Relative Threshold-Based Event-Triggered Control for Nonlinear Constrained Systems With Application to Aircraft Wing Rock Motion

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***Abstract*—This paper concentrates upon the event-driven controller design problem for a class of nonlinear single input single output (SISO) parametric systems with full state constraints. A varying threshold for the triggering mechanism is exploited, which makes the communication more flexible. Moreover, from the viewpoint of energy conservation and consumption reduction, the system capability becomes better owing to the contribution of the proposed event triggered mechanism. In the meantime, the developed control strategy can avoid the Zeno behavior since the lower bound of the sample time is provided. The considered plant is in a lower-triangular form, in which the match condition is not satisfied. To ensure that all the states retain in a predefined region, a barrier Lyapunov function (BLF) based adaptive control law is developed. Due to the existence of the parametric uncertainties, an adaptive algorithm is presented as an estimated tool. All the signals appearing in the closed-loop systems are then proven to be bounded. Meanwhile, the output of the system can track a given signal as far as possible. In the end, the effectiveness of the proposed approach is validated by an aircraft wing rock motion system.**

***Index Terms*—Barrier Lyapunov function, full state constraints, event-driven control, parametric systems, varying trigger threshold**

I. INTRODUCTION

IN the past few years, the study of controlling nonlinear systems received a great attention due to its various applications both in theory and in reality [1]-[6]. Among those literatures, a good deal of adaptive control approaches have been established by integrating the parameter estimator with the Lyapunov stability theory [7]-[9]. Meanwhile, from a practical point of view, considerable adaptive algorithms [10] [11] [12] [13] have been also proposed for several real world systems, such as multi-variable industrial processes, two-link planar robot arms [14], bouncing ball systems and flexible air-breathing hypersonic vehicles [15]. Nonetheless, the above strategies are all realized on the premise that the time-triggered control approach is necessary, in which the controller implementation is periodic [16] [17]. That is to say, the control law is adjusted simultaneously at every operation instant. Obviously, it is unreasonable when the considered plants are resource-limited. In all of aforementioned results, the control inputs have to be transmitted to the executing agency consecutively. And then, it may occupy large amount of capacity of the communication channel [18]. As a matter of fact, when the communication resource is restricted, how to alleviate the communication burden of the controlled systems become very important as a result. The event-triggered control (ETC), as pointed out in [19]-[21] and the references therein, has the potential to overcome the mentioned problem.

In the event-triggered systems, the control signal is tuned at some certain time instants, which is not continuous. In addition, the control law in the frame of the ETC approach updates itself aperiodic when a defined event occurs [22]-[24]. In this case, the energy consumption of the controller component is mitigated, which causes the actuator wear to slow down. At the same time, the computational burden and the communication resource may be also reduced [2] [25]. Based on these characteristics, a great deal of representative contributions for linear or nonlinear systems have been reported [19]-[29]. Specifically speaking, the problem of event-based adaptive controller design is realized for a class of nonlinear strict-feedback systems in [30] via the switching threshold method, in which the assumption of input-to-state stability is not required. It is worth mentioning that the studied systems in [19]-[31] are required to be known in advance. When the system uncertainty exists, such as unknown internal dynamics, the results cannot be exploited to deal with this issue. In addition, in all the control strategies mentioned above, the output constraint or the state constraints are not taken into account. This point has not aroused the attention of scholars, which motivates us to present this paper.

The constraints are omnipresent in real world applications, such as electrostatic microactuator [32], active suspension [33], continuous stirred tank reactor [34], robotic manipulator [36], and etc. Clearly, the research of constraint issue is of great importance, and how to compensate for the constraint problem generated by the physical devices is a challenge objective [37]-[43]. To cope with the constraint problem, the barrier Lyapunov function (BLF) based adaptive control scheme for nonlinear strict feedback systems was proposed by the pioneer, Professor S. S. Ge, in 2009 [44]. The value of BLF will increase to infinity while some elements are close

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to corresponding limits [45] [46]. And then, by applying the control signal to the closed-loop systems, the constraints are never violated [47] [48]. However, it should be noted that the previous BLF-based control strategies neglect the problem of how to decrease the overspending of communication resources. If the bandwidth of signal communication channel is restricted under some practical circumstances, the aforementioned methods are limited to be local and will be out at elbows. Hence, it is meaningful to combine the ETC with the BLF together to solve the state constraint problems for a class of nonlinear parametric systems to decrease the numbers of update for the controllers.

As discussed in the above contents, this paper proposes a BLF-based event-triggered controller for a class of nonlinear single input single output (SISO) strict-feedback parametric systems. All the states are needed to stay in the constrained interval. By handling the constrained problem, we put forward a novel control approach to get the feasibility constraint solution. Noting that the lower bound of the inter-implementing scope is larger than a certain constant, which implies that the Zeno behavior is avoided. Then, it is proved that all the signals appearing in the closed-loop systems are bounded while none of the states violate their constrained boundary. Besides, the tracking errors converge to a small neighborhood around the origin. The main contributions are as follows

- In contrast to the existing event based control approaches [19]-[26], in which the state constraint problems are neglected, this paper develops a BLF based constraint control strategy to keep the states remaining in the predefined interval all the time. Not only the event trigger mechanism is set up to reduce bandwidth utilization, but also the constraint issues are resolved simultaneously.
- Because the systems contain unknown parameters, it is hard to satisfy the assumption of the input-to-state stability (this assumption is a basic requirement in the conventional event driven control) in regard to the measurement errors. The problem is solved by designing an adaptive control strategy based on the relative threshold approach.

II. Problem Formulation

In this paper, we consider the following nonlinear parametric systems in the strict-feedback form

$$\begin{cases} \dot{\xi}_1(t) = \xi_2(t) + \omega_1^T(t)\varphi_1(\bar{\xi}_1(t)), \\ \dot{\xi}_2(t) = \xi_3(t) + \omega_2^T(t)\varphi_2(\bar{\xi}_2(t)), \\ \quad \vdots \\ \dot{\xi}_n(t) = u(t) + \omega_n^T(t)\varphi_n(\bar{\xi}_n(t)), \\ y(t) = \xi_1(t), \end{cases} \tag{1}$$

where $\bar{\xi}_i(t) = [\xi_1(t), \ldots, \xi_i(t)]^T,\ i = 1, \ldots, n$, are the system state vectors, $\omega_i^T(t)$ are unknown constant vectors, $\varphi_i(\bar{\xi}_i(t))$ stand for known smooth vector functions which are assumed to be bounded, $u(t)$ represents the system input and $y(t)$ is the system output, respectively. In the meantime, all the states are required to remain in the predefined intervals, i.e. $|\xi_i(t)| < \kappa_i$ with κ_i being given constants.

The control target is to propose an event-based adaptive state feedback controller to guarantee the following three aspects: firstly, none of the states violate the constrained sets; secondly, the output $y(t)$ is driven to approach the desired trajectory y_r as far as possible; thirdly, all the variable signals appeared in the closed-loop systems are bounded.

Throughout this paper, to obtain the above control target, the following assumptions are needed in the controller design.

Assumption 1: The desired trajectory $y_r(t)$ is bounded, i.e. $0 \leq y_r(t) \leq Y_0$ with y_0 being a positive constant.

Assumption 2: The $j-$th order derivatives of the desired signals, $y_r^{(j)}$, are assumed to satisfy $|y_r^{(j)}| < \bar{\kappa}_j$, where $\bar{\kappa}_j$ are positive constants.

Remark 1: The adaptive backstepping control issue of lower-triangular systems with output constraint or state constraints has been studied extensively in [34]-[46], [48]. Nevertheless, these control strategies execute their operation periodically all the time even when all the control objectives are realized in a sufficient accuracy. As we know, such control methods are serviceable under the assumption that the communication resources are enough and the communication burden is not taken into account. As a result, deterioration of system performances may occur when the communication resources are limited. Therefore, this paper is focused on the aforementioned problem in constrained systems. In the following part, the event-based control scheme is developed in the sense of full state constraints, which avoids the waste of the communication resources.

III. Event-Triggered Adaptive Controller Design

In this section, the detailed design procedure will be developed, which synthesizes the event-triggered approach and the backstepping technique. In the following, in order to facilitate the analysis and design, the time t would be omitted later on.

Firstly, the coordinate transformation is defined, which will cause our control strategy and main results,

$$\begin{cases} z_1 = \xi_1 - y_r, \\ z_i = \xi_i - \beta_{i-1} \end{cases} \tag{2}$$

where z_1 is the tracking error, z_i stand for the transformation errors, and β_i are stabilizing functions to be structured in the later contents. All the designed stabilizing functions β_i are required to be bounded, i.e., $\beta_i < \bar{\beta}_i$ with $\bar{\beta}_i$ being positive constants (see the contents above Eq.2 in [54]).

The design procedure from step 1 to step n is shown in the following.

Step 1: Considering the first sub-equations of (1) and (2), and noting $z_2 = \xi_2 - \beta_1$, one deduces

$$\begin{aligned} \dot{z}_1 =& \dot{\xi}_1 - \dot{y}_r \\ =& \xi_2 + \omega_1^T\varphi_1(\bar{\xi}_1) - \dot{y}_r \\ =& z_2 + \beta_1 + \omega_1^T\varphi_1(\bar{\xi}_1) - \dot{y}_r. \end{aligned} \tag{3}$$

Consider the BLF as follows

$$V_1 = \frac{1}{2}\log\frac{\kappa_1^2}{\kappa_1^2 - z_1^2} + \frac{1}{2}\tilde{\omega}_1^T\Gamma_1^{-1}\tilde{\omega}_1, \tag{4}$$

where $\log(\cdot)$ is the logarithmic function, $\kappa_1 = \kappa_{s1} - Y_0$, $\tilde{\omega}_1 = \omega_1 - \hat{\omega}_1$ is the estimation error in which $\hat{\omega}_1$ is the estimation of ω_1, and $\Gamma_1 = \Gamma_1^T$ is a positive definite matrix. According to the properties of $\log(\cdot)$, it is obvious that the BLF V_1 is continuous in $\Omega_{z_1} = \{z_1 : |z_1| < \kappa_1\}$.

Then, the derivative of BLF V_1 is obtained as

$$\dot{V}_1 = \frac{z_1 \dot{z}_1}{\kappa_1^2 - z_1^2} - \tilde{\omega}_1^T \Gamma_1^{-1} \dot{\hat{\omega}}_1. \tag{5}$$

In (4), based on the properties of antilogarithm or logarithmic function, the term $\frac{\kappa_1^2}{\kappa_1^2 - z_1^2}$ is positive, which implies that the denominator $\kappa_1^2 - z_1^2$ is positive. Therefore, in (5), the term $\kappa_1^2 - z_1^2$ is not equal to zero. Similar approaches can refer to [36] [44]. In fact, it is difficult to guarantee the system state in a given range at the beginning. Hence, in practical applications, the definition and choice of κ_1 is very careful. In general, a slightly larger value of κ_1 is an effective method to realize the control objective. The focus of this paper is the asymptotic behavior and the design of event-driven mechanism. We do not put much attention on the state guarantee at the beginning. Maybe, the potential and effective schemes are the remove feasibility condition method [39] and the nonlinear mapping method [43].

Substituting (3) into $\dot{V}_1$, one gets

$$\dot{V}_1 = \frac{z_1}{\kappa_1^2 - z_1^2}\left(z_2 + \beta_1 + \omega_1^T \varphi_1(\bar{\xi}_1) - \dot{y}_r\right) - \tilde{\omega}_1^T \Gamma_1^{-1} \dot{\hat{\omega}}_1. \tag{6}$$

Design the virtual controller as

$$\beta_1 = -\gamma_1 z_1 - \hat{\omega}_1^T \varphi_1(\bar{\xi}_1) + \dot{y}_r. \tag{7}$$

where $\gamma_1 > 0$ is a constant.

The adaptive law in this step is set up as

$$\dot{\hat{\omega}}_1 = -\Gamma_1 \left[\tau_1 \hat{\omega}_1 - \frac{z_1}{\kappa_1^2 - z_1^2} \varphi_1(\bar{\xi}_1)\right]. \tag{8}$$

where $\tau_1 > 0$ is a design parameter.

Then, it yields

$$\dot{V}_1 \leq -\gamma_1 \frac{z_1^2}{\kappa_1^2 - z_1^2} + \frac{z_1 z_2}{\kappa_1^2 - z_1^2} + \tau_1 \tilde{\omega}_1^T \hat{\omega}_1. \tag{9}$$

The following fact is deduced

$$\tau_1 \tilde{\omega}_1^T \hat{\omega}_1 \leq -\frac{\tau_1}{2} \tilde{\omega}_1^T \tilde{\omega}_1 + \frac{\tau_1}{2} \omega_1^T \omega_1. \tag{10}$$

Thus, $\dot{V}_1$ can be given by

$$\begin{aligned} \dot{V}_1 \leq & -\gamma_1 \frac{z_1^2}{\kappa_1^2 - z_1^2} - \frac{\tau_1}{2} \tilde{\omega}_1^T \tilde{\omega}_1 \\ & + \frac{\tau_1}{2} \omega_1^T \omega_1 + \frac{z_1 z_2}{\kappa_1^2 - z_1^2}. \end{aligned} \tag{11}$$

As stated in [44], it generates

$$\frac{z_1^2}{\kappa_1^2 - z_1^2} < \log \frac{\kappa_1^2}{\kappa_1^2 - z_1^2}. \tag{12}$$

Thus, it results in

$$\begin{aligned} \dot{V}_1 \leq & -\gamma_1 \log \frac{\kappa_1^2}{\kappa_1^2 - z_1^2} - \frac{\tau_1}{2} \tilde{\omega}_1^T \tilde{\omega}_1 \\ & + \frac{\tau_1}{2} \omega_1^T \omega_1 + \frac{z_1 z_2}{\kappa_1^2 - z_1^2}. \end{aligned} \tag{13}$$

Define

$$\mu_1 = \min\left\{2\gamma_1, \tau_1/\lambda_{\max}(\Gamma_1^{-1})\right\}, \tag{14}$$

$\dot{V}_1$ can be finally expressed as

$$\dot{V}_1 \leq -\mu_1 V_1 + \frac{\tau_1}{2} \omega_1^T \omega_1 + \frac{z_1 z_2}{\kappa_1^2 - z_1^2}. \tag{15}$$

Step $i (i = 2, \ldots, n-1)$: Considering the $i-$th equation of (1) and (2), and noticing $z_{i+1} = \xi_{i+1} - \beta_i$, one deduces that

$$\begin{aligned} \dot{z}_i = & \dot{\xi}_i - \dot{\beta}_{i-1} = \xi_{i+1} + \omega_i^T \varphi_i(\bar{\xi}_i) - \dot{\beta}_{i-1} \\ = & z_{i+1} + \beta_i + \omega_i^T \varphi_i(\bar{\xi}_i) - \dot{\beta}_{i-1}. \end{aligned} \tag{16}$$

Consider the following Lyapunov function

$$V_i = V_{i-1} + \frac{1}{2} \log \frac{\kappa_i^2}{\kappa_i^2 - z_i^2} + \frac{1}{2} \tilde{\omega}_i^T \Gamma_i^{-1} \tilde{\omega}_i, \tag{17}$$

where $\Gamma_i = \Gamma_i^T$ is a positive definite matrix, $\tilde{\omega}_i = \omega_i - \hat{\omega}_i$ is the estimation error with $\hat{\omega}_i$ being the estimate of ω_i. Moreover, κ_i will be determined in the subsequent contents. Obviously, the function V_i is continuous in $\Omega_{z_i} = \{z_i : |z_i| < \kappa_i\}$.

The derivative of V_i along time is concluded that

$$\begin{aligned} \dot{V}_i = & \dot{V}_{i-1} + \frac{z_i \dot{z}_i}{\kappa_i^2 - z_i^2} - \tilde{\omega}_i^T \Gamma_i^{-1} \dot{\hat{\omega}}_i, \\ \leq & -\mu_{i-1} V_{i-1} + \sum_{j=1}^{i-1} \frac{\tau_j}{2} \omega_j^T \omega_j + \frac{z_{i-1} z_i}{\kappa_{i-1}^2 - z_{i-1}^2} \\ & + \frac{z_i}{\kappa_i^2 - z_i^2}\big(z_{i+1} + \beta_i + \omega_i^T \varphi_i(\bar{\xi}_i) \\ & - \dot{\beta}_{i-1}\big) - \tilde{\omega}_i^T \Gamma_i^{-1} \dot{\hat{\omega}}_i. \end{aligned} \tag{18}$$

Define $-\dot{\beta}_{i-1} = \alpha_{i-1}$, one gets

$$\frac{z_i}{\kappa_i^2 - z_i^2} \alpha_{i-1} \leq \frac{1}{2} \bar{\alpha}_{i-1}^2 + \frac{z_i^2}{2(\kappa_i^2 - z_i^2)^2}$$

where $\bar{\alpha}_{i-1}$ is the bound of α_{i-1} (this is an immediate conclusion from the descriptions above Eq.68 in [13]).

Then, design the virtual controller and the adaptive law as

$$\begin{aligned} \beta_i = & -\gamma_i z_i - \hat{\omega}_i^T \varphi_i(\bar{\xi}_i) - \frac{z_i}{2(\kappa_i^2 - z_i^2)} \\ & - \frac{\kappa_i^2 - z_i^2}{\kappa_{i-1}^2 - z_{i-1}^2} z_{i-1}, \end{aligned} \tag{19}$$

$$\dot{\hat{\omega}}_i = -\Gamma_i \left[\tau_i \hat{\omega}_i - \frac{z_i}{\kappa_i^2 - z_i^2} \varphi_i(\bar{\xi}_i)\right], \tag{20}$$

where $\gamma_i > 0$ and $\tau_i > 0$ are design parameters.

Then, it is deduced that

$$\begin{aligned} \dot{V}_i \leq & -\mu_{i-1} V_{i-1} + \sum_{j=1}^{i-1} \frac{\tau_j}{2} \omega_j^T \omega_j + \sum_{j=1}^{i-1} \frac{1}{2} \bar{\alpha}_j^2 \\ & - \gamma_i \frac{z_i^2}{\kappa_i^2 - z_i^2} + \frac{z_i z_{i+1}}{\kappa_i^2 - z_i^2} + \tau_i \tilde{\omega}_i^T \hat{\omega}_i. \end{aligned} \tag{21}$$

The following fact holds

$$\tau_i \tilde{\omega}_i^T \hat{\omega}_i \leq -\frac{\tau_i}{2} \tilde{\omega}_i^T \tilde{\omega}_i + \frac{\tau_i}{2} \omega_i^T \omega_i. \tag{22}$$

Therefore, $\dot{V}_i$ is further expressed as

$$\dot{V}_i \leq -\mu_{i-1}V_{i-1} + \sum_{j=1}^{i}\frac{\tau_j}{2}\omega_j^T\omega_j + \sum_{j=1}^{i-1}\frac{1}{2}\bar{\alpha}_j^2 - \gamma_i\frac{z_i^2}{\kappa_i^2 - z_i^2} - \frac{\tau_i}{2}\tilde{\omega}_i^T\tilde{\omega}_i + \frac{z_i z_{i+1}}{\kappa_i^2 - z_i^2}. \tag{23}$$

It holds that

$$\frac{z_i^2}{\kappa_i^2 - z_i^2} \geq \log\frac{\kappa_i^2}{\kappa_i^2 - z_i^2}. \tag{24}$$

Thus, it leads to

$$\dot{V}_i \leq -\mu_{i-1}V_{i-1} + \sum_{j=1}^{i}\frac{\tau_j}{2}\omega_j^T\omega_j - \frac{\tau_i}{2}\tilde{\omega}_i^T\tilde{\omega}_i - \gamma_i\log\frac{\kappa_i^2}{\kappa_i^2 - z_i^2} + \frac{z_i z_{i+1}}{\kappa_i^2 - z_i^2} + \sum_{j=1}^{i-1}\frac{1}{2}\bar{\alpha}_j^2. \tag{25}$$

Based on (25), the time derivative $\dot{V}_i$ is

$$\dot{V}_i \leq -\mu_i V_i + \sum_{j=1}^{i}\frac{\tau_j}{2}\omega_j^T\omega_j + \frac{z_i z_{i+1}}{\kappa_i^2 - z_i^2} + \sum_{j=1}^{i-1}\frac{1}{2}\bar{\alpha}_j^2, \tag{26}$$

where

$$\mu_i = \min\left\{\mu_{i-1}, 2\gamma_i, \tau_i/\lambda_{\max}(\Gamma_i^{-1})\right\}.$$

Step n: Consider the $n-$th equation of (2), it causes

$$\dot{z}_n = \dot{\xi}_n - \dot{\beta}_{n-1} = u(t) + \omega_n^T\varphi_n(\bar{\xi}_n) - \dot{\beta}_{n-1}. \tag{27}$$

In the $n-$th step, we design the following event-triggered controller

$$v(t) = -(1+\delta)\left[v_0(t)\tanh\left(\frac{\frac{z_n}{\kappa_n^2 - z_n^2}v_0(t)}{\varepsilon}\right) + \bar{m}\tanh\left(\frac{\frac{z_n}{\kappa_n^2 - z_n^2}\bar{m}}{\varepsilon}\right)\right], \tag{28}$$

where

$$v_0(t) = -\gamma_n z_n - \hat{\omega}_n^T\varphi_n(\bar{\xi}_n) - \frac{z_n}{2(\kappa_n^2 - z_n^2)} - \frac{\kappa_n^2 - z_n^2}{\kappa_{n-1}^2 - z_{n-1}^2}z_{n-1},$$

$\gamma_n > 0$, $\delta \in (0,1)$, $\varepsilon > 0$, $\bar{m} > m/(1-\delta)$ and $m > 0$ (m will appear in (33)) are all design parameters.

The adaptive law in the last stage is constructed as

$$\dot{\hat{\omega}}_n = -\Gamma_n\left[\tau_n\hat{\omega}_n - \frac{z_n}{\kappa_n^2 - z_n^2}\varphi_n(\bar{\xi}_n)\right], \tag{29}$$

where $\tau_n > 0$ is a design parameter.

Define the triggering event as

$$u(t) = v(t_k),\ \forall t \in [t_k, t_{k+1}), \tag{30}$$

$$t_{k+1} = \inf\left\{t \in R\big||e(t)| \geq \delta|u(t)| + m\right\},\ t_1 = 0, \tag{31}$$

where $v(t_k)$ represents a medium control signal which will be given in the subsequent, $t_k, k \in z^+$ stands for the updated time of the system input $u(t)$, $e(t) = v(t) - u(t)$ is the measurement error, $\delta|u(t)| + m$ denotes the relative threshold, with $0 < \delta < 1$ and m being positive constants.

According to (31), one obtains that $|v(t) - u(t)| < \delta|u(t)| + m$, for $\forall t \in [t_k, t_{k+1})$. Then, there must exist two parameters $|\lambda_1(t)| \leq 1$ and $|\lambda_2(t)| \leq 1$ [30], such that

$$v(t) = [1 + \lambda_1(t)\delta]u(t) + \lambda_2(t)m. \tag{32}$$

This indicates that

$$u(t) = \frac{v(t)}{1 + \lambda_1(t)\delta} - \frac{\lambda_2(t)m}{1 + \lambda_1(t)\delta}. \tag{33}$$

Substituting (33) into (27) leads to

$$\dot{z}_n = \frac{v(t)}{1 + \lambda_1(t)\delta} - \frac{\lambda_2(t)m}{1 + \lambda_1(t)\delta} + \omega_n^T\varphi_n(\bar{\xi}_n) - \dot{\beta}_{n-1}. \tag{34}$$

Choose the BLF as

$$V_n = V_{n-1} + \frac{1}{2}\log\frac{\kappa_n^2}{\kappa_n^2 - z_n^2} + \frac{1}{2}\tilde{\omega}_n^T\Gamma_n^{-1}\tilde{\omega}_n, \tag{35}$$

where $\hat{\omega}_n$ is the estimation of ω_n with the corresponding estimation error being defined as $\tilde{\omega}_n = \hat{\omega}_n - \omega_n$, κ_n will be given in the stability analysis, and $\Gamma_n = \Gamma_n^T$ is a positive definite matrix decided by users. It is quite clear that the BLF V_n is continuous in $\Omega_{z_n} = \{z_n : |z_n| < \kappa_n\}$.

The derivative of V_n is obtained as

$$\begin{aligned}\dot{V}_n =& \dot{V}_{n-1} + \frac{z_n\dot{z}_n}{\kappa_n^2 - z_n^2} - \tilde{\omega}_n^T\Gamma_n^{-1}\dot{\hat{\omega}}_n,\\ \leq& -\mu_{n-1}V_{n-1} + \sum_{j=1}^{n-1}\frac{\tau_j}{2}\omega_j^T\omega_j + \frac{z_{n-1}z_n}{\kappa_{n-1}^2 - z_{n-1}^2}\\ &+ \frac{z_n}{\kappa_n^2 - z_n^2}\Big(\frac{v(t)}{1 + \lambda_1(t)\delta} - \frac{\lambda_2(t)m}{1 + \lambda_1(t)\delta}\\ &+ \omega_n^T\varphi_n(\bar{\xi}_n) + \alpha_{n-1}\Big) - \tilde{\omega}_n^T\Gamma_n^{-1}\dot{\hat{\omega}}_n.\end{aligned} \tag{36}$$

where $\alpha_{n-1} = -\dot{\beta}_{n-1}$. On the basis of the statements above Eq.27 in [12], it holds that α_{n-1} is bounded, i.e., $\alpha_{n-1} < \bar{\alpha}_{n-1}$ with $\bar{\alpha}_{n-1}$ being a positive constant.

Note that, for $\forall a \in R$ and $\varepsilon > 0$, there has $-a\tanh(a/\varepsilon) \leq 0$. Then, based on (28), one has

$$\frac{z_n}{\kappa_n^2 - z_n^2}v(t) \leq 0.$$

Further, due to $|\lambda_1(t)| \leq 1$ and $|\lambda_2(t)| \leq 1$, it holds that

$$\frac{\frac{z_n}{\kappa_n^2 - z_n^2}v(t)}{1 + \lambda_1(t)\delta} \leq \frac{\frac{z_n}{\kappa_n^2 - z_n^2}v(t)}{1 + \delta}, \tag{37}$$

$$\left|\frac{\lambda_2(t)m}{1 + \lambda_1(t)\delta}\right| \leq \frac{m}{1 - \delta}. \tag{38}$$

Thus, according to (28), one gets

$$\frac{\frac{z_n}{\kappa_n^2 - z_n^2}v(t)}{1 + \lambda_1(t)\delta} = -\frac{z_n}{\kappa_n^2 - z_n^2}v_0(t)\tanh\left(\frac{\frac{z_n}{\kappa_n^2 - z_n^2}v_0(t)}{\varepsilon}\right) - \frac{z_n}{\kappa_n^2 - z_n^2}\bar{m}\tanh\left(\frac{\frac{z_n}{\kappa_n^2 - z_n^2}\bar{m}}{\varepsilon}\right). \tag{39}$$

Adding and subtracting both $\frac{z_n}{\kappa_n^2-z_n^2}v_0(t)$ and $\left|\frac{z_n}{\kappa_n^2-z_n^2}\bar{m}\right|$, one obtains

$$\begin{aligned}\frac{\frac{z_n}{\kappa_n^2-z_n^2}v(t)}{1+\lambda_1(t)\delta}\leq&\left|\frac{z_nv_0(t)}{\kappa_n^2-z_n^2}\right|-\frac{z_nv_0(t)}{\kappa_n^2-z_n^2}\tanh\left(\frac{\frac{z_nv_0(t)}{\kappa_n^2-z_n^2}}{\varepsilon}\right)\\&+\left|\frac{z_n\bar{m}}{\kappa_n^2-z_n^2}\right|-\frac{z_n\bar{m}}{\kappa_n^2-z_n^2}\tanh\left(\frac{\frac{z_n\bar{m}}{\kappa_n^2-z_n^2}}{\varepsilon}\right)\\&-\left|\frac{z_n\bar{m}}{\kappa_n^2-z_n^2}\right|+\frac{z_nv_0(t)}{\kappa_n^2-z_n^2}.\end{aligned}\tag{40}$$

The function $\tanh(\cdot)$ has the property that [30]

$$0\leq|\rho|-\rho\tanh\left(\frac{\rho}{\delta_0}\right)\leq0.2785\delta_0,\tag{41}$$

with $\rho\in R$ and $\delta_0>0$. Therefore, it has

$$\frac{\frac{z_n}{\kappa_n^2-z_n^2}v(t)}{1+\lambda_1(t)\delta}\leq0.557\varepsilon-\left|\frac{z_n\bar{m}}{\kappa_n^2-z_n^2}\right|+\frac{z_nv_0(t)}{\kappa_n^2-z_n^2}.\tag{42}$$

Substituting (42) into $\dot{V}_n$, one has

$$\begin{aligned}\dot{V}_n\leq&-\mu_{n-1}V_{n-1}+\sum_{j=1}^{n-1}\frac{\tau_j}{2}\omega_j^T\omega_j+0.557\varepsilon\\&-\left|\frac{z_n\bar{m}}{\kappa_n^2-z_n^2}\right|-\frac{z_n}{\kappa_n^2-z_n^2}\frac{\lambda_2(t)m}{1+\lambda_1(t)\delta}\\&-\tilde{\omega}_n^T\Gamma_n^{-1}\dot{\hat{\omega}}_n+\frac{z_n}{\kappa_n^2-z_n^2}\Bigg(\omega_n^T\varphi_n(\bar{\xi}_n)\\&+\alpha_{n-1}+\frac{\kappa_n^2-z_n^2}{\kappa_{n-1}^2-z_{n-1}^2}z_{n-1}+v_0(t)\Bigg).\end{aligned}\tag{43}$$

It gets

$$\frac{z_n}{\kappa_n^2-z_n^2}\alpha_{n-1}\leq\frac{1}{2}\bar{\alpha}_{n-1}^2+\frac{z_n^2}{2(\kappa_n^2-z_n^2)^2}$$

Then, based on (38), it leads to

$$\begin{aligned}\dot{V}_n\leq&-\mu_{n-1}V_{n-1}+\sum_{j=1}^{n-1}\frac{\tau_j}{2}\omega_j^T\omega_j+0.557\varepsilon\\&-\left|\frac{z_n\bar{m}}{\kappa_n^2-z_n^2}\right|+\left|\frac{z_n}{\kappa_n^2-z_n^2}\frac{m}{1-\delta}\right|+\tau_n\tilde{\omega}_n^T\hat{\omega}_n\\&-\gamma_n\frac{z_n^2}{\kappa_n^2-z_n^2}+\sum_{j=1}^{n-1}\frac{1}{2}\bar{\alpha}_j^2.\end{aligned}\tag{44}$$

The following fact holds

$$\tau_n\tilde{\omega}_n^T\hat{\omega}_n\leq-\frac{\tau_n}{2}\tilde{\omega}_n^T\tilde{\omega}_n+\frac{\tau_n}{2}\omega_n^T\omega_n.\tag{45}$$

Because $\bar{m}>m/(1-\delta)$, it implies that

$$-\left|\frac{z_n\bar{m}}{\kappa_n^2-z_n^2}\right|+\left|\frac{z_n}{\kappa_n^2-z_n^2}\frac{m}{1-\delta}\right|\leq0.$$

Then, $\dot{V}_n$ is further expressed as

$$\begin{aligned}\dot{V}_n\leq&-\mu_{n-1}V_{n-1}+\sum_{j=1}^{n}\frac{\tau_j}{2}\omega_j^T\omega_j+0.557\varepsilon\\&-\gamma_n\frac{z_n^2}{\kappa_n^2-z_n^2}-\frac{\tau_n}{2}\tilde{\omega}_n^T\tilde{\omega}_n+\sum_{j=1}^{n-1}\frac{1}{2}\bar{\alpha}_j^2.\end{aligned}\tag{46}$$

The following fact holds

$$\frac{z_n^2}{\kappa_n^2-z_n^2}\geq\log\frac{\kappa_n^2}{\kappa_n^2-z_n^2}.\tag{47}$$

Hence, it causes

$$\begin{aligned}\dot{V}_n\leq&-\mu_{n-1}V_{n-1}+\sum_{j=1}^{n}\frac{\tau_j}{2}\omega_j^T\omega_j+0.557\varepsilon\\&-\gamma_n\log\frac{\kappa_n^2}{\kappa_n^2-z_n^2}-\frac{\tau_n}{2}\tilde{\omega}_n^T\tilde{\omega}_n+\sum_{j=1}^{n-1}\frac{1}{2}\bar{\alpha}_j^2.\end{aligned}\tag{48}$$

Define

$$\mu_n=\min\left\{\mu_{n-1},2\gamma_n,\tau_n/\lambda_{\max}(\Gamma_n^{-1})\right\},$$

then, $\dot{V}_n$ can be finally expressed as

$$\dot{V}_n\leq-\mu_nV_n+\sum_{j=1}^{n}\frac{\tau_j}{2}\omega_j^T\omega_j+0.557\varepsilon+\sum_{j=1}^{n-1}\frac{1}{2}\bar{\alpha}_j^2.\tag{49}$$

Theorem 1: Consider the nonlinear parametric systems as shown in (1), under Assumptions 1 and 2, the adaptive event-triggered controller is proposed in (28), and the adaptive laws are established in (8) (20) (29). Then, the presented approach guarantees that 1) the full state constraints are never transgressed, 2) all the variable signals appearing in the closed-loop systems are bounded, 3) the tracking error converges to a small neighborhood of the origin. Simultaneously, the Zeno behavior can also be avoided, i.e., there exists a lower bound $T>0$ such that $t_{k+1}-t_k\geq T$, for $\forall k\in z^+$.

Proof: Please see the appendix A.

In this paper, the assumption that the controlled system states remain in the given intervals at the early stage is needed. This assumption is necessary, which excludes the occurrence of larger initial-values of system states. It may save energy and increase the smoothness of the closed-loop system. In some real-world applications, such as chemical process [34] and manipulator systems [36], the assumption is satisfied. In addition, it is better to define the constraint functions as decreasing exponential functions (ae^{-bt} for example). If the assumption is not satisfied in some special cases, the alternative approaches are the remove feasibility condition method [39] and the nonlinear mapping method [43].

Remark 2: It should be mentioned that the event-driven control design is established on the basis of the relative threshold approach, while the fixed threshold approach is developed in [21] [26]-[29]. However, when the size of the trigger condition is too large, a bigger adjusted range for the control signal is set up, which makes the control behavior not so accurate. Nevertheless, the proposed relative threshold approach can improve the control performance. On this occasion, the triggering event threshold is associated with the magnitude of the controller itself, and a smaller trigger threshold can cause a more precise system performance.

Remark 3: The significance of the equation (49) is shown here, which implies that the boundedness of all signals appearing in the systems are ensured. Details are given in the Section VI of the Supplementary Material.

In order to overcome the drawback of the traditional periodic control approaches, it is necessary to reduce the unexpected waste of the communications and computer memory resources. For a class of strict feedback nonlinear constrained systems, we put forward a relative threshold-based event trigger control scheme, which is carried out effectively. Compared with the existing control schemes, the event trigger control has evident advantages, such as reducing the execution times of control task, saving energy and communication resources, etc. In theory, this paper gives the detailed recursive process. In addition, to verify the effectiveness of the proposed scheme, it is applied to the aircraft wing rock motion in the simulation. At present, it is only realized on the data platform. The control and design of aircraft wing rock motion are mainly from the perspective of reducing energy consumption and saving resources. In addition, the event trigger control has potential applications for any practical systems, including safety-critical systems, which is worth being investigated. Of course, reliability issue should be considered simultaneously in practical applications of the safety critical systems, which is beyond the scope of the paper.

In addition, a further discussion of the asymptotic stability-like is given in the following Inference.

Inference: Consider the nonlinear parametric systems as shown in (1), the following virtual controllers and event-triggered controller are designed:

$$\beta_1 = -(\gamma_1 + c_1)z_1 - \hat{\omega}_1^T \varphi_1(\bar{\xi}_1) + \dot{y}_r. \tag{50}$$

$$\begin{aligned}\beta_i = &-(\gamma_i + c_i)z_i - \hat{\omega}_i^T \varphi_i(\bar{\xi}_i) - \frac{z_i}{2(k_i^2 - z_i^2)} \\ &- \frac{\kappa_i^2 - z_i^2}{4c_{i-1,1}(\kappa_{i-1}^2 - z_{i-1}^2)} z_i\end{aligned} \tag{51}$$

$$\begin{aligned}v(t) = &-(1+\delta)\left[v_0(t)\tanh\left(\frac{\frac{z_n}{\kappa_n^2 - z_n^2}v_0(t)}{\varepsilon}\right)\right. \\ &\left. + \bar{m}\tanh\left(\frac{\frac{z_n}{\kappa_n^2 - z_n^2}\bar{m}}{\varepsilon}\right)\right],\end{aligned} \tag{52}$$

$$\begin{aligned}v_0(t) = &-(\gamma_n + c_n)z_n - \hat{\omega}_n^T \varphi_n(\bar{\xi}_n) \\ &- \frac{z_n}{2(k_n^2 - z_n^2)} - \frac{\kappa_n^2 - z_n^2}{4c_{n-1,1}(\kappa_{n-1}^2 - z_{n-1}^2)} z_n\end{aligned} \tag{53}$$

$$u(t) = v(t_k),\ \forall t \in [t_k, t_{k+1}), \tag{54}$$

$$t_{k+1} = \inf\left\{t \in R \big| |e(t)| \geq \delta|u(t)| + m\right\},\ t_1 = 0, \tag{55}$$

where $c_i = c_{i,1} + c_{i,2}, c_n = c_{n,2}, i = 1, \dots, n-1$ with $c_{i,1} > 0, c_{i,2} > 0, i = 1, \dots, n$.

Then, i) the full state constraints are never transgressed, ii) all the variable signals appearing in the closed-loop systems converge to the origin asymptotically, iii) the Zeno behavior is avoided.

Proof: This proof is divided into two parts. The first part is to illustrate that the full state constraints are never transgressed and all the variable signals appearing in the closed-loop systems are bounded. The second part is to show that all the error signals can exponential converge to a small neighborhood of the origin. The proof of the avoidance of Zeno behavior is similar to the proof of Theorem 1. Due to the limited space, the detailed proof is shown in the Supplementary Materials.

The presented result can also be expanded to a class of uncertain strict-feedback nonlinear systems and uncertain pure feedback nonlinear systems, whose unknown dynamics donot satisfy the linear parameterization conditions. Details please see the Extension 1 and Extension 2 in the Supplementary Materials.

The results can also be extended to the case of removing feasibility condition, which is shown in the Extension 3 in the Supplementary Materials.

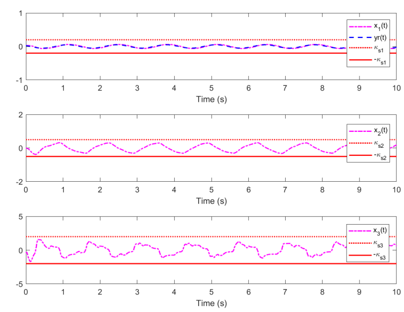

Fig. 1. The trajectories of $x_1(t)$, $x_2(t)$, $x_3(t)$ and their corresponding constraints.

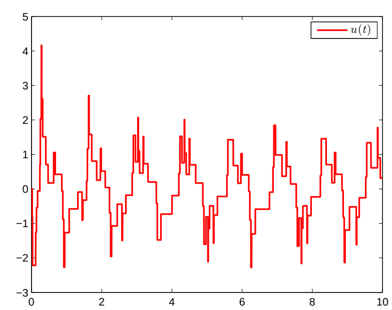

Fig. 2. The curves of control signals.

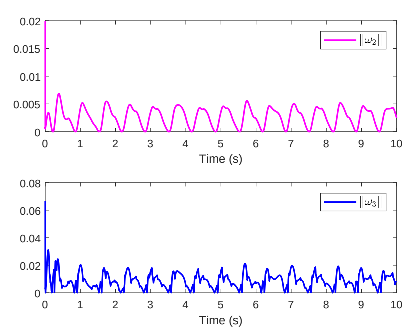

Fig. 3. The trajectories of adaptive laws for $\|\omega_2\|$ and $\|\omega_2\|$.

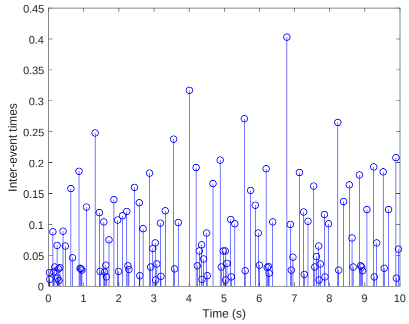

Fig. 4. Time interval of triggering events.

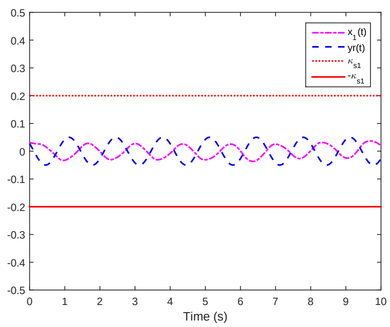

Fig. 5. The tracking performance of x_1 and y_r in [31].

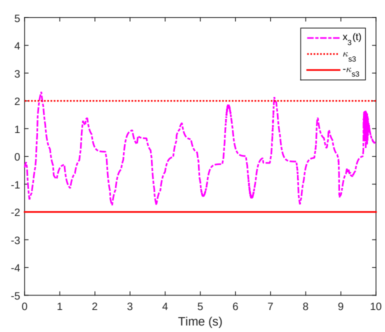

Fig. 6. The curves of x_3 in [31].

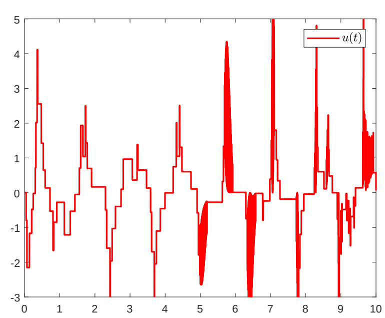

Fig. 7. The curves of u in [31].

IV. Validation for an Aircraft Wing Motion System

In this section, to illustrate the efficacy of the constructed controller scheme, we take the aircraft wing rock motion into account. The model is established by the identification technique of wind tunnel experimental data [6]. Here, the case that all the states are constrained in corresponding interval is considered. The dynamic is given as

$$\begin{cases} \dot{\xi}_1 = \xi_2 \\ \dot{\xi}_2 = b\xi_3 + \omega_2^T \varphi_2(\bar{\xi}_2) \\ \dot{\xi}_3 = -\frac{1}{\eta} u + \omega_3^T \varphi_3(\bar{\xi}_3) \end{cases} \tag{56}$$

where ξ_1 stands for the aircraft roll angle (rad), ξ_2 represents the roll rate (rad/s), ξ_3 is the aileron deflection angle (rad), u is the actuator output, $\omega_2 = [\theta_0, \theta_1, \theta_2, \theta_3, \theta_4, \theta_5]^T$, and $\varphi_2(\bar{\xi}_2) = [1, x_1, x_2, |x_1|x_2, |x_2|x_2, x_1^3]^T$. Furthermore, $\omega_3 = \frac{1}{\eta} = \frac{1}{15}$, $\varphi_3(\bar{\xi}_3) = x_3$. The aerodynamic parameters of delta wing for 25° angle of attack [6] are given by $b = 1.5, \theta_0 = 0, \theta_1 = -0.2, \theta_2 = 0015, \theta_3 = -0.06, \theta_4 = 0.009, \theta_5 = 0.02$. The control target is to guarantee that all the variables in the wing rock motion are bounded, while the states are constrained in $|\xi_1| < 0.2$, $|\xi_2| < 0.5$, $|\xi_3| < 2$, respectively. The aircraft roll angle (expressed as ξ_1) is a critical variable affected the rolling moment seriously. If the roll angle exceeds a safe operating range, the balance of the aircraft may be destroyed. At high speeds, the aeroelastic torsion occurs on the wings, which is related to the roll rate (expressed as ξ_2). In this case, it reduces the aileron efficiency relative to the rigid wing. Due to the specific situation, it is necessary to keep the roll rate in a certain restriction. When the aileron deflection angle (expressed as ξ_3) is excessive, the aerodynamic load maybe too big, which causes the adverse yaw phenomenon likely happen. Thus, the designers should give a constrain on the deflection angle to decrease the inverse deflection of aileron.

Although the model of aircraft wing motion system is established based on the identification technique by the wind tunnel experimental data as shown in [6], all the key dynamic information is taken into account. The simplified model is enough to represent the essential characteristics of the prototypical aircraft wing motion system. The external disturbance is not considered in this paper, since we think that there are special equipment and control technology to solve related problem, which is not the major concern of this paper.

The initial values of states are selected as $\xi_1(0) = 0.03$, $\xi_2(0) = 0$, $\xi_3(0) = -0.4$. In this example, the reference signal is chosen as

$$y_r = 0.05 \cos(0.15\pi t/100 + 1) \tag{57}$$

The related parameters are given as $\Gamma_{21} = 0.5$, $\Gamma_{22} = 2$, $\Gamma_{23} = 0.01$, $\Gamma_{24} = 4$, $\Gamma_{25} = 2$, $\Gamma_3 = 5$, $\tau_{21} = 0.5$, $\tau_{22} = 0.01$, $\tau_{23} = 0.01$, $\tau_{24} = 0.01$, $\tau_{25} = 0.01$, $\tau_3 = 0.01$, $m = 0.45$, $\bar{m} = 1$, $\varepsilon = 2$, $\delta = 0.4$, $k_1 = 20$, $k_2 = 1$, $k_3 = 10$.

Fig.1-Fig.6 show the simulation results. Fig.1 displays the trajectories of $x_1(t)$, $x_2(t)$, $x_3(t)$ and their constrained bounds. It implies that the output signal follows the reference signal y_r, without violating its constrained bound, and the constraints of $x_2(t)$ and $x_3(t)$ are also not overstepped. Fig.2 is given

to describe the control signals $u(t)$. Based on this figure, it is obtained that the boundedness of $u(t)$ is guaranteed. The curves of adaptive laws for $\|\omega_2\|$ and $\|\omega_3\|$ are depicted in Fig.3. It indicates that the adaptive laws developed are also bounded. The time interval of triggering events are shown in Fig.4, in which the trigger points are clearly described. To highlight the developed approach, a comparison with the result [31] without considering the state constraints is given in Fig.5-Fig.7. The original values and parameters are the same as previous. According to Fig.5, one concludes that the tracking performance is not so well by applying the method in [31] directly. From Fig.6, although the boundedness of x_3 is ensured, one gets that the constraint is violated. Fig.7 shows the trajectory of the control input in [31]. We see that the control input proposed in this paper is smaller than that in [31], which further shows that both the smaller tracking error and smaller control input are obtained by using the proposed method.

In [49], an adaptive event-driven controller is proposed for a class of nonlinear systems based on the batch least-squares identifier and the event-triggered control is applied to a wing-rock model. The asymmetric barrier Lyapunov function (ABLF) based adaptive event-driven control approach is developed in [50] for a 6-DOF quadrotor with time-varying output constraints, while the event-triggered control problem for full state constraints is solved for aircraft wing rock motion system in this paper. The authors in [51] address the event-driven control problem for a class of strict-feedback systems with unknown injection data and the theoretical method is executed in a wing rock model. Besides, two event-driven control methods are presented for a switched linear parameter varying systems in [52], and the control strategies are used to an aircraft engines. In contrast to [49], [51] and [52], our event-triggered control method pays close attention to the constraints of the roll angle, roll rate and aileron deflection angle.

Since the model of aircraft wing motion systems is built based on the identification technique by the wind tunnel experimental data, the simulation results are well carried out. In reality, it is interesting to apply the method of real-time monitoring and control industrial cyberphysical systems to the aircraft wing motion systems. In this case, the data-driven technique is a potential and high-efficiency approach. This point is not the main focus of this paper, but it is a very interesting topic. Interested readers can refer to [4].

V. Conclusion

In this paper, a BLF based event-driven control approach is presented for a class of uncertain SISO parametric systems, in which each state should be strictly limited in a predefined constraint space. The trigger mechanism is structured on the basis of a relative threshold. This indicates that the control accuracy becomes higher, the system performance is better, and the communication resource is saved. The considered system is viewed as a lower-triangular system that the matching condition is no longer satisfied. This makes the ETC more complex in the sense of full state constraints. The BLF based control method guarantees that the full state constraints are satisfied. It has been proved that all the variable quantities in the resulting closed-loop system are bounded. At the same time, the tracking error converges to a small compact set near the origin and the Zeno phenomenon does not exist in the developed approach. In the future, the proposed scheme can be extended to multiple input single output (MISO) systems or multiple input multiple output (MIMO) systems.

Appendix A
Proof of the Theorem 1

Proof: The proof of the Theorem 1 is divided into two parts. The first part is to show the stability of the closed-loop system and the full constraint conditions can be ensured. The second part shows the interpretation that the developed scheme indeed avoid the Zeno behavior.

Part 1: Consider the Barrier Lyapunov function as

$$V = V_n = \sum_{j=1}^{n} \frac{1}{2} \log \frac{\kappa_j^2}{\kappa_j^2 - z_j^2} + \sum_{j=1}^{n} \frac{1}{2} \tilde{\omega}_j^T \Gamma_j^{-1} \tilde{\omega}_j. \tag{58}$$

On the basis of the above backstepping scheme, the first-order derivative of V is obtained as

$$\dot{V} \leq -\mu V + \Pi, \tag{59}$$

where

$$\mu = \min\left\{2\gamma_j, \gamma_j/\lambda_{\max}(\Gamma_j^{-1}), j = 1, \dots, n\right\},$$

and

$$\Pi = \sum_{j=1}^{n} \frac{\tau_n}{2} \omega_n^T \omega_n + 0.557\varepsilon + \sum_{j=1}^{n-1} \frac{1}{2} \bar{\alpha}_j^2.$$

Therefore, according to the analysis procedure [47], one concludes that the terms $\log \frac{\kappa_j^2}{\kappa_j^2 - z_j^2}$ and $\tilde{\omega_j}$ are bounded. Then, the boundedness of the errors z_i are guaranteed , i.e. $z_j < \kappa_j$. Because the ideal weight ω_j are bounded, the adaptive parameters $\hat{\omega}_j$ are bounded. Since $\xi_1 = z_1 + y_r$ and $|y_r| < Y_0$, the boundedness of ξ_1 is obtained. That is to say, $|\xi_1| \leq |z_1| + |y_r| < \kappa_1 + Y_0$. Based on the Assumption 1, if the parameter κ_1 is chosen as $\kappa_1 = \kappa_{s1} - Y_0$, and then, one gets that $|\xi_1| < k_{s1}$, which implies that the constrained condition for ξ_1 is not violated. Recall (50), the virtual controller β_1 is the function of z_1, $\hat{\omega}_1$ and $\dot{y}_r$. Based on the Assumption 2 and the results in [46], the continuous function β_1 is bounded. Hence, there exists a constant $\bar{\beta}_1 > 0$ such that $\beta_1 < \bar{\beta}_1$. One can repeat the above process from the second step to the last step, and then, the boundedness of β_i can be ensured. Because $\xi_2 = z_2 + \beta_1$, one obtains $|\xi_2| \leq |z_2| + |\beta_1| < \kappa_2 + \bar{\beta}_1$. When the parameter κ_2 is designed as $\kappa_2 = \kappa_{s2} - \bar{\beta}_1$, it holds that $|\xi_2| < \kappa_{s2}$. In a similar way, one can in turn obtain that $|\xi_i| < \kappa_{si}, i = 3, \cdots, n$. Hence, all the states are not violated corresponding constrained conditions.

Part 2: Recall the definition of $e(t)$, i.e. $e(t) = v(t) - u(t)$, where $u(t) = v(t_k)$ (see Eq.(54) for details). For $\forall t \in [t_k, t_{k+1}), k \in z^+$, one has

$$\dot{e}\left(t\right) = \dot{v}\left(t\right) - \dot{v}\left(t_k\right).$$

Here $\dot{v}(t_k)$ can be viewed as a constant in the fixed interval $[t_k, t_{k+1})$, which means that $\dot{v}(t_k) = 0$. Then, we have

$$\frac{d}{dt}|e| = \text{sign}(e)\dot{e} \leq |\dot{v}| \tag{60}$$

According to the definition of $v(t)$ in (28), we know that $\dot{v}$ is the function of z_n and $\hat{\omega}_n$. Since all the signals in the closed-loop system are bounded known from the result in Part 1, $\dot{v}$ is bounded [30]. Here, we assume that $|\dot{v}| \leq \bar{v}$. In addition, $e(t_k) = 0$ and $\lim_{t\to t_{k+1}} e(t_{k+1}) = \delta|u(t)| + m$. Then, by integrating (60) on both sides, we obtain $t_{k+1} - t_k \geq T = (\delta|u(t)| + m)/\bar{v}$. Thus, the Zeno behavior is avoided.

This completes the proof.

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