Standard Histogram Test Precision of ADC Gain and Offset Error Estimation

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***Abstract*—The quality of measurements made in any system is quantified by supplying the expanded uncertainty of the result, as recommended in the Guide to Uncertainty in Measurement. In a system that involves analog-to-digital converters (ADCs), one of the sources of uncertainty is the converter's gain and offset error. The uncertainty of these two parameters should be known in order to compute the uncertainty of the measurements made with the system. In this paper, we study the uncertainty of terminal-based defined gain and offset error that are estimated using a standard histogram test in the presence of an additive noise.**

***Index Terms*—Analog-to-digital converter (ADC), gain, histogram test, offset error, precision.**

I. INTRODUCTION

A STANDARD histogram test is extensively used in the area of analog-to-digital converter (ADC) testing to obtain their transfer function and, consequently, several parameters of interest, namely, the integral nonlinearity (INL), differential NL (DNL), gain, and offset error, among others. All these parameters attest to the capacity of the ADC to perform its intended function. ADCs are rarely used alone but are often included in more elaborate systems. The performance of the ADCs will affect the performance of the system where it is included, and the precision with which the ADC parameters are known is necessary to compute the precision of the final results of the system using it. This, in turn, is extremely important in accessing the quality of the system and, ultimately, the application it serves.

The authors have extensively worked to determine the precision of the estimates of the ADC characteristics that are obtained with the standard histogram test and other ADC test methods [1]–[5]. Other authors have also published valuable contributions in this area [6]–[9], which is a very active field of research. This paper focuses on the ADC gain and offset error, which are characteristics that have not received as much study as the other parameters, like INL and DNL, for instance. We consider the influence of three factors, namely, the amplitude of the stimulus signal, the amount of additive noise that is present in the ADC itself and in the test setup, and the number of acquired samples when performing the test. Other factors, like phase noise and frequency error, can also potentially affect the gain and offset error estimates uncertainty, but the study of their influence will be relegated to a separate publication.

In Section II, we will present the definitions of ADC gain and offset error as they are usually understood. Next, in Section III, we determine the variance of the ADC gain and offset error estimates, and in Section IV, we present experimental results that would attest to the validity of the derived expressions. Finally, in Section VI, we present some concluding remarks.

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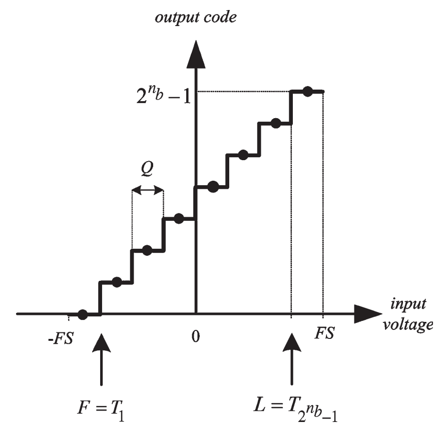

Fig. 1. Transfer function of a bipolar ADC. This type of transfer function is known as with no true zero.

II. TERMINAL-BASED GAIN AND OFFSET ERROR

The purpose of an ADC is to convert the values of a current or voltage that is present at the input, which is a continuous variable, into a digital word that should represent that input. The relationship between the input variable and output digital words (or codes) is known as the ADC transfer function and is determined by the ADC manufacturer. In the rest of the text, we will consider that the input variable is a voltage. There are different types of transfer functions. One of them, which is used with bipolar ADCs, is the one represented in Fig. 1 and is known as "with no true zero." Variable n_b represents the ADC number of bits, and FS represents the full-scale voltage.

Each output code corresponds to a range of input voltage values (horizontal lines). Given an output code, one cannot exactly determine which was the input voltage at the time of the ADC. It is a convention to adopt the middle point of the ranges that is mentioned as the value of the input voltage for a given output code (black circles).

The transition voltages T_k define the ADC transfer function, i.e., the relation between the input voltage and output code k. For an ideal ADC, the transition voltages of the transfer function, which is defined as in Fig. 1, are

$$T_k^{\text{ideal}} = -\text{FS} + k \cdot Q. \tag{1}$$

They are equally spaced by an amount Q that is given, from the definition of the transfer function, by

$$Q = \frac{2 \cdot \text{FS}}{2^{n_b}}. \tag{2}$$

In an actual ADC, the real transition voltages will be different from the ideal ones. To express those differences, several parameters are used. Two of those are the ADC gain and offset error. They can be defined in different ways. Two of the most commonly used definition are the terminal-based definition and the independently based definition [10]. In this paper, we will focus our attention on the first one, leaving the other one for another publication due to its complexity. According to the terminal-based definition, the offset error, plus the product of the gain by the first and last real transition voltages, results in the first and last ideal transition voltages, respectively. Hence, the designation "terminal-based" refers to the fact that the definition is based on the extremes of the transfer function, i.e., on the value of the first (lowest) and last (highest) transition voltages. The gain (G) and offset error (O) will satisfy the definition if they are computed using the following expressions [1]:

$$G = \frac{L_{\text{ideal}} - F_{\text{ideal}}}{L - F} \quad \text{and} \quad O = F_{\text{ideal}} - G \cdot F. \tag{3}$$

To simplify the notation, we introduced the variables $F = T_1$ and $L = T_{2^{n_b}-1}$.

When testing an ADC with the standard histogram test, we obtain an estimate of the transition voltages (not the real transition voltages). From those estimates, we can compute the estimated ADC gain and offset error

$$\widehat{G} = \frac{L_{\text{ideal}} - F_{\text{ideal}}}{\widehat{L} - \widehat{F}} \quad \text{and} \quad \widehat{O} = F_{\text{ideal}} - \widehat{G} \cdot \widehat{F}. \tag{4}$$

The hat over the symbols signifies that they are an estimate and not the actual values of the ADC under test.

III. Estimates Precision

Since the terminal-based gain and offset error are a function of two random variables, namely, the first and last estimated transition voltages $\widehat{F}$ and $\widehat{L}$, they will also be random variables. We will now determine the standard deviation of the estimated gain and offset error.

The variance of the estimated gain and offset error can be approximated by [11, p. 156]

$$\begin{aligned}\sigma^2_{\widehat{G}} &\approx \left(\frac{\partial \widehat{G}}{\partial \widehat{F}}\right)^2_{\widehat{F}=\mu_{\widehat{F}}} \sigma^2_{\widehat{F}} + \left(\frac{\partial \widehat{G}}{\partial \widehat{L}}\right)^2_{\widehat{L}=\mu_{\widehat{L}}} \sigma^2_{\widehat{L}} \\ \sigma^2_{\widehat{O}} &\approx \left(\frac{\partial \widehat{O}}{\partial \widehat{F}}\right)^2_{\widehat{F}=\mu_{\widehat{F}}} \sigma^2_{\widehat{F}} + \left(\frac{\partial \widehat{O}}{\partial \widehat{L}}\right)^2_{\widehat{L}=\mu_{\widehat{L}}} \sigma^2_{\widehat{L}}\end{aligned} \tag{5}$$

which, in this case, by using (4), leads to

$$\begin{aligned}\sigma^2_{\widehat{G}} &\approx \left(\frac{L_{\text{ideal}} - F_{\text{ideal}}}{(\mu_{\widehat{L}} - \mu_{\widehat{F}})^2}\right)^2 \sigma^2_{\widehat{F}} + \left(\frac{L_{\text{ideal}} - F_{\text{ideal}}}{(\mu_{\widehat{L}} - \mu_{\widehat{F}})^2}\right)^2 \sigma^2_{\widehat{L}} \\ \sigma^2_{\widehat{O}} &\approx \left(\mu_{\widehat{L}} \frac{L_{\text{ideal}} - F_{\text{ideal}}}{(\mu_{\widehat{L}} - \mu_{\widehat{F}})^2}\right)^2 \sigma^2_{\widehat{F}} + \left(\mu_{\widehat{F}} \frac{L_{\text{ideal}} - F_{\text{ideal}}}{(\mu_{\widehat{L}} - \mu_{\widehat{F}})^2}\right)^2 \sigma^2_{\widehat{L}}.\end{aligned} \tag{6}$$

Approximating the mean of the estimated values of $\widehat{F}$ and $\widehat{L}$ by their ideal values leads to

$$\begin{aligned}\sigma^2_{\widehat{G}} &\approx \left(\frac{1}{L_{\text{ideal}} - F_{\text{ideal}}}\right)^2 \sigma^2_{\widehat{F}} + \left(\frac{1}{L_{\text{ideal}} - F_{\text{ideal}}}\right)^2 \sigma^2_{\widehat{L}} \\ \sigma^2_{\widehat{O}} &\approx \left(\frac{L_{\text{ideal}}}{L_{\text{ideal}} - F_{\text{ideal}}}\right)^2 \sigma^2_{\widehat{F}} + \left(\frac{F_{\text{ideal}}}{L_{\text{ideal}} - F_{\text{ideal}}}\right)^2 \sigma^2_{\widehat{L}}.\end{aligned} \tag{7}$$

On a bipolar ADC, with a "no-true-zero" transfer function, one has, from (1), $L_{\text{ideal}} = -F_{\text{ideal}} = \text{FS} - Q$. Equation (7) simplifies to

$$\begin{aligned}\sigma^2_{\widehat{G}} &\approx \frac{1}{4(\text{FS} - Q)^2} \left(\sigma^2_{\widehat{F}} + \sigma^2_{\widehat{L}}\right) \\ \sigma^2_{\widehat{O}} &\approx \frac{1}{4} \left(\sigma^2_{\widehat{F}} + \sigma^2_{\widehat{L}}\right).\end{aligned} \tag{8}$$

Equation (8) expresses the variance of the estimated gain and offset error as a function of the variances of the first and last estimated transition voltages. The precision of the estimated transition voltages using the standard histogram method has been a subject of a previous work in [3]. We will now briefly review the results adapted to the situation considered here.

The standard histogram method involves the application of a sinusoidal stimulus signal, with frequency f, amplitude A, and offset C, to the ADC under test, and the acquisition of a predefined number of samples M with a sampling frequency f_s. The output codes that are obtained are then grouped into classes that form a histogram—more precisely, a cumulative histogram—since the number of elements c_k in each class k is the number of samples with output codes that are equal to or lower than k. From the cumulative histogram, the transition voltages are estimated using [3], [6]

$$\widehat{T}_{k+1} = C - A \cdot \cos\left(\pi \frac{c_k}{M}\right) \cdot \tag{9}$$

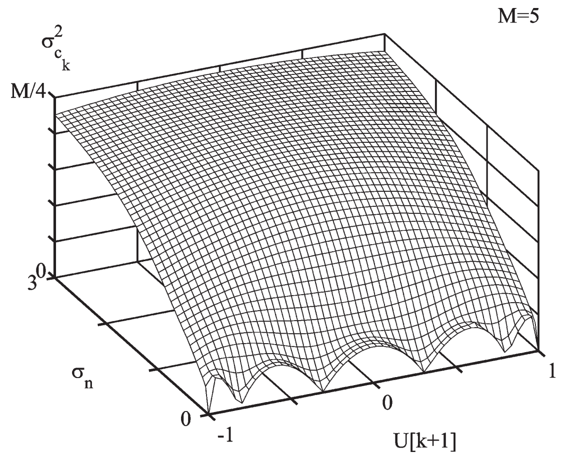

Fig. 2. Variance of the number of counts of cumulative histogram as a function of the normalized additive noise (σ_n) and normalized transition voltages (U).

The number of counts in the cumulative histogram is a random variable. From (9), one can determine the variance of the estimated transition voltages [11, p. 113]

$$\sigma^2_{\widehat{T}_{k+1}} \approx \left(\frac{A\pi}{M}\right)^2 \left(1 - \left(\frac{T_{k+1} - C}{A}\right)^2\right) \sigma^2_{c_k}. \tag{10}$$

The variance of the number of counts of the cumulative histogram depends on the real transition voltages (T), additive-noise standard deviation (σ), and number of samples (M). In [3], we presented its mathematical derivation, which lead to the following result:

$$\begin{aligned}
\sigma^2_{c_k} &= \mu_{\sigma^2_{c_k|\varphi}} + \sigma^2_{\mu_{c_k|\varphi}} \\
\mu_{\sigma^2_{c_k|\varphi}} &= \frac{M}{2\pi} \int_{-\pi}^{\pi} p_k(\gamma)\left[1 - p_k(\gamma)\right] d\gamma \\
\sigma^2_{\mu_{c_k|\varphi}} &= \frac{M}{2\pi} \int_{0}^{\frac{2\cdot\pi}{M}} \left(\sum_{j=0}^{M-1} p_k\left(j\frac{2\pi}{M} + \varphi\right)\right)^2 d\varphi \\
&\quad - \left(\frac{M}{2\pi}\int_{-\pi}^{\pi} p_k(\gamma) d\gamma\right)^2
\end{aligned} \tag{11}$$

where

$$p_k(\gamma) = \frac{1}{2} + \frac{1}{2}\,\mathrm{erf}\left(\frac{U_{k+1} + \cos(\gamma)}{\sqrt{2}\cdot\sigma_n}\right) \tag{12}$$

and

$$U_k = \frac{T_k - C}{A}, \quad \sigma_n = \frac{\sigma}{A}. \tag{13}$$

This variance is graphically represented in Fig. 2 as a function of the normalized transition voltage (U), normalized additive-noise standard deviation (σ_n), which is given by (13), and the number of acquired samples (M).

Note that the number of arcs observed for $\sigma_n = 0$ is the same as the number of acquired samples. In addition, note that the variance of the number of counts of the cumulative histogram approaches $M/4$ when the additive-noise standard deviation approaches infinity. The presented example is for $M = 5$, just to illustrate the influence of the different variables. In practice, the number of samples will be much greater, and the normalized standard deviation is smaller (< 0.1).

There is no closed-form expression that can be derived to calculate the variance of the number of counts of the cumulative histogram. In [3], an approximate expression was proposed for the maximum value of the variance of the number of counts of the cumulative histogram for every transition voltage. Here, we are not interested in the maximum value of the variance for all transition voltages, but we are interested, instead, in the variance of the number of counts of the cumulative histogram for the first and last transition voltages. The approximate expression that we will use here is

$$\sigma^2_{c_k} \approx \max\left(\frac{1}{4}, \frac{M}{\pi\sqrt{\pi}}\frac{\sigma_n}{\sqrt{1 - U^2_{k+1}}}\right) \tag{14}$$

which is a good approximation for the practical values of $\sigma_n < 0.1$.

By using (10) and (14), we can derive an approximate expression for the variance of the estimated transition voltages

$$\sigma^2_{\widehat{T}_{k+1}} \approx \left(\frac{A\pi}{M}\right)^2 \left(1 - U^2_{k+1}\right) \max\left(\frac{1}{4}, \frac{M}{\pi\sqrt{\pi}}\frac{\sigma_n}{\sqrt{1 - U^2_{k+1}}}\right). \tag{15}$$

From (8) and by considering that, for a bipolar ADC with a no-true-zero transfer function, the first and last transition voltages are symmetric and their estimate has the same variance, we can write

$$\sigma_{\widehat{G}} \approx \frac{\sigma_{\widehat{T}[0]}}{\sqrt{2}(\mathrm{FS} - Q)} \quad \text{and} \quad \sigma_{\widehat{O}} \approx \frac{\sigma_{\widehat{T}[0]}}{\sqrt{2}}. \tag{16}$$

By introducing (15) into (16), we have an approximate expression for the variance of the estimated gain and offset error, as desired:

$$\begin{aligned}
\sigma_{\widehat{O}} &= (\mathrm{FS} - Q)\sigma_{\widehat{G}} \\
&\approx \frac{A\pi}{M}\frac{1}{\sqrt{2}}\sqrt{1 - U_0^2}\sqrt{\max\left(\frac{1}{4}, \frac{M}{\pi\sqrt{\pi}}\frac{\sigma_n}{\sqrt{1 - U_0^2}}\right)}.
\end{aligned} \tag{17}$$

IV. Experimental Results

To demonstrate the validity of the expressions presented here, we tested a 12-bit ADC using the standard histogram test. Only the eight most significant bits were used so that the ADC could be considered ideal. By using a Monte Carlo procedure with 1000 repetitions (N), we computed the standard deviation of the estimated gain and offset error.

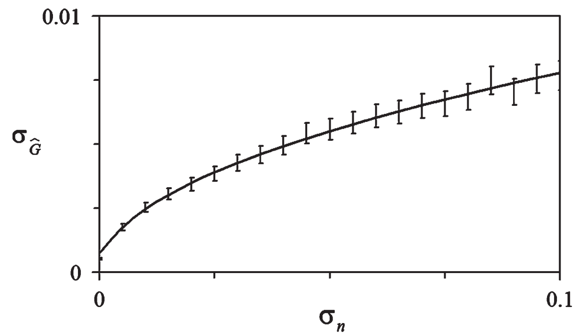

Fig. 3. Estimated gain as a function of the normalized additive-noise standard deviation. We considered $n_b = 8$, $A = 1.2$ V, FS = 1 V, $M = 1000$, $f = 200$ Hz, $f_s = 200$ kHz, and $N = 1000$. The solid line represents the approximation given by (17), and the vertical bars represent the experimental results.

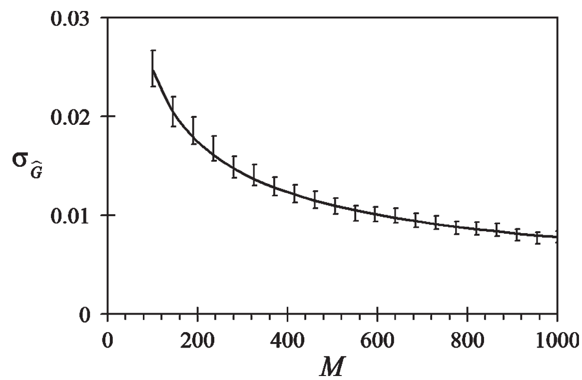

Fig. 4. Estimated gain as a function of the number of samples. We considered $n_b = 8$, $A = 1.1$ V, FS = 1 V, $\sigma_n = 0.1$, $f = f_s/M$, $f_s = 200$ kHz, and $N = 1000$. The solid line represents the approximation given by (17), and the vertical bars represent the experimental results.

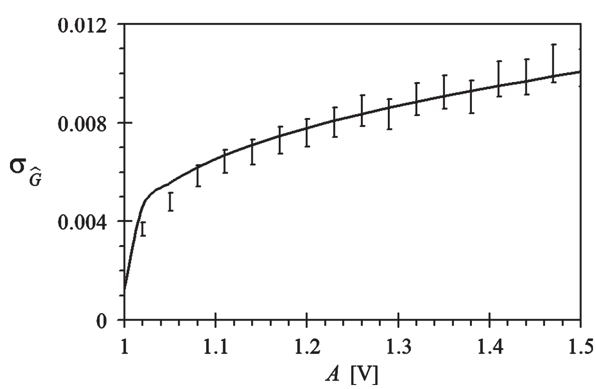

Fig. 5. Estimated gain as a function of the stimulus signal amplitude. We considered $n_b = 8$, $M = 1000$, FS = 1 V, $\sigma_n = 0.1$, $f = 200$ Hz, $f_s = 200$ kHz, and $N = 1000$. The solid line represents the approximation given by (17), and the vertical bars represent the experimental results.

We varied the additive-noise standard deviation, number of samples, and stimulus signal amplitude in order to observe their influence on the standard deviation of the estimated gain and offset error. The results for the estimated gain are shown in Figs. 3–5. Since the variance of the estimated offset error is just a scaled version of the variance of the estimated gain, as seen in (8), we refrained from showing them here.

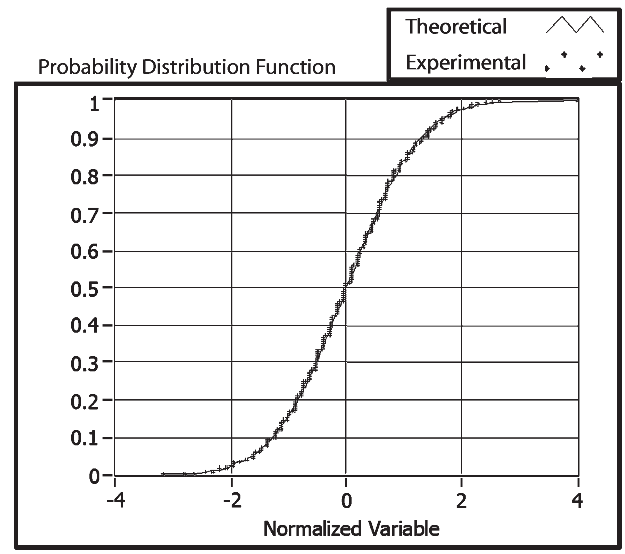

Fig. 6. Estimated probability distribution function of the terminal gain that is estimated using the histogram method. We considered $n_b = 8$, $A = 1.2$ V, FS = 1 V, $\sigma_n = 0.05$, $M = 1000$, $f = 200$ Hz, and $f_s = 200$ kHz. The number of repetitions (N) was 1000.

The vertical bars in the previous figures translate the 99.9% confidence interval to account for the Monte Carlo uncertainty due to the 1000 test repetitions (N) that are being carried out.

It can be seen, by observing the previous figures, that the experimental results, which are represented by vertical bars, are below the value given by the approximate expression (17). This validates the derivations presented here and justifies the use of those expressions to determine an upper bound for the estimated ADC gain and offset error.

V. Expanded Uncertainty

An accepted way to express the quality of measurements is through a confidence interval that is obtained by multiplying the standard uncertainty with the coverage factor [12]. Usually, the chosen value is two, which corresponds to a confidence level of approximately 95% if the probability distribution of the measurement result is normal. In order to verify that this is the case for the estimation of the ADC gain using the histogram method, we repeated the test 1000 times and constructed the probability distribution function that is shown in Fig. 6. The experimental values are very close to the theoretical ones for a normal distribution.

The tests were carried out with an additive-noise value of 5% of the stimulus signal amplitude and 1000 samples. Further studies with different test parameters will be carried out and will be a subject of another publication that is dedicated to verifying if the probability distribution can always be considered to be normal.

VI. Conclusion

In this paper, we analyzed the precision of the estimates of ADC gain and offset error that are obtained with the standard histogram method. The main results were shown in (17), which can be used to determine the expanded uncertainty and the corresponding uncertainty interval for the estimated gain and offset error. This has the same importance as in any measurement system, where the quality of the measurements should be expressed in terms of confidence intervals for the results.

The case considered here was for an ADC with a bipolar no-true-zero transfer function. The results for a bipolar ADC with a "true zero" transfer function would approximately be the same since the variance of the first estimated transition would be lower, but the variance of the last would be higher, leading to approximately the same sum in (8). The presented derivations can also be easily done for the unipolar ADCs.

Future work will show the independently based definition of ADC gain and offset error and the effect of other contributions to the uncertainty of the test results, namely, phase noise and frequency error.

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