**Combination of Images Reconstructed by Two Algorithms   
based on Graph Cut**

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**Abstract:** Image reconstruction plays a key role in the application of electrical capacitance tomography (ECT). Although many different algorithms have been developed in the past, it is often difficult to obtain satisfactory images in all imaging regions by a single algorithm due to the the soft-field nature of ECT. To address this issue, this paper presents a method to combine images reconstructed by two algorithms. More specifically, a concept of graph cut is proposed. To verify the method, images reconstructed by two single-step algorithms, i.e. linear back projection (LBP) and Tikhonov regularization, are combined. Preliminary tests show that the proposed method can retain the advantages of both algorithms. In numerical simulation, both data without noise and with noise are examined. Good experimental results are also obtained.

**Keywords:** Electrical capacitance tomography, image reconstruction algorithm, linear back projection, Tikhonov regularization, graph cut

1. **Introduction**

Since 1980s, process tomography has attracted considerable interest due to its ability to visualize and interpret complex two-phase flows moving within processes [1, 2]. There are many different process tomography modalities due to different sensing techniques, such as x-ray tomography [3], Gamma-ray tomography [4], ultrasound tomography [5], electrical capacitance tomography (ECT), and microwave tomography [6]. Among them, ECT is the most mature and has advantages of high temporal resolution, robustness, withstanding high temperature and high pressure, non-intrusive and non-invasive, and no radiation, which make it an ideal tool for measurement of highly dynamic two-phase flows encountered in many industries [7, 8]. So far, ECT has been successfully applied to the measurement of gas-solids fluidized beds [9-11], oil pipelines [12], combustion flame [13] and other industrial processes.

The principle of ECT is to reconstruct the permittivity distribution as a representation of the material distribution in the region of interest from the measured inter-electrode capacitance via a specific image reconstruction algorithm. With the reconstructed images, some key hydrodynamic parameters, such as the bubble size in a gas-solids fluidized bed and oil fraction in an oil pipeline, can be obtained. Therefore, image reconstruction algorithm plays a key role in the application of ECT [14]. However, there are two main difficulties, which are associated with image reconstruction. First, the inverse problem is severely under-determined due to the number of capacitance measurements is far less than the number of pixels that need to be derived from the capacitance measurements. Second, the ill-posed and ill-conditioned property of the sensitivity matrix make the reconstructed images be sensitive to measurement noise.

To address the above problems, many image reconstruction algorithms have been developed in the last two decades [9, 15-23]. For example, Xie et al. [22] proposed a simple linear back projection (LBP) method. Peng et al. [21] introduced Tikhonov regularization (TR), as a universal tool for ill-posed inverse problems. Yang et al. developed Landweber iteration [20]. Recently, Lei et al. used deep learning to improve reconstruction accuracy [16].

However, due to the soft-field nature of ECT, it is difficult to obtain satisfactory images in all imaging regions by a single image reconstruction algorithm. Some algorithms are good at reconstructing permittivity distributions in a specific region while some other algorithms perform well in other regions. Considering this fact, an intuitionistic idea is to combine the images reconstructed by two different algorithms. However, in literature, researchers attempted to develop new image reconstruction algorithms to improve the image quality of ECT. To our best knowledge, to date, little work has been attempted to combine two existing algorithms, which may take advantages of the two algorithms. We propose a new method to combine two images reconstructed by different algorithms to improve image quality in different regions.

Simply stacking images or weighted superposition has a particular difficulty in selecting appropriate weighting factor for each image pixel. Directly synthesizing two images to a new one can overcome this difficulty by making use of the concept of computer graphics. To find similarity in a pixel and implement image segmentation and synthesis, a widely used computer graphics technique known as graph cut has been used recently. A variety of methods based on graph cut have been developed to construct digital images in the fields of remote sensing [24, 25], medical science [26], geophysics and geostatistical modeling [27, 28]. Similarly, ECT images reconstructed by different algorithms may be combined by making use of graph cut.

This paper presents a method based on graph cut to combine images reconstructed by two single-step algorithms, LBP and TR, which are widely used for on-line image reconstruction due to their simplicity and fast speed. However, the limitation of these two algorithms is that neither of them can provide satisfactory images. Images reconstructed by LBP show no artifacts in the near-wall region but are blurred in the central region [9, 14, 29]. TR can obtain good results in the central region, but there are always artifacts in the near-wall region [9, 14, 23]. Therefore, these two algorithms are complimentary and can be combined together to improve the image quality.

1. **Fundamentals of ECT and image reconstruction**

**2.1 ECT sensor model**

Peng et al. [29] investigated the effect of the number of measurement electrodes on image quality and they recommended 12-electrode sensors for most applications. Therefore, a circular 12-electrode sensor with the electrode covering ratio of 0.9 is modeled. It has been confirmed by Ye et al. [30] that such an electrode covering ratio can achieve good image quality. Figure 1 shows the detailed dimensions of the modeled sensor with ?? distribution displayed. Usually, one of the electrodes is selected in turn as the excitation electrode and others as detection electrodes to obtain the inter-electrode capacitance between all possible electrode pairs. With this measurement strategy, the number of independent capacitance measurements is 66.

There are two major computational problems in ECT, i.e., the forward problem and the inverse problem [9, 14].

The forward problem is to determine the inter-electrode capacitance from a predefined sensor and permittivity distribution. The relationship between them is governed by

(1)

where *ε0* is the permittivity of vacuum, *V* is the potential difference between two electrodes forming the capacitance, *εr*(*x,y*) and *φ*(*x,y*)are the relative permittivity and potential distributions in the sensing domain, respectively, and Γ is the electrode surface.

To simplify calculation, a linear equation in a normalized form is usually used to be approximation to equation (1).

(2)

where *g* is the normalized permittivity and *λ* is the normalized capacitance defined as

(3)

where *CM* indicates the inter-electrode capacitance for an arbitrary permittivity distribution and *CH* and *CL* are the capacitances when the sensor is full of high- and low-permittivity materials, respectively.

In real measurement, the capacitance data contain noise. Therefore, equation (2) changes to

(4)

where *e* is the measurement noise. White Gaussian noise is usually assumed for simulation.

The parameter *S* in equations (2) and (4) is the normalized sensitivity matrix, which represents the change in the normalized capacitance of each electrode pair in response to a perturbation in the normalized permittivity distribution. In this work, a grid of 64×64 square elements is used, which results in 3228 effective pixels in the circular imaging area.

The sensitivity matrix is usually calculated by numerical simulation of potential distribution in a vacuum permittivity distribution based on the quasi-static field assumption and then by dot multiplying two potential distributions.

(5)

where is the sensitivity between the *i*th and *j*th electrodes at the pixel *p*(*x,y*) and *φi*(*x,y*) and *φj*(*x,y*) are the potential distributions when the *i*th and *j*th electrodes are excited by applying voltages of *Vi* and *Vj*, respectively.

Then, *S\** is normalized as

(6)

where *Smn* and *S\* mn* are the elements in the *m*th row and *n*th column of *S* and *S\**, respectively, and *N* is the number of pixels in the imaging area.

**2.2 Image reconstruction algorithms**

The inverse problem of ECT is to reconstruct the permittivity distribution from the measured inter-electrode capacitance via a specific image reconstruction algorithm. In this section, two commonly used single-step algorithms, i.e., LBP and TR, are introduced.

* + 1. **LBP**

LBP was the first developed algorithm for ECT [31]. Its principle is to replace the inverse of *S*, which does not exist, with the transpose of *S*, as formulated by

(7)

where the reconstructed normalized permittivity and *uλ* is a vector of ones with the same dimension as *λ*.

* + 1. **TR**

TR is a well-established technique to solve ill-posed problems and has been extensively used in ECT image reconstruction [9, 18, 23]. Its formula is

(8)

where *μ* is a regularization parameter and *I* isan *N*×*N* identity matrix. In general, a small value of *μ* can give a reliable approximation to the solution, but a too small value of *μ* may lead to a singularity. As suggested by Guo et al. [9], *μ* takes the value of 0.0001 in this work.

1. **Graph cut-based combination strategy**

In animation movies and video games, a large number of new images showing similar features as the sampled or training images are needed to describe continuing movements or background landscapes. Image synthesis technique is usually used to generate new images by assembling irregular pieces of the sampled images and adjust them to create seamless transitions [32]. From this point view, the computer graphics technology has the potential to combine images reconstructed by two algorithms. The synthesized image can keep good pieces of the two images with the local artifacts in one image replaced by another image from the other algorithm.

Graph theory deconstructs a graph or digital image to nodes and edges that connect each node and its nearest nodes. A graph can be partitioned into two disjoint subsets by a cut, and the subsets are disjoint when they do not share any elements. As long as two images overlap, graph cut techniques can analyze the similarity of the overlap and identify the optimal cut to seam the two images together along the most similar passage way. Consequently, the graph cut problem is also known as a min-cut problem. Figure 2 shows the theorem and process of the graph cut method.

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**Figure 2.** Graph cut method.

The graph cut combination (GCC) method considers the overlap region *δ* between images obtained by Algorithms A and B as a graph containing two terminals (denoted by *T1* and *T2*, respectively), a set of nonterminal nodes (denoted by hollow circle), and edges connecting neighbor nodes (denoted by solid line). The terminal *T1* denotes the nodes connecting to image reconstructed by Algorithm A and the terminal *T2* denotes the nodes connecting to image reconstructed by Algorithm B. The values at nonterminal nodes (*NT*) are the absolute difference of the two images. For example, the value of node *n1* is denoted as *δ*(*n1*):

(13)

where and are the values at node *n1* from images reconstructed by Algorithms A and B, respectively. The capacity of the edges connecting *n1* and *n2* denoted as *e*(*n1, n2*) is calculated by

(14)

Once an edge is cut, the cut cost is assigned as the capacity of the edge. Graph cut techniques, also known as the min-cut theorem, find the cut that has the minimum total cost among all possible cuts throughout the graph to separate the non-terminal nodes into two sets: one set attached to *T1* and the other attached to *T2*. According to the attaching label, the new image is constructed by valuing the pieces from images reconstructed by Algorithms A and B together. The graph cut-based method combines images from different sources to a new image in a way as seamless as possible. Many algorithms have been developed for graph cut problems and the fast augmenting path algorithm proposed by Boykov and Kolmogorov [33] is used in this work.

1. **Implementation on images reconstructed by the LBP and TR algorithms**

In this work, the proposed graph cut-based combination strategy is implemented on images reconstructed by the LBP and TR algorithms to take advantages of both algorithms. Figure 3 shows the process of a graph cut implementation. The true distribution is shown in Figure 3a. Figures 3b and 3c show the corresponding images reconstructed by the LBP and TR algorithms, respectively. It is obvious that the image reconstructed by LBP shows a good result in the near-wall region with poor accuracy in the central region, while TR can perfectly present the central region with artifacts around the near-wall region. As a result, the overlap of these two images, as shown in Figure 3d, has several pieces with a relatively large difference. The large difference part can be identified with a user defined threshold. Figure 3e shows the difference pieces with a threshold of 0.85 (i.e. the value above 85% nodes). These pieces are divided into three kinds according to the location: (1) the pieces around the boundary are defined as terminal *T1* attaching with the image reconstructed by the LBP algorithm, (2) the pieces in the center are defined as terminal *T2* attaching with the image reconstructed by the TR algorithm, and (3) the pieces throughout the imaging area as long as other nodes are the *NT* nodes. The graph cut-based method labels these *NT* nodes as shown in Figure 3f and seam the corresponding images together to a new image as shown in Figure 3g. As can be seen, the combined image improves the results by taking advantages of the two algorithms.

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**Figure 3.** Implementation of graph cut method on images reconstructed by LBP and TR.

Note that the user defined threshold and the definition of terminals are the key for final results. The principle in determining the threshold is the identification of the most obvious difference of the overlap and a value of 0.85 is used in this work for the combination of images reconstructed by the LBP and TR algorithms.

1. **Evaluation results and discussion**

Both numerical simulation with and without noise and experiments were performed to evaluate the effectiveness of the graph cut-based combination method with the LBP and TR algorithms.

The ECT sensor used in numerical simulation has been shown in Figure 1. The simulation procedure is as follows. First, a specific permittivity distribution is defined in the imaging area. The low and high permittivity values of the materials used are 1 and 3, respectively. Then, the forward problem is solved to obtain the inter-electrode capacitance. Finally, the obtained capacitance is converted to the reconstructed permittivity distribution using a specific image reconstruction algorithm. The correlation coefficient (CC) is calculated, which reflects the spatial similarity between the true and reconstructed distributions, to evaluate the performance of different algorithms quantitatively. The definition of CC is

(15)

whereand are the mean values of and , respectively. A larger CC indicates a better image quality [9, 14].

Figures 4a and 4b show respectively the axial and cross-sectional views of the ECT sensor used in experiments. 12 measurement electrodes made of self-adhesive copper sheet are stuck onto the outside wall of a quartz glass tube with the inner and outer diameter of 8 and 8.9 cm, respectively. The vertical height of the electrodes is 4 cm and the width is specified to keep the electrode covering ratio the same as that in the simulation. Two axial end screens located at both axial ends of the measurement electrodes and an outer screen wrapped around the tube are connected to ground to eliminate external interference. An AC-based ECT system [34] is used for capacitance measurement. Besides some simple distributions with stationary objects, the ECT sensor can also be installed on a fluidized bed, as shown in Figure 4c, to measure complex distributions in a highly dynamic system. The gas and particles used in experiments are gas and Al2O3 powder, which have the permittivity of 1 and 4, respectively. Considering the particle packing concentration is about 0.63, the high-permittivity used for calibration is 2.89 ().



**Figure 4.** ECT sensor used in experiments: (a) axial view, (b) cross-sectional view, and (c) installed on a fluidized bed.

* 1. **Evaluation by numerical simulation**

**Case 1**

In Case 1, six simple permittivity distributions of 0 and 1, as shown in Figure 5, were used as the true normalized permittivity distributions to evaluate the images reconstructed by different methods, in which cases 1a-1c represent bubble flows and cases 1d-1f represent stratified flows.

ands.

Quantitative comparison associated with CC for different methods is shown in Figure 6. As can be seen, for the three bubble flows, the images reconstructed by LBP are blurred in the central region and CC is also lowest in all algorithms, which make it difficult to identify the number of bubbles in the imaging area. By TR, although there are artifacts in the near-wall region, the bubbles in the central region can all be well reconstructed. For the three stratified flows, LBP has the highest CC, which is in agreement with the conclusion by Peng et al. [29] that LBP can generate good images for stratified distribution. Even so, it is noted that the boundary between the high- and low-permittivity materials by LBP is indistinct. In contrast, by TR, a clear boundary can be obtained. But still, the artifacts in the near-wall region worsen the overall image quality, as indicated by the lowest CC for these three distributions in Figure 6.

The GCC method proposed in this work can combine the advantages of both LBP and TR. Therefore, images reconstructed by the GCC method are all satisfied. As shown in Figure 5, the artifacts in the near-wall region are all removed and good images in the central region reconstructed by TR are obtained.

Figure 6 also shows the average CC for all tested distributions with different methods. Clearly, the average CC obtained by the GCC method is higher than both LBP and TR.

ands

**Case 2**

To evaluate the performance of different image reconstruction algorithms, it is a common practice to perform numerical simulations and/or experiments with stationary objects [35]. In this way, only some simple distributions with the 0-1 model like those shown in Figure 5 can be tested. However, real distribution in a two-phase flow system is much more complex due to the so-called chaotic behavior [36]. Therefore, it is necessary to introduce the two-phase flow characteristics to the evaluation of an image reconstruction algorithm. Recently, Guo et al. [9] reported such a framework based on computational fluid dynamic (CFD) and electrostatic simulation, by which the reconstructed images can be compared to the material distributions extracted from CFD simulation results that are used to analog the true distributions in a two-phase system.

sCCCGCC GCC IsGCC

Figures 7 and 8 show the results by the Landweber iteration algorithm [20], which is the most popular iterative algorithm for ECT, to assess the GCC method. Clearly, the image quality obtained by the GCC method is similar to that by the Landweber iteration algorithm (see Figure 8). In the cases of stratified distributions shown in Cases 2e-2f, the results by the GCC method are even better. To obtain similar image quality, the time needed for the Landweber iteration algorithm on a PC with an Intel Core i5 3.30 GHz is about 0.3 s, while the time cost by the GCC method is only 8 ms. Note that one of the most attractive advantages of ECT is its high temporal resolution and the typical measuring speed of an ECT system is about 100 frames per second [8]. Therefore, an algorithm that can reconstruct images at a speed faster than 100 frames per second is desirable to achieve on-line real-time measurement. Obviously, the reconstruction time by the GCC method can meet such requirement, i.e. reconstruct images at a speed faster than 100 frames per second, which is fast enough to characterize the hydrodynamic behavior in most two-phase flow systems, such as gas-solids fluidized beds [11].

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**Figure 7.** Images reconstructed by different methods using CFD simulation data.

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**Figure 8.** Correlation coefficient of different methods using CFD simulation data.

**Case 3**

Image reconstruction with ECT is a typical ill-posed problem, and is sensitive to measurement noise. The typical signal-to-noise ratio (SNR) of an ECT system is usually higher than 50 dB [37, 38]. Therefore, to evaluate the noise immunity of the proposed method, 50 and 60 dB white Gaussian noise were added to the inter-electrode capacitance for distributions in Case 1 and Case 2. Figures 9 shows some examples reconstructed by different algorithms using the data with noise. The average CC for all 12 distributions in Case 1 and Case 2 is shown in Figure 10.

As can be seen, the added noise has no significant effect on image quality by LBP for all distributions, which is consist with previous study [35]. For other methods, with the increase in the noise level, image quality becomes worse. Nevertheless, the GCC method is always superior to LBP and TR, indicating that the GCC method can be effectively used in noisy environments.



**Figure 9.** Images reconstructed by different methods using simulation data with noise.

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**Figure 10.** Correlation coefficient of different methods using simulation data with noise.

* 1. **Evaluation by experiments**

**Case 4**

To validate the simulation results and further verify the feasibility as well as the noise immunity of the GCC method, experiments with both stationary object distributions and typical gas-solids flows in a gas-solids fluidized bed were performed. In this case, the reconstructed images with stationary object distributions by different methods are shown in Figure 11. The SNR of the used ECT system is about 58 dB, between the two SNR levels used in numerical simulation. It can be clearly seen in Figure 11 that the images reconstructed using LBP and TR show similar feature to numerical simulation, i.e. the images by LBP are blurred in the central region and the images by TR show artifacts in the near-wall region. Finally, satisfactory image quality can be obtained by the GCC method with respect to the number and shape of the objects in the imaging area.



**Figure 11.** Images reconstructed by different methods using experimental data with stationary object distributions.

**Case 5**

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In Case 5, the reconstructed images from a real gas-solids fluidized bed by different methods are shown in Figure 12. Although true distributions are unknown, it is still clear in Figure 12 that the GCC method can extract the distribution reconstructed by TR in the central region and the distribution reconstructed by LBP in the near-wall region to form a new image. The results confirm the feasibility of the GCC method in reconstructing material distributions in a gas–solids two-phase flow system.

1. **Conclusions**

A new graph cut-based combination method is proposed to combine images reconstructed by two algorithms for ECT. As an example, the method is implemented on images reconstructed by two single-step algorithms, i.e. LBP and TR. Both numerical simulation and experiments associated with stationary object distributions and gas-solids flow patterns in a gas-solids fluidized bed were performed to show the effectiveness of the proposed method. The results demonstrate that the new method can retain the good image in the central region reconstructed by TR and at the same time avoid artifacts by LBP in the near-wall region. In this way, satisfactory results can be obtained by the graph cut-based combination method. Numerical simulation data with noise and experiments show that the overall performance obtained by the graph cut-based combination method is always superior to LBP and TR, indicating that the graph cut-based combination method has good noise immunity. In addition, the typical reconstruction time by the graph cut-based combination method is about 8 ms, which can achieve 100 frames per second. Therefore, the graph cut-based combination method has the potential to be used for on-line measurement.

Due to the difficulty in image reconstruction with ECT, it is difficult to obtain satisfactory images in all imaging regions by a single algorithm. Although only some examples of the combination of LBP and TR are presented, this work opens a new promising way of combining different image reconstruction algorithms to make use of the advantages of different algorithms. Therefore, the proposed method may also be used to combine other image reconstruction algorithms if they are complementary. Moreover, the proposed method has potential for 3D ECT image reconstruction.

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**References**

[1] J. Yao and M. Takei, "Application of Process Tomography to Multiphase Flow Measurement in Industrial and Biomedical Fields: A Review," *IEEE Sens. J.,* vol. 17, no. 24, pp. 8196-8205, 2017.

[2] T. Dyakowski, "Process tomography applied to multi-phase flow measurement," *Meas. Sci. Technol.,* vol. 7, no. 3, pp. 343-353, 1996.

[3] R. F. Mudde, "Time-resolved X-ray tomography of a fluidized bed," *Powder Technol.,* vol. 199, no. 1, pp. 55-59, 2010.

[4] C. Boyer, A. Koudil, P. Chen, and M. P. Dudukovic, "Study of liquid spreading from a point source in a trickle bed via gamma-ray tomography and CFD simulation," *Chem. Eng. Sci.,* vol. 60, no. 22, pp. 6279-6288, 2005.

[5] L. J. Xu and L. A. Xu, "Gas/liquid two-phase flow regime identification by ultrasonic tomography," *Flow Meas. Instrum.,* vol. 8, no. 3-4, pp. 145-155, 1997.

[6] H. Q. Che, H. G. Wang, J. M. Ye, W. Q. Yang, and Z. P. Wu, "Application of microwave tomography to investigation the wet gas-solids flow hydrodynamic characteristics in a fluidized bed," *Chem. Eng. Sci.,* vol. 180, pp. 20-32, 2018.

[7] W. Zhang, C. Wang, W. Yang, and C.-H. Wang, "Application of electrical capacitance tomography in particulate process measurement – A review," *Adv. Powder Technol.,* vol. 25, no. 1, pp. 174-188, 2014.

[8] W. Yang, "Design of electrical capacitance tomography sensors," *Meas. Sci. Technol.,* vol. 21, no. 4, p. 042001, 2010.

[9] Q. Guo, S. Meng, D. Wang, Y. Zhao, M. Ye, W. Yang, and Z. Liu, "Investigation of gas-solid bubbling fluidized beds using ECT with a modified Tikhonov regularization technique," *AIChE J.,* vol. 64, no. 1, pp. 29-41, 2018.

[10] Q. Guo, S. Meng, Y. Zhao, L. Ma, D. Wang, M. Ye, W. Yang, and Z. Liu, "Experimental Verification of Solid-like and Fluid-like States in the Homogeneous Fluidization Regime of Geldart A Particles," *Ind. Eng. Chem. Res.,* vol. 57, no. 7, pp. 2670-2686, 2018.

[11] K. Huang, S. Meng, Q. Guo, M. Ye, J. Shen, T. Zhang, W. Yang, and Z. Liu, "High-temperature electrical capacitance tomography for gas–solid fluidised beds," *Meas. Sci. Technol.,* vol. 29, no. 10, p. 104002, 2018.

[12] J. C. Gamio, J. Castro, L. Rivera, J. Alamilla, F. Garcia-Nocetti, and L. Aguilar, "Visualisation of gas–oil two-phase flows in pressurised pipes using electrical capacitance tomography," *Flow Meas. Instrum.,* vol. 16, no. 2-3, pp. 129-134, 2005.

[13] Z. Gut and P. Wolanski, "Flame Imaging Using 3D Electrical Capacitance Tomography," *Combust. Sci. Technol.,* vol. 182, no. 11-12, pp. 1580-1585, 2010.

[14] W. Q. Yang and L. Peng, "Image reconstruction algorithms for electrical capacitance tomography," *Meas. Sci. Technol.,* vol. 14, no. 1, pp. R1-R13, 2003.

[15] J. Ye, H. Wang, and W. Yang, "Image Reconstruction for Electrical Capacitance Tomography Based on Sparse Representation," *IEEE Trans. Instrum. Meas.,* vol. 64, no. 1, pp. 89-102, 2015.

[16] J. Lei, Q. Liu, and X. Wang, "Deep Learning-Based Inversion Method for Imaging Problems in Electrical Capacitance Tomography," *IEEE Trans. Instrum. Meas.,* vol. 67, no. 9, pp. 2107-2118, 2018.

[17] M. Soleimani, P. K. Yalavarthy, and H. Dehghani, "Helmholtz-Type Regularization Method for Permittivity Reconstruction Using Experimental Phantom Data of Electrical Capacitance Tomography," *IEEE Trans. Instrum. Meas.,* vol. 59, no. 1, pp. 78-83, 2010.

[18] J. Lei, S. Liu, Z. Li, and M. Sun, "An image reconstruction algorithm based on the extended Tikhonov regularization method for electrical capacitance tomography," *Measurement,* vol. 42, no. 3, pp. 368-376, 2009.

[19] J. Zhao, L. Xu, and Z. Cao, "Direct Image Reconstruction for Electrical Capacitance Tomography Using Shortcut D-Bar Method," *IEEE Trans. Instrum. Meas.,* pp. 1-10, 2018.

[20] W. Q. Yang, D. M. Spink, T. A. York, and H. McCann, "An image-reconstruction algorithm based on Landweber's iteration method for electrical-capacitance tomography," *Meas. Sci. Technol.,* vol. 10, no. 11, pp. 1065-1069, 1999.

[21] L. Peng, H. Merkus, and B. Scarlett, "Using Regularization Methods for Image Reconstruction of Electrical Capacitance Tomography," *Part. Part. Syst. Charact.,* vol. 17, no. 3, pp. 96-104, 2000.

[22] C. G. Xie, A. Plaskowski, and M. S. Beck, "8-electrode capacitance system for two-component flow identification. I. Tomographic flow imaging," *IEE Proc. A,* vol. 136, no. 4, pp. 173-183, 1989.

[23] Q. Xue, H. Wang, Z. Cui, and C. Yang, "Electrical capacitance tomography using an accelerated proximal gradient algorithm," *Rev. Sci. Instrum.,* vol. 83, no. 4, p. 043704, Apr 2012.

[24] Y. Wang, H. Song, and Y. Zhang, "Spectral-Spatial Classification of Hyperspectral Images Using Joint Bilateral Filter and Graph Cut Based Model," *Remote Sens.,* vol. 8, no. 9, p. 748, 2016.

[25] D. Cheng, G. Meng, S. Xiang, and C. Pan, "Efficient sea–land segmentation using seeds learning and edge directed graph cut," *Neurocomputing,* vol. 207, pp. 36-47, 2016.

[26] M. Unberath, S. Achenbach, R. Fahrig, and A. Maier, "Exhaustive graph cut-based vasculature reconstruction," in *2016 IEEE 13th International Symposium on Biomedical Imaging*, date, 2016, place, pp. 1143-1146.

[27] T. Zahner, T. Lochbühler, G. Mariethoz, and N. Linde, "Image synthesis with graph cuts: a fast model proposal mechanism in probabilistic inversion," *Geophys. J. Int.,* vol. 204, no. 2, pp. 1179-1190, 2015.

[28] X. Li, G. Mariethoz, D. Lu, and N. Linde, "Patch-based iterative conditional geostatistical simulation using graph cuts," *Water Resour. Res.,* vol. 52, no. 8, pp. 6297-6320, 2016.

[29] L. Peng, J. Ye, G. Lu, and W. Yang, "Evaluation of effect of number of electrodes in ECT sensors on image quality," *IEEE Sens. J.,* vol. no. pp. 2011.

[30] J. Ye, H. Wang, and W. Yang, "Evaluation of electrical capacitance tomography sensor based on the coupling of fluid field and electrostatic field," *Meas. Sci. Technol.,* vol. 27, no. 7, p. 074003, 2016.

[31] C. G. Xie, S. M. Huang, M. S. Beck, B. S. Hoyle, R. Thorn, C. Lenn, and D. Snowden, "Electrical capacitance tomography for flow imaging: system model for development of image reconstruction algorithms and design of primary sensors," *IEE Proc. G,* vol. 139, no. 1, p. 89, 1992.

[32] V. Kwatra, A. Schödl, I. Essa, G. Turk, and A. Bobick, "Graphcut textures: image and video synthesis using graph cuts," *ACM Trans. Graph.,* vol. 22, no. 3, p. 277, 2003.

[33] Y. Boykov and V. Kolmogorov, "An experimental comparison of min-cut/max-flow algorithms for energy minimization in vision," *IEEE Trans. Pattern Analysis and Machine Intelligence,* vol. 26, no. 9, pp. 1124-37, Sep 2004.

[34] W. Q. Yang and T. A. York, "New AC-based capacitance tomography system," *IEE Proc. A,* vol. 146, no. 1, pp. 47-53, 1999.

[35] J. Ye, H. Wang, Y. Li, and W. Yang, "Coupling of Fluid Field and Electrostatic Field for Electrical Capacitance Tomography," *IEEE Trans. Instrum. Meas.,* vol. 64, no. 12, pp. 3334-3353, 2015.

[36] Z.-K. Gao, P.-C. Fang, M.-S. Ding, and N.-D. Jin, "Multivariate weighted complex network analysis for characterizing nonlinear dynamic behavior in two-phase flow," *Exp. Therm. Fluid Sci.,* vol. 60, pp. 157-164, 2015.

[37] Y. Li and D. J. Holland, "Fast and robust 3D electrical capacitance tomography," *Meas. Sci. Technol.,* vol. 24, no. 10, p. 105406, 2013.

[38] R. K. Rasel, C. E. Zuccarelli, Q. M. Marashdeh, L.-S. Fan, and F. L. TeixeiraIeee, "Toward Multiphase Flow Decomposition Based on Electrical Capacitance Tomography Sensors," *IEEE Sens. J.,* vol. 17, no. 24, pp. 8027-8036, 2017.