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A Novel Algorithm for Quantized Particle Filtering with Multiple Degrading Sensors: Degradation Estimation and Target Tracking

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***Abstract*—This paper addresses the particle filtering problem for a class of nonlinear/non-Gaussian systems with quantized measurements and multiple degrading sensors. A degradation variable described by the Wiener process is proposed to describe the phenomenon of sensor degradation that is often encountered in engineering practice. The measurement output of each sensor is quantized by a uniform quantizer before being sent to the remote filter. An augmented system is constructed which aggregates the original system state and the degradation variables. In the presence of the sensor degradation and the quantization errors, a new likelihood function at the remote filter is calculated by resorting to all the transmitted measurements. According to the mathematical characterization of the likelihood function, a novel particle filtering algorithm is developed where the parameters of both the degradation processes and the quantization functions are exploited to obtain the modified importance weights. Finally, the effectiveness of the proposed method is shown via a target tracking example with bearing measurements.**

***Index Terms*—Particle filter; sensor degradation; multiple sensors; uniform quantization; Wiener process.**

I. Introduction

For decades, the sequential Bayesian filter algorithm has been well recognized to be an effective way of estimating the real value of an observed variable that evolves in time. For a class of linear systems with additive Gaussian noises, the renowned Kalman filter serves as a kind of optimal sequential Bayesian filter and, to deal with more comprehensive systems, many minimum-variance filters [4], [10], [12], [25] have been developed as variants of the Kalman filter. In practice, however, it is often the case that the underlying systems are inherently nonlinear/uncertain that undergo non-Gaussian noises. In these systems, it is prohibitively complicated, or even intractable, to calculate the probability density functions (PDFs) of the system states conditioned on the measurements, and an alternative would then be to use approximations and suboptimal solutions to obtain the PDFs.

Among those approximation-based filters developed in the literature, the particle filter (PF), also known as sequential Monte Carlo (SMC) filter, has been extensively investigated [1] due to its design flexibility, ease of implementation, and the capability of handling a wide range of practical systems. In PF algorithms, a group of particles with importance weights is used to approximate the posterior PDF of the system state, and such an approximation enables PF to deal with nonlinear/uncertain/non-Gaussian systems. In recent years, there have been fruitful results on the analysis and improvement of PF methods. For example, an auxiliary PF has been designed in [14] for nonlinear/non-Gaussian stochastic systems under inequality state constraints, where the weights of the particles which are remote from feasible areas have been lowered in a probabilistic sense. A novel event generator has been constructed in [16] such that only the informative measurements can be selected and used by the PF, thereby reducing unnecessary transmission burden.

Performance deteriorations of sensors are inevitable in many practical situations due to reasons such as aging, abrasions and environmental disturbances, and such deteriorations may lead to eventually complete failures [2], [19]. Because of its adverse effects on the system performance, the phenomenon of sensor degradation has received considerable research attention in many disciplines. For example, much effort has been devoted to predicting the remaining useful life of sensors with different degradation models [26], and some fault-tolerant control methods have been proposed to compensate the negative influences from the degradations [17].

When it comes to the filtering problem for systems with sensor degradations, there have been some initial results available in the literature [7]. In [30], stochastic variables obeying time-invariant distributions have been employed to characterize the stochastic sensor gain degradations, and some statistical information of the variables has been used to design the filter in the minimum mean-square sense. Such mathematical formulations of the sensor gain degradations appear to be a bit overly simplified because of the negligence of the relationship between the actual degradation processes and the operating time. In fact, the degradation should evolve in time and its dynamics should be captured by using differential/difference equations, and this motivates us to design the filter that reflects

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certain dynamic properties of the degradation variables.

It is worth mentioning that, the introduction of dynamical degradation models would pose significant challenges to the filter analysis and synthesis because of the difficulties in quantifying degradation-induced changes on system parameters. As such, many classical filters (e.g. minimum-variance, H_∞, set-membership filters) are no longer applicable to systems with sensor degradations. By contrast, the approximation-based PF method emerges as a particularly suitable candidate to handle the degrading sensors, where the weights of particles can be determined according to the dynamics of the degradation variables. In this regard, a novel nonlinear state-dependent model has been established in [19] for the actuator degradation process, and the PF has been designed to simultaneously estimate the system state and the degradation variable. It is worth mentioning that only actuator degradations have been considered in [19] for systems with *linear* measurement functions and *Gaussian* noises, and the PF design problem for systems experiencing sensor degradations and nonlinear measurements still remains open.

With advances in sensor and wireless communication technologies, it has become more and more common to monitor a system of interest with *multiple* sensors, which collect information from the system and transmit the data to a remote monitor/filter [5], [15], [27], [29]. A typical example is the localization and tracking problem with spatially distributed sensors [9], [11]. In the multi-sensor situations, some PF algorithms have been developed to cope with measurements from different sensor nodes, where the PF has been combined with the probability hypothesis density filter to solve the multi-target tracking problem, and the data association problem has been circumvented via estimating all the potential targets [13]. In the context of developing PF algorithms under multiple and possibly heterogeneous sensors, a novel independent Metropolis Hasting resampling algorithm has been adopted in the distributed PF proposed in [23] to improve the robustness and efficiency of the signal tracking via avoiding the particle degeneracy and impoverishment. In [24], the distributed PF and the interacting multiple model have been used to track a speaker in distributed microphone networks, and multiple time differences of arrivals have been used to determine the weights of the particles to eliminate the adverse influences from reverberations and noises. Unfortunately, all the existing PF methodologies with multiple sensors have overlooked the sensor degradation phenomenon, and one of our motivations is therefore to shorten such a gap, which is technically non-trivial because the appearance of the degradation variables would influence the likelihood functions and importance weights in the desired remote PF.

In communication channels with finite bandwidth, it is ubiquitous that the word size for the transmitted data is limited and, as a result, quantization is needed to map the original signal to a set with finite elements, thereby bringing in unavoidable quantization errors at the receiving end which might degrade the system performance. So far, there has been a rich body of literature on the control/filtering problems with quantization effects [18], but the PF-related results have been really scattered. In [21], a PF has been designed to process multiple measurements with companded quantizers. A modified Kalman-like PF has been established in [20] with a new likelihood function calculated based on the quantization process. With quantized measurements, PF approaches have also been developed to estimate the phase information in a digital phase-locked loop [3] and identify parameters for controlled auto-regressive systems [6]. Note that the likelihood functions in these papers have been revised in the presence of the quantization errors. Yet, the degradation processes have not been considered in existing results, and this has necessitated the current investigation on the particle filtering problem for systems with quantizations and multiple degrading sensors.

Motivated by the discussions above, in this paper, a novel PF is to be designed for a class of systems with quantized measurements and multiple degrading sensors. The Wiener process is employed to describe the dynamics of the sensor degradation variables. The measurement outputs are subject to uniform quantization effects before being sent to the remote filter. The likelihood functions are calculated in consideration of sensor degradations and quantizations, and the weights of the particles are updated according to the novel likelihood functions. The effectiveness of the proposed method is illustrated with a target tracking example.

The main contributions of this paper are highlighted as follows: 1) sensor degradations are considered to reflect engineering practice in the framework of particle filtering; 2) novel likelihood functions are obtained by incorporating degradation variables and quantization functions; and 3) a modified PF algorithm is proposed for a class of nonlinear/non-Gaussian systems in the simultaneous presence of multiple degrading sensors and uniform quantizations.

The rest of the paper is organized as follows. Section II formulates the particle filtering problem for nonlinear/non-Gaussian systems with multiple degrading sensors and uniform quantizations. The PF design problem is investigated based on newly obtained likelihood functions in Section III. A target tracking simulation example is presented in Section IV to show the validness of the developed methods, and the paper is concluded in Section V.

Notations. The notation used in the paper is fairly standard except where otherwise stated. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. $\mathbb{E}\{x\}$ is the expectation of the stochastic variable x. $p_x(\cdot)$ denotes the PDF of the random variable x. $p(x|y)$ stands for the conditional PDF of a stochastic variable x given y. $x_{a:b}$ represents the trajectory of x from time step a to time step b. $N(\mu, \Sigma)$ denotes the Gaussian PDF with mean μ and covariance Σ. $\|x\|$ is the Euclidean norm of vector x. The notation $\propto$ stands for "be proportional to".

II. Problem Formulation

Consider the following nonlinear system with N sensor nodes:

$$\begin{cases} x_{k+1} = f(x_k) + w_k, \\ y_{i,k} = h_i(\phi_{i,k}, x_k) + v_{i,k}, \quad i = 1, \dots, N \end{cases} \tag{1}$$

where $x_k \in \mathbb{R}^n$ is the system state, $y_{i,k} \in \mathbb{R}^{m_i}$ is the measurement output of the ith sensor node, and $\phi_{i,k} \in \mathbb{R}$ is the time-varying degradation variable that reflects the health

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degree of the ith sensor at time step k. $w_k \in \mathbb{R}^n$ and $v_{i,k} \in \mathbb{R}^{m_i}$ are the plant noise and the measurement noise of the ith sensor node, respectively. It is assumed that w_k, $v_{i,k}$, and the initial condition x_0 are independent of each other with known prior densities $p_{w_k}(\cdot)$, $p_{v_{i,k}}(\cdot)$ and $p_{x_0}(\cdot)$. $f(\cdot) : \mathbb{R}^n \rightarrow \mathbb{R}^n$ and $h_i(\cdot,\cdot) : \mathbb{R} \times \mathbb{R}^n \rightarrow \mathbb{R}^{m_i}$ denote, respectively, the state transition function and the measurement function of the ith sensor node.

The actual degradation process for a sensor is generally continuous, and the following widely adopted Wiener-process-based degradation model is used to formulate the continuous degradation variable $\phi_i(t)$ for the ith sensor node:

$$\phi_i(t) = \phi_i(0) + \mu_i t + \sigma_i B(t), \tag{2}$$

where $\phi_i(0)$ is the initial value, $\mu_i > 0$ is the drift coefficient, $\sigma_i > 0$ denotes the diffusion coefficient, and $B(t)$ represents standard Brownian motion. The drift coefficient μ_i stands for the degradation rate because

$$\frac{dE\{\phi_i(t)\}}{dt} = \mu_i.$$

The term $\sigma_i B(t)$ characterizes the stochasticity in the degradation process. $\phi_i(0)$, μ_i, and σ_i are all assumed to be available.

Since a discrete-time degradation variable $\phi_{i,k}$ is needed in (1), let us discretize the continuous model (2) with sampling time Δ_t and establish the following iterative equation for $\phi_{i,k}$:

$$\phi_{i,k+1} = \phi_{i,k} + \mu_i \Delta_t + \epsilon_{i,k}, \tag{3}$$

where $\epsilon_{i,k} \sim N(0, \sigma_i \sqrt{\Delta_t})$.

To estimate the degradation variable and the system state at each time step, an augmented state is constructed as follows:

$$z_k = \begin{bmatrix} x_k \\ \phi_{1,k} \\ \vdots \\ \phi_{N,k} \end{bmatrix}. \tag{4}$$

Based on (1) and (3), we have

$$\begin{cases} z_{k+1} = \hat{f}(z_k) + \hat{w}_k, \\ y_{i,k} = \hat{h}_i(z_k) + v_{i,k}, \end{cases} \tag{5}$$

where

$$\hat{f}(z_k) = \begin{bmatrix} f(x_k) \\ \phi_{1,k} + \mu_1 \Delta_t \\ \vdots \\ \phi_{N,k} + \mu_N \Delta_t \end{bmatrix}, \ \hat{w}_k = \begin{bmatrix} w_k \\ \epsilon_{1,k} \\ \vdots \\ \epsilon_{N,k} \end{bmatrix},$$
$$\hat{h}_i(z_k) = h_i(\phi_{i,k}, x_k).$$

As illustrated in Fig. 1, in this paper, each sensor sends the quantized measurement to the remote filter, where a uniform quantizer is used at each sensor in the proposed data transmission process. For the original measurement output of the ith sensor node

$$y_{i,k} = \left[y_{i,k}^{(1)}, \ldots, y_{i,k}^{(m_i)} \right]^T,$$

a given scaling parameter $s_i > 0$, and a positive integer d_i, the quantization region is set to be

$$\mathcal{R} = \left\{ y_{i,k} \in \mathbb{R}^{m_i} : \left\| y_{i,k}^{(j)} \right\| \leq s_i, j = 1, \ldots, m_1 \right\},$$

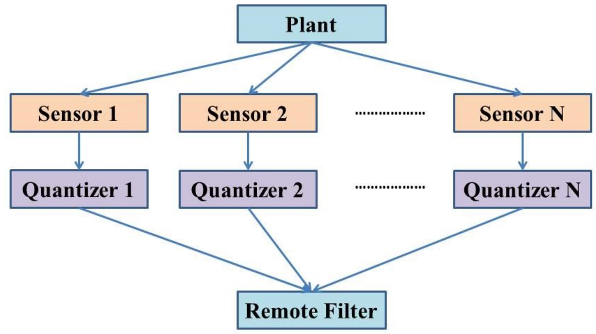

Fig. 1. Block diagram of the system with multiple sensors and quantizers

and the quantization process $q_i(\cdot) : \mathbb{R}^{m_i} \rightarrow \mathbb{R}^{m_i}$ is defined as:

$$q_i(y_{i,k}) = \left[q_i^{(1)}\left(y_{i,k}^{(1)}\right), \ldots, q_i^{(m_i)}\left(y_{i,k}^{(m_i)}\right) \right]^T, \tag{6}$$

where

$$q_i^{(j)}\left(y_{i,k}^{(j)}\right) = \begin{cases} \rho_{i,h}, & \rho_{i,h} - \frac{s_i}{d_i} \leq y_{i,k}^{(j)} < \rho_{i,h} + \frac{s_i}{d_i}, \\ \rho_{i,d_i-1}, & y_{i,k}^{(j)} = s_i, \end{cases} \tag{7}$$

and the quantization levels are given by

$$\mathcal{L}_i = \left\{ \rho_{i,h} : \rho_{i,h} = -s_i + \frac{(2h+1)s_i}{d_i}, h = 0, \ldots, d_i - 1 \right\}, \tag{8}$$

for $h = 0, \ldots, d_i - 1$ and $j = 1, \ldots, m_i$.

At the remote filter end, the signal after quantization can be received. Denote the received signal as $\hat{y}_k = \left[\hat{y}_{1,k}^T, \ldots, \hat{y}_{N,k}^T \right]^T$, where $\hat{y}_{i,k} = q_i(y_{i,k})$.

Remark 1: Due to its advantage of formulating nonmonotonic and nonlinear processes, the diffusion process has been extensively employed in existing literature to describe degradation processes in practical cases such as rolling element bearings [28], dielectric strength of insulators [22], aircraft piston pumps [8], and so on. In this paper, the Wiener process, which is a typical diffusion process, is selected to model the sensor degradation process. The degradation variable $\phi_{i,k}$ makes it difficult to analyze the variation of the measurement function since $\phi_{i,k}$ is usually unavailable and subject to stochastic disturbances. As a consequence, for the addressed nonlinear/non-Gaussian system (5) (even if the measurement noise $v_{i,k}$ is Gaussian distributed, the sensor degradation would render the system non-Gaussian), the approximation-based PF method is adopted to tackle the filtering problem for systems with degrading sensors.

Remark 2: In the paper, measurement data are collected from multiple sensors, which can provide abundant information for the system monitoring. The value of the scaling parameter s_i can be determined based on the normal working condition and the measurement equation of the addressed system. A uniform quantizer is considered for each sensor, and the quantization error will influence the calculation of the importance weights in the PF method to be developed. The consideration of the multiple degrading sensors and quantizations leads to some extra challenges in the design of the PF.

The purpose of this paper is to develop a particle filtering algorithm for the multi-sensor system with quantizations and sensor degradations. With the quantized measurement information $\hat{y}_{1:k}$, the system state and the degradation variables will be jointly estimated at the remote filter in the sense of minimum mean-square error (MMSE).

III. Algorithm Design and Discussion

In practical situations, it is almost intractable to calculate the marginal posterior PDF $p(z_k|\hat{y}_{1:k})$ for nonlinear (uncertain or non-Gaussian) systems, and thus the MMSE estimate cannot be accurately obtained. To solve this problem, a set of weighted particles is used in the PF algorithm to approximate the posterior PDF expression as follows:

$$p(z_{0:k}|\hat{y}_{1:k}) = \sum_{m=1}^{M} W_k^m \delta\left(z_{0:k} - z_{0:k}^m\right), \tag{9}$$

where M is the number of the particles, and $\delta(\cdot)$ denotes Dirac delta function. The particles $\{z_{0:k}^m\}$ are drawn from an importance density $q(z_{0:k}|\hat{y}_{1:k})$, and the importance weights W_k^m can be determined as

$$W_k^m \propto W_{k-1}^m \frac{p(\hat{y}_k|z_k^m)\, p\left(z_k^m|z_{k-1}^m\right)}{q\left(z_k^m|z_{0:k-1}^m, \hat{y}_{1:k}\right)}. \tag{10}$$

Since the measurement noises $v_{i,k}$ are independent from each other, we have

$$p(\hat{y}_k|z_k^m) = \prod_{i=1}^{N} p(\hat{y}_{i,k}|z_k^m). \tag{11}$$

Substituting (11) into (10) yields

$$W_k^m \propto W_{k-1}^m \frac{\left(\prod_{i=1}^{N} p(\hat{y}_{i,k}|z_k^m)\right) p\left(z_k^m|z_{k-1}^m\right)}{q\left(z_k^m|z_{0:k-1}^m, \hat{y}_{1:k}\right)}. \tag{12}$$

As a frequently adopted proposal distribution, the importance density in this paper is selected as

$$q\left(z_k^m|z_{0:k-1}^m, \hat{y}_{1:k}\right) = p\left(z_k^m|z_{k-1}^m\right), \tag{13}$$

and (12) becomes

$$W_k^m \propto W_{k-1}^m \left(\prod_{i=1}^{N} p(\hat{y}_{i,k}|z_k^m)\right). \tag{14}$$

It is obvious that the measurement $\hat{y}_{i,k}$ received by the remote sensor is dependent on both the sensor degradation and the uniform quantization. As a result, the likelihood function $p(\hat{y}_{i,k}|z_k^m)$, which is necessary in the calculation of W_k^m, is related to the degradation variable $\phi_{i,k}$ and the quantization process $q_i(\cdot)$. In the sequel, the likelihood function will be extensively investigated under the effects of $\phi_{i,k}$ and $q_i(\cdot)$. The consideration of the degradation and the quantization distinguishes the proposed method and the conventional sequential importance sampling approaches.

With the nonlinear model (5) and the quantization process (7), $p(\hat{y}_{i,k}|z_k)$ is given by

$$p(\hat{y}_{i,k}|z_k) = p\left(\bigcap_{j=1}^{m_i}\left(\hat{y}_{i,k}^{(j)} - \frac{s_i}{d_i} \le y_{i,k}^{(j)} < \hat{y}_{i,k}^{(j)} + \frac{s_i}{d_i}\right)\middle| z_k\right), \tag{15}$$

where $\hat{y}_{i,k}^{(j)}$ is the jth component of $\hat{y}_{i,k}$.

Algorithm 1 The proposed quantized particle filtering with multiple degrading sensors

Step 1. Particle initialization
Draw M particles from the initial prior probability density function $z_0^m \sim p_{z_0}(\cdot)$ and all importance weights are set to be $\frac{1}{M}$. Furthermore, set the maximum simulation step K.

Step 2. Measurement collection
Collect the quantized measurement $\hat{y}_{i,k}$ from each sensor at the current time instant.

Step 3. Importance sampling
For each $m = 1, \ldots, M$, draw particle z_k^m from the transition probability density function $p\left(z_k^m|z_{k-1}^m\right)$.

Step 4. Weight calculation
According to (12), calculate the importance weights $\hat{W}_k^m$ as $\hat{W}_k^m = W_{k-1}^m \left(\prod_{i=1}^{N} p(\hat{y}_{i,k}|z_k^m)\right)$ for each $m = 1, \ldots, M$, and normalize the weights as $W_k^m = \dfrac{\hat{W}_k^m}{\sum_{l=1}^{M} \hat{W}_k^l}$.

Step 5. State estimate update
Calculate the state estimate $\hat{z}_k = \sum_{m=1}^{M} W_k^m z_k^m$.

Step 6. Resampling
Resample a new set of particles with equal weights from $\sum_{m=1}^{M} W_k^m \delta(z_k - z_k^m)$.

Step 7. If $k < K$, then set $k = k + 1$ and go to Step 2; otherwise go to Step 8.

Step 8. Stop.

The analytical expression of the $p(\hat{y}_{i,k}|z_k)$ can be obtained only in some special situations. For example, when $\hat{y}_{i,k}$ is a scalar and $v_{i,k} \sim N(\mu_{i,k}, \sigma_{i,k}^2)$, we have

$$\begin{aligned} p(\hat{y}_{i,k}|z_k) &= p\left(\hat{y}_{i,k} - \frac{s_i}{d_i} \le y_{i,k} < \hat{y}_{i,k} + \frac{s_i}{d_i}\middle| z_k\right) \\ &= p\left(\underline{\eta}_{i,k} \le v_{i,k} < \bar{\eta}_{i,k}\middle| z_k\right) \\ &= \Phi\left(\frac{\bar{\eta}_{i,k} - \mu_{i,k}}{\sigma_{i,k}}\right) - \Phi\left(\frac{\underline{\eta}_{i,k} - \mu_{i,k}}{\sigma_{i,k}}\right), \end{aligned} \tag{16}$$

where

$$\underline{\eta}_{i,k} = \hat{y}_{i,k} - \hat{h}_i(z_k) - \frac{s_i}{d_i},$$
$$\bar{\eta}_{i,k} = \hat{y}_{i,k} - \hat{h}_i(z_k) + \frac{s_i}{d_i},$$

and $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution. The above calculation can be extended to

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deal with multi-dimensional measurements, each component of which is subject to uncorrelated Gaussian noise. When the measurement $\hat{y}_{i,k}$ is a vector and the noise $v_{i.k}$ is non-Gaussian, the approach described in the above example is no longer suitable, and a Monte-Carlo method proposed in [16] can be adopted here to approximate the likelihood function $p\left(\hat{y}_{i,k}|z_k\right)$. Specifically, for each particle z_k^m, H samples denoted as $\left\{\hat{y}_{i,k}^{m,h}\right\}(h = 1,\ldots,H)$ can be drawn from measurement equation (5) and the quantization process (7). With these samples, $p\left(\hat{y}_{i,k}|z_k^m\right)$ can be approximated as

$$p\left(\hat{y}_{i,k}|z_k^m\right) = \frac{1}{H}\sum_{h=1}^{H} 1_{\left\{\hat{y}_{i,k}^{m,h}=\hat{y}_{i,k}\right\}}, \tag{17}$$

where $1_{\left\{\hat{y}_{i,k}^{m,h}=\hat{y}_{i,k}\right\}}$ is an indicator function defined as

$$1_{\left\{\hat{y}_{i,k}^{m,h}=\hat{y}_{i,k}\right\}} = \begin{cases} 1, \text{ if } & \hat{y}_{i,k}^{m,h} = \hat{y}_{i,k}, \\ 0, \text{ otherwise.} \end{cases} \tag{18}$$

The implementation of the proposed particle filtering with multiple degrading sensors and quantized measurements is outlined in Algorithm 1 on the previous page.

Remark 3: The PF method has been designed for a class of nonlinear systems with multiple degrading sensors and quantizations. The effects of the sensor degradation have been reflected by $\phi_{i,k}(i = 1,\ldots,N)$ in the augmented system state z_k, and the quantization process has been considered in the calculation of the likelihood function $p\left(\hat{y}_{i,k}|z_k\right)$. The proposed method can also handle other types of quantization processes by appropriately adjusting the likelihood function, and thus the applicability of the presented strategy can be enhanced. The position of the degradation and the quantization-induced modification of the likelihood function constitute the main difference between the filter developed in this paper and that designed in [19]. If we neglect the variable $\phi_{i,k}$ in the problem formulation (1), then the proposed algorithm is also applicable to the situation where the sensor degradation is not considered. W_k^m is the actual weight after the normalization of $\hat{W}_k^m$, and the normalization can guarantee that $\sum_{m=1}^{M} W_k^m = 1$. In addition, if the addressed plant is subject to degradation as well, i.e., $f(\cdot)$ is also affected by a degradation variable, a new system state can be constructed by augmenting the original system state and actuator/sensor degradation variables, and corresponding PF algorithm can be developed analog to Algorithm 1.

Remark 4: When the approximation-based method (17) is adopted to calculate the likelihood function, it can be seen that $H \times M$ samples are needed at each time step, which may result in heavy calculation burden. To guarantee the feasibility of the proposed method in practice, the values of H and M cannot be set to be overly large. To effectively execute our filtering algorithm, the computational resources of the remote filter are assumed to be sufficient. This assumption is fairly reasonable for a filter which needs to monitor the addressed system with multiple sensors.

IV. Numerical Study

In this section, the effectiveness of the proposed quantized PF algorithm with multiple degrading sensors is illustrated in a two-dimensional target tracking example. The system state represents the coordinate value of the target and is defined as $x_k = \left[x_k^p, y_k^p\right]^T$, where x_k^p and y_k^p are the X- and Y-axis positions, respectively. A uniform circular motion trajectory under disturbance is considered, and the discrete-time model is established as follows:

$$x_{k+1} = c + r\begin{bmatrix} \cos\left[\arctan\left(\dfrac{y_k^p - c_y}{x_k^p - c_x}\right) + \dfrac{2\pi}{T}\right] \\ \sin\left[\arctan\left(\dfrac{y_k^p - c_y}{x_k^p - c_x}\right) + \dfrac{2\pi}{T}\right] \end{bmatrix} + w_k, \tag{19}$$

where $c = \left[c_x, c_y\right]^T$ is the center of the circle, $r > 0$ is the radius, $T > 0$ is the period of the circular motion, and w_k is an additive noise.

Bearing-only measurements are considered in our simulation. Set the coordinate of the ith sensor to be

$$\left[x_{i,k}^{\text{sensor}}, y_{i,k}^{\text{sensor}}\right]^T.$$

Due to the existence of the sensor degradation, the target-sensor distance in the calculation of the bearing is assumed to be subject to an additive disturbance, whose coefficient is the degradation variable. Then, the bearing measurement of the ith sensor can be formulated as follows:

$$y_{i,k} = \arctan\left(\frac{y_k^p - y_{i,k}^{\text{sensor}} + \phi_{i,k}s_{x,i,k}}{x_k^p - x_{i,k}^{\text{sensor}} + \phi_{i,k}s_{y,i,k}}\right) + v_{i,k}, \tag{20}$$

where $s_{x,i,k}$, $s_{y,i,k}$, and $v_{i,k}$ are noises which are independent of each other and all satisfy standard normal distribution.

The following root mean-square error (RMSE) is adopted to evaluate the accuracy of the filtering scheme:

$$\text{RMSE}_k = \sqrt{\frac{1}{L}\sum_{l=1}^{L}\left[\left(x_k^{p,l} - \hat{x}_k^{p,l}\right)^2 + \left(y_k^{p,l} - \hat{y}_k^{p,l}\right)^2\right]}, \tag{21}$$

where L is the total number of the Monte Carlo runs, $\left(x_k^{p,l}, y_k^{p,l}\right)$ is the realization of (x_k^p, y_k^p) in the lth Monte Carlo run, and the respective estimate is denoted by $\left(\hat{x}_k^{p,l}, \hat{y}_k^{p,l}\right)$.

In each independent Monte Carlo run, the initial value of the actual trajectory is drawn from $x_0 \sim N([200, -250]^T, I_2)$. The maximum simulation time K and the sampling period Δ_t are set to be 100s and 1s, respectively. The period of the circular motion is $T = 100$s. The center of the circle is $c = [150, -250]^T$m, and the radius of the trajectory $r = 50$m. The additive noise satisfies $w_k \sim N(0_{2\times1}, I_2)$.

In our example, the number of the sensors is three, and the parameters of the sensors and quantizers are listed in Table I. The noises $v_{i,k}$ all obey standard normal distribution for $i = 1, 2, 3$. The particle number is selected to be 100. Because all the three measurements are scalars and the noises are assumed to satisfy Gaussian distribution, (16) can be directly used to calculate $p\left(\hat{y}_{i,k}|z_k\right)$ in our simulation.

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TABLE I
PARAMETERS OF THE SENSORS AND QUANTIZERS

Parameters	Values		
i	1	2	3
$\phi_{i,0}$	0	0	0
μ_i	0.2	0.4	0.6
σ_i	5	3	1
s_i	π	π	π
d_i	2	3	4

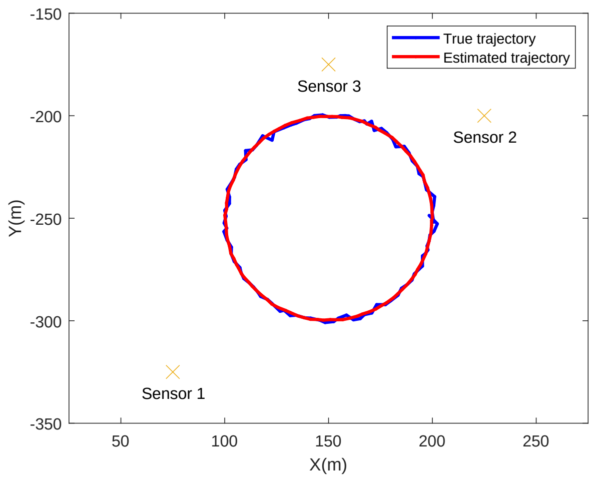

Fig. 2. True target trajectory and its estimate obtained by PF-SD-UQ in one trial.

In the simulation, the following four algorithms are evaluated and compared:

1) PF-SD-UQ: the proposed PF with sensor degradation and uniform quantization;
2) PF-SD: a PF considering the sensor degradation but neglecting the quantization;
3) PF-UQ: a PF considering the quantization but neglecting the sensor degradation; and
4) PF: a conventional PF neglecting the quantization and the sensor degradation.

In Fig. 2, one realization of the true circular trajectory, its estimate obtained by the proposed PF-SD-UQ, as well as the sensor positions are depicted. To better present the tracking performance in a run, the tracking results for X- and Y-axis positions are illustrated in Figs. 3 and 4, respectively. It can be observed that tracking performance of the PF-SD-UQ is satisfactory. The original and quantized measurements from the three sensors are shown in Figs. 5-7. The effects of the uniform quantization on the measurements are clearly reflected in the figures. In the presence of the information loss in the quantization process, the PF method has been revised with a novel likelihood function in our algorithm.

The RMSEs obtained with the PF-SD-UQ, PF-SD, PF-UQ, and PF are demonstrated in Fig. 8 with 20 Monte Carlo runs. It can be seen that the proposed PF-SD-UQ method has the best target tracking performance among the four approaches. This is due to our way of calculating the weights for each particle in consideration of sensor degradations and

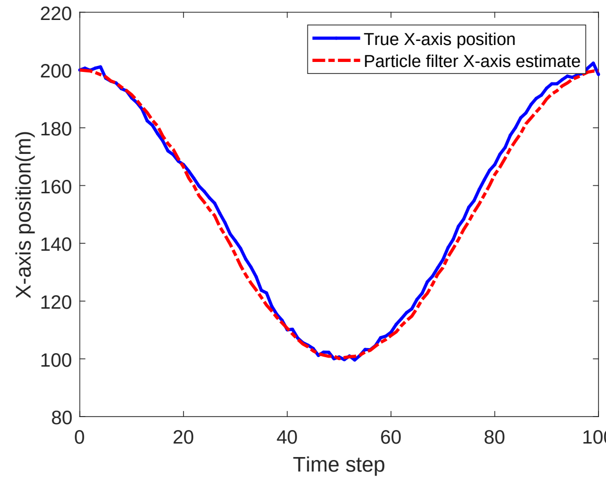

Fig. 3. True X-axis position and its estimate in one trial.

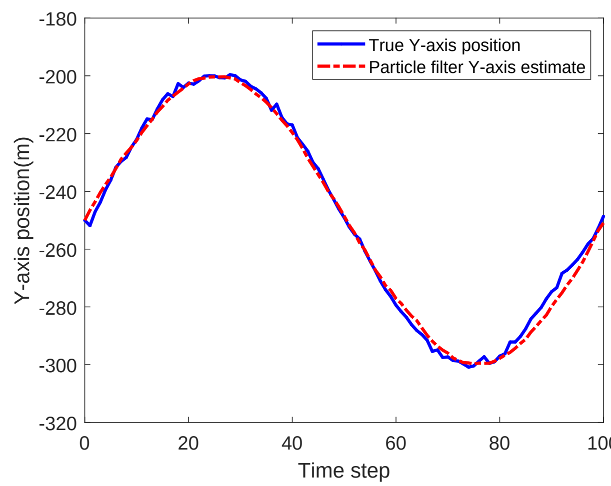

Fig. 4. True Y-axis position and its estimate in one trial.

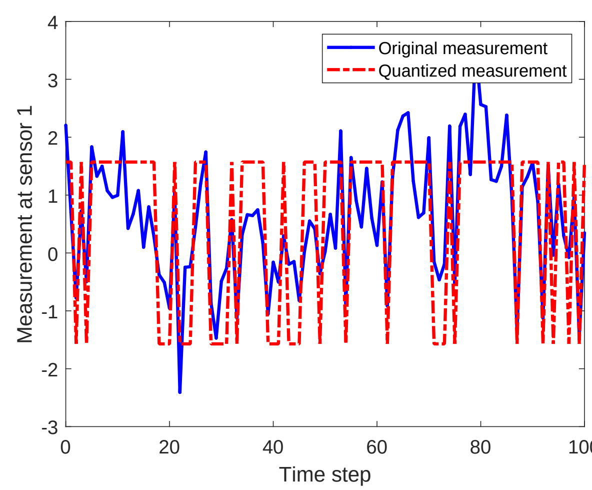

Fig. 5. Original and quantized measurements at sensor 1.

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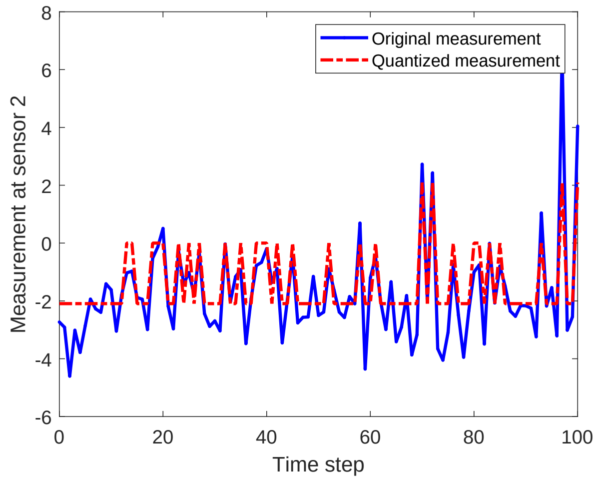

Fig. 6. Original and quantized measurements at sensor 2.

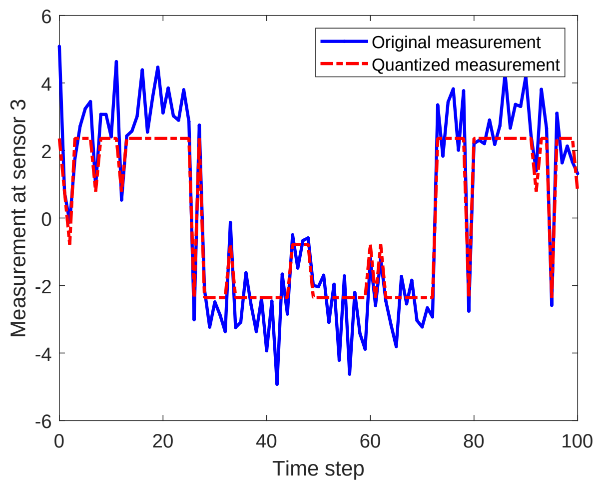

Fig. 7. Original and quantized measurements at sensor 3.

uniform quantizations. The increase of the RMSE stems from degradation-induced noises, which become larger with time.

Now, let us discuss the effects of the sensor degradation on the filtering performance.In the comparative simulations, the variable σ is set to be the same for the three sensors. Fig. 9 depicts the RMSEs obtained with different σs from 20 Monte Carlo runs. It is clear that a smaller σ leads to a better tracking result. This is natural since the degradation variable with a small variance can be estimated accurately, and the estimation performance for the system states can also be improved.

The relationship between the RMSEs and the drift coefficients is reflected Fig. 10 from 20 Monte Carlo runs. A larger μ, which means a larger average value of the noise in the $\arctan$ function of the measurements, results in a larger filtering error.

Finally, we are going to investigate the relationship between the quantization process and the filtering performance. From Figs. 5-7, it can already be seen that dense quantization levels can reduce the quantization error between the original

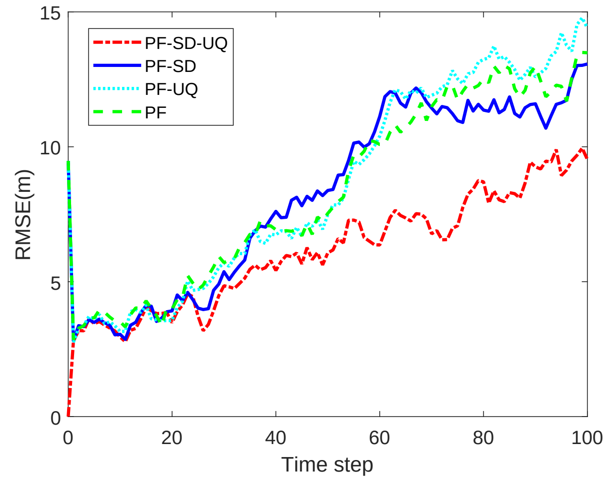

Fig. 8. RMSEs obtained with the PF-SD-UQ, PF-SD, PF-UQ, and PF.

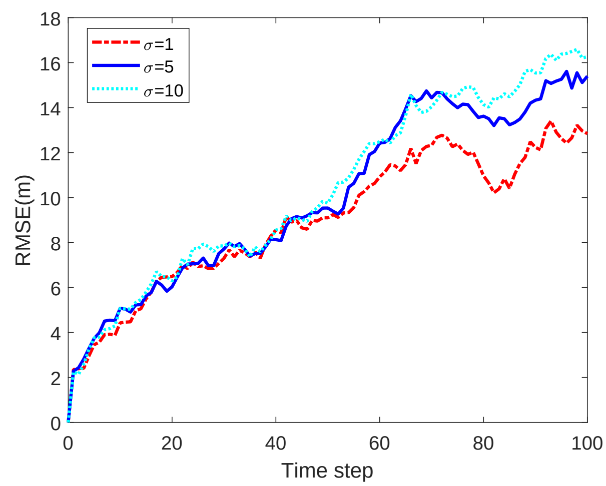

Fig. 9. RMSEs obtained with different σs.

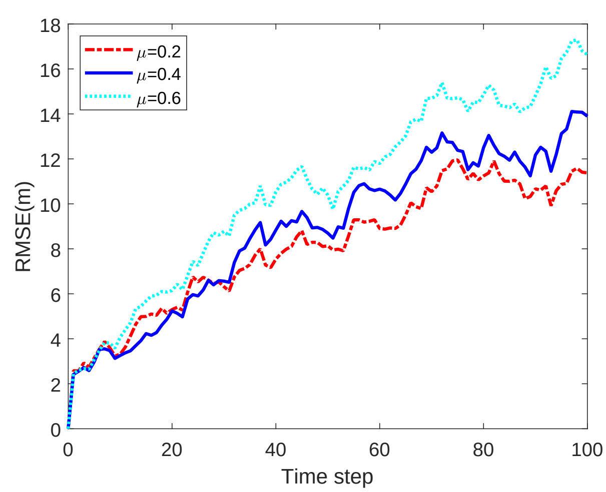

Fig. 10. RMSEs obtained with different μs.

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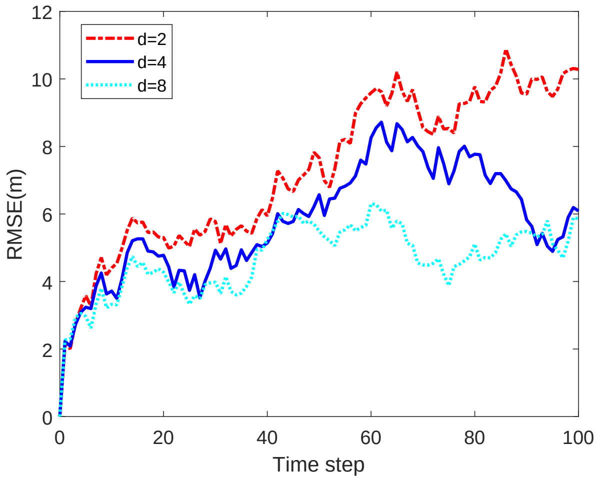

Fig. 11. RMSEs obtained with different ds.

measurement and the quantized measurement. With the same d for each sensor, Fig. 11 illustrates the RMSEs obtained with different ds from 20 Monte Carlo runs. It can be readily concluded that a larger d gives rise to a smaller quantization error and a more accurate tracking result. It should also be noted that, a larger d also brings in a larger word length for each transmitted measurement, so a trade-off between the filtering accuracy and the transmission load needs to be achieved by adjusting the quantization parameters.

V. Conclusion

In this paper, the quantized PF design problem has been studied for a class of nonlinear/non-Gaussian systems with multiple degrading sensors. The Wiener process has been used to formulate the dynamics of the degradation variables. A uniform quantizer has been introduced at each sensor to reduce the word length of the transmitted signal and thus can lower the transmission burden in the links. A new likelihood function has been obtained under the influences of the sensor degradation and the uniform quantization, and the recursion for importance weights has been established in the proposed PF. Finally, a target tracking example has been provided to illustrate the effectiveness of the proposed algorithm. Because of our effort in dealing with the sensor degradation and the uniform quantization, the tracking performance of the proposed PF-SD-UQ algorithm has shown to be satisfactory. One possible future task is to apply the developed filtering approach to systems with more network-induced phenomena and more general degradations (such as nonlinear degradation and multi-phase degradation).

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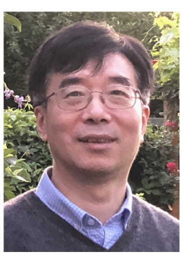

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