)-(13)-(14); it ensures the accuracy and chattering limitation of the delay estimation.

B. TDE with slow-varying artificial delay

After estimating the transmission delay, some HIL tests will be done to deal with long artificial time-delays. To produce an arbitrarily long time-delay, $h_s(t)$ is no longer zero. However, if $h_s(t)$ is not zero, the round-trip delay is difficult to be directly measured by using a ping test. To overcome this problem, one sets $h_s(t)$ much larger than $h_i(t)$ and $h_o(t)$, then the round-trip delay (6) can be approximated by

$$\tilde{h}(t)=h_i(t)+h_s(t)+h_o(t). \tag{27}$$

In sections IV-B-IV-D, one uses the approximation (27) to verify the performance of the delay estimators. In this subsection, the following slow-varying artificial time-delay is introduced:

$$h_s(t)=\begin{cases}4, & \text{for} \quad 0\le t<5\\ 2, & \text{for} \quad 5\le t<10\\ 4, & \text{for} \quad 10\le t<15\\ 10-0.4t, & \text{for} \quad 15\le t<20\\ 2+\sin(0.4\pi(t-17.5)), & \text{for} \quad t\ge 20\end{cases}. \tag{28}$$

Note that $h_s(t)\ge 1$ for all $t\ge 0$; it means that the artificial delay is much larger than the transmission delays. Then, it is possible to use approximation (27) to verify the accuracy of the TDE algorithms. The parameters are defined by $\alpha=10$, $\lambda=15$ and $K=2.3$. The initial conditions of two delay estimators are set to $\hat{h}(0)=0.5$s, and the external signal is set to $s(t)=t$.

Due to the fact that the transmission delays are much smaller than the time-varying delay $h_s(t)$, it is possible to use the boundedness of $h_s(t)$ to determine the parameters. Since the external signal is $s(t)=t$, then one has $\varepsilon=1$ and $\varepsilon'=0$. Following (28), the bounds given in (7)-(8) read as $\delta=1.2566$ and $\delta'=1.5791$. Inequalities (24)-(25) are satisfied by choosing the parameters $C=1.5792$, $\alpha=10$ and $\lambda=15$. Finally, the convergence conditions (24)-(25) of the super-twisting algorithm are satisfied. In addition, the standard sliding mode based delay estimator is also designed with the well-tuned gain $K>\varepsilon(1+\delta)$.

The results are presented in Figure 4. In Figure 4b, the round-trip delay $h(t)$ is estimated with delay estimator (11)-(12)-(13)-(14) whereas the results obtained by (11)-(13)-(26) are displayed by Figure 4a. As shown by Figure 4c, if one uses the standard sliding mode delay estimator (26), the convergence speed is slower and the chattering effect is more serious. This HIL test highlights the performances of the super-twisting algorithm in practice. It provides better results than the standard sliding mode algorithm.

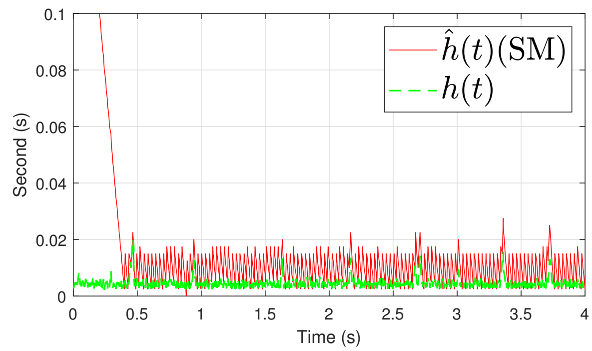
C. TDE with fast-varying delay

In many engineering systems, time-delays are no longer slow-varying (*i.e.* the derivative of the time-delay is larger than 1 [36, p.273]). The control of such systems is challenging because:

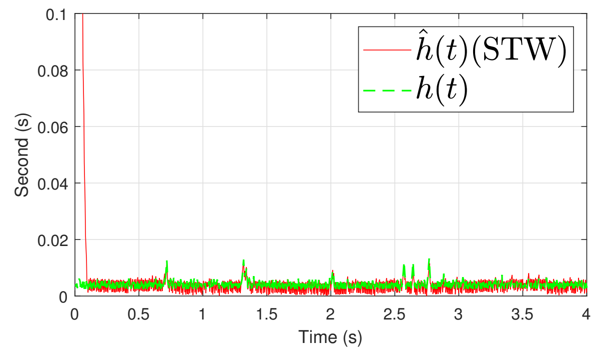
- for the continuous-time TDS, the Lyapunov-Krasovskii theorem [1, Theorem 3.1] is difficult to deal with this case [36, p.273];
- for the networked control systems, packet reordering (older packet arrives at the destination after the new one) may arise when the time-delay is fast-varying, and this makes the control strategies more complicated [37].

Moreover, as stated in [20, p.1765], estimating such delays is more challenging than slow-varying delays for many existing

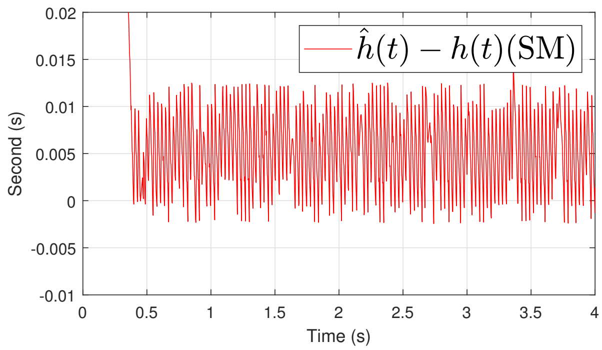
[3] Ping [35] is a computer network administration software that measures the round-trip time for messages sent from the originating host to a destination computer that is echoed back to the source.



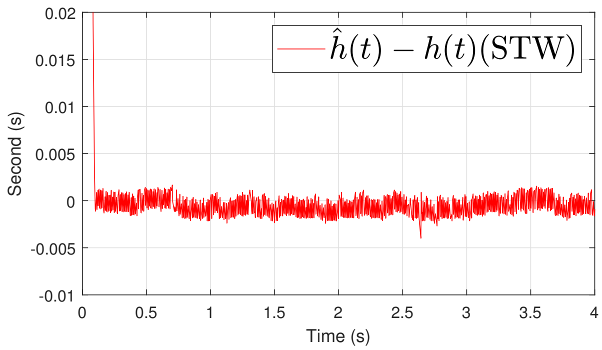
(a) Delay estimation $\hat{h}(t)$ with sliding mode method.



(b) Delay estimation $\hat{h}(t)$ with super-twisting method.



(c) Estimation error with sliding mode method.



(d) Estimation error with super-twisting method.

Fig. 3: Comparison between the two approaches (SM and STW) for transmission delay estimation.

approaches as [22] and [18, Theorems 4-6]. Thus, TDE for fast-varying delay is important to consider, and this technique is also helpful for the stabilization of TDS with unknown fast-varying delay.

In this subsection, a time-varying artificial time-delay

$$h_s(t) = \begin{cases} 3+2t, & \text{for} \quad 0 \le t < 1 \\ 7-2t, & \text{for} \quad 1 \le t < 3 \\ 2t-5, & \text{for} \quad 3 \le t < 5 \\ 2t-9, & \text{for} \quad 5 \le t < 7 \\ 2t-13, & \text{for} \quad 7 \le t < 9 \\ 4+\cos(\pi(t-9)), & \text{for} \quad t \ge 9 \end{cases} \tag{29}$$

is now introduced. Similarly, approximation (27) still holds in this subsection. The round-trip delay is going to be estimated by two delay estimators with parameters $\alpha = 20$, $\lambda = 15$, $K = 5$ and initial condition $\hat{h}(0) = 0.5$s. The same analysis as presented in subsection IV-B can be done in order to check that the convergence conditions (24)-(25) are verified.

The results are presented in Figure 5. Firstly, Figures 5a and 5b show that the super-twisting algorithm based delay estimator (11)-(12)-(13)-(14) converges faster than the standard sliding mode one (11)-(13)-(26). Secondly, Figure 5c illustrates that the super-twisting algorithm has less chattering than the other. In conclusion, although the time-delay is no longer slow-varying, the proposed method is still able to estimate it, and the proposed method still has faster convergence speed and better chattering limitation.

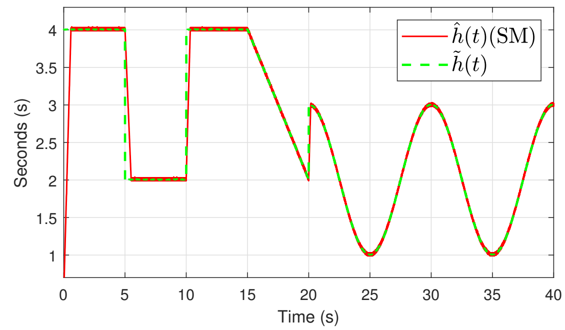
Remark 4. *As stated in [38, p.1907], if one increases the gain K in* (26)*, then it results in larger chattering that makes the delay estimation worse. However, if one decreases the gain K in order to reduce the chattering, the analysis in [25, equations (1.10), (1.17)] shows that the sliding variable converges to zero more slowly. This discussion shows that the standard sliding mode estimator cannot converge fastly and reduce the chattering at the same time. However, as shown in Figures 4 and 5, the super-twisting algorithm ensures a high convergence speed and a chattering limitation at the same time.*

D. Discrete-time random delay estimation

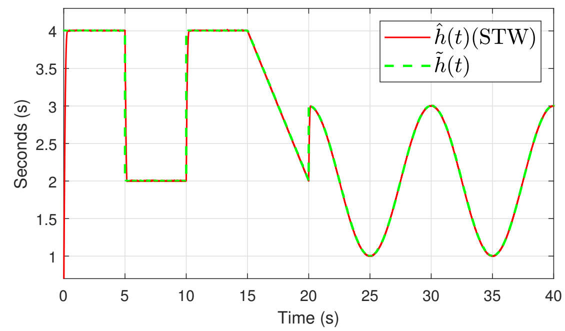
In practice, the time-varying delay is not always differentiable, that makes the TDE task difficult. For example, the arguments in [6, p.231] show that the communication routing of a networked control system can be changed to keep data queuing lines below some acceptable value, that causes delay jump phenomena. Therefore, the ability of estimating discontinuous time-varying delay is important to consider.

As the example given in [6, Figure 6(b)], the artificial time-varying delay $h_s(t)$ can be modeled as a discrete-time random process $D(n,T_d)$ that changes value for each T_d seconds. In this subsection, $D(n,T_d)$ is chosen as a uniform random variable on $[0.8, 1.2]$ with $T_d = 0.15$s. The parameters of the delay estimator are set to $\alpha = 10$ and $\lambda = 15$.

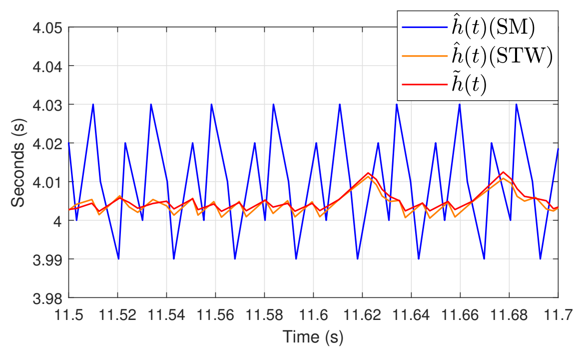
Figure 6 illustrates that the discrete time-varying delay $h(t) = D(n,T_d)$ is well estimated. Indeed, the proposed method can estimate piecewise-continuous time-delay, and this HIL test



(a) Slow-varying delay estimation with SM method.

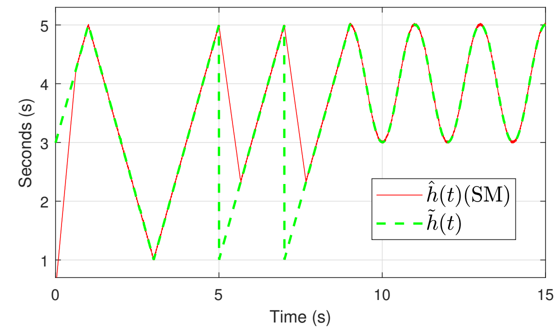


(b) Slow-varying delay estimation with STW method.

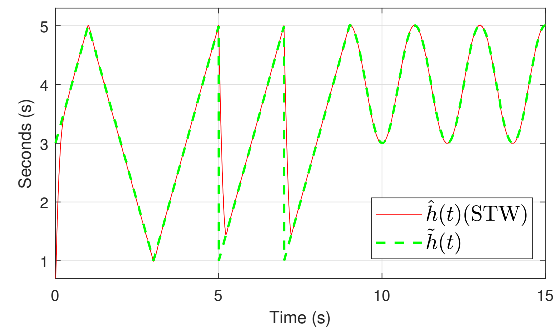


(c) Chattering analysis for slow-varying estimation on interval $t \in [11.5, 11.7]$.

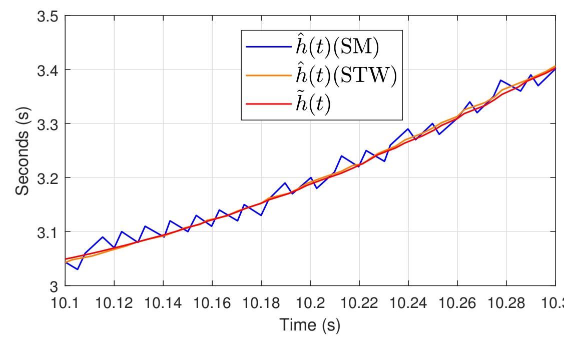
Fig. 4: Comparison between the two approaches (SM and STW) for slow-varying delay estimation.



(a) fast-varying delay estimation with SM method.



(b) fast-varying delay estimation with STW method.



(c) Chattering analysis for fast-varying delay estimation on interval $t \in [10.1, 10.3]$.

Fig. 5: Comparison between the two approaches (SM and STW) for fast-varying delay estimation.

highlights the practical use of the proposed delay estimation approach.

E. Comparison with the measurement approach in the presence of channel inherent noise

This subsection considers now the noise and perturbation in the communication channel. As stated in [39], there may exist noise (usually modeled as Gaussian noise [40, p.173]) and distortion in a communication channel or at the receiving terminal. Then, it is necessary to consider the robustness of the algorithms with respect to the noise and the perturbation. The motivation of this test is to compare the estimation performance of the three methods (the measurement approach [22], the standard sliding mode approach, and the proposed method).

Firstly, on assumes that the delayed signal $s(t-h(t))$ cannot be perfectly known due to the noise; only the following perturbed signal is available:

$$s_n(t-h(t)) = s(t-h(t)) + n(t) \tag{30}$$

with $n(t)$ a Gaussian noise. Thus, one can only use the noised signal $s_n(t-h(t))$ to calculate $\sigma(t)$ and $\hat{h}(t)$. Secondly, one uses the $\mathscr{L}_2$ norm of the estimation error $e(t) = h(t) - \hat{h}(t)$

$$\|e\|_{[t_1,t_2]} = \left(\int_{t_1}^{t_2} |e(s)|^2 \mathrm{d}s \right)^{1/2} \tag{31}$$

to evaluate the accuracy and the variation of the estimation algorithm on $[t_1, t_2]$. If $\|e\|$ is large, it means that the algorithm is not accurate or the estimation error oscillates a lot. Consider the round-trip delay

$$h(t) = 1 + 0.3\sin(2t) + 0.2\cos(t) \tag{32}$$

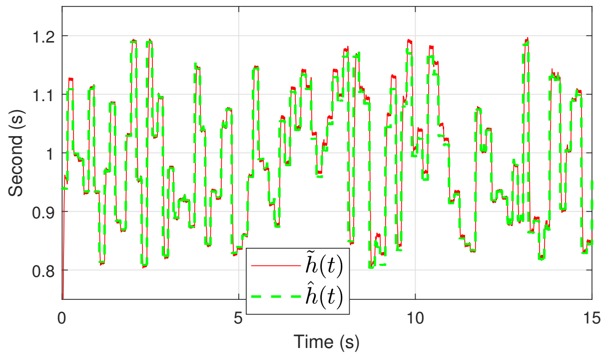
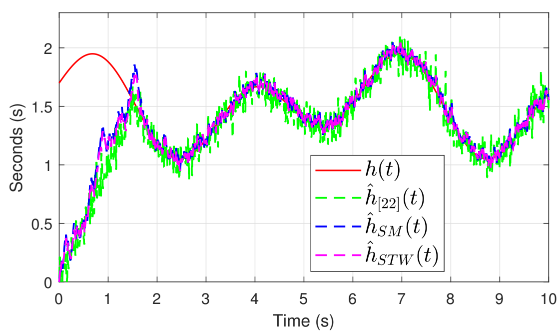


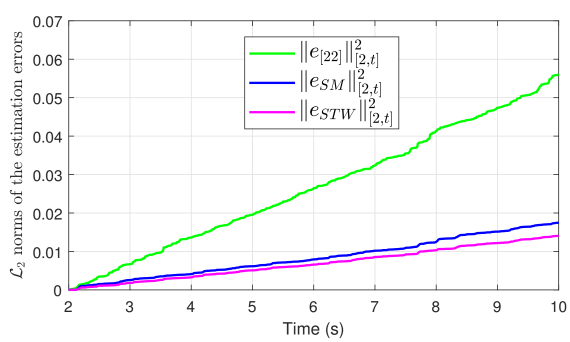
Fig. 6: Discrete-time random delay $h(t) = D(n, T_d)$ and its estimation $\hat{h}(t)$ by using the proposed method.

and the external signal $s(t) = t$ as in [22]. The parameters are designed as $K = 5$, $\alpha = 10$ and $\lambda = 15$. The channel inherent noise $n(t)$ is a Gaussian white noise with power $P = 7 \times 10^{-5}$ and switching time $T_{ns} = 0.01$s.

Figure 7b illustrates that the estimation error of the proposed



(a) Delay estimation via the three methods (delay measurement approach: $\hat{h}_{[22]}(t)$, standard sliding mode: $\hat{h}_{SM}(t)$ and super-twisting method: $\hat{h}_{STW}(t)$) in the presence of channel inherent noises.



(b) $\mathscr{L}_2$ norms of the three estimation errors in the presence of channel inherent noises.

Fig. 7: Comparison between the measurement approach [22], the SM method and STW method in the presence of channel inherent noise.

method has the minimum $\mathscr{L}_2$ norm among the three methods. It shows that the channel inherent noise has lower effects on the sliding mode based techniques, especially on the proposed method.

V. Conclusion

In this paper, an external signal is introduced to estimate the time-varying delays. The super-twisting algorithm is used to improve the performances. The delay estimator is implemented on an experimental set-up composed of two computers, and the performances are illustrated by HIL tests and simulations. Two improvements will be considered for future works:

- the adaptive-gain super-twisting algorithm [41] will be considered in the future. With this method, the gains α and λ are dynamically adapted, and they are no longer overestimated (mentioned in Remark 1) *i.e.* they will just be the smallest value that ensures the estimation accuracy.
- The stabilization of time-varying delay systems by using the proposed delay estimator and the predictor-based controller (for time-varying delays) [29], [42] will also be considered for future researches.

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