On Ergodic Secrecy Capacity of Multiple Input Wiretap Channel with Statistical CSIT

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Abstract—We consider the secure transmission in ergodic fast-Rayleigh fading multiple-input single-output single-antennaeavesdropper (MISOSE) wiretap channels. We assume that the statistics of both the legitimate and eavesdropper channels is the only available channel state information at the transmitter (CSIT). By introducing a new secrecy capacity upper bound, we prove that the secrecy capacity is achieved by Gaussian input without prefixing. To attain this, we form another MISOSE channel for upper-bounding, and tighten the bound by finding the worst correlations between the legitimate and eavesdropper channel coefficients. The resulting upper bound is tighter than the others in the literature which are based on modifying the correlation between the noises at the legitimate receiver and eavesdropper. Next, we fully characterize the ergodic secrecy capacity by showing that the optimal channel input covariance matrix is a scaled identity matrix, with the transmit power allocated uniformly among the antennas. The key to solve such a complicated stochastic optimization problem is by exploiting the completely monotone property of the ergodic secrecy capacity to use the stochastic ordering theory. Finally, our simulation results show that for the considered channel setting, the secrecy capacity is bounded in both the high signal-to-noise ratio and large number of transmit antenna regimes.

I. INTRODUCTION

The secrecy capacity of a wiretap channel is the maximum achievable secrecy rate between the transmitter and a legitimate receiver, and a perfect secrecy constraint is imposed to make no information be attainable by an eavesdropper [1] [2]. In the wireless environments, the time-varying characteristic of fading channels can also be exploited to enhance the secrecy capacity [3]. Further enhancements are attainable by employing multiple antennas at each node, e.g., in [4] [5]. However, these secrecy capacity results [3]–[5] rely on perfect knowledge of the legitimate receiver's channel state information at the transmitter (CSIT). Because of the limited feedback bandwidth and the delay caused by channel estimation, it may be hard to track the channel coefficients if they vary rapidly. Thus for fast-fading channels, it is more practical to consider the case with only partial CSIT of the legitimate channel. However, in this case, only some lower and upper bounds of the secrecy capacity are known [6] [7], and the secrecy capacity is unknown. Although the general secrecy capacity formula is reported in [1], the optimal auxiliary random variable for prefixing in this formula is still unknown.

In this letter, we consider one important scenario of partial CSIT, i.e., the transmitter only knows the statistics of both the

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legitimate and eavesdropper channels but not the realizations of them. Under this scenario, we derive the secrecy capacity of the fast-fading, multiple-input single-output single-antennaeavesdropper (MISOSE) wiretap channels, where the transmitter has multiple antennas while the legitimate receiver and eavesdropper each have single antenna. Both the coefficients of the legitimate and eavesdropper channels are Rayleigh faded. We first propose a new secrecy capacity upper bound to show that the transmission scheme in [6] is secrecy-capacity achieving, which is based on [1] with Gaussian input but without prefixing. Then we find the optimal channel input covariance matrix analytically to fully characterize the ergodic secrecy capacity, while such a optimization problem is solved numerically in [6] without guaranteeing the optimality. The key is to exploit the completely monotone property of the ergodic secrecy capacity, then invoking the stochastic ordering theory [8].

To obtain a tighter secrecy capacity upper bound than that reported in [7], we introduce another MISOSE channel with a relaxed secrecy constraint for upper-bounding, while finding the worst correlations between the coefficients of the legitimate and eavesdropper channels to tighten the bound. In [7], the upper bound is obtained by directly applying the concepts from [3] [4] where the correlation is only introduced between the noises at the legitimate receiver and eavesdropper and the secrecy constraint is left unchanged. Note that the secrecy capacity lower bound in [7] is indeed not achievable. In order to achieve such a bound, the variable-rate coding in [3] must be invoked, where the full CSIT of the legitimate channel must be used to vary the transmission rate in every channel fading state. This can not be done with only statistical CSIT of the legitimate channel as in our setting. In addition to the CSIT assumptions, the secrecy capacity result of [3] is builded on the ergodic slow fading channel assumption where coding among lots of slow fading channel blocks (each block with lots of coded symbols) is used. This assumption may be unrealistic owing to the long latency. For fast fading channels with full CSIT of legitimate channel and statistical CSIT of the eavesdropper channel, only some achievability results are known [9]. In contrast to our results, in [9], the prefixing in [1] may be useful to increase the secrecy rate. More detailed comparisons between our results and those in [7] [6] [3] can be found in Remarks 1 and 2.

II. SYSTEM MODEL

In the considered MISOSE wiretap channel, we study the problem of reliably communicating a secret message w from the transmitter to the legitimate receiver subject to a constraint on the information attainable by the eavesdropper (in

upcoming (4)). The received signals y and z at the legitimate receiver and eavesdropper (each with single antenna) from the transmitter equipped with multiple-antenna, can be represented respectively as *

$$y = \mathbf{h}^H \mathbf{x} + n_{\mathbf{v}},\tag{1}$$

$$z = \mathbf{g}^H \mathbf{x} + n_z, \tag{2}$$

where \mathbf{x} is a $N_t \times 1$ complex vector representing the transmitted vector signal, N_t is the number of transmit antennas, while n_y and n_z are independent and identically distributed (i.i.d.) circularly symmetric additive white Gaussian noise with zero mean and unit variance at the legitimate receiver and eavesdropper, respectively. In (1) and (2), \mathbf{h} and \mathbf{g} are both $N_t \times 1$ complex vectors, and representing the channels from the transmitter to the legitimate receiver and eavesdropper, respectively.

In this work, the channels are assumed to be fast Rayleigh fading. That is, $\mathbf{h} \sim CN(0, \sigma_{\mathbf{h}}^2 \mathbf{I})$ and $\mathbf{g} \sim CN(0, \sigma_{\mathbf{g}}^2 \mathbf{I})$, respectively, while \mathbf{h} , \mathbf{g} , n_y and n_z are independent. The channel coefficients change in every symbol time. We assume that the legitimate receiver knows the instantaneous channel state information of \mathbf{h} perfectly, while the eavesdropper knows those of \mathbf{h} and \mathbf{g} perfectly. As for the CSIT, only the distributions of \mathbf{h} and \mathbf{g} are known at the transmitter, while the realizations of \mathbf{h} and \mathbf{g} are unknown. Thus the transmitter is subjected to a power constraint as

$$\operatorname{Tr}(\Sigma_{\mathbf{x}}) \le P,$$
 (3)

where $\Sigma_{\mathbf{x}}$ is the covariance matrix of \mathbf{x} in (1) and (2).

The perfect secrecy and secrecy capacity are defined as follows. Consider a $(2^{NR}, N)$ -code with an encoder that maps the message $w \in \mathcal{W}_N = \{1, 2, \dots, 2^{NR}\}$ into a length-N codeword; and a decoder at the legitimate receiver that maps the received sequence y^N (the collections of y over the code length N) from the legitimate channel (1) to an estimated message $\hat{w} \in \mathcal{W}_N$. We then have the following definitions.

Definition 1 (Secrecy Capacity [3]) Perfect secrecy is achievable with rate R if, for any $\varepsilon' > 0$, there exists a sequence of $(2^{NR}, N)$ -codes and an integer N_0 such that for any $N > N_0$

$$H(w|z^N, \mathbf{h}^N, \mathbf{g}^N)/N > R - \varepsilon', \text{ and } \Pr(\hat{w} \neq w) < \varepsilon',$$
 (4)

where w is the secret message, z^N , \mathbf{h}^N , and \mathbf{g}^N are the collections of z, \mathbf{h} , and \mathbf{g} over code length N, respectively. The secrecy capacity C_s is the supremum of all achievable secrecy rates.

III. SECRECY CAPACITY OF THE MISOSE FAST RAYLEIGH FADING WIRETAP CHANNEL

In this section, we fully characterize the secrecy capacity of the MISOSE fast Rayleigh fading channel in the upcoming Theorem 2. Before that, we present the following Theorem 1 which shows that the scheme in [6], which uses Gaussian \mathbf{x} without prefixing in [1], is capacity achieving. By introducing new bounding techniques, we obtain tighter secrecy capacity upper bound than that in [7] to attain the secrecy capacity. For such a upper bound, we form a better degraded MISOSE channel of (1)(2) with a less stringent perfect secrecy constraint than (4) (in the upcoming (6)), and tighten the upper bound by carefully introducing correlations to the channels \mathbf{h} and \mathbf{g} (in the upcoming (8)).

Theorem 1 For the MISOSE fast Rayleigh fading wiretap channel (1)(2) with the statistical CSIT of **h** and **g**, using Gaussain **x** without prefixing is the optimal transmission strategy. And the non-zero secrecy capacity C_s is obtained only when $\sigma_h > \sigma_g$, which is

$$C_s = \max_{\Sigma_{\mathbf{x}}} (\mathbb{E}_{\mathbf{h}} [\log (1 + \mathbf{h}^H \Sigma_{\mathbf{x}} \mathbf{h})] - \mathbb{E}_{\mathbf{g}} [\log (1 + \mathbf{g}^H \Sigma_{\mathbf{x}} \mathbf{g})]), \quad (5)$$

where Σ_x is the covariance matrix of the Gaussian channel input x and subject to (3), while $h \sim \text{CN}(0, \sigma_h^2 I)$ and $g \sim \text{CN}(0, \sigma_g^2 I)$.

Proof: From [6], we know that the right-hand-side (RHS) of (5) is achievable and serves as a secrecy capacity lower-bound. Now we present our new secrecy capacity upper bound which matches the RHS of (5). The key for establishing such an upper-bound is first forming a better MISOSE channel than (1)(2) in terms of higher secrecy capacity, and applying the results in [1]. First, we consider a better scenario where the eavesdropper does not know the realizations of **h**, and (4) becomes

$$H(w|z^N, \mathbf{g}^N)/N > R - \varepsilon', \text{ and } \Pr(\hat{w} \neq w) < \varepsilon'.$$
 (6)

As in [10], equivalently, we can respectively treat the output of the legitimate channel as (y, \mathbf{h}) while that of the eavesdropper channel as (\mathbf{z}, \mathbf{g}) . From [11], the secrecy capacity under constraint (6) is only related to the marginal distributions $p(y, \mathbf{h} | \mathbf{x})$ and $p(z, \mathbf{g} | \mathbf{x})$. Then for any joint conditional PDF $p_{y', \mathbf{h}', z, \mathbf{g} | \mathbf{x}}$ such that $p(y', \mathbf{h}' | \mathbf{x}) = p(y, \mathbf{h} | \mathbf{x})$, the secrecy capacity under constraint (6) is the same. Now we introduce our same marginal legitimate channel $p(y', \mathbf{h}' | \mathbf{x})$ for (1), which is formed by replacing \mathbf{h} in \mathbf{y} with $\mathbf{h}' = (\sigma_{\mathbf{h}}/\sigma_{\mathbf{g}})\mathbf{g}$ as

$$y' = (\mathbf{h}')^H \mathbf{x} + n_y = ((\sigma_{\mathbf{h}}/\sigma_{\mathbf{g}})\mathbf{g})^H \mathbf{x} + n_y.$$
 (7)

Since \mathbf{x} is independent of \mathbf{h} and \mathbf{g} due to our CSIT assumption, both \mathbf{h}' and \mathbf{h} have the same conditional distributions condition on \mathbf{x} (which equal to $CN(0, \sigma_{\mathbf{h}}^2\mathbf{I})$). Then we know that $p(y', \mathbf{h}'|\mathbf{x}) = p(y, \mathbf{h}|\mathbf{x})$. From (7), we will focus on the MISOSE channel in the following,

$$y'' = \mathbf{g}^H \mathbf{x} + (\sigma_{\mathbf{g}}/\sigma_{\mathbf{h}})n_{y}, \quad z = \mathbf{g}^H \mathbf{x} + n_{z}.$$
 (8)

We now use the secrecy capacity of the MISOSE channel (8), under constraint (6), to upper-bound that of the original channel (1)(2). Again, we can treat as (y'', \mathbf{g}) and (z, \mathbf{g}) as the output of the legitimate channel and that of the eavesdropper channel in (8), respectively. As in [9], we apply this fact into

^{*} In this letter, $\|\mathbf{a}\|$ is the vector norm of vector \mathbf{a} . The trace and complex conjugate transpose of matrix \mathbf{A} is denoted by $\mathrm{Tr}(\mathbf{A})$ and \mathbf{A}^{H} , respectively. The diagonal matrix is denoted by diag(.). The zero-mean complex Gaussian random vector with covariance matrix Σ is denoted as $CN(0,\Sigma)$. For random variables (vectors) A and B, p(A) is the probability distribution function (PDF) of A, I(A;B) denotes the mutual information between them while H(A|B) denotes the conditional differential entropy. We use $A \to B \to C$ to represent that A,B, and C form a Markov chain. All the logarithm operations are of base 2 such that the unit of rates is in bit.

the secrecy capacity formula of [1],

$$C_s \le \max_{P(U,\mathbf{x})} I(U; y'', \mathbf{g}) - I(U; z, \mathbf{g})$$
(9)

$$= \max_{p_{(U,\mathbf{x})}} I(U;y''|\mathbf{g}) - I(U;z|\mathbf{g}), \tag{10}$$

$$\leq \max_{p_{\mathbf{x}}} I(\mathbf{x}; y''|z, \mathbf{g}). \tag{11}$$

where U in (9) is an auxiliary random variable for prefixing, which forms the Markov chain $U \to \mathbf{x} \to (y'', z, \mathbf{g})$; the equality (10) follows from [10] by the independence of U and \mathbf{g} due to our CSIT assumptions; and the inequality (11) is from [5]. When $\sigma_{\mathbf{g}} < \sigma_{\mathbf{h}}$, the equivalent channel (8) is degraded, and $\mathbf{x} \to y'' \to z$ given \mathbf{g} . From [12], apply the Markov chain property to (11)

$$C_s \le \max_{p_{\mathbf{x}}} I(\mathbf{x}; \mathbf{y}''|\mathbf{g}) - I(\mathbf{x}; z|\mathbf{g}). \tag{12}$$

From [4], we know that Gaussian \mathbf{x} is optimal for the upper bound in (12), and the upper-bound in (12) matches the RHS of (5) when $\sigma_{\mathbf{g}} < \sigma_{\mathbf{h}}$. Note that when $\sigma_{\mathbf{g}} < \sigma_{\mathbf{h}}$, the RHS of (5) is positive. In contrast, when $\sigma_{\mathbf{g}} \geq \sigma_{\mathbf{h}}$, the upper bound in (11) is zero since from (8), $\mathbf{x} \rightarrow z \rightarrow y''$ given \mathbf{g} . And it concludes the proof.

Remark 1: When the transmitter additionally knows the realizations of \mathbf{h} , e. g. [3], the legitimate channel (7) is not a same marginal channel of (1). In this case, given \mathbf{x} , \mathbf{h} may not be Gaussian but $\mathbf{h}' = (\sigma_{\mathbf{h}}/\sigma_{\mathbf{g}})\mathbf{g}$ is Gaussian, and $p(y',\mathbf{h}'|\mathbf{x})$ from (7) may not equal to $p(y,\mathbf{h}|\mathbf{x})$ from (1), In our case, the CSIT assumption makes \mathbf{x} independent of \mathbf{h} and \mathbf{g} , then the legitimate channel in (7) has the same marginal as that in (1). Then we can get rid of the unrealistic ergodic slow fading assumptions in [3] and find the secrecy capacity in fast fading channel. Note that the upper-bound in [7] is obtained by directly applying the derivations in [3] to fast fading channel, and is looser than the upper-bound (11) which is based on the channel (8) and is tightened by the "worst" correlation between \mathbf{h} and \mathbf{g} ($\mathbf{h} = \mathbf{g}$).

Now we show that the optimal $\Sigma_{\mathbf{x}}$ of (5) is diag $\{P/N_T, \dots, P/N_T\}$, and fully characterize the secrecy capacity as follows.

Theorem 2 For the MISOSE fast Rayleigh fading wiretap channel (1)(2) with the statistical CSIT of **h** and **g**, under power constraint P, the non-zero secrecy capacity C_s is obtained only when $\sigma_h > \sigma_g$, which is

$$C_s = \mathbb{E}_{\mathbf{h}} \left[\log \left(1 + P \frac{||\mathbf{h}||^2}{N_T} \right) \right] - \mathbb{E}_{\mathbf{g}} \left[\log \left(1 + P \frac{||\mathbf{g}||^2}{N_T} \right) \right], \quad (13)$$

where $\mathbf{h} \sim CN(0, \sigma_{\mathbf{h}}^2 \mathbf{I})$, $\mathbf{g} \sim CN(0, \sigma_{\mathbf{g}}^2 \mathbf{I})$, and N_T is the number of transmit antennas.

Proof: Subjecting to (3), after substituting $\mathbf{h} \sim CN(0, \sigma_{\mathbf{h}}^2 \mathbf{I})$ and $\mathbf{g} \sim CN(0, \sigma_{\mathbf{g}}^2 \mathbf{I})$ into the optimization problem in (5), it becomes

$$\max_{\Sigma_{\mathbf{x}}} \left(\mathbb{E}_{\mathbf{g}} \left[\log \frac{\sigma_{\mathbf{g}}^{2} / \sigma_{\mathbf{h}}^{2} + \mathbf{g}^{H} \Sigma_{\mathbf{x}} \mathbf{g}}{\sigma_{\mathbf{g}}^{2} / \sigma_{\mathbf{h}}^{2}} \right] - \mathbb{E}_{\mathbf{g}} \left[\log (1 + \mathbf{g}^{H} \Sigma_{\mathbf{x}} \mathbf{g}) \right] \right). \tag{14}$$

By using the eigenvalue decomposition $\Sigma_{\mathbf{x}} = \mathbf{U}\mathbf{D}\mathbf{U}^H$, where \mathbf{U} is unitary and \mathbf{D} is diagonal, finding the optimal $\Sigma_{\mathbf{x}}^*$ of (14) is equivalent to solving

$$\max_{\mathbf{U},\mathbf{D}} (\mathbb{E}_{\mathbf{g}}[\log (\sigma_{\mathbf{g}}^2/\sigma_{\mathbf{h}}^2 + \mathbf{g}^H \mathbf{U} \mathbf{D} \mathbf{U}^H \mathbf{g})] - \mathbb{E}_{\mathbf{g}}[\log (1 + \mathbf{g}^H \mathbf{U} \mathbf{D} \mathbf{U}^H \mathbf{g})]),$$

$$= \max_{\mathbf{D}} \left(\mathbb{E}_{\mathbf{g}} \left[\log \left(\sigma_{\mathbf{g}}^{2} / \sigma_{\mathbf{h}}^{2} + \mathbf{g}^{H} \mathbf{D} \mathbf{g} \right) \right] - \mathbb{E}_{\mathbf{g}} \left[\log (1 + \mathbf{g}^{H} \mathbf{D} \mathbf{g}) \right] \right), \tag{15}$$

where the equality comes from the fact that the distribution of $\mathbf{g} \sim CN(0, \sigma_{\mathbf{g}}^2 \mathbf{I})$ is unchanged by the rotation of unitary \mathbf{U} , and we can set $\Sigma_{\mathbf{x}} = \mathbf{D}$ ($\mathbf{U} = \mathbf{I}$) without loss of optimality.

In the following, we show that subjecting to $Tr(\mathbf{D}) \leq P$, the optimal **D** for (15) is

$$\mathbf{D}^* = \operatorname{diag}\{P/N_T, P/N_T, \cdots, P/N_T\}. \tag{16}$$

First of all, from [6, Section V], the optimal **D** for (15) satisfies $\text{Tr}(\mathbf{D}) = P$. For any $\mathbf{D} = [d_1, d_2, \cdots, d_{N_T}]$ where $\sum d_i = P$ and $d_i \geq 0, \forall i$, we want to prove that for \mathbf{D}^* defined in (16)

$$\mathbb{E}_{\mathbf{g}} \left[\log \left(a + \mathbf{g}^{H} \mathbf{D} \mathbf{g} \right) \right] - \mathbb{E}_{\mathbf{g}} \left[\log \left(1 + \mathbf{g}^{H} \mathbf{D} \mathbf{g} \right) \right]$$

$$\leq \mathbb{E}_{\mathbf{g}} \left[\log \left(a + \mathbf{g}^{H} \mathbf{D}^{*} \mathbf{g} \right) \right] - \mathbb{E}_{\mathbf{g}} \left[\log \left(1 + \mathbf{g}^{H} \mathbf{D}^{*} \mathbf{g} \right) \right],$$
 (17)

where we denote $\sigma_{\mathbf{g}}^2/\sigma_{\mathbf{h}}^2$ by a, which belongs to [0,1). Here we introduce some results from the stochastic ordering theory [8] to proceed.

Definition 2 [8, p.234] A function $\psi(x) : [0, \infty) \to \mathbb{R}$ is completely monotone if for all x > 0 and $n = 0, 1, 2, \cdots$, its derivative $\psi^{(n)}$ exists and $(-1)^n \psi^{(n)}(x) \ge 0$.

Definition 3 [8, (5.A.1)] Let B_1 and B_2 be two nonnegative random variables such that $\mathbb{E}[e^{-sB_1}] \ge \mathbb{E}[e^{-sB_2}]$, for all s > 0. Then B_1 is said to be smaller than B_2 in the Laplace transform order, denoted as $B_1 \le_{LT} B_2$.

Lemma 1 [8, Th. 5.A.4] Let B_1 and B_2 be two nonnegative random variables. If $B_1 \leq_{LT} B_2$ then $\mathbb{E}[f(B_1)] \leq \mathbb{E}[f(B_2)]$, where the first derivative of a differentiable function f on $[0,\infty)$ is completely monotone, provided that the expectations exist.

To prove (17), we let $B_1 = \mathbf{g}^H \mathbf{D} \mathbf{g}$, $B_2 = \mathbf{g}^H \mathbf{D}^* \mathbf{g}$, and $f(x) = \log(a+x) - \log(1+x)$ to invoke Lemma 1. It can be easily verified that $\psi(x)$, the first derivative of f(x), satisfies Definition 2. More specifically, the *n*th derivative of ψ meets

$$\Psi^{(n)}(x) = \begin{cases}
\frac{n!}{(a+x)^{n+1}} - \frac{n!}{(1+x)^{n+1}} > 0, & \text{if } n \text{ is even,} \\
-\frac{n!}{(a+x)^{n+1}} + \frac{n!}{(1+x)^{n+1}} < 0, & \text{if } n \text{ is odd,}
\end{cases}$$
(18)

when x > 0, since $a \in [0,1)$. Now from Lemma 1 and Definition 3, we know that to prove (17) is equivalent to proving $\mathbb{E}[e^{-sB_1}] \geq \mathbb{E}[e^{-sB_2}]$ or $\log(\mathbb{E}[e^{-sB_1}]/\mathbb{E}[e^{-sB_2}]) \geq 0, \forall s > 0$. From [13, p.40], we know that

$$\log\left(\frac{\mathbb{E}[e^{-sB_1}]}{\mathbb{E}[e^{-sB_2}]}\right) = \sum_{k=1}^{N_T} \log(1 + 2d_k^* s) - \sum_{k=1}^{N_T} \log(1 + 2d_k s). \quad (19)$$

To show that the above is nonnegative, we resort to the majorization theory. Note that $\sum_{k=1}^{N_T} \log(1+2\check{d}_k s)$ is a Schurconcave function [14] in $(\check{d}_1,\ldots,\check{d}_{N_T})$, $\forall s>0$, and by the definition of majorization [14]

$$(d_1^*, \dots, d_{N_T}^*) = (P/N_T, P/N_T, \dots, P/N_T) \prec (d_1, d_2, \dots d_{N_T}),$$

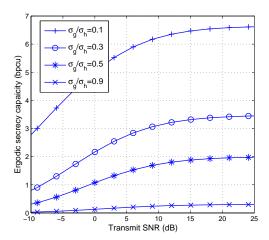


Fig. 1. Comparison of the ergodic secrecy capacities under different eavesdropper-to-legitimate-channel qualities σ_g/σ_h .

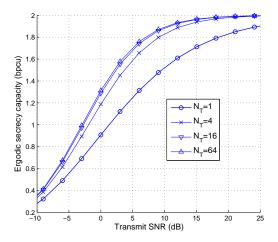


Fig. 2. Comparison of the ergodic secrecy capacities under different numbers of transmit antennas N_T .

where $\mathbf{b} \prec \mathbf{a}$ means that \mathbf{b} is majorized by \mathbf{a} . Thus from [14], we know that the RHS of (19) is nonnegative, $\forall s > 0$. Then (17) is valid, and \mathbf{D}^* is the optimal \mathbf{D} for (15). Note that \mathbf{D}^* is also the optimal $\Sigma_{\mathbf{x}}$ of (5) according to the discussion under (15). Substituting \mathbf{D}^* in (16) as the optimal $\Sigma_{\mathbf{x}}$ into the target function of (5), we have (13).

Remark 2: In [6, Sec. VII], the optimal $\Sigma_{\mathbf{x}}$ for (13) is found by an iterative algorithm without guaranteeing the optimality. The contribution of Theorem 2 is analytically finding the optimal $\Sigma_{\mathbf{x}}$, which equals to \mathbf{D}^* in (16), by exploiting the completely monotone property of the ergodic secrecy capacity and invoking Lemma 1. Finally, as discussed in Section I, the secrecy rate lower-bound in [7] is not achievable, thus the conclusion in [7] that uniform power allocation among transmit antennas as (16) is *not* secrecy capacity achieving is *wrong*.

IV. SIMULATION RESULTS

In this section we compare the secrecy capacities under different channel conditions. The transmit signal-to-noise ratio (SNR) is defined as P in dB scale since n_y and n_y both have

unit variances. In Fig. 1 we compare the secrecy capacities with $N_T=2$ under different $\sigma_{\bf g}/\sigma_{\bf h}$. The secrecy capacity increases with decreasing $\sigma_{\bf g}/\sigma_{\bf h}$. The capacity converges to $2\log(\sigma_{\bf h}/\sigma_{\bf g})$ when the SNR is high, which meets (13) with large P. In Fig. 2, we compare the secrecy capacities with different numbers of transmit antennas N_T . We can also find that the capacity converges when N_T is large enough. This results can be easily seen by letting $N_T \to \infty$ in (13), and applying the central limit theorem on $||{\bf h}||^2/N_T$ and $||{\bf g}||^2/N_T$, respectively.

V. CONCLUSION

In this paper, we derived the secrecy capacity of the MISOSE ergodic fast Rayliegh fading wiretap channel, where only the statistical CSIT of the legitimate and eavesdropper channels is known. By introducing a new secrecy capacity upper bound, we first showed that Gaussian input without prefixing is secrecy capacity achieving. Then we analytically found the optimal channel input covariance matrix, and fully characterized the secrecy capacity.

REFERENCES

- I. Csiszár and J. Körner, "Broadcast channels with confidential messages," *IEEE Trans. Inform. Theory*, vol. 24, no. 3, pp. 339–348, May 1978.
- [2] A. D. Wyner, "The wiretap channel," Bell Syst. Tech. J., vol. 54, pp. 1355–1387, 1975.
- [3] P. Gopala, L. Lai, and H. El Gamal, "On the secrecy capacity of fading channels," *IEEE Trans. Inform. Theory*, vol. 54, no. 10, pp. 4687–4698, Oct. 2008.
- [4] F. Oggier and B. Hassibi, "The secrecy capacity of the MIMO wiretap channel," *IEEE Trans. Inform. Theory*, vol. 57, no. 8, Aug. 2011.
- [5] T. Liu and S. Shamai, "A note on the secrecy capacity of the multiantenna wiretap channel," *IEEE Trans. Inform. Theory*, vol. 55, no. 6, pp. 2547–2553, Jun. 2009.
- [6] J. Li and A. P. Petropulu, "On ergodic secrecy rate for Gaussian MISO wiretap channels," *IEEE Trans. Wireless Commun.*, vol. 10, no. 4, pp. 1176–1187, April 2011.
- [7] Z. Rezki, F. Gagnon, and V. Bhargava, "The ergodic capacity of the MIMO wire-tap channel," http://arxiv.org/abs/0902.0189, 2009.
- [8] M. Shaked and J. G. Shanthikumar, Stochastic Orders. Springer, 2007.
- [9] Z. Li, R. Yates, and W. Trappe, "Achieving secret communication for fast Rayleigh fading channels," *IEEE Trans. Wireless Commun.*, vol. 9, no. 9, pp. 2792 – 2799, Sep. 2010.
- [10] G. Caire and S. Shamai, "On the capacity of some channels with channel state information," *IEEE Trans. Inform. Theory*, vol. 45, no. 6, pp. 2007– 2019, Sept. 1999.
- [11] Y. Liang, H. V. Poor, and S. S. Shamai, "Information theoretic security," Foundations and Trends in Communications and Information Theory, vol. 5, no. 4-5, pp. 355–580, Apr. 2009.
- [12] T. M. Cover and J. A. Thomas, *Elements of information theory*, 2nd ed. Wiley-Interscience, 2006.
- [13] A. M. Mathai and S. B. Provost, Quadratic forms in random variables. Marcel Dekker, New York, 1992.
- [14] A. W. Marshall and I. Olkin, Inequalities: theory of majorization and its application. Academic Press, 1980.