

Single-Leader-Multiple-Followers Stackelberg Security Game with Hypergame Framework

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Abstract—In this paper, we employ a hypergame framework to analyze the single-leader-multiple-followers (SLMF) Stackelberg security game with two typical misinformed situations: misperception and deception. We provide a stability criterion with the help of hyper Nash equilibrium (HNE) to investigate both strategic stability and cognitive stability of equilibria in SLMF games with misinformation. In fact, we find mild stable conditions such that the equilibria with misperception and deception can become HNE. Moreover, we discuss the robustness of the equilibria to reveal whether players have the ability to keep their profits under the influence of some misinformation.

Index Terms—Stackelberg Security Game, Hypergame, Misperception, Deception, Cognition, Stability.

I. INTRODUCTION

SECURITY games describe the situation between defenders and attackers, with applications in many fields such as cyber-physical system (CPS), infrastructure protection, and counterterrorism problems [1]–[5]. The Stackelberg security game is one of the significant categories to characterize practical conflict [1]–[3]. As a fundamental model in [1], the leader is a defender to prevent invading, while the follower is an attacker to implement malicious behaviors after observing the leader's action. In addition, security models with multiple followers are also important since followers can not be usually treated as a monolithic party, considering that they may have different preferences, capabilities, and operational strategies [4].

Misinformation occurs in lots of security games [6]–[9], which may lead to players' different observations. Specifically, misperception and deception are two typical misinformed situations [10]. On the one hand, misperception is usually caused by external disturbances, bounded rationality, or accidental errors [11]–[13]. For instance, limited attention of players in the Internet of Things (IoT) increases cyber risks of the community [12]. Accordingly, the equilibrium of Stackelberg

security games with misperception can be described by the misperception strong Stackelberg equilibrium (MSSE) [9], where no player can change others' observations. On the other hand, deception usually results from belief manipulation, concealment, or camouflages [14]–[16]. For instance, in CPS, the network administrator might change the system's TCP/IP stack and obfuscate the services running on the port, while the hacker is probing the system [6]. Accordingly, the equilibrium of Stackelberg security games with deception can be defined as the deception strong Stackelberg equilibrium (DSSE) [17], and the hoaxer can manipulate others' observations, which is different from the situation with misperception.

In fact, both DSSE and MSSE reflect players' strategic stability, that is, each player has no will to change its own strategy unilaterally. However, due to misinformation, players' cognitive stability [18], [19] is crucial for whether players trust their observations of the game. Actually, players' suspicions of their cognitions may ruin the balance or even lead to the collapse of the model. For example, players may realize the biased misperception and intend to explore the truth [12], [20], or the hoaxer does not prefer the current deception along with unsatisfactory benefits [21], [22]. The most existent works have not paid enough attention to cognitive stability of equilibrium analysis in security games, including players changing their communication neighbors [23], keeping their current cooperators [24], or maintaining power systems [25].

Fortunately, hypergames provide an effective tool to analyze both strategic and cognitive stability of games with misinformation. Roughly speaking, hypergames describe complex situations when players have different understandings by decomposing a game into multiple subjective games [26]. Hyper Nash equilibrium (HNE) [27] is a core concept in hypergames, which represents the best response in each player's subjective game. Once achieving a HNE, each player not only rejects changing its strategy unilaterally, but also trusts its observation of the game since others' strategies are consistent with its own cognition. Such analogous discussions on cognitive stability with HNE have been applied in various circumstances, including resource allocation, military conflicts, and economics [22], [26].

Therefore, the motivation of this paper is to employ a hypergame framework and HNE to investigate the strategic stability and cognitive stability of MSSE and DSSE in SLMF Stackelberg security games with misinformation.

Related Works: Security games with misinformation have been widely investigated. Misperception is one of the typical situations with imperfect observations among players, such as in the infrastructure protection with disturbed invaders

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[1], [28], [29], and in CPS with small random fluctuations [11], [30]. Deception is another typical situation where some players mislead others' observations. In online negotiations, participants aim to interfere others' beliefs [8], [31], while in cyber deception games, the network administrator obfuscates the services running on the port to deceive hackers [6], [8], [9]. Additionally, in signaling games, a defender sends deceptive signals to a receiver [31], [32].

Different properties of equilibria appear in security games. As we know, the Nash equilibrium (NE) is one of the well-known concepts, and [23], [24] analyzed the conditions when players prefer to keep their current cooperators by NE in wireless security problems. With the Stackelberg equilibrium (SE), [17], [33], [34] analyzed counterterrorism problems with imitative criminals or different attack types in the single-leader-single-follower (SLSF) security game, while [25] used the cost-based SE for the optimal allocation of players' investment resources in power systems. Moreover, robustness in equilibria with misinformation was also widely discussed in security games. [35] considered the sensor networks' robustness capacity for resisting interference signals in Denial-of-Service (DoS) attack problems, while [14] analyzed the defender's robustness against the attacker's manipulation in CPS.

Besides strategic stability of equilibria, cognitive stability has also attracted extensive attention. [19] utilized cognitive stability to describe whether each player trusts its own cognition in financial markets without awareness. [21], [22] showed that the misinformation might ruin the balance among players' cognitions of the game, and players' preference changes with the deception in economics and psychology with deception. Also, [18], [27] revealed a relationship between HNE and cognitive stability in human interactive situations.

Contributions: The main contributions are as follows:

- We provide a novel second-level hypergame model for SLMF Stackelberg security games with misperception and deception, and present a stability evaluation criterion by HNE. Moreover, compared with the current stability analysis in security games [23]–[25], the stability criterion based on HNE reflects that players not only avoid changing their strategies unilaterally, but also tend to believe their own cognitions of the game.
- We show two different stable conditions such that MSSE and DSSE can become HNE. In such stable conditions, a HNE as an evaluation criterion covers the stable states when players do not realize the inherent misperception [27] or the hoaxer has no will to change its manipulation under deception [22]. Furthermore, we show the broad applicability of the obtained stable conditions by verifying them in typical circumstances [1], [28], [29].
- With the help of HNE, we also investigate the robustness of MSSE and DSSE to reveal players' capacities to keep their profits. We give lower bounds of MSSE and DSSE to describe whether players can safely ignore the misperception and easily implement deception in different misinformed situations, respectively.

The rest of this paper is organized as follows: Section II introduces the SLMF Stackelberg security game with misper-

ception and deception, and also describes a second-level Stackelberg hypergame model. Section III provides equilibrium analysis with a stability evaluation criterion. Then Section IV gives sufficient conditions such that MSSE and DSSE are HNE, while Section V analyzes the robustness of the derived equilibria. Additionally, Section VI gives numerical simulations to illustrate our results. Finally, Section VII summarizes this paper. A summary of important notations is provided in Table I.

TABLE I
IMPORTANT NOTATIONS

Notations	Description
n	Number of followers.
l	The leader.
\mathbf{P}	Set of followers.
Ω_l	Strategy set of the leader.
Ω_f	Strategy set of followers.
U_l	Utility function of the leader.
\mathbf{U}_f	Set of followers' utility functions.
\mathcal{G}	SLMF Stackelberg security game.
θ_0	True value of a parameter in \mathcal{G} .
θ'	Misinformation of θ_0 .
Θ	Set of all possible θ' .
θ^*	Optimal deception of θ' by the leader.
\mathcal{H}^1	First-level hypergame.
$\mathcal{H}^2(\theta')$	Second-level hypergame with misperception θ' .
$\mathcal{H}^2(\Theta)$	Second-level hypergame with deception in Θ .
BR_i	Best response of the i th follower.
$\mathbf{1}_n$	Row vector with all elements of one.
I_n	$n \times n$ identity matrix.
$\text{int}(\cdot)$	Interior of the set.
$\chi(\cdot)$	Indicative function.
$\ \cdot\ $	Euclidean norm.

II. SECURITY GAME AND HYPERGAME

In this section, we formulate two kinds of SLMF Stackelberg security games with misperception and deception, respectively, and we provide a second-level Stackelberg hypergame for SLMF games with misinformation.

A. SLMF Stackelberg Security Game

In the Stackelberg security game model [1], the leader is a defender, and the follower is an attacker to attack K targets. The attacker chooses to attack a certain target, while the defender tries to prevent attacks by covering targets with resources from a feasible set. In practice, there may be multiple followers with different preferences, capabilities, and strategies. For instance, there are multiple layers [2] and edge caching devices [4] in wireless networks. Also, governments may be confronted with numerous attackers in counterterrorism problems [5].

Define the SLMF Stackelberg security game by $\mathcal{G} = \{\{l\} \cup \mathbf{P}, \Omega_l \times \Omega_f, \{U_l\} \cup \mathbf{U}_f\}$, where l is the leader and $\mathbf{P} = \{1, \dots, n\}$ is the set of followers. $\Omega_l \subseteq \mathbb{R}^K$ is the strategy set of the leader and $\Omega_f = \Omega_1 \times \dots \times \Omega_n$, where $\Omega_i \subseteq \mathbb{R}^K$ is the

strategy set of the i th follower. Also, $U_l : \Omega_l \times \Omega_f \rightarrow \mathbb{R}$ is the leader's utility function and $\mathbf{U}_f = \{U_1, \dots, U_n\}$, where $U_i : \Omega_l \times \Omega_i \rightarrow \mathbb{R}$ is the utility function of the i th follower. The leader's utility is influenced by all players actions, while each follower's utility only relies on its own action and the leader's action. On this basis, each player aims at maximizing its own utility function. Denote the target set as $T = \{t_1, \dots, t_K\}$, which each follower (attacker) aims at attacking but the leader (defender) tries to protect. Then the SLMF game model \mathcal{G} can be presented concisely in Fig. 1. Suppose that the leader has a resource $R_l \in \mathbb{R}$ to assign on each target, i.e., x^k on target k with $\sum_{k=1}^K x^k = R_l$. Then the leader's strategy is $x = [x^1, \dots, x^K]^T$ and its strategy set is denoted by

$$\Omega_l = \{x \mid \sum_{k=1}^K x^k = R_l, x^k \geq 0, \forall k = 1 \dots, K\}. \quad (1)$$

Similarly, the i th follower has a resource $R_i \in \mathbb{R}$ with the strategy $y_i = [y_i^1, \dots, y_i^K]^T$, where $\sum_{k=1}^K y_i^k = R_i$. Then the strategy set is denoted by

$$\Omega_i = \{y_i \mid \sum_{k=1}^K y_i^k = R_i, y_i^k \geq 0, \forall k = 1 \dots, K\}, \forall i \in \mathbf{P}. \quad (2)$$

Moreover, let $\mathbf{y} = [y_1^T, \dots, y_n^T]^T$ be the strategy profile of all followers.

Remark 1 Many attack-defense mechanisms can be modeled by the SLMF Stackelberg security game \mathcal{G} . For example, in CPS [36], the invasion type is the DoS attack, while the intrusion detection system (IDS), as a defender, monitors the network with a probability distribution. In counterterrorism problems [29], terrorists select different attacking options, like assassination, armed assault, or hijacking, while the government, as a defender, allocates budgets on cities to defend against terrorists.

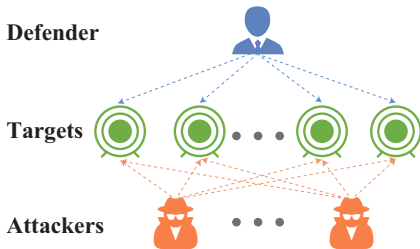


Fig. 1. SLMF Stackelberg security game model.

As discussed in [1], let $U_l^c(t_k)$ be the leader's utility when the leader allocates per unit of resource to target t_k with per unit attacking resource. $U_l^u(t_k)$ is the leader's utility when the leader does not allocate per unit of resources to target t_k with per unit attacking resource. Given a strategy profile (x, \mathbf{y}) , the leader's utility function is

$$U_l(x, \mathbf{y}) = \sum_{k=1}^K \left(\sum_{i=1}^n y_i^k \right) (x^k U_l^c(t_k) + (R_l - x^k) U_l^u(t_k)), \quad (3)$$

where $\sum_{i=1}^n y_i^k$ reflects the influence of all followers on target k . Similarly, the i th follower's utility consists of $U_i^c(t_k)$ and $U_i^u(t_k)$. Given the strategy profile (x, \mathbf{y}) , the i th follower's utility function is

$$U_i(x, y_i) = \sum_{k=1}^K y_i^k (x^k U_i^c(t_k) + (R_l - x^k) U_i^u(t_k)). \quad (4)$$

If the followers cannot observe the actions of the leader and all players make decisions simultaneously, then we can consider the Nash Equilibrium (NE) [1], [37].

Definition 1 A strategy profile (x^*, \mathbf{y}^*) is said to be a NE of the SLMF game \mathcal{G} if

$$\begin{aligned} x^* &\in \operatorname{argmax}_{x \in \Omega_l} U_l(x, \mathbf{y}^*), \\ y_i^* &\in \operatorname{argmax}_{y_i \in \Omega_i} U_i(x^*, y_i), \forall i \in \mathbf{P}. \end{aligned}$$

On the other hand, when the leader implements an allocation, followers determine their strategies after observing the leader's strategy. Denote the i th follower's best response to the leader's strategy x by

$$\mathbf{BR}_i(x) = \operatorname{argmax}_{y_i \in \Omega_i} U_i(x, y_i), \forall i \in \mathbf{P},$$

and $\mathbf{BR}(x) = \mathbf{BR}_1(x) \times \dots \times \mathbf{BR}_n(x)$. Without loss of generality, followers can break ties optimally for the leader if there are multiple best responses. In this case, we introduce the Strong Stackelberg Equilibrium (SSE) [38].

Definition 2 A strategy profile (x^*, \mathbf{y}^*) is said to be a SSE of the SLMF game \mathcal{G} if

$$(x^*, \mathbf{y}^*) \in \operatorname{argmax}_{x \in \Omega_l, \mathbf{y} \in \mathbf{BR}(x)} U_l(x, \mathbf{y}).$$

Remark 2 According to [17], the SSE in the SLSF security game can be reducible to $\{x^*, k^*\}$, where the leader's SSE strategy is x^* and the follower attacks target t_{k^*} . However, multiple followers may attack different targets, and their SSE strategies can not be reduced to a single target. Additionally, in the SLMF game, each follower's strategy set is a subset of \mathbb{R}^K , and is more complex than a subset of \mathbb{R} in [2]. Although our model has similar utility functions as [29], we focus on that each follower has its own resource, i.e., $\sum_{k=1}^K y_i^k = R_i$, instead of that all followers allocate a total resource $\sum_{i=1}^n \sum_{k=1}^K y_i^k = R$.

Then we discuss SLMF games with misinformation, when players have different cognitions. Specifically, misperception and deception are two typical misinformed situations.

B. Misperception and Deception

We first consider misperception for a situation when there are imperfect or prejudiced observations/understandings of the game among players. It is caused by passive factors with players' biased cognitions. For example, in communication channels such as sensor systems, external disturbances may cause imperfect observations [11], while players with bounded

rationality may have prejudiced observations in the IoT [12]. Moreover, in the computer and information security, accidental errors from small random fluctuations also bring players imprecise observations [30].

Consider that the followers have prejudiced observations of the security game \mathcal{G} , while the leader realizes the situation. To describe different observations, we consider a parameter $\theta_0 \in \mathbb{R}^m$ in the SLMF game \mathcal{G} , and followers' prejudiced observations of θ_0 are $\theta' \in \mathbb{R}^m$, where θ_0 and θ' only affect followers' utility functions. Then the SLMF game with misperception can be denoted by $\mathcal{G}_M(\theta_0, \theta') = \{\{l\} \cup \mathbf{P}, \Omega_l \times \Omega_f, \{U_l\} \cup \mathbf{U}_f, \{\theta_0, \theta'\}\}$. Specifically, $\mathbf{U}_f = \{U_1, \dots, U_n\}$, where $U_i : \Omega_l \times \Omega_i \times \{\theta_0, \theta'\} \rightarrow \mathbb{R}$ is the utility function of the i th follower. Here we rewrite the security model without misperception as $\mathcal{G} = \mathcal{G}_M(\theta_0, \theta_0)$.

In addition, given the strategy profile (x, \mathbf{y}) , the leader's actual utility function is $U_l(x, \mathbf{y})$, which is the same as (3). Also the i th follower's actual utility function is $U_i(x, y_i, \theta_0)$, which is exactly (4). However, known to the leader, the i th follower believes that its own utility function is as follow:

$$U_i(x, y_i, \theta') = \sum_{k=1}^K y_i^k (x^k U_i^c(\theta', t_k) + (R_l - x^k) U_i^u(\theta', t_k)). \quad (5)$$

Correspondingly, the i th follower's best response to the leader strategy x under the prejudiced observation θ' is

$$\text{BR}_i(x, \theta') = \underset{y_i \in \Omega_i}{\operatorname{argmax}} U_i(x, y_i, \theta'), \forall i \in \mathbf{P},$$

and $\mathbf{BR}(x, \theta') = \text{BR}_1(x, \theta') \times \dots \times \text{BR}_n(x, \theta')$. Similar to SSE, the leader implements an allocation, and afterward, followers determine their strategies after observing the leader's strategy. Therefore, following the security game with misperception [9], the equilibrium with misperception can be denoted by the Misperception Strong Stackelberg Equilibrium (MSSE).

Definition 3 A strategy profile (x^*, \mathbf{y}^*) is said to be a MSSE of the SLMF game with misperception $\mathcal{G}_M(\theta_0, \theta')$ if

$$(x^*, \mathbf{y}^*) \in \underset{x \in \Omega_l, \mathbf{y} \in \mathbf{BR}(x, \theta')}{\operatorname{argmax}} U_l(x, \mathbf{y}).$$

Next, we address deception for another situation when some players mislead others' cognitions with selfish or malevolent motivation. Unlike misperception, deception is caused by active factors among players with players' manipulated cognitions. For instance, each player is explicitly interested in convincing the others to hold some particular beliefs such as in authentication protocols and online negotiations [14], [31]. Moreover, the leader may tend to deceive followers like a network administrator (leader) and a hacker (follower) in CPS [6], while the network administrator might change a system's TCP/IP stack and obfuscate the services running on the port [8], [9].

In this situation, the leader can manipulate the followers' observation, while followers are not aware. Set $\theta_0 \in \mathbb{R}^m$ as the true value of the parameter in \mathcal{G} . Take $\Theta \subseteq \mathbb{R}^m$ as the deceptive set, while the leader manipulates followers' observations of the parameter as $\theta' \in \Theta$ to maximize its own utility function. Denote the SLMF Stackelberg security game with

deception by $\mathcal{G}_D(\Theta) = \{\{l\} \cup \mathbf{P}, \Omega_l \times \Omega_f \times \Theta, \{U_l\} \cup \mathbf{U}_f, \theta_0\}$. Specifically, $\mathbf{U}_f = \{U_1, \dots, U_n\}$, where $U_i : \Omega_l \times \Omega_i \times \Theta \rightarrow \mathbb{R}$ is the utility function of the i th follower. Here we rewrite the security model without deception as $\mathcal{G} = \mathcal{G}_D(\{\theta_0\})$.

Given the strategy profile (x, \mathbf{y}, θ') , players' actual utility functions are $U_l(x, \mathbf{y})$ and $U_i(x, y_i, \theta_0)$, $\forall i \in \mathbf{P}$. Since followers' observations are manipulated as θ' , the i th follower regard its own utility function as $U_i(x, y_i, \theta')$, which is generated by (5) and the domain of U_i contains Θ instead of $\{\theta_0, \theta'\}$.

Therefore, the leader manipulates followers' observations of the parameter as $\theta^* \in \Theta$ to maximize its own utility function at first. Then, similar to SSE and MSSE, the leader provides its own strategy, and afterward, each follower acts according to the observation θ^* and the leader's strategy. Thus, based on the SLSF security game with deception [6], [17], the equilibrium with deception can be defined as the Deception Strong Stackelberg Equilibrium (DSSE).

Definition 4 A strategy profile $(x^*, \mathbf{y}^*, \theta^*)$ is said to be a DSSE of the SLMF game with deception $\mathcal{G}_D(\Theta)$ if

$$(x^*, \mathbf{y}^*) \in \underset{x \in \Omega_l, \mathbf{y} \in \mathbf{BR}(x, \theta^*)}{\operatorname{argmax}} U_l(x, \mathbf{y}),$$

where $\theta^* \in \underset{\theta' \in \Theta}{\operatorname{argmax}} \max_{x \in \Omega_l, \mathbf{y} \in \mathbf{BR}(x, \theta')} U_l(x, \mathbf{y})$ is the optimal deception of the leader.

Different from MSSE, DSSE describes a decision with players' manipulated cognitions, and the leader can manipulate followers' observations of the parameter θ_0 . The following assumptions have been widely employed in security games with deception [1], [6], [17], [32], [33], [36], [39]–[42].

Assumption 1 Θ is compact and convex, while $\text{int}(\Theta)$ is nonempty and $\theta_0 \in \Theta$.

Assumption 2 For $i \in \mathbf{P}, k = 1, \dots, K$, $U_i^c(\theta', t_k)$ and $U_i^u(\theta', t_k)$ are differentiable in $\theta' \in \Theta$.

Assumption 3 For $k = 1, \dots, K$, $U_l^c(t_k) > U_l^u(t_k)$.

Assumption 4 For $\theta' \in \Theta, i \in \mathbf{P}, k = 1, \dots, K$, $U_i^c(\theta', t_k) < U_i^u(\theta', t_k)$.

Assumption 5 There exists k , such that for $i \in \mathbf{P}, l \neq k$, $U_i^c(\theta_0, t_k) \geq U_i^u(\theta_0, t_l)$.

Assumptions 1 and 2 guarantee the existence of a DSSE of $\mathcal{G}_D(\Theta)$ [6], [39], which are also adopted in real-world security problems such as unmanned aerial vehicles (UAVs) security games [40], [41] and moving target defense (MTD) problems [32]. Furthermore, Assumption 3 indicates that, for the leader, the unit utility for defending a target is larger than that without defending. Assumption 4 indicates that, for each follower, the unit utility for attacking a target is larger than that without attacking. They are consistent with the fact that the leader tends to resist attacks and followers tend to implement invasions [1], [17], [33]. Moreover, Assumption 5 refers to the situation when there exists a most attractive target for followers [36], [42], which describes a relationship among different targets, different from Assumption 4.

Remark 3 In many practical situations, the leader has direct access to others' cognition. For example, in CPS, a network

administrator can obfuscate the services running on the port, while the administrator knows all the information of the services [8], [9]. In UAVs security problems, the defender may show wrong targets' locations to UAVs, where both true and wrong locations are detected by the defender [40]. Additionally, in an industrial control system, Stuxnet, as a leader, can directly obtain the access to the system and feed fake data to disguise malicious actions [14].

C. Hypergame

The hypergame theory describes different cognitions among players for the strategic interactions in situations with misinformation. It covers misperception or deception, where players may have biased misperception or can manipulate others' observations. The main idea of the hypergame is to decompose complex situations with misinformation into multiple subjective games. According to [26], each player in a hypergame may

- have a misled or false understanding of other players' preferences or utility functions;
- have an incorrect comprehension of other players' strategy sets;
- be not aware of every one of all players;
- have any combination of the above.

In fact, the hypergame theory has been applied in various circumstances, such as CPS and economic behaviors [14], [18], [22]. Benefiting from the subjective games decomposed by hypergames, the relationship among players' strategies, incorporation of opponents' cognitions, and fears of being out-guessed are further explored in situations with misinformation.

Remark 4 *Standard Stackelberg games describe players' different acting sequences, where the leader implements strategy first and followers act after observing the leader's actions. Hypergames focus on players' different cognitions with multiple subjective games. Players play different subjective games and may also know others' cognition. Actually, standard Stackelberg games may be regarded as a special hypergame, where followers know the leader's game and follow its action, and the leader also knows this fact. Since the hypergame is good at describing complex situations with misinformation, it helps us analyze both the strategic and cognitive stability of equilibria with misperception and deception.*

There are different levels for describing different cognitive environments [43]. For instance, the first-level hypergame describes the situation when players are playing different games, but no one realizes the fact. Correspondingly, the second-level hypergame occurs when at least one player is aware that different games are played. Then we aim at employing the second-level Stackelberg hypergame to analyze SLMF games with misperception and deception and providing a criterion for evaluating the stability of the equilibrium with misinformation.

As we know, both misperception and deception can cause observation errors of the parameter θ_0 in the game. Take $\Theta \subseteq \mathbb{R}^m$ as the cognitive set of all followers' possible observations and $\theta_0 \in \mathbb{R}^m$ as the true value of the parameter in \mathcal{G} . Denote the game under the observation $\theta' \in \Theta$ by $\mathcal{G}(\theta') =$

$\{\{U\} \cup \mathbf{P}, \Omega_l \times \Omega_f, \{U_l\} \cup \mathbf{U}_f, \theta'\}$ with $\mathbf{U}_f = \{U_1, \dots, U_n\}$, where $U_i : \Omega_l \times \Omega_i \times \Theta \rightarrow \mathbb{R}$ is the utility function of the i th follower. In addition, given the strategy profile (x, \mathbf{y}) , players' actual utility functions are $U_l(x, \mathbf{y})$ and $U_i(x, y_i, \theta_0)$, $\forall i \in \mathbf{P}$. However, in all players' views, the i th follower's utility function is $U_i(x, y_i, \theta')$. Here we rewrite the SLMF game model without any observation error in Section II as $\mathcal{G} = \mathcal{G}(\theta_0)$.

Consider the first-level hypergame to describe a complex situation in the SLMF game \mathcal{G} when there are observed differences among players, but no one is aware. Concretely, suppose that all followers' observations of the parameter are $\theta' \in \Theta$, while the leader's observation is θ_0 . For the leader, denote $\mathcal{G}_l = \mathcal{G}(\theta_0)$ as the game of the leader's self-cognition. For $i \in \mathbf{P}$, denote $\mathcal{G}_i = \mathcal{G}(\theta')$ as the game of the i th follower's self-cognition. Then the situation can be defined as $\mathcal{H}^1 = \{\mathcal{G}(\theta_0), (\mathcal{G}(\theta'))_{i \in \mathbf{P}}\}$, which is a first-level hypergame as shown in Fig. 2.

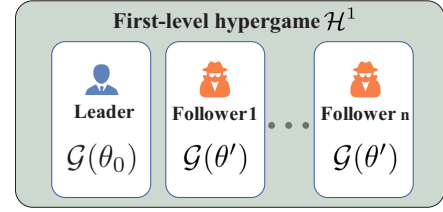


Fig. 2. First-level hypergame \mathcal{H}^1 .

Moreover, we employ the second-level hypergame to describe different misinformed situations when all followers do not realize the observed differences and the leader is aware of the fact. On the one hand, in the leader's view, for $i \in \mathbf{P}$, denote \mathcal{G}_{il} as the i th follower's game under the leader's perception. Thus, $\mathcal{G}_{il} = \mathcal{G}(\theta')$ since the leader knows the i th follower's observation is θ' . Also, denote \mathcal{G}_{ll} as the leader's game under its own perception. Then $\mathcal{G}_{ll} = \mathcal{G}(\theta_0)$ since the leader's own observation is θ_0 . Thus, $\mathcal{H}_l^1 = \{\mathcal{G}_{ll}, (\mathcal{G}_{il})_{i \in \mathbf{P}}\}$ is a novel first-level hypergame perceived by the leader. On the other hand, in the view of the i th follower, denote \mathcal{G}_{li} as the leader's game and \mathcal{G}_{ii} as its own game under the i th follower's perception. Thus, $\mathcal{G}_{li} = \mathcal{G}(\theta')$ and $\mathcal{G}_{ii} = \mathcal{G}(\theta')$ since the i th follower is not conscious with $\theta' \neq \theta_0$. Thus, $\mathcal{H}_i^1 = \{\mathcal{G}_{li}, \mathcal{G}_{ii}\}$ is another first-level hypergame perceived by the i th follower. Notice that, for all $i \in \mathbf{P}$, \mathcal{H}_l^1 and \mathcal{H}_i^1 are different since the leader notices the cognitive set Θ but followers do not. Therefore, the different first-level hypergames perceived by all players form a Stackelberg hypergame $\mathcal{H}^2(\Theta) = \{\mathcal{H}_l^1, (\mathcal{H}_i^1)_{i \in \mathbf{P}}\}$, which is also a second-level hypergame as shown in Fig. 3.

Clearly, $\mathcal{H}^2(\Theta)$ can describe SLMF games with both deception and misperception. For instance, regarding Θ as the deceptive set, we can rewrite the SLMF game with deception in Section II as $\mathcal{G}_D(\Theta) = \mathcal{H}^2(\Theta)$. Especially, let $\Theta = \{\theta'\}$, and then the SLMF game with misperception in Section II can be written as $\mathcal{G}_M(\theta_0, \theta') = \mathcal{H}^2(\theta')$.

In all players' views, the leader chooses a strategy with the utility function $U_l(x, \mathbf{y})$, while the i th follower makes a

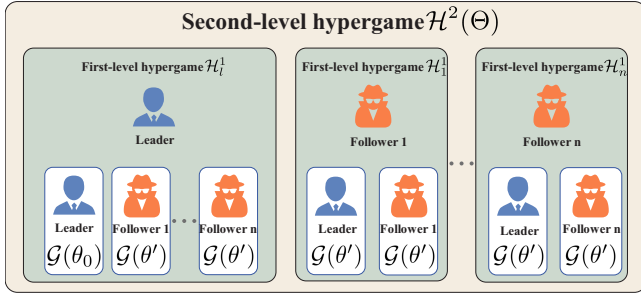


Fig. 3. Second-level hypergame $\mathcal{H}^2(\Theta)$.

decision with the utility function $U_i(x, y_i, \theta')$, which leads to a concept called Hyper Nash Equilibrium (HNE) [27].

Definition 5 A strategy profile (x^*, \mathbf{y}^*) is said to be a HNE of $\mathcal{H}^2(\Theta)$ with any fixed $\theta' \in \Theta$ if

$$\begin{aligned} x^* &= \operatorname{argmax}_{x \in \Omega_l} U_l(x, \mathbf{y}^*), \\ y_i^* &= \operatorname{argmax}_{y_i \in \Omega_i} U_i(y_i, x^*, \theta'), \forall i \in \mathbf{P}. \end{aligned}$$

In fact, HNE is such a strategy profile that is the best response strategy in everyone's subjective game. Each player does not change its strategy unilaterally if all the players play HNE strategies. It is the same as NE if there is no cognitive difference. In addition, at HNE, each player can not realize that its cognition is different from others, since others' strategies are consistent with its own anticipation. Then all players have no incentive to update their observations of the parameter θ_0 . Hence, the HNE in our hypergame is a desired equilibrium with cognitive stability, which was similarly described in [18], [19]. Compared with previous discussions on equilibrium stability in security games [23]–[25], HNE helps analyze both the strategic stability and cognitive stability of SLMF games with misinformation, and provides a unified framework for misperception and deception.

Remark 5 The signaling game is a leader-follower game with deception, where the leader sends deceptive signals to followers [31], [32]. It usually focuses on whether players can achieve the equilibrium with deception, which is actually a learning process to update cognition for followers. Different from the signaling game, the hypergame concerns the cognitive stability of games with misinformation [18], [19]. In the cases of misperception or deception, the cognitive stability plays an important role since it shows the conditions that each player trusts its own current cognition. Otherwise, the player's anticipation may not be consistent with others' strategies, and thus, the player may be suspicious about its cognition. For instance, the hoaxer may not believe in the current deception along with benefits, while the victim (follower) may be aware of the biased cognition if the opponents' best response strategies are "impractical". Thus, we adopt the hypergame to analyze the cognitive stability of the SLMF game with misinformation.

III. EQUILIBRIUM ANALYSIS

In this section, we analyze the equilibria of the proposed formulations in security games.

With complete information, the following lemma verifies the existence of NE and SSE in the SLMF game \mathcal{G} .

Lemma 1 There exists a NE and a SSE of \mathcal{G} .

Proof: Recalling (1) and (2), for $i \in \mathbf{P}$, Ω_l and Ω_i are compact and convex. $U_l(\cdot, \mathbf{y})$ and $U_i(x, \cdot)$ are linear for fixed $\mathbf{y} \in \Omega_f$ and $x \in \Omega_l$, respectively. By Theorem 2.1 in [44], there exists a NE of \mathcal{G} . Also, based on [1], since $\mathbf{BR}(x)$ is compact and U_l and U_i is continuous, there exists a SSE of \mathcal{G} . \square

With misperception, the following lemma shows the existence of a MSSE of $\mathcal{H}^2(\theta')$, whose proof can be given by replacing $U_i(x, y_i)$ with $U_i(x, y_i, \theta')$ in Lemma 1.

Lemma 2 For any $\theta' \in \mathbb{R}^m$, there exists a MSSE of $\mathcal{H}^2(\theta')$.

Moreover, with deception, the next lemma shows the existence of a DSSE in $\mathcal{H}^2(\Theta)$.

Lemma 3 Under Assumptions 1 and 2, there exists a DSSE of $\mathcal{H}^2(\Theta)$.

Proof: Denote $(x(\theta'), \mathbf{y}(\theta')) = \operatorname{argmax}_{x \in \Omega_l, \mathbf{y} \in \mathbf{BR}(x, \theta')} U_l(x, \mathbf{y})$ for any $\theta' \in \Theta$. Take $L(\theta') = U_l(x(\theta'), \mathbf{y}(\theta'))$ and $L^* = \sup_{\theta' \in \Theta} L(\theta')$. Then there exists a sequence $\{\theta_j\}_{j=1}^\infty$ such that $L(\theta_j) > L^* - \frac{1}{j}$. Since Θ is compact, there exists a convergent subsequence $\{\theta_{j_m}\}_{m=1}^\infty$, where $\lim_{m \rightarrow \infty} \theta_{j_m} = \theta^* \in \Theta$. Thus, $L^* \geq U_l(x(\theta_{j_m}), \mathbf{y}(\theta_{j_m})) > L^* - \frac{1}{j_m}$. By the continuity of U_l , $\lim_{m \rightarrow \infty} U_l(x(\theta_{j_m}), \mathbf{y}(\theta_{j_m})) = L^*$. Also, there is a convergent subsequence $\{(x(\theta_{j_m_q}), (\theta_{j_m_q}))\}_{q=1}^\infty$, where $\lim_{q \rightarrow \infty} (x(\theta_{j_m_q}), (\theta_{j_m_q})) = (x^*, \mathbf{y}^*)$. Then $U_l(x^*, \mathbf{y}^*) = \max_{\theta' \in \Theta} \max_{x \in \Omega_l, \mathbf{y} \in \mathbf{BR}(x, \theta')} U_l(x, \mathbf{y})$. By Lemma 17.30 in [45], $\mathbf{y}^* \in \mathbf{BR}(x^*, \theta^*)$. Then $(x^*, \mathbf{y}^*, \theta^*)$ is a DSSE of $\mathcal{H}^2(\Theta)$. \square

The following example indicates that Assumptions 1 and 2 are fundamental in Lemma 3, since there may be no existence of DSSE without Assumptions 1 and 2.

Example 1 Consider a SLSF game with $R_l = R_1 = 1$, $K = 2$, and $\Theta = (0, 1)$. Take $U_l^c(t_1) = 1$, $U_1^c(\theta', t_1) = \theta' - 1$, $U_1^u(\theta', t_1) = \theta'$, and $U_l^u(t_1) = U_l^c(t_2) = U_l^u(t_2) = U_1^c(\theta', t_2) = U_1^u(\theta', t_2) = 0$. Then for any $\theta' \in \Theta$, players take $x = [\theta', 1 - \theta']^T$ and $y_1 = [1, 0]^T$. Then, the leader's profit is $U_l(x, y_1) = \theta'$. Since Θ is not closed, there is no DSSE.

Furthermore, the following theorem shows the existence of a HNE in $\mathcal{H}^2(\Theta)$.

Theorem 1 Under Assumptions 1 and 2, there exists a HNE of $\mathcal{H}^2(\Theta)$.

Proof: For any fixed $\theta' \in \Theta$, in the leader's view, the i th follower acts with $U_i(x, y_i, \theta')$. For any $x \in \Omega_l$ and $\mathbf{y} \in \Omega_f$, denote $F(x, \mathbf{y}) = \{(\hat{x}, \hat{\mathbf{y}}) | \hat{x} \in \operatorname{argmax}_{x' \in \Omega_l} U_l(x', \mathbf{y}), \hat{\mathbf{y}} \in \mathbf{BR}(x, \theta')\}$. Take (x_j, \mathbf{y}_j) as a convergent sequence, where $\lim_{j \rightarrow \infty} (x_j, (\mathbf{y}_j)) = (x^*, \mathbf{y}^*)$. There exists $(\hat{x}_j, \hat{\mathbf{y}}_j) \in F(x_j, (\mathbf{y}_j))$. Since $\Omega_l \times \Omega_f$ is compact, there exists a convergent subsequence $\{(\hat{x}_{j_m}, (\hat{\mathbf{y}}_{j_m}))\}_{m=1}^\infty$, where

$\lim_{m \rightarrow \infty} (\hat{x}_{j_m}, (\hat{\mathbf{y}})_{j_m}) = (\hat{x}^*, \hat{\mathbf{y}}^*)$. By the continuity of U_l ,

$$U_l(\hat{x}^*, \mathbf{y}^*) = \lim_{m \rightarrow \infty} U_l(\hat{x}_{j_m}, (\mathbf{y})_{j_m}) = \lim_{m \rightarrow \infty} \max_{x' \in \Omega_l} U_l(x', (\mathbf{y})_{j_m}).$$

According to Lemma 17.30 in [45],

$$U_l(\hat{x}^*, \mathbf{y}^*) = \max_{x' \in \Omega_l} \lim_{m \rightarrow \infty} U_l(x', (\hat{\mathbf{y}})_{j_m}) = \max_{x' \in \Omega_l} U_l(x', (\mathbf{y})^*).$$

Thus, $\hat{x}^* \in \operatorname{argmax}_{x' \in \Omega_l} U_l(x', (\mathbf{y})^*)$. Similarly, $\hat{\mathbf{y}}^* \in \mathbf{BR}(x^*, \theta')$.

Then $(\hat{x}^*, \hat{\mathbf{y}}^*) \in F(x^*, \mathbf{y}^*)$. According to Theorem A.14 in [44], there exists (x', \mathbf{y}') such that $(x', \mathbf{y}') \in F(x', \mathbf{y}')$. Then (x', \mathbf{y}') is the best response strategy for everyone in the leader's view. Also, in the i th follower's view, $x^* \in \operatorname{argmax}_{x \in \Omega_l} U_l(x, \mathbf{y}^*)$, and $\mathbf{y}'_i \in \mathbf{BR}_i(x', \theta')$. Thus, (x', \mathbf{y}') is a HNE of $\mathcal{H}^2(\Theta)$. \square

The following example indicates that misinformation may lead to some players' suspicions on the observation of the game, since others' strategies do not match their cognitions.

Example 2 Consider a SLMF game with $n = K = 2$, $R_l = R_1 = R_2 = 1$, $\Theta = \{0, 1\}$, and $\theta_0 = 0$. Take $U_l^u(t_1) = 2$, $U_l^u(t_2) = 3$, $U_2^u(\theta', t_1) = U_2^u(\theta', t_2) = 0$, and $U_l^c(t_1) = U_l^c(t_2) = U_2^c(\theta', t_1) = U_2^c(\theta', t_2) = 1$. Denote $U_1^c(\theta', t_1) = U_1^u(\theta', t_1) = \theta'$ and $U_1^c(\theta', t_2) = U_1^u(\theta', t_2) = 1 - \theta'$. Then the leader's optimal deception is $\theta^* = 1$, and players take $x^* = [0, 1]^T$, $\mathbf{y}_1^* = [1, 0]^T$, and $\mathbf{y}_2^* = [0, 1]^T$. Notice that the first follower may not observe the leader's strategy before attacking [1]. In the first follower's view, x^* is not the best response strategy to \mathbf{y}^* , and players should have taken $x' = [0, 1]^T$, $\mathbf{y}_1' = [0, 1]^T$ with $\theta_0 = 0$. Clearly, θ^* brings more benefits to the leader since $U_l(x^*, \mathbf{y}_1^*, \mathbf{y}_2^*) > U_l(x', \mathbf{y}_1', \mathbf{y}_2')$. Then the first follower realizes the misinformation and tends to update its observation.

According to Example 2, players' suspicions on their cognitions may cause them to update their observations, and even make the game model collapse. Thus, cognitive stability is crucial for SLMF games with misinformation. To this end, we aim at analyzing the cognitive stability and strategic stability of MSSE and DSSE with the help of HNE.

IV. STABILITY ANALYSIS

It is known that HNE describes a stable state when each player does not update its own cognitions and strategies. In this section, we explore conditions to reveal how the MSSE and DSSE can become a HNE in the Stackelberg hypergame.

A. Stable Conditions

With misperception, MSSE is called stable when players do not realize the inherent misperception, while, with deception, DSSE is called stable when the leader has no will to change its manipulation on followers' cognitions.

First, in order to evaluate the stability of MSSE, given $\mathbf{y} \in \Omega_f$, $\theta' \in \mathbb{R}^m$, define

$$\operatorname{SOL}(\mathbf{y}, \theta') = \{\mathbf{y}' \in \Omega_f, \lambda > 0 | A_1(\theta')\mathbf{y}' = \lambda B\mathbf{y}, A_2\mathbf{y}' = 0\},$$

where

$$\begin{aligned} A_1(\theta') &= [A_1(\theta', 1), \dots, A_1(\theta', n)], \\ A_1(\theta', i) &= \operatorname{diag}\left(\frac{U_1^u(\theta', t_1) - U_1^c(\theta', t_1)}{U_l^c(t_1) - U_l^u(t_1)}, \dots, \frac{U_1^u(\theta', t_K) - U_1^c(\theta', t_K)}{U_l^c(t_K) - U_l^u(t_K)}\right), \\ A_2 &= \operatorname{diag}(\mathbf{1}_{nK} - \chi(\mathbf{y})), \quad B = [I_K, I_K, \dots, I_K]. \end{aligned}$$

Notice that $A_2\mathbf{y}' = 0$ is equivalent to $(\mathbf{y}')_i^k = 0$ if $y_i^k = 0$ for any $i \in \mathbf{P}, k = 1, \dots, K$. Here, $\chi(\cdot)$ is the indicative function where $\chi(x) = 0$ iff $x = 0$.

Let $(x_{\text{MSSE}}, \mathbf{y}_{\text{MSSE}})$ be a MSSE of $\mathcal{H}^2(\theta')$. In the following, we give a result about the MSSE of $\mathcal{H}^2(\theta')$, whose proof can be found in Appendix A.

Theorem 2 Under Assumptions 3 and 4, if $\operatorname{SOL}(\mathbf{y}_{\text{MSSE}}, \theta')$ is nonempty, then $(x_{\text{MSSE}}, \mathbf{y}_{\text{MSSE}})$ is also a HNE.

Theorem 2 implies that a MSSE strategy is stable in such a condition, since such a decision-making process prevents players from realizing the inherent misperception. Concretely, for each follower, the leader's strategy is consistent with each follower's anticipation, and its MSSE strategy is also the best response strategy in its own subjective game. Thus, they can not be aware of their cognitive errors in $\mathcal{H}^2(\theta')$, which also conforms with the two-players game model in [27]. Additionally, for the leader, Theorem 2 also indicates that, even if followers cannot observe the consequences of the leader's strategy, the leader can safely play a MSSE strategy since it is still the best response strategy. Furthermore, no matter how many followers move simultaneously, the conclusion holds in the SLMF game with misperception $\mathcal{H}^2(\theta')$, which covers the situation in [1].

Second, in the view of deception, our major concern is the stability of DSSE. The following theorem gives a sufficient condition to guarantee that a DSSE of $\mathcal{H}^2(\Theta)$ is a HNE, whose proof can be found in Appendix B.

Theorem 3 Under Assumptions 1-4, if $K_{\max} \in \operatorname{argmax}_{k=1, \dots, K} U_l^c(t_k)$ and the leader is able to trick followers into attacking target $t_{K_{\max}}$, then the corresponding DSSE of $\mathcal{H}^2(\Theta)$ is a HNE.

Theorem 3 indicates that a DSSE strategy is stable in such a sufficient condition, since the leader has no will to change its manipulation on followers' cognitions. The deception brings the leader the most benefit since its DSSE strategy is the best response strategy. Also, similar to Theorem 2, followers are not able to find that their observations of θ_0 are misled, because the leader acts as they expect. Thus, followers do not update their observations, and the leader can safely deceive followers without being found out. Then the deception result is stable, as the discussion in [22]. Additionally, no matter how many followers cannot observe the consequence of the leader's strategy, the leader can select a DSSE strategy because it is at least a HNE strategy for itself. Thus, the leader has no need to worry whether followers can observe its decision, which also consists with the analysis of two-players games in [46].

Remark 6 Clearly, MSSE describes a situation caused by passive factors with players' biased cognitions, while DSSE

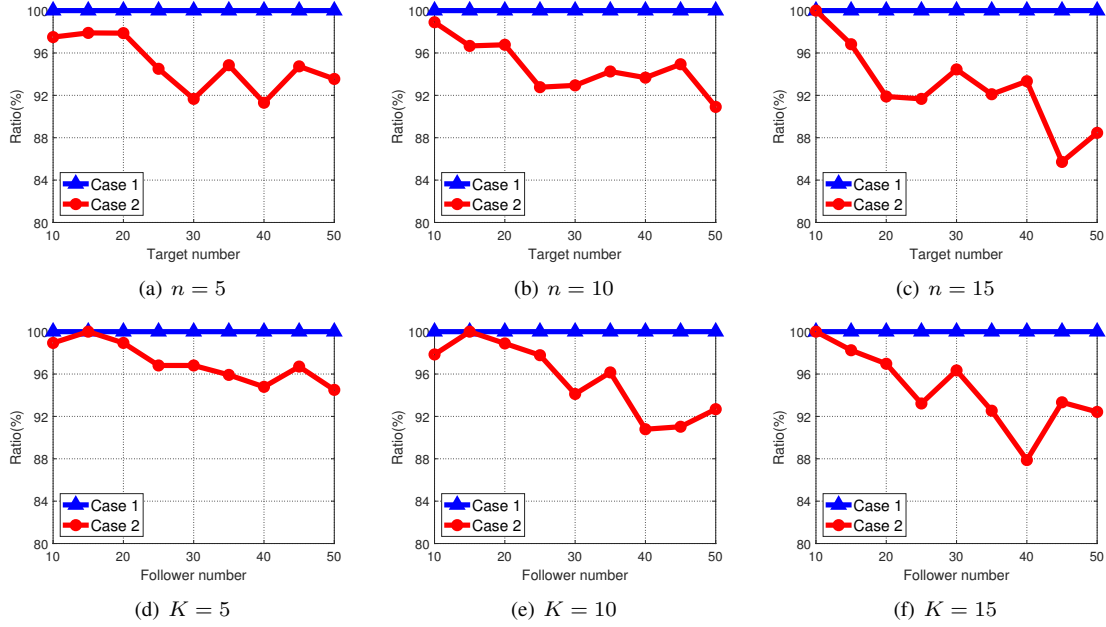


Fig. 4. Ratios of the two cases in misperception. The x -axis is for the target number in (a)-(c), and for the follower number in (d)-(f). y -axis is for ratios of Case 1 and Case 2 in 100 instances, which are depicted in blue and red, respectively.

describes another situation caused by active factors among players with players' manipulated cognitions. Additionally, it is called *stable with misperception* when players do not realize the inherent misperception, while it is called *stable with deception* when the leader has no will to change its manipulation on followers' cognitions. On the other hand, the evaluation processes of stability with misperception and deception are different. Since deception happens in a cognitive set, we use the deception's influences on players to investigate the stability of DSSE, while for misperception, $\text{SOL}(\mathbf{y}, \theta')$ gives how the misperception affects players' strategies.

B. Typical Cases

Here we investigate several typical cases to further explain the proposed stable conditions in Theorems 2 and 3, in order to show them can be widely applied to many practical problems.

1) *Players' Perspective*: Consider the SLSF game in [1]. The follower takes strategy $y_1 \in \Omega_1$, and its utility function is $U_1(x, y, \theta')$. For any $\theta' \in \mathbb{R}^m$, $\text{SOL}(y, \theta')$ can be converted to $\text{SOL}(y, \theta') = \{y' \in \Omega_f, \lambda > 0 | A_1(\theta', 1)y' = \lambda y\}$. It is easy to verify that $\text{SOL}(y, \theta')$ is always nonempty. Then we have the following result, regarded as an extension of the Theorem 3.9 in [1].

Corollary 1 *Under Assumptions 3 and 4, for any $\theta' \in \mathbb{R}^m$, any MSSE is a HNE of the SLSF game $\mathcal{H}^2(\theta')$.*

In addition, players' DSSE strategies are with a certain θ^* , and the following result follows directly.

Corollary 2 *Under Assumptions 1-4, any DSSE is a HNE of the SLSF game $\mathcal{H}^2(\Theta)$.*

2) *Targets' Perspective*: Consider the case that all followers prefer to attack the same target. Notice that our SLMF game

is with independent targets. Attacks on one target do not affect others. If followers attack the same target independently, $A_1(\theta')y' = \lambda B y$ covers the solution to $A_2 y' = 0$. Thus, $\text{SOL}(y, \theta')$ can be converted to

$$\text{SOL}(y, \theta') = \{y' \in \Omega_f, \lambda > 0 | A_1(\theta')y' = \lambda B y\}. \quad (6)$$

Also, it is easy to see that (6) is always nonempty. Then we get the following result, which consists with the 'near decomposability' [28].

Corollary 3 *Under Assumptions 3 and 4, for any $\theta' \in \mathbb{R}^m$, a MSSE of $\mathcal{H}^2(\theta')$ is also a HNE if all followers attack the same target.*

3) *With Same Perception*: If there is no cognitive difference, followers' observations of θ_0 are true, i.e., $\theta' = \theta_0$, and all players are involved in an identical game \mathcal{G} . Then the model turns into the SLMF game in [34]. Take $(x_{\text{SSE}}, y_{\text{SSE}})$ as the SSE of \mathcal{G} . The next result reveals a relationship between SSE and NE with the same perception.

Corollary 4 *Under Assumptions 1, 3 and 4, if $\text{SOL}(y_{\text{SSE}}, \theta_0)$ is nonempty, then $(x_{\text{SSE}}, y_{\text{SSE}})$ is also a NE of \mathcal{G} .*

V. ROBUSTNESS ANALYSIS

In this section, we discuss the robustness of MSSE and DSSE. As a complement to HNE, we focus on the misinformation's influence on players' actual utility functions, which refers to players' capacities to keep their profits.

Conveniently, for any $x \in \Omega_l, \theta \in \Theta, i \in \mathbf{P}, k = 1, \dots, K$, let $g_i(x, \theta, k) = x^k U_i^c(\theta, t_k) + (R_l - x^k) U_i^u(\theta, t_k)$. Correspondingly, denote

$$\Gamma_i^1(x, \theta) = \underset{k=1, \dots, K}{\operatorname{argmax}} g_i(x, \theta, k),$$

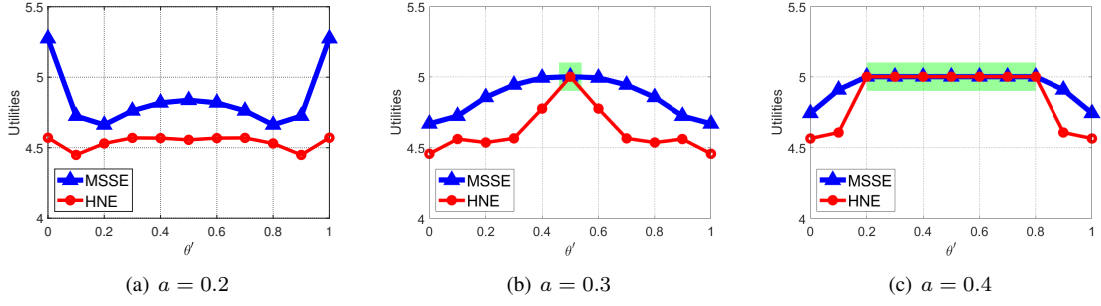


Fig. 5. To find cognitively stable MSSE with different environment settings. The blue line describes the leader's utility U_L of MSSE with different $\theta' \in \Theta$, while the red line stands for the leader's utility U_L of HNE with different $\theta' \in \Theta$. The light green region shows when MSSE is HNE.

$$\Gamma_i^2(x, \theta) = \underset{k=1, \dots, K, k \notin \Gamma_i^1(x, \theta)}{\operatorname{argmax}} g_i(x, \theta, k).$$

Intuitively, $\Gamma_i^1(x, \theta)$ and $\Gamma_i^2(x, \theta)$ represent the corresponding target sets of the two most attractive utilities to the i th follower under the leader's strategy x and the observation θ . Moreover, let

$$\hat{g}_i^r = g_i(x_{SSE}, \theta_0, k), k \in \Gamma_i^r(x_{SSE}, \theta_0), r \in \{1, 2\}, \quad (7)$$

$$\nabla_i^* = \max_{k \in \Gamma_i^1(x_{SSE}, \theta_0)} \|\nabla_{\theta} g_i(x_{SSE}, \theta_0, k)\|. \quad (8)$$

A. For MSSE

In this situation, the imprecise observation mainly affects followers, and additionally, the followers' decisions under different observations also reflect the game's performance of resisting misperception. For instance, in cyber-physical security problems, external perturbation influences the followers' profits through their observations [14]. If the followers' profits do not change under the perturbation, the game has a strong anti-jamming capacity. In the view of bounded rationality such as computational constraints, emotion, and habitual thoughts, players have an inherent observation error [12]. If the profits of followers remain unchanged under bounded rationality, the game is said to be robust to the inherent systemic uncertainty. Moreover, for the accidental error, it reveals the tolerance of the model for the random internal uncertainty, if the accidental error does not change the followers' profits [30].

For $\theta' \in \mathbb{R}^m$, let $\mathbf{y}_{MSSE}(\theta')$ be the followers' the MSSE strategy of $\mathcal{H}^2(\theta')$ and $(x_{SSE}, \mathbf{y}_{SSE})$ be the SSE of $\mathcal{G}(\theta_0)$. We are interested in the subset $\delta_{\theta} \subseteq \Theta$ such that

$$U_i(x_{SSE}, (\mathbf{y}_{SSE})_i, \theta_0) = U_i(x_{SSE}, (\mathbf{y}_{MSSE})_i(\theta'), \theta_0), \forall \theta \in \delta_{\theta}, i \in \mathbf{P}, \quad (9)$$

which is regarded as the robustness set of MSSE. The following result shows the robustness of MSSE, whose proof can be found in Appendix C.

Theorem 4 Under Assumptions 1-4,

- 1) there exists a convex subset $\delta_{\theta} \subseteq \Theta$ satisfying (9) with nonempty $\operatorname{int}(\delta_{\theta})$;
- 2) moreover, if $U_i^c(\theta, t_k)$ and $U_i^u(\theta, t_k)$ are convex and ς -Lipschitz continuous in $\theta \in \Theta$ for all $i \in \mathbf{P}, k = 1, \dots, K$,

there exists $\delta_{\theta} = \{\theta \in \Theta : \|\theta - \theta_0\| < \Delta\theta\}$ satisfying (9) such that

$$\Delta\theta = \min_{i \in \mathbf{P}} \frac{\hat{g}_i^1 - \hat{g}_i^2}{\nabla_i^* + \varsigma R_l},$$

where \hat{g}_i^1 and \hat{g}_i^2 are from (7), and ∇_i^* is according to (8).

Conclusion 1) of Theorem 4 shows that there is always a nonempty subset of the observation parameter such that the MSSE is robust for the followers. Additionally, 2) of Theorem 4 gives a lower bound for players to ignore the misperception if the game model satisfies the convexity and Lipschitz continuity.

B. For DSSE

In the deception situation, the leader deceives followers by manipulating followers' observations, and followers are unaware of the deception. Obviously, the leader will not deceive if the implementation does not increase its own profit [33], and it always needs to spend energy for deception [35]. Therefore, the deceiver decides to cheat when the rewards exceed the lower bound of the deceptive energy. Moreover, the ridiculous and outrageous deception may cause followers' suspicions, which may lead to the collapse of the model [21].

Let $(x_{DSSE}, \mathbf{y}_{DSSE}, \theta^*)$ be the DSSE of $\mathcal{H}^2(\delta_{\theta})$ for the deceptive set δ_{θ} . We are interested in the subset $\delta_{\theta} \subseteq \Theta$ such that

$$U_l(x_{SSE}, \mathbf{y}_{SSE}) = U_l(x_{DSSE}, \mathbf{y}_{DSSE}), \forall \theta \in \delta_{\theta}, \quad (10)$$

which is regarded as the robustness set of DSSE. The following theorem reveals the robustness of DSSE, whose proof can be found in Appendix D.

Theorem 5 Under Assumptions 1-5,

- 1) there exists a convex subset $\delta_{\theta} \in \Theta$ satisfying (10) with nonempty $\operatorname{int}(\delta_{\theta})$;
- 2) moreover, if $U_i^c(\theta, t_k)$ and $U_i^u(\theta, t_k)$ are convex and ς -Lipschitz continuous in $\theta \in \Theta$ for all $k = 1, \dots, K, i \in \mathbf{P}$, there exists $\delta_{\theta} = \{\theta \in \Theta : \|\theta - \theta_0\| < \Delta\theta\}$ satisfying (10) such that

$$\Delta\theta = \min_{i \in \mathbf{P}} \frac{\hat{g}_i^1 - \hat{g}_i^2}{2\varsigma R_l},$$

where \hat{g}_i^1 and \hat{g}_i^2 are from (7), and ∇_i^* is according to (8).

Conclusion 1) of Theorem 5 shows that there is always a nonempty subset of the observation parameter such that the

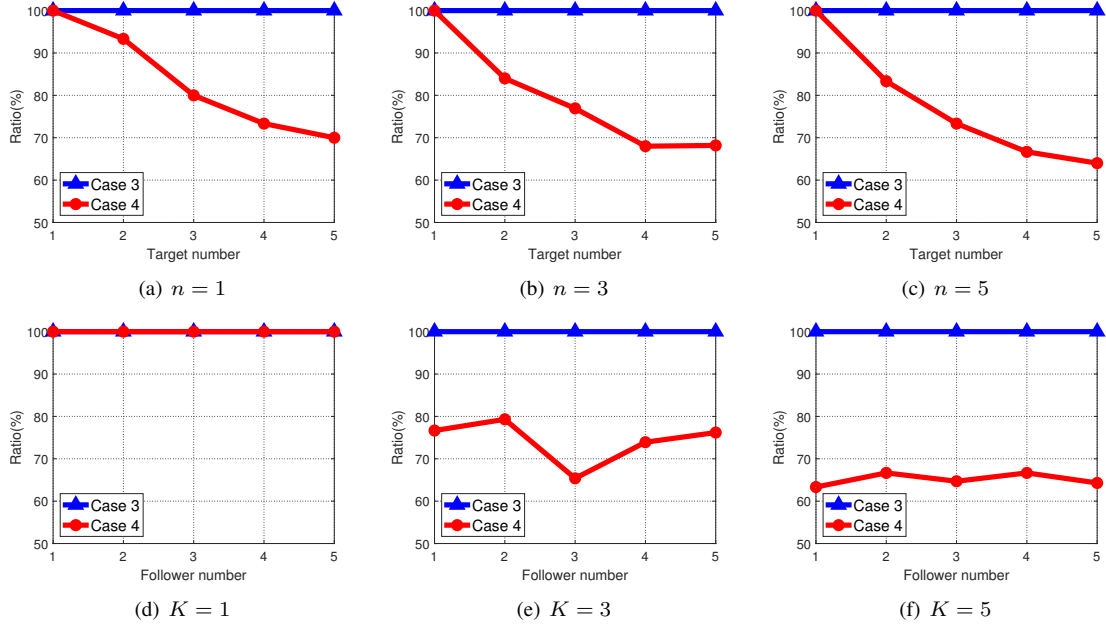


Fig. 6. Ratios of the two cases in deception. The x -axis is for the target number in (a)-(c), and for the follower number in (d)-(f). y -axis is for ratios of *Case 3* and *Case 4* in 30 instances, which are depicted in blue and red, respectively.

leader does not implement deception, since tiny deception does not bring the leader more benefits. Moreover, 2) of Theorem 5 gives a lower bound if utility functions satisfy the convexity and Lipschitz continuity. From the perspective of energy, if the leader wants more benefits from deception, it needs to pay energy no less than the lower bound $\Delta\theta$. Therefore, it can be regarded as a tradeoff for the leader.

Remark 7 The robustness of the MSSE indicates that followers can ignore the misperception, while the robustness of the DSSE means the leader does not implement deception in this region. On the other hand, the proofs of robustness under misperception and deception are different. We consider Assumption 5 in the robustness analysis of deception since the deceptive strategy is affected by followers' utility functions in different targets. Moreover, the proof for the misperception focuses on followers' profits under the fixed leader's strategy, while the proof to handle the deception counts in the influence of the leader's deceive strategy on followers' actions.

VI. EXPERIMENT

In this section, we provide numerical simulations for the stability and robustness of MSSE and DSSE.

A. Stable Condition for MSSE

1) Inspired by the single-leader-single-follower game in infrastructures protection problems [1], with misperception θ' , we verify Theorem 2 by a numerical simulation. We consider models for $K = 10, 15, \dots, 50$ when $n = 5, 10, 15$, and other models for $n = 10, 15, \dots, 50$ when $K = 5, 10, 15$, respectively. In each model, we randomly generate 100 instances as follows. For the leader, $U_l^c(t_k)$ and $U_l^u(t_k)$ are uniformly generated in the ranges $[5, 10]$ and $[0, 5]$, while for the i th

follower, $U_i^c(\theta', t_k)$ and $U_i^u(\theta', t_k)$ are uniformly generated in the ranges $[0, 5]$ and $[5, 10]$. R_l and R_i are uniformly generated in the range $[1, 5]$. Moreover, we compute MSSE of $\mathcal{H}^2(\theta')$ by the extension of the mixed-integer linear program [1]:

$$\begin{aligned} \max_{x, y, a} \quad & \sum_{k=1}^K \sum_{i=1}^n y_i^k R_i (x^k U_i^c(t_k) + (R_l - x^k) U_l^u(t_k)), \\ \text{s.t.} \quad & 0 \leq a_i - R_i g_i(x, \theta', k) \leq (1 - y_i^k) M, \\ & \sum_{k=1}^K x^k = R_l, x^k \geq 0, \sum_{k=1}^K y_i^k = 1, y_i^k \in \{0, 1\}, \\ & a = [a_1, \dots, a_n]^T \in \mathbb{R}^n, \forall i \in \mathbf{P}, k = 1, \dots, K, \end{aligned} \quad (11)$$

where $M = 10^9$ is a sufficiently large number. Take the MATLAB toolbox YALMIP [48] to solve (11) with the terminal condition $\frac{\bar{U}^q - \underline{U}^q}{|\underline{U}^q|} < 10^{-6}$, where \bar{U}^q and \underline{U}^q are the upper and lower bounds of the objective function in q th iteration. Set $(x_{\text{MSSE}}, y_{\text{MSSE}})$ as the MSSE strategy of each instance.

Case 1: $(x_{\text{MSSE}}, y_{\text{MSSE}})$ is a HNE when $\text{SOL}(y_{\text{MSSE}}, \theta')$ is nonempty.

Case 2: $\text{SOL}(y_{\text{MSSE}}, \theta')$ is nonempty when $(x_{\text{MSSE}}, y_{\text{MSSE}})$ is a HNE.

In Fig. 4, the ratio of *Case 1* is always 100%, which verifies Theorem 2. Also, the ratio of *Case 2* is always larger than 85%. Therefore, when $(x_{\text{MSSE}}, y_{\text{MSSE}})$ is a HNE, the stable condition of Theorem 2 can cover most instances.

2) Consider a single-leader-two-followers model in MTD problems [32]. Take $n = K = 2$, $R_l = R_i = 1$, $\Theta = [0, 1]$, $U_2^c(\theta', t_1) = 0.041((\theta' - 0.5)^2 - 10 + a)^2 + 4.305$, $U_2^c(\theta', t_2) = 0$, $U_2^u(\theta', t_1) = -0.05((\theta' - 0.5)^2 + 10 - a)^2 + 5.1532$, and $U_2^u(\theta', t_2) = -0.004((\theta' - 0.5)^2 - 10 + a)^2 + 0.82$, where $a \in \mathbb{R}$ is a parameter in attackers' migration cost. Set $a = 0.2, 0.3$, and 0.4 in Fig. 5(a), 5(b), and 5(c), respectively. In Fig. 5(a), MSSE is always not HNE, and no cognitively stable MSSE

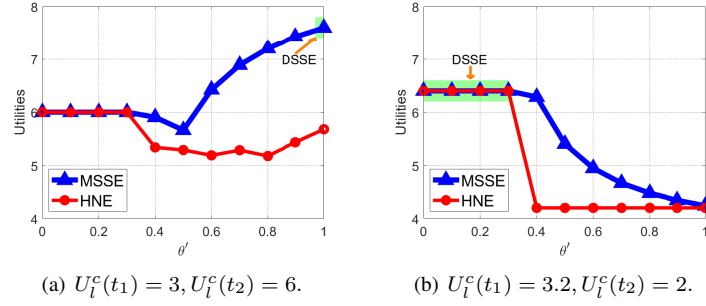


Fig. 7. To find cognitively stable DSSE with different environment settings. The blue line describes for the leader's utility U_L of MSSE with different $\theta' \in \Theta$, while the red line describes the leader's utility U_L of HNE with different $\theta' \in \Theta$. Besides, the light green region exhibits U_L of DSSE since the leader aims to maximize its own utility among all possible θ' .

can be found by Theorem 2. Further, in Fig. 5(b), there is only one cognitively stable MSSE when $\theta' = 0.5$. In this case, it is usually hard for the player to reach the cognitively stable MSSE in MTD problems [32], since the probability for finding such a singleton is zero. However, by verifying the stable condition in Theorem 2, we obtain a stable MSSE precisely and conveniently. Fig. 5(c) shows a similar result, and we can improve the efficiency to find a cognitively stable MSSE once the stable condition in Theorem 2 is verified.

B. Stable Condition for DSSE

1) Similar to security problems in deployed systems [17], we verify Theorem 3 by a numerical simulation. We consider models for $K = 1, \dots, 5$ when $n = 1, 3, 5$, and other models for $n = 1, \dots, 5$ when $K = 1, 3, 5$, respectively. In each model, we randomly generate 30 instances as follows. $U_L^u(t_k), U_L^c(t_k), R_i$, and R_l are uniformly generated in the range $[5, 10]$, and $U_L^c(t_k), U_i^u(t_k)$ are uniformly generated in the range $[0, 5]$. Take $\Theta = [0, 5]^{nK} \subset \mathbf{R}^{nK}$. Concretely, for any $\theta' \in \Theta$, $\theta' = [\theta'_{1,1}, \dots, \theta'_{1,K}, \dots, \theta'_{n,1}, \dots, \theta'_{n,K}]^T$, where $\theta'_{i,k} \in [0, 5]$. For the followers under the observation θ' , set $U_i^c(\theta', t_k) = U_i^c(t_k) + \theta'_{i,k}$ and $U_i^u(\theta', t_k) = U_i^u(t_k) + \theta'_{i,k}$. We compute DSSE of $\mathcal{H}^2(\Theta)$, similar to (11). Take $(x_{DSSE}, y_{DSSE}, \theta^*)$ as the DSSE strategy of each instance.

Case 3: (x_{DSSE}, y_{DSSE}) is a HNE when the leader is able to trick followers into attacking target $t_{K_{max}}$, where $K_{max} \in \arg\max_{k=1, \dots, K} U_L^c(t_k)$.

Case 4: the leader is able to trick followers into attacking target $t_{K_{max}}$ when the DSSE strategy is a HNE.

The above two cases are represented in blue lines and red lines.

In Fig. 6, the ratio of Case 3 is always 100%, which verifies Theorem 3. Also, the ratio of Case 4 is always larger than 60%. Hence, when the DSSE is a HNE, the stable condition of Theorem 3 can cover many instances.

2) Consider a single-leader-two-followers model in infrastructures protection problems [33]. Take $n=K=2, R_l=R_i=1, \Theta=[0, 1], U_L^u(t_1)=2, U_L^u(t_2)=1, U_L^c(\theta', t_1)=3, U_L^c(\theta', t_2)=1, U_L^u(\theta', t_1)=4, U_L^u(\theta', t_2)=2, U_2^c(\theta', t_1)=-2.52\theta'+1.428, U_2^c(\theta', t_2)=0, U_2^u(\theta', t_1)=-0.4\theta'+2, \text{ and } U_2^u(\theta', t_2)=-0.16\theta'+0.504$. Also, take $U_L^c(t_1)=3, U_L^c(t_2)=6$ in Fig. 7(a) and $U_L^c(t_1)=3.2, U_L^c(t_2)=2$ in Fig. 7(b), where $U_L^c(t_k)$,

the reward for protecting t_k , is different in situations with different leader's forms. In the environment setting of Fig. 7(a), no DSSE is HNE. Neither can the previous work [33] find the cognitively stable DSSE, nor can our proposed condition in Theorem 3 be verified. However, the phenomenon changes in Fig. 7(b), because we can find a cognitively stable DSSE, i.e., HNE, once the stable condition in Theorem 3 is satisfied. Thus, our proposed framework and conclusion in Theorem 3 actually provide a way to tell the differences among various environment settings when DSSE is HNE.

C. Robustness of MSSE: in Counterterrorism Problems

Inspired by the counterterrorism problems with multiple attack forms [29], we consider that the American government wants to defend against the criminals with different attack forms, including armed assaults, bombing/explosion, assassinations, facility/infrastructure attacks, hijackings, and hostage taking. Regard the government as a leader and the criminals with 6 attack forms as followers. Besides, 'New York City', 'Los Angeles', 'San Francisco', 'Washington, D.C.', and 'Chicago' are ranked as the top five risky urban areas in America. Then we regard the 5 cities as targets such as the first target for 'New York City'. Suppose that all players have \$1 million budgets, i.e., $R_l=1$ and $R_i=1$ for $i \in \mathbf{P}$. Take $U_L^c(t_k), U_L^u(t_k), U_i^c(t_k), U_i^u(t_k) \in [0, 0.7]$ as utilities under the true observation. Also, followers have a success probability of 0.2, considering that the United States can interdict some attack plots [29]. Therefore, followers have a false observation of the success rate as $p_{i,k}(\theta') = d_{i,k}\theta' + 0.2$, where $\theta' \in \Theta = [-0.2, 0.2]$, $\theta_0 = 0$ and $d_{i,k}$ is generated in the range $[-1, 1]$. Hence, $U_i^c(\theta', t_k) = p_{i,k}(\theta')U_i^c(t_k)$ and $U_i^u(\theta', t_k) = p_{i,k}(\theta')U_i^u(t_k)$ are the utilities perceived by the i th follower.

Fig. 8 shows the utilities of all followers, where the x -axis represents the value of θ' and the y -axis is for the true utility of each follower under the observation θ' . The blue cylinders are followers' true utilities when they select the MSSE strategy under different θ' . In Fig. 8, the light blue region is in $|\theta'| \leq 0.045$ and the light red region is in $|\theta'| \leq 0.075$. Actually, the utility under $\theta' = 0$ is the real one with no misperception. Notice that all followers' utilities are unchanged when $|\theta'| \leq 0.075$, which is consistent with Theorem 4 since $\Delta\theta = 0.045 < 0.075$.

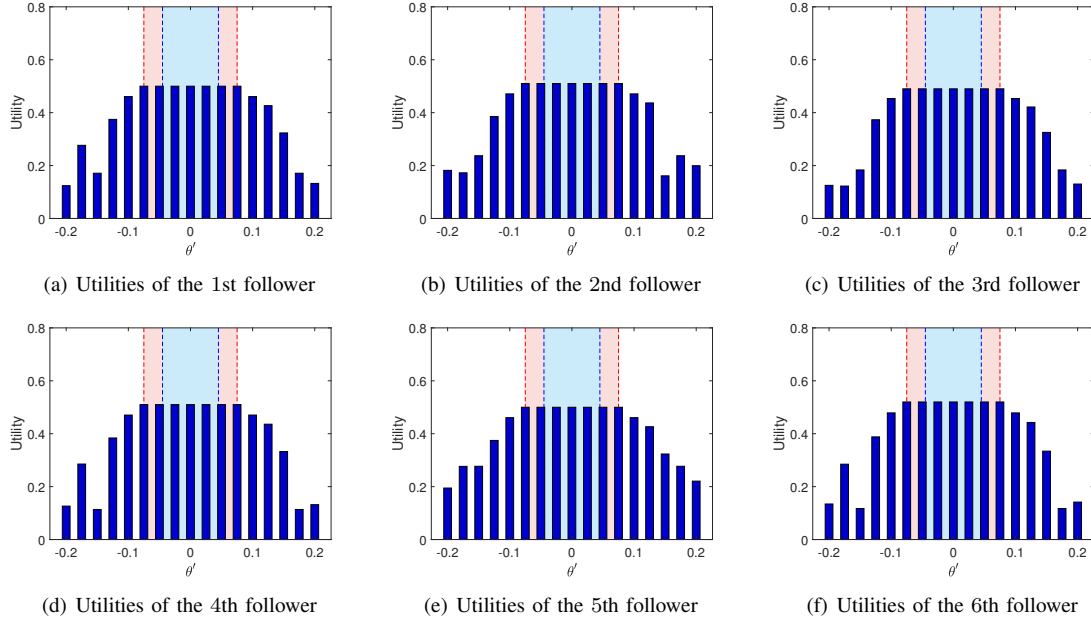


Fig. 8. Utilities of followers in the counterterrorism problem. θ' denotes the followers' false observation of the parameter, which affects success rate $p_{i,k}(\theta')$ for all i, k . The light blue region describes robust bounds of MSSE according to Theorem 4, and the light red region shows bounds referring to θ' , where all followers' utilities are invariant in this instance.

D. Robustness of DSSE: in CPS

Similar to the CPS with two players [6], we consider one network administrator (leader) and one hacker (follower). The follower invades the leader with many attack methods such as 'malware', 'web-based attacks', 'denial-of-service', 'malicious insiders', 'phishing and social engineering', 'malicious code', 'stolen devices', 'ransomware', and 'botnets'. Regard the 9 attack methods as 9 targets such as the first target for 'malware'. Each player has \$1 million budgets, i.e., $R_l = 1$, $R_f = 1$ and they allocate funds to the 9 targets. Denote $U_l^c(t_k), U_l^u(t_k), U_f^c(t_k), U_f^u(t_k) \in [0, 2.5]$ as the values for different targets. Moreover, the network administrator makes some observable properties of a system such as TCP/IP stack appear different from what it actually is, and then the hacker probes the system. Concretely, denote $\Theta = [-1, 1]$ as the deceptive set and $\theta_0 = 0$ as the true value. For $\theta' \in \Theta$, $U_f^u(\theta', t_k) = U_f^u(t_k) + d_k \theta'^2$ and $U_f^c(\theta', t_k) = U_f^c(t_k)$ are utilities perceived by the follower, where d_k is generated in the range $D \subset \mathbb{R}$.

Fig. 9 shows the utilities of the leader with $D = (0, 1), (0, 2)$, and $(0, 3)$, respectively. The blue cylinders are the leader's utilities if the leader deceives as θ' . The light blue regions are in $|\theta'| \leq 0.45$ in 9(a), $|\theta'| \leq 0.225$ in 9(b) and $|\theta'| \leq 0.15$ in 9(c). Also, the light red regions are in $|\theta'| \leq 0.9$, $|\theta'| \leq 0.6$, and $|\theta'| \leq 0.5$, respectively. Besides, the blue cylinder under $\theta' = 0$ is the utility if the leader does not deceive. Notice that in 9(a), the leader's utility under $|\theta'| \leq 0.9$ is no larger than that under $\theta' = 0$. It is consistent with Theorem 5 that the DSSE strategy is robust for the leader since $\Delta\theta = 0.45 < 0.9$. Moreover, in Fig. 9(a), if the leader wishes to benefit more from deception, the deception strategy needs to exceed $|\theta'| \geq 0.9 > \Delta\theta$. Similar conclusions can

be found in Fig. 9(b) and 9(c). In fact, the robust boundary decreases as the bound of the parameter set D increases, which is also consistent with Theorem 5.

VII. CONCLUSIONS

In this paper, we have investigated the SLMF Stackelberg security game by virtue of the second-level Stackelberg hypergame. We have provided a novel criterion to evaluate both the strategic and cognitive stability of games with misinformation based on HNE. Moreover, we have provided two different stable conditions to connect MSSE and DSSE with HNE. Also, we have analyzed the influences of misperception and deception by the robustness of the MSSE and DSSE strategies. Finally, we have presented numerical experiments for the validity and broad applicability of our results.

APPENDIX A PROOF OF THEOREM 2

Denote $E_i(x, \theta') = \max_{y_i \in \Omega_i} U_i(x, y_i, \theta')$, $E(x, \theta') = \sum_{i=1}^n E_i(x, \theta')$, and $E^*(\theta') = \min_{x \in \Omega_l} E(x, \theta')$. The leader's strategy $x \in \Omega_l$ is said to be a *Minimax Strategy* if $E(x, \theta') = E^*(\theta')$. Then the following proof consists of three steps. Step 1 shows the relationship between the leader's utility function and followers' ones. Step 2 reveals that x_{MSSE} is a leader's *Minimax Strategy*. Step 3 shows that (x_{MSSE}, y_{MSSE}) is HNE.

Step 1: Since $(y', \lambda) \in \text{SOL}(y, \theta')$, $\lambda > 0$, and $A_1(\theta')y' = \lambda B y$. Thus,

$$\sum_{i=1}^n (y')_i^k \frac{U_i^u(\theta', t_k) - U_i^c(\theta', t_k)}{U_l^c(t_k) - U_l^u(t_k)} = \lambda \sum_{i=1}^n y_i^k, \quad \forall k = 1, \dots, K.$$

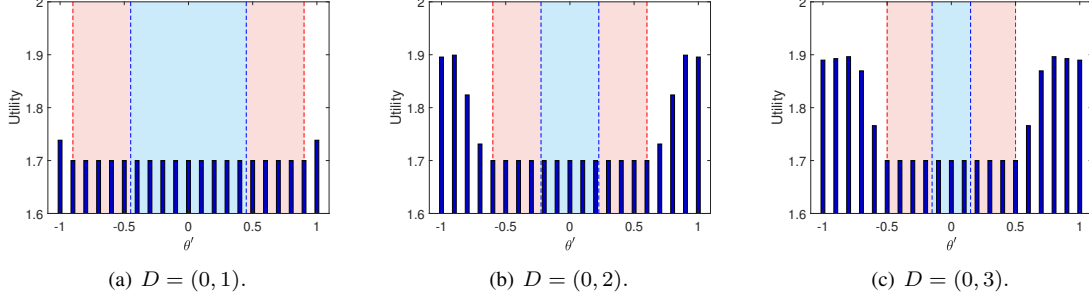


Fig. 9. Utilities of the leader in CPS. θ' leads to followers' perceived utility $U_f^u(\theta', t_k)$ for all k . The light blue region describes robust bounds of DSSE according to Theorem 5, and the light red region describes bounds referring to θ' , where all followers' utilities are invariant in this instance.

Then

$$\begin{aligned}
 & U_l(x, \mathbf{y}) - U_l(x', \mathbf{y}) \\
 &= \sum_{k=1}^K \sum_{i=1}^n y_i^k (x^k - (x')^k) (U_l^c(t_k) - U_l^u(t_k)) \\
 &= -\frac{1}{\lambda} \sum_{k=1}^K \sum_{i=1}^n (\mathbf{y}')_i^k (x^k - (x')^k) (U_i^c(\theta', t_k) - U_i^u(\theta', t_k)).
 \end{aligned}$$

Clearly,

$$\begin{aligned}
 & \sum_{i=1}^n U_i(x, \mathbf{y}', \theta') - \sum_{i=1}^n U_i(x', \mathbf{y}', \theta') \\
 &= \sum_{k=1}^K \sum_{i=1}^n (\mathbf{y}')_i^k (x^k - (x')^k) (U_i^c(\theta', t_k) - U_i^u(\theta', t_k)).
 \end{aligned}$$

Therefore,

$$U_l(x, \mathbf{y}) - U_l(x', \mathbf{y}) = \frac{1}{\lambda} \left(\sum_{i=1}^n U_i(x', (\mathbf{y}')_i, \theta') - \sum_{i=1}^n U_i(x, (\mathbf{y}')_i, \theta') \right).$$

Since $\lambda > 0$, $U_l(x, \mathbf{y}) > (=) U_l(x', \mathbf{y})$ for $x, x' \in \Omega_l$, and $\mathbf{y} \in \Omega_f$ if and only if $\sum_{i=1}^n U_i(x, \mathbf{y}', \theta') < (=) \sum_{i=1}^n U_i(x', \mathbf{y}', \theta')$ for $x, x' \in \Omega_l$, $\mathbf{y} \in \Omega_f$, $\theta' \in \mathbb{R}^m$, and $(\mathbf{y}', \lambda) \in \text{SOL}(\mathbf{y}, \theta')$.

Step 2: By the definition of E^* , $E(x_{MSSE}, \theta') \geq E^*(\theta')$ is always true. Suppose

$$E(x_{MSSE}, \theta') > E^*(\theta'). \quad (12)$$

Consider x^* as the leader's *Minimax Strategy*, where $E(x^*, \theta') = E^*(\theta')$. For $(\mathbf{y}', \lambda) \in \text{SOL}(\mathbf{y}_{MSSE}, \theta')$,

$$\sum_{i=1}^n U_i(x^*, \mathbf{y}', \theta') \leq E^*(\theta').$$

Denote $S(\mathbf{y}) = \{(i, k) | y_i^k \neq 0, i \in \mathbf{P}, k = 1, \dots, K\}$. For any $(i, k_1), (i, k_2) \in S(\mathbf{y}_{MSSE})$, the target k_1 and target k_2 have the same appeal to the i th follower. For any $(i, k) \in S(\mathbf{y}_{MSSE})$, set

$$M_i(x_{MSSE}) = x_{MSSE}^k U_i^c(\theta', t_k) + (R_l - x_{MSSE}^k) U_i^u(\theta', t_k). \quad (13)$$

Additionally, $A_2 \mathbf{y}' = 0$ implies $(\mathbf{y}')_i^k = 0$ if $(\mathbf{y}_{MSSE})_i^k = 0$. Then $S(\mathbf{y}') \subseteq S(\mathbf{y}_{MSSE})$. Therefore, for any $(i, k) \in S(\mathbf{y}')$, (13) also holds. Then

$$\begin{aligned}
 & U_i(x_{MSSE}, \mathbf{y}', \theta') \\
 &= \sum_{k=1}^n (\mathbf{y}')_i^k (x_{MSSE}^k U_i^c(\theta', t_k) + (R_l - x_{MSSE}^k) U_i^u(\theta', t_k)) \\
 &= \sum_{(\mathbf{y}')_i^k \neq 0} M_i(x_{MSSE}) = R_i M_i(x_{MSSE}).
 \end{aligned}$$

Similarly, $U_i(x_{MSSE}, \mathbf{y}_{MSSE}, \theta') = R_i M_i(x_{MSSE})$. Thus,

$$U_i(x_{MSSE}, \mathbf{y}', \theta') = U_i(x_{MSSE}, (\mathbf{y}_{MSSE})_i, \theta'). \quad (14)$$

As a result, \mathbf{y}' is the followers' best response strategy to x_{MSSE} under the observation θ' . Then

$$E(x_{MSSE}, \theta') = \sum_{i=1}^n \max_{y_i \in \Omega_i} U_i(x_{MSSE}, y_i, \theta') = \sum_{i=1}^n U_i(x_{MSSE}, \mathbf{y}', \theta').$$

Consequently, $\sum_{i=1}^n U_i(x^*, \mathbf{y}', \theta') < \sum_{i=1}^n U_i(x_{MSSE}, \mathbf{y}', \theta')$. According to [1], there exists x' such that $\mathbf{y}_{MSSE} \in \mathbf{BR}(x', \theta')$ and $\sum_{i=1}^n U_i(x', \mathbf{y}', \theta') < \sum_{i=1}^n U_i(x_{MSSE}, \mathbf{y}', \theta')$. Recalling **Step 1**, $U_l(x', \mathbf{y}_{MSSE}) > U_l(x_{MSSE}, \mathbf{y}_{MSSE})$, which contradicts that x_{MSSE} is the leader's MSSE strategy. Thus, (12) does not hold. As a result, $E(x_{MSSE}, \theta') = E^*(\theta')$, which indicates that x_{MSSE} is the leader's *Minimax Strategy* and \mathbf{y}_{MSSE} is the corresponding strategies of followers.

Step 3: Note that $E(x_{MSSE}, \theta') = E^*(\theta')$ and $\mathbf{y}_{MSSE} \in \mathbf{BR}(x_{MSSE}, \theta')$. By (14), $\mathbf{y}' \in \mathbf{BR}(x_{MSSE}, \theta')$. Define another associated zero-sum game $\bar{\mathcal{G}}$ with two players denoted as $\{1, 2\}$ in $\bar{\mathcal{G}}$. The strategy set of player 1 is Ω_l and the strategy set of player 2 is Ω_f . For any $x \in \Omega_l$, $\mathbf{y} \in \Omega_f$, $\bar{U}_1(x, \mathbf{y}, \theta') = -\sum_{i=1}^n U_i(x, y_i, \theta')$ and $\bar{U}_2(x, \mathbf{y}, \theta') = \sum_{i=1}^n U_i(x, y_i, \theta')$ are the utility functions of player 1 and player 2, respectively. Each player aims at maximizing its utility functions.

For any $\mathbf{y} \in \Omega_f$, since $\mathbf{y}'_i \in \mathbf{BR}_i(x_{MSSE}, \theta')$, \mathbf{y}' is the best response strategy to x_{MSSE} in $\bar{\mathcal{G}}$. Moreover,

$$E(x, \theta') = \sum_{i=1}^n \max_{y_i \in \Omega_i} U_i(x, y_i, \theta') = \max_{y \in \Omega_f} \bar{U}_2(x, \mathbf{y}, \theta').$$

Since $E(x_{MSSE}, \theta') = E^*(\theta')$,

$$x_{MSSE} \in \operatorname{argmin}_{x \in \Omega_l} \max_{y \in \Omega_f} \bar{U}_2(x, y, \theta').$$

Then x_{MSSE} is the *Minimax Strategy* in $\bar{\mathcal{G}}$. By Theorem 3.2 in [47], (x_{MSSE}, y') is also a NE of $\bar{\mathcal{G}}$. Then x_{MSSE} is also the best response strategy to y' in $\bar{\mathcal{G}}$. For any $x \in \Omega_l$, $\bar{U}_1(x_{MSSE}, y', \theta') \geq \bar{U}_1(x, y', \theta')$. Therefore, $\sum_{i=1}^n U_i(x_{MSSE}, y'_i, \theta') \leq \sum_{i=1}^n U_i(x, y'_i, \theta')$. Then by **Step 1**,

$$U_l(x_{MSSE}, y_{MSSE}, \theta') \geq U_l(x, y_{MSSE}, \theta'). \quad (15)$$

Because (15) holds for any $x \in \Omega_l$, x_{MSSE} is the best response strategy to y_{MSSE} in $\mathcal{H}^2(\theta')$. Then (x_{MSSE}, y_{MSSE}) is HNE of $\mathcal{H}^2(\theta')$. \square

APPENDIX B PROOF OF THEOREM 3

Clearly, there exists $\theta^* \in \Theta$ such that $(y^*)_i^{K_{max}} = R_i$, $x^* \in \Omega_l$, and $y^* \in \mathbf{BR}(x^*, \theta^*)$, where (x^*, y^*) is the decision result under the observation θ . Thus, $\text{SOL}(y^*, \theta^*)$ has a solution

$$\begin{cases} \lambda = \frac{\sum_{i=1}^n R_i \frac{U_i^u(\theta^*, t_{K_{max}}) - U_i^c(\theta^*, t_{K_{max}})}{U_i^c(t_{K_{max}}) - U_i^u(t_{K_{max}})}}{\sum_{i=1}^n R_i}, \\ (y^*)_i^{K_{max}} = R_i, \forall i \in \mathbf{P}, \\ (y^*)_i^k = 0, \forall l \neq K_{max}, i \in \mathbf{P}. \end{cases}$$

By Theorem 2, x^* is the best response strategy to y^* and $(x^*)^{K_{max}} = R_l$. Thus,

$$\begin{aligned} & U_l(x^*, y^*) \\ &= \sum_{k=1}^K \left(\sum_{i=1}^n (y^*)_i^k \right) ((x^*)^k U_l^c(t_k) + (R_l - (x^*)^k) U_l^u(t_k)) \\ &= \sum_{i=1}^n R_i R_l U_l^c(t_{K_{max}}) = R_l U_l^c(t_{K_{max}}) \left(\sum_{i=1}^n R_i \right). \end{aligned}$$

Since $K_{max} \in \operatorname{argmax}_{k \in K} U_l^c(t_k)$, for all $k = 1, \dots, K$, $U_l^c(t_{K_{max}}) \geq U_l^c(t_k)$. By Assumption 3, $U_l^c(t_{K_{max}}) \geq U_l^u(t_k)$. Then, for any $\theta \in \Theta$, $y \in \Omega_f$, $x \in \Omega_l$, we have

$$\begin{aligned} & U_l(x, y) \\ &\leq \sum_{k=1}^K \sum_{i=1}^n y_i^k (x^k U_l^c(t_{K_{max}}) + (R_l - x^k) U_l^c(t_{K_{max}})) \\ &= \sum_{k=1}^K (x^k U_l^c(t_{K_{max}}) + (R_l - x^k) U_l^c(t_{K_{max}})) \sum_{i=1}^n y_i^k \\ &= \sum_{k=1}^K R_l U_l^c(t_{K_{max}}) \sum_{i=1}^n y_i^k = R_l U_l^c(t_{K_{max}}) \left(\sum_{i=1}^n R_i \right). \end{aligned}$$

Thus, $U_l(x, y) \leq U_l(x^*, y^*)$ for any $x \in \Omega_l$, $y \in \Omega_f$. Therefore, $\theta^* \in \operatorname{argmax}_{\theta' \in \Theta} \max_{x \in \Omega_l, y \in \mathbf{BR}(x, \theta')} U_l(x, y)$ is the optimal deception. Also, (x^*, y^*) is a DSSE of $\mathcal{H}^2(\Theta)$. Besides, x^* is the best response strategy to y^* since $U_l(x, y^*) \leq U_l(x^*, y^*)$ for any $x \in \Omega_l$. Thus, the conclusion follows. \square

APPENDIX C PROOF OF THEOREM 4

1) For any $\alpha_i \in \Gamma_i^1(x_{SSE}, \theta_0)$, $l \notin \Gamma_i^1(x_{SSE}, \theta_0)$, $g_i(x_{SSE}, \theta_0, \alpha_i) > g_i(x_{SSE}, \theta_0, l)$. Since $U_i^c(\theta, t_{\alpha_i})$ and $U_i^c(\theta, t_{\alpha_i})$ are differentiable in $\theta \in \Theta$ by Assumption 2, there is a convex set δ_θ^i such that, for all $\theta \in \delta_\theta^i$, $g_i(x_{SSE}, \theta, \alpha_i) > g_i(x_{SSE}, \theta, l)$. Let $\delta_\theta = \cap_{i=1}^n \delta_\theta^i$. Then $\text{int}(\delta_\theta)$ is nonempty. For any $\theta' \in \delta_\theta$, the i th follower attacks the target in Γ_i , which leads to the same profit as y_{SSE} . Thus, $U_i(x_{SSE}, (y_{SSE})_i, \theta_0) = U_i(x_{SSE}, (y_{MSSE})_i(\theta'), \theta_0)$.

2) For any $\alpha_i \in \Gamma_i^1(x_{SSE}, \theta_0)$, $\beta_i \in \Gamma_i^2(x_{SSE}, \theta_0)$, $l \notin \Gamma_i^1(x_{SSE}, \theta_0)$, we have $g_i(x_{SSE}, \theta_0, \alpha_i) > g_i(x_{SSE}, \theta_0, l)$ and $g_i(x_{SSE}, \theta_0, \beta_i) \geq g_i(x_{SSE}, \theta_0, l)$. Since $U_i^c(\theta, t_k)$ and $U_i^u(\theta, t_k)$ are ς -Lipschitz continuous in $\theta \in \Theta$, for any $\theta, \theta' \in \Theta$,

$$\begin{aligned} |U_i^c(\theta, t_k) - U_i^c(\theta', t_k)| &\leq \varsigma \|\theta - \theta'\|, \\ |U_i^u(\theta, t_k) - U_i^u(\theta', t_k)| &\leq \varsigma \|\theta - \theta'\|. \end{aligned}$$

Thus,

$$\begin{aligned} & |g_i(x_{SSE}, \theta, k) - g_i(x_{SSE}, \theta', k)| \\ &= |x_{SSE}^k (U_i^c(\theta, t_k) - U_i^c(\theta', t_k)) \\ &\quad + (R_l - x_{SSE}^k) (U_i^u(\theta, t_k) - U_i^u(\theta', t_k))| \\ &\leq x_{SSE}^k \varsigma \|\theta - \theta'\| + (R_l - x_{SSE}^k) \varsigma \|\theta - \theta'\| \\ &= \varsigma R_l \|\theta - \theta'\|. \end{aligned}$$

Therefore, for any k , $g_i(x_{SSE}, \theta, k)$ is ςR_l -Lipschitz continuous in $\theta \in \Theta$. Then

$$\begin{aligned} g_i(x_{SSE}, \theta, l) &\leq g_i(x_{SSE}, \theta_0, l) + \varsigma R_l \|\theta - \theta'\| \\ &\leq g_i(x_{SSE}, \theta_0, \beta_i) + \varsigma R_l \|\theta - \theta'\|. \end{aligned} \quad (16)$$

Also, since $U_i^c(\theta, t_k)$ and $U_i^u(\theta, t_k)$ are convex and differentiable in θ , $g_i(x_{SSE}, \theta, k)$ is convex in $\theta \in \Theta$. Thus,

$$g_i(x_{SSE}, \theta, k) - g_i(x_{SSE}, \theta_0, k) \geq \nabla_\theta g_i(x_{SSE}, \theta_0, k)^T (\theta - \theta_0).$$

Take $\nabla_i^{\alpha_i} = \nabla_\theta g_i(x_{SSE}, \theta_0, \alpha_i)$. If $(\nabla_i^{\alpha_i})^T \nabla_i^{\alpha_i} \neq 0$, then, with taking $q_\theta = \frac{(\nabla_i^{\alpha_i})^T (\theta - \theta_0)}{(\nabla_i^{\alpha_i})^T \nabla_i^{\alpha_i}}$, $|q_\theta| \leq \|\theta - \theta_0\|$. Thus,

$$\begin{aligned} & g_i(x_{SSE}, \theta, \alpha_i) - g_i(x_{SSE}, \theta_0, \alpha_i) \\ &\geq \nabla_\theta g_i(x_{SSE}, \theta_0, \alpha_i)^T (\theta - \theta_0) \\ &= -q_\theta (\nabla_i^{\alpha_i})^T \nabla_i^{\alpha_i} \\ &\geq -\|\theta - \theta_0\| (\nabla_i^{\alpha_i})^T \nabla_i^{\alpha_i}. \end{aligned}$$

Obviously, if $(\nabla_i^{\alpha_i})^T \nabla_i^{\alpha_i} = 0$,

$$g_i(x_{SSE}, \theta, \alpha_i) - g_i(x_{SSE}, \theta_0, \alpha_i) \geq 0 = -\|\theta - \theta_0\| (\nabla_i^{\alpha_i})^T \nabla_i^{\alpha_i}.$$

Recalling (16),

$$\begin{aligned} & g_i(x_{SSE}, \theta, \alpha_i) - g_i(x_{SSE}, \theta, l) \\ &\geq g_i(x_{SSE}, \theta_0, \alpha_i) - g_i(x_{SSE}, \theta_0, \beta_i) - ((\nabla_i^{\alpha_i})^T \nabla_i^{\alpha_i} + \varsigma R_l) \|\theta - \theta'\|. \end{aligned}$$

Since $\|\theta - \theta'\| < \Delta\theta = \min_{i \in \mathbf{P}} \frac{\hat{g}_i^1 - \hat{g}_i^2}{\nabla_i^{\alpha_i} + \varsigma R_l}$,

$$\|\theta - \theta'\| < \frac{g_i(x_{SSE}, \theta_0, \alpha_i) - g_i(x_{SSE}, \theta_0, \beta_i)}{(\nabla_i^{\alpha_i})^T \nabla_i^{\alpha_i} + \varsigma R_l}.$$

For any $\alpha_i \in \Gamma_i^1(x_{SSE}, \theta_0)$, $l \notin \Gamma_i^1(x_{SSE}, \theta_0)$,

$$g_i(x_{SSE}, \theta, \alpha_i) > g_i(x_{SSE}, \theta, l).$$

For $i \in \mathbf{P}$, $U_i(x_{SSE}, (y_{SSE})_i, \theta_0) = U_i(x_{SSE}, (y_{MSSE})_i(\theta'), \theta_0)$. \square

APPENDIX D

PROOF OF THEOREM 5

1) By Theorem 4, the leader does not change its strategy under δ_θ according to [17]. Thus, the leader's profit does not change, and $U_l(x_{SSE}, \mathbf{y}_{SSE}) = U_l(x_{DSSE}, \mathbf{y}_{DSSE})$.

2) By Assumption 5, $\Gamma_i^1(x_{SSE}, \theta_0)$ has the unique element. Take $\alpha_i \in \Gamma_i^1(x_{SSE}, \theta_0)$, $\beta_i \in \Gamma_i^2(x_{SSE}, \theta_0)$, $l \notin \Gamma_i^1(x_{SSE}, \theta_0)$. As shown in the proof of Theorem 4, $g_i(x_{SSE}, \theta, l)$ is ςR_l -Lipschitz continuous in $\theta \in \Theta$. Then

$$g_i(x_{SSE}, \theta, l) \leq g_i(x_{SSE}, \theta_0, l) + \varsigma R_l \|\theta - \theta'\|.$$

Since $\beta_i \in \Gamma_i^2(x_{SSE}, \theta_0)$, $g_i(x_{SSE}, \theta_0, \beta_i) \geq g_i(x_{SSE}, \theta_0, l)$. Then $g_i(x_{SSE}, \theta, l) \leq g_i(x_{SSE}, \theta_0, \beta_i) + \varsigma R_l \|\theta - \theta'\|$. Also,

$$g_i(x_{SSE}, \theta, \alpha_i) \geq g_i(x_{SSE}, \theta_0, \alpha_i) - \varsigma R_l \|\theta - \theta'\|.$$

Therefore,

$$\begin{aligned} & g_i(x_{SSE}, \theta, \alpha_i) - g_i(x_{SSE}, \theta, l) \\ & \geq g_i(x_{SSE}, \theta_0, \alpha_i) - g_i(x_{SSE}, \theta_0, \beta_i) - 2\varsigma R_l \|\theta - \theta'\|. \end{aligned}$$

For any θ with $\|\theta - \theta_0\| < \Delta\theta$, since $\Delta\theta = \min_{i \in \mathbf{P}} \frac{\hat{g}_i^1 - \hat{g}_i^2}{2\varsigma R_l}$,

$$\|\theta - \theta_0\| < \frac{g_i(x_{SSE}, \theta_0, \alpha_i) - g_i(x_{SSE}, \theta_0, \beta_i)}{2\varsigma R_l}.$$

Therefore, for $i \in \mathbf{P}$,

$$g_i(x_{SSE}, \theta, l) < g_i(x_{SSE}, \theta, \alpha_i). \quad (17)$$

According to [17] and Assumption 5, (17) holds for any $x_{DSSE} \in \Omega_l$. Thus, the leader does not change its strategy under δ_θ , and $U_l(x_{SSE}, \mathbf{y}_{SSE}) = U_l(x_{DSSE}, \mathbf{y}_{DSSE})$. \square

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