[[1]](#footnote-1)

Determining the Electromagnetic Polarizability Tensors of Metal Objects during In-line Scanning

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*Abstract*— In metal detection systems, the response of a detector to a metal object may be approximated from the electromagnetic polarizability tensor of the object. Conversely, the tensors may be determined from multi-position measurements as the detector and object are moved relative to each other. The paper introduces and sets out the general approach to determining the tensor during in-line scanning. Two common application scenarios are considered, which share similar consideration in the calculation of the tensor components. The first is the case for detectors of landmines or explosive remnants of war, where the detector is scanned on a surface above the object. The condition of this inverse problem depends on the geometry of the coil(s) and the measurement protocol, which at present is not fully understood. Our results consider two cases namely a single line scan over the object as two extreme cases. The results suggest that tensor inversion is possible for the 2D raster scan, but not for a single line scan. The second application is a conveyor type metal detector, which is used with a typical detector for industrial process lines. Here, a new rotation measurement method is proposed and examined for the case of simple coaxial sensor coils and in-line scanning. Finally, different inverse methods are analyzed for the new rotation measurement method.

*Index Terms*— Metal detection; inverse problem; electromagnetic polarizability tensor; dipole moment; finite element model

# Introduction

T

he electromagnetic polarizability tensor is widely used to approximate the electromagnetic responses of metal detectors to metal objects. There are a number of metal detector configurations, which are widely used in the field for applications such as detection of buried objects, detection of metal contamination on industrial conveyors, and walk through metal detector for personnel screening. This paper concentrates on the first two as these have different types of sensor coils with different measurement protocols. This first one is the planar sensor coils for the landmine/UXO detectors [[1-5](#_ENREF_1)]. The objects are inspected at a distance to the sensor coil (Fig. 1.). The second one is the symmetric coaxial sensor coils for the in-line metal detectors [[6-10](#_ENREF_6)]. The objects are transported through the sensor coils for inspection (Fig.3.). Walk through metal detectors contain two panels of coils; their design is somewhat different to the former two and has already has been considered elsewhere [[11-13](#_ENREF_11)] in detail.

In landmine/UXO detection and geophysics, the electromagnetic polarizability tensors of metal objects are employed to discriminate of metal objects, e.g. landmines or UXO from other unwanted metal signals, e.g. metal clutters [[1](#_ENREF_1), [14-26](#_ENREF_14)]. These electromagnetic polarizability tensors are inverted from the measurements of the metal targets at various locations below a planar magnetic sensor [[1](#_ENREF_1), [3](#_ENREF_3), [14-17](#_ENREF_14)]. The multi-position measurements are theoretically independent only when the metal targets are exposed to incident magnetic fields from different directions at different measured positions, as illustrated in Fig. 1.. These independent measurements are determined by both the geometry of sensor coils and measurement protocol.



Experimental configurations for multi-position measurements of landmine/UXO detection and geophysics

The in-line type of metal detector is widely utilized in food and pharmaceutical manufacture lines to prevent hidden metal contaminants [[27](#_ENREF_27)]. They discriminate the metal fragments from unwanted product signals and provide compliance with food and pharmacy safety inspection standards, industry guidance and legislations [[28](#_ENREF_28)].



Balanced sensor coil array [[9](#_ENREF_9)]

In-line metal detectors employ a balanced sensor coil array to eliminate the background signals as shown in Fig. 2.. The metal target is transported through the aperture of the detector by a conveyor. The target is exposed to incident magnetic fields from mainly one direction, e.g. *x* direction in Fig. 3.. Therefore, the multi-position measurements for landmine/UXO detectors may not be directly applied to in-line metal detectors to measure the electromagnetic polarizability tensors [[29](#_ENREF_29)]. Therefore, a different approach is needed such as rotating the metal target and passing it several times through the detector with different angles, α, β, and γ as depicted in Fig.3., between the global and local coordinates.



Experimental configurations for rotation measurements of in-line metal detectors

In this paper, first, the theories of both the current multi-position measurement method and the new rotation measurement method will be fully elaborated. The independence of these measurements is evaluated by determining the ranks of their sensitivity matrices.

Second, both the current multi-position measurements for the landmine detectors and the new rotation measurements for the in-line metal detector will be analyzed by the simulations in a finite element method (FEM) solver.

In the multi-position measurement simulation, two types of planar sensor coils with different measurement protocols in the landmine detectors are simulated as undesired and desired cases for the electromagnetic tensor inversion. The undesired case is a simple co-axial sensor coils and in-line scanning (Fig. 4). The desired case is a complex sensor coils and 2D raster scanning (Fig. 5). It concluded that the independence of measurements can be evaluated by the calculated rank of inverse matrix. Furthermore, the multi-position measurement cannot be applied to in-line metal detectors for tensor inversion.

In the rotation measurement simulation, a metal target is placed and rotated in a balanced sensor coil array, which is similar to the in-line metal detector. It concluded that the rotation measurements can accurately determine the electromagnetic polarizability tensor from an undesired case for tensor inversion with multi-position measurement.

Third, the rotation measurements are implemented on a commercial in-line detector to further verify this method. The electromagnetic polarizability tensors of metal targets are directly inversed from the rotation measurements by Gaussian elimination. These inversion results are compared to the calculated tensors from our previous paper [[9](#_ENREF_9)] and demonstrated a high coherence.

Last, different direct and iterative inversion methods are employed for the electromagnetic polarizability tensor inversions, in addition to Gaussian elimination.

# Theory

The dipole approximation equation was derived to analyze the electromagnetic responses of small metal objects, e.g. landmine/UXO with the dimension of a few centimeters and metal contaminants with the dimension of a few millimeters [[9](#_ENREF_9), [14](#_ENREF_14), [25](#_ENREF_25)]. The secondary magnetic fields from a metal target are represented by a multi-pole expansion [[14](#_ENREF_14), [20](#_ENREF_20), [30](#_ENREF_30)]. The first term from this expansion is dominant for small objects and therefore the magnetic dipole moment is used to approximate the secondary magnetic fields. Then the induced voltages on the receivers from the secondary fields are derived by the reciprocity principle [[25](#_ENREF_25), [26](#_ENREF_26)].

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Here is the electromagnetic polarizability tensor, which is a 3 x 3 complex matrix and mainly determined by the characteristics of the metal target. and are the incident magnetic fields from the transmitter coil and receiver coils respectively. Their values at the observation points can be obtained from numerical methods, by defining the exciting current and the assumed receiver current as 1 A amplitude. In landmine/UXO detection, these observation points are located a few centimeters below the planar sensor coil [[31](#_ENREF_31)] (Fig. 1.). For in-line metal detectors, these observation points are placed at trajectories through the aperture of detector [[27](#_ENREF_27)] (Fig. 3.).

From [[32](#_ENREF_32)], the dot product of and is defined as the sensitivity of the metal detector, which is only determined by the coil geometry instead of the targets. So the metal targets are not included in the simulations of incident magnetic fields. As usual, is the imaginary unit, is the angular frequency of the coil currents and is the magnetic permeability of vacuum.

## Electromagnetic Tensor Inversion from Multi-position Measurements

Due to the reciprocity theorem, the electromagnetic tensor matrix in equation (3) is always symmetric. So , and . Then equation (2) can be simplified to the equation below [[12](#_ENREF_12), [30](#_ENREF_30), [33](#_ENREF_33)].

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Here is an unknown vector of electromagnetic tensor elements. is a known vector of incident magnetic fields obtained from FEM simulations or in-situ measurements. The superscript describes the matrix transpose.

Equation (4) can be solved by minimizing the least squares problem below.

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Here is the measured response signal of metal object and is the approximated metal object response from equation (4). The inversion of electromagnetic tensor vector from equation (5) can be implemented by iterative methods, e.g. regularized Gauss Newton [[13](#_ENREF_13), [30](#_ENREF_30)] or direct methods, e.g. singular value decomposition [[14](#_ENREF_14)]. Here, the direct Gauss elimination method is used to implement the electromagnetic tensor inversion.

If *N* measurements are made at *N* different positions, the optimized 6 elements in for equation (5) are expected to fulfil the equation below. Here is an *N x 6* matrix and is an *N x 1* matrix.

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Then by using Gauss elimination, the electromagnetic tensor vector in equation (6) can be directly solved from the equation below.

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Theoretically at least 6 independent measurements (*N* >=6) are required to compute the vector , which has 6 unknown elements.

## Electromagnetic Tensor Inversion from Rotation Measurements

From [[9](#_ENREF_9), [14-16](#_ENREF_14), [29](#_ENREF_29), [34](#_ENREF_34)], the electromagnetic polarizability tensor of a metal target can be represented by its eigenvalue matrix and a rotation matrix .

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|  |  |

Here is the eigenvalue matrix of , which is demonstrated as . is a 3 x 3 unitary matrix, which can be characterized by the Euler rotation matrix in *x-y-z* rotation sequence. In equation (9), and are the rotation angles around *x, y* and *z* axes in the local coordinate of the metal target.

By substituting equation (8) into (2), the electromagnetic responses of metal targets can be represented by the equation below.

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Here is the unknown vector of the eigenvalues of electromagnetic polarizability tensor matrix for a metal object. is the known vector of incident magnetic fields and rotation matrix. The elements of vector are shown below. Here (:,1), (:,2) and (:,3) means first, second and third columns of the matrix.

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In equation (10), only three unknown variables in the eigenvalue vector need to be inverted from the measured response signals at the rotation angles, and . Equation (10) can be solved by minimizing the least squares problem below.

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If *N* measurements are made at *N* different orientations, the optimized 3 elements in for equation (12) are expected to fulfil the equation below. Here is an *N x 3* matrix and is an *N x 1* matrix.

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By using Gaussian elimination, for instance, the electromagnetic tensor vector in equation (13) can be directly solved from the equation below.

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Theoretically at least 3 independent measurements (N>=3) are required to compute the vector , which has 3 unknown elements.

## Independence of Measurements Determination

The electromagnetic tensor vectors in equation (6) and in equation (13) can be inverted as follows.

|  |  |
| --- | --- |
|  | (15) |
|  | (16) |

The inverse matrices and are 6 x 6 and 3 x 3 square matrices separately. The independence of multi-position measurements and rotation measurements can be directly determined by calculating the ranks of these two matrices without the need to inverse the electromagnetic tensor of metal objects.

These ranks are equal to the number of singular values of these inverse matrices, which are larger than the minimum tolerances . These singular values are calculated from singular value decomposition (SVD). The minimum tolerance is given by the equation below [[35](#_ENREF_35)].

|  |  |
| --- | --- |
|  | (17) |

Here is the maximum size of the inverse matrix. is the maximum singular value of the inverse matrix and is the number of bits for the fractional part of a floating number. For the double precision floating-point numbers in this paper, equals 52 bits [[36](#_ENREF_36)].

# Simulation Results

Both the multi-position measurement method and rotation measurement method were analyzed by FEM simulations, using a commercial FEM solver, Ansys Maxwell *v16* to provide synthetic data to test the methods.

## Target Objects

In landmine and UXO detection, the target objects can mostly be grouped in few categories, i.e. rings, spheres, cylinders and spheroids [[19](#_ENREF_19)]. For in-line metal detectors, the spherical and cylindrical objects are of most concern to evaluate the sensitivity [[9](#_ENREF_9), [27](#_ENREF_27)]. As the electromagnetic polarizability tensors of metal sphere can be easily calculated [[9](#_ENREF_9), [20](#_ENREF_20), [37](#_ENREF_37)], the cylindrical objects are focused for the tensor determinations from landmine/UXO and in-line metal detector systems in this paper.

The trans-impedances between the transmitter and receiver coils are simulated with and without the metal target objects. The simulated target object responses are the differential values of these two trans-impedances.

As the electromagnetic responses of metal objects are strong at high frequency, the FEM models are simulated at 800 kHz for the converged and accurate results.

## Multi-position Measurement Method

The current multi-position measurement method is analyzed in the undesired and desired cases for the tensor inversion.

The first FEM model represents an undesired case for tensor inversion. It has simple co-axial sensor coils with magnetic field sensitivity mainly in *z* direction and a simple measurement protocol along a 1D line.

It has a 10 cm radius red outer transmitter coil and a 6 cm radius green inner receiver coil as shown in Fig.4.. The electromagnetic responses of a metal object are simulated along a radial straight line at a distance *D* below the sensor coils, as shown in blue connected stars line.

 

FEM model 1 for the multi-position measurement method

(left: FEM model; right: simulated locations of metal objects)

The second FEM model represents a desired case for tensor inversion. It has two-receiver sensor coils with magnetic field sensitivity in various directions and a complex measurement protocol in a 2D plane.

It consists of a 13 cm radius red outer transmitter coil and two green inner opposite wired receiver coils in 6 cm radius. The metal object was simulated at various locations at a distance *D* below the sensor coil, as shown in blue connected stars line.

 

FEM model 2 for the multi-position measurement method

(left: FEM model; right: simulated locations of metal objects)

A brass cylindrical wire with 1.25 mm diameter and 40 mm length is vertically placed at *D*=30 cm below these two sensor coils. Its electromagnetic response signals at various locations (shown in blue dots) were simulated at an arbitrary selected frequency, in this case 800 kHz and used to invert its electromagnetic tensor matrix by using equations (4)-(7).

The phases and magnitudes of the eigenvalues of the inverted electromagnetic tensors are compared to calculated results from previous paper [[9](#_ENREF_9)] and shown in the tables below.

Phases of eigenvalues of simulated and calculated electromagnetic tensor matrices of brass wire 1.25 x 40 mm (Diameter x Length) at 800 KHz for the multi-position measurement method

| Sources | Phases of Eigenvalues of Electromagnetic Tensor Matrix : (Degree) | | |
| --- | --- | --- | --- |
|  |  |  |
| Calculation [[9](#_ENREF_9)] | 165.38 | 165.40 | 165.40 |
| FEM model 1 | 126.45 | 125.77 | 125.88 |
| FEM model 2 | 168.20 | 168.88 | 168.75 |

Magnitudes of eigenvalues of simulated and calculated electromagnetic tensor matrices of brass wire 1.25 x 40 mm (Diameter x Length) at 800 KHz for the multi-position measurement method

| Sources | Magnitudes of Eigenvalues of Electromagnetic Tensor Matrix : | | |
| --- | --- | --- | --- |
|  |  |  |
| Calculation [[9](#_ENREF_9)] | 4.15E-04 | 8.94E-04 | 8.95E-04 |
| FEM model 1 | 2.38E+01 | 6.45E-04 | 5.54E-04 |
| FEM model 2 | 4.65E-04 | 8.58E-04 | 8.40E-04 |

In Table I and II, only the FEM model 2 provides correct phases and magnitudes, which are close to the previous calculated results. In Table II, the magnitudes of simulated eigenvalues are scaled to measured eigenvalues by a constant value, i.e. 3.80E+01 for FEM model 1 and 2.04E+03 for FEM model 2. These scaling factors are caused by the differences in magnetic field strengths and gains in the electronic system.

Ranks of the inverse matrices of FEM model 1 and 2

| Sources | Tolerance | Singular Values of | Rank |
| --- | --- | --- | --- |
| FEM model 1 | 6.51E-19 | 8.06E-04 , 8.54E-05, 4.08E-06 , 7.13E-12, 5.60E-14, 3.89E-21 | 5 |
| FEM model 2 | 2.60E-18 | 3.11E-03, 1.07E-03, 4.51E-04, 1.44E-04 , 1.91E-05, 1.04E-05 | 6 |

In Table III, the inverse matrix in FEM model 1 is not full rank, i.e. less than 6. It indicates that there are insufficient independent measurements. The reason is that the brass wire in FEM model 1 is not exposed to any tangential field components. This indicates the difficulties of reconstructing a tensor matrix from a single line passing over the sensor. In general a 2D scan of the area is required.

In addition, the in-line metal detector has some characteristics similar to the FEM model 1, i.e. simple co-axial sensor coils with magnetic field sensitivity mainly in one direction and a simple measurement protocol along a 1D line. So the electromagnetic tensors cannot be accurately calculated for in-line metal detectors from the multi-position measurement along a single line passing through the sensor.

## Rotation Measurement Method

The proposed rotation measurement method was analyzed on a balanced sensor coil array characteristic of the type used for in-line metal detector as shown in Fig. 6.. This is a similar geometry to an actual test set-up used by other works in the field [[38](#_ENREF_38)].

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FEM model 3 for the rotation measurement method

(left: FEM model; right: simulated orientations of metal objects)

This sensor coil array has a 35 mm outer radius transmitter coil with the cross section *t* = 2 mm by *H* = 100 mm and two 40 mm outer radius receiver coils with the cross section *t* = 2 mm by *h*= 40 mm. The magnetic fields inside this coil array are mainly in *z* direction.

A brass cylindrical wire with 1.25 mm diameter and 40 mm length is placed at the center of the upper receiver coil. Its electromagnetic response signals at various orientations, i.e. 0º, 15º, 45º, 60º, 90º, 105º, 135º and 150º around *x* axis are simulated at an arbitrary selected frequency, in this case 800 kHz and used to invert its electromagnetic tensor matrix by using equations (10) – (14).

Phases of eigenvalues of simulated and calculated electromagnetic tensor matrices of brass wire 1.25 x 40 mm (Diameter x Length) at 800 KHz for the rotation measurement method

| Sources | Phases of Eigenvalues of Electromagnetic Tensor Matrix : (Degree) | | |
| --- | --- | --- | --- |
|  |  |  |
| Calculation [[9](#_ENREF_9)] | 165.38 | 165.40 | 165.40 |
| FEM model 3 | 165.76 | 165.97 | 165.97 |

Magnitudes of eigenvalues of simulated and calculated electromagnetic tensor matrices of brass wire 1.25 x 40 mm (Diameter x Length) at 800 KHz for the rotation measurement method

| Sources | Magnitudes of Eigenvalues of Electromagnetic Tensor Matrix : | | |
| --- | --- | --- | --- |
|  |  |  |
| Calculation [[9](#_ENREF_9)] | 4.15E-04 | 8.94E-04 | 8.95E-04 |
| FEM model 3 | 4.07E-04 | 9.03E-04 | 9.03E-04 |

Ranks of the inverse matrices of FEM model 3

| Sources | Tolerance | Singular Values of | Rank |
| --- | --- | --- | --- |
| FEM model 3 | 2.79E-09 | 5.11E+06, 1.90E+06, 1.17E+06 | 3 |

In Table IV and Table V, the phases and magnitudes of inversion electromagnetic tensors from the FEM simulations of rotation measurement method are very close to the previous calculated results. The scaling factor for the simulated magnitudes results in Table V is 1.22E+03. In Table VI, the inverse matrix in FEM model 3 is full rank, which indicates the independent measurements.

As can be seen, the proposed rotation measurement can accurately determine the electromagnetic polarizability tensor from a magnetic sensor with magnetic fields mainly in one direction and a simple measurement protocol.

# Experiments Results

The rotation measurements were implemented on a magnetic sensor with magnetic fields mainly in one direction, i.e. an in-line metal detector. The utilized commercial in-line metal detector system is shown in Fig. 7. The size of the aperture is 350 x 175 mm (*Wap x Hap*). The distance between two receiver coils is 100 mm (CP). The size of the case is 620 x 420 x 275 mm (*Wca x Hca x Lca*). The case is constructed from stainless steel with 3 mm thickness.



3D computer model of an in-line metal detector in the experiment. (The transmitter coil is in red. The two receiver coils are in blue.)[[9](#_ENREF_9)]

## Experiment Configurations and Data Processing Method

The response signals from the receiver coils are demodulated on a receiver circuit board in the in-line metal detector. The demodulated signals and the trigger signal from a position sensor are acquired by a 24-bit data acquisition card (National Instruments, NI-9239). The sampling rate is configured to 4000 samples per second. The signal is recorded over 3 seconds, during which time the metal test pieces are scanned through the detector on plastic conveyor belt. Optical beam breaks are used to trigger the start of the acquisition. The acquisition time is long enough for the target to pass through the detector at a nominal speed of a conveyor of around 20 m/min. At this conveyor speed, the demodulated response signals of this in-line metal detector to metal objects are around 2 Hz. This frequency is mainly determined by the conveyor speed, distance between two receiver coils, the size of the case and the size of metal object. A block diagram of the experimental arrangement is given in Fig. 8..



Data acquisition setup. (I: In-phase; Q: Quadra-phase) [[9](#_ENREF_9)]

In Fig. 9., the acquired data are first subtracted by a fitted linear equation to eliminate the high frequency noise, DC offsets and drifts. Then a 3rd order Butterworth low pass filter with cut-off frequency at 50 Hz is placed to remove the high frequency noises. Additionally, a calibration method is introduced to align the phase response and correct for gain errors at different operating frequencies. The calibration is performed using a small MnZn ferrite test object.



Data processing method [[9](#_ENREF_9)]

The signal-to-noise ratios (SNRs) of the captured response signals of a 2.5 mm diameter cobalt sphere are represented before and after the data processing method. In general, the SNR is higher at high frequency, as the response signals of metal objects are linear to frequency in equation (2).

Signal-to-noise ratio (SNR) of the captured response signals of a 2.5 mm diameter cobalt sphere before and after the data processing method

| Frequency | SNR: (dB) | |
| --- | --- | --- |
| Before the data processing method in Fig. 9. | After the data processing method in Fig. 9. |
| 100 kHz | 27.0 | 28.9 |
| 300 kHz | 37.8 | 38.9 |
| 800 kHz | 37.5 | 41.0 |

## Rotation Measurements and Electromagnetic Tensor Inversion

The metal wires were passed through the in-line metal detector at least 3 times with different orientations from 0º to 150º to the incident magnetic fields.



Orientations of metal wires [[9](#_ENREF_9)]

The electromagnetic tensors of the metal objects are inverted from the rotation measurements by using equation (10) - (14). The eigenvalues of the inverted electromagnetic tensors are compared to the calculated results from previous paper [[9](#_ENREF_9)]. The tested metal samples includes brass wire 0.8 x 40 mm (diameter x length), brass wire 1.25 x 40 mm, stainless steel wire 0.8 x 40 mm and iron wire 0.9 x 40 mm.

For metal cylindrical wires, the eigenvalue element of tensor matrix equals the element , which characterize the transverse responses. The eigenvalue element of characterizes the longitude responses [[14](#_ENREF_14), [39](#_ENREF_39)]. The ferrous wire, i.e. iron has a stronger response to longitudinal magnetic fields, i.e. . The non-ferrous wire, i.e. brass and stainless steel has a stronger response to transverse magnetic fields, i.e. [[27](#_ENREF_27)].

The inverted eigenvalues of tensor matrices from the rotation measurements by using Gauss elimination (Ss: stainless steel, wire diameter x length in (mm))

| Sample | 100 kHz | | 300 kHz | | 800 kHz | |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |
| Brass  0.8x40mm | 5.60E-4-1.73E-3\*j | 1.25E-3-3.86E-3\*j | 6.97E-3-8.29E-3\*j | 1.54E-2-1.80E-2\*j | 4.31E-2-1.95E-2\*j | 9.43E-2-4.26E-2\*j |
| Brass  1.25x40mm | 5.54E-3-7.02E-3\*j | 1.20E-2-1.51E-2\*j | 3.21E-2-1.71E-2\*j | 6.92E-2-3.67E-2\*j | 1.26E-1-3.30E-2\*j | 2.72E-1-7.10E-2\*j |
| Ss  0.8x40mm | -3.12E-5-1.55E-4\*j | -2.32E-5-3.14E-4\*j | -6.62E-5-1.24E-3\*j | 9.37E-5-2.75E-3\*j | 1.98E-3-1.07E-2\*j | 4.44E-3-2.37E-2\*j |
| Iron  0.9x40mm | -1.10E-1-1.19E-1\*j | -2.02E-2-4.88E-3\*j | -1.72E-1-2.02E-1\*j | -4.84E-2-1.86E-2\*j | -2.69E-1-3.41E-1\*j | -1.00E-1-7.03E-2\*j |

The phases of and above are compared to the calculated results from previous paper [[9](#_ENREF_9)]. The ratios of magnitudes of only and above are compared to the calculated results from previous paper [[9](#_ENREF_9)].

Phases differences between the inverted eigenvalues of tensor matrices by using Gauss elimination and the calculated eigenvalues of tensor matrices from previous paper (Ss: stainless steel, wire diameter x length in (mm))

| Sample | Phase Difference | | | | | |
| --- | --- | --- | --- | --- | --- | --- |
| 100 kHz | | 300 kHz | | 800 kHz | |
|  |  |  |  |  |  |
| Brass  0.8x40mm | 0.30% | -0.40% | 0.03% | -0.21% | 0.09% | -0.15% |
| Brass  1.25x40mm | -0.01% | 0.05% | -0.02% | 0.02% | -0.04% | -0.01% |
| Ss  0.8x40mm | -15.89% | -0.54% | -1.26% | -0.16% | 0.71% | 0.17% |
| Iron  0.9x40mm | -0.11% | 18.76% | -0.06% | 9.79% | -0.06% | 1.07% |

Magnitude differences ( between the inverted eigenvalues of tensor matrices by using Gauss elimination and the calculated eigenvalues of tensor matrices from previous paper (Ss: stainless steel, wire diameter x length in (mm))

| Sample | Magnitude Difference ( | | |
| --- | --- | --- | --- |
| 100 kHz | 300 kHz | 800 kHz |
| Brass 0.8x40 mm | 0.45% | 0.93% | 0.55% |
| Brass 1.25x40 mm | 0.29% | 1.25% | -0.19% |
| Ss 0.8x40 mm | 17.59% | 1.13% | 1.03% |
| Iron 0.9x40 mm | -6.74% | -5.18% | -2.04% |

From Table IX and X, the phases and magnitudes differences are generally less than 10%. The differences are significant only for small and low conductivity metal wires, e.g. ss 0.8x40 mm and iron 0.9x40 mm at low frequency, e.g. 100 kHz.

The inverse matrices, of the results above are all full rank. So these rotation measurements are independent. From equation (16) and (17), these differences are caused by the factors within the measured response signals .

First, the response signals of these low conductivity samples at low frequencies are weak and noisy, comparing with the other samples at higher frequencies. Second, the iron wire may be partially magnetized in the experiments.

# Inversion Methods

In equation (14), the eigenvalues of the electromagnetic polarizability tensors are directly inverted from rotation measurements by Gaussian elimination. In this section, two additional inversion methods, i.e. singular value decomposition (SVD) and regularized Gauss Newton are applied to the inversion of electromagnetic polarizability tensors from rotation measurement.

## Singular Value Decomposition (SVD) Method

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Here is a matrix containing the values of vector from the total number of *N* wireorientation measurements. and are two unitary matrices and is a diagonal matrix with singular values of the matrix .

By using equation (18), equation (16) can be simplified to the equation below for the inversion of eigenvalues of electromagnetic polarizability tensors.

|  |  |
| --- | --- |
|  | (19) |

Phases differences between the inverted eigenvalues of tensor matrices by using the singular value decomposition method and the calculated eigenvalues of tensor matrices from previous paper (Ss: stainless steel, wire diameter x length in (mm))

| Sample | Phase Difference | | | | | |
| --- | --- | --- | --- | --- | --- | --- |
| 100 kHz | | 300 kHz | | 800 kHz | |
|  |  |  |  |  |  |
| Brass  0.8x40mm | -0.44% | 0.63% | -0.09% | 0.58% | -0.14% | 0.56% |
| Brass  1.25x40mm | 0.02% | -0.11% | 0.12% | -0.09% | 0.40% | 0.14% |
| Ss  0.8x40mm | 17.15% | -6.24% | 1.21% | 0.10% | -0.88% | -0.11% |
| Iron  0.9x40mm | 0.04% | -2.27% | 0.02% | -1.46% | 0.02% | -0.44% |

Magnitude differences ( between the inverted eigenvalues of tensor matrices by using the singular value decomposition method and the calculated eigenvalues of tensor matrices from previous paper (Ss: stainless steel, wire diameter x length in (mm))

| Sample | Magnitude Difference *(* | | |
| --- | --- | --- | --- |
| 100 kHz | 300 kHz | 800 kHz |
| Brass 0.8x40 mm | 0.45% | 0.93% | 0.34% |
| Brass 1.25x40 mm | 0.28% | 1.25% | -0.19% |
| Ss 0.8x40 mm | 17.58% | 1.12% | 1.03% |
| Iron 0.9x40 mm | -6.77% | -5.20% | -2.05% |

The inversed tensor results from the singular value decomposition method are also compared to the calculated results in previous paper [[9](#_ENREF_9)]. From Table XI and XII, the differences in phases and magnitudes from the singular value decomposition method are similar to the direct Gauss elimination method in Table IX and X.

## Regularized Gauss Newton Method

The regularized Gauss Newton method is utilized to find the minimum of equation (12) after a number of iterations. The equation to be iterated is shown below.

|  |  |
| --- | --- |
|  | (20) |

Here *k* is the number of iteration. is the constant value for regulation. is the diagonal regulation matrix. From equation (10), the Jacobian matrix is given below.

|  |  |
| --- | --- |
|  | (21) |

By taking equation (21) into equation (20), the equation for regularized Gauss Newton iteration is shown below.

|  |  |
| --- | --- |
|  | (22) |

In the regularized Gauss Newton iterations, is set as a fixed constant at 1E-04. is set as a diagonal matrix with three diagonal elements equal to . The initial value for is set as .

In general, the differences between the values of from the last two iterations are lower than (1E-13)% after at least 7 iterations.

Phases differences between the inverted eigenvalues of tensor matrices by using the regularized Gauss Newton method and the calculated eigenvalues of tensor matrices from previous paper (Ss: stainless steel, wire diameter x length in (mm))

| Sample | Phase Difference | | | | | |
| --- | --- | --- | --- | --- | --- | --- |
| 100 kHz | | 300 kHz | | 800 kHz | |
|  |  |  |  |  |  |
| Brass  0.8x40mm | -0.39% | 0.69% | -0.06% | 0.61% | -0.12% | 0.58% |
| Brass  1.25x40mm | 0.10% | -0.03% | 0.17% | -0.04% | 0.43% | 0.17% |
| Ss  0.8x40mm | 17.21% | -6.20% | 1.23% | 0.11% | -0.87% | -0.10% |
| Iron  0.9x40mm | 0.07% | -2.20% | 0.03% | -1.43% | 0.03% | -0.42% |

Magnitude differences ( between the inverted eigenvalues of tensor matrices by using the regularized Gauss Newton method and the calculated eigenvalues of tensor matrices from previous paper (Ss: stainless steel, wire diameter x length in (mm))

| Sample | Magnitude Difference ( | | |
| --- | --- | --- | --- |
| 100 kHz | 300 kHz | 800 kHz |
| Brass 0.8x40 mm | 0.39% | 0.87% | 0.28% |
| Brass 1.25x40 mm | 0.23% | 1.19% | -0.25% |
| Ss 0.8x40 mm | 17.53% | 1.06% | 0.97% |
| Iron 0.9x40 mm | -6.57% | -5.06% | -1.95% |

The inversed tensor results from the regularized Gauss Newton method are also compared to the calculated results in previous paper [[9](#_ENREF_9)]. From Table XIII and XIV, the differences in phases and magnitudes from the regularized Gauss Newton method are similar to the direct Gauss elimination method in Table IX and X.

In conclusion, the inversed tensor results from the singular value decomposition method and regularized Gauss Newton method are similar to the results from direct Gauss elimination method.

Therefore, the electromagnetic tensor matrix and the incident magnetic fields and in equation (2) are very independent in the experiments on an in-line metal detector. So the direct Gauss elimination is sufficient for the tensor inversion from a linear equation.

# Discussion

The proposed rotation measurements determine the electromagnetic tensors of metal objects with known positions and orientations. In practice, the magnetic field vector in equation (4) can be represented by Biot-Savart law, where an unknown vector of target position is included and inverted along with the electromagnetic tensors. However, this approach assumes that there is no metal case around the sensor coil, which generates secondary magnetic fields from eddy currents. So for the metal detectors with metal cases, e.g. in-line metal detector, this approach is not applicable. The incident magnetic fields can only be measured or simulated at known positions.

On the practical target orientation estimation, the axis-symmetrical metal objects, i.e. cylindrical metal wires, can be rotated over two orthogonal planes to determine their orientations, i.e. eigenvectors. But for non-symmetrical metal objects, a more complicated rotational scanning protocol is needed, i.e. rotations over three orthogonal planes.

# Conclusion

From this paper, the accuracy of the inversed electromagnetic polarizability tensor is mainly determined by the sensor coil, the measurement protocol and the measured response signals of metal objects. The first two factors can be analyzed by the calculated ranks of inverse matrices. The last factor depends on the SNR of response signals and the characteristics of metal samples, e.g. magnetization.

The rotation measurement is proposed to determine the electromagnetic polarizability tensors from an undesired case for current multi-position measurement method, i.e. simple co-axial coils and in-line scanning. The experiments are implemented on a commercial in-line metal detector to prove the feasibility of the rotation measurement.

Last, two addition inverse methods, i.e. the singular value decomposition method and regularized Gauss Newton method, are analysed in determining the electromagnetic tensors. In conclusion, for the overdetermined system of electromagnetic tensor inverse from rotation measurements on in-line metal detectors, the influence of the inversed method is less significant.

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